Pradeep's

FUNDAMENTAL PHYSICS

Vol. I

Strictly according to new syllabus & examination pattern of CBSE, ISCE and state boards of Punjab, Haryana, H.P., J & K, Odisha, Karnataka, Kerala, Nagaland, Manipur, Rajasthan, Jharkhand, Bihar, U.P., Uttarakhand, M.P., Chhatisgarh, Assam, Gujarat, West Bengal, A.P. etc.
for **class XI of +2 stage.**

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2023-2024

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JALANDHAR (INDIA)

Syllabus

[CBSE, Delhi]

CLASS–XI (THEORY)

Time : 3 Hours **Total Marks 70**

		No. of Periods	Marks
Unit–I	**Physical World and Measurement**	08	23
	Chapter–1: Units and Measurements		
Unit–II	**Kinematics**	24	
	Chapter–2: Motion in a Straight Line		
	Chapter–3: Motion in a Plane		
Unit–III	**Laws of Motion**	14	
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	Chapter–5: Work, Energy and Power		
Unit–V	**Motion of System of Particles and Rigid Body**	18	
	Chapter–6: System of Particles and Rotational Motion		
Unit–VI	**Gravitation**	12	
	Chapter–7: Gravitation		
Unit–VII	**Properties of Bulk Matter**	24	20
	Chapter–8: Mechanical Properties of Solids		
	Chapter–9: Mechanical Properties of Fluids		
	Chapter–10: Thermal Properties of Matter		
Unit–VIII	**Thermodynamics**	12	
	Chapter–11: Thermodynamics		
Unit–IX	**Behavior of Perfect Gases and Kinetic Theory of Gases**	08	
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Unit–X	**Oscillations and Waves**	26	10
	Chapter–13: Oscillations		
	Chapter–14: Waves		
	Total	160	70

Unit I: Physical World and Measurement 08 Periods

Chapter–1: Units and Measurements

Need for measurement: Units of measurement; systems of units; SI units, fundamental and derived units. significant figures. Dimensions of physical quantities, dimensional analysis and its applications.

Unit II: Kinematics 24 Periods

Chapter–2: Motion in a Straight Line

Frame of reference, Motion in a straight line, Elementary concepts of differentiation and integration for describing motion, uniform and non- uniform motion, and instantaneous velocity, uniformly accelerated motion, velocity - time and position-time graphs. Relations for uniformly accelerated motion (graphical treatment).

Chapter–3: Motion in a Plane

Scalar and vector quantities; position and displacement vectors, general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors, Unit vector; resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors.

Motion in a plane, cases of uniform velocity and uniform acceleration- projectile motion, uniform circular motion.

Unit III: Laws of Motion 14 Periods

Chapter–4: Laws of Motion

Intuitive concept of force, Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion.

Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication.

Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on a banked road).

Unit IV: Work, Energy and Power

14 Periods

Chapter–5: Work, Energy and Power

Work done by a constant force and a variable force; kinetic energy, work- energy theorem, power.

Notion of potential energy, potential energy of a spring, conservative forces: non- conservative forces, motion in a vertical circle; elastic and inelastic collisions in one and two dimensions.

Unit V: Motion of System of Particles and Rigid Body

18Periods

Chapter–6: System of Particles and Rotational Motion

Centre of mass of a two-particle system, momentum conservation and Centre of mass motion. Centre of mass of a rigid body; centre of mass of a uniform rod.

Moment of a force, torque, angular momentum, law of conservation of angular momentum and its applications.

Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions.

Moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects (no derivation).

Unit VI: Gravitation

12 Periods

Chapter–7: Gravitation

Kepler's laws of planetary motion, universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential, escape velocity, orbital velocity of a satellite.

Unit VII: Properties of Bulk Matter

24 Periods

Chapter–8: Mechanical Properties of Solids

Elasticity, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity (qualitative idea only), Poisson's ratio; elastic energy.

Chapter–9: Mechanical Properties of Fluids

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes), effect of gravity on fluid pressure.

Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its simple applications.

Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

Chapter–10: Thermal Properties of Matter

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity.

Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law.

Unit VIII: Thermodynamics

12 Periods

Chapter–11: Thermodynamics

Thermal equilibrium and definition of temperature zeroth law of thermodynamics, heat, work and internal energy. First law of thermodynamics, Second law of thermodynamics: gaseous state of matter, change of condition of gaseous state -isothermal, adiabatic, reversible, irreversible, and cyclic processes.

Unit IX: Behavior of Perfect Gases and Kinetic Theory of Gases

08 Periods

Chapter–12: Kinetic Theory

Equation of state of a perfect gas, work done in compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

Unit X: Oscillations and Waves

26 Periods

Chapter–13: Oscillations

Periodic motion - time period, frequency, displacement as a function of time, periodic functions and their application.

Simple harmonic motion (S.H.M) and its equations of motion; phase; oscillations of a loaded spring- restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Chapter–14: Waves

Wave motion: Transverse and longitudinal waves, speed of travelling wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats.

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USEFUL INFORMATION

A. DERIVED SI UNITS

Physical quantity	Name	Symbol
frequency	hertz	Hz
energy	Joule	J
force	newton	N
power	watt	W
pressure	pascal	Pa
electric charge	coulomb	C
electric current	ampere	A
electric potential	volt	V
electric resistance	ohm	Ω
capacitance	farad	F
magnetic flux	weber	Wb
inductance	henry	H
magnetic induction	tesla	T

B. FUNDAMENTAL CONSTANTS

Constant	Symbol	Value [with estimated error]
Speed of light	c	$3 \times 10^{8} ms^{-1}$
Gravitational constant	G	$6{\cdot}67 \times 10^{-11} Nm^{2}kg^{-2}$
Acceleration of free fall	g	$9{\cdot}81 ms^{-2}$
Atmospheric pressure at sea level	atm	$1{\cdot}01 \times 10^{5} Nm^{-2}$

Fundamental Constants (Contd.)

Constant	Symbol	Value [with estimated error]
Absolute zero	K	$-273{\cdot}15^{\circ}C$
Boltzmann's constant	k	$1{\cdot}38\text{x}10^{-23}JK^{-1}$
Avogadro's number	N	$6{\cdot}023\text{x}10^{23}mol^{-1}$
Universal gas constant	R	$8{\cdot}31\ J\ mol^{-1}k^{-1}$
Stefan's constant	σ	$5{\cdot}67\text{x}10^{-8}\ Wm^{-2}K^{-4}$
Permittivity of free space	ϵ_0	$8{\cdot}85\text{x}10^{-12}C^2N^{-1}m^{-2}$
Permeability of free space	μ_0	$4\pi\text{x}10^{-7}TmA^{-1}(WbA^{-1}m^{-1})$
Electronic charge	e	$1{\cdot}60\ \text{x}10^{-19}C$
Electron rest mass	m_e	$9{\cdot}1\text{x}10^{-31}kg$
Specific charge	e/m	$1{\cdot}76\text{x}10^{11}Ckg^{-1}$
Proton rest mass	m_p	$1{\cdot}6726\text{x}10^{-27}kg$
Neutron rest mass	m_n	$1{\cdot}6749\text{x}10^{-27}kg$
Atomic mass unit	u	$1{\cdot}67\text{x}10^{-27}kg = 931MeV$
Planck's constant	h	$6{\cdot}63\text{x}10^{-34}\ Js$
Electronic Radius	r_e	$2{\cdot}8\text{x}10^{-15}m$
Rydberg constant	R_H	$1{\cdot}09678\text{x}10^{7}m^{-1}$
Bohr Magneton	μ_B	$9{\cdot}27\text{x}10^{-24}Am^2$

C. CONVERSION FACTORS

1 mile = $1{\cdot}61$ km

1 radian = $57{\cdot}30^{\circ} = 57^{\circ}\ 18'$

1 amu = $1{\cdot}66 \times 10^{-27}kg = 931MeV$

1 electron volt [eV] = $1{\cdot}6 \times 10^{-19}J$

$\log_e X = 2{\cdot}3026 \log_{10} x$

$\pi = 3{\cdot}1416, \pi^2 = 9{\cdot}8696$

1 emu of charge = 3×10^{10}esu of charge

1 emu of current = 3×10^{10}esu of current

1 emu of current = 10 ampere

1 emu of potential = 10^{-8}volt

1 esu of potential = 300 volt

UNIT-0

MATHEMATICAL TOOLS

CHAPTER 0.

MATHEMATICAL TOOLS

0

MATHEMATICAL TOOLS

A knowledge of elementary mathematics is essential for understanding Physics. In this unit on Mathematical Tools, we shall summarise the various mathematical formulae and techniques with which a student of Physics must be familiar.

SECTION 1—ALGEBRA

1.1. COMMON FORMULAE

(i) $(a+b)^2 = a^2 + b^2 + 2\,ab$

(ii) $(a-b)^2 = a^2 + b^2 - 2\,ab$

(iii) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2\,ab + 2\,bc + 2\,ca$

(iv) $(a+b)\,(a-b) = a^2 - b^2$

(v) $(a+b)^3 = a^3 + b^3 + 3\,ab\,(a+b)$

(vi) $(a-b)^3 = a^3 - b^3 - 3\,ab\,(a-b)$

(vii) $(a+b)^2 - (a-b)^2 = 4\,ab$

(viii) $(a+b)^2 + (a-b)^2 = 2\,(a^2+b^2)$

(ix) $a^3 + b^3 = (a+b)\,(a^2 - ab + b^2) = (a+b)^3 - 3\,ab\,(a+b)$

(x) $a^3 - b^3 = (a-b)\,(a^2 + ab + b^2) = (a-b)^3 + 3\,ab\,(a-b)$

1.2. QUADRATIC EQUATION

An equation of second degree is called a quadratic equation. A standard quadratic equation is of the form

$$ax^2 + bx + c = 0$$

where a is coefficient of x^2, b is coefficient of x, and c is a constant term.

Any value of the variable (x), for which the quadratic equation is satisfied is called the ***root*** of the equation.

Every quadratic equation has two and only two roots. These may be equal or may not be equal.

1.3. ROOTS OF A QUADRATIC EQUATION

The two roots (α and β) of a standard quadratic equation, $ax^2 + bx + c = 0$ are given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

In general, we may write the roots as

$$x = \frac{-\text{coeff of } x \pm \sqrt{(\text{coeff. of } x)^2 - 4(\text{coeff. of } x^2) \times (\text{constant term})}}{2(\text{coeff. of } x^2)}$$

Here, $\alpha + \beta = -\dfrac{b}{a}$, $\quad \alpha . \beta = \dfrac{c}{a}$, $\quad \alpha - \beta = \dfrac{\sqrt{b^2 - 4ac}}{a}$

Example 1 **Solve : $6x^2 - 13x + 6 = 0$**

Solution. Here $a = 6$, $b = -13$ and $c = 6$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+13 \pm \sqrt{(-13)^2 - 4 \times 6 \times 6}}{2 \times 6}$$

or $$x = \frac{13 \pm \sqrt{169 - 144}}{12} = \frac{13 \pm 5}{12} = \frac{18}{12}; \frac{8}{12}, \quad \textit{i.e.,} \quad x = \frac{3}{2}; \frac{2}{3}$$

These are the roots of the given equation.

Example 2 **Solve : $x^2 + x - 2 = 0$.**

Solution. Here, $a = 1$, $b = 1$, $c = -2$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 1 \times (-2)}}{2 \times 1} = \frac{-1 \pm 3}{2} = 1 \text{ or } -2 \quad \therefore \quad x = \mathbf{1} \text{ or } \mathbf{-2}$$

PROBLEMS FOR PRACTICE

1. Solve the followings : $10x^2 - 27x + 5 = 0$ **[Ans. 5/2, 1/5]**
2. Solve the following : $3x^2 - 8x + 5 = 0$ **[Ans. 1 or 5/3]**
3. Solve the followings : $9x^2 + 15x + 4 = 0$ **[Ans. – 1/3, – 4/3]**
4. Solve the followings : $x^2 + 4x - 5 = 0$ **[Ans. 1 or – 5]**
5. Solve the following equation ; $3x^2 + 6x - 9 = 0$ **[Ans. 1 or – 3]**
6. Solve the followings : $5x^2 + 4x - 7 = 0$ **[Ans. 0·849 or – 1·649]**
7. Solve the equation for x : $4x^2 - 4ax + (a^2 - b^2) = 0$. $\left[\textbf{Ans. } \dfrac{(a+b)}{2}, \dfrac{(a-b)}{2}\right]$

1.4. BINOMIAL THEOREM (FOR POSITIVE INTEGRAL INDEX)

According to this theorem, when n is a positive integer and x and a are any real numbers, then

$$(x + a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} . a + {}^nC_2\, x^{n-2} . a^2 + \ldots\ldots + {}^nC_r\, x^{n-r} . a^r + \ldots\ldots + {}^nC_n\, a^n$$

Clearly :

(*i*) Total number of terms in the expansion = $(n + 1)$, *i.e.,* one more than the index of Binomial.

(*ii*) In every successive term in the expansion, the power of x goes on decreasing by 1 and power of a goes on increasing by 1, so that the sum of powers of x and a in each term is always equal to n.

(*iii*) ${}^nC_0, {}^nC_1, {}^nC_2 \ldots\ldots {}^nC_n$ are called **Binomial coefficients.** Here, ${}^nC_0 = 1$, ${}^nC_1 = \dfrac{n!}{1!(n-1)!} = n$

Here $n!$ is read as factorial n, and $n! = n(n-1)(n-2) \ldots\ldots \times 3 \times 2 \times 1$

$$ {}^nC_1 = \frac{n!}{1!(n-1)!} = \frac{n(n-1)(n-2)\ldots 3\times2\times1}{1(n-1)(n-2)\ldots 3\times2\times1} = n $$

Similarly, $\quad {}^nC_2 = \dfrac{n!}{2!(n-2)!} = \dfrac{n(n-1)}{2\times1}$ and so on, $\quad {}^nC_n = 1$.

1.5. BINOMIAL THEOREM FOR ANY INDEX

If n is any number, positive, negative or fraction and x is any real number, such that $x < 1$, *i.e.*, x lies between -1 and $+1$, then according to Binomial Theorem

$$ (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \text{ to } \infty $$

Here, $2! = 2 \times 1$, $\quad 3! = 3 \times 2 \times 1, \ldots,$ $\quad n! = n(n-1)(n-2)\ldots \times 3 \times 2 \times 1$.

Note. 1. If n is a positive integer, then the expansion will have $(n + 1)$ terms.

2. If n is a negative integer or a fraction, then the number of terms in the expansion will be infinite, *i.e., there is no last term.*

3. If $|x| << 1$, then only the first two terms of the expansion are significant. It is so because the values of second and the higher order terms being very very small, can be neglected. In this case, the expansion reduces to the following simplified forms

$$ (1+x)^n = 1 + nx \quad ; \quad (1+x)^{-n} = 1 - nx $$
$$ (1-x)^n = 1 - nx \quad \text{and} \quad (1-x)^{-n} = 1 + nx $$

Example 1 **Evaluate $(1001)^{1/3}$ upto six places of decimal.**

Solution. $(1001)^{1/3} = (1000+1)^{1/3} = \left[1000\left(1 + \dfrac{1}{1000}\right)\right]^{1/3} = 10\,(1 + \cdot 001)^{1/3}$

$$ = 10\left[1 + \frac{1}{3}(\cdot 001) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(\cdot 001)^2 + \ldots\right] = 10\,[1 + 0\cdot0003333 - \frac{1}{9}(0\cdot000001) + \ldots\ldots $$

$= 10\,[1 + 0\cdot0003333 - 0\cdot0000001]$

$= \mathbf{10\cdot003332}$

PROBLEMS FOR PRACTICE

1. Expand using binomial $(1 + 3x)^5$ upto the term having x^3. **[Ans. $1 + 15x + 90x^2 + 270x^3$]**
2. Evaluate $(999)^{1/3}$ upto 4 places of decimal. **[Ans. 9·9967]**
3. Evaluate $\sqrt{26}$ correct upto four places of decimal. **[Ans. 5·0990]**
4. The acceleration due to gravity g' at height h from the surface of earth of radius R is given by

$$ g' = \frac{g R^2}{(R+h)^2} $$

where g is the acceleration due to gravity on the earth's surface. Find out the value of g' using the Binomial theorem, if $h << R$. **[Ans. $g\,(1 - 2\,h/R)$]**

5. Simplify the following, using Binomial theorem.

$$E = K\left[\left(1+\frac{\Delta\theta}{\theta_0}\right)^4 - 1\right], \text{ where } \Delta\theta << \theta_0. \qquad \left[\text{Ans. } \frac{4\,K\,\Delta\theta}{\theta_0}\right]$$

6. Evaluate $(1{\cdot}056)^{1/3}$ correct up to the four places of decimal. **[Ans. 1·0155]**

SECTION 2—GRAPH

2.1. GRAPH

> ***A graph** is a line, straight or curved which shows the variation of one quantity w.r.t. other, which are interrelated with eachother.*

In a relation of two quantities, the quantity which is made to alter at will, is called the **independent variable** and the other quantity which varies as a result of this change is called the **dependent variable.** For example, in a relation $F = ma$,

if mass is taken as constant, then $F \propto a$. It means, the acceleration is determined by the force. So, to plot a graph, force is taken as independent variable and acceleration is the dependent variable.

Conventionally, in any graph, the independent variable (i.e., the cause) is represented along x-axis and dependent variable (i.e., the effect) is represented along y-axis. Thus in the relation, $F = ma$, the force will be plotted along x- axis and the acceleration will be plotted along y-axis.

In order to plot a graph, we must have sufficient data, *i.e.*, a set of values for the independent variables and the corresponding values of dependent variables. This data can be obtained either theoretically or experimentally. The theoretical data is obtained by putting the various values of one quantity in a theoretical relationship and calculating the corresponding magnitudes of other quantity.

While plotting a graph for the variation of two quantities, we have to select the suitable scale for the two axes along which those two quantities are to be represented. The scale chosen must be simple, convenient to operate and to represent the graph to its best advantage. The scales should not be very small so that all the points of the data may not get crowded in a very small portion of the graph. Also, the scale should not be so large that the entire data may not be represented on that graph.

2.2. USES OF GRAPH

1. The shape of the graph plotted for two quantities gives us the clear idea about the relationship between those two quantities. For example, if the graph is a straight line and passes through the origin, it may be represented by $y = mx$. If the graph is a straight line having positive or negative intercepts on y-axis, it is represented by $y = mx \pm c$.

2. The slope of the graph and its intercepts on the x or y-axis give us the values of the physical quantities.

3. A graph plotted between two quantities helps us in identifying the accuracy in the experimental data.

4. With the help of graph, we can find out the mean value easily from a large number of observations.

2.3. SOME GRAPHS FOR VARIOUS EQUATIONS

(*i*) For equation, $y = mx$, the graph between x and y is a straight line OA passing through origin (Fig. 0.1). Here slope of the straight line with x-axis $= m\ (= \tan\theta)$.

(*ii*) For equation, $y = mx + c$, the graph between x and y is a straight line BC with positive intercept on y-axis (Fig. 0.2). Here $OB = c$ and slope of straight line $BC = m\ (= \tan\theta)$.

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(*iii*) For equation, $y = mx - c$, the graph between x and y is a straight line DE, with negative intercept on Y-axis (Fig. 0.3), where $OD = -c$ and slope of straight line $DE = m$ (= tan θ).

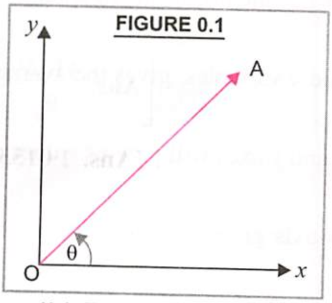
FIGURE 0.1

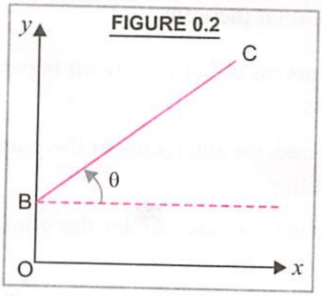
FIGURE 0.2

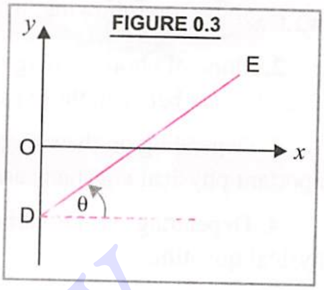
FIGURE 0.3

(*iv*) For equation $y = -mx + c$, the graph between x and y is a straight line FG (Fig. 0.4). Here $OF = c$ and slope of straight line $= -m$ (= tan θ).

(*v*) For equations (*a*) $y^2 = kx$ (*b*) $y^2 = -kx$ and (*c*) $x^2 = ky$, the graphs between x and y will be symmetric parabolic curves as shown in Fig. 0.5 (a), 0.5 (b) and 0.5 (c) respectively.

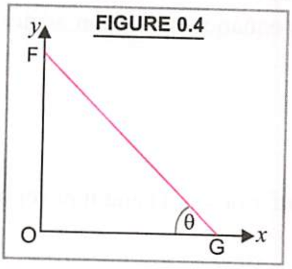
FIGURE 0.4

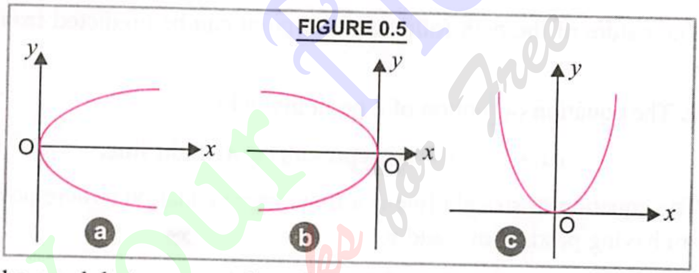
FIGURE 0.5

(*vi*) For equation, $y = ax + bx^2$, the graph between x and y will be asymmetric parabola. If both a and b are positive constants, the graph is as shown in Fig. 0.6 (a). If a is positive and b is negative, the graph is as shown in Fig. 0.6 (b).

(*vii*) For equation $xy = c$, where c is a constant, the graph between x and y is a rectangular hyperbola (Fig. 0.7).

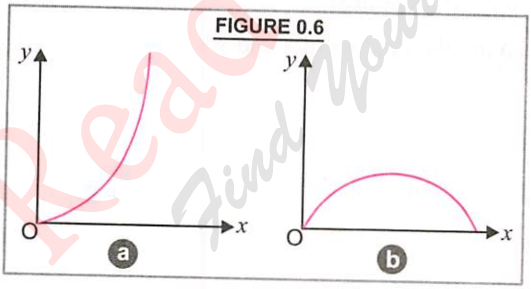
FIGURE 0.6

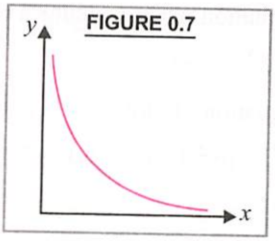
FIGURE 0.7

(*viii*) For equations (*a*) $x^2 + y^2 = a^2$ and (*b*) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the graph between x and y is (*a*) a circle, (*b*) an ellipse as shown in Fig. 0.8(a) and 0.8(b) respectively.

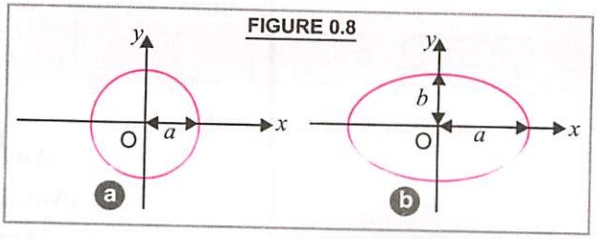
FIGURE 0.8

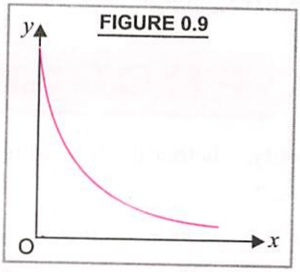
FIGURE 0.9

(*ix*) For equation, $y = e^{-kx}$, the graph between x and y is as shown in Fig. 0.9, which is an exponential curve.

Importance of these Graphs

1. Slope of a graph, drawn between x and y variables, at any point gives instantaneous rate of change of y w.r.t. x.

2. Slope of chord joining two points on the graph drawn between x and y variables, gives the average value of dy/dx between those two points.

3. Depending on the variables plotted, the intercepts of the graph on x and y axes will give us values of important physical constants and quantities.

4. Depending on the variable plotted, the area under the graph and x-axis gives us values of various physical quantities.

2.4. LOCUS OF A POINT

It is the path followed by a point which is constrained to move under different conditions. The path of the point may be straight line, circular, parabolic, elliptical or hyperbolic.

The nature of the path followed by a point can be predicted from its equation of motion as discussed below :

1. The equation of motion of a point given by

$ax + by + c = 0$; represents a **straight line.**

The equation of straight line is a first degree equation (where power of x or y is 1) and it never contain the term having product of x and y.

2. The equation of motion of a point given by

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

represents **a circle**. The equation of circle is second degree equation in x and y in such a way so that

(*i*) the coefficient of x^2 is always equal to coefficient of y^2

(*ii*) the equation never contains a term having the product of x and y

(*iii*) $g^2 + f^3 - c > 0$

3. The equation of motion of a point given by

$$ax^2 + by^2 + 2gx + 2fy + 2\,hxy + c = 0$$

represents

(*a*) **a parabola** if $h^2 - ab = 0$

(*b*) **an ellipse** if $h^2 - ab < 0$

(*c*) **a hyperbola** if $h^2 - ab > 0$.

PROBLEMS FOR PRACTICE

Identify, whether the following equations represent a straight line, parabola or circle :

1. $y = 7$ **[Ans.** St. line**]**

2. $y^2 - 6\,x + 3y + 5 = 0$ **[Ans.** Parabola**]**

3. $x^2 + y^2 = 16$ **[Ans.** Circle**]**

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SECTION 3—TRIGONOMETRY

3.1. ANGLE

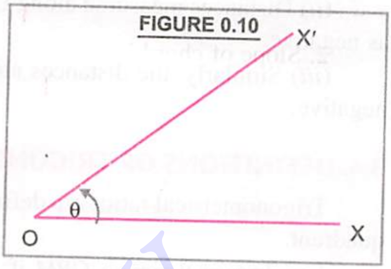


FIGURE 0.10

Suppose OX is a straight line revolving about its one end O. After some time, let it reach the position OX', Fig. 0.10. *The amount of revolution of the line is called the angle traced by the line.* We represent it as $\angle XOX' = \theta$

θ is taken as ***positive,*** when rotation is in the anticlockwise direction ; and it is ***negative,*** when rotation is in the clockwise direction.

3.2. SYSTEMS OF MEASUREMENT OF AN ANGLE

Following are the three systems of measurement of an angle.

1. Sexagesimal system, in which

1 right angle = 90 degrees (90°)
1 degree = 60 minutes (60′)
1 minute = 60 seconds (60″)

2. Centesimal system, in which

1 right angle = 100 grades (100 g)
1 grade = 100 minutes (100′)
1 minute = 100 seconds (100″)

3. Circular system. In this system, angle is measured in ***radian.***

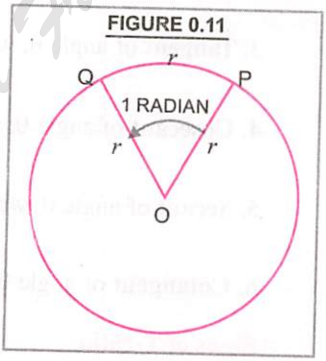


FIGURE 0.11

One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. **In Fig. 0.11,** ***if*** $PQ = l = r$;

then $\angle POQ = 1$ radian

A radian is a constant angle.

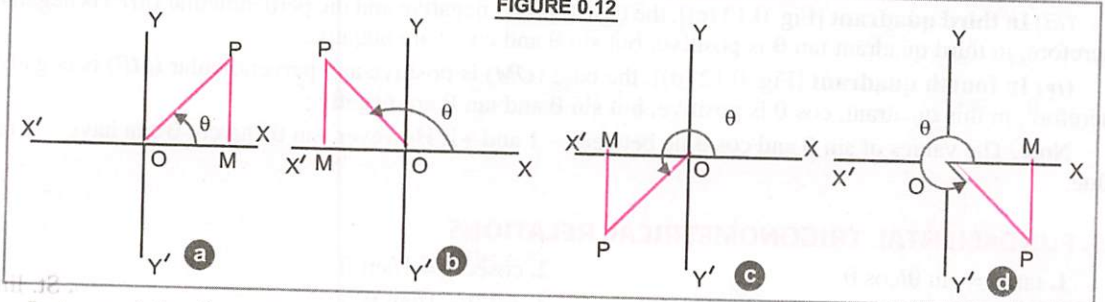
$$2\,\pi \text{ radians} = 360^\circ \quad \text{or} \quad 1\,\text{radian} = \frac{180^\circ}{\pi} = 57{\cdot}27^\circ$$

If l is the length of an arc, r is the radius of the circle, θ is the angle (in radians) subtended by the arc at the centre of the circle, then $\theta = l/r$

3.3. FOUR QUADRANTS AND SIGN CONVENTIONS

Let XOX' and YOY' be two mutually perpendicular lines intersecting each other at O. These lines divide the plane of the paper into four parts, Fig. 0.12. Each part is called a ***quadrant.*** XOY is called the *first quadrant*, YOX' is the *second quadrant*, $X'OY'$ is the *third quadrant* and $Y'OX$ is the *fourth quadrant*.

FIGURE 0.12



Let a revolving line OP coinciding initially with OX revolve about O in the anticlockwise direction and trace an angle θ. The values of θ in all the four quadrants have been shown in Fig. 0.12. From P, draw $PM \perp XOX'$. Thus we obtain a right angled triangle, OPM.

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The sign conventions used for measuring the distances are :

(*i*) All distances are measured from the origin O.

(*ii*) Distances measured along OX are taken as positive and the distances measured along OX' are taken as negative.

(*iii*) Similarly, the distances along OY are taken as positive and the distances along OY' are taken as negative.

3.4. DEFINITIONS OF TRIGONOMETRICAL RATIOS

Trigonometrical ratios are defined in terms of the ratios of the three sides of a right angle triangle in any quadrant.

In right angled triangle OPM, if $\angle POM = \theta$, then the side MP (opposite to θ) is called the **perpendicular** ; the longest side OP is called the **hypotenuse** and the third side OM is called the **base** of the triangle.

1. **Sine** of angle θ, written as $\sin\theta = \dfrac{\text{perp.}}{\text{hypt.}} = \dfrac{MP}{OP}$

2. **Cosine** of angle θ written as $\cos\theta = \dfrac{\text{base}}{\text{hypt.}} = \dfrac{OM}{OP}$

3. **Tangent** of angle θ, written as $\tan\theta = \dfrac{\text{perp.}}{\text{base}} = \dfrac{MP}{OM}$

4. **Cosecant** of angle θ, written as $\text{cosec}\ \theta = \dfrac{\text{hypt.}}{\text{perp.}} = \dfrac{OP}{MP}$

5. **Secant** of angle θ, written as, $\sec\ \theta = \dfrac{\text{hypt.}}{\text{base}} = \dfrac{OP}{OM}$

6. **Cotangent** of angle θ, written as $\cot\ \theta = \dfrac{\text{base}}{\text{perp.}} = \dfrac{OM}{MP}$

Signs of T-ratio

The sign of T-ratio depends upon the sign of base and perpendicular of a right angle triangle and not on the hypotenuse in a right angle triangle.

Using the sign conventions mentioned in Art. 3.3, we observe that

(*i*) **In first quadrant** [Fig. 0.12 (a)], the base (OM) is positive, the perpendicular (MP) is also positive. Therefore, in first quadrant, **all the T-ratios are positive.**

(*ii*) **In second quadrant** [Fig. 0.12 (b)], the base (OM) is negative and the perpendicular (MP) is positive. Therefore, in second quadrant, sin θ is positive but cos θ and tan θ are negative.

(*iii*) **In third quadrant** [Fig. 0.12 (c)], the base (OM) is negative and the perpendicular (MP) is negative. Therefore, in third quadrant tan θ is positive, but sin θ and cos θ are negative.

(*iv*) **In fourth quadrant** [Fig. 0.12 (d)], the base (OM) is positive and perpendicular (MP) is negative. Therefore, in this quadrant, cos θ is positive, but sin θ and tan θ are negative.

Note. The values of sin θ and cos θ lie between – 1 and +1. However, tan θ and cot θ can have any real value.

3.5. FUNDAMENTAL TRIGONOMETRICAL RELATIONS

1. $\tan\theta = \sin\theta/\cos\theta$

2. $\text{cosec}\ \theta = 1/\sin\theta$

3. $\sec\theta = 1/\cos\theta$

4. $\cot\theta = 1/\tan\theta$

5. $\sin^2\theta + \cos^2\theta = 1$ or $\sin^2\theta = 1 - \cos^2\theta$

6. $\sec^2\theta - \tan^2\theta = 1$ or $\sec^2\theta = 1 + \tan^2\theta$

7. $\text{cosec}^2\theta - \cot^2\theta = 1$ or $\text{cosec}^2\theta = 1 + \cot^2\theta$

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3.6. T-RATIOS OF ALLIED ANGLES

When the sum of two angles is equal to 90°, they are called **complementary angles.** When the sum of two angles is equal to 180°, they are called **supplementary angles.**

The angles whose sum or difference with angle θ is zero or a multiple of 90° are called angle allied to θ.

We give below the T-ratios of some of the allied angles, which are commonly used.

1. $\sin(-\theta) = -\sin\theta$, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$, $\cos(-\theta) = \cos\theta$, $\sec(-\theta) = \sec\theta$,
$\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$

For example. $\sin(-30°) = -\sin 30°$ and $\cos(-30°) = +\cos 30°$

2. $\sin(90° - \theta) = \cos\theta$, $\operatorname{cosec}(90° - \theta) = \sec\theta$, $\cos(90° - \theta) = \sin\theta$, $\sec(90° - \theta) = \operatorname{cosec}\theta$,
$\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

Note. As angle $(90° - \theta)$ lies in first quadrant, **all T-ratios are positive.**

For example : (*i*) $\sin 30° = \cos(90° - 30°) = \cos 60°$ and $\cos 60° = \sin(90° - 60°) = \sin 30°$.

3. $\sin(90° + \theta) = \cos\theta$, $\operatorname{cosec}(90° + \theta) = \sec\theta$, $\cos(90° + \theta) = -\sin\theta$,
$\sec(90° + \theta) = -\operatorname{cosec}\theta$, $\tan(90° + \theta) = -\cot\theta$, $\cot(90° + \theta) = -\tan\theta$

[$\because$ $(90° + \theta)$ lies in 2nd quadrant and $\cos\theta$ and $\tan\theta$ are negative in 2nd quadrant]

For example : $\sin 120° = \sin(90° + 30°) = \cos 30°$ and $\cos 150° = \cos(90° + 60°) = -\sin 60°$

4. $\sin(180° - \theta) = \sin\theta$, $\operatorname{cosec}(180° - \theta) = \operatorname{cosec}\theta$, $\cos(180° - \theta) = -\cos\theta$,
$\sec(180° - \theta) = -\sec\theta$, $\tan(180° - \theta) = -\tan\theta$, $\cot(180° - \theta) = -\cot\theta$

Note. As angle $(180° - \theta)$ lies in 2nd quadrant, therefore, **only $\sin\theta$ is positive.**

For example : $\sin 150° = \sin(180° - 30°) = \sin 30°$ and $\cos 135° = \cos(180° - 45°) = -\cos 45°$.

5. $\sin(180° + \theta) = -\sin\theta$, $\cos(180° + \theta) = -\cos\theta$, $\tan(180° + \theta) = \tan\theta$

Note. As angle $(180° + \theta)$ lies in 3rd quadrant, therefore **only $\tan\theta$ is positive.**

For example : (*i*) $\tan 210° = \tan(180° + 30°) = \tan 30°$

(*ii*) $\sin 240° = \sin(180° + 60°) = -\sin 60°$

6. $\sin(270° + \theta) = -\cos\theta$, $\cos(270° + \theta) = \sin\theta$, $\tan(270° + \theta) = -\cot\theta$

Note. As angle $(270° + \theta)$ lies in 4th quadrant, therefore, **only $\cos\theta$ is positive**

For example : $\sin 300° = \sin(270° + 30°) = -\cos 30°$, $\cos 290° = \cos(270° + 20°) = \sin 20°$

7. $\sin(360° - \theta) = -\sin\theta$, $\cos(360° - \theta) = \cos\theta$, $\tan(360° - \theta) = -\tan\theta$

Note that, angle $(360° - \theta)$ lies in the 4th quadrant, therefore only $\cos\theta$ is positive.

8. $\sin(360° + \theta) = \sin\theta$, $\cos(360° + \theta) = \cos\theta$, $\tan(360° + \theta) = \tan\theta$

Note that, angle $(360° + \theta)$ lies in the first quadrant, therefore, **all the T-ratios are positive.**

For example : $\sin 400° = \sin(360° + 40°) = \sin 40°$, $\cos 425° = \cos(360° + 65°) = \cos 65°$
$\tan 395° = \tan(360° + 35°) = \tan 35°$.

3.7. VALUES OF T-RATIOS OF SOME ANGLES

The values of T-ratios of some standard angles, which are commonly used are given in the following table :

Angle θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	1	0	∞
120°	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$

135°	$1/\sqrt{2}$	$-1/\sqrt{2}$	– 1
150°	1/2	$-\sqrt{3}/2$	$-1/\sqrt{3}$
180°	0	– 1	0
270°	– 1	0	– ∞
360°	0	1	0

PROBLEMS FOR PRACTICE

Find the value of the followings :

1. (*a*) sin (– 30°) (*b*) cos (– 60°) (*c*) tan (– 45°) **[Ans. (*a*) – 1/2, (*b*) 1/2 (*c*) – 1]**

2. (*a*) sin 120° (*b*) cos 135° (*c*) tan 150° **[Ans. (*a*) $\sqrt{3}/2$ (*b*) $-1/\sqrt{2}$ (*c*) $-1/\sqrt{3}$]**

3. (*a*) sin 210° (*b*) cos 220° (*c*) tan 245° **[Ans. (*a*) – 1/2 (*b*) – 0·7660 (*c*) 2·1445]**

4. If sin θ = 3/5, find the values of cosine θ and tangent θ. **[Ans. 4/5 ; 3/4]**

5. An inclined plane rises 1 in 10. If the length of the inclined plane is 5 m, calculate the height of the raised end above the horizontal. **[Ans. 0·5 m]**

3.8. ADDITION FORMULAE

(*i*) $\sin (A + B) = \sin A \cos B + \cos A \sin B$ (*ii*) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

(*iii*) $\tan (A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Putting $B = A$ in these formulae, we get

(*iv*) $\sin 2A = 2 \sin A \cos A$ (*v*) $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

(*vi*) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Thus, $\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2 \sin^2 \theta$

Again from, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \cos^2 \theta - (1 - \cos^2 \theta) = 2\cos^2 \theta - 1$ and $\tan 2\theta = \dfrac{2 \tan \theta}{1 - \tan^2 \theta}$

Replacing 2 θ by θ and θ by θ/2 in the above relations, we may rewrite, $\sin \theta = 2 \sin \theta/2 \cos \theta/2$;

$$\cos \theta = \cos^2 \theta/2 - \sin^2 \theta/2 = 1 - 2\sin^2 \theta/2 = 2\cos^2 \theta/2 - 1, \qquad \tan \theta = \frac{2 \tan \theta/2}{1 - \tan^2 \theta/2}$$

3.9. DIFFERENCE FORMULAE

(*i*) $\sin (A - B) = \sin A \cos B - \cos A \sin B$ (*ii*) $\cos (A - B) = \cos A \cos B + \sin A \sin B$

(*iii*) $\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

3.10. TRANSFORMATION FORMULAE

On adding the addition and subtraction formulae of sine, we get

$$\sin (A + B) + \sin (A - B) = 2 \sin A \cos B \quad ...(i)$$

Similarly
$$\cos (A + B) + \cos (A - B) = 2 \cos A \cos B \quad ...(ii)$$

On subtracting the subtraction formula from addition formula of sine, we get

$$\sin (A + B) - \sin (A - B) = 2 \cos A \sin B \quad ...(iii)$$

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Similarly, $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$ ...(*iv*)

If we put $(A+B) = C$ and $(A-B) = D$

then on adding and subtracting, we get $A = \dfrac{C+D}{2}$ and $B = \dfrac{C-D}{2}$

Putting these values in (*i*) to (*iv*), we obtain

(*i*) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(*ii*) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(*iii*) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(*iv*) $\cos D - \cos C = 2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

or $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

3.11. THE SINE AND COSINE FORMULAE FOR A TRIANGLE

In a triangle ABC, of sides *a*, *b*, *c* and angles A, B and C, Fig. 0.13. The following formulae hold good.

(*i*) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

(*ii*) $a^2 = b^2 + c^2 - 2\,bc\cos A$

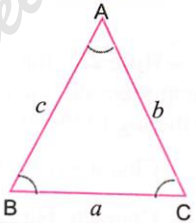
(*iii*) $b^2 = c^2 + a^2 - 2\,ca\cos B$

(*iv*) $c^2 = a^2 + b^2 - 2\,ab\cos C$

(*v*) Area of a triangle ABC $= \sqrt{S(S-a)(S-b)(S-c)}$

where, $S = (a+b+c)/3$

FIGURE 0.13



SECTION 4 — LOGARITHM

4.1. DEFINITION OF LOGARITHM

Logarithm of a number with respect to a given base is the power to which the base must be raised to represent that number.

For example : $1000 = 10^3$ $\therefore$ $\log_{10} 1000 = \log_{10}(10)^3 = 3$

In general, if $N = a^x$ then $\log_a N = \log_a a^x = x$

It follow that $\log_a 1 = 0$ $(\because a^o = 1)$

and $\log_a a = 1$ $(\because a^1 = a)$

4.2. FOUR STANDARD FORMULAE OF LOGARITHM

These four formulae are commonly used. These are mentioned below without proof.

(*i*) $\log_a mn = \log_a m + \log_a n$ ...(product formula)

(*ii*) $\log_a \dfrac{m}{n} = \log_a m - \log_a n$ ...(quotient formula)

(*iii*) $\log_a m^n = n\log_a m$ ...(power formula)

(*iv*) $\log_a m = \log_b m \times \log_a b$ ...(base change formula)

4.3. TWO SYSTEMS OF LOGARITHM

The two systems of Logarithm in common use are :

(*i*) **Neperian log or Natural log.** Here the base is *e*, where $e = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \ldots = 2{\cdot}718$ (approx). In all theoretical calculations, we use Natural logarithm

(*ii*) **Common Logarithm.** Here, the base of the log is 10. In all **numerical** calculations, we use common log.

We can convert Natural log into common log, as follows :

$$\log_e m = \log_{10} m \times \log_e 10$$

As $\log_e 10 = 2{\cdot}3026 \log_{10} 10 = 2{\cdot}3026 \times 1 = 2{\cdot}3026$ $[\because \log_{10} 10 = 1]$

$\therefore$ $\log_e m = 2{\cdot}3026 \log_{10} m$

4.4. LOG TABLES

Log Tables are the standard common log tables which are available to us for calculations. These are usually four digit tables and are given at the end of the book.

The logarithm of any given number can be read from these tables. The logarithm of a number consists of two parts : (*i*) integral part, called **characteristic.** (*ii*) decimal part, called **mantissa.**

(*a*) Characteristic. It may be positive or zero or negative. We use the following two rules for finding the characteristic :

Rule 1. If the given number > 1, its characteristic is positive. Count the number of digits to the left of the decimal point. Subtract 1 from this number. This is the characteristic of the log of given number.

For example : Characteristics of log of 1795·2 is $4 - 1 = 3$
Characteristics of log of 16·78 is $2 - 1 = 1$
Characteristics of log of 1·923 is $1 - 1 = 0.$

Rule 2. If the given number is less than one, its characteristic is **negative.** It is put under the bar. Count the number of zeros immediately after the decimal point. Add 1 to this number. It gives us the characteristic of the log of the given number. **For example** :

Characteristic of log of $0{\cdot}7821 = -(0 + 1) = \bar{1}$, Characteristic of log of $0{\cdot}0618 = -(1 + 1) = \bar{2}$

Characteristic of log of $0{\cdot}000345 = -(3 + 1) = \bar{4}$

(*b*) Mantissa. As mentioned above, it is the decimal part of log of a given number. It is always positive, and can be read from the log tables as follows:

(*i*) In the given number, omit the decimal point and zeros in the beginning and at the end.

For example, in the log of 0·001184, we have to look up the mantissa of 1184.

(*ii*) Take first two digits, *i.e.*, 11 and locate it in the first vertical column of the four figure log table.

(*iii*) Go through the horizontal row beginning with 11 and look up the value under the column headed by the third digit, *i.e.*, 8. From the log tables, we find this number as 0719.

(*iv*) Continue moving in the same horizontal row and note the number in the small differences column headed by the fourth digit, *i.e.*, 4. This number is 15

(*v*) Add 15 to 0719. We get 0734. $\therefore$ The mantissa of 0·001184 = ·0734.

Note : If the number consists of a single digit, put zeroes at the end of the number to complete four digits. Read the mantissa as explained above.

For example : log 7 = log 7·000 = 0·8451

Mantissa for 7, 70, 700, 0·7 etc. is the same.

4.5. ANTILOGARITHM

Antilogrithm is the reverse process of logarithm. Thus antilogarithm to logarithm is as division to multiplication.

For example, if log 7 = 0·8451, then antilog 0·8451 = 7

Standard antilog tables are available for calculations. We follow the rules given below for reading antilog tables :

(*i*) Read the antilog of mantissa only (*i.e.*, of the decimal part) ; locating the four digits the way we did in reading mantissa from log tables.

(*ii*) If the characteristic is positive say *n*, the decimal is placed after $(n + 1)$ digits in the value read.

(*iii*) If the characteristic is negative, say $\bar{n}$, then $(n - 1)$ zeroes are placed before the left side of the number read ; and then decimal point is placed.

For example : antilog $\bar{1}{\cdot}3478 = 0{\cdot}2227$, antilong $2{\cdot}7192 = 523{\cdot}8$, antilog $0 = 1{\cdot}000$

4.6. USE OF LOGARITHM

Logarithm is practically very useful in simplifying the complicated calculations. Following examples will enable the students to use the log tables for that purpose.

Example 1 **Find the fifth root of 0·0076.**

Solution. Let, $x = (0{\cdot}0076)^{1/5}$

Taking log of both sides, we get $\log x = \frac{1}{5}\log(0{\cdot}0076) = \frac{1}{5}(\bar{3}{\cdot}8808) = \frac{1}{5}(\bar{5}+2{\cdot}8808) = \bar{1}{\cdot}5762$

Taking antilog of both the sides, we get $\mathbf{x = 0{\cdot}3769}$

Example 2 **Simplify :** $\dfrac{\mathbf{1{\cdot}792\times 77{\cdot}4}}{\mathbf{(129{\cdot}7)^{2/3}}}$

Solution. Let, $x = \dfrac{1{\cdot}792\times 77{\cdot}4}{(129{\cdot}7)^{2/3}}$

Taking log of both the sides, we get

$$\log x = \log 1{\cdot}792 + \log 77{\cdot}4 - \frac{2}{3}\log 129{\cdot}7 = 0{\cdot}2534 + 1{\cdot}8887 - \frac{2}{3}(2{\cdot}1130)$$

$$= 0{\cdot}2534 + 1{\cdot}8887 - 1{\cdot}4087 = 2{\cdot}1421 - 1{\cdot}4087 = 0{\cdot}7334$$

$$\therefore \quad x = \text{antilong } 0{\cdot}7334 = \mathbf{1{\cdot}5413}$$

PROBLEMS FOR PRACTICE

1. Simplify the followings :

(*i*) $(0{\cdot}0036)^{1/5}$ (*ii*) $(0{\cdot}056)^{2/3}$ (*iii*) $10^{-1/5}$ **[Ans. (*i*) 0·3245 (*ii*) 0·1463 (*iii*) 0·6310]**

2. Simplify the followings :

(*i*) $(0{\cdot}05246)^{1/8} - 2{\cdot}6055$ (*ii*) $\dfrac{3{\cdot}142\times(80{\cdot}2)^{1/2}}{(9{\cdot}8)^{1/2}}$ **[Ans. (*i*) – 1·9137 (*ii*) 8·989]**

3. Find antilog of (*i*) – 2·6674 (*ii*) – 3·01 (*iii*) – 0·3467 **[Ans. (*i*) ·002151 (*ii*) 0·0009772]**

HINTS

For Difficult Problems

1. (*iii*) $x = 10^{-1/5}$

$$\log x = -\frac{1}{5}\log 10 = -\frac{1}{5}\times 1 = -0{\cdot}2 = \bar{1}+1-0{\cdot}2 = \bar{1}{\cdot}8$$

Taking antilog on both the sides, we get $\mathbf{x = 0{\cdot}6310}$

3. (*i*) Given, $\log x = -\,2{\cdot}6674 = \bar{3}+3-2{\cdot}6674 = \bar{3}{\cdot}3326$

Taking antilog on both the sides, we get $\mathbf{x = 0{\cdot}002151}$

(*ii*) Given, $\log x = -\,3{\cdot}01 = \bar{4}+4-3{\cdot}01 = \bar{4}{\cdot}99$

Taking antilog on both the sides, we get $\mathbf{x = 0{\cdot}0009772}$

(*iii*) Given $\log x = -\,0{\cdot}3467 = \bar{1}+1-0{\cdot}3467 = \bar{1}{\cdot}6533$

Taking antilog on both the sides, we get $\mathbf{x = 0{\cdot}4501}$

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SECTION 5 — FUNCTIONS AND LIMITS

5.1. FEW IMPORTANT TERMS RELATED TO FUNCTIONS

1. Quantity. It is a thing which can be measured.

2. Constant and variables. A quantity whose value does not change during mathematical operations is called **a constant.** *e.g.*, π, *e*, integers, fractions etc. On the other hand, a quantity which can take up different values during mathematical operations is called **a variables.** A variable is generally represented by *x*, *y* or *z* etc.

3. Independent and dependent variables. A variable quantity which can have any value within the specified limits is called as **independent variable,** whereas a variable quantity whose value depends upon the numerical value assigned to the independent variable is called as **dependent variable.**

4. Function.

> ***If the value of a variable quantity y depends upon the value of some other variable quantity x, through some relation, such that there exists only one finite value of y for each value of x, then y is called a function of x.***

Mathematically, it is represented as : $y = f(x)$

Here, $f(x)$ is a symbol, representing a function of x. It is not of multiplied by x.

In a relation, $y = 2x^2 + 3x + 4$; if $x = 0$ then $y = 4$; $x = 2$, $y = 18$; $x = 3$, $y = 31$ and so on. It means, for every value of x, there is a definite value of y. Hence, $y = 2x^2 + 3x + 4$ represents a function of x.

5. Types of elementary functions

In physics, we generally use the following elementary functions :

(*i*) Power function ; *e.g.*, $y = x^n$

where n is a constant real number. For $n = 0$, power function is a constant quantity, *i.e.*, $y = 1$.

(*ii*) Trignometric functions, *e.g.*, $y = \sin x$; $y = \cos x$; $y = \cos x \cot x + \cos^2 x$.

(*iii*) Exponential function, *e.g.*, $y = a^x$ where a is the positive number, called the base number.

(*iv*) Logarithmic function, *e.g.*, $y = \log_e x^2$.

6. Limit of a function. A function $f(x)$ is said to tend to a limit l as its variable x approaches a constant a, $(x \to a)$ such that the absolute difference between $f(x)$ and l, as $x \to a$ is negligible small but not zero. It is represented as $\underset{x \to a}{\text{Lt}}\ f(x) = l$

To understand it, consider a function, $y = f(x) = \dfrac{x^3 - 8}{x - 2}$. This function can be defined at all points except at $x = 2$. If we use $x = 2$, then $f(x) = \dfrac{2^3 - 8}{2 - 2} = \dfrac{0}{0}$ which is **indeterminate,** *i.e.*, **cannot be defined.** However, for all values of x very close to 2 more or less than 2, the function tends to 12 as shown below :

When, $x = 1{\cdot}98$, $f(x) = \dfrac{(1{\cdot}98)^3 - 8}{1{\cdot}98 - 2} = 11{\cdot}88$, When $x = 1{\cdot}99$, $f(x) = \dfrac{(1{\cdot}99)^3 - 8}{1{\cdot}99 - 2} = 11{\cdot}94$

When $x = 2{\cdot}01$, $f(x) = \dfrac{(2{\cdot}01)^3 - 8}{2{\cdot}01 - 2} = 12{\cdot}06$, When $x = 2{\cdot}02$, $f(x) = \dfrac{(2{\cdot}02)^3 - 8}{2{\cdot}02 - 2} = 12{\cdot}12$

From all values of x very close to 2 but not exactly 2, $\dfrac{x^3 - 8}{x - 2} \approx 12$

So, we write $\underset{x \to 2}{\text{Lt}}\ \dfrac{x^3 - 8}{x - 2} = \mathbf{12}$

5.2. SOME IMPORTANT LIMITS USED IN PHYSICS ARE GIVEN BELOW

(*i*) $\underset{\theta \to 0}{\text{Lt}} \sin\theta = \theta$ (in radians) (*ii*) $\underset{\theta \to 0}{\text{Lt}} \dfrac{\sin\theta}{\theta} = 1$ (*iii*) $\underset{\theta \to 0}{\text{Lt}} \cos\theta = 1$

(*iv*) $\underset{\theta \to 0}{\text{Lt}} \tan\theta = \theta$ (in radians) (*v*) $\underset{\theta \to 0}{\text{Lt}} \dfrac{\tan\theta}{\theta} = 1$ (*vi*) $\text{Lt}\left(x + \dfrac{1}{x}\right)^x = \underset{y \to 0}{\text{Lt}} (1+y)^{\frac{1}{y}} = e = 2{\cdot}718$

SECTION 6 — DIFFERENTIATION

6.1. DIFFERENTIAL COEFFICIENT OR DERIVATIVE OF A FUNCTION

The differential coefficient or derivative of variable y with respect to variable x is defined as the instantaneous rate of change of y w.r.t. x. It is denoted by $\frac{dy}{dx}$.

Let y be a function of x, *i.e.*, $y = f(x)$

Here, x is the independent variable and y is the dependent variable. Therefore, by changing x, the value of y changes. Let x be increased by a small amount Δx, called increment in x. Let the corresponding increment in y be Δy. Then, $y + \Delta y = f(x + \Delta x)$

or $\Delta y = (y + \Delta y) - y = f(x + \Delta x) - f(x)$ or $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$ ...(*i*)

Here $\dfrac{\Delta y}{\Delta x}$ is the average change or increment ratio in the value of the function $f(x)$. In order to obtain the instantaneous rate of change of y with respect to x, we calculate the limit of $\dfrac{\Delta y}{\Delta x}$ as $\Delta x \to 0$

$$\therefore \quad \underset{\Delta x \to 0}{\text{Lt}} \frac{\Delta y}{\Delta x} = \underset{\Delta x \to 0}{\text{Lt}} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The quantity $\underset{\Delta x \to 0}{\text{Lt}} \dfrac{\Delta y}{\Delta x}$ is represented by $\dfrac{dy}{dx}$ and is called differential coefficient or derivative of y w.r.t. x. This differential coefficient or derivative of variable y w.r.t. variable x is the **instantaneous rate of change of *y* w.r.t. *x*.** Hence, $\dfrac{dy}{dx} = \underset{\Delta x \to 0}{\text{Lt}} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

Note. 1. The symbol $\dfrac{dy}{dx}$ should not be taken as quotient of dy and dx.

2. The symbol $\dfrac{d}{dx}$ represents a symbol for the rate of change w.r.t. x or the derivative w.r.t. x.

6.2. GEOMETRICAL INTERPRETATION OF THE DERIVATIVE OF A FUNCTION

Let the function ; $y = f(x)$ be represented graphically by a curve AB (Fig. 0.14). Take two close points $P(x, y)$ and $Q(x + \Delta x, y + \Delta y)$ on this curve. Join P and Q, we get a chord PQ. Let α be the angle which the chord PQ makes with the positive x-axis. Draw a perpendicular PS on the straight line QQ_1. From the figure, it is clear that

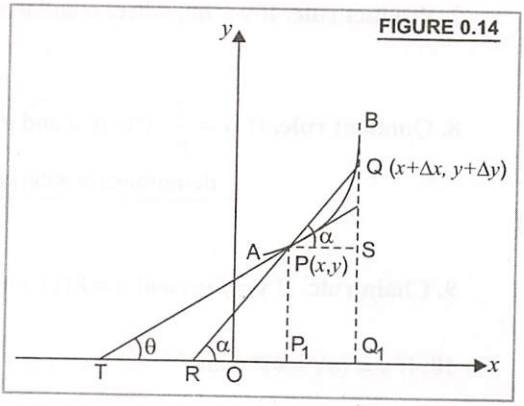
$$PS = P_1Q_1 = OQ_1 - OP_1 = (x + \Delta x) - x = \Delta x$$

Similarly, $QS = \Delta y$

Slope of chord $PQ = \tan\alpha = \dfrac{QS}{PS}$

$$= \frac{\Delta y}{\Delta x} = \frac{(y + \Delta y) - y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

FIGURE 0.14



As point Q approaches point P, both Δx and Δy decrease continuously. In the limiting case, when the point Q will almost coincide with point P, the chord PQ becomes almost a tangent to the curve at P. Let the tangent to the curve at P make an angle θ with the positive x-axis. The slope of this tangent will be $\tan\theta$.

Thus, we have $$\underset{\Delta x \to 0}{\text{Lt}} \frac{\Delta y}{\Delta x} = \tan\theta \quad \text{or} \quad \frac{dy}{dx} = \tan\theta$$

Thus **geometrically, the differential coefficient of $y = f(x)$, with respect to x at any point is equal to the slope of the tangent to the curve representing $y = f(x)$ at that point.**

6.3. PHYSICAL EXAMPLES OF DIFFERENTIAL COEFFICIENT AS RATE OF MEASUREMENT

1. Instantaneous speed of a body, v = rate of change of distance s with time t at the given instant

i.e., $$v = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

2. The instantaneous acceleration, a = rate of change of velocity v of the body with time t, at the given instant *i.e.,* $$a = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

3. Force F is equal to the rate of change of linear momentum (mv) with time t, *i.e.,* $F = \dfrac{d\,(mv)}{dt}$.

6.4. FUNDAMENTAL FORMULAE OF DIFFERENTIATION

1. If c is some constant, then $$\frac{d}{dx}(c) = 0$$

2. If $y = cx$, where c is a constant, then $$\frac{dy}{dx} = \frac{d}{dx}(cx) = c\frac{dx}{dx} = c$$

3. If $y = cu$, where c is a constant and u is a function of x, then $$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$$

4. If $y = x^n$, where n is a real number, then $$\frac{dy}{dx} = nx^{n-1}$$

5. If $y = u^n$, where n is a real number and u is a function of x, then $$\frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$$

6. If $y = u + v$, where u and v are the functions of x, then $$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

7. Product rule. If $y = uv$, where u and v are the functions of x, then $$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

8. Quotient rule. If $y = \dfrac{u}{v}$, where u and v are the functions of x, then $$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{\text{demominator} \times \text{derivative of numerator} - \text{numerator} \times \text{derivative of demoninator}}{(\text{demoninator})^2}$$

9. Chain rule. If $y = f(u)$ and $u = f(x)$, then $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

10. If $y = (ax + b)^n$ then $$\frac{dy}{dx} = n(ax+b)^{n-1} \times \frac{d}{dx}(ax+b)$$

11. If y depends on u and u depends on x, then $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

6.5. DERIVATIVES OF TRIGONOMETRICAL FUNCTIONS

12. If $y = \sin x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(\sin x) = \cos x$

13. If $y = \cos x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(\cos x) = -\sin x$

14. If $y = \tan x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(\tan x) = \sec^2 x$

15. If $y = \cot x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

16. If $y = \sec x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(\sec x) = \tan x \sec x$

17. If $y = \operatorname{cosec} x$, then $\dfrac{dy}{dx} = \dfrac{d}{dx}(\operatorname{cosec} x) = -\cot x \operatorname{cosec} x$

6.6. DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

18. If $y = \log_e x$, then $\dfrac{dy}{dx} = \dfrac{1}{x}\log_e e = \dfrac{1}{x}$

19. If $y = \log_e u$, then $\dfrac{dy}{dx} = \dfrac{1}{u} \times \dfrac{d\,(u)}{dx}$

20. If $y = e^x$, then $\dfrac{dy}{dx} = e^x \log_e e = e^x$

21. If $y = e^{ax}$, then $\dfrac{dy}{dx} = ae^{ax}$

6.7. CONDITIONS FOR MAXIMA OR MINIMA

If $y = f(x)$, then for maximum or minimum value of y for a value of x the first differential coefficient of y w.r.t. x should be zero, *i.e.*, $\dfrac{dy}{dx} = 0$. Find the value of x from here.

Now, find the second derivative of y w.r.t. x, *i.e.*, $\dfrac{d^2y}{dx^2}$ and put the value of x.

(*i*) If the value of $\dfrac{d^2y}{dx^2}$ is negative, then y is maximum for a given value of x.

(*ii*) If the value of $\dfrac{d^2y}{dx^2}$ is positive, then y is minimum for a given value of x.

SOLVED EXAMPLES

Example 1 **Differentiate the following w.r.t. x. (i) 2002 (ii) e^{-1} (iii) π^2**

Solution. Since these are constant quantities, so (*i*) $\dfrac{d}{dx}(2002) = \mathbf{0}$ (*ii*) $\dfrac{d}{dx}(e^{-1}) = \mathbf{0}$ (*iii*) $\dfrac{d}{dx}(\pi^2) = \mathbf{0}$

Example 2 **Differentiate the following with respect to x.**

(i) x^6 **(ii)** $\dfrac{1}{x^3}$ **(iii)** $\sqrt{x}$

Solution. (*i*) $y = x^6$; $\dfrac{dy}{dx} = 6\,x^{6-1} = \mathbf{6\,x^5}$ (*ii*) $y = \dfrac{1}{x^3} = x^{-3}$; $\dfrac{dy}{dx} = (-3)\,x^{-3-1} = -3\,x^{-4} = \mathbf{\dfrac{-3}{x^4}}$

(*iii*) $y = \sqrt{x} = x^{\frac{1}{2}}$; $\dfrac{dy}{dx} = \dfrac{1}{2}x^{\left(\frac{1}{2}-1\right)} = \dfrac{1}{2}x^{-\frac{1}{2}} = \mathbf{\dfrac{1}{2\sqrt{x}}}$

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Example 3 **Differentiate the following with respect to x.**

(i) $4x^3 - 3x^2 + \frac{4}{x^2} - 8$ **(ii)** $5x^4 + 4x^{3/4} - 3x^2 + 2x$

Solution. *(i)* $y = 4x^3 - 3x^2 + \frac{4}{x^2} - 8$

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^3 - 3x^2 + \frac{4}{x^2} - 8\right) = \frac{d}{dx}(4x^3) - \frac{d}{dx}(3x^2) + \frac{d}{dx}\left(\frac{4}{x^2}\right) - \frac{d}{dx}(8)$$

$$= 4 \times 3\, x^{3-1} - 3 \times 2\, x^{2-1} + 4\,(-2)\, x^{-2-1} - 0 = 12x^2 - 6x - 8x^{-3} = \mathbf{12x^2 - 6x - \frac{8}{x^3}}$$

(ii) $y = 5x^4 + 4x^{3/4} - 3x^2 + 2x$

$$\frac{dy}{dx} = \frac{d}{dx}(5x^4 + 4x^{3/4} - 3x^2 + 2x) = \frac{d}{dx}(5x^4) + \frac{d}{dx}(4x^{3/4}) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(2x)$$

$$= 5 \times 4\, x^{4-1} + 4 \times \frac{3}{4} x^{\frac{3}{4}-1} - 3 \times 2\, x^{2-1} + 2 \times 1 = \mathbf{20x^3 + 3x^{-1/4} - 6x + 2}$$

Example 4 **Differentiate the following with respect to x.**

(i) $(4x+2)(5x^2+4)$ **(ii)** $(2x^3+3)(2x^{-3}+1)$

Solution. *(i)* $y = (4x+2)(5x^2+4)$

Using product rule

$$\frac{dy}{dx} = (4x+2)\frac{d}{dx}(5x^2+4) + (5x^2+4)\frac{d}{dx}(4x+2)$$

$$= (4x+2) \times (5 \times 2\, x^{2-1} + 0) + (5x^2+4)(4 \times 1\, x^{1-1} + 0)$$

$$= (4x+2)\, 10x + (5x^2+4)\, 4 = 40x^2 + 20x + 20x^2 + 16$$

$$= \mathbf{60x^2 + 20x + 16}$$

(ii) $y = (2x^3+3)(2x^{-3}+1)$

Using product rule

$$\frac{dy}{dx} = (2x^3+3)\frac{d}{dx}(2x^{-3}+1) + (2x^{-3}+1)\frac{d}{dx}(2x^3+3)$$

$$= (2x^3+3)\,[2 \times (-3)\, x^{-3-1} + 0] + (2x^{-3}+1)(2 \times 3\, x^{3-1} + 0)$$

$$= (2x^3+3)\left(\frac{-6}{x^4}\right) + \left(\frac{2}{x^3}+1\right)6x^2 = \frac{-12}{x} - \frac{18}{x^4} + \frac{12}{x} + 6x^2 = \mathbf{6x^2 - \frac{18}{x^4}}$$

Example 5 **Differentiate the following with respect to x.**

(i) $(x^3 + 2x^2 - 4)(4x^5 - x^2 + 1)$ **(ii)** $\frac{7x^5}{x+3}$

Solution. *(i)* $y = (x^3 + 2x^2 - 4)(4x^5 - x^2 + 1)$

Using product rule

$$\frac{dy}{dx} = (x^3 + 2x^2 - 4)\frac{d}{dx}(4x^5 - x^2 + 1) + (4x^5 - x^2 + 1)\frac{d}{dx}(x^3 + 2x^2 - 4)$$

$$= (x^3 + 2x^2 - 4)\,[20x^4 - 2x] + (4x^5 - x^2 + 1)\,[3x^2 + 4x]$$

$$= (20x^7 + 40x^6 - 80x^4 - 2x^4 - 4x^3 + 8x) + (12x^7 - 3x^4 + 3x^2 + 16x^6 - 4x^3 + 4x)$$

$$= \mathbf{32x^7 + 56x^6 - 85x^4 - 8x^3 + 3x^2 + 12x}$$

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(*ii*) $y = \dfrac{7x^5}{(x+3)}$

Using quotient rule

$$\frac{dy}{dx} = \frac{(x+3)\frac{d}{dx}(7x^5) - 7x^5\frac{d}{dx}(x+3)}{(x+3)^2} = \frac{(x+3)\times 35x^4 - 7x^5(1+0)}{(x+3)^2}$$

$$= \frac{35x^5 + 105x^4 - 7x^5}{(x+3)^2} = \frac{28x^5 + 105x^4}{(x+3)^2} = \boldsymbol{\frac{7x^4(4x+15)}{(x+3)^2}}$$

Example 6 **Differentiate the followings with respect to x**

(*i*) sin $2x$ **(*ii*) cos $3x$** **(*iii*) tan $4x$** **(*iv*) $\sin^3 x$** **(*v*) $x^2/\tan x$**

Solution. (*i*) $y = \sin 2x$; $\dfrac{dy}{dx} = \cos 2x \dfrac{d}{dx}(2x) = (\cos 2x)\times 2$ $= \mathbf{2 \cos 2x}$

(*ii*) $y = \cos 3x$; $\dfrac{dy}{dx} = -\sin 3x \dfrac{d}{dx}(3x)$ $= \mathbf{-3 \sin 3x}$

(*iii*) $y = \tan 4x$; $\dfrac{dy}{dx} = \sec^2 4x \dfrac{d}{dx}(4x)$ $= \mathbf{4 \sec^2 4x}$

(*iv*) $y = \sin^3 x$; $\dfrac{dy}{dx} = \dfrac{d}{dx}(\sin x)^3 = 3(\sin x)^2 \dfrac{d}{dx}(\sin x)$ $= \mathbf{3 \sin^2 x \cos x}$

(*v*) $y = x^2/\tan x = x^2 \cot x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(x^2) = x^2(-\text{cosec}^2 x) + \cot x\,(2x) = \mathbf{-x^2\,\text{cosec}^2 x + 2x \cot x}$$

Example 7 **A particle starts from rest with a uniform acceleration. Its displacement x after t seconds is given in metres by the relation** $\mathbf{x = 5 + 6t + 7t^2}$

Calculate the magnitude of its (i) initial velocity (ii) velocity at t = 3 s (iii) uniform acceleration and (iv) displacement at t = 5 s.

Solution. The velocity v at any instant is given by $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(5 + 6t + 7t^2)$

$$= 0 + 6\times 1 + 7\times 2t = 6 + 14t$$

(*i*) For initial velocity, $t = 0$ $\therefore$ Initial velocity = 6 + 14 × 0 = **6 m/s**

(*ii*) When $t = 3$, velocity, v = 6 + 14 × 3 = 6 + 42 = **48 m/s**

(*iii*) Acceleration, a at any time t is, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(6 + 14t) = 0 + 14 \times 1 =$ **14 m/s²**

(*iv*) At t = 5 s, displacement, $x = 5 + 6 \times 5 + 7 \times 5^2 = 5 + 30 + 175 =$ **210 m**

Example 8 **The mass of a body is 2.5 kg. It is in motion and its velocity v after time t is**

$$v = \frac{t^3}{3} + \frac{t^2}{2} + 1$$

Calculate the force acting on the body at the time t = 3 s.

Solution. Acceleration, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(\dfrac{t^3}{3} + \dfrac{t^2}{2} + 1\right) = \dfrac{d}{dt}\left(\dfrac{t^3}{3}\right) + \dfrac{d}{dt}\left(\dfrac{t^2}{2}\right) + \dfrac{d}{dt}(1)$

$$= \frac{1}{3}\times 3t^{3-1} + \frac{1}{2}2t^{2-1} + 0 = t^2 + t$$

When $t = 3$ s, $a = 3^2 + 3 = 12\ \text{m/s}^2$

As, force = mass × acceleration = $2{\cdot}5 \times 12 =$ **30 N**

Example 9 The air is filled in a balloon and the volume of balloon increases gradually. Find the rate of increase of volume of balloon with radius when radius of balloon becomes 30 cm.

Solution. Here, radius of balloon, $r = 30$ cm

Volume of balloon, $V = \frac{4}{3}\pi r^3$

Rate of increase of volume of balloon w.r.t. radius

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi \times 3r^2 = 4\pi r^2$$

When $r = 30$ cm $= 0{\cdot}30$ m

$$\frac{dV}{dr} = 4 \times \frac{22}{7} \times (0{\cdot}30)^2 = \mathbf{1{\cdot}13\ m^2}$$

Example 10 A particle is at rest. It starts rotating about a fixed point. Its angle of rotation (θ) with time (*t*) is given by the relation :

$$\theta = \frac{6t^3}{15} - \frac{t^2}{2}$$

where θ is in radian and *t* is seconds. Find the angular velocity and angular acceleration of a particle at the end of 6 second.

Solution. Here, $\theta = \frac{6t^3}{15} - \frac{t^2}{2}$

Angular velocity, $\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left[\frac{6t^3}{15} - \frac{t^2}{2}\right] = \frac{6}{5}t^2 - t$

Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{6}{5}t^2 - t\right) = \frac{12}{5}t - 1$

When $t = 6$s, $\omega = \frac{6}{5} \times 6^2 - 6 = 43{\cdot}2 - 6 = 37{\cdot}2$ **rad/s**

$$\alpha = \frac{12}{5} \times 6 - 1 = \mathbf{13{\cdot}4\ rad/s^2}$$

PROBLEMS FOR PRACTICE

BASED ON DIFFERENTIATION

1. Differentiate the following w.r.t. x
   (*i*) π^2 (*ii*) e^2 (*iii*) -14 **[Ans. (*i*) 0 (*ii*) 0 (*iii*) 0]**
2. Differentiate the following w.r.t. x
   (*i*) $(3x + 5)$ (*ii*) x^{-2} (*iii*) $x^{3/2}$ (*iv*) $\sqrt{x} - \frac{1}{\sqrt{x}}$ (*v*) $\frac{1}{(x+2)}$

$$\left[\text{Ans. } (i)\,3\ (ii)\,\frac{-2}{x^3}\ (iii)\,\frac{3}{2}x^{1/2}\ (iv)\,\frac{1}{2\sqrt{x}}\left(\frac{x+1}{x}\right)\ (v) -\frac{1}{(x+2)^2}\right]$$

3. Differentiate the following w.r.t. x

(*i*) $3\,x^{5/2}$ (*ii*) $(4\,x)^{1/3}$ (*iii*) $\dfrac{6}{\sqrt{x}}$ (*iv*) $x \sin nx$

[Ans. (*i*) $\dfrac{15}{2}x^{3/2}$ (*ii*) $\dfrac{4^{1/3}}{3}x^{-2/3}$ (*iii*) $-3\,x^{-3/2}$ (*iv*) $\sin nx + nx \cos nx$]

4. Differentiate the following w.r.t. x

(*i*) $(5\,x^2+6)\,(2\,x^3+4)$ (*ii*) $\sqrt{x}\,(x^2+7)$ (*iii*) $(x^2+3)\,(x^4-9)$

[Ans. (*i*) $50\,x^4+36\,x^2+40\,x$ (*ii*) $\dfrac{5}{2}x^{3/2}+\dfrac{7}{2}x^{-1/2}$ (*iii*) $6\,x\,(x^4+2\,x^2-3)$]

5. Differentiate the following *w.r.t.* x

(*i*) $\dfrac{2}{(3x+1)}$ (*ii*) $\dfrac{3x+5}{4x+6}$ (*iii*) $\dfrac{x^2+1}{x+2}$ **[Ans. (*i*) $\dfrac{-6}{(3x+1)^2}$ (*ii*) $\dfrac{-2}{(4x+6)^2}$ (*iii*) $\dfrac{x^2+4x-1}{(x+2)^2}$]**

6. Differentiate the followings w.r.t. x

(*i*) $\sin 3\,x$ (*ii*) $\cos^2 x$ (*iii*) $\sin x^3$ **[Ans. (*i*) $3\cos 3\,x$ (*ii*) $-\sin 2\,x$ (*iii*) $3\,x^2 \cos x^3$]**

7. Differentiate the following w.r.t. x and find the value when $x = 9$.

(*i*) $\sin 5\,x$ (*ii*) $\tan 5\,x$ (*iii*) $\cos 5\,x$ **[Ans. (*i*) $5/\sqrt{2}$ (*ii*) 10 (*iii*) $-5/\sqrt{2}$]**

8. Differentiate the followings w.r.t., x

(*i*) $\tan^3 x$ (*ii*) $\dfrac{\cos x}{(1-\sin x)}$ (*iii*) $\sin\,(ax+b)^2$

[Ans. (*i*) $3\tan^2 x \sec^2 x$ (*ii*) $\dfrac{1}{(1-\sin x)}$ (*iii*) $2a\,(ax+b)\cos\,(ax+b)^2$]

9. If $x = a\,(\theta + \sin\theta)$ and $y = a\,(1-\cos\theta)$, find $\dfrac{dy}{dx}$. **[Ans. $\tan\theta/2$]**

10. If $x = at^3$ and $y = bt^2$, find $\dfrac{dy}{dx}$. **$\left[\text{Ans. } \dfrac{2b}{3at}\right]$**

11. If $x = a\cos\theta$ and $y = b\sin\theta$, find $\dfrac{dy}{dx}$. **$\left[\text{Ans. } -\dfrac{b}{a}\cot\theta\right]$**

12. If the displacement x of a particle (in metre) is related with time (in second) according to relation

$$x = 2\,t^3 - 3\,t^2 + 2\,t + 2$$

find the position, velocity and acceleration of a particle at the end of 2 seconds.

[Ans. 10 m ; 14 m/s ; 18 m/s^2]

13. A particle starts from rest and its angular displacement (in rad) is given by $\theta = \dfrac{t^2}{20} + \dfrac{t}{5}$; calculate the angular velocity at the end of $t = 4$ second. **[Ans. 0·6 rad/s]**

14. A metallic disc is being heated. Its area A (in m^2) at any time t (in sec) is given by

$$A = 5\,t^2 + 4\,t + 8$$

Calculate the rate of increase of area at $t = 3$ s. **[Ans. 34 m^2/s]**

15. A particle starts from origin with uniform acceleration. Its displacement after t seconds is given in metres by the relation $x = 2 + 5\,t + 7\,t^2$

Calculate the magnitude of its (*i*) initial velocity (*ii*) velocity at $t = 4$ s (*iii*) uniform acceleration (*iv*) displacement at $t = 5$ s. **[Ans. (*i*) 5 m/s (*ii*) 61 m/s (*iii*) 14 m/s^2 (*iv*) 202 m]**

16. If the displacement of the particle at an instant is given by

$$y = r\sin(\omega t - \theta)$$

where r is amplitude of oscillation, ω is the angular velocity and $-\theta$ is the initial phase of the particle, then find the particle velocity and particle acceleration. **[Ans. $r\,\omega\cos(\omega t - \theta)$; $-\omega^2 y$]**

SECTION 7—INTEGRATION

7.1. WHAT IS INTEGRATION

The process of integration is just the reverse of differentiation. The symbol $\int$ is used to denote integration. If $f(x)$ is the differential coefficient of a function $F(x)$ with respect to x, then

$$\frac{dy}{dx} = \frac{d}{dx}[F(x)] = f(x) \qquad \text{or} \qquad dy = f(x)\,dx = d\,[F(x)]$$

Taking summation of all the differentials, we have

$$\Sigma\,dy = \Sigma f(x)\,dx = \Sigma\,d\,[F(x)] \qquad \text{or} \qquad y = \Sigma f(x)\,dx = F(x)$$

When dx approaches to zero, the summation is replaced by integral. So $y = \int f(x)\,dx = F(x)$ ...(*i*)

Here $\int f(x)\,dx$ is read as integral of $f(x)\,dx$. In relation (*i*), $f(x)$ is called integrand and $F(x)$ is called integral or permitive function of $f(x)$.

Thus the integration of $f(x)$ with respect to x is $F(x)$.

It is important to note that sign Σ is used for summation of discrete values, while sign $\int$ is used for summation of continuous function. It means the method of integration is used to sum up the effect of a continuous varying function. *e.g.*, workdone against a variable force. It is to be noted that ordinary algebra does not provide any method to sum up the effect of a varying function.

7.2. FUNDAMENTAL FORMULAE OF INTEGRATION

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, provided $n \neq -1$ **2.** $\int dx = \int x^0\,dx = \frac{x^{0+1}}{0+1} = x + C$

3. $\int (u+v)\,dx = \int u\,dx + \int v\,dx$ **4.** $\int cu\,dx = c\int u\,dx$ where c is a constant and u is a function of x.

5. $\int cx^n\,dx = c\frac{x^{n+1}}{n+1} + C$ **6.** $\int x^{-1}\,dx = \int \frac{dx}{x} = \log_e x + C$ **7.** $\int e^x\,dx = e^x + C$

8. $\int e^{ax}dx = \frac{e^{ax}}{a} + C$ **9.** $\int a^x dx = \frac{a^x}{\log_e a} + C$

10. (*i*) $\int \sin x\,dx = -\cos x + C$ (*ii*) $\int \sin nx\,dx = \frac{-\cos nx}{n} + C$

11. (*i*) $\int \cos x\,dx = \sin x + C$ (*ii*) $\int \cos nx\,dx = \frac{\sin nx}{n} + C.$ **12.** $\int \sec^2 x\,dx = \tan x + C$

13. $\int \text{cosec}^2\,x\,dx = -\cot x + C$ **14.** $\int \sec x\tan x\,dx = \sec x + C$

15. $\int \text{cosec}\,x\cot x\,dx = -\text{cosec}\,x + C$ **16.** $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)\frac{d}{dx}(ax+b)} = \frac{(ax+b)^{n+1}}{a\,(n+1)} + C$

17. $\int \frac{a}{(ax+b)}\,dx = \frac{a\log_e(ax+b)}{\frac{d}{dx}(ax+b)} = \log_e(ax+b) + C$

7.3. INDEFINITE INTEGRALS

If a function of x is given by $y = x^5 + C$, where C is a constant of integration then

$$\frac{dy}{dx} = \frac{d}{dx}(x^5 + C) = 5x^4 + 0 = 5x^4$$

As integration is reverse of differentiation, so $\int 5x^4\,dx$ should be equal to $x^5 + C$. But using the standard integral formulae as discussed earlier, we have $\int 5x^4\,dx = x^5$

It means during integration, a constant does not appear. It is due to the fact that the differential coefficient of a constant is zero. Hence, we cannot say definitely whether $\int 5x^4\,dx = x^5$ or $x^5 + C$. Such an integration is known as **Indefinite integration.** Therefore, in all the indefinite integrals, a constant of integration C is supposed to be present, if the condition of integration (*i.e.*, limit of integration) is not mentioned. **Therefore, we should add a constant C in the result of all indefinite integrals.**

7.4. METHODS OF INTEGRATION

Sometimes, we come across some functions which cannot be integrated directly by using the standard integrals. In such cases, the integral of a function can be obtained by using one or more of the following methods :

(*a*) Integration by substitution

(*b*) Integration by parts.

(*a*) Integration by substitution

Those functions which cannot be integrated directly can be reduced to standard integrand by making a suitable substitution and then can be integrated directly by using the standard integrals. To understand the method, we take the few examples.

Example 1 **Evaluate ;** $\int \left(\frac{1}{ax+b}\right) dx$.

Solution. Put $ax + b = z$. Then, $\frac{d}{dx}(ax+b) = \frac{dz}{dx}$ or $a\,dx = dz$ or $dx = \frac{dz}{a}$

$$\therefore \quad \int \frac{1}{(ax+b)}\,dx = \int \frac{1}{z}\frac{dz}{a} = \frac{1}{a}\int \frac{dz}{z} = \frac{1}{a}\log z + C = \frac{1}{a}\log_e(ax+b) + C$$

Example 2 **Evaluate :** $\int \sec^4 x \tan x\, dx$.

Solution. Put $\sec x = z$ so, $\frac{d}{dx}(\sec x) = \frac{dz}{dx}$ or $\sec x \tan x\, dx = dz$

$$\therefore \int \sec^4 x \tan x\, dx = \int \sec^3 x\,(\sec x \tan x)\, dx$$

$$\int z^3 dz = \frac{z^4}{4} + C = \frac{\sec^4 x}{4} + C = \frac{1}{4}\sec^4 x + C$$

Example 3 **Evaluate** $\int (x - \cos x + 1/x)$.

Solution. $\int\left(x - \cos x + \frac{1}{x}\right) dx = \int x\,dx - \int \cos x\,dx + \int \frac{dx}{x} = \frac{x^2}{2} - \sin x + \log_e x + C$

(*b*) Integration by parts. This method of integration is based on the following rule :

Integral of a product of two functions = first function × integral of second function – integral of (differential coefficient of first function × integral of second function).

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Thus, if u and v are the functions of x, then $\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\times\int v\,dx\right]dx$

To illustrate the method we take the few examples.

Example 1 **Evaluate $\int x\cos x\,dx$.**

Solution. $\int x\cos x\,dx = x\int\cos x\,dx - \int\left[\frac{d}{dx}(x)\int\cos x\,dx\right]dx = x\sin x - \int 1.\sin x\,dx$

$$= \boldsymbol{x\sin x + \cos x + C}$$

Example 2 **Evaluate : $\int x^n \log x\,dx$.**

Solution. $\int x^n \log x\,dx = \int \log x.x^n\,dx = \log x\int x^n\,dx - \int\left[\frac{d}{dx}(\log x)\int x^n\,dx\right]dx$

$$= \log x\times\frac{x^{n+1}}{n+1} - \int\frac{1}{x}.\frac{x^{n+1}}{n+1}\,dx = \frac{x^{n+1}\log x}{n+1} - \frac{1}{n+1}\int x^n\,dx$$

$$= \frac{x^{n+1}\log x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C = \boldsymbol{\frac{x^{n+1}}{n+1}\left[\log x - \frac{1}{(n+1)}\right] + C}$$

7.5. DEFINITE INTEGRALS

> ***When a function is integrated between definite limits, the integral is called definite integral.***

For example, $\int_a^b f(x)\,dx$ is a definite integral of $f(x)$ between the limits a and b and is written as

$$\int_a^b f(x)\,dx = \left| F(x)\right|_a^b = F(b) - F(a)$$

Here a is called the lower limit and b is called the upper limit of integration.
It is to be noted that no constant of integration is involved in the integration of definite integrals.

Example 1 **Evaluate $\int_1^4 x^3\,dx$**

Solution. $\int_1^4 x^3\,dx = \left(\frac{x^{3+1}}{3+1}\right)_1^4 = \left(\frac{x^4}{4}\right)_1^4 = \left(\frac{4^4}{4} - \frac{1^4}{4}\right) = 64 - 0{\cdot}25 = \boldsymbol{63{\cdot}75}$

Example 2 **Evaluate : $\int_0^2 \frac{1}{(1+3x)}\,dx$.**

Solution. $\int_0^2 \frac{1}{(1+3x)}\,dx = \left[\frac{\log(1+3x)}{3}\right]_0^2 = \frac{1}{3}\,[\log(1+3\times 2) - \log(1+3\times 0)]$

$$= \frac{1}{3}\,[\log 7 - \log 1] = \frac{1}{3}\,(\log 7 - 0) = \boldsymbol{\frac{1}{3}\log 7}$$

Example 3 Evaluate : $\displaystyle\int_a^b \frac{\log x}{x}\,dx$.

Solution. Let $\log x = z$; so $\frac{1}{x}\,dx = dz$.

When $x = a$, $z = \log a$ and when $x = b$, $z = \log b$

Hence, $$\int_a^b \frac{\log x}{x}\,dx = \int_{\log a}^{\log b} z\,dz = \left(\frac{z^2}{2}\right)_{\log a}^{\log b} = \frac{1}{2}[(\log b)^2 - (\log a)^2]$$

Example 4 Evaluate $\displaystyle\int_{40^\circ}^{60^\circ} \frac{d\theta}{(\theta-\theta_0)}$; where $\theta_0 = 20^\circ$

Solution. $$\int_{40^\circ}^{60^\circ} \frac{d\theta}{(\theta-\theta_0)} = \left[\log_e(\theta-\theta_0)\right]_{40}^{60} = \log_e(60-20) - \log_e(40-20)$$

$$= \log_e \frac{40}{20} = \log_e 2 = 2{\cdot}3026 \log_{10} 2 = 2{\cdot}3026 \times 0{\cdot}3010 = \mathbf{0{\cdot}6931}$$

PROBLEMS FOR PRACTICE

BASED ON INTEGRATION

1. Integrate the following with respect to x

(*i*) $\sqrt[3]{x^5}$ (*ii*) $\sqrt{x} - \frac{1}{\sqrt{x}}$ (*iii*) $\sqrt{(3x-4)^3}$ (*iv*) $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ (*v*) $\left(x + \frac{1}{x}\right)^3$ (*vi*) $\left(3x^2 + \frac{1}{x^2} - 2x\right)$

[**Ans.** (*i*) $\frac{3}{8}x^{8/3} + C$ (*ii*) $\frac{2}{3}x^{3/2} - 2x^{1/2} + C$ (*iii*) $\frac{2}{15}(3x-4)^{5/2} + C$ (*iv*) $\frac{x^2}{2} + \log_e x + 2x + C$
(*v*) $\frac{x^4}{4} + \frac{3x^2}{2} + 3\log_e x - \frac{1}{2x^2} + C$ (*vi*) $x^3 - \frac{1}{x} - x^2 + C$]

2. Integrate the following functions w.r.t. x and find within the limits $x = 2$ to 4.

(*i*) x^3 (*ii*) $\sqrt{x}$ (*iii*) x^{-1} (*iv*) $x^{-1/2}$ [**Ans.** (*i*) **60** (*ii*) **3·448** (*iii*) **0·6931** (*iv*) $\mathbf{4 - 2\sqrt{2}}$]

3. Evaluate the following integrals

(*i*) $\displaystyle\int_{-\pi/2}^{+\pi/2} \cos x\,dx$ (*ii*) $\displaystyle\int_0^{\pi/3} \sec x \tan x\,dx$ (*iii*) $\displaystyle\int_0^{\pi/6} \sec x^2\,x\,dx$ [**Ans.** (*i*) **2** (*ii*) **1** (*iii*) $\mathbf{1/\sqrt{3}}$]

4. Evaluate the following integrals

(*i*) $\displaystyle\int_{15}^{30} \cos(4x-3)\,dx$ (*ii*) $\displaystyle\int_0^{30} \sin 7x\,dx$ (*iii*) $\displaystyle\int_0^{30} \cos 5x\,dx$ (*iv*) $\displaystyle\int_0^{10} \sec^2(3x+6)\,dx$

[**Ans.** (*i*) $\frac{1}{4}$ [**sin 117° – sin 57°**] (*ii*) **0·2666** (*iii*) $\frac{1}{10}$ (*iv*) $\frac{1}{3}$ **(tan 36° – tan 6°)**]

5. Evaluate w.r.t. x ; (*i*) $\int x \log x \, dx$ (*ii*) $\int \left(x^2 - \cos x + \frac{1}{x} \right) dx$

$$\left[\textbf{Ans. } (i) \frac{x^2}{2} \left(\log x - \frac{1}{2} \right) + C \; (ii) \frac{x^3}{3} - \sin x + \log_e x + C \right]$$

6. Evaluate

(*i*) $\int_0^{\pi/4} \sin x \cos x \, dx$ (*ii*) $\int_0^{\pi/2} (1 + \cos x)^{1/2} \, dx$ (*iii*) $\int_0^{\pi/2} (1 + \sin x)^{1/2} \, dx$ (*iv*) $\int_0^{\pi/4} (1 - \cos 2x)^{1/2} \, dx$

$$\left[\textbf{Ans. } (i) \frac{1}{4} \; (ii)\, 2 \; (iii)\, 2 \; (iv)\, 0{\cdot}414 \right]$$

7. Evaluate $\int_R^{\infty} \frac{GM\,m}{x^2} \, dx$. $\left[\textbf{Ans. } \frac{\mathbf{GM\,m}}{\mathbf{R}} \right]$

8. Evaluate $\int_{50}^{40} \frac{d\theta}{\theta - \theta_0}$, where $\theta_0 = 30°C$, a constant temperature of surroundings. **[Ans. – 0·6931]**

9. Evaluate : (*a*) $\int_u^v dv = \int_0^t a\,dt$ (*b*) $\int_0^s ds = \int_0^t u\,dt + \int_0^t at\,dt$ (*c*) $\int_u^v v\,dv = \int_0^s a\,ds$

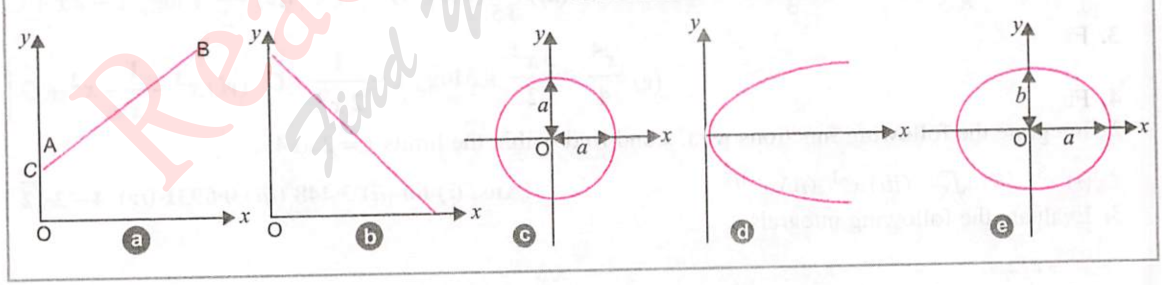
$$\left[\textbf{Ans. (a) } v = u + at \textbf{ (b) } s = ut + \frac{1}{2} at^2 \textbf{ (c) } v^2 = u^2 + 2\,as \right]$$

SECTION 8 — GENERAL

8.1. EQUATIONS OF SOME STANDARD CURVES

1. Straight line : $y = mx + c$; where m and c are constants. Fig. 0.15(a).

FIGURE 0.15



The equation of straight line can also be put in the form ; $\frac{x}{a} + \frac{y}{b} = 1$

where a and b are constants. Fig. 0.15(b).

2. Circle : $x^2 + y^2 = a^2$; Its centre is (0, 0) and radius = a. Fig. 0.15(c).

3. Parabola : $y^2 = 4\,ax$; Fig. 0.15(d).

4. Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; where a = semi major axis of the ellipse, b = semi minor axis of the ellipse. Fig. 0.15(e).

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8.2. FORMULAE FOR DETERMINATION OF AREA

1. Area of a square = $(\text{side})^2$
2. Area of a rectangle = length × breadth
3. Area of a triangle = $\frac{1}{2}$ base × height
4. Area enclosed by a circle = πr^2, where r is radius
5. Surface area of a sphere = $4\pi r^2$, where r is radius
6. Area of a parallelogram = base × height
7. Area of curved surface of cylinder = $2\pi r.l$ (r is radius, l is length).
8. Total surface area of right circular cylinder = $2\times\pi r^2 + 2\pi r l = 2\pi r(r+l)$ where r is the radius and l is the length of cylinder.

8.3. FORMULAE FOR DETERMINATION OF VOLUME

1. Volume of a rectangular slab = length × breadth × height
2. Volume of a cube = $(\text{side})^3$
3. Volume of a sphere $=\frac{4}{3}\pi r^3$, where r is radius.
4. Volume of a cylinder = $\pi r^2 l$, where r is radius, l is length
5. Volume of a cone $=\frac{1}{3}\pi r^2 h$, where r is radius, h is height.

Note, $\pi = \frac{22}{7} = 3{\cdot}142$; $\pi^2 = 9{\cdot}8776$ and $\frac{1}{\pi} = 0{\cdot}3182$.

PROBLEMS FOR PRACTICE

1. Find the (*i*) surface area (*ii*) volume of the cylinder of length 10 cm and radius 2 cm. **[Ans. (*i*) 48 π cm^2 (*ii*) 40 π cm^3]**
2. Find the (*i*) surface area and (*ii*) volume of a sphere of radius 5 cm. **[Ans. (*i*) 100 π cm^2 (*ii*) 166·7 π cm^3]**
3. Find the (*i*) surface area (*ii*) volume of the rectangular body of dimension 30 cm × 10 cm × 5 cm. **[Ans. (*i*) 1000 cm^2 (*ii*) 1500 cm^3]**
4. Find the volume of the cone of height 10 cm and radius of the base 3 cm. **[Ans. 30 π cm^3]**

8.4. FORMULAE OF EXPANSIONS FOR SOME QUANTITIES

(*a*) **Exponential Expansion ;** $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

(*b*) **Logarithmic Expansion ;** $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots.\ (x<1)$

(*c*) **Trignometric Expansion ;** (when θ is in radians),

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots.$$

HINTS

For Difficult Problems

BASED ON DIFFERENTIATION

1. All are constant quantities, hence, their differentiation is zero.

2. (*iv*) $$\frac{d}{dx}(\sqrt{x})-\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)=\frac{d}{dx}(x^{1/2})-\frac{d}{dx}(x^{-1/2})=\frac{1}{2}x^{-1/2}+\frac{1}{2}x^{-3/2}=\frac{1}{2\sqrt{x}}\left(1+\frac{1}{x}\right)=\mathbf{\frac{1}{2\sqrt{x}}\left(\frac{x+1}{x}\right)}$$

(*v*) $$\frac{d}{dx}\left(\frac{1}{x+2}\right)=\frac{d}{dx}(x+2)^{-1}=(-1)(x+2)^{-2}\frac{d}{dx}(x+2)=-\frac{1}{(x+2)^2}[1+0]=\mathbf{-\frac{1}{(x+2)^2}}$$

3. (*i*) $$y=\frac{d}{dx}(3x^{5/2})=3\frac{d}{dx}(x^{5/2})=3\times\frac{5}{2}x^{3/2}=\mathbf{\frac{15}{2}x^{3/2}}$$ (*ii*) $$y=\frac{d}{dx}(4x)^{\frac{1}{3}}=4^{\frac{1}{3}}\frac{d}{dx}(x)^{\frac{1}{3}}=\mathbf{4^{\frac{1}{3}}\times\frac{1}{3}x^{-2/3}}$$

(*iii*) $$y=\frac{d}{dx}\left(\frac{6}{\sqrt{x}}\right)=6\frac{d}{dx}(x^{-\frac{1}{2}})=6\times\left(-\frac{1}{2}x^{-3/2}\right)=\mathbf{-3\,x^{-3/2}}$$

(*iv*) $$\frac{d}{dx}(x\sin nx)=x\frac{d}{dx}(\sin nx)+\sin nx\frac{d}{dx}(x)=x\,(\cos nx)\,n+\sin nx=\mathbf{nx\cos nx+\sin nx}$$

4. Using product rule.

(*i*) $$\frac{dy}{dx}=(5x^2+6)\frac{d}{dx}(2x^3+4)+(2x^3+4)\frac{d}{dx}(5x^2+6)=(5x^2+6)\times 6x^2+(2x^3+4)\times 10x$$
$$=30x^4+36x^2+20x^4+40x=\mathbf{50x^4+36x^2+40x}$$

(*ii*) $$\frac{dy}{dx}=x^{\frac{1}{2}}\frac{d}{dx}(x^2+7)+(x^2+7)\frac{d}{dx}(x^{\frac{1}{2}})=x^{\frac{1}{2}}(2x+0)+(x^2+7)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$
$$=2x^{3/2}+\frac{1}{2}x^{3/2}+\frac{7}{2}x^{-\frac{1}{2}}=\mathbf{\frac{5}{2}x^{3/2}+\frac{7}{2}x^{-1/2}}$$

(*iii*) $$\frac{dy}{dx}=(x^2+3)\frac{d}{dx}(x^4-9)+(x^4-9)\frac{d}{dx}(x^2+3)=(x^2+3)[4x^3-0]+(x^4-9)[2x-0]$$
$$=4x^5+12x^3+2x^5-18x=6x^5+12x^3-18x=\mathbf{6x[x^4+2x^2-3]}$$

5. Using quotien rule. (*i*) $$\frac{dy}{dx}=\frac{(3x+1)\frac{d}{dx}(2)-2\frac{d}{dx}(3x+1)}{(3x+1)^2}=\frac{(3x+1)\times 0-2(3+0)}{(3x+1)^2}=\mathbf{\frac{-6}{(3x+1)^2}}$$

(*ii*) $$\frac{dy}{dx}=\frac{(4x+6)\frac{d}{dx}(3x+5)-(3x+5)\frac{d}{dx}(4x+6)}{(4x+6)^2}=\frac{(4x+6)(3+0)-(3x+5)(4+0)}{(4x+6)^2}$$
$$=\frac{12x+18-12x-20}{(4x+6)^2}=\mathbf{\frac{-2}{(4x+6)^2}}$$

(*iii*) $$\frac{dy}{dx}=\frac{(x+2)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x+2)}{(x+2)^2}=\frac{(x+2)(2x+0)-(x^2+1)(1+0)}{(x+2)^2}$$
$$=\frac{2x^2+4x-x^2-1}{(x+2)^2}=\mathbf{\frac{x^2+4x-1}{(x+2)^2}}$$

6. (*i*) $y = \sin 3x$; Let $u = 3x$; $\dfrac{du}{dx} = 3$

Then, $y = \sin u$; $\dfrac{dy}{du} = \cos u = \cos 3x$ $\therefore$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = (\cos 3x) \times 3 = \mathbf{3 \cos 3x}$

(*ii*) Let $u = \cos x$; $\dfrac{du}{dx} = -\sin x$, $y = \cos^2 x = u^2$; $\dfrac{dy}{du} = 2u = 2\cos x$

Using chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 2 \cos x\,(-\sin x) = \mathbf{-\sin 2x}$$

(*iii*) Let $u = x^3$; $\dfrac{du}{dx} = 3x^2$, $y = \sin u$; $\dfrac{dy}{dx} = \cos u = \cos x^3$

Using chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \mathbf{3x^2 \cos x^3}$$

7. (*i*) Let $u = 5x$, $\dfrac{du}{dx} = 5$ so, $y = \sin u$ and $\dfrac{dy}{du} = \cos u = \cos 5x$

Using chain rule

$\therefore$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = \cos 5x \times 5 = 5\cos 5x$ when $x = 9$, $\dfrac{dy}{dx} = 5\cos 5 \times 9 = 5\cos 45° = 5 \times \dfrac{1}{\sqrt{2}}$

(*ii*) $\dfrac{dy}{dx} = \dfrac{d}{dx}(\tan 5x) = \sec^2 5x \times \dfrac{d}{dx}(5x) = \sec^2 5x \times 5$ when $x = 9$, $\dfrac{dy}{dx} = 5\sec^2 5 \times 9 = 5 \times (\sqrt{2})^2 = \mathbf{10}$

(*iii*) $\dfrac{dy}{dx} = \dfrac{d}{dx}(\cos 5x) = -5\sin 5x = -5\sin 5 \times 9 = -\dfrac{\mathbf{5}}{\mathbf{\sqrt{2}}}$

8. (*i*) $y = \tan^3 x = (\tan x)^3$; $\dfrac{dy}{dx} = \dfrac{d}{dx}(\tan x)^3 = 3(\tan x)^2 \dfrac{d}{dx}(\tan x) = \mathbf{3 \tan^2 x \sec^2 x}$

(*ii*) $y = \dfrac{\cos x}{1 - \sin x}$; $\dfrac{dy}{dx} = \dfrac{(1 - \sin x)\dfrac{d}{dx}\cos x - \cos x \dfrac{d}{dx}(1 - \sin x)}{(1 - \sin x)^2}$

$$\frac{dy}{dx} = \frac{(1 - \sin x)(-\sin x) - \cos x \times (-\cos x)}{(1 - \sin x)^2} = \frac{-\sin x + \sin^2 x + \cos^2 x}{(1 - \sin x)^2} = \frac{-\sin x + 1}{(1 - \sin x)^2} = \frac{\mathbf{1}}{\mathbf{(1 - \sin x)}}$$

(*iii*) $y = \sin (ax + b)^2$; $\dfrac{dy}{dx} = \dfrac{d}{dx}[\sin (ax + b)^2] = \cos (ax + b)^2 \dfrac{d}{dx}(ax + b)^2$

$$= \cos (ax + b)^2 \times 2(ax + b) \frac{d}{dx}(ax + b) = \cos (ax + b)^2 \times 2(ax + b) \times a$$

$$= \mathbf{2a\,(ax + b) \cos (ax + b)^2}$$

9. $\dfrac{dx}{d\theta} = a[1 - \cos\theta]$; $\dfrac{dy}{d\theta} = a(0 + \sin\theta) = a\sin\theta.$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{a\sin\theta}{a(1 + \cos\theta)} = \dfrac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2} = \mathbf{\tan \theta/2}$

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10. $\frac{dx}{dt} = 3at^2$; $\frac{dy}{dt} = 2bt$ $\quad \therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2bt}{3at^2} = \mathbf{\frac{2b}{3at}}$

11. $\frac{dx}{d\theta} = -a\sin\theta$; $\frac{dy}{d\theta} = b\cos\theta$ $\quad \therefore \frac{dy}{dx} = \frac{dy}{d\theta}\Big/\frac{dx}{d\theta} = \frac{b\cos\theta}{-a\sin\theta} = \mathbf{-\frac{b}{a}\cot\theta}$

12. $x = 2t^3 - 3t^2 + 2t + 2$ ...(*i*)

velocity $= \frac{dx}{dt} = 6t^2 - 6t + 2$, Acceleration $= \frac{d}{dt}\left(\frac{dx}{dt}\right) = 12t - 6$

When $t = 2$ sec, $x = 2 \times 2^3 - 3 \times 2^2 + 2 \times 2 + 2 = 16 - 12 + 4 + 2 = \mathbf{10\ m}$

Velocity $= 6 \times 2^2 - 6 \times 2 + 2 = \mathbf{14\ m/s}$, Acceleration $= 12 \times 2 - 6 = \mathbf{18\ m/s^2}$

13. Angular velocity, $\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left(\frac{t^2}{20} + \frac{t}{5}\right) = \frac{2t}{20} + \frac{1}{5}$ When $t = 4$ s, $\omega = \frac{2\times4}{20} + \frac{1}{5} = \mathbf{0{\cdot}6\ rad/s}$

14. $\frac{dA}{dt} = \frac{d}{dt}(5t^2 + 4t + 8) = 10t + 4$ When $t = 3$ s ; $\frac{dA}{dt} = 10 \times 3 + 4 = \mathbf{34\ m^2/s}$

15. Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(2 + 5t + 7t^2) = (5 + 14t)$

(*i*) When $t = 0$, $v = \mathbf{5\ m/s}$ (*ii*) When $t = 4$ s, $v = 5 + 14 \times 4 = \mathbf{61\ m/s}$

(*iii*) Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(5 + 14t) = \mathbf{14\ m/s^2}$

(*iv*) When $t = 5$ s, $x = 2 + 5 \times 5 + 7 \times 5^2 = 2 + 25 + 175 = \mathbf{202\ m}$

16. $y = r\sin(\omega t - \theta)$

Velocity, $v = \frac{dy}{dt} = r\cos(\omega t - \theta)\frac{d}{dt}(\omega t - \theta) = r\cos(\omega t - \theta) \times \omega = \mathbf{r\,\omega\cos(\omega t - \theta)}$

Acceleration, $A = \frac{dv}{dt} = r\omega[-\sin(\omega t - \theta)] \times \omega = -\omega^2 r\sin(\omega t - \theta) = \mathbf{-\omega^2 y}$

BASED ON INTEGRATION

1. (*i*) $\int \sqrt[3]{x^5}\,dx = \int x^{5/3}\,dx = \frac{x^{5/3+1}}{\frac{5}{3}+1} = \frac{x^{8/3}}{8/3} = \mathbf{\frac{3}{8}x^{8/3} + C}$

(*ii*) $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx = \int (x^{1/2} - x^{-1/2})\,dx = \int x^{1/2}\,dx - \int x^{-1/2}\,dx = \frac{x^{(1/2)+1}}{\frac{1}{2}+1} - \frac{x^{(-1/2)+1}}{-\frac{1}{2}+1} + C$

$$= \frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} + C = \mathbf{\frac{2}{3}x^{3/2} - 2x^{1/2} + C}$$

(*iii*) $\int \sqrt{(3x-4)^3}\,dx = \int (3x-4)^{3/2}\,dx = \frac{(3x-4)^{(3/2)+1}}{\left(\frac{3}{2}+1\right)\frac{d}{dx}(3x-4)} + C$

$$= \frac{(3x-4)^{5/2}}{\frac{5}{2}\times3} + C = \mathbf{\frac{2}{15}(3x-4)^{5/2} + C}$$

(iv) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx=\int\left(x+\frac{1}{x}+2\right)dx \;=\frac{x^2}{2}+\log_e x+2x+C$

(v) $\int\left(x+\frac{1}{x}\right)^3 dx=\int\left(x^3+3x+\frac{3}{x}+\frac{1}{x^3}\right)dx \;=\frac{x^4}{4}+\frac{3x^2}{2}+3\log_e x+\frac{x^{-3+1}}{-3+1}$

$$=\frac{x^4}{4}+\frac{3x^2}{2}+3\log_e x-\frac{1}{2x^2}+C$$

(vi) $\int(3x^2+x^{-2}-2x)dx \;=3\frac{x^3}{3}+\frac{x^{-2+1}}{-2+1}-2\frac{x^2}{2}+C=x^3-\frac{1}{x}-x^2+C$

2. (i) $\int_2^4 x^3 dx=\left(\frac{x^4}{4}\right)_2^4=\frac{4^4}{4}-\frac{2^4}{4}=\mathbf{60}$

(ii) $\int_2^4 \sqrt{x}\,dx=\int_2^4 x^{1/2}\,dx=\left(\frac{x^{3/2}}{3/2}\right)_2^4=\frac{2}{3}\left[4^{3/2}-2^{3/2}\right] \;=\frac{2}{3}\left[8-2\sqrt{2}\right]=\mathbf{3{\cdot}448}$

(iii) $\int_2^4 x^{-1}\,dx=\int_2^4\frac{dx}{x}=(\log_e x)_2^4=\log_e 4-\log_e 2=\log_e\frac{4}{2}=\log_e 2=2{\cdot}3026\log_{10}2$

$$=2{\cdot}3026\times 0{\cdot}3010=\mathbf{0{\cdot}6931}$$

(iv) $\int_2^4 x^{-1/2}\,dx=\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right)_2^4=2(x^{1/2})_2^4=2[4^{1/2}-2^{1/2}]=2(2-\sqrt{2}) \;=\mathbf{4-2\sqrt{2}}$

3. (i) $\int_{-\pi/2}^{+\pi/2}\cos x\,dx=(\sin x)_{-\pi/2}^{+\pi/2}=\sin\frac{\pi}{2}-\sin\left(-\frac{\pi}{2}\right)$

$$=\sin\frac{\pi}{2}+\sin\frac{\pi}{2}=1+1=\mathbf{2}$$

(ii) $\int_0^{\pi/3}\sec x\tan x\,dx=(\sec x)_0^{\pi/3}=\sec(\pi/3)-\sec 0 \;=2-1=\mathbf{1}$

(iii) $\int_0^{\pi/6}\sec^2 x\,dx=(\tan x)_0^{\pi/6}=\tan(\pi/6)-\tan 0=\frac{1}{\sqrt{3}}-0=\mathbf{\frac{1}{\sqrt{3}}}.$

4. (i) Put $4x-3=z$; so $4\,dx=dz$ or $dx=dz/4$

when $x=15$; $z=4\times15-3=57$ when $x=30$; $z=4\times30-3+117$

$\therefore \int_{15}^{30}\cos(4x-3)\,dx=\int_{57}^{117}\cos z\frac{dz}{4} \;=\frac{1}{4}(\sin z)_{57}^{117} \;=\mathbf{\frac{1}{4}(\sin 117°-\sin 57°)}$

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(ii) $$\int_0^{30} \sin 7x\,dx = \left(\frac{-\cos 7x}{7}\right)_0^{30} = -\frac{1}{7}[\cos 7\times 30 - \cos 7\times 0]$$

$$= -\frac{1}{7}[\cos 210 - \cos 0] = -\frac{1}{7}[\cos (180+30) - 1]$$

$$= -\frac{1}{7}[-\cos 30 - 1] = \frac{1}{7}\left[\frac{\sqrt{3}}{2}+1\right] = \mathbf{0{\cdot}2666}$$

(iii) $$\int_0^{30} \cos 5x\,dx = \left(\frac{\sin 5x}{5}\right)_0^{30} = \frac{1}{5}[\sin 5\times 30 - \sin 5\times 0] = \frac{1}{5}\sin 150^\circ$$

$$= \frac{1}{5}\sin(180^\circ - 30^\circ) = \frac{1}{5}\sin 30^\circ = \frac{1}{5}\times\frac{1}{2} = \mathbf{\frac{1}{10}}$$

(iv) $$\int_0^{10} \sec^2(3x+6)\,dx = \left[\frac{\tan(3x+6)}{3}\right]_0^{10} = \frac{1}{3}\ [\tan(3\times 10+6) - \tan 6]$$

$$= \mathbf{\frac{1}{3}\ [\tan 36^\circ - \tan 6^\circ]}$$

5. (i) $$\int x\log x\,dx = \int \log x . x\,dx = \log x \int x\,dx - \int\left[\frac{d}{dx}(\log x)\int x\,dx\right]dx$$

$$= \log x \times \frac{x^2}{2} - \int \frac{1}{x}\times\frac{x^2}{2}\,dx = \frac{x^2}{2}\log x - \frac{x^2}{4} + C$$

$$= \mathbf{\frac{x^2}{2}\left(\log x - \frac{1}{2}\right) + C}$$

(ii) $$\int\left(x^2 - \cos x + \frac{1}{x}\right)dx = \int x^2 dx - \int \cos x\,dx + \int \frac{1}{x}\,dx = \mathbf{\frac{x^3}{3} - \sin x + \log_e x + C}$$

6. (i) $$\int_0^{\pi/4} \sin x\cos x\,dx = \frac{1}{2}\int_0^{\pi/4}\sin 2x\,dx = \frac{1}{2}\left[\frac{-\cos 2x}{2}\right]_0^{\pi/4} = \mathbf{\frac{1}{4}}$$

(ii) $$\int_0^{\pi/2} (1+\cos x)^{\frac{1}{2}}\,dx = \int_0^{\pi/2} \sqrt{2}\cos\frac{x}{2}\,dx = \sqrt{2}\left(\frac{\sin\frac{x}{2}}{\frac{1}{2}}\right)_0^{\pi/2} = 2\sqrt{2}\left[\sin\frac{\pi/2}{2} - \frac{\sin 0}{2}\right]$$

$$= 2\sqrt{2}\left[\sin\frac{\pi}{4}\right] = 2\sqrt{2}\times\frac{1}{\sqrt{2}} = \mathbf{2}$$

(*iii*) $$\int_0^{\pi/2} (1+\sin x)^{1/2}\,dx = \int_0^{\pi/2}\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}\right)^{1/2} dx$$

$$= \int_0^{\pi/2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right) dx = \left[-2\cos\frac{x}{2}+2\sin\frac{x}{2}\right]_0^{\pi/2}$$

$$= 2\left[\sin\frac{x}{2}-\cos\frac{x}{2}\right]_0^{\pi/2}$$

$$= 2\,[(\sin \pi/4 - \cos \pi/4) - (\sin 0 - \cos 0)] = 2\left[\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-(0-1)\right] = \mathbf{2}$$

(*iv*) $$\int_0^{\pi/4} (1-\cos 2x)^{1/2}\,dx = \int_0^{\pi/4} (2\sin^2 x)^{1/2}\,dx$$

$$= \sqrt{2}\int_0^{\pi/4} \sin x\,dx = \sqrt{2}\,(-\cos x)_0^{\pi/4}$$

$$= -\sqrt{2}\,[\cos \pi/4 - \cos 0] = -\sqrt{2}\left[\frac{1}{\sqrt{2}}-1\right]$$

$$= -1+\sqrt{2} = \mathbf{0{\cdot}414}$$

7. $$\int_R^{\infty} \frac{GM\,m}{x^2}\,dx = GM\,m\int_R^{\infty} x^{-2}\,dx = GM\,m\left[\frac{x^{-2+1}}{-2+1}\right]_R^{\infty} = \frac{\mathbf{GM\,m}}{\mathbf{R}}$$

8. $$\int_{50}^{40} \frac{d\theta}{\theta-\theta_0} = \left[\log_e(\theta-\theta_0)\right]_{50}^{40} = \log_e\frac{40-30}{50-30} = \log_e\frac{1}{2}$$

$$= 2{\cdot}3026\,[\log 1 - \log 2] = 2{\cdot}3026\,[0 - 0{\cdot}3010] = \mathbf{-\,0{\cdot}6931.}$$

9. (*a*) $\int_u^v dv = \int_0^t a\,dt$ or $(v)_u^v = a(t)_0^t$ or $\boldsymbol{v-u=at}$

(*b*) $$\int_0^s ds = \int_0^t u\,dt + \int_0^t at\,dt$$

$(s)_0^s = u(t)_0^t + a\left(\frac{t^2}{2}\right)_0^t$ or $(s-0) = u(t-0)+\frac{1}{2}a(t^2-0^2)$ or $\boldsymbol{s = ut+\frac{1}{2}at^2}$

(*c*) $\int_u^v v\,dv = \int_0^s a\,ds$ or $\left(\frac{v^2}{2}\right)_u^v = a(s)_0^s$ or $\frac{v^2}{2}-\frac{u^2}{2} = as$ or $\boldsymbol{v^2 = u^2+2\,as}$

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. In the quadratic equation, $pqx^2 - (p^2 + q^2)x + pq = 0$, the values of x are

(*a*) $\frac{p+q}{p}, \frac{p+q}{q}$ (*b*) $\frac{p-q}{p}, \frac{p-q}{q}$

(*c*) $\frac{p}{q}, \frac{q}{p}$ (*d*) $\frac{pq}{p+q}, \frac{pq}{p-q}$

2. An inclined plane rises in 1 in 10. If length of the inclined plane is 5 m, the height of the raised end above the horizontal is

(*a*) 0·1 m (*b*) 0·25 m

(*c*) 0·5 m (*d*) 1·0 m

3. The equation $x^2 - 8x + 2y + 7 = 0$, represents

(*a*) a straight line (*b*) parabola

(*c*) circle (*d*) ellipse

4. When $x = 4$, the derivative of $\sqrt{x}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$

(*c*) 2 (*d*) 4

5. The value of $\int_0^{10} \left(x + \frac{1}{x}\right) dx$ is

(*a*) 51·0 (*b*) 52·3

(*c*) 101·0 (*d*) 103·3

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both A and R are true, and R is the correct explanation of A.

(*b*) Both A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

6. Assertion. Total surface area of cylinder of length 10 cm and radius 4 cm is $150\ \pi$ cm^3.

Reason. Total surface area of cylinder of length l and radius r is $(\pi r^2 + 2\pi r l)$.

7. Assertion. Surface area of sphere of radius 10 cm is $100\ \pi$ cm^2.

Reason. Surface area of sphere $= \pi r^2$.

8. Assertion. $\int_0^{\pi/2} \sin x\, dx = 1$.

Reason. $\int_a^b \sin x\, dx = (-\cos x)_a^b$

$$= -[\cos b - \cos a].$$

ANSWERS

I. Multiple Choice Questions

1. (*c*) **2.** (*c*) **3.** (*b*) **4.** (*b*) **5.** (*b*)

II. Assertion-Reason Type Questions

6. (*c*) **7.** (*d*) **8.** (*a*)

HINTS/EXPLANATIONS For Difficult Questions

II. Assertion-Reason Type Questions

6. Total surface area of cylinder is $2\pi r^2 + 2\pi r l$.

Thus **Reason is false.** Now total surface area when $r = 4$ cm and $l = 10$ cm is

Area $= 2\pi \times 5^2 + 2\pi \times 5 \times 10 = 150\ \pi$ cm^3

Thus **Assertion is true.**

7. Both Assertion and Reason are false as surface area of sphere $= 4\pi r^2$

$$= 4\pi \times 10^2$$
$$= 400\ \pi \text{ cm}^2.$$

8. Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

UNIT-1

PHYSICAL WORLD AND MEASUREMENT

CHAPTER 1.

PHYSICAL WORLD

CHAPTER 2.

UNITS OF MEASUREMENT

1*

PHYSICAL WORLD

1.1. WHAT IS SCIENCE ?

Ours is the age of science. The computer ; the television ; video, audio systems ; the cell phone ; *e*-mail ; internet ; fast means of travel ; health, nutrition and so on, all are the gifts of science to humans. There is not even a single field of human activity, which has not been influenced by science.

The word **'Science'** originates from the Latin verb **Scientia,** meaning 'to know'. The Sanskrit word **Vijnana** and the Arabic word **Ilm** convey similar meaning, namely 'knowledge'. Humans have always been curious about the world around them. The night sky with its bright celestial objects ; the regular repetitions of the day and night ; the annual cycle of seasons, the eclipses, the tides, the volcanoes, the rainbow have always been a source of wonder. The inquiring and imaginative human mind observes all these wonders carefully and looks for any meaningful patterns and relations in these phenomena. New tools are built up and used for further studies. ***This knowledge which humans have gained through observations and experiments, when organised systematically is called Science.***

The knowledge so collected has become so vast today that it has been divided into many branches.

The sciences which deal with non living things are called **Physical Sciences**. For example, Physics, Chemistry, Geology, Geography, Astronomy, Astrology, Oceanology, etc.

The sciences which deal with living things are called **Biological Sciences**. For example, Botany, Zoology, Ornithology, Anthropology, Entomology, Forensic science etc.

According to **Albert Einstein**, one of the most brilliant scientists of all times,

'Science is not just a collection of laws, a catalogue of unrelated facts. It is a creation of human mind, with its freely invented ideas and concepts.' The same scientist once remarked : *'The most incomprehensible thing about the world is that it is comprehensible.'* This statement is based on a simple fact that the behaviour of everything in this vast universe is governed by a few laws, which are well defined.

According to **Bruce Lindsay**, *'Science is a method for describing, creating and understanding human experience'*.

In the words of a famous scientist **Neils Bohr**, *'the task of science is both to extend the range of our experience and to reduce it to order'*.

According to **Gerald Holton** 'Science is ever unfinished quest to discover all facts, the relationship between things and the laws by which the world runs.'

In the words of **Phillips Feyman**, 'I would like not to underestimate the value of the world view, which is the result of scientific effort'.

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* Not included in CBSE syllabus. This chapter has been given only for the preparation of competitive examinations.

According to **Heisenberg**, *'The two processes, that of Science and that of Art are not very different. Both Science and Art, form in the course of centuries, a human language, by which we can speak about the more real part of reality'.*

A famous Philosopher, **Bertrand Russel** comments on Science as follows : *'We know very little and yet it is astonishing that we know so much, and still more astonishing that so little knowledge (of science) can give us so much power'.*

1.2. THE SCIENTIFIC METHOD AND THE SCIENTIFIC THEORY

We know that science is a systematic attempt to explore and understand natural phenomena in as much detail and depth as possible. The knowledge so gained is used to predict, modify and control the phenomena. The scientific method involves the following steps :

(i) Taking a large number of systematic observations through controlled experiments.

(ii) Studying these observations and looking for their logical behaviour based on qualitative and quantitative reasoning.

(iii) Mathematical modelling, i.e., suggesting some model to account for the observed behaviour.

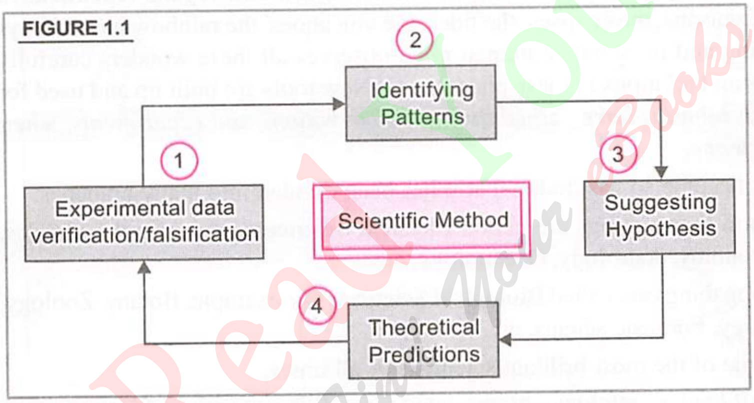
(iv) Theoretical prediction of what is not actually observed on the basis of the suggested model.

(v) Verification or falsification of the model.

Speculation and conjecturing also have a place in the scientific method.

Fig. 1.1 represents the synopsis of the scientific method.

FIGURE 1.1



DO YOU KNOW ?

The Italian Physicist Galileo and English philospher Francis Bacon are usually credited as the principal founders of the scientific method.

A hypothesis is a supposition without assuming that it is true.

An axiom is a self evident truth.

A model is a theory proposed to explain the observed phenomena.

Science is ever dynamic. There is no final theory in science and no unquestioned authority amongst scientists. Infact, interplay of theory and experiments is basic to progress of science. As observations improve in detail and precision or experiments yield new results, theories must be modified, if necessary, to account for them. The reverse is also true. Just as a new experiment may suggest an alternative theoretical model, a theoretical advance may suggest what to look for in some experiments.

Occasionally, existing theory is simply unable to explain new observations. This causes major upheaval in science leading to the development of entirely new theories.

Some of the striking examples where small discrepancies have led to new theories in Physics are :

(*i*) The concept of flat earth was replaced by the concept of spherical earth, from the observations of distant ships in a sea.

(*ii*) Geocentric theory imagining earth to be at the centre of the universe was replaced by Heliocentric theory imagining sun to be stationary and all planets revolving around it.

(*iii*) The corpuscular theory of light given by Newton was replaced first by Huygens wave theory of light, which was replaced later by Planck's quantum theory of light. Finally, dual theory of light was given by de-Broglie to account for all the phenomena observed in case of light.

Thus, in science, the approach is always ***'open-minded'***, in which no points of view are overlooked without logical reasons. Infact, all theories of science have to be updated, whenever required, so that they are consistent with all the experimental data collected by that time.

1.3. WHAT IS PHYSICS ?

Physics is a basic discipline in the category of **'Natural Sciences'.** The word 'Physics' comes from the Greek word **'fusis'** meaning **nature.** Its Sanskrit equivalent is **'Bhautiki'** that is used to refer to the study of the physical world. Hence,

Physics is the branch of science which is devoted to the study of nature and natural phenomena.

Thus, Physics is the most basic of all sciences.

In the study of Physics, there are **two principal thrusts : Unification and Reductionism.**

Unification means attempting to explain diverse physical phenomena in terms of a few concepts and laws. For example, the same law of gravitation given by Newton accounts for (*i*) fall of an apple to the ground (*ii*) motion of satellites around the planets (*iii*) motion of planets around the sun. Similarly, the basic laws of electromagnetism in the form of Maxwell's equations explain all the electric and magnetic phenomena.

Reductionism means attempting to derive the properties of a bigger, more complex system from the properties of its constituent simpler parts.

For example, the subject of thermodynamics deals with bulk systems in terms of macroscopic quantities like temperature, internal energy, entropy etc. The later developments of kinetic theory and statistical mechanics interpreted these quantities in terms of the properties of the molecular constituents of the bulk systems. For example, temperature of the bulk system was related to the average kinetic energy of the molecules of the system.

1.4. SCOPE AND EXCITEMENT OF PHYSICS

The two domains of interest in Physics are : Macroscopic and Microscopic.

The macroscopic domain includes the study of phenomena involving objects of finite size on terrestrial scale and even on astronomical scale. This makes up **Classical Physics.** Most of it was developed upto the year 1900.

The microscopic domain includes the study of phenomena involving molecules, atoms, nuclei, electrons and other elementary particles. This makes up **Modern Physics.** Most of it was developed after the year 1900.

The **Classical Physics** includes subjects like Mechanics, Thermodynamics, Electrodynamics and Optics.

Mechanics deals with the study of general system of particles, rigid bodies, deformable bodies, liquids etc., propulsion of rockets, propagation of water waves, sound waves etc.

Thermodynamics deals with the study of gaseous systems. The changes in temperature, internal energy and entropy of the system through external work are investigated. Modes of transfer of heat, efficiency of heat engines and refrigerators are also included in thermodynamics.

Electrodynamics deals with the study of electric and magnetic phenomena associated with charged particles and magnetic materials. The basic laws governing these phenomena were given by Coulomb, Oersted, Ampere and Faraday. These laws were encapsulated by Maxwell in his famous set of equations. The generation of electric power, response of a circuit to alternating current, propagation of electromagnetic waves etc. also come under electrodynamics.

Optics involves the study of various phenomena connected with light and optical instruments like microscope, telescope etc.

The Classical Physics is inadequate to handle the microscopic domain, where we deal with the constitution and structure of matter at the minute scales of atoms, nuclei, and even smaller scales of length. **Quantum Theory** is currently accepted as the proper framework for explaining microscopic domain.

From what we have studied above, you can realize that *the scope of Physics is truly vast.* It covers a very wide range of magnitudes of physical quantities like length, mass, time, energy etc. At one end, Physics includes the study of electron, proton, nuclei etc. of size 10^{-14} m or even less. And at the other end, it deals with astronomical phenomena involving galaxies and even the entire universe of size $\approx 10^{26}$ m. The two length scales differ by a factor of 10^{40} or even more.

The corresponding range of time scales involved in Physics is obtained by dividing the length scales by the speed of light ($\approx 10^8$ m/s). The time scales would range from $\dfrac{10^{-14}}{10^8} = 10^{-22}$ s to $\dfrac{10^{26}}{10^8} = 10^{18}$ s. The range of masses involved in the study of Physics varies from 10^{-30} kg (mass of an electron) to 10^{55} kg (mass of known observable universe). The terrestrial phenomena lie somewhere in the middle of these ranges.

The phenomenal progress of Physics in the last few centuries is due to the following three reasons :

(*i*) Quantitative measurement is central to the growth of Physics as the laws of nature can be expressed in precise mathematical equations.

(*ii*) The basic laws of Physics are universal, *i.e.*, the same basic laws can explain diverse physical phenomena.

(*iii*) The strategy of approximation is very successful. Most of the observed phenomena in daily life are rather complex manifestations of the simple basic laws. Therefore, it is good to focus first on the essential features, discover the basic principles and then introduce modifications to build a more refined theory of the phenomena.

The study of Physics is exciting in many ways. For example :

(*i*) A few basic concepts and laws can explain diverse physical phenomena.

(*ii*) Carrying out imaginative new experiments to unlock secrets of nature by verifying or falsifying the existing theories.

(*iii*) The most interesting part is designing useful devices based on the physical laws.

For a layman, the study of Physics is exciting. For example :

(*i*) live transmission of events thousands of kilometres away on the television,

(*ii*) S.T.D. ; I.S.D. ; Fax ; Pager ; Cellular phone etc.

(*iii*) the speed and memory of the fifth generation of computers,

(*iv*) use of robots,

(*v*) journey to moon and to some nearby planets with controls from the ground,

(*vi*) technological advances in health sciences,

(*vii*) lasers and their ever increasing applications.

(*viii*) exploring the new sources of energy

(*ix*) study of various types of forces in nature, and so on.

1.5. PHYSICS IN RELATION TO OTHER SCIENCES

As Physics involves a basic study of the various natural phenomena, it can rightly be regarded as the most fundamental of all sciences. Physics has played a key role in the development of many other branches of science. For example :

(*a*) Physics in relation to Chemistry

The study of structure of atoms, radioactivity, *X*-ray diffraction etc. in Physics have enabled chemists to rearrange elements in periodic table on the basis of atomic number ; have better understanding of chemical bonding and complex chemical structures.

(*b*) Physics in relation to Biological Sciences

The optical microscopes developed in Physics are extensively used in the study of biological samples. Using an electron microscope, the structure of biological cells is studied. *X*-rays discovered in Physics have a variety of applications in biological sciences. The radio-isotopes are used in the treatment of cancer and so on.

(*c*) Physics in relation to Astronomy

The giant astronomical telescopes developed in Physics are used for observing planets and other heavenly bodies in the sky. Radio telescopes have enabled the astronomers to observe distant limits of the universe.

(*d*) Physics in relation to Mathematics

Theories in Physics often make use of mathematical concepts. Mathematics has infact served as a powerful tool in the development of modern theoretical Physics.

(*e*) Physics in relation to Geology

The diffraction techniques developed in Physics help in studying crystal structures of various rocks. The phenomenon of radioactivity is used to estimate the age of rocks and fossils.

(*f*) Physics in relation to Seismology

The study of movement of the earth's crust and the types of waves generated help us a lot in the study of earthquakes.

(*g*) Physics in relation to meteorology

Forecasting of weather (meteorology) is based primarily on the variation of pressure with temperature.

1.6. PHYSICS RELATED TO SOCIETY

Most of the developments in Physics have a direct impact on society. For example :

(*i*) The development of telephone, telegraph, telex enable us to transmit messages instantly.

(*ii*) The development of radio, television, satellites have revolutionised the means of communication.

(*iii*) Advances in electronics, computers, lasers have greatly enriched the society.

(*iv*) Rapid means of transport are no less important for the society.

(*v*) Exploration of the new sources of energy is of great significance to the society ; and so on.

1.7. PHYSICS IN RELATION TO TECHNOLOGY

Technology is the application of the principles of Physics for practical purposes. For example, steam engine which played key role in the Industrial Revolution in England in 18th century, was developed on the laws of thermodynamics.

A survey of development of Physics shows that sometimes technology gives rise to new Physics and the other times, Physics generates new technology. For example, the technology of wireless communication was based on the fundamental laws of electricity and magnetism. More than often, it is not easy to foresee the application of Physics. For example, in 1933, Rutherford dismissed the idea of energy obtainable from atoms. And, in 1938, Hahn and Meitner carried out fission of ${}_{92}U^{235}$, which served as the basis of nuclear power reactors and nuclear weapons. Yet another example is the discovery of 'silicon chip' that has trigerred the computer revolution.

Table 1.1 lists some of the important technologies and the principles of Physics on which they are based.

TABLE 1.1. Link between Technology and Physics

S. No.	Technology	S. No.	Basic Principle(s)
1.	Steam engine	1.	Laws of Thermodynamics.
2.	Rocket propulsion	2.	Newton's Laws of motion.
3.	Aeroplane	3.	Bernoulli's principle in fluid dynamics.
4.	Sonar	4.	Reflection of ultrasonic waves.
5.	Optical fibres	5.	Total internal reflection of light.
6.	Electric generator	6.	Faraday's laws of electromagnetic induction.
7.	Hydroelectric power	7.	Conversion of gravitational potential energy into electrical energy.
8.	Particle accelerators	8.	Motion of charged particles in electromagnetic fields.
9.	Radio and Television	9.	Generation, propagation and detection of electromagnetic waves.
10.	Photocell	10.	Photoelectric effect.
11.	Nuclear Reactor	11.	Controlled nuclear chain reaction.
12.	Fusion Test reactor (Tokamak)	12.	Magnetic confinement of plasma.
13.	Electron microscope	13.	Wave nature of electrons.
14.	Lasers	14.	Light amplification by stimulated emission of radiation.
15.	Computers	15.	Digital logic
16.	Production of ultra high magnetic fields.	16.	Super conductivity
17.	Non-reflecting coatings	17.	Thin film optical interference.
18.	Giant metrewave radio telescope (GMRT)	18.	Detection of cosmic radio waves.
19.	Bose Einstein condensate	19.	Trapping and cooling of atoms by laser beams and magnetic fields.
20.	Genetic engineering	20.	Role of DNA in heredity.

1.8. SOME GREAT PHYSICISTS AND THEIR CONTRIBUTIONS

The present status of Physics is due to the contributions of a number of brilliant Physicists of the past and the present.

The greatest scientist who ever lived on earth was Sir Issac Newton (1642–1727). He was born in England. Amongst his contributions are Newton's three laws of motion, universal law of gravitation, a reflecting telescope and so on. His work on calculus – differential and integral led to the development of a new language of Physics. The famous poet **Alexander Pope** said of Newton :

"Nature and Nature's Laws lay hid in night ; God said, Let Newton be ! and all was light". Inspite of his greatness, Newton remained a simple man. Shortly before his death, **Newton** remarked :

"I do not know what I may appear to the world, but to myself, I seem to have been only like a boy playing on the sea-shore and diverting myself every now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay undiscovered before me". Table 1.2 gives only a few selected names of great physicists, alongwith the name of the discovery they made and country of their origin.

TABLE 1.2. Some Great Physicists with their country of origin and major contribution/discovery

S.No.	Name	Country of origin	Major contribution/discovery
1.	Archimedes	Greece	Principle of buoyancy
2.	Galileo Galilei	Italy	Law of inertia
3.	Sir Isaac Newton	U.K.	Universal law of gravitation and others
4.	Huygens	Holland	Wave Theory of light
5.	Michael Faraday	U.K.	Laws of electromagnetic induction
6.	James Clerk Maxwell	U.K.	Electromagnetic theory
7.	Heinrich Rudolf Hertz	Germany	Generation of electromagnetic waves
8.	J.J. Thomson	U.K.	Electron
9.	W.K. Roentgen	Germany	*X*-rays
10.	J.C. Bose	India	Ultra short radio waves
11.	Marie Curie	Poland	Studies of natural radioactivity
12.	Albert Einstein	Germany	Explanation of Photoelectric effect ; Theory of Relativity
13.	R.A. Millikan	U.S.A.	Measurement of electronic charge
14.	Ernest Rutherford	New Zealand	Nuclear model of atom
15.	Niels Bohr	Denmark	Quantum model of hydrogen atom
16.	C.V. Raman	India	Inelastic scattering of light by molecules
17.	M.N. Saha	India	Thermal ionisation
18.	S.N. Bose	India	Quantum statistics
19.	Victor Francis Hess	Austria	Cosmic Radiation
20.	Louis de Broglie	France	Wave nature of matter
21.	Wolfgang Pauli	Austria	Exclusion Principle
22.	Enrico Fermi	Italy	Controlled nuclear fission
23.	Paul Dirac	U.K.	Relativistic theory of electron ; quantum statistics
24.	Werner Heisenberg	Germany	Uncertainty principle ; Quantum Mechanics
25.	James Chadwick	U.K.	Neutron
26.	Hideki Yukawa	Japan	Theory of nuclear forces
27.	Edwin Hubble	U.S.A.	Expanding universe
28.	S. Chandra shekhar	India	Structure and evolution of stars
29.	Ernest orlando Lawrence	U.S.A.	Cyclotron
30.	Homi Jehangir Bhabha	India	Cascade process of cosmic radiation
31.	Lev Davidovich Landau	Russia	Theory of condensed matter, liquid helium
32.	John Bardeen	U.S.A.	Transistors ; Theory of superconductivity
33.	C.H. Townes	U.S.A.	Maser : Laser
34.	Abdus Salam	Pakistan	Unification of weak and electromagnetic interactions

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IMPORTANT NOTE

A review of Tables 1.2 and 1.1 reveals multicultural, international character of scientific endevour. We shall urge our students to consult their teachers, some good books and some websites on science, and add on to Tables 1.2 and 1.1. This exercise would be highly educative with probably no end, as the progress of science is unstoppable.

Curiosity Questions

Q. 1. What is a discovery ?

Ans. According to Abbert Szent-Gyorgyi (1893-1986), discovery consists of seeing that everybody has seen, but thinking what nobody has thought.

Q. 2. Can you visualize the growth and development of a useful scientific tool ?

Ans. Yes. For example, E.O. Lawrence was the first to build an operating cyclotron for the study of subatomic particles in the year 1930. This cyclotron had a circumference of about *13 inches.*

In contrast, the latest cyclotron developed is a super conducting super collider (SSC), which has a circumference of about *52 miles.* The machine has cost about 10 billion dollars and can give particles 100 million times more energy. The researches from SSC have led us to new methods of treating cancer and AIDS ; and development of more advanced computers.

1.9. FUNDAMENTAL FORCES IN NATURE

We know that Physics is the study of nature and natural phenomena. It deals with certain basic laws which govern the natural world. We shall now discuss the nature of fundamental forces and the nature of laws that govern diverse phenomena of physical world.

Intuitively, we know that *force is an effort in the form of push or pull, required to move a body or stop a moving body or deform/break the body.* We realize the impact of force when somebody hits us. In macroscopic world, we come across gravitational forces ; muscular forces ; tension in springs, ropes ; forces of friction ; forces due to pressure ; surface tension ; force due to buoyancy ; viscous forces etc. In addition, there are forces between charged particles and magnetic bodies.

Physics has revealed that all the forces occuring in different contexts arise from a small number of fundamental forces in nature. Though the origin of derived forces is complex, yet they can be understood in terms of the following ***four fundamental forces ;*** which govern the macroscopic as well as microscopic world :

1. Gravitational forces ;

2. Electromagnetic forces ;

3. Strong nuclear forces ;

4. Weak nuclear forces.

(*a*) Gravitational forces

The gravitational force is the force of mutual attraction between any two objects by virtue of their masses.

As every object has some mass, therefore, gravitational forces are universal forces, *i.e.*, every object in this universe attracts every other object at all distances. For example, all objects on earth experience the force of gravity due to the earth. This force governs the motion of bodies falling towards the earth. The revolution of moon and other artificial satellites around the earth is also governed by the force of gravity of earth. The revolution of earth and other planets around the sun is explained in terms of pull of gravity of sun on the planets. Infact, all large scale phenomena of the universe such as formation and evolution of stars, galaxies and galactic clusters are explained in terms of gravitational forces.

The magnitude of the force exerted by a particle of mass m_1 on another particle of mass m_2 at a distance r from it is given by *Newton's law of gravitation :*

$$\boxed{F = \frac{G\,m_1\,m_2}{r^2}}$$

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where G is *universal gravitational constant* and has the value $G = 6{\cdot}67 \times 10^{-11}$ Nm^2/kg^2. The direction of the force on m_2 is towards m_1. An equal but opposite force is exerted by mass m_2 on mass m_1, in accordance with Newton's third law of motion. Thus the two particles exert equal forces of attraction on eachother.

Some of the **important features/properties of gravitational forces** are :

1. Gravitational forces are universal attractive forces, *i.e.*, they exist between microscopic as well as macroscopic objects irrespective of their size, shape, separation and intervening medium. *These forces are never repulsive.*

2. Gravitational force between any two bodies is directly proportional to the product of masses of the two bodies.

3. Gravitational force between any two bodies does not depend upon the presence or absence of other bodies.

4. These are the weakest forces in nature.

5. They operate over very long distances especially when the bodies are massive. For example, rotation of earth around the sun is due to gravitational pull of sun on earth.

6. Gravitational forces obey inverse square law, *i.e.*, they vary inversely as the square of the distance between the two bodies.

7. Gravitational forces are central forces, *i.e.*, they act along the line joining the centres of two bodies.

8. Gravitational forces are conservative forces*, *i.e.*, work done in moving a body by or against the gravitational force is independent of the path followed.

9. The field particle of gravitational forces is called '*graviton*'. The concept of exchange of field particle between two bodies explains how the two bodies interact from a distance.

(b) Electromagnetic Forces

The force acting between two static electric charges is called **electrostatic force.**

The force acting between two magnetic poles is called **magnetic force.**

Infact, electrostatic and magnetic forces are closely inter-related. For example, a moving charge produces a magnetic field, and this magnetic field exerts a force on the moving charge. This force depends on the charge, and its velocity. Thus the electrostatic and magnetic forces are inseparable. These are considered to be the two facets of a general force, called **electromagnetic force.**

The magnitude of electrostatic force F between two static point charges q_1 and q_2 separated by a distance r in air/vacuum is given by ***Coulomb's law* :**

$$F = \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_2}{r^2}$$

where $\dfrac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m^2/C^2 is a constant

and $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$ C^2 N^{-1} m^{-2},

is absolute electrical permittivity of free space.

The smallest magnitude of charge found in nature is the charge on an electron or a proton represented by

$$e = 1{\cdot}6 \times 10^{-19}\ \text{C}$$

The proton has charge $+\,e$ and electron has charge $-\,e$. All charges are found to have magnitude ne, where n is an integer. Electric charge is, therefore, said to be **quantized.**

As is known, matter consists of elementary particles like neutrons, electrons and protons. The electrons and protons are charged. As electromagnetic force is much stronger than the gravitational force, it dominates all phenomena on atomic and molecular scales. The structure of atoms and molecules, the dynamics of chemical reactions etc. are governed by electromagnetic forces. Some of the macroscopic forces, like the forces between two surfaces in contact, force of friction, tension in a spring/rope, forces due to springs etc. are also explained in terms of electromagnetic forces.

For example :

(*i*) Forces between two surfaces in contact.

When we put two bodies in contact, the atoms of the two surfaces come close to eachother. The charged constituents of the atoms of two bodies exert strong forces between them.

Generally, the forces between the two bodies in contact are along the common normal, *i.e.*, perpendicular to the surface of contact. Also, these forces are in the form of a push or repulsion. For example, when a book is placed on a table, the book pushes the table downwards (away from it) and the table pushes the book upwards (again away from it).

(*ii*) Force of Friction.

The two bodies in contact may have a component of force parallel to the surface of contact. This component is called friction. Obviously, the surfaces in contact will be frictionless, when they exert forces only along the direction perpendicular to them. Bodies with smooth surfaces in contact will exert only small amount of forces parallel to the surface of contact.

For example, we can cling on to the trunk of a tree, which is quite rough. The trunk exerts an upward frictional force parallel to the surface of the trunk, on us to hold us there. This is the reaction of an equal frictional downward force, we exert on the tree due to our weight. On the contrary, it is difficult to stay on a smooth metallic lamp post, as it cannot exert enough force parallel to the surface of the lamp post.

DO YOU KNOW ?

The Classical Physics is a good description of nature when we confine ourselves to the particles of linear size $> 10^{-6}$ m, moving with velocities $< 10^8$ m/s. The size restriction automatically excludes any appreciable effects of nuclear forces or weak forces. Thus in Classical Physics, we need to consider only the gravitational forces and electromagnetic forces.

The formulation of Classical Physics is quite accurate for heavenly bodies like the earth, the moon and the sun. Further, it is equally good for the behaviour of rain drops, grains of sand etc. The difficulty arises only when we deal with subatomic particles. The behaviour of such particles is governed by 'Quantum Physics'.

(*iii*) Tension in a string/rope.

When a heavy block hangs from a ceiling by a string, the string is in a state of tension. The electrons and protons of the string near the lower end exert forces on the electrons and protons of the block. The resultant of these forces is the force exerted by the string on the block, that supports the block.

Infact, a string or rope under tension exerts electromagnetic forces on the bodies attached at the two ends to pull them.

(*iv*) Forces due to a spring.

When extention or compression is not too large, the force exerted by the spring (F) is proportional to change in its length (l), *i.e.*, $F \propto l$ or $F = kl$, where k is spring constant. If the spring is extended, the force will be directed towards its centre, and if compressed, the force will be directed away from the centre. This force comes into picture due to the electromagnetic forces between the atoms of the material of the spring.

Note that two bodies–not in contact with eachother, do not exert appreciable electromagnetic forces though both the bodies contain a very large number of electrons and protons. This is because the charged particles of one body and those of the other body have both attractive and repulsive nature. Therefore, these forces largely cancel out.

Some of the **salient features of electromagnetic forces** are :

1. These forces may be attractive or repulsive. Like charges repel eachother and unlike charges attract eachother.

2. These forces are governed by Coulomb's laws which are similar to Newton's law of gravitation.

3. They obey inverse square law.

4. Electrostatic forces (between two protons) are 10^{36} times stronger than gravitational forces between them, for any fixed distance.

5. They operate over distances which are not very large.

6. They are central forces.

7. They are also conservative forces.

8. The field particle of electromagnetic forces is *photon*, which carries no charge and has zero rest mass.

(c) Strong Nuclear Forces

The forces that bind the neutrons and protons together in a nucleus are called the strong nuclear forces.

The strong nuclear forces are of three types :

(*i*) *n-n* forces between two neutrons ;

(*ii*) *p-p* forces between two protons and

(*iii*) *n-p* forces between a neutron and a proton.

When two nucleons are within 1 fermi (10^{-15} m) of eachother, the strong nuclear attractive force is about 100 times stronger than the respulsive electric force between two protons at this separation. That is why a nucleus is a stable structure inspite of strong electrostatic forces of repulsion between protons.

However, the nuclear forces have a very short range, *i.e.*, their strength decreases very rapidly with increasing separation (much more rapidly than the inverse square decrease). At a separation of about 15 fermi, nuclear forces become negligibly small compared to electrical force.

DO YOU KNOW ?

Between two neutral objects, the gravitational force goes on adding over all pairs of particles of the two objects. The electromagnetic forces being equally attractive and repulsive add upto zero. That is why the large scale motion in the universe is controlled by the gravitational force, which is otherwise the weakest force.

The strong nuclear forces cannot be electrostatic forces because positively charged protons in the nucleus repel each other very strongly. The gravitational attraction between two protons being much weaker, cannot overcome the strong electrostatic repulsion between the protons.

Being short ranged, nuclear forces come into picture only if we consider charges within the nucleus. As bare nuclei are hardly encountered in daily life, we are generally not aware of nuclear forces. However, the phenomena like radioactivity, nuclear fission, nuclear fusion etc. involve nuclear forces.

Some of the **salient features of nuclear forces** are :

1. Nuclear forces are the strongest forces in nature. They are 10^{38} times stronger than gravitational forces, 10^{2} times stronger than electrostatic forces and 10^{13} times stronger than the weak forces.

2. Nuclear forces have the shortest range. They operate within the nucleus only, *i.e.*, upto distances of the order of 10^{-14} metre.

3. Nuclear forces do not depend on charge on the nucleon.

4. Nuclear forces do not obey inverse square law. They vary inversely as some higher power of distance between nucleons.

5. They are basically attractive forces. Only when distance between nucleons is less than 0·8 fermi, nuclear forces become repulsive.

6. Nuclear forces are non central forces.

7. They are also non-conservative forces

8. The field particle for nuclear forces is the 'π- meson'.

(d) Weak Nuclear Forces

These are the forces that appear only between elementary particles of short life times, involved in a nuclear process such as β-decay of a nucleus.

In β-decay, electrons are emitted from a radioactive nucleus. It was believed that β decay occurs when a neutron in the nucleus is converted into a proton and an electron. According to theoretical considerations, all β particles from a radioactive nucleus should possess the same energy and hence same velocity. However, experiments show that energy of an electron emitted from radioactive nucleus varies continuously from zero to a certain maximum value.

To account for the variable energy of electron, **Pauli** in 1930, postulated the existence of an uncharged particles, called anti-neutrino $(\bar{\nu})$, which is emitted along with the electron.

The energy emitted during β-decay is shared between the electron and antineutrino. When antineutrino carries maximum energy, the energy of electron is minimum and vice-versa. This particle (antineutrino) was later discovered experimentally.

Hence it was concluded that an electron and an antineutrino interact with each other through weak nuclear forces.

Some of the **important properties of weak nuclear forces are :**

1. Weak nuclear forces exist between leptons* and leptons ; and also between leptons and mesons*.

2. Weak nuclear forces are 10^{25} times stronger than the gravitational forces.

3. Weak nuclear forces operate only through a range of nuclear size ($0 \approx 10^{-15}$ m).

4. The field particles that transmit weak nuclear force between elementary particles are the massive vector bosons ($W^{\pm}$; Z^{o}).

Retain in Memory

1. The relative strengths of four types of basic forces in nature can be represented as

$$F_G : F_W : F_E : F_N = 1 : 10^{25} : 10^{36} : 10^{38}$$

2. Each fundamental force arises due to exchange of characteristic particle called the *messenger particle* or the *field particle.*

(*i*) For gravitational forces, the field particle is *graviton*,

(*ii*) For electromagnetic forces, the field particle is *photon*,

(*iii*) For strong nuclear forces, the field particle is *π meson*, and

(*iv*) For weak nuclear forces, the field particle is *vector boson* ($W^{\pm}$, Z^{o}).

Table 1.3 gives a summary of the fundamental forces in nature.

TABLE 1.3. Fundamental forces in nature

S.No.	Name	Relative strength	Range	Operates among
1.	Gravitational force	1	Infinite	All objects in universe
2.	Electromagnetic force	10^{36}	Not very large	Charged particles
3.	Strong nuclear force	10^{38}	Very short, nuclear size ($\approx 10^{-15}$ m)	Nucleons, heavier elementary particles
4.	Weak nuclear force	10^{25}	Very short subnuclear size ($\approx 10^{-15}$ m)	Some elementary particles like electron and neutrino

DO YOU KNOW ?

Elementary particles have been grouped as

(*i*) **Baryons**, which include nucleons (n, p) and their higher mass counterparts.

(*ii*) **Mesons** like pions and kaons.

(*iii*) **Leptons** like electrons, neutrino and their higher mass counterparts.

Whereas Baryons interact through strong nuclear forces ; Mesons and Leptons interact only through Weak Nuclear Forces.

---

*leptons and mesons are kinds of elementary particles. For details, refer to some Text Book on Nuclear Physics.

1.10. TOWARDS UNIFICATION OF FORCES

By unification of forces, we mean that there exists a relationship between the various forces of nature.

A lot of efforts have been made towards unification of different forces and domains of Physics. Some of the highlights are :

(*i*) In 1687, **Newton** unified terrestrial and celestial mechanics showing that the same laws of motion and the law of gravitation apply to both.

(*ii*) **Oersted** (1820) and **Faraday** (1830) showed that electric and magnetic phenomena are inseparable.

(*iii*) **Maxwell** (1873) unified electricity, magnetism and optics showing that light is an electromagnetic wave.

(*iv*) **Abdus Salam** and associates showed in 1979 that 'electromagnetic force' and 'weak' nuclear force are different aspects of a single electro-weak force. The predictions of this theory were verified experimentally by Rubia et al in the year 1984.

Table 1.4 shows the progress in unification of different forces/domains in nature.

TABLE 1.4. Progress in unification of different forces/domains in nature

S. No.	Name of Physicist	Year	Achievement in unification
1.	Isaac Newton	1687	Unified terrestrial and celestial mechanics
2.	Haris Christian Oersted	1820	Electric and magnetic phenomena are inseparable aspects of a unified domain : electromagnetism.
3.	Michael Faraday	1830	
4.	James Clerk Maxwell	1873	Unified electricity, magnetism and optics ; showed that light is an electromagnetic wave.
5.	Sheldon Glashow, Abdus Salam, Steven Weinberg	1979	Weak nuclear force and the electromagnetic force could be viewed as different aspects of a single electro-weak force.
6.	Carlo Rubia, Simon Vander Meer	1984	Verified exptally the prediction of the theory of electro-weak forces.

1.11. NATURE OF PHYSICAL LAWS

Physics is the study of nature and natural phenomena. The keen observations and experiments lead the physicists to certain facts. An attempt is made to explain these facts on the basis of certain laws.

A remarkable fact is that ***some special physical quantities remain constant in time. These are called the conserved quantities of nature.*** Understanding the conservation principles is very important in describing the natural phenomena quantitatively. Conservation laws are infact important tools of analysis.

In Classical Physics, we often deal with the following conservation laws :

1. *Law of conservation of energy.*
2. *Law of conservation of linear momentum.*
3. *Law of conservation of angular momentum.*
4. *Law of conservation of charge.*

A brief discussion of these laws is given here :

(*a*) Law of conservation of energy

The concept of energy is central to Physics. The expressions for energy can be written for every physical system. According to this law,

> ***The sum total of energy of all kinds in this universe remains constant. Energy can be transformed from one form to the other or transported from one place to the other, but the total amount of energy never changes.***

The law of conservation of energy is valid across all domains of nature, from the microscopic to the macroscopic. It is applied in the analysis of atomic, nuclear and elementary particle processes. Though all kinds of violent phenomena occur in the universe all the time, yet the total energy of the universe remains constant always.

The general law of conservation of energy is valid for all forces and for any kind of transformation between different forms of energy. For example, when an object is falling freely under the action of gravity, both the kinetic energy and potential energy of the object change continuously with time, but their sum remains constant at all times. If the object is released from rest from a height, the initial potential energy of the object is converted completely into the kinetic energy of the object just before it hits the ground. On striking the ground, kinetic energy of the object gets transformed into other forms of energy : heat and sound. The total energy of the system (object plus the surroundings) remains unchanged.

Until the advent of Einstein's theory of relativity, the law of conservation of mass was regarded as another basic conservation law of nature. According to Einstein's theory, mass m is equivalent to energy E given by the relation $E = mc^2$, where c is the speed of light in vacuum. Energy can be obtained at the cost of mass and vice-versa. In a nuclear process, mass gets converted into energy. As c is very large ($= 3 \times 10^8$ m/s), therefore, the energy released is tremendous even when m is small. This is the basis of energy released in nuclear power reactors and nuclear explosions of atom bomb and hydrogen bomb. Consequently, the law of conservation of mass was included in the law of conservation of energy itself. For example, the mass of a bound system like a nucleus is not exactly equal to the sum of the masses of its constituents, the protons and neutrons. It is slightly less, by an amount equal to the binding energy (B.E.) divided by the square of the speed of light. Thus, mass of a bound system = sum of the masses of its constituents – B.E./c^2.

However, in a chemical reaction, there occurs basically a rearrangement of atoms among different molecules. As the atoms are merely rearranged, but not destroyed, therefore, total mass of the reactants is the same as the total mass of the products in a chemical reaction. The changes in binding energy are too small to be measured as changes in mass.

(*b*) Law of conservation of linear momentum

According to this law,

> ***"in the absence of an extrnal force, the linear momentum of a system remains unchanged."***

The law means that in order to change the momentum of a system, we have to exert an impulse/force on the system. Further, the force or impulse must be exerted on the system by something outside the system. The internal forces won't work. This is because internal forces come in balanced pairs that cancel within the object. To change the momentum, an outside push or pull is required. For example :

(*i*) When a gun is fired, force on the bullet inside the gun barrel is equal and opposite to the force on the gun. These forces are internal to the system comprising the gun and the bullet. Before the firing, the system is at rest and the momentum is zero. *On firing, the total momentum of gun and bullet is still zero.* On firing, the bullet gains momentum in the forward direction and the gun gains an equal momentum in the backward direction so that the gun-bullet system gains none. That is why the gun recoils on firing.

(*ii*) Similarly, when atomic nuclei undergo radioactive decay or a star explodes, the forces involved are internal. Therefore, net momentum of the system before and after the event is the same.

(*c*) Law of conservation of angular momentum

We know that a rotating object has inertia. Therefore, such an object also possesses momentum associated with its rotation. This momentum is called 'angular momentum'. We shall prove later that

Angular momentum (L) = moment of inertia (I) × angular speed (ω)

According to the law of conservation of angular momentum :

> ***If the total external torque acting on a system is zero, angular momentum of the system remains constant.***

As angular momentum is a vector quantity, both the magnitude as well as direction of angular momentum shall stay constant in the absence of an external torque. For example, planets revolving around the sun in elliptical orbits maintain their orbital plane and angular momentum in this plane constant, due to angular momentum conservation.

The shapes of galaxies (such as our milky way) have much to do with the conservation of angular momentum. Consider a globular mass of gas in space that begins to contract under the influence of its own gravity. As the gas contracts, its rotational inertia (I) decreases. Because angular momentum is conserved, the angular speed (ω) of the galaxy increases turning it into a spiral galaxy.

In many cases, conservation laws simplify the problems. For example, during a collision of two automobiles, the law of conservation of linear momentum enables us to predict or rule out possible outcomes of the collision. Further, using conservation laws of energy and momentum for beta decay, Pauli predicted in 1931, the existence of a new particle called **neutrino** emitted alongwith electron in β-decay.

(*d*) Law of conservation of charge

Matter is made of atoms and atoms are made of electrons and protons (and neutrons as well). An object that has equal number of electrons and protons has no net electric charge. But if there is an imbalance in the numbers, the object is then electrically charged. An imbalance comes about by adding or removing electrons. If an object has more electrons than protons, the object is negatively charged. If it has fewer electrons than protons, then it is positively charged.

In every event, whether at large scale or at the atomic or nuclear level, the *principle of conservation of charge* applies.

> ***It means that charges (in the form of electrons) are neither created nor destroyed, but are simply transferred from one body to another.***

Therefore, two bodies of an isolated system can have charges which will always be equal and opposite.

DO YOU KNOW ?

Conservation laws have a deep connection with symmetries of nature. For example, *laws of nature do not change with time.* This symmetry of nature with respect to translation (*i.e.*, displacement) in time is equivalent to the law of conservation of energy.

Further, the laws of nature are the same everywhere in the universe. This symmetry of nature with respect to translation in space gives rise to the law of conservation of linear momentum.

Similarly, isotropy of space (*i.e.*, no intrinsically preferred direction in space) underlines the law of conservation of angular momentum.

DO YOU KNOW ?

A conservation law cannot be proved. It can only be verified or disproved by experiments.

Retain in Memory

The scientists often rely on conservation laws to determine the behaviour of objects they cannot observe directly. For example, we cannot see actually the atomic scale particles. But we can predict their behaviour by applying conservation laws.

Further, note that all quantities in Physics are not conserved. For example, there is no conservation of force and no conservation of velocity, acceleration etc.

REVISION AT A GLANCE

1. **Physics** is that branch of science, which deals with the study of nature and natural phenomena. The study of Physics has been divided broadly into Mechanics, Heat, Sound, Properties of Matter, Optics, Electricity, Magnetism, Atomic Physics, Nuclear Physics, Electronics etc.
2. **Basic forces in nature** are of four types : Gravitational forces ; Electromagnetic forces ; Weak (nuclear) forces and (strong) Nuclear forces. Unification of different forces/domains in nature is a basic quest in Physics.
3. **Conservation Laws** that we come across in classical Physics are : Laws of conservation of linear momentum ; energy ; angular momentum and charge.

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I. Multiple Choice Questions (with One Correct Answer)

1. The most incomprehensible thing about the world is that it is comprehensible. Who made these remarks ?

(*a*) Einstein
(*b*) Bohr
(*c*) Rutherford
(*d*) Madam Curie

2. Science is the art of soluble. It implies that

(*a*) different phenomena can be explained in terms of only a few fundamental laws.
(*b*) every great physical theory ends as a dogma
(*c*) every physical theory starts as a hear say
(*d*) none of the above

3. The factors which have hindered the progress of science in India are :

(*a*) Lack of education
(*b*) poverty
(*c*) uncontrolled population
(*d*) all the above

4. No physicist has even seen an electron

(*a*) true
(*b*) false
(*c*) can not say

5. Industrial revolution in England was trigerred by

(*a*) development of steam engine
(*b*) development of blast furnace
(*c*) development of power looms
(*d*) all the above

6. Present day scientific revolution is due to

(*a*) development of superconducting materials
(*b*) development of superfast computers
(*c*) development of robots
(*d*) all the above

7. Which of the following is not a good use of science ?

(*a*) development of computers
(*b*) development of artificial satellites
(*c*) development of radio and television
(*d*) prenatal sex determination

8. Who made these remarks : It is important to have beauty in the equations of Physics than to have them agree with experiments ?

(*a*) PAM Dirac
(*b*) Niels Bohr
(*c*) Newton
(*d*) C.V. Raman

9. Who is the author of the famous book : Physics for the inquiring mind ?

(*a*) EM Rogers (*b*) G. Gamow
(*c*) Feynman (*d*) None of the above

ANSWERS

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*a*) **5.** (*d*) **6.** (*d*) **7.** (*d*) **8.** (*a*) **9.** (*a*)

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II. Multiple Choice Questions (with One or More than One Correct Answers)

10. Given equal opportunity, women are at par with men.

(*a*) Madam curie won Nobel Prize in Physics

(*b*) Mother Teresa proved herself a saint.

(*c*) Mrs. Indira Gandhi excelled others in politics

(*d*) Mrs. Margaret Thatcher excelled as an administrator

11. Which of the following can be treated as a good use of science ?

(*a*) development of nuclear weapons

(*b*) development of techniques of biological warfare.

(*c*) computers for increase in work efficiency

(*d*) rapid means of transport

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension **Ptolemy postulated that earth is stationary and all the heavenly bodies like sum, stars, planets, moon etc. revolve around the earth. Galileo postulated that sun is stationary and earth alongwtih other planets revolve around the sun. Galileo was punished for spreading wrong concepts. Later on Newton and Keplar supported Galileo's theory.**

Read the above passage and answer the following questions :

12. Who postulated that earth is stationary ?

(*a*) Newton (*b*) Keplar

(*c*) Galileo (*d*) Ptolemy

13. Who postulated that sun is stationary and planets revolve around the sun ?

(*a*) Newton

(*b*) Galileo

(*c*) Keplar

(*d*) Ptolemy

14. Who was punished for spreading wrong concept ?

(*a*) Keplar

(*b*) Ptolemy

(*c*) Newton

(*d*) Galileo

IV. Matching Type Questions

DIRECTIONS. In each of the following question, match column I and column II and select the correct match out of the four given choices.

15. Match appropriately column I and column II

Column I	Column II
(A) Planck	(*p*) dual theory
(B) de Broglie	(*q*) vapour pressure lowering
(C) Einstein	(*r*) quantum theory
(D) Raoult	(*s*) photoelectric effect

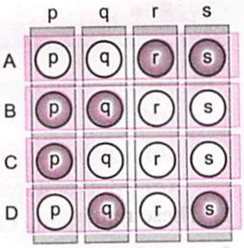
(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*p* ; C-*r* ; D-*s* (*c*) A-*p* ; B-*q* ; C-*s* ; D-*r* (*d*) A-*q* ; B-*p* ; C-*s* ; D-*r*

ANSWERS

10. (*a,b,c,d*) **11.** (*c,d*) **12.** (*d*) **13.** (*b*) **14.** (*d*)

15. (A) → (*r*) ; (B) → (*p*) ; (C) → (*s*) ; (D) → (*q*)

V. Matrix-Match Type Questions

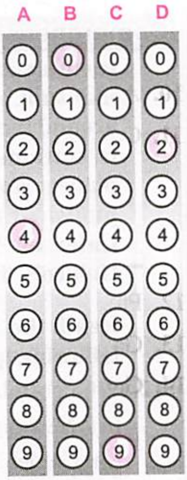


DIRECTIONS. Each of the following question contains statements given in two columns, which have to be matched. The answer to these questions have to be appropriately bubbled. If the correct matches are A-*r* ; A-*s* ; B-*p* ; B-*q* ; C-*p*, and D-*q* ; D-*s*, then the correctly bubbled matrix will look like the one shown here.

16.

Column I (Technology)		Column II (Basic principle/Inventor)
(A) Rocket propulsion	(*p*)	Total internal reflection
(B) Optical fibers	(*q*)	Controlled nuclear chain reaction
(C) nuclear reactor	(*r*)	Role of DNA in heredity
(D) Genetic engineering	(*s*)	Newton's laws of motion
	(*t*)	Enrico Fermi

VI. Integer Type Questions



DIRECTIONS. The answer to each of the following question is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

17. Though the origin of derived forces is complex, they can be understood in terms of how many types of fundamental forces ?

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 1

VII. Assertion-Reason Type Questions

DIRECTIONS. In each of the following questions read the two statements and choose if

(A) both, Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) both, Assertion and Reason are true but the Reason is not a correct explanation of the Assertion.

(C) Assertion is true but the Reason is false.

(D) both, Assertion and Reason are false.

18. Assertion. Nuclear forces have the shortest range.

Reason. Nuclear forces operate within the nucleus only

(*a*) A (*b*) B (*c*) C (*d*) D

19. Assertion. Weak nuclear forces appear only between elementary particles of short life times.

Reason. They operate only through a range of nuclear size.

(*a*) A (*b*) B (*c*) C (*d*) D

ANSWERS

16. (A) → (*s*) ; (B) → (*p*) ; (C) → (*q*, *t*) ; (D) → (*r*) **17.** (*a*) **18.** (*a*) **19.** (*a*)

HINTS/EXPLANATIONS.....

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. These remarks were made by Einstein.

2. Art of soluble implies that different phenomena dissolve into one that can be explained in terms of only a few fundamental laws.

7. Prenatal sex determination is not a good use of science, because the information obtained can be misused.

8. *PAM* Dirac made these remarks.

9. Physics for the inquiring mind is written by *EM* Rogers.

II. Multiple Choice Questions (with One or More than One Correct Answers)

10. All the four choices are correct.

11. Computers and rapid means of transport are two good uses of science.

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

12. Ptolemy postulated that earth is stationary.

13. Galileo postulated that sun is stationary.

14. Galileo was punished for spreading wrong concepts.

IV. Matching Type Questions

15. Quantum theory was given by Planck. de Broglie postulated dual theory. Einstein was awarded Nobel prize for discovery of photoelectric effect.

Raoult worked on lowering vapour pressure.

V. Matrix-Match Type Questions

16. Rocket propulsion is based on Newton's laws of motion.

Optical fibers are based on total internal reflection.

Nuclear reactor is based on controlled nuclear chain reaction, which was inverted by Enrico Fermi.

Genetic engineering is based on role of *DNA* in heredity.

VI. Integer Type Questions

17. Fundamental forces in nature are of four types.

HINTS/EXPLANATIONS....

For Difficult Questions

VII. Assertion-Reason Type Questions

18. Both, the Assertion and Reason are true, and reason is correct explanation of the assertion.

19. Both, the Assertion and Reason are true, and reason is correct explanation of the assertion.

2

UNITS AND MEASUREMENT

2.1. NEED FOR MEASUREMENT

Physics as we know, is a branch of Science which deals with the study of nature and natural phenomena. For a precise description of any such phenomenon, measurement of the quantities involved is essential.

For example, every body, when free, falls down towards the surface of earth. To understand this natural phenomenon, we must know why the body falls. With what velocity does the body fall ? Is this velocity a constant ? What is the time taken by the body to reach the ground ? Does this time depend upon the mass of the body ? Is the time a function of height of the body ? And so on. To answer all these questions, measurements of distance, time, mass etc. are essential.

> ***A quantity that can be measured, and in term of which, laws of Physics are described is called a physical quantity. For example, length, mass, time, force etc.***

2.2. THE MEASURING PROCESS

The process of measurement is basically a process of comparison. It involves the selection of a unit of measurement and comparing the quantity with the standard unit. *The standard chosen should obviously be of the same nature as that of the quantity to be measured.* For example, unit of length has to be in terms of length only. Similarly, unit of mass has to be in terms of mass and so on.

> ***An arbitrarily chosen standard of measurement of a quantity, which is accepted internationally is called unit of the quantity.***

In comparing the quantity with the standard unit, we have to find the *number of times* this unit is contained in the quantity.

In general, magnitude of a quantity Q = numerical value × size of its unit. Thus when a unit u of the quantity is contained n times in the quantity, we write

$$\boxed{Q = n\,u}$$

2.3. CONCEPT OF MASS

> ***Mass of a body is defined as the quantity of matter in the body, which can never be zero.***

The most common unit of mass is **kilogram.**

2.4. CONCEPT OF LENGTH

The concept of length in Physics is the concept of distance in every day life.

> ***Length of** an object may be defined as the distance of separation between any two points at the extreme ends of the object.*

The most common unit of length is **metre.**

2.5. CONCEPT OF TIME

The concept of time is very old. The idea of passage of time occurred first from the motion of moon across the sky, then from the formation of day and night as a result of rotation of earth around its axis, and so on. Today, even a child is familiar with what time is. But it is very difficult to define time precisely.

> ***According to Einstein, 'time is what a clock reads'.***

Infact, time is a measure of duration between the occurrence of two events. We may also define time as a measure of the duration for which an event lasts.

The most common unit of time is **second.**

2.6. FUNDAMENTAL UNITS AND DERIVED UNITS

As the number of physical quantities to be measured is very large, it is not feasible to define a separate unit for each quantity. To simplify the things, we make use of relation between different physical quantities.

In mechanics, we treat length, mass and time as the three *basic* or *fundamental quantities*, because

(i) they represent our basic scientific notions,

(ii) there is no other quantity which is simpler to them,

(iii) length, mass and time cannot be obtained from one another,

(iv) all other physical quantities in mechanics can be obtained from them.

> ***The units of measurement of length, mass and time are, therefore called fundamental units or base units, in mechanics.***

> ***The units of measurement of all other physical quantities, which can be obtained by suitable multiplication or division of powers of fundamental units are called derived units.***

For example, unit of speed (*i.e.*, ms^{-1}) is a derived unit.

As speed = distance/time

$$\therefore \quad \text{unit of speed} = \frac{\text{unit of distance}}{\text{unit of time}} = \frac{m}{s} = ms^{-1}$$

Similarly, derived unit of area is m^2 ; derived unit of volume is m^3 and so on.

The units of physical quantities like density, momentum, acceleration, force, work, power, energy, pressure etc. can be expressed in terms of fundamental units of length, mass and time. Their units are derived units.

2.7. CHOICE OF A STANDARD UNIT

The unit chosen for measuring any physical quantity should meet the following essential requirements :

(*i*) *It should be of suitable size* ;

(*ii*) *It should be accurately defined* ;

(*iii*) *It should be easily accessible* ;

(*iv*) *It should be easily reproducible, i.e., replicas of the unit should be available easily* ;

(*v*) *It should not change with time*, and

(*vi*) *It should not change with the changing physical conditions like temperature, pressure etc.*

DO YOU KNOW ?

The National Bureau of standards is now known as the National Institute of Standards and Technology (NIST). A body named General Conference on Weights and Measures has been authorised to decide the units.

Who Decides the Units ?

The first essential thing about the choice of a standard unit for a physical quantity is its **international acceptance.** If everyone were to choose his/her own unit for the quantity, it will become a mess to communicate freely among the scientists all over the world.

A body named **'General Conference on Weights and Measures'** was set-up and given the authority to decide the units by international agreement. It holds its meetings and any changes in standard units are communicated through the publications of the conference.

2.8. SYSTEMS OF UNITS

A system of units is the complete set of units, both fundamental and derived, for all kinds of physical quantities. Each system is named in terms of fundamental units on which it is based. The common systems of units used in mechanics are given below :

(*a*) **The f.p.s. system** is the British Engineering system of units, which uses *foot* as the unit of length, *pound* as the unit of mass and *second* as the unit of time.

(*b*) **The c.g.s. system** is the Gaussian system which uses *centimetre, gram* and *second* as the three basic units for length, mass and time respectively.

(*c*) **The m.k.s. system** is based on *metre*, *kilogram* and *second* as the fundamental units of length, mass and time respectively.

The c.g.s. and m.k.s. systems are called *metric systems* or *decimal systems* because multiples and sub multiples of basic units are related to the practical units by powers of ten.

The **drawback of c.g.s. system** is that many of the derived units on this system are inconveniently small. The *advantages* of m.k.s. system are that some of the derived units are of convenient size. Also, the m.k.s system leads us to electrical units in practical use.

The f.p.s. system is losing its popularity because of inconvenient multiples and submultiples involved in it for conversion.

Retain in Memory

1. When m.k.s. system is extended to electricity, then

(*i*) with current as fundamental quantity and ampere (A) as its unit, it is called MKSA system.

(*ii*) with charge (Q) as fundamental quantity and coulomb as its unit, it is called MKSQ system.

2. The c.g.s. system and m.k.s system are **metric or decimal systems** as the multiples and submultiples of their basic units are related to the practical units by powers of 10. Obviously, f.p.s. system is not a decimal system.

(*d*) International System of Units (SI)

The system of units, which is accepted internationally for measurement is abbreviated as SI. The SI, with standard scheme of symbols, units and abbreviations, was developed and recommended by *General conference on Weights and Measures* in 1971 for international usage in scientific, technical, industrial and commercial work.

The SI is based on the following **seven fundamental units and two supplementary units** as listed in Table 2.1.

TABLE 2.1. Fundamental Units and Supplementary units on SI

S. No.	Basic Physical Quantities	Fundamental unit	Symbol used
1.	Mass (*M*)	kilogram	kg
2.	Length (*L*)	metre	*m*
3.	Time (*T*)	second	*s*
4.	Temperature (θ)	kelvin	*K*
5.	Electric current (*I*)	ampere	*A*
6.	Luminous intensity (*cd*)	candela	cd
7.	Quantity of matter (mol)	mole	mol

S. No.	Supplementary physical quantities	Supplementary unit	Symbol used
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

This system of units is essentially a modification over the m.k.s. system and is, therefore, called **rationalised m.k.s. system.** This rationalisation was essential to obtain the units of all the physical quantities in Physics. The fundamental units of mass, length and time on m.k.s. system could be used to obtain the units of physical quantities in mechanics only.

In addition to the fundamental units and the supplementary units, there are a large number of derived units on this system. Some of the derived units are listed in Table 2.2.

TABLE 2.2. Some derived units on SI

S.No.	Physical Quantitiy	Derived Unit	Symbol used
1.	Force	newton	N
2.	Work, Energy	joule	J
3.	Power	watt	W
4.	Electric potential	volt	V
5.	Electric capacity	farad	F
6.	Magnetic flux	weber	Wb , and so on.

2.9. DEFINITIONS OF FUNDAMENTAL UNITS AND SUPPLEMENTARY UNITS OF SI

The seven fundamental units and two supplementary units on the SI have been defined as under, by the General Conference of Weights and Measures :

1. The metre

The standard metre is exactly equal to 1650763.73 wavelengths in vacuum, of the radiation from Krypton isotope of mass 86.

Krypton – 86 emits light of different wavelengths. The light emitted by Kr-86 due to transition between 2 p_{10} and 5 d_5 levels is orange red in colour and has wavelengths = 6.0578021×10^{-7} m. The number of these wavelengths in 1 m comes out to be 1650763.73.

DO YOU KNOW ?

The iodine stabilized helium neon laser has been used to realize the latest definition of one metre of length. One metre is equal to 1579778·84 wavelengths of He-Ne laser light of wavelength 633 nm.

The number of wavelengths of light in a given distance is measured by an *'optical interferometer'* with an accuracy of 1 in 10^{12}.

The **advantages** of defining standard metre in terms of wavelength of light are :

(*i*) Kr-86 isotope can be obtained relatively easily and cheaply in rather pure form. So the standard is easily reproducible.

(*ii*) Wavelength of light is not affected by time or variation of any other physical condition.

(*iii*) The unit has an accuracy of 1 in 10^9 parts.

Yet a new definition of metre was adopted in 1983 at the 17th General Conference of Weights and Measures, taking velocity of light in vacuum c = 299, 792, 458 ms^{-1}.

One metre is the length of the path travelled by light in vacuum in 1/299, 792, 458 of a second.

2. The kilogram

One kilogram is defined as the mass of the international prototype of the kilogram (which is a standard block of Platinum Iridium alloy 3·9 cm in height and in diameter preserved in the International Bureau of Weights and Measures at Sevres, near Paris, France.

3. The second

One second is the duration of 9, 192, 631, 770 periods of radiation corresponding to unperturbed transition between the two hyperfine levels (F = 4, M = 0 and F = 3, M = 0) of the ground state of Cs–133 atom.

Atomic clocks are based on this definition. They ensure an accuracy of 1 in 10^{11} sec, *i.e.*, an error of one sec. only in 5000 years.

4. The ampere

It was adopted as the unit of electric current. One ampere is the constant current, which when maintained in each of the two straight parallel conductors of infinite length and negligible cross section, held one metre apart in vacuum, shall produce a force per unit length of 2 × 10^{-7} N/m between them.

5. The kelvin

It was adopted as the unit of temperature. One degree Kelvin is the fraction $\left(\dfrac{1}{273{\cdot}16}\right)$ ***of the thermodynamical temperature of the triple point * of water.***

---

*Triple point of water is the temperature at which ice, the water and water vapours coexist. It has a fixed value of 273.16 K.

6. The candela

One candela is the luminous intensity in a given direction, of a source that emits monochromatic radiation of frequency $5\cdot4 \times 10^{14}$ Hz and that has a radiant intensity of $\frac{1}{683}$ watt/steradian in that direction.

7. The mole

It was adopted as the unit of quantity of matter. One mole is the amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of pure carbon-12.

Note that number of atoms in one mole = number of atoms in 0·012 kg of carbon-12 is called *Avogadro constant.* The best value of Avogardro constant is $6\cdot022045 \times 10^{23}$ with an error of $\pm\, 0\cdot000031 \times 10^{23}$.

The **two supplementary units** on the SI are defined as under :

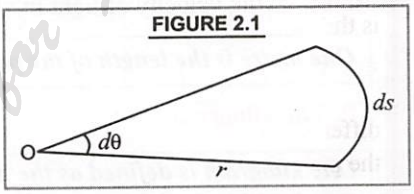
1. The radian (rad)

It is the unit of angle in a plane

one radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

If an arc of length ds subtends an angle $d\theta$ at the centre O of a circle of radius r, Fig. 2.1, then

FIGURE 2.1



$$\boxed{d\theta = \frac{ds}{r} \text{ radian}}$$

It is known that

$$\pi \text{ radian} = 180^\circ \text{ ; } 1 \text{ radian } = \frac{180^\circ}{\pi} = \frac{180 \times 7^\circ}{22} = \frac{630^\circ}{11} = 57\cdot7^\circ$$

Also, 1° (degree of arc) = 60′ (minute of arc) and 1′ (minute of arc) = 60″ (seconds of arc)

2. The Steradian (sr)

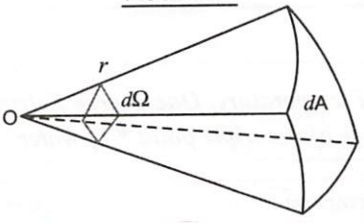
It is the unit of solid angle (*i.e.*, angle in space).

One steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere, which is equal in area, to the square of radius of the sphere.

If an area dA of a spherical surface subtends a solid angle $d\Omega$ at the centre of the sphere of radius r, Fig. 2.2, then

$$\boxed{d\Omega = \frac{dA}{r^2} \text{ steradian}}$$

FIGURE 2.2



Curiosity Question

Q. Why is the use of common units throughout the world desirable ?

Ans. The use of same units of measurement all over the world will certainly improve trade among nations, as the products would be standardized, and inventories will be reduced. The information exchange among the scientists of the world will also be facilitated.

The use of different units of measurement may prove disastrous sometimes. For example, a pilot or an air controller is accustomed to reporting altitude in feet, not in metres. For example, if altitude of a plane is 3000 metres, is this about 10000 feet or 3000 feet or 1000 feet ? The difference might mean disaster. Thus air travel everywhere would become safer with the use of common units throughout the world.

2.10. ADVANTAGES OF SI

Following are the main advantages of SI over the other systems of units :

1. *SI is a coherent system of units, i.e.,* a system based on a certain set of fundamental units, from which all derived units are obtained by multiplication or division without introducing numerical factors.

2. *SI is a rational system of units,* as it assigns only one unit to a particular physical quantity. For example, joule is the unit for all types of energy. This is not so in other systems of units. For example, in mks system, mechanical energy is in joule, heat energy is in calorie and electric energy is in watt hour.

3. *SI is an absolute system of units.* There are no gravitational units on the system. The use of factor 'g' is thus eliminated.

4. *SI is a metric system, i.e.,* the multiples and submultiples of units are expressed as powers of 10.

5. In current electricity, the absolute units on the SI, like *ampere* (A) for current, *volt* (V) for potential difference, *ohm* (Ω) for resistance, *henry* (H) for inductance, *farad* (F) for capacity and so on, happen to be the practical units for measurement of these quantities.

Retain in Memory

Remember that international system of units (SI) applies to all branches of science, whereas *m.k.s.* system is confined to mechanics only.

In our country, the responsibility of maintenance of physical standards of length, mass and time etc. has been given to National Physical Laboratory, New Delhi.

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2.11. SOME IMPORTANT PRACTICAL UNITS

(*a*) In the MACRO-COSM MEASUREMENTS, *i.e.,* in the *measurement of very large distances,* we use the following three units :

1. Astronomical Unit (AU)

It is the average distance of the centre of the sun from the centre of the earth.

$$1\ AU = 1.496 \times 10^{11}\ \text{m} \simeq 1.5 \times 10^{11}\ \text{m}$$

2. Light Year (*ly*)

One light year is the distance travelled by light in vacuum in one year.

As velocity of light in vacuum is $3 \times 10^8\ \text{ms}^{-1}$ and 1 year = 365 × 24 × 60 × 60 second, therefore,

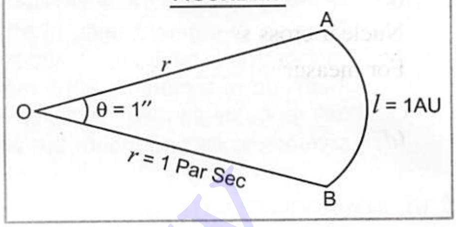
1 light year = $3 \times 10^8 \times (365 \times 24 \times 60 \times 60)$ metre

$$1\ ly = 9.46 \times 10^{15}\ \text{m}$$

3. Par sec

One Par sec is the radius of a circle at the centre of which an arc of the circle, 1 AU long subtends an angle of 1″, Fig. 2.3.

FIGURE 2.3



As $l = 1\,AU = 1.496 \times 10^{11}$ m

and $\theta = 1\,\text{sec} = \dfrac{1}{60}\,\text{min} = \dfrac{1}{60\times 60}\,\text{degree}$

$$= \frac{1}{60\times 60}\times\frac{\pi}{180}\ \text{radian}$$

then as $r = \dfrac{l}{\theta}$, from Fig. 2.4,

$$\therefore\ 1\,\text{par sec} = \frac{1\,\text{AU}}{1\,\text{sec}} = \frac{1.496\times 10^{11}\ \text{m}}{\pi/(60\times 60\times 180)\ \text{rad}} = 3.084\times 10^{16}\ \text{m}$$

$$\boxed{1\ \text{par sec} = 3.1\times 10^{16}\ \text{m}}$$

Relation between AU, *ly* and par sec

As $1\ \text{AU} = 1.5\times 10^{11}$ m, $1\ ly = 9.46\times 10^{15}$ m

and $1\ \text{par sec} = 3.1\times 10^{16}$ m

$$\therefore\ \frac{1\,ly}{1\,\text{AU}} = \frac{9.46\times 10^{15}}{1.5\times 10^{11}} = 6.3\times 10^4$$

Hence, $\boxed{1\ ly = 6.3\times 10^4\ \text{AU}}$

Also, $\dfrac{1\,\text{par sec}}{1\,ly} = \dfrac{3.1\times 10^{16}}{9.46\times 10^{15}} = 3{\cdot}26 \quad \therefore \boxed{1\ \text{par sec} = 3.26\ ly}$

Clearly, 1 par sec is bigger than 1 *ly*, which is bigger than 1 AU. It may be of interest to note that size of universe is nearly 10^{10} *ly* The nearest star, Alpha centauri, outside our solar system is 4.3 *ly* away from the earth.

Sample Problem **Calculate the number of astronomical units in one metre.**

Sol. We know, $1\ \text{AU} = 1{\cdot}5\times 10^{11}$ m or $1{\cdot}5\times 10^{11}\ \text{m} = 1$ AU

$$\therefore\quad 1\,\text{m} = \frac{1}{1{\cdot}5\times 10^{11}}\,\text{AU} = \mathbf{6{\cdot}67\times 10^{-12}\ AU}$$

(*b*) *In the MICRO-COSM MEASUREMENTS, i.e.,* in the *measurement of small distances*, we use the following four units :

(*i*) 1 **micron** = 1 μ or 1 μm = 10^{-6} m

(*ii*) 1 **nanometre** = 1 nm = 10^{-9} m

(*iii*) 1 **angstrom** = 1A° = 10^{-10} m

(*iv*) 1 **fermi** = 1 femtometre = 1 *fm* = 10^{-15} m

For example, the radius of proton is 1.2 fm and radius of hydrogen atom is 0.5 Å.

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Some other useful units of length are

1 inch = 0·0254 m ; 1 foot = 0·3048 m

1 yard = 0·9144 m ; 1 mile = $1{\cdot}609 \times 10^3$ m

1 nautical mile = $1{\cdot}852 \times 10^3$ m

(c) *For measuring very small areas,* the unit used is : 1 barn = 10^{-28} m^2

Nuclear cross sections are measured in barns.

For measuring large areas, the units used are :

1 acre = 4047 m^2, 1 are (*a*) = 10^2 m^2, 1 hactare = 10^4 m^2

(d) *For measuring heavy masses,* the units used are

(*i*) 1 tonne or 1 metric ton = 1000 kg

(*ii*) 1 quintal = 100 kg

(*iii*) 1 slug = 14.57 kg

(*iv*) 1 lb = 0·4536 kg

The largest practical unit of mass is **Chandra Shekhar Limit** (C.S.L)

1 C.S.L = 1·4 times the mass of sun.

(e) *For measuring very small masses,* the unit used is

1 atomic mass unit = 1 a.m.u. or 1 u = 1.66×10^{-27} kg. The atomic and nuclear masses are measured in this unit.

Yet another unit of small mass is *carat*, where 1 *c* = 200 mg

(*f*) *Some practical units of standard of time are :*

(*i*) **Solar day.** It is the time interval between two successive passages of the sun across the meridian.

(*ii*) **Sedrial day.** It is the time interval between two successive passages of a fixed star across the meridian.

(*iii*) **Solar year** (or year) is the time taken by the earth to complete one revolution around the sun in its orbit.

1 solar year = 365·25 average solar days = 366·25 sedrial days

The year in which there is total solar eclipse is called a **tropical year.** The year which is divisible by 4, and in which month of February has 29 days, is called **a leap year.**

One hundred years make up one **century.**

(*iv*) **Lunar month.** It is the time taken by moon to complete one revolution around the earth in its orbit.

1 Lunar month = 27·3 days.

(*v*) **Shake.** It is the smallest practical unit of time.

1 shake = 10^{-8} s

(g) *For measuring pressure,* the units used are :

(*i*) 1 bar = 1 atmospheric pressure = 10^5 N/m^2

1 bar = 10^5 N/m^2 = 10^5 Pascal (Pa)

1 millibar = 10^2 Pa

(*ii*) 1 torr = 1 mm. of Hg column

As 1 standard atmospheric pressure = 760 mm of Hg col.

$\therefore \boxed{1 \text{ bar} = 760 \text{ torr}}$

(h) *Two more units retained for general use are :*

1 curie (*Ci*) = $3{\cdot}7 \times 10^{10}$ disintegrations/sec

1 roentgen (*R*) = $2{\cdot}58 \times 10^{-4}$ C/kg

2.12. METRIC PREFIXES FOR POWERS OF 10

The physical quantities whose magnitude is either too large or too small can be expressed more compactly by the use of certain prefixes.

For example, the distance of Delhi from Ambala is 200,000 metres. It is more compact to represent this distance as 200 km. Similarly, weight of an average man is 60000 gm. We prefer to say it as 60 kg. Earth takes 31000000 second to complete one revolution around the sun. It is more convenient to mention this time as 31 mega seconds.

The prefixes we commonly use for powers of 10 are listed below in Table 2.3.

TABLE 2.3. Metric Prefixes

S. No.	Power of 10	Prefix	Symbol
1.	10^{-1}	deci	*d*
2.	10^{-2}	centi	*c*
3.	10^{-3}	milli	*m*
4.	10^{-6}	micro	μ
5.	10^{-9}	nano	*n*
6.	10^{-12}	pico	*p*
7.	10^{-15}	femto	*f*
8.	10^{-18}	atto	*a*
9.	10^{-21}	zepto	*z*
10.	10^{-24}	yocto	*y*
11.	10^{1}	deca	*da*
12.	10^{2}	hecto	*h*
13.	10^{3}	kilo	*k*
14.	10^{6}	mega	*M*
15.	10^{9}	giga	*G*
16.	10^{12}	tera	*T*
17.	10^{15}	peta	*P*
18.	10^{18}	exa	*E*
19.	10^{21}	zetta	*Z*
20.	10^{24}	yotta	*Y*

2.13. CONVENTIONS FOR WRITING OF UNITS AND THEIR SYMBOLS

(*i*) Unit is never written with capital initial letter, even if it is named after a scientist. For example, SI unit of force is newton.

(*ii*) For a unit named after a scientist, the symbol is a capital letter. For other units, the symbol is not a capital letter. For example, SI unit of force is newton (N), and SI unit of length is metre (m).

(*iii*) The units or symbols are never written in plural form. For example $l = 10$ cm, $F = 20$ dyne. '*s*' is not added to show the plural form.

(*iv*) In a symbol, we prefer not to use more than one solidus. For example,

$$\text{Intensity} = \text{energy per second per unit area} = \frac{\text{joule}}{\text{sec metre}^2} = \text{W m}^{-2} \text{ (and not J/s/m}^2\text{)}$$

(*v*) Punctuation marks *e.g.*, full stops are not written after the symbols. For example, 1 litre = 1000 cc (and not c.c).

Some of the *most commonly used units* with prefixes are :

$$1 \text{ micrometre} = 1\ \mu\text{ m} = 10^{-6}\text{ m}, \quad 1 \text{ millimetre} = 1 \text{ mm} = 10^{-3}\text{ m}$$

$$1 \text{ centimetre} = 1 \text{ cm} = 10^{-2}\text{ m}, \quad 1 \text{ kilometre} = 1 \text{ km} = 10^{3}\text{ m}$$

Similarly, $1\ \mu\text{ g} = 10^{-6}\text{ g}$; $1 \text{ mg} = 10^{-3}\text{ g}$; $1 \text{ kg} = 10^{3}\text{ g}$

Retain in Memory

The general rules for calculations that involve the multiplication and division of powers of 10 are :

(*i*) $\dfrac{1}{10^n} = 10^{-n}$

(*ii*) $10^n \times 10^m = 10^{(n+m)}$ ...(exponents are added in multiplication)

(*iii*) $\dfrac{10^n}{10^m} = 10^{(n-m)}$ ...(exponents are subtracted in division)

2.14. ORDER OF MAGNITUDE

The order of magnitude of a quantity does not tell us the absolute value or exact value of the quantity. It gives us the value nearest to the actual value, in terms of suitable powers of 10. For this, a number less than 5 is treated as 1 and a number between 5 and 10 is treated as 10. ***The power or exponent of 10 so obtained represents the order of magnitude of the quantity.*** For example, radius of earth is 6400 km = $6{\cdot}4 \times 10^6$ m. Taking 6·4 as 10, size of earth = 10^7 m. Hence, the order of magnitude of size of earth is 7. Again, the average height of a man is 1·8 m = $1{\cdot}8 \times 10^0$ metre. Taking 1·8 as 1, height of a man is 10^0 m. Hence, the order of magnitude of height of man is zero. Some other examples indicating order of magnitude are given in Table 2.4.

TABLE 2.4. Order of magnitude

Number	Expressed in nearest power of 10	Order of magnitude
1	10^0	Zero
49	$4{\cdot}9 \times 10^1 \to 1 \times 10^1 \ (\because 4{\cdot}9 < 5)$	1
51	$5{\cdot}1 \times 10^1 \to 10 \times 10^1 = 10^2 \ (\because 5{\cdot}1 > 5)$	2
499	$4{\cdot}99 \times 10^2 \to 1 \times 10^2$	2
501	$5{\cdot}01 \times 10^2 \to 10 \times 10^2 = 10^3$	3
999	$9{\cdot}99 \times 10^2 \to 10 \times 10^2 = 10^3$	3
1000	$1 \times 10^3 \to 10^3$	3
0·049	$4{\cdot}9 \times 10^{-2} \to 1 \times 10^{-2}$	–2
0·050	$5{\cdot}0 \times 10^{-2} \to 10 \times 10^{-2} = 10^{-1}$	–1

We may define

Order of magnitude of a quantity as the power of 10, which gives us a value nearest to the actual value of the quantity.

Sample Problem **Write the order of magnitude of the following measurements :**

(*i*) 38240000 m **(*ii*) 0·000520 kg**

Sol. (*i*) Here, 38240000 m = $3{\cdot}824 \times 10^7$

As $3{\cdot}824 < 5$, $\therefore$ order of magnitude = 7

(*ii*) 0·000520 kg = $5{\cdot}2 \times 10^{-4}$

As $5{\cdot}2 > 5$ $\therefore$ $0{\cdot}000520 = 10 \times 10^{-4} = 10^{-3}$

The order of magnitude = – 3

2.15. RANGE OF VARIATION OF LENGTH

The objects of our interest vary widely in sizes. For example, large objects like galaxy, stars, sun, earth, moon etc. and their distances constitute **macrocosm,** *i.e.,* **the large world.**

On the contrary, objects like molecules, atoms, proton, neutron, electron, bacteria etc. constitute **microcosm,** *i.e.,* **the small world.** Table 2.5 gives us an idea of the enormous range over which lengths can vary. These may vary from the size of proton ($\approx 10^{-15}$ m) to the size of observable universe ($\approx 10^{26}$ m).

TABLE 2.5. Range of lengths

S. No.	Size of object or distance measured	Order of length (m)
1.	Diameter of electron	10^{-16}
2.	Radius of a proton	10^{-15}
3.	Size of atomic nucleus	10^{-14}
4.	Size of hydrogen atom	10^{-10}
5.	Size of poliomylitis virus	10^{-8}
6.	Wavelength of visible light	10^{-7}
7.	Size of red blood corpuscle	10^{-5}
8.	Thickness of paper	10^{-4}
9.	Height of a person	10^{0}
10.	Height of mount everest	10^{4}
11.	Distance between Ambala and Delhi.	10^{5}
12.	Radius of moon	10^{6}
13.	Radius of earth	10^{7}
14.	Mean distance of moon from earth	10^{8}
15.	Radius of sun	10^{9}
16.	Mean distance of sun from earth	10^{11}
17.	Distance of farthest planet pluto from sun	10^{13}
18.	Distance of nearest star from earth	10^{16}
19.	Diameter of our galaxy	10^{20}
20.	Distance of nearest galaxy (andromedia)	10^{22}
21.	Estimated size of observable universe	10^{26}

2.16. MEASUREMENT OF LENGTH

The **direct methods** for the measurement of length involve the use of

(*i*) a *metre scale* for distances from 10^{-3} m to 10^{2} m.

(*ii*) a *vernier callipers* for distances upto 10^{-4} m

(*iii*) a *screw gauge* and a *spherometer* for distances upto 10^{-5} m.

For all other distances beyond these ranges, we use **indirect methods.**

2.17. INDIRECT METHODS FOR MEASUREMENT OF LARGE DISTANCES

1. ECHO METHOD OR REFLECTION METHOD

An echo is the phenomenon of repetition of sound on reflection from an obstacle. That is why echo method is also called *reflection method.*

Suppose we have to find the distance (x) of a hill from an observation point P. Let a shot be fired in air at P. The sound of the shot travels a distance x towards the hill, gets reflected from the hill, the reflected sound travels the same distance x to the point P, when an echo of the shot is heard. The time interval (t) between the

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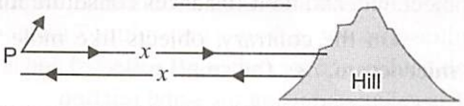
firing of the shot and hearing of its echo at P is measured. Let v be the velocity of sound in air at the temperature during the experiment.

FIGURE 2.4



As distance = velocity × time

$$\therefore \quad x + x = v \times t$$

or $$\boxed{x = \frac{v \times t}{2}} \qquad ...(1)$$

As v is known, therefore x can be calculated by measuring the time t.

2. LASER METHOD

LASER is a very intense, monochromatic* and undirectional** beam of light. The distance of moon from earth has been calculated by this method, using the same principle of echo. The LASER beam transmitted from earth is received back on earth after reflection from the moon. The time interval (t) between transmission and reception of the beam is measured accurately. If c is velocity of LASER beam in air/vacuum, then as

distance = velocity × time

$$\therefore \quad x + x = c \times t \quad \text{or} \quad \boxed{x = \frac{c \times t}{2}} \qquad ...(2)$$

Now, $c = 3 \times 10^8$ m/s. By measuring t accurately, the distance x of the moon from earth can be calculated.

3. RADAR METHOD

RADAR is an abbreviation for **Radio Detection and Ranging.** It is a powerful equipment which is used for detection of an object like aeroplane and measuring its distance (and even its elevation and velocity) using radio waves. This method is also based on the principle of echo, *i.e.*, reflection of radio waves.

The radio waves are transmitted in space all around from the RADAR station. When they are intercepted by an object like aeroplane, they get relfected and are received back at the RADAR station. The time interval (t) between transmission and reception of the radio waves is measured accurately. Proceeding as explained above, the distance x of the aeroplane can be calculated from the relation :

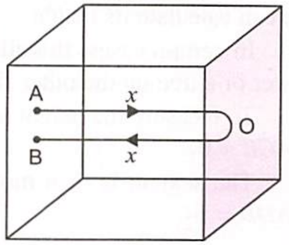
$$\boxed{x = \frac{c \times t}{2}}$$

Note that here x is the distance of aeroplane at the instant radio wave strikes it. The arrangement for measuring elevation and velocity of aeroplane are made in sophisticated RADAR systems.

IMPORTANT NOTE

This very method can be used for measuring small thickness of sheets of matter. The signal transmitted from any point A on the front face of the sheet is received back at B on the front face, after reflection at the point O on the back face of the sheet, Fig. 2.5.

FIGURE 2.5



Thickness x of the sheet is calculated from the same relation $x = \frac{c \times t}{2}$

where c is velocity of the signal in the material of the sheet and t is the time interval between transmission and reception of signal. As x is small, t shall be extremely small. Therefore, the arrangement for measuring time interval t should be very precise.

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*Monochromatic beam means a beam of single colour, *i.e.*, of one wavelength.

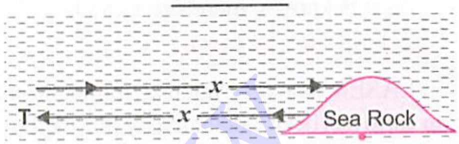
**Unidirectional beam means that it is travelling along the same straight line with least angular spread.

4. SONAR METHOD

SONAR is an abbreviation for **Sound Navigation and Ranging.** This method is used for detecting an object under water (*e.g.*, a submarine, ship, sea rock, etc.) and measuring its distance using ultrasonics*. The ultrasonic waves from a transmitter T under water travel in a given direction. When they are intercepted by an object like sea rock, they get reflected and are received back at T, Fig. 2.6. The distance x of the sea rock from T is calculated using the same relation :

$$\boxed{x = \frac{v \times t}{2}}$$

FIGURE 2.6



where v is velocity of ultrasonic waves in sea water and t is the time interval between transmission and reception of ultrasonic signal.

IMPORTANT NOTE

The SONAR method is also used for detection of flaws in materials and in the mechanical testing of materials without damaging them. This is carried out by measuring the change in the echo time, because velocity of the signal in different materials is different.

Sample Problem **The distance of nearest star from earth is 10^{13} km. Calculate the time taken by a laser beam to return to earth after reflection from the star.**

Sol. Here, $x = 10^{13}$ km $= 10^{16}$ m

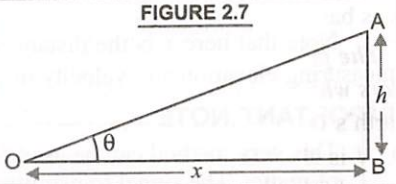
Velocity of laser beam, $c = 3 \times 10^8$ m/s

As $x = \dfrac{c \times t}{2}$. $\quad \therefore \; t = \dfrac{2x}{c} = \dfrac{2 \times 10^{16}}{3 \times 10^8}$ s $= \mathbf{6{\cdot}67 \times 10^7}$ **s**

5. TRIANGULATION METHOD

This method uses the geometry of triangles for measuring the height of a distant object. In Fig. 2.7, AB is an object whose height h is to be measured. Let O be the observation point. Using an apparatus called **'sextant'** at O, we measure $\angle AOB = \theta$. This is called *angle of elevation of the object.*

FIGURE 2.7



In ΔAOB, $\quad \tan\theta = \dfrac{AB}{OB} = \dfrac{h}{x}$

$$\therefore \qquad \boxed{h = x \tan\theta} \qquad \textbf{...(3)}$$

If distance (x) of object from the observation point is known, we can calculate its height.

In certain cases, **this distance x is not known.** The object is said to be **inaccessible.** For example, a tower or a tree on the other side of a river is inaccessible.

To measure the height h of an inaccessible object AB, we hold the *sextant* at any point C and measure $\angle ACB = \theta_1$.

The sextent is then moved to any other position D where $CD = x$ is measured. Again, we measure $\angle ADB = \theta_2$.

In ΔABC, $\quad \cot\theta_1 = \dfrac{CB}{AB} \qquad$ **...(4)**

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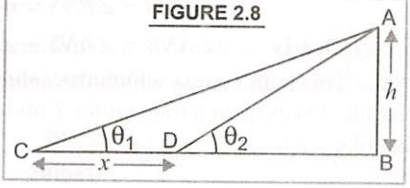
*Ultrasonics are the waves (of sound) which cannot be heard, because their frequency is more than 20000 hertz.

FIGURE 2.8



In ΔABD, $\cot\theta_2 = \frac{DB}{AB}$ ...(5)

Subtract (5) from (4)

$$\cot\theta_1 - \cot\theta_2 = \frac{CB}{AB} - \frac{DB}{AB} = \frac{CB - DB}{AB} = \frac{CD}{AB} = \frac{x}{h}$$

or $$\boxed{h = \frac{x}{\cot\theta_1 - \cot\theta_2}} \quad ...(6)$$

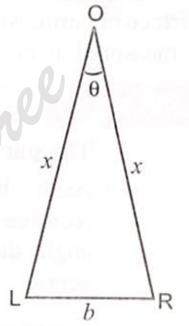
Hence h can be calculated.

6. PARALLAX METHOD

Large distances, such as the distance of a planet or a star from earth can be measured by the **parallax method.**

Parallax is the name given to change in the position of an object with respect to the background, when the object is seen from two different positions. The distance between the two positions (*i.e.,* points of observation) is called the **basis.**

FIGURE 2.9

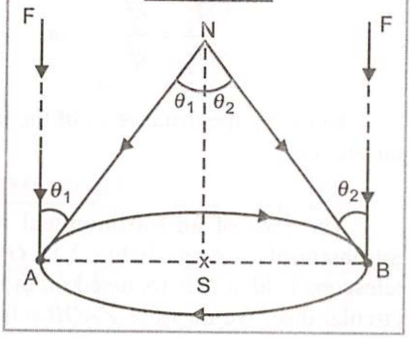


For example, in Fig. 2.9, when we hold a pen in front of us against the background of a wall and look at the pen first by our left eye L (closing the right eye) and then by our right eye R (closing the left eye), we find that position of pen changes with respect to the background wall. *The shift in position of an object (say, a pen) when viewed from two eyes, keeping one eye closed at a time is called* **parallax.** The distance between the left eye (L) and right eye (R) in this case is the **basis.** $\angle LOR$ is called the *parallax angle* or *parallactic angle.*

Taking LR as an arc of length b and radius $LO = RO = x$, we get, $\theta = \frac{b}{x}$

Knowing b and measuring θ, we can calculate x.

FIGURE 2.10



As astronomical distances are too large, parallex angle θ is too small to be measured. Hence, instead of taking distance between two eyes as basis, we may broaden the base suitably.

The parallax method has been used for measuring distances of stars which are less than 100 light years away. The diameter AB of earth's orbit around the sun (S) is chosen as the base line, Fig. 2.10. N is the nearby star whose distance (d) from earth is to be measured. F is far off star whose direction is taken practically the same at all positions of earth in its orbital motion.

Suppose A is the position of earth in its orbital motion at any time. Using an astronomical telescope, we measure $\angle FAN = \theta_1$, between the directions of light from distant star F and nearby star N.

As is clear from Fig. 2.10,

$$\angle FAN = \angle ANS = \theta_1$$

After six months, earth is at B, a position diametrically opposite to the position A on the orbit. $\angle FBN = \theta_2$, between the directions of light from distant star F and nearby star N is measured again*.

---

*The distance of nearby star N is so large that any change in the position of N during six months is negligible.

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$\therefore \qquad \angle FBN = \angle BNS = \theta_2$

Clearly, $\qquad \angle ANB = \angle ANS + \angle BNS = \theta_1 + \theta_2$

This is the angle which the nearby star N subtends on the orbital diameter of earth.

As $\qquad$ angle $= \dfrac{\text{arc}}{\text{radius}} \qquad \therefore \qquad \boxed{\theta_1 + \theta_2 = \dfrac{AB}{AN}}$ ...(7)

or $\qquad AN = \dfrac{AB}{\theta_1 + \theta_2}$ ...(8)

Clearly, $\qquad AB = 2\,AS,$ where AS is distance between sun and earth.

$\therefore \qquad AB = 2\,AU = 2 \times 1{\cdot}5 \times 10^{11}\ \text{m} = 3 \times 10^{11}\ \text{m}$

If $(\theta_1 + \theta_2)$ is known (in radian), then AN can be calculated.

The parallax method is also used for determining distance of moon from earth. Two places on the surface of earth, which are far removed from eachother form the base line. Parallax angle (w.r.t a far off star) is measured using an astronomical telescope.

Retain in Memory

The parallax method is used for measuring distances of nearby stars only.

As the distance of star increases, the parallax angle decreases, and a great degree of accuracy is required for its measurement. Keeping in view the practical limitation in measuring the parallax angle, the maximum distance of a star we can measure by parallax method is limited to 100 light years.

7. SPECTROSCOPIC METHOD

This method is used for measuring the distance of a star which is more than 100 light years away from earth. Here, we compare the intensity I_1 of the faint image of the far off star on a photographic plate to the intensity I_2 of the bright image of a nearby star (say, sun) on the photographic plate. Suppose r_1 and r_2 are the respective distances of far off star and nearby star from earth.

As intensity is inversely proportional to the square of the distance, therefore,

$$\boxed{\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}} \quad \text{or} \quad r_1^2 = r_2^2\,\frac{I_2}{I_1} \quad \text{or} \quad r_1 = r_2\sqrt{I_2/I_1}$$

Knowing the distance r_2 of the nearby star and the ratio (I_2/I_1), we can calculate r_1, the distance of the far off star.

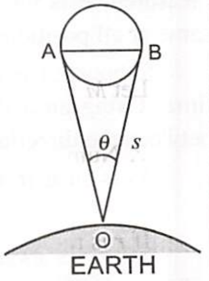
8. SIZE OF AN ASTRONOMICAL OBJECT

The size of an astronomical object like moon can be measured using an astronomical telescope. In Fig. 2.11, O is an observation point on earth. An astronomical telescope held at O is focussed on moon, when we observe an image in the form of a circular disc. We measure $\angle AOB = \theta$, say. It is the angle between the two directions when two diametrically opposite points of the moon are viewed through the telescope.

Let s be the average distance of moon from the surface of earth. As s is very large compared to the diameter AB of moon, therefore

$AB \simeq$ length of circular arc of radius $s = s\,\theta$ ...(3)

FIGURE 2.11



Hence, AB can be calculated, when s is known and θ is measured.

The angle subtended by the two diametrically opposite ends of the moon at a point on the earth is called the **angular diameter of the moon.** Its value is about 0·5°.

Sample Problem 1 **The parallax of a heavenly body measured from two points diametrically opposite on equator of earth is 2·0 minute. If radius of earth is 6400 km, calculate distance of heavenly body.**

Sol. Here, $\theta = 2{\cdot}0' = \frac{2^\circ}{60} = \frac{2}{60} \times \frac{\pi}{180}$ radian

l = diameter of earth = 2 × 6400 km

$= 1{\cdot}28 \times 10^4$ km $= 1{\cdot}28 \times 10^7$ m.

$r = ?$

From $l = r\,\theta$,

$$r = \frac{l}{\theta} = \frac{1{\cdot}28 \times 10^7}{\pi/30 \times 180} = \mathbf{2{\cdot}2 \times 10^{10}\ m}$$

DO YOU KNOW ?

You can watch the sunset over a calm ocean, once while you lie on a beach, and then once again, when you stand up. Surprisingly, by measuring the time interval between the two sunsets, you can calculate approximately, the radius of the earth. This is one of the indirect methods for measuring size of earth.

Sample Problem 2 **The moon subtends an angle of 57 minutes at the base line equal to radius of earth. What is the distance of moon from earth. Given radius of earth is 6400 km.**

Sol. Here, $\theta = 57$ min. $= \frac{57^\circ}{60} = \frac{19}{20} \times \frac{\pi}{180}$ rad

l = radius of earth = 6400 km = $6{\cdot}4 \times 10^6$ m $\quad r = ?$

From $l = r\,\theta$, $r = \frac{l}{\theta} = \frac{6{\cdot}4 \times 10^6 \times 20 \times 180}{19 \times \pi}$ m $= \mathbf{3{\cdot}86 \times 10^8\ m}$

2.18. INDIRECT METHODS FOR ESTIMATION OF VERY SMALL DISTANCES : SIZE OF A MOLECULE

1. AVOGADRO'S METHOD

This method is used to measure distances of the order of 10^{-10} m, which is the size of an atom. An atom is a tiny sphere. When such atoms lie packed in any substance, empty spaces are left inbetween. According to Avogadro's hypothesis, *volume of all atoms in any mass of the substance is 2/3 of the volume occupied by that mass of the substance.*

Suppose m = mass of a substance

V = volume occupied by the substance

ρ = density of the substance

$$\therefore \quad \rho = \frac{m}{V} \quad \text{or} \quad V = \frac{m}{\rho} \qquad \text{...(9)}$$

Let M be the atomic weight of the substance and N be the Avogadro number.

$\therefore$ Number of atoms in mass m of the substance $= \frac{Nm}{M}$

If r is radius of each atom, then volume of each atom $= \frac{4}{3}\pi r^3$

Volume of all the atoms in the substance $= \frac{4}{3}\pi r^3 \times \frac{Nm}{M}$

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According to Avogadro's hypothesis,

$$\text{Volume of all the atoms} = \frac{2}{3}\,(\text{Volume of substance})$$

$$\frac{4}{3}\pi r^3 \frac{Nm}{M} = \frac{2}{3}V = \frac{2}{3}\frac{m}{\rho} \quad \text{or} \quad r^3 = \frac{M}{2\pi N\rho} \qquad \boxed{r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}} \qquad ...(10)$$

The value of r, so calculated is of the order of 10^{-10} m.

2. RUTHERFORD METHOD

This method is used for measuring nuclear sizes $\approx 10^{-14}$ m to 10^{-15} m. When a collimated beam of alpha particles is made to hit an extremely thin metal foil, some of the alpha particles retrace their path, turning through 180°. This led Rutherford to the discovery of **atomic nucleus** in which the entire positive charge and almost entire mass of the atom are supposed to be concentrated.

An alpha particle carries two units of positive charge. When it tends to hit the nucleus, it is repelled constantly by the positive charge on the nucleus. The velocity and hence kinetic energy of α-particle go on decreasing and in turn, potential energy of α-particle goes on increasing. At a particular point, KE of α particle becomes zero. It cannot go further. This is the point of closest approach of α particle to the nucleus. The alpha particle retraces its path from this point. The distance of closest approach of α particle to the nucleus gives us an approximate size of the nucleus.

3. SIZE OF MOLECULE OF OLEIC ACID

Oleic acid ($C_{17}H_{32}COOH$) is a soapy liquid with large molecular size ($\simeq 10^{-9}$ m). To measure this size, we proceed in following steps :

(*i*) Take 1 cc of oleic acid, mix it with alcohol to prepare 25 cc of the solution.

(*ii*) Take 1 cc of this solution and mix it again with alcohol to prepare 20 cc of the solution. In this way, solution of oleic acid in alcohol with concentration equal to (1/500) is prepared.

(*iii*) Sprinkle some lycopodium powder on the surface of water in a large trough. Pour n drops of this solution of volume v each on the surface of water in the trough.

Volume of n drops of the solution $= n\,v$

$$\text{Volume of oleic acid in this solution} = \frac{n\,v}{500} \qquad ...(11)$$

(*iv*) This solution of oleic acid spreads very fast on the surface of water and forms a roughly circular thin film of thickness t, say. The alcohol evaporates and oleic acid layer is left on the surface of water.

(*v*) Find the area A occupied by the oleic acid layer using a microscope.

$$\text{Volume of oleic acid in the layer} = A \times t \qquad ...(12)$$

$$\text{From (4) and (5),} \quad A \times t = \frac{n\,v}{500} \quad \text{or} \quad \boxed{t = \frac{n\,v}{500\,A}} \qquad ...(13)$$

Hence, t can be calculated. If the thin layer is assumed to be one molecule thick, the value of t would give us the size (*i.e.*, diameter) of a molecule of oleic acid. The value of t comes out to be of the order of 10^{-9} m.

DO YOU KNOW ?

To measure very small sizes like that of a molecule (10^{-8} m to 10^{-10} m), we cannot use *an optical microscope*. This is because the optical microscope uses visible light of wavelength ranging from 4×10^{-7} m to 8×10^{-7} m and it cannot resolve particles with sizes smaller than this.

We shall study later that electrons can also behave as waves of wavelength of the order of $0{\cdot}6 \times 10^{-10}$ m *An electron microscope* uses electron beams which can be focussed by properly designed electric and magnetic fields. Such microscopes can almost resolve atoms and molecules in a material. Recently, *tunnelling microscopy* has been developed which has still better resolution.

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Sample Problem **A drop of olive oil of diameter $0{\cdot}6 \times 10^{-3}$ m spreads into a circular film of radius 12 cm. Estimate the molecular size of olive oil.**

Sol. Here, radius of oil drop $= r = \dfrac{0{\cdot}6\times10^{-3}}{2}$ m $= 0{\cdot}3 \times 10^{-3}$ m

$$R = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$$

$$\text{Thickness of oil film} = \frac{\text{Volume of oil drop}}{\text{Area of film}} = \frac{\frac{4}{3}\pi r^3}{\pi R^2}$$

$$t = \frac{\frac{4}{3}\pi(0{\cdot}3\times10^{-3})^3}{\pi R^2} = \frac{4\,(0{\cdot}3\times10^{-3})^3}{3\times(12\times10^{-2})^2} = \frac{4\times0{\cdot}027\times10^{-5}}{3\times12\times12}\text{ m} = 2{\cdot}5\times10^{-9}\text{ m}$$

If we assume that the film is one molecule thick, then molecular size of oleic acid $= \mathbf{2{\cdot}5 \times 10^{-9}}$ **m**

2.19. RANGE OF VARIATION OF MASS

The masses of objects which we shall study in this course vary over a very wide range. These may vary from tiny mass of electron ($\approx 10^{-30}$ kg) to huge mass of the known universe ($\approx 10^{55}$ kg).

The order of masses of various objects is shown in Table 2.6.

TABLE 2.6. Range of masses

S.No.	Object	Order of mass (kg)
1.	Electron	10^{-30}
2.	Proton or Neutron	10^{-27}
3.	Uranium atom	10^{-24}
4.	Red blood corpuscle	10^{-14}
5.	A cell	10^{-10}
6.	Dust particle	10^{-9}
7.	Rain drop	10^{-6}
8.	Mosquito	10^{-5}
9.	Grape	10^{-3}
10.	Frog	10^{-1}
11.	Human	10^{2}
12.	Car	10^{3}
13.	Elephant	10^{3}
14.	Ship	10^{5}
15.	Boeing 747 air craft	10^{8}
16.	Moon	10^{23}
17.	Earth	10^{25}
18.	Sun	10^{30}
19.	Milky way	10^{42}
20.	Observable Universe	10^{55}

2.20. MEASUREMENT OF MASS

As is known, mass of a body is the quantity of matter in the body, which does not depend upon temperature, pressure or location of the body in space.

We have studied already in Art. 2.3 that mass is of two types : inertial mass and gravitational mass. We shall now study how the two types of masses are measured.

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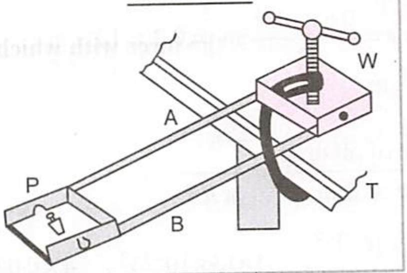
MEASUREMENT OF INERTIAL MASS

Inertial mass of a body is measured using an *inertial balance*. It consists of a block of wood (*W*) and a pan (*P*) joined with the help of two flat metallic strips *A* and *B* as shown in Fig. 2.12. The wooden block is clamped firmly to the top of a table (*T*). Any body whose inertial mass is to be found can be put in the pan. When the pan is displaced slightly in the *horizontal direction*, it executes vibratory motion.

FIGURE 2.12



The time period of vibration may depend upon length of inertial balance, coefficient of elasticity of the material of the strips and total mass of the pan and the body placed on it.

DO YOU KNOW ?

For measuring large masses like that of planets, stars etc., we make use of **gravitational methods** (See chapter 8, for details).

For measurement of small masses of atomic/subatomic particles etc., we make use of *a* **mass spectro-graph.** In mass spectrograph, the radius of the path followed by a charged particle moving in uniform electric and magnetic field is proportional to the mass of the particle. Such masses are measured in atomic mass units.

It is found that the *square of time period of vibration is directly proportional to total mass of the pan and the body placed on it*

i.e., $t^2 \propto m \quad \therefore \quad \frac{t_2^2}{t_1^2} = \frac{m_2}{m_1}$ or $\boxed{m_2 = m_1 \frac{t_2^2}{t_1^2}}$.

If inertial mass of one of the objects is known, *i.e.*, m_1 is known, we can calculate inertial mass m_2 of another object by measuring the periods of vibration in the two cases.

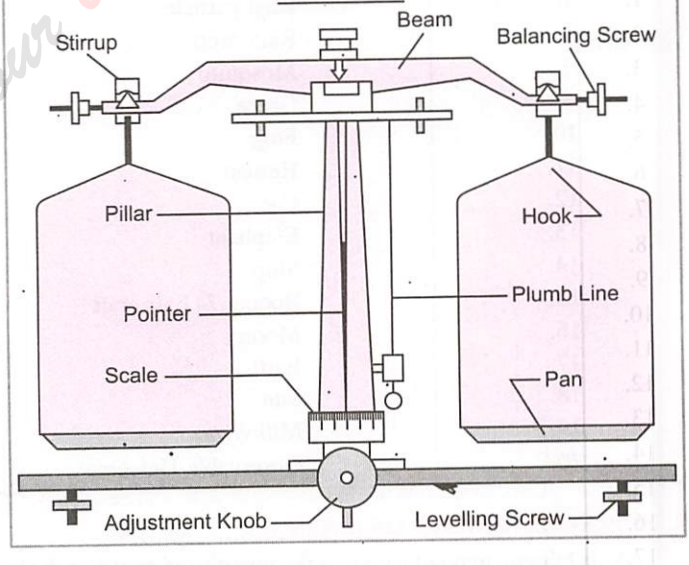
MEASUREMENT OF GRAVITATIONAL MASS

The gravitational mass of a body is measured using a *physical balance* or a *common balance*, with which most of us are familiar.

A common balance is based on the **principle of moments of forces.** According to this principle, *when a beam is in equilibrium, the algebraic sum of moments of all the forces about a fixed point is always zero.*

The essential parts of a physical balance are shown in Fig. 2.13.

FIGURE 2.13



To measure the gravitational mass of a body, we proceed in the following steps :

(*i*) Adjust the positions of balancing screws so that when the lever is turned on, the pointer vibrates equally about the central zero mark on the ivory scale, when both the pans are empty. The side to which pointer has a larger swing is lighter. For adjustment, balancing screw on the lighter side has to be moved outwards and vice-versa.

(*ii*) The body to be weighed is put in the left pan and standard weights are put in the right pan. The forceps are often used to handle the weights. The weights are adjusted till the beam is horizontal as is indicated by equal swings of the pointer on either side of the central zero mark on ivory scale.

At this stage, gravitational force on the body is equal to the gravitational force on the standard weights. Hence gravitational mass of the body will be equal to that of the standard weights.

MEASUREMENT OF WEIGHT

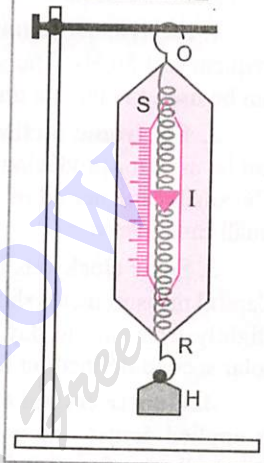
We know from chapter 2 that **weight of a body is the force with which the body is attracted towards the centre of earth.** It is equal to the product of mass of the body and acceleration due to gravity (*g*) of earth on the body. As the value of *g* on moon is less, a body will weigh less on moon than it weighs on earth. In unit 6, we shall study further, the points of distinction between mass and weight of a body.

To measure the weight of a body, we make use of a **spring balance** or a platform balance or a chemical balance.

A spring balance is based on **Hook's law of elasticity.** According to this law, within elastic limits, the extension produced in a spring is directly proportional to the force applied on the spring. Initially, we use standard (known) weights and calibrate the elongation of the spring in terms of these weights.

A spring balance consists of a helical steel spring *S*. The upper end of the spring is fixed at *O* and a hook *H* is attached to the lower end, with the help of a metallic rod *R*, Fig. 2.15. The spring and rod are enclosed in a metallic cover with a vertical slit. An indicator *I* attached to the spring moves in the slit, when a body is attached to the hook. The weight of the body can be read from the position of indicator on the scale graduated along the slit.

FIGURE 2.14



2.21. RANGE OF VARIATION OF TIME

The time interval of events which we shall come across in Physics vary over a very wide range. Table 2.7 shows the order of time intervals of some important events.

TABLE 2.7. Order of time intervals

S. No.	Event	Order of time interval (s)
1.	Life span of most unstable particle	10^{-24}
2.	Time taken by light to cross a distance of nuclear size	10^{-22}
3.	Period of X-rays	10^{-19}
4.	Time period of electron in hydrogen atom	10^{-15}
5.	Period of visible light waves	10^{-15}
6.	Time taken by visible light to cross through a window pane	10^{-11}
7.	Life time of an excited state of an atom	10^{-8}
8.	Period of radiowaves	10^{-6}
9.	Time period of audible sound waves	10^{-3}
10.	Wink of eye	10^{-1}
11.	Time interval between two successive heart beats	10^{0}
12.	Travel time of light from moon to earth	10^{0}
13.	Travel time of light from sun to earth	10^{2}
14.	Half life of a free neutron	10^{3}
15.	Time period of a satellite	10^{4}
16.	Time period of rotation of earth around its axis (one day)	10^{5}
17.	Time period of revolution of earth around the sun (one year)	10^{7}
18.	Average life of a human being	10^{9}
19.	Age of Egyptian pyramids	10^{11}
20.	Age of universe	10^{17}

2.22. MEASUREMENT OF TIME INTERVALS

To measure any time interval, we need a clock. Any phenomenon that repeats itself regularly can serve as a clock.

The Italian Physicist Galileo was the first to attempt a simple method of measuring time. Galileo timed the oscillations of a lamp hanging on a long chain in a church using his own pulse as a watch. Some of the clocks developed later are :

1. Electric oscillators. The main a.c. supply in our country has a frequency of 50 Hz. The synchronous rotations of a motor run on a.c. can be used to obtain a time scale.

2. Electronic oscillators. Vacuum tubes or junction transistors can be used for producing electromagnetic waves of high frequencies. The small time period of such oscillations can be used for measuring small time intervals.

3. Solar clock was based on revolution of earth around the sun. Careful measurements show that the period of rotation of earth changes slightly from day to day, year to year. Therefore, accuracy of mean solar second defined on this basis becomes questionable.

DO YOU KNOW ?

From Table 2.5, the ratio of the longest and shortest sizes of objects in our universe is about 10^{41}. And from Table 2.7, the ratio of longest and shortest time intervals associated with the events in our universe is also about 10^{41}. Is this magic number 10^{41} purely accidental ?

4. Quartz crystal clock. A quartz crystal shows *piezo electric effect*, *i.e.*, when fluctuating pressure is applied across one pair of faces of the crystal, an oscillating e.m.f. develops across another pair of perpendicular faces and vice-versa. The oscillations so produced are used for measuring time. Quartz crystal clocks have an accuracy of 1 sec in every 10^9 sec.

5. Atomic Clock. It is an atomic standard of time, which is based on the periodic vibrations produced in a cesium atom. The atomic clock is, therefore, also called **cesium clock.**

The efficient cesium atomic clocks are so accurate that they impart uncertainty in time realisation as ± (1×10^{-13}) second, *i.e.*, 1 part in 10^{13} second. It means that such clocks gain or lose no more than 3 μs in one year.

The Indian Standard Time (IST) is maintained by the National Physical Laboratory (NPL) situated at New Delhi, using a cesium atomic clock.

6. Decay of elementary particles using photographic emulsion techniques has led to measurement of time intervals ranging from 10^{-16} sec. to 10^{-24} sec.

7. Radioactive dating technique is used for measuring long time intervals of the order of 10^{17} sec.

DO YOU KNOW ?

The atomic clock NIST-F1 is considered one of the world's most accurate clocks. It keeps time with an accuracy of about one second in twenty million years.

Age of fossils, rocks, earth etc. is estimated by studying the ratio of number of radioactive atoms decayed to the number of surviving atoms in the specimen.

Curiosity Questions

Q. 1. Give one example where precise measurement of length is important.

Ans. Precise measurements of length are very important. For example, parts used in high performance car engines must be measured to an accuracy of 7 μm, *i.e.*, 0·000007 metre.

Q. 2. From where do we get highly accurate time signals in U.S. ?

Ans. Highly accurate time signals are obtained from the U.S. Naval Observatory Master Clock. When we have a radio capable of receiving 5, 10 or 15 MHz signals, we can pick up co-ordinated universal time signals coming from National Institute of Standards and Technology (NIST) station in Boulder, Colorado, USA.

2.23. DIMENSIONS OF A PHYSICAL QUANTITY

The seven fundamental or base quantities chosen in SI are called seven dimensions of the physical world. They are denoted with square brackets [ ]. Thus, length is represented by $[L]$, mass by $[M]$, time by $[T]$, electric current by $[A]$, thermodynamical temperature by $[K]$, luminous intensity by $[Cd]$ and amount of substance by [mol].

> ***The dimensions** of a physical quantity are the powers (or exponents) to which the units of base quantities are raised to represent a derived unit of that quantity.*

Note that using the square brackets [ ] round a quantity means that we are dealing with the dimensions of the quantity.

In mechanics, all the physical quantities can be written in terms of the dimensions of $[L]$, $[M]$ and $[T]$. For example :

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height} \quad \therefore \quad \text{Volume} = [L] \times [L] \times [L] = [L^3]$$

Thus to represent volume, we have to raise $[L]$ to the power 3. Therefore, volume is said to have *three* dimensions in length.

As units of mass and time are *not required* in representing volume, we write

Volume $= [M^0 L^3 T^0]$, and say that volume has zero dimension in mass and zero dimension in time in addition to 3 dimensions in length.

Similarly, for velocity, we write, $\quad \text{Velocity} = \dfrac{\text{displacement}}{\text{time}} = \dfrac{[L]}{[T]}$

$$\boxed{\text{Velocity} = [L^1 T^{-1}] = [M^0 L^1 T^{-1}]}$$

Hence, the dimensions of velocity are : zero in mass, +1 in length and – 1 in time.

Note that in dimensional representation, the magnitudes are not considered. It is the quality of the physical quantity that matters. For example, speed, velocity, initial velocity, final velocity, change in velocity, instantaneous velocity, relative velocity – all are equivalent in this context having dimensions $[M^0 L^1 T^{-1}]$.

2.24. DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

> ***The expression which shows how and which of the base quantities represent a physical quantity is called the dimensional formula of the given physical quantity.***

For example, as deduced above, $[M^0 L^1 T^{-1}]$ is the dimensional formula of velocity. It reveals that unit of velocity depends on $[L]$ and $[T]$. It does not depend on $[M]$. Further, unit of velocity varies directly as unit of length and inversely as unit of time.

Similarly, $[M^0 L^3 T^0]$ is dimensional formula of volume.

Further, if we represent velocity by $[v]$, then $[v] = [M^0 L^1 T^{-1}]$

is called *dimensional equation* of velocity. Thus,

> ***When a physical quantity is equated to its dimensional formula, what we obtain is the dimensional equation of the physical quantity.***

Similarly, dimensional equation of volume (V) is represented as $[V] = [M^0 L^3 T^0]$

2.25. DIMENSIONAL FORMULAE/EQUATIONS OF SOME QUANTITIES

As is known, acceleration $= \dfrac{\text{change in velocity}}{\text{time taken}} = \dfrac{L/T}{T} = LT^{-2} = [M^0 L^1 T^{-2}]$.

This is dimensional formula of acceleration.

Again, force = mass × acceleration, therefore, $F = [M] \times [L^1 T^{-2}] = [M^1 L^1 T^{-2}]$, and so on

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The dimensional formulae of some of the important mechanical quantities are derived below and are listed in Table 2.8. The SI units of all these quantities are also given in the table.

TABLE 2.8. **Dimensional formulae and SI units of some mechanical Quantities**

S.No.	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
1.	*Area*	length × breadth	$L \times L = L^2 = [M^0 L^2 T^0]$	m^2
2.	*Volume*	length × breadth × height	$L \times L \times L = L^3 = [M^0 L^3 T^0]$	m^3
3.	*Density*	$\dfrac{\text{mass}}{\text{volume}}$	$\dfrac{M}{L^3} = [M^1 L^{-3} T^0]$,	kg m^{-3}
4.	*Specific gravity*	$\dfrac{\text{density of body}}{\text{density of water at 4°C}}$	$\dfrac{M/L^3}{M/L^3} = 1 = [M^0 L^0 T^0]$ → no dimensions	*No units*
5.	*Speed or velocity*	$\dfrac{\text{distance or displacement}}{\text{time}}$	$\dfrac{L}{T} = LT^{-1} = [M^0 L^1 T^{-1}]$	ms^{-1}
6.	*Linear momentum*	mass × velocity	$M \times LT^{-1} = [M^1 L^1 T^{-1}]$	kg ms^{-1}
7.	*Acceleration*	$\dfrac{\text{change in velocity}}{\text{time taken}}$	$\dfrac{L/T}{T} = LT^{-2} = [M^0 L^1 T^{-2}]$	ms^{-2}
8.	*Acceleration due to gravity* (*g*)	$\dfrac{\text{change in velocity}}{\text{time taken}}$	$\dfrac{L/T}{T} = LT^{-2} = [M^0 L^1 T^{-2}]$	ms^{-2}
9.	*Force*	mass × acc.	$M \times LT^{-2} = [M^1 L^1 T^{-2}]$	*N* (newton)
10.	*Impulse*	force × time	$MLT^{-2} \times T = [M^1 L^1 T^{-1}]$	*Ns*
11.	*Pressure*	force/area	$\dfrac{MLT^{-2}}{L^2} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
12.	*Universal constant of gravitation* (*G*)	From Newton's law of gravitation. $F = \dfrac{Gm_1 m_2}{r^2}$ or $G = \dfrac{F r^2}{m_1 m_2}$, where *F* is force between masses m_1, m_2 at a distance *r*	$G = \dfrac{(MLT^{-2}) L^2}{MM}$ $= [M^{-1} L^3 T^{-2}]$	$Nm^2\ kg^{-2}$
13.	*Work*	force × distance	$MLT^{-2} \times L = [M^1 L^2 T^{-2}]$	*J* (joule)
14.	*Energy* (All types)	work	$[M^1 L^2 T^{-2}]$	*J* (joule)
15.	*Moment of force*	force × distance	$MLT^{-2} \times L = [M^1 L^2 T^{-2}]$	*N-m*
16.	*Power*	$\dfrac{\text{work}}{\text{time}}$	$\dfrac{ML^2 T^{-2}}{T} = [M^1 L^2 T^{-3}]$	*W* (watt)
17.	*Surface tension*	$\dfrac{\text{force}}{\text{length}}$	$\dfrac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$	Nm^{-1}
18.	*Surface energy*	Energy of free surface	$[M^1 L^2 T^{-2}]$	*J*
19.	*Force constant*	$\dfrac{\text{force}}{\text{displacement}}$	$\dfrac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$	Nm^{-1}

...contd.

20.	*Thrust*	force	$[M^1 L^1 T^{-2}]$	N (newton)
21.	*Tension*	force	$[M^1 L^1 T^{-2}]$	N (newton)
22.	*Stress, Pressure*	$\dfrac{\text{force}}{\text{area}}$	$\dfrac{MLT^{-2}}{L^2} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
23.	*Strain*	$\dfrac{\text{change in dimension}}{\text{original dimension}}$	$\dfrac{L}{L} = 1 = [M^0 L^0 T^0]$	*No units*
24.	*Coefficient of elasticity*	$\dfrac{\text{stress}}{\text{strain}}$	$\dfrac{M^1 L^{-1} T^{-2}}{1} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
25.	*Radius of gyration* (*K*)	distance	$L = [M^0 L^1 T^0]$	m
26.	*Moment of inertia* (*I*)	mass (radius of gyration)^2	$ML^2 = [M^1 L^2 T^0]$	$kg\ m^2$
27.	*Angle* (θ) or *Angular displacement* (θ)	$\dfrac{\text{length}\,(l)}{\text{radius}\,(r)}$	$\dfrac{L}{L} = 1 = [M^0 L^0 T^0]$	*radian*
28.	*Angular velocity* (ω)	$\dfrac{\text{angle}\,(\theta)}{\text{time}\,(t)}$	$\dfrac{1}{T} = T^{-1} = [M^0 L^0 T^{-1}]$	$rad\ s^{-1}$
29.	*Angular acc.* (α)	$\dfrac{\text{change in angular vel.}}{\text{time taken}}$	$\dfrac{1/T}{T} = T^{-2} = [M^0 L^0 T^{-2}]$	$rad\ s^{-2}$
30.	*Angular momentum*	$I\,\omega$	$(ML^2)\,(T^{-1}) = [M^1 L^2 T^{-1}]$	$kg\ m^2\ s^{-1}$
31.	*Torque*	$I\,\alpha$	$(ML^2)\,(T^{-2}) = [M^1 L^2 T^{-2}]$	N-m
32.	*Wavelength* (λ)	length of one wave, *i.e.*, distance	$L = [M^0 L^1 T^0]$	m
33.	*Frequency* (ν)	number of vibrations/sec	$1/T = T^{-1} = [M^0 L^0 T^{-1}]$	s^{-1} or *Hz* (*hertz*)
34.	*Angular frequency* (ω)	$2\pi \times$ frequency	$T^{-1} = [M^0 L^0 T^{-1}]$	*radian*/sec
35.	*Velocity of light in vacuum* (*c*)	$\dfrac{\text{distance travelled}}{\text{time taken}}$	$\dfrac{L}{T} = [M^0 L^1 T^{-1}]$	ms^{-1}
36.	*Velocity gradient*	$\dfrac{\text{velocity}}{\text{distance}}$	$\dfrac{LT^{-1}}{L} = T^{-1} = [M^0 L^0 T^{-1}]$	s^{-1}
37.	*Rate of flow*	$\dfrac{\text{volume}}{\text{time}}$	$\dfrac{L^3}{T} = L^3 T^{-1} = [M^0 L^3 T^{-1}]$	$m^3\ s^{-1}$
38.	*Planck's constant* (*h*)	$\dfrac{\text{energy (E)}}{\text{frequency}\,(\nu)}$	$\dfrac{ML^2 T^{-2}}{T^{-1}} = [M^1 L^2 T^{-1}]$	J-s
39.	*Linear mass density* (*m*)	$\dfrac{\text{mass}}{\text{length}}$	$\dfrac{M}{L} = [M^1 L^{-1} T^0]$	$kg\ m^{-1}$
40.	*Distance travelled in nth second*	$\dfrac{\text{distance}}{\text{time}}$	$\dfrac{L}{T} = [M^0 L^1 T^{-1}]$	ms^{-1}

...contd.

41.	***Avogadro's number (N)***	Number of atoms/ molecules in one gram atom/mole	$[M^0 L^0 T^0]$	*mole*$^{-1}$
42.	***Reynold number (N_R)***	$\frac{\rho D v}{\eta}$	$\frac{(ML^{-3})(L)(LT^{-1})}{(ML^{-1}T^{-1})} = [M^0 L^0 T^0]$	*No units*
43.	***Rydberg constant (R)***	$\frac{2\pi^2 m K^2 e^4}{ch^3}$	$[M^0 L^{-1} T^0]$	m^{-1}
44.	***Coefficient of viscosity (η)***	$\frac{F \times r}{A \times v}$	$[M^1 L^{-1} T^{-1}]$	*Pascal-second*
45.	***Kinematic viscosity***	square metre per second	$[M^0 L^2 T^{-1}]$	$m^2 s^{-1}$
46.	***Specific volume***	cubic metre per kg	$[M^{-1} L^3 T^0]$	$m^3 kg^{-1}$
47.	***Surface potential***	joule per kilogram	$[M^0 L^2 T^{-2}]$	$m^2 s^{-2}$
48.	***Refractive index (μ)***	$\mu = \frac{c}{v}$	$[M^0 L^0 T^0]$	*No units*
49.	***Kinetic energy***	$\frac{1}{2}mv^2$	$M(LT^{-1})^2 = [ML^2 T^{-2}]$	*J (joule)*
50.	***Potential energy***	mgh	$M(LT^{-2})\,L = [ML^2 T^{-2}]$	*J* (joule)
51.	***Rotational kinetic energy***	$\frac{1}{2}I\omega^2$	$(ML^2)(T^{-1})^2 = [ML^2 T^{-2}]$	*J* (joule)
52.	***Pressure energy***	Pressure × volume	$(ML^{-1}T^{-2})(L^3) = [ML^2 T^{-2}]$	*J* (joule)
53.	***Pressure gradient***	$\frac{\text{pressure}}{\text{distance}}$	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$	$N\,m^{-3}$
54.	***Trigonometric ratio (sin θ, cos θ, tan θ etc.)***	$\frac{\text{length}}{\text{length}}$	$\frac{L}{L} = 1 = [M^0 L^0 T^0]$	*No units*
55.	***Moment of force, moment of couple***	force × distance	$(MLT^{-2}) \times L = [M^1 L^2 T^{-2}]$	$N\,m$
56.	***Escape velocity***	$\sqrt{2gR}$	$\sqrt{LT^{-2}L} = LT^{-1} = [M^0 L^1 T^{-1}]$	ms^{-1}
57.	***Critical velocity***	$\frac{N_R\,\eta}{\rho D}$, where N_R is Reynold number	$\frac{(M^0 L^0 T^0)(ML^{-1}T^{-1})}{(ML^{-3})(L)} = [M^0 LT^{-1}]$	ms^{-1}
58.	***Energy density***	$\frac{\text{Energy}}{\text{Volume}}$	$\frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$	$J\,m^{-3}$
59.	***Intensity of wave***	energy/sec/area	$\frac{ML^2T^{-2}}{T.L^2} = [ML^0T^{-3}]$	$W\,m^{-2}$

...contd.

60.	***Radiation pressure***	$\frac{\text{Intensity of wave}}{\text{speed of light}}$	$\frac{ML^0T^{-3}}{LT^{-1}}=[ML^{-1}T^{-2}]$	$N\,m^{-2}$
61.	***Hubble constant***	$\frac{\text{Recession speed}}{\text{distance}}$	$\frac{LT^{-1}}{L}=T^{-1}=[M^0L^0T^{-1}]$	sec^{-1}
62.	***Efficiency***	$\frac{\text{output energy}}{\text{input energy}}$	$\frac{ML^2T^{-2}}{ML^2T^{-2}}=[M^0L^0T^0]$	*No units*
63.	***Angular impulse***	torque × time	$(ML^2T^{-2})(T)=[M^1L^2T^{-1}]$	$kg\,m^2\,s^{-1}$
64.	***Centripetal acceleration***	$\frac{(\text{velocity})^2}{\text{radius}}$	$\frac{(LT^{-1})^2}{L}=LT^{-2}=[M^0L^1T^{-2}]$	ms^{-2}
65.	***Wave number (k)***	$\frac{2\pi}{\lambda}$	$\frac{1}{L}=L^{-1}=[M^0L^{-1}T^0]$	m^{-1}
66.	***Mass defect (Δm)***	Sum of masses of nucleons – mass of nucleus	$[M^1L^0T^0]$	*kg*
67.	***Binding energy of nucleus***	(mass defect) × (speed of light)^2	$M(LT^{-1})^2=[ML^2T^{-2}]$	*J*

In Table 2.9, we have derived and listed the dimensional formulae and SI units of some of the important non-mechanical quantities.

TABLE 2.9 Dimensional Formulae and SI units of some Thermal Quantities

S.No.	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
1.	***Heat*** **(Q)**	Energy	$[M^1L^2T^{-2}]$	*J* (joule)
2.	***Temperature*** **(θ)**	—	$[M^0L^0T^0K^1]$	*K*
3.	***Latent heat*** **(L)**	$\frac{\text{heat (Q)}}{\text{mass (m)}}$	$\frac{ML^2T^{-2}}{M}=[M^0L^2T^{-2}]$	$J\,kg^{-1}$
4.	***Specific heat capacity (s)***	$\frac{Q}{m\theta}$	$\frac{ML^2T^{-2}}{MK}=[M^0L^2T^{-2}K^{-1}]$	$J\,kg^{-1}\,K^{-1}$
5.	***Internal energy (U)***	Energy	$[M^1L^2T^{-2}]$	*J*
6.	***Mechanical equivalent of heat (J)***	$\frac{W}{Q}$	$\frac{[M^1L^2T^{-2}]}{[M^1L^2T^{-2}]}=[M^0L^0T^0]$	$J\,cal^{-1}$
7.	***Gas constant (R)***	$\frac{PV}{T}$	$\frac{(ML^{-1}T^{-2})L^3}{K}=[M^1L^2T^{-2}K^{-1}]$	JK^{-1}

...contd.

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8.	***Boltzmann constant (k)***	$\frac{R}{N}$, where N is Avogadro's number	$[M^1 L^2 T^{-2} K^{-1}]$	JK^{-1}
9.	***Coefficient of thermal conductivity (K)***	From $\frac{\Delta Q}{\Delta t} = KA\left(\frac{\Delta T}{\Delta x}\right)$ $K = \frac{\Delta Q}{\Delta t}\left(\frac{\Delta x}{\Delta T}\right)\times\frac{1}{A}$	$\frac{(ML^2T^{-2})(L)}{(T)(K)L^2} = [M^1 L^1 T^{-3} K^{-1}]$	$Wm^{-1} K^{-1}$
10.	***Wein's constant (b)***	$b = \lambda_m \times T$	$[M^0 L^1 T^0 K^1]$	$m\ K$
11.	***Stefan's constant***	$\sigma = \frac{E}{T^4}$ where E is energy/sec/area	$\frac{ML^2T^{-2}}{TL^2K^4} = [M^1 L^0 T^{-3} K^{-4}]$	$W\,m^{-2} K^{-4}$
12.	***Heat capacity or entropy***	joule per kelvin	$ML^2 T^{-2} K^{-1}$	JK^{-1}
13.	***Coefficient of thermal expansivity***	$\frac{\text{change in dimension}}{\text{original dimension} \times \text{temp.}}$	$\frac{L}{L(K)} = K^{-1} = [M^0L^0T^0K^{-1}]$	K^{-1}

TABLE 2.10. Dimensional Formulae and SI units of some Electrical Quantities

S.No.	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
1.	***Electric current (I)***	ampere (A) is fundamental unit of current	$[M^0 L^0 T^0 A^1]$	A (ampere)
2.	***Current density***	Current per unit area	$A/L^2 = [M^0 L^{-2} T^0 A^1]$	A/m^2
3.	***Charge (q)***	current × time	$[M^0 L^0 A^1 T^1]$	C (coulomb)
4.	***Electric potential or voltage or emf***	$\frac{\text{work}}{\text{charge}}$	$\frac{ML^2T^{-2}}{AT} = [M^1 L^2 T^{-3} A^{-1}]$	V (volt)
5.	***Electric intensity (E) or Electric field***	$= \frac{\text{Force}}{\text{Charge}}$	$= \frac{MLT^{-2}}{AT} = [M^1 L^1 T^{-3} A^{-1}]$	NC^{-1}
6.	***Electric Permittivity of free space (ϵ_0)***	From $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ $\epsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$	$\lvert\epsilon_0\rvert = \frac{\lvert q\rvert^2}{F r^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]}$ $= [M^{-1} L^{-3} T^4 A^2]$	$C^2 J^{-1} m^{-1}$
7.	***Dielectric constant (K) or Relative electric permittivity (ϵ_r)***	K or $\epsilon_r = \frac{\epsilon}{\epsilon_0}$	$= [M^0 L^0 T^0]$	*No units*

...contd.

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8.	***Coefficient of self inductance (L)***	From $e = \frac{L\,dI}{dt}$ $L = \frac{e\,dt}{dI} = \frac{(W/q)\,dt}{dI}$	$\frac{ML^2T^{-2}}{AT}\cdot\frac{T}{A} = [M^1L^2T^{-2}A^{-2}]$	H (henry)
9.	***Coefficient of mutual inductance (M)***	From $e = M\frac{dI}{dt}$ $M = \frac{e\,dt}{dI}$	$[M^1L^2T^{-2}A^{-2}]$	H (henry)
10.	***Resistance (R)***	$\frac{\text{potential diff.}}{\text{current}}$	$\frac{ML^2T^{-3}A^{-1}}{A} = [M^1L^2T^{-3}A^{-2}]$	Ω (ohm)
11.	***Capacitance (C)***	$\frac{\text{charge}}{\text{potential diff.}}$	$\frac{AT}{ML^2T^{-3}A^{-1}} = [M^{-1}L^{-2}T^4A^2]$	f (farad)
12.	***Surface density of charge***	$\sigma = \frac{\text{charge}}{\text{area}}$	$\frac{AT}{L^2} = [M^0L^{-2}T^1A^1]$	$C\,m^{-2}$
13.	***Electric dipole moment (p)***	$p = q\,(2\,a)$	$AT\,(L) = [M^0L^1T^1A^1]$	$C\,m$
14.	***Specific Resistance or resistivity (ρ)***	$\frac{Ra}{l}$	$\frac{(ML^2T^{-3}A^{-2})(L^2)}{L}$ $= [M^1L^3T^{-3}A^{-2}]$	*ohm-m*
15.	***Conductance (G)***	$\frac{1}{R}$	$\frac{1}{[ML^2T^{-3}A^{-2}]} = [M^{-1}L^{-2}T^3A^2]$	ohm^{-1}
16.	***Conductivity (σ)***	$\frac{1}{\rho}$	$\frac{1}{[M^1L^3T^{-3}A^{-2}]} = [M^{-1}L^{-3}T^3A^2]$	$ohm^{-1}\,m^{-1}$
17.	***Electric flux***	Electric field × area	$(M^1L^1T^{-3}A^{-1})\,(L^2)$ $= [M^1L^3T^{-3}A^{-1}]$	$N\,m^2\,C^{-1}$
18.	***Faraday constant***	Avogadro number × elementary charge	$\frac{1}{\text{mol}} \times AT = [M^0L^0T^1A^1\,\text{mol}^{-1}]$	C
19.	***Decay constant (λ)***	$\frac{0{\cdot}693}{\text{half life}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	sec^{-1}
20.	***Inductive reactance (X_L)***	Angular frequency × Inductance	$(T^{-1})\,(M^1L^2T^{-2}A^{-2})$ $= [M^1L^2T^{-3}A^{-2}]$	*ohm*
21.	***Capacitative reactance (X_C)***	(Angular frequency × Capacitance)$^{-1}$	$[(T^{-1})\,(M^{-1}L^{-2}T^4A^2)]^{-1}$ $= [M^1L^2T^{-3}A^{-2}]$	*ohm*
22.	***Resonant frequency***	$\frac{1}{2\pi\sqrt{\text{inductance} \times \text{capacitance}}}$	$= [(M^1L^2T^{-2}A^{-2})$ $(M^{-1}L^{-2}T^4A^2)]^{-1/2}$ $= [M^0L^0T^{-1}]$	*hertz*
23.	***Quality factor or Q factor of coil***	$\frac{\text{Resonant frequency} \times \text{inductance}}{\text{Resistance}}$	$\frac{(T^{-1})(M^1L^2T^{-2}A^{-2})}{(M^1L^2T^{-3}A^{-2})} = [M^0L^0T^0]$	*No units*

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TABLE 2.11. Dimensional Formulae and SI units of some Magnetic Quantities

S.No.	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
1.	*Magnetic flux* (φ)	From $e = \frac{d\phi}{dt}$ $d\phi = e.dt = \frac{W}{q}dt$	$\frac{[ML^2T^{-2}][T]}{AT} = [ML^2T^{-2}A^{-1}]$	*Wb* (weber)
2.	*Magnetic induction* (*B*) ; *Magnetic field* ; *Magnetic flux density*	From $F = B\,q\,v$ $B = \frac{F}{q\,v}$	$\frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [MT^{-2}A^{-1}]$	*T* (tesla)
3.	*Magnetic intensity* (*H*)	From Biot Savart law, $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$ and $B = \mu_0 H$, we get $H = \left[\frac{I\,dl}{r^2}\right]$	$= \frac{AL}{L^2} = [AL^{-1}]$	$A\,m^{-1}$
4.	*Magnetic dipole moment* (*M*)	$M = IA$	$AL^2 = [M^0L^2T^0A^1]$	$A\,m^2$
5.	*Pole strength* (*m*)	$m = \frac{M}{2l}$	$\frac{AL^2}{L} = AL = [M^0L^1T^0A^1]$	*A m*
6.	*Magnetic Permeability of free space* (μ_0)	From Coulomb's law in magnetism, $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$, where m_1, m_2 are strengths of two poles ; $[\mu_0] = \frac{4\pi[F][r^2]}{m_1 m_2}$	$\frac{[MLT^{-2}][L^2]}{[AL]^2} = [M^1L^1T^{-2}A^{-2}]$	$H\,m^{-1}$

TABLE 2.12. Dimensional Formulae and SI units of some Optical Quantities

S.No.	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
1.	*Luminous flux or luminous power*	visible light energy emitted/sec.	$\frac{[M^1L^2T^{-2}]}{T} = [M^1L^2T^{-3}]$	*lumen*
2.	*Luminance or Brightness*	Luminous flux from unit area	$\frac{M^1L^2T^{-3}}{L^2} = [M^1L^0T^{-3}]$	*lambert*

...contd.

3.	*Illuminance or intensity of illumination*	Luminous flux on unit area	$\frac{M^1 L^2 T^{-3}}{L^2} = [M^1 L^0 T^{-3}]$	*lux or* *lm/m²*
4.	*Power of lens (P)*	(focal length)$^{-1}$	$L^{-1} = [M^0 L^{-1} T^0]$	*dioptre*
5.	*Magnification (m)*	$\frac{\text{size of image}}{\text{size of object}}$	$\frac{L}{L} = 1 = [M^0 L^0 T^0]$	*No units*
6.	*Radiant flux, Radiant power*	Energy emitted/time	$\frac{M^1 L^2 T^{-2}}{T} = [M^1 L^2 T^{-3}]$	*watt*
7.	*Luminosity of radiant flux or radiant intensity*	$\frac{\text{Radiant flux}}{\text{solid angle}}$	$\frac{(M^1 L^2 T^{-3})}{(M^0 L^0 T^0)} = [M^1 L^2 T^{-3}]$	*watt/sr*
8.	*Luminous intensity or illuminating power of source*	$\frac{\text{luminous flux}}{\text{solid angle}}$	$\frac{(ML^2 T^{-3})}{(M^0 L^0 T^0)} = [M^1 L^2 T^{-3}]$	*watt/sr*
9.	*Luminous efficiency*	$\frac{\text{total luminous flux}}{\text{total radiant flux}}$	$\frac{ML^2 T^{-3}}{ML^2 T^{-3}} = [M^0 L^0 T^0]$	*No units*
10.	*Relative luminosity*	$\frac{\text{luminous flux of a source of given wavelength}}{\text{luminous flux of peak sensitivity wavelength source of same power}}$	$\frac{ML^2 T^{-3}}{ML^2 T^{-3}} = [M^0 L^0 T^0]$	*No units*

2.26. FOUR TYPES OF QUANTITIES

From the study of dimensional formulae of various quantities, we can easily divide them into four types :

1. **Dimensional constants.** These are the quantities whose values are constant and they possess dimensions. *For example,* velocity of light in vacuum, universal gravitational constant, universal gas constant, Boltzmann constant, Planck's constant etc.

2. **Dimensional variables.** These are the quantities whose values are variable, and they possess dimensions. *For example*, area, volume, density, velocity, acceleration, force etc.

3. **Dimensionless constants.** These are the quantities, whose values are constant, but they do not possess dimensions. For example, pure numbers 1, 2, 3, 4, ..., mathematical constants π and '*e*' etc.

4. **Dimensionless variables.** These are the quantities, whose values are variable, and they do not have dimensions. For example, angle, strain, specific gravity etc.

Sample Problem **Show that angular momentum has the same dimensions as the Planck's constant.**

Sol. Angular momentum = moment of momentum = momentum × distance

$$= [M LT^{-1}] \times L = [M^1 L^2 T^{-1}]$$

Planck's constant. $h = \frac{E}{\nu} = \frac{M\,L^2\,T^{-2}}{T^{-1}} = [M\,L^2\,T^{-1}]$

Hence both have the same dimensions.

2.27. USES OF DIMENSIONAL EQUATIONS

Following are the three uses of dimensional equations :

1. *Conversion of one system of units into another* ; 2. *Checking the correctness of various formulae* ; 3. *Derivation of formulae.*

Let us now discuss each advantage in detail:

2.28. CONVERSION OF ONE SYSTEM OF UNITS INTO ANOTHER

This is based on the fact that magnitude of a physical quantity remains the same, whatever be the system of its measurement, i.e.,

$$\boxed{Q = n_1\,u_1 = n_2\,u_2} \qquad \text{...(14)}$$

where u_1 and u_2 are two units of measurement of the quantity Q and n_1, n_2 are their respective numerical values.

Suppose M_1, L_1, T_1 are the fundamental units of mass, length and time in one system ; M_2, L_2, T_2 are the fundamental units of mass, length and time in the other system ; a, b, c are the respective dimensions of the quantity in mass, length and time on both the systems.

The units of measurement u_1 and u_2 of the quantity on the two systems would be

$$u_1 = [M_1^a\,L_1^b\,T_1^c] \quad \text{and} \quad u_2 = [M_2^a\,L_2^b\,T_2^c] \qquad \text{...(15)}$$

From (14), $\quad n_2 = \frac{n_1\,u_1}{u_2}$

Using (15), we get $\quad n_2 = \frac{n_1\,[M_1^a\,L_1^b\,T_1^c]}{[M_2^a\,L_2^b\,T_2^c]} \qquad \boxed{n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c} \qquad \text{...(16)}$

Knowing (M_1, L_1, T_1) ; (M_2, L_2, T_2) ; (a, b, c) and n_1, we can calculate n_2, the numerical value of the quantity in the other system.

Retain in Memory

It should be clearly understood that eqn. (16) is to be applied only after expressing the quantity in its absolute units. This is because, when M, L, T are in absolute units, the derived unit of the quantity involved will be in absolute units only. The gravitational or practical units of any quantity are obtained only by proper conversion subsequently.

The gravitational or other practical units must be converted first into absolute units before using the conversion technique of eqn. (16).

Sample Problem **Surface tension of mercury is 540 dyne/cm. What will be its value when unit of mass is 1 kg, unit of length is 1m and unit of time is 1 minute ?**

Sol. Here, $n_1 = 540$ dyne/cm, $M_1 = 1$ g, $L_1 = 1$ cm, $T_1 = 1$ sec

$n_2 = ?$, $M_2 = 1$ kg, $L_2 = 1$ m, $T_2 = 1$ min $= 60$ s

$$\text{Surface tension} = \frac{\text{force}}{\text{length}} = \frac{M\,LT^{-2}}{L} = [M^1\,L^0\,T^{-2}]$$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^0\left(\frac{T_1}{T_2}\right)^{-2} = 540\left(\frac{1\,\text{g}}{1\,\text{kg}}\right)^1 \times 1\left(\frac{1\,\text{s}}{60\,\text{s}}\right)^{-2}$$

$n_2 = 540 \times 10^{-3} \times 60 \times 60 =$ **1944 new units of surface tension**

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IMPORTANT NOTE

Sometimes, it is more convenient to carry out the conversions directly. *For example* : To convert $g = 9{\cdot}8\ \text{m/s}^2$ into km/h^2, we write

$$g = \frac{9{\cdot}8\ \text{m}}{(1\,\text{s})^2} = \frac{9{\cdot}8 \times (10^{-3}\ \text{km})}{\left(\dfrac{1}{60 \times 60}\,h\right)^2} = 9{\cdot}8 \times 10^{-3} \times 3600 \times 3600\ \text{km h}^{-2} = \mathbf{1{\cdot}27 \times 10^5\ km\ h^{-2}}$$

2.29. CHECKING THE CORRECTNESS OF FORMULAE

Just as mass cannot be added to velocity or an electric current cannot be subtracted from time, therefore, we conclude that quantities having different dimensions cannot be added or subtracted from one another. This simple fact leads us to the *principle of homogeneity of dimensions* which is used in checking the correctness of the formulae, and also in deducing relation amongst different physical quantities. According to this principle, ***only that formula is correct, in which the dimensions of various fundamental units on one side of the relation are equal to their respective dimensions on the other side of the relation.***

Retain in Memory

It should be clearly understood that powers of all the fundamental units, *i.e.*, *M*, *L*, *T*, *A*, *K* etc. must match on either side of the formula. When power of even one of them fails to match, the formula is wrong.

To check the correctness of the given relation, we shall write the dimensions of the quantities on both sides of the relation. If the principle of homogeneity of dimensions is obeyed, the formula is correct.

Sample Problem **Check the dimesional consistency of the relation $v = \frac{1}{l}\sqrt{\frac{P}{\rho}}$ where l is length, v is velocity, P is pressure and ρ is density,**

Sol. The given relation is $v = \dfrac{1}{l}\sqrt{\dfrac{P}{\rho}}$

$$\text{LHS} = v = L\,T^{-1} = [M^0\,L^1\,T^{-1}]$$

$$\text{RHS} = \frac{1}{l}\sqrt{\frac{P}{\rho}} = \frac{1}{L}\sqrt{\frac{M\,L^{-1}\,T^{-2}}{M\,L^{-3}}} = \frac{1}{L}\,(LT^{-1}) = [T^{-1}]$$

As LHS ≠ RHS, ∴ The relation is **not dimensionally consistent.**

DO YOU KNOW ?

That mere dimensional correctness of an equation does not ensure its physical correctness. For example, work = torque, is dimensionally correct, but not physically correct.

Retain in Memory

1. Only those quantities can be added or subtracted from each other which have the same dimensions. For example, distance can be added or subtracted from distance only and not from any other quantity. Thus, $L + L = L$ and $L - L = L$, *i.e.*, when one distance is subtracted from some other distance, what is left is some distance only, whatever be its magnitude. The numerical value of distance is irrelevant because we are considering dimensions only. That is why we do not write $L + L = 2\,L$ and $L - L = 0$.
2. If an equation fails this consistency test, it is proved wrong. But if the equation passes this consistency test, it is not necessarily proved right. Thus, a dimensionally correct equation may not actually be a correct equation. But a dimensionally incorrect/inconsistent equation must be wrong. For example, the equation of motion : $s = ut + at^2$ is dimensionally correct, but it is numerically wrong.

2.30. DERIVATION OF FORMULAE

Using the same principle of homogeneity of dimensions, we can derive the formula of a physical quantity, provided we know the factors on which the physical quantity depends.

We suppose the dimensions of the given physical quantity in terms of these factors, combine them to form an equation ; write the dimensions of various quantities in terms of mass, length and time on either side of the equation.

Using principle of homogeneity of dimensions, equate the powers of M, L and T on both sides of the dimensional equation. The three equations so obtained are solved to obtain the values of three unknown powers/dimensions. On substituting these values in the equation we formed, we obtain the preliminary form of the relation/formula.

Sample Problem **The velocity v of sound waves in a medium may depend upon modulus of elasticity (E), density (d) and wavelength (λ) of the waves. Use method of dimensions to derive the formula for v.**

Sol. Let $v \propto E^a \, d^b \, \lambda^c$ ...(*i*)

where K is dimensionless constant of proportionality.

Writing the dimensions on either side of (*i*), we get

$$[M^0 L^1 T^{-1}] = [M L^{-1} T^{-2}]^a \, [M L^{-3}]^b \, L^c = M^{a+b} \, L^{-a-3b+c} \, T^{-2a}$$

Applying the principle of homogeneity of dimensions, we get

$$a + b = 0 \qquad \text{...(ii)}$$

$$-a - 3b + c = 1 \; ;$$

$$-2a = -1, \; a = \frac{1}{2} \qquad \text{...(iii)}$$

From (*ii*), $b = -a = -\dfrac{1}{2}$ From (*iii*), $c = 1 + a + 3b = 1 + \dfrac{1}{2} - \dfrac{3}{2} = 0$

Put in (*i*), $v = KE^{1/2} \cdot d^{-1/2} \cdot \lambda^0 = K\sqrt{\dfrac{E}{d}}$

2.31. LIMITATIONS OF DIMENSIONAL ANALYSIS

In conversion of one system of units into another and in checking the accuracy of formulae, there are no limitations of the method of dimensions. But *in the derivation of formulae by the method of dimensions, we come across the following limitations/problems :*

1. This method gives us *no information about the dimensionless constants* in the formula, *e.g.*, 1, 2, 3 .... π, *e*, etc.

2. If a quantity depends on *more than three factors, having dimensions, the formula cannot be derived.* This is because on equating the powers of M, L and T on either side of the dimensional equation, we can obtain three equations, from which only three unknown dimensions can be calculated.

3. *We cannot derive the formulae containing trigonometrical functions, exponential functions, log functions etc., which have no dimensions.*

4. The method of dimensions cannot be used to derive an exact form of relation, when it *consists of more than one part on any side.* For example, the exact form of the formula $s = ut + \dfrac{1}{2}\, at^2$ cannot be obtained.

5. It gives no information whether a physical quantity is a scalar or a vector.

Curiosity Question

Q. Principle of homogeneity of dimensions is the consistency test for any equation. If an equation fails this test, it is proved wrong. But if the equation passes this consistency test, it is not necessarily proved right. Why ?

Ans. The consistency test for any equation is the principle of homogeneity of dimensions, *i.e.,* an equation is correct, when dimensions/powers of M, L, T on one side of the equation are equal to their respective dimensions/powers on the other side of the equation. When power of even one of M, L, T fails to match, the equation is wrong.

However, if an equation passes this consistency test, it may not still be right. This is because mere dimensional correctness of an equation does not ensure its physical correctness. For example, work = torque is dimensionally correct, but not physically correct. Work is a scalar which is equivalent to energy. Torque is a vector, which represents turning effect of a force.

2.32. ERRORS OF MEASUREMENT

Measurement is the foundation of all experimental science and technology.

The measuring process is essentially a process of comparison. To measure any physical quantity, we compare it with a standard (unit) of that quantity. No measurement is perfect as the errors involved in the process cannot be removed completely. Hence, inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value.

This difference in the true value and the measured value of a quantity is called *error of measurement.*

i.e., error in a quantity = (True value – Measured value) of the quantity.

Types of Errors

The errors in measurement can be broadly classified as

1. Systematic Errors
2. Random Errors
3. Gross Errors

1. Systematic Errors

Systematic errors are those which *occur according to a definite pattern.* These errors affect the measurement alike, *i.e.*, in the same way. ***The causes of systematic errors are known. Therefore, such errors can be minimised.***

Some of the sources of systematic errors are :

(*i*) **Instrumental errors** which arise from the errors due to imperfect design or manufacture or calibration of the measuring instrument. For example, an ordinary metre scale may be worn off at one end ; in a vernier callipers, the zero mark of vernier scale may not coincide with the zero mark of the main scale ; the temperature graduations of a thermometer may not be accurately calibrated and so on.

The instrumental errors can be reduced by using more accurate instruments and applying zero correction etc. when required.

(*ii*) **Imperfection in experimental technique or procedure.** For example, to determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature. Again, buoyancy error in weighing is usually ignored. Similarly, radiation error in experiments on heat is usually ignored.

(*iii*) **Personal errors** arise due to inexperience of the observer or defect in the eyes of observer. For example, lack of proper setting of the apparatus, reading an instrument without setting it properly ; taking observations without observing proper precautions etc.

(*iv*) **Errors due to external causes.** The external conditions such as changes in temperature, pressure, humidity, wind velocity etc. during the experiment may affect the measurements.

(*v*) **Least Count Error.** *The smallest value that can be measured by a measuring instrument is called the least count of the instrument.* For example, a metre scale having graduations at 1 mm division scale spacing has a least count of 1 mm or 0·1 cm. Similarly, a vernier calliper usually has a least count of 0·01 cm and a spherometer or a screw gauge usually has a least count of 0·001 cm. All the readings or values measured by any measuring instrument are good only upto its least count.

Thus in the measurement of length, using a metre scale, reliability is upto $\frac{1}{10}$ cm ; using a vernier calliper, reliability is upto $\frac{1}{100}$ cm ; and using a screw gauge, reliability pushes upto $\frac{1}{1000}$ cm .

Therefore, the least count error is the error associated with the resolution of the instrument. *Using instruments of higher resolution and by improving experimental techniques, we can reduce the least count error.*

Retain in Memory

Smaller the least count of a measuring device, higher is the accuracy of measurement.

Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal errors as far as possible.

For a given experimental set up, systematic errors may be estimated to a certain extent. The necessary corrections may then be applied to the observations.

2. Random Errors

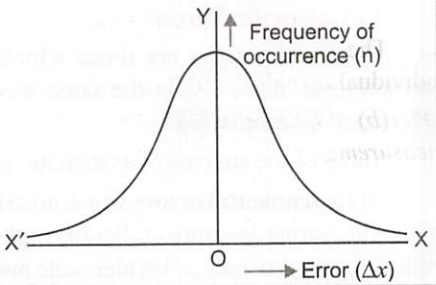
The random errors are the errors which occur irregularly due to unpredictable variations in experimental conditions. They are random with respect to sign and size, *i.e.*, sometimes, random errors may be positive and the other time, they may be negative. Similarly, sometimes, the random errors may be big and the other time, they may be small.

The random errors may arise due to random and unpredictable variations in experimental conditions *e.g.*, temperature, pressure, voltage supply, mechanical vibrations etc. These may also be due to personal errors by the observer doing the experiment.

The random errors are sometimes called the **'chance errors.'** For example, when the same person repeats the same observation, he may get different readings every time.

Random errors often follow the well known ***'Gaussian Law of Normal Distribution'***. According to this law :

FIGURE 2.15



(*i*) In any measurement x, probability of an error $(+\Delta x)$ is the same as the probability of an error $(-\Delta x)$.

(*ii*) In any measurement, small magnitudes of error are more likely than larger magnitudes of error.

The Gaussian law of Normal Distribution is represented graphically in Fig. 2.1.

From the facts stated above and nature of graph, we come to the conclusion that *arithmetic mean* of a large number of observations is likely to be closer to its 'true value' than any of the individual observations.

Hence, the random errors can be minimised by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the true value.

When number of observations is made n times, the random error reduces to $(1/n)$ time. For example, if random error in the arithmetic mean of 100 observations is x, then the random error in the arithmetic mean of 400 observations would be $x/4$.

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3. Gross Errors

These errors arise on account of shear carelessness of the observer. For example :

(*i*) Reading an instrument without setting it properly.

(*ii*) Taking the observations wrongly without caring for the sources of errors and the precautions.

(*iii*) Recording the observations wrongly.

(*iv*) Using wrong values of the observations in calculations.

These errors can be minimised only when the observer is sincere and mentally alert.

2.33. ABSOLUTE ERROR, MEAN ABSOLUTE ERROR, RELATIVE ERROR OR FRACTIONAL ERROR

(*a*) **Absolute error** *in the measurement of a physical quantity is the magnitude of the difference between the true value and the individual measured value of the quantity.*

Let a physical quantity be measured n times. Let the measured values be $a_1, a_2, a_3, ...a_n$. The arithmetic mean of these values is

$$a_m = \frac{a_1 + a_2 + ... a_n}{n} \qquad ...(19)$$

or

$$\boxed{a_m = \frac{1}{n} \sum_{i=1}^{i=n} a_i} \qquad ...(20)$$

Usually, *the arithmetic mean a_m is taken as the best possible/true value of the quantity, if the same is not known otherwise.*

By definition, **absolute errors in the individual measured values of the quantity are**

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

....

$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases. This is because individual measurements are as likely to overestimate as to underestimate the true value of the quantity.

(*b*) **Mean absolute error.** *It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity**. It is represented by Δa_{mean}. Thus,

$$\boxed{\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + ... |\Delta a_n|}{n}}$$

If $|\Delta a_i|$ is magnitude of absolute error in ith measurement, then

$$\boxed{\Delta a_{\text{mean}} = \frac{1}{n} \times \sum_{i=1}^{i=n} |\Delta a_i|} \qquad ...(21)$$

Hence, the final result of measurement may be written as $\boxed{a = a_m \pm \Delta a_{\text{mean}}}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \Delta a_{\text{mean}})$ and $(a_m - \Delta a_{\text{mean}})$.

---

*While taking arithmetic mean, we ignore the sign (±) of absolute errors.

(*c*) Relative error or Fractional error

The relative error or fractional error of measurement (δa) *is defined as the ratio of mean absolute error to the mean value of the quantity measured.* Thus,

$$\boxed{\text{Relative error or Fractional error, } \delta a = \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\Delta a_{\text{mean}}}{a_m}}$$

When the relative/fractional error is expressed in percentage, we call it **percentage error.** Thus,

$$\boxed{\text{Percentage error, } \delta a = \frac{\Delta a_{\text{mean}}}{a_m} \times 100\%}$$

IMPORTANT NOTE

1. Unit of absolute error is same as that of the quantity being measured.
2. Accuracy of measurement is expressed either as relative error or fractional error or percentage error.

Sample Problem **The refractive index of water is found to have the values 1·29, 1·33, 1·34, 1·35, 1·32, 1·36, 1·30 and 1·33. Calculate the mean value, absolute error, the relative error and the percentage error.**

Sol. Here, mean value of refractive index, $\mu = \frac{1{\cdot}29 + 1{\cdot}33 + 1{\cdot}34 + 1{\cdot}35 + 1{\cdot}32 + 1{\cdot}36 + 1{\cdot}30 + 1{\cdot}33}{8}$

$\mu = 1{\cdot}3275 = \mathbf{1{\cdot}33}$ (rounded off to two places of decimal).

Absolute errors in measurment are :

$\Delta\mu_1 = 1{\cdot}33 - 1{\cdot}29 = 0{\cdot}04$ $\quad\Delta\mu_2 = 1{\cdot}33 - 1{\cdot}33 = 0{\cdot}00$ $\quad\Delta\mu_3 = 1{\cdot}33 - 1{\cdot}34 = -\,0{\cdot}01$

$\Delta\mu_4 = 1{\cdot}33 - 1{\cdot}35 = -\,0{\cdot}02$ $\quad\Delta\mu_5 = 1{\cdot}33 - 1{\cdot}32 = +\,0{\cdot}01$ $\quad\Delta\mu_6 = 1{\cdot}33 - 1{\cdot}36 = -\,0{\cdot}03$

$\Delta\mu_7 = 1{\cdot}33 - 1{\cdot}30 = +\,0{\cdot}03$ $\quad\Delta\mu_8 = 1{\cdot}33 - 1{\cdot}33 = 0{\cdot}00.$

$$\text{Mean absolute error, } \Delta\mu_{\text{mean}} = \frac{\sum_{i=1}^{i=n} |(\Delta\mu)_i|}{n}$$

$$= \frac{0{\cdot}04 + 0{\cdot}00 + 0{\cdot}01 + 0{\cdot}02 + 0{\cdot}01 + 0{\cdot}03 + 0{\cdot}03 + 0{\cdot}00}{8}$$

$$= \frac{0{\cdot}14}{8} = 0{\cdot}0175 = \mathbf{0{\cdot}02}$$

$$\text{Relative error } = \pm\frac{\Delta\mu_{\text{mean}}}{\mu} = \pm\frac{0{\cdot}02}{1{\cdot}33} = \pm\,0{\cdot}015 = \pm\,\mathbf{0{\cdot}02}$$

Percentage error = $\pm\,0{\cdot}015 \times 100 = \pm\,\mathbf{1{\cdot}5\%}$

2.34. PROPAGATION OR COMBINATION OF ERRORS

As is known, the result of an experiment is calculated by performing mathematical operations (like addition, subtraction, multiplication, division, raising to some power etc.) on several measurements, which have different degrees of accuracy. To calculate the net error in the result, we should study how errors propagate in different mathematical operations. We discuss below the propagation of errors in the five different mathematical operations :

(A) ERROR IN A SUM

Suppose $x = a + b$ ...(22)

Let Δa = absolute error in measurement of a, Δb = absolute error in measurement of b

Δx = absolute error in calculation of x, *i.e.*, sum of a and b.

$\therefore$ from (22), $x \pm \Delta x = (a \pm \Delta a) + (b \pm \Delta b) = (a + b) \pm \Delta a \pm \Delta b = x \pm \Delta a \pm \Delta b$ or $\pm \Delta x = \pm \Delta a \pm \Delta b$

The four possible values of Δx are $(+\Delta a + \Delta b)$; $(+\Delta a - \Delta b)$; $(-\Delta a + \Delta b)$; $(-\Delta a - \Delta b)$

Therefore, the maximum absolute error in x is $\boxed{\Delta x = \pm(\Delta a + \Delta b)}$

Hence, ***maximum absolute error in sum of the two quantities is equal to sum of the absolute errors in the individual quantities.***

(B) ERROR IN A DIFFERENCE

Let $x = a - b$ ...(23)

Let Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b

Δx = absolute error in calculation of x, *i.e.*, the difference of a and b.

From (5), $x \pm \Delta x = (a \pm \Delta a) - (b \pm \Delta b) = a \pm \Delta a - b \mp \Delta b = (a - b) \pm \Delta a \mp \Delta b$

$x \pm \Delta x = x \pm \Delta a \mp \Delta b$ or $\pm \Delta x = \pm \Delta a \mp \Delta b$

The four possible values of Δx are $(+\Delta a - \Delta b)$; $(+\Delta a + \Delta b)$; $(-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$.

Therefore, the maximum absolute error in x is $\boxed{\Delta x = \pm(\Delta a + \Delta b)}$

i.e., ***maximum absolute error in difference of two quantities is equal to sum of the absolute errors in the individual quantities.***

Sample Problem 1 **In an experiment, two capacities measured are $(1{\cdot}3 \pm 0{\cdot}1)$ μF and $(2{\cdot}4 \pm 0{\cdot}2)$ μF. Calculate the total capacity in parallel with percentage error.**

Sol. Here, $C_1 = (1{\cdot}3 \pm 0{\cdot}1)$ μF and $C_2 = (2{\cdot}4 \pm 0{\cdot}2)$ μF.

In parallel, $C_p = C_1 + C_2 = 1{\cdot}3 + 2{\cdot}4 = 3{\cdot}7$ μF, $\Delta C_p = \pm(\Delta C_1 + \Delta C_2) = \pm(0{\cdot}1 + 0{\cdot}2) = \pm 0{\cdot}3$

$$\%\text{age error} = \pm\frac{0{\cdot}3}{3{\cdot}7}\times 100 = \pm 8{\cdot}1\%$$

Hence, $C_p = $ **(3·7 ± 0·3) μF = 3·7 μF ± 8·1%**

Sample Problem 2 **The lengths of two cylinders are measured to be $l_1 = (5{\cdot}62 \pm 0{\cdot}01)$ cm and $l_2 = (4{\cdot}34 \pm 0{\cdot}02)$ cm. Calculate difference in lengths with error limits.**

Sol. Here, $l_1 = (5{\cdot}62 \pm 0{\cdot}01)$ cm, $l_2 = (4{\cdot}34 \pm 0{\cdot}02)$ cm

$l' = l_1 - l_2 = 5{\cdot}62 - 4{\cdot}34 = 1{\cdot}28$ cm., $\Delta l' = \pm(\Delta l_1 + \Delta l_2) = \pm(0{\cdot}01 + 0{\cdot}02) = \pm 0{\cdot}03$

$$\text{Percentage error} = \frac{\pm 0{\cdot}03}{1{\cdot}28}\times 100 = \pm 2{\cdot}34\%$$

Hence, difference in lengths = **(1·28 ± 0·03) cm = 1·28 cm ± 2·34%**

(C) ERROR IN A PRODUCT

Let $x = a \times b$ ...(24)

Let Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b

Δx = absolute error in calculation of x, *i.e.*, the product of a and b

$\therefore$ From (24), $(x \pm \Delta x) = (a \pm \Delta a) \times (b \pm \Delta b)$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = a\left(1 \pm \frac{\Delta a}{a}\right) b\left(1 \pm \frac{\Delta b}{b}\right) = a\,b\left(1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\,\frac{\Delta b}{b}\right)$$

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$$x\left(1\pm\frac{\Delta x}{x}\right)=x\left(1\pm\frac{\Delta a}{a}\pm\frac{\Delta b}{b}\pm\frac{\Delta a}{a}\frac{\Delta b}{b}\right)$$

or $$1\pm\frac{\Delta x}{x}=1\pm\frac{\Delta a}{a}\pm\frac{\Delta b}{b}\pm\frac{\Delta a}{a}\frac{\Delta b}{b}$$ or $$\pm\frac{\Delta x}{x}=\pm\frac{\Delta a}{a}\pm\frac{\Delta b}{b}\pm\frac{\Delta a}{a}\frac{\Delta b}{b}$$

As $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$ both, are small, their product is still smaller and can be neglected.

$$\therefore\ \pm\frac{\Delta x}{x}=\pm\frac{\Delta a}{a}\pm\frac{\Delta b}{b}$$

Four possible values of $\frac{\Delta x}{x}$ are $\left(+\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$; $\left(+\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$; $\left(-\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$.

Hence, maximum possible value of $$\boxed{\frac{\Delta x}{x}=\pm\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)}$$

Hence, ***maximum fractional error or relative error in product of quantities is equal to sum of the fractional or relative errors in the individual quantities.***

(D) ERROR IN A QUOTIENT

Let $$x=\frac{a}{b} \quad ...(25)$$

Suppose Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b

Δx = absolute error in the calculation of division of a and b

$$\therefore \text{ from (25), } x\pm\Delta x=\frac{a\pm\Delta a}{b\pm\Delta b}$$

$$x\left(1\pm\frac{\Delta x}{x}\right)=\frac{a\,(1\pm\Delta a/a)}{b\,(1\pm\Delta b/b)} \text{ or } x\left(1\pm\frac{\Delta x}{x}\right)=\frac{a}{b}\left(1\pm\frac{\Delta a}{a}\right)\left(1\pm\frac{\Delta b}{b}\right)^{-1}$$

As $\frac{\Delta b}{b}<<1$, therefore expanding Binomially, we get

$$x\left(1\pm\frac{\Delta x}{x}\right)=x\left(1\pm\frac{\Delta a}{a}\right)\left(1\mp\frac{\Delta b}{b}\right) \text{ or } 1\pm\frac{\Delta x}{x}=1\pm\frac{\Delta a}{a}\mp\frac{\Delta b}{b}\pm\frac{\Delta a}{a}\frac{\Delta b}{b} \quad ...(26)$$

As $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$, both are small, therefore their product $\left(\frac{\Delta a}{a}\right)\left(\frac{\Delta b}{b}\right)$ may be taken as negligibly small.

$$\therefore \text{ from (26), } \pm\frac{\Delta x}{x}=\pm\frac{\Delta a}{a}\mp\frac{\Delta b}{b}$$

The four possible values of $\frac{\Delta x}{x}$ are $\left(+\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$; $\left(+\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$; $\left(-\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$

Therefore, the maximum value of $$\boxed{\frac{\Delta x}{x}=\pm\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)}$$

Hence, ***the maximum value of fractional or relative error in division of quantities is equal to sum of the fractional or relative errors in the individual quantities.***

(E) ERROR IN CASE OF A MEASURED QUANTITY RAISED TO A POWER

Let
$$x = a^n/b^m = a^n\,b^{-m} \qquad \text{...(27)}$$

Suppose Δa = absolute error in the measurement of a,
Δb = absolute error in the measurement of b,
Δx = absolute error in calculation of x

Differentiating (27) w.r.t. to independent variables, we have

$$\Delta x = (n\,a^{n-1}\,\Delta a)\,b^{-m} + a^n\,(-\,m\,b^{-m-1}\,\Delta b) = \frac{a^n}{b^m}\left[\frac{n\Delta a}{a} - \frac{m\Delta b}{b}\right] = x\left[\frac{n\Delta a}{a} - \frac{m\Delta b}{b}\right]$$

or
$$\frac{\Delta x}{x} = \frac{n\Delta a}{a} - \frac{m\Delta b}{b}$$

Fractional error or relative error in x is
$$\pm\frac{\Delta x}{x} = \pm\frac{\Delta a}{a} \mp \frac{m\Delta b}{b}$$

Therefore, maximum fractional error in x is
$$\boxed{\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)} \qquad \text{...(28)}$$

It follows from (28) that, ***fractional error or relative error in a quantity raised to power (n) is n times the fractional or relative error in the individual quantity.***

As the error multiplies n times, therefore, in any formula, ***the quantity with maximum power should be measured with highest degree of accuracy, i.e., with least error.***

In general, suppose $X = \dfrac{a^l\,b^m}{c^n}$ $\quad\therefore\quad$ $\dfrac{\Delta X}{X} = l\left(\dfrac{\Delta a}{a}\right) + m\left(\dfrac{\Delta b}{b}\right) + n\left(\dfrac{\Delta c}{c}\right)$

i.e., maximum relative error or percentage error in X = (l times the relative error or percentage error in a) + (m times the relative error or percentage error in b) + (n times the relative error or percentage error in c).

IMPORTANT NOTE

If a quantity appears with a power less than one in an expression, then its contribution to error in the final result is reduced.

Sample Problem **Calculate percentage error in the determination of $g = 4\,\pi^2\,l/t^2$, when l and t are measured with ± 2% and ± 3% errors respectively.**

Sol. Here, $g = 4\,\pi^2\,l/t^2$ $\quad\therefore\quad$ $\dfrac{\Delta g}{g}\times 100 = \pm\left(\dfrac{\Delta l}{l}\times 100 + 2\,\dfrac{\Delta t}{t}\times 100\right) = \pm\,(2 \pm 2\times 3)\% = \mathbf{\pm\,8\%}$

2.35. ACCURACY AND PRECISION

The two terms : accuracy and precision are often used interchangeably, but they have specific meanings. Let us distinguish between them.

The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

Precision tells us to what resolution or limit, the quantity is measured by a measuring instrument.

Thus, precision describes the limitation of the measuring instrument.

Infact, ***precision is determined by the least count of the measuring instrument. Smaller the least count, greater is the precision.***

For example, least count of a vernier calliper is often 0·01 cm and least count of a screw gauge or spherometer is often 0·001 cm. Therefore, measurement of small length using a screw gauge or a spherometer will be more precise than the same measurement using a vernier calliper. Similarly, a screw gauge or spherometer with least count = 0·0005 cm will be more precise than the one with least count = 0·001 cm.

The accuracy in measurement may depend on several factors like personal errors ; imperfection in technique/procedure ; instrumental errors and errors due to external causes, random errors and so on. ***Poor accuracy involves errors that can often be corrected. As we reduce the errors, measurement's accuracy increases.***

Further, ***accuracy of measurement shall also depend upon the resolution of the measuring instrument.*** For example, suppose the true value of a certain length is near 3·678 cm. In one experiment, using a measuring instrument of resolution 0·1 cm, the measured value is found to be 3·5 cm. In another experiment, using a measuring instrument of greater resolution, say 0·01 cm, the length is found to be 3·38 cm. We find that the first measurement is **more accurate** as it is closer to the true value, but it has **lesser precision** (because its resolution is only 0·1 cm). On the contrary, the second measurement is less accurate, but it is more precise.

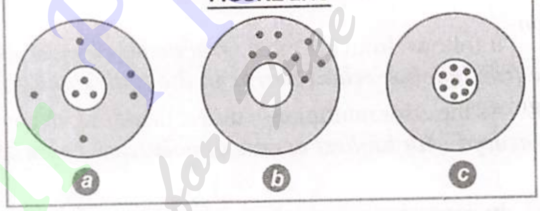
Curiosity Question

Q. Give some analogy to differentiate between accuracy and precision.

Ans. A good analogy to differentiate between accuracy and precision is the *target analogy.*

Fig. 2.16. shows three targets on a chalkboard screen. The dots represent bullet holes in the target. **Target (*a*)** shows *good accuracy and poor precision.* **Target (*b*)** shows *good precision and poor accuracy.* **Target (*c*)** represents *good accuracy and good precision.*

FIGURE 2.16



2.36. SIGNIFICANT FIGURES

We have studied above that every measurement involves some errors. Therefore, it is reasonable to report the result of any measurement in such a way that it indicates the accuracy and precision of measurement.

Normally, the degree of accuracy or precision is shown by the *number of digits* upto which the measurement is recorded.

For example: suppose the length of an object is 12·4 cm. It shows that the measurement has been made to the nearest of (1/10)th of a centimetre. It also means that the length lies between 12·35 cm and 12·45 cm. It means the digits 1 and 2 are absolutely correct but digit 4 is only approximately correct.

Again, if the length recorded is 12·47 cm, it shows that measurement has been made correctly upto $\left(\frac{1}{100}\right)$th of a cm. In this case, the digits 1, 2, 4 are reliable but the digit 7 is uncertain. The reported result of measurement in this case has four significant digits (1, 2, 4, 7) out of which three are reliable (1, 2, 4) and the last digit 7 is uncertain. Hence,

The digits beyond the significant digits reported in any result are superfluous.

> ***The digits that are known reliably plus the first uncertain digit are known as significant digits or significant figures.***

Normally, larger the number of significant figures in a measurement, higher is the accuracy of measurement, and vice-versa. For example, suppose thickness of a glass slide measured using a vernier calliper is 0·23 cm. When the same thickness is measured using a spherometer, we may obtain $t = 0{\cdot}237$ cm.

If we take last digit to be unreliable, then maximum error in $t = 0{\cdot}23$ cm is ± 0·01 cm. Percentage error in this observation $= \frac{0{\cdot}01}{0{\cdot}23} \times 100 = 4{\cdot}34\%$. Similarly, in the second observation, $t = 0{\cdot}237$ cm, max. error is ± 0·001 cm.

$\therefore$ Percentage error in second observation $= \dfrac{0{\cdot}001}{0{\cdot}237} \times 100 = 0{\cdot}422\%$

Clearly, 2nd observation is more precise than the first observation. This proves the point.

Thus stating the result of measurement that includes more digits than the significant digits is misleading as it would give a wrong idea about the precision of measurement.

2.37. COMMON RULES FOR COUNTING SIGNIFICANT FIGURES

Following are some of the common rules for counting significant figures in a reported measurement :

Rule 1. *All non zero digits are significant.*

For example : $x = 1234$ has four significant figures. Again $x = 189$ has only three significant digits.

Rule 2. *All zeros occurring inbetween two non zero digits are significant no matter where the decimal point is, if at all.*

For example : $x = 1007$ has four significant figures. Again $x = 1{\cdot}0809$ has five significant figures.

Rule 3. *If the number is less than one, the zero(s) on the right of decimal point and to the left of first non-zero digit are not significant.*

For example in $\underline{0}\cdot\underline{00}5704$, the underlined zeros are not significant. The zero between 7 and 4 is significant. The number of significant figures is 4.

Rule 4. *In a number without a decimal point, the terminal or trailing zeros are not significant.*

For example, $x = 3210$ metre has three significant figures, the trailing zero being not significant.

Rule 5. *However, in a number with decimal point, the trailing zeros are significant.* For example, $x = 3{\cdot}500$ has four significant figures. Similarly, $x = \underline{0}\cdot\underline{00}69000$ has five significant figures. The underlined zeros are not significant as per Rule 3.

Rule 6. *Change of units does not change the number of significant figures in a measurement.*

For example, the length $x = 5{\cdot}608$ cm has four significant digits. In different units, the same length can be written as $x = 56{\cdot}08$ mm, $x = 0{\cdot}05608$ m ; $x = 0{\cdot}00005608$ km ; $x = 56080$ μm. All these numbers have the same number of significant figures, namely four, the digits 5, 6, 0 and 8. This shows that *the location of decimal point is of no consequence in determining the number of significant figures.*

Rule 7. *The digit 0 conventionally, put on the left of a decimal for a number less than 1 is never significant.*

For example, $\underline{0}\cdot 3570$ has four significant digits. The underlined zero is not significant.

Rule 8. *The multiplying or dividing factors, which are neither rounded numbers nor numbers representing measured values, are exact. They have infinite number of significant digits, as per the situation. For example* in radius $(r) = \dfrac{\text{diameter}\,(d)}{2}$ and circumference, $s = 2\,\pi\, r$, the factor 2 is an exact number. It can be written as 2·0, 2·00 or 2·000 as required.

Note. Referring to Rules 4, 5 and 6, there can be some confusion regarding the terminal or trailing zeros.

Suppose a measured length is reported as $x = 4{\cdot}700$ m. Clearly, the zeros are meant to convey the precision of measurement and are therefore significant. As such, there are *four* significant figures.

Let us now change the units. We can rewrite the same length as $x = 0{\cdot}004700$ km ; $x = 470{\cdot}0$ cm ; $x = 4700$ mm.

As per Rule 4, we would erroneously conclude that $x = 4700$ mm has two significant digits. The fact is that $x = 4700$ mm has four significant figures and a mere change of units cannot change the number of significant figures (Rule 6).

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To remove such ambiguities in determining the number of significant digits, the best way is to report every measurement in scientific notation (i.e., in power of 10, often putting the decimal point after the first digit). The confusion mentioned above disappears :

$$x = 4{\cdot}700\ \text{m} = 4{\cdot}700 \times 10^2\ \text{cm} = 4{\cdot}700 \times 10^3\ \text{mm} = 4{\cdot}700 \times 10^{-3}\ \text{km}$$

In each case, the number of significant figures is 4, **as the power of 10 is irrelevant to the determination of significant figures.**

2.38. ROUNDING OFF

The result of computation with approximate numbers, which contain more than one uncertain digit, should be rounded off.

While rounding off measurements, we use the following rules by convention :

Rule 1. If *the digit to be dropped is less than 5, then the preceding digit is left unchaged.* For example, $x = 7{\cdot}82$ is rounded off to 7·8. Again, $x = 3{\cdot}94$ is rounded off to 3·9.

Rule 2. *If the digit to be dropped is more than 5, then the preceding digit is raised by one.* For example, $x = 6{\cdot}87$ is rounded off to 6·9. Again, $x = 12{\cdot}78$ is rounded off to 12·8.

Rule 3. *If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.*

For example, $x = 16{\cdot}351$ is rounded off to 16·4. Again $x = 6{\cdot}758$ is rounded off to 6·8.

Rule 4. *If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is left unchanged, if it is even.*

For example, $x = 3{\cdot}250$ becomes 3·2 on rounding off, Again $x = 12{\cdot}650$ becomes 12·6 on rounding off.

Rule 5. *If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.*

For example, $x = 3{\cdot}750$ is rounded off to 3·8. Again, $x = 16{\cdot}150$ is rounded off to 16·2.

Retain in Memory

1. For calculation, a number known accurately to many significant digits can be rounded off to an approximate value. For example, speed of light in vacuum is $c = 2{\cdot}99792458 \times 10^8$ m/s. It is rounded off to $c = 3 \times 10^8$ m/s.
2. The value of $\pi = 3{\cdot}1415926$ is known to a large number of significant figures. However, in calculations, we may take $\pi = 3{\cdot}142$ or 3·14, as per our requirement.
3. Only the final result should be rounded off using the rules stated here. Intermediate results should never be rounded off.

Sample Problem **Round off the following to two decimal places :**

(*i*) 12·3650 **(*ii*) 787·5350**

Sol. Rounding off to two decimal places as per Rule 4 and Rule 5 stated above, we get

(*i*) 12·3650 = **12·36**, and (*ii*) 787·5350 = **787·54**

2.39. ARITHMETICAL OPERATIONS WITH SIGNIFICANT FIGURES

Any result is calculated by compounding (*i.e.*, adding/subtracting/multiplying/dividing) two or more variables, which might have been measured with different degrees of accuracy. Inaccuracy in the measurement of any one variable affects the accuracy of the final result. Hence, in general, ***the final result of compounding any number of variables should not have more significant figures than the original data from which it was obtained.***

For example, suppose mass of an object is measured to be 4·237 g (four significant digits) and its volume is measured to be 2·51 cm³ (three significant figures). As density $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{4{\cdot}237}{2{\cdot}51}$

∴ density = 1·68804780876 g/cm^3

Clearly, it is absurd and irrelevant to record the calculated value of density to such a precision, when the actual measurements on which density is based have much less precision. To understand this further, let us consider a gold chain of which all links are strong except the one. The chain will obviously break at the weakest link. Thus the strength of the chain cannot be more than the strength of the weakest link in the chain.

Therefore, the results of arithmetical operations performed with measurements cannot be more accurate than the original measurments themselves.

The following rules for arithmetic operations with significant figures ensure that the final result of a calculation is shown with the precision that is consistent with the precision of the measured values :

(*a*) ADDITION AND SUBTRACTION

In both, addition and subtraction, the final result should retain as many decimal places as are there in the number with the smallest number of decimal places.

For example, the sum of three measurements of length ; 2·1 m, 1·78 m and 2·046 m is 5·926 m, which is rounded off to 5·9 m (upto smallest number of decimal places in the three measurements).

Similarly, if $x = 12{\cdot}587$ m and $y = 12{\cdot}5$ m, then $(x - y) = 12{\cdot}587 - 12{\cdot}5 = 0{\cdot}087$ m,

which is rounded off to 0·1 m, upto smallest number of decimal places in y.

Note that *in the subtraction of quantities of nearly equal magnitudes, accuracy is almost destroyed.* For example, if $x = 12{\cdot}87$ m and $y = 12{\cdot}86$ m, then $x - y = 12{\cdot}87 - 12{\cdot}86 = 0{\cdot}01$ m. The differerence has only one significant figure, whereas x and y have four significant digits each.

To avoid this, we try to **measure directly** the difference between two nearly equal quantities – instead of measuring the quantities and calculating their difference.

(*b*) MULTIPLICATION AND DIVISION

In multiplication or division, the final result should *retain as many significant figures as are there in the original number with smallest number of significant figures.*

For example, suppose $x = 3{\cdot}8$ and $y = 0{\cdot}125$. Therefore, $x\,y = (3{\cdot}8)\,(0{\cdot}125) = 0{\cdot}475$. As least number of significant figures is 2 (in $x = 3{\cdot}8$), therefore, $x\,y = 0{\cdot}475 = 0{\cdot}48$ is rounded off to two significant figures.

Again, in the example given above, when mass = 4·237 g and volume = 2·51 cm^3,

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{4{\cdot}237}{2{\cdot}51} = 1{\cdot}68804780876 \text{ g/cm}^3 = 1{\cdot}69 \text{ g/cm}^3$$

Thus, the density is reported to three significant figures only.

Sample Problem 1 **Write the results of the following with regard to significant figures.**

(i) 876 + 0·4382 (ii) 8·0 – 0·42

Sol. Here, (*i*) 876 + 0·4382 = 876·4382 = **876**

As there is no decimal point in 876 therefore, result of addition has been rounded off to no decimal point.

(*ii*) 8·0 – 0·42 = 7·58 = **7·6,** rounded off to one place of decimal, as in 8·0.

Sample Problem 2 **Compute the following with regard to significant figures**

(i) 4·6 × 0·128 (ii) $\mathbf{\dfrac{0{\cdot}9995 \times 1{\cdot}53}{1{\cdot}592}}$

Sol. Here, (*i*) 4·6 × 0·128 = 0·5888 = **0·59**

The result has been rounded off to have two significant digits (as in 4·6)

(*ii*) $\dfrac{0{\cdot}9995 \times 1{\cdot}53}{1{\cdot}592} = 0{\cdot}96057 = \mathbf{0{\cdot}961}$

The result has been rounded off to three significant digits (as in 1·53).

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2.40. PERMISSIBLE ERROR IN A RESULT

> ***The permissible error is the error which arises on account of limitations on the measuring abilities of various instruments used in an experiment.***

This error persists even when we carefully avoid gross errors and select the most accurate instruments for measurement.

For example, suppose length of an object is measured with a metre rod with least count equal to 0·1 cm. If the measured length is 62·5 cm., it has to be recorded as (62·5 ± 0·1) cm, stating the *limits of error.* Similarly, suppose time period of a pendulum is measured to be 2·0 sec, using a stop watch of least count 0·1 sec., it has to be recorded as (2·0 ± 0·1) sec, meaning thereby that time period is in the range of 1·9 sec. and 2·1 sec.

Permissible error in a result can be calculated by combining suitably the errors of various instruments used in a particular set up, as per rules discussed already.

Sample Problem **If the errors involved in the measurement of mass and length of one side of a cube are 4% and 3% respectively. What is the maximum permissible relative error in calculation of density of material of the cube ?**

Sol. Here, $\frac{\Delta M}{M} = \pm 4\%$ and $\frac{\Delta L}{L} = \pm 3\%$ $\frac{\Delta \rho}{\rho} = ?$

As density $\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{L^3}$ $\therefore$ $\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{3\,\Delta L}{L} = \pm\, 4\% + 3\,(\pm\, 3\%) = \pm\, \mathbf{13\%}$

2.41. RULES FOR DETERMINING UNCERTAINTY IN THE RESULTS OF ARITHMETIC CALCULATIONS

The following examples illustrate the rules for determination of uncertainty/error in arithmetical operations on the measured quantities :

Rule 1. To understand this rule, suppose we use a metre scale to measure length and breadth of a thin rectangular sheet as 15·4 cm and 10·2 cm respectively. Each measurement has three significant figures and a precision upto first place of decimal. Therefore, we can write

$$\text{length, } l = (15{\cdot}4 \pm 0{\cdot}1)\text{ cm} = 15{\cdot}4\text{ cm} \pm \left(\frac{0{\cdot}1}{15{\cdot}4} \times 100\right)\% = 15{\cdot}4\text{ cm} \pm 0{\cdot}6\%$$

$$\text{Similarly, we can write breadth, } b = (10{\cdot}2 \pm 0{\cdot}1)\text{ cm} = 10{\cdot}2\text{ cm} \pm \left(\frac{0{\cdot}1 \times 100}{10{\cdot}2}\right)\% = 10{\cdot}2\text{ cm} \pm 1\%$$

Now, area of the sheet, $A = l \times b = 15{\cdot}4 \times 10{\cdot}2\text{ cm}^2 = 157{\cdot}08\text{ cm}^2$

As per rules, percentage error in area $= \pm\,(0{\cdot}6 + 1{\cdot}0)\% = \pm\, 1{\cdot}6\%$

$$\text{We may write, } A = 157{\cdot}08\text{ cm}^2 \pm 1{\cdot}6\% = 157{\cdot}08\text{ cm}^2 \pm \left(\frac{1{\cdot}6}{100} \times 157{\cdot}08\right)\text{cm}^2 = (157{\cdot}08 \pm 2{\cdot}51)\text{ cm}^2$$

As per rule, the final value of area can contain only three significant figures and error can contain only one significant figure, therefore, we may write the final result as $A = (157 \pm 3)\text{ cm}^2$.

Rule 2. *If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures. However, if the data are subtracted, the number of significant figures would reduce.* **For example**

$x = 13{\cdot}7$ m and $y = 8{\cdot}08$ m, both have three significant figures.

Now $x - y = 13{\cdot}7 - 8{\cdot}08 = 5{\cdot}62$ m

The result has three significant figures. But in addition and subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places. Therefore, rounding off to one place of decimal, we get $x - y = 5{\cdot}6$ m.

Rule 3. *The relative error of a value of number specified to n significant figures depends not only on n, but also on the number itself.*

For example, suppose $m_1 = (1{\cdot}04 \pm 0{\cdot}01)$ kg and $m_2 = (9{\cdot}24 \pm 0{\cdot}01)$ kg

Both the measured masses have an error or uncertainty of 0·01 kg.

The relative error in 1·04 kg is $= \pm\left(\dfrac{0{\cdot}01}{1{\cdot}04}\right)\times 100\% = \pm 1\%$

Similarly, the relative error in 9·24 kg is $= \pm\left(\dfrac{0{\cdot}01}{9{\cdot}24}\right)\times 100 = \pm 0{\cdot}1\%$

Thus, the relative error depends on the number itself.

Rule 4. *In a multi-step computation, the intermediate results should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.*

For example, $x = 9{\cdot}58$ has three significant digits. Now, reciprocal of x is $\dfrac{1}{x} = \dfrac{1}{9{\cdot}58} = 0{\cdot}104$, rounded off to three significant digits. When we take reciprocal of 0·104, we get 9·62, rounded off to three significant digits.

However, if we calculate $\dfrac{1}{x} = \dfrac{1}{9{\cdot}58} = 0{\cdot}1044$, rounded off to *four* significant figures (*i.e.*, one more than the number of significant digits), then $\dfrac{1}{0{\cdot}1044} = 9{\cdot}58$, rounded off to three significant digits.

Thus, retaining one more extra digit in intermediate steps of complex calculations would avoid additional errors in the process of rounding off the numbers.

SOLVED EXAMPLES

TYPE I. SIMPLE CONVERSION OF UNITS

Formulae used

1 AU	$= 1.496 \times 10^{11}$ m
1 *ly*	$= 9.46 \times 10^{15}$ m
1 par sec	$= 3.1 \times 10^{16}$ m
1 A°	$= 10^{-10}$ m. ; 1 nm $= 10^{-9}$ m ;
1 μm	$= 10^{-6}$ m
1 mm	$= 10^{-3}$ m, and so on.
60″ seconds (of arc)	= 1 minute (of arc)
60′ minute (of arc)	= 1 degree (of arc)
180° degree (of arc)	= π radian

Example 1 **How many par sec are there in one light year ?**

Solution. As is known, 1 ly $= 9{\cdot}46 \times 10^{15}$ m

and 1 par sec $= 3{\cdot}1 \times 10^{16}$ m

$$\therefore \quad \frac{1\,\text{ly}}{1\,\text{par sec}} = \frac{9{\cdot}46\times10^{15}}{3{\cdot}1\times10^{16}} = 0{\cdot}305$$

$\therefore$ 1 ly = **0·305 par sec**

Example 2 **The mass of a proton is 1·67 × 10⁻²⁷ kg. How many protons would make 1 gram ?**

Solution. Here, $m_p = 1{\cdot}67 \times 10^{-27}$ kg ;

total mass, $m = 1$ g $= 10^{-3}$ kg

$$\therefore \text{ Number of protons} = \frac{m}{m_p} = \frac{10^{-3}\text{ kg}}{1{\cdot}67\times10^{-27}\text{ kg}}$$

$$= \mathbf{5{\cdot}99 \times 10^{23}}$$

Example 3 **Calculate the angle of (a) 1° (degree) (b) 1′ (minute of arc or arc min) and (c) 1″ (second of arc or arc sec) in radian. Use 360° = 2 π rad., 1° = 60′ and 1′ = 60″.**

Solution. (*a*) $1° = \dfrac{2\pi}{360}\text{ rad} = \dfrac{\pi}{180}\text{ rad}$

$$= \frac{22}{7\times180}\text{ rad} = \mathbf{1{\cdot}746 \times 10^{-2}\ rad}$$

(*b*) 1 arc min $= 1' = \dfrac{1°}{60} = \dfrac{1}{60}\times\dfrac{\pi}{180}$ rad

$$= 2{\cdot}91 \times 10^{-4}\text{ rad}$$

(*c*) 1 arc sec $= 1'' = \dfrac{1'}{60} = \dfrac{1^\circ}{60 \times 60}$

$$= \frac{1}{60 \times 60} \times \frac{\pi}{180} \text{ rad} = \mathbf{4{\cdot}85 \times 10^{-6}\ rad}$$

TYPE II. ORDER OF MAGNITUDE AND INDIRECT METHODS FOR MEASURING DISTANCES

Formulae used

1. Order of magnitude of a quantity is the power of 10, which gives us a value nearest to the actual value.
2. Angular diameter, $\theta = \dfrac{l}{r}$
3. Echo Method : $x + x = v \times t$
4. Keplar's Law : $T^2 \propto R^3$
5. Size of molecule of oleic acid,

$$d = \frac{nv}{500\,A} = \frac{nv}{500\,\pi R^2}$$

where n is number of oleic acid drops each of volume v ; 1/500 is concentration of oleic acid in alcohol, A is area of oleic acid layer.

Units used,

l, r, x, R, d in metre ; v in m^3 ; t, T in sec.

Example 4 **The moon is observed from two diametrically opposite points *A* and *B* on earth. The angle θ subtended at the moon by the two directions of observation is 1° 54′. Given the diameter of earth to be about 1·276 × 10⁷ m, calculate the distance of moon from earth.**

Solution. Here, $\theta = 1^\circ\,54' = 114' = 114 \times 60''$

As $1'' = 4{\cdot}85 \times 10^{-6}$ radian,

$\therefore \quad \theta = 114 \times 60 \times 4{\cdot}85 \times 10^{-6}$ rad

$= 3{\cdot}32 \times 10^{-2}$ rad

Diameter of earth, $l = 1{\cdot}276 \times 10^7$ m,

earth-moon distance, $r = ?$

As $\quad l = r\,\theta$

$$\therefore \quad r = \frac{l}{\theta} = \frac{1{\cdot}276 \times 10^7}{3{\cdot}32 \times 10^{-2}} = \mathbf{3{\cdot}84 \times 10^8\ m}$$

Example 5 **When the planet Jupiter is at a distance of 824·7 million kilometres from the Earth, its angular diameter is measured to be 35·72″ of arc. Calculate the diameter of Jupiter ?**

Solution. Here, $r = 824{\cdot}7 \times 10^6$ km.

$$\theta = 35{\cdot}72'' = \frac{35{\cdot}72}{60 \times 60} \times \frac{\pi}{180} \text{ radian}$$

Diameter, $\quad l = ?$

As $\quad l = r\,\theta$

$$\therefore \quad l = 824{\cdot}7 \times 10^6 \times \frac{35{\cdot}72 \times \pi}{60 \times 60 \times 180} \text{ km}$$

$$= \mathbf{1{\cdot}429 \times 10^5\ km}$$

Example 6 **The sun's angular diameter is measured to be 1920″. The distance of the sun from the earth is 1·496 × 10¹¹ m. What is the diameter of the sun ?** (NCERT Solved Example)

Solution. Here, $\theta = 1920''$

$$= \frac{1920}{60 \times 60} \times \frac{\pi}{180} \text{ radian},$$

$r = 1{\cdot}496 \times 10^{11}$ m, $l = ?$

As $\quad l = r\,\theta$

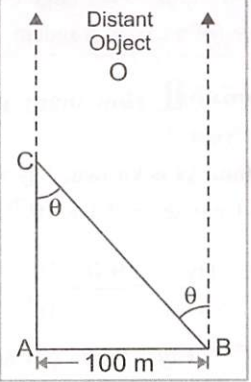
$$\therefore \quad l = 1{\cdot}496 \times 10^{11} \times \frac{1920}{60 \times 60} \times \frac{\pi}{180}$$

$$= \mathbf{1{\cdot}39 \times 10^9\ m}$$

Example 7 **A man wishes to estimate the distance of a nearby tower from him. He stands at a point *A* in front of the tower *C* and spots a very distant object *O* in line with *AC*. He then walks perpendicular to *AC* upto *B*, a distance of 100 m and looks at *O* and C again. Since *O* is very distant, the direction of *BO* is practically the same as *AO*, but he finds the line of sight of *C* shifted from the original line of sight by an angle θ = 40° (θ is known as parallax). Estimate the distance of the tower *C* from his original position *A*.**

Solution. As is clear from Fig. 2.17, parallax angle, $\quad \theta = 40^\circ$, $AB = 100$ m, $AC = ?$

FIGURE 2.17



In ΔABC, $\tan\theta = \dfrac{AB}{AC}$

$$AC = \frac{AB}{\tan\theta} = \frac{100}{\tan 40°}$$

$$AC = \frac{100}{0{\cdot}8391} = \mathbf{119\ m}$$

Example 8 **A drop of olive oil of radius 0·25 mm spreads into a circular film of radius 10 cm on the water surface. Estimate the size of molecule of oleic acid.**

Solution. Here, $r = 0{\cdot}25$ mm $= 0{\cdot}25 \times 10^{-3}$ m,

$$R = 10 \text{ cm} = 10^{-1} \text{ m}$$

$$\text{Thickness of oil film} = \frac{\text{volume of oil drop}}{\text{area of the film}}$$

$$= \frac{\frac{4}{3}\pi r^3}{\pi R^2} = \frac{4}{3}\frac{r^3}{R^2}$$

$$= \frac{4}{3}\frac{(0{\cdot}25 \times 10^{-3})^3}{(10^{-1})^2}$$

$$= 2{\cdot}08 \times 10^{-9} \text{ m}$$

If we assume the oil film to be one molecule thick, then size of molecule of oleic acid

$$= \mathbf{2{\cdot}08 \times 10^{-9}\ m}$$

Example 9 **A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth ?**

Solution. Here, $t = 2{\cdot}56$ s,

velocity of laser light in vacuum,

$$c = 3 \times 10^8 \text{ m/s}$$

The radius of lunar orbit is the distance of moon from earth. Let it be x

As $x = \dfrac{c \times t}{2}$

$$\therefore \quad x = \frac{3 \times 10^8 \times 2{\cdot}56}{2} = \mathbf{3{\cdot}84 \times 10^8\ m}$$

Example 10 **A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine ? (Speed of sound in water = 1450 ms^{-1}).**

Solution. Here, $t = 77{\cdot}0$ s, $x = ?$ $v = 1450$ ms^{-1}.

As $x = \dfrac{v \times t}{2}$

$$\therefore \quad x = \frac{1450 \times 77{\cdot}0}{2} \text{ m} = \mathbf{55825\,m}$$

Example 11 **The farthest objects in our universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the earth. These objects (known as quasers) have many puzzling features, which have yet not been satisfactorily explained. What is the distance in km of a quasar from which light takes 3·0 billion years to reach us ?**

Solution. Here, $x = ?$, time taken,

$t = 3{\cdot}0$ billion years $= 3 \times 10^9$ yr

$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60$ s

vel. of light in vacuum, $c = 3 \times 10^8$ m/s

$= 3 \times 10^5$ km/s

As distance = velocity × time

$\therefore$ $x = (3 \times 10^5) \times 3 \times 10^9 \times 365 \times 24 \times 60 \times 60$ km

$= \mathbf{2{\cdot}84 \times 10^{22}}$ **km**

Example 12 **It is a well known fact that during a total solar eclipse, the disc of the moon almost completely covers the disc of the Sun. From this fact and from the information you can gather from Solved Examples 5 and 6 on page 1/50, determine the approximate diameter of the moon.**

Solution. Solved Examples 5 and 6 on page 1/50 give us the following data :

Distance of moon from earth,

$$ME = 3{\cdot}84 \times 10^8 \text{ m}$$

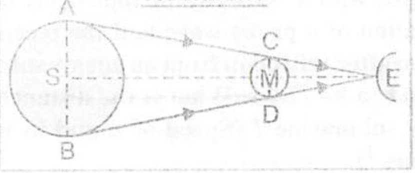
Distance of sun from earth,

$$SE = 1{\cdot}496 \times 10^{11} \text{ m.}$$

Diameter of sun $AB = 1{\cdot}39 \times 10^9$ m.

The situation during total solar eclipse is shown in Fig. 2.18.

FIGURE 2.18



As Δ^S ABE of CDE are similar, therefore,

$$CD = AB \times \frac{ME}{SE} = \frac{1{\cdot}39 \times 10^9 \times 3{\cdot}84 \times 10^8}{1{\cdot}496 \times 10^{11}}$$

$$= 3{\cdot}5679 \times 10^6 \text{ m}$$

$$= \mathbf{3567{\cdot}9 \text{ km}}$$

This is the diameter of the moon.

TYPE III. MEASUREMENT OF TIME

Formula used. Fractional error

$$= \frac{\text{Difference in time}}{\text{time interval}}$$

Unit used. Time is measured in sec.

Example 13 **Age of the universe is about 10^{10} years whereas the mankind has existed for 10^6 years. How many seconds would the man have existed if age of universe were one day.**

Solution. Here,

$$\frac{\text{Age of mankind}}{\text{Age of universe}} = \frac{10^6 \text{ yrs}}{10^{10} \text{ yrs}} = \frac{x}{1 \text{ day}}$$

$$x = 10^{-4} \text{ day} = 10^{-4} \times 86400 \text{ s}$$

$$= 8{\cdot}64 \text{ s}$$

Example 14 **Two atomic clocks allowed to run for average life of an Indian (say, 70 years) differ by 0·2 s only. Calculate the accuracy of standard atomic clock in measuring a time interval of 1 sec.**

Solution. Time interval = 70 yrs

= (70 × 365 + 17) days (*17 additional days are on account of 17 leap years during 70 yrs*).

= 25567 days

= 25567 × 24 × 60 × 60 sec.

Difference = 0·2 s

$$\therefore \text{ fractional error} = \frac{0{\cdot}2}{25567 \times 24 \times 60 \times 60}$$

$$= 9{\cdot}04 \times 10^{-11} \approx 10^{-10}$$

Hence, accuracy shown by atomic clock is **10^{-10} s in 1 s or 1 s in 10^{10} s**

Example 15 **The mean life of an elementary particle pion is 2×10^{-7} ns. The age of the universe is about 4×10^9 years. Identify a physically meaning time that is approximately half way between these two on a logarithmic scale.**

Solution. Here, $t_1 = 2 \times 10^{-7} \text{ ns} = 2 \times 10^{-16}$ s

$$t_2 = 4 \times 10^9 \text{ years} = 4 \times 10^9 \times 3 \times 10^7 \text{ s}$$

$$= 1{\cdot}2 \times 10^{17} \text{ s}$$

Let the time interval t be half way between t_1 and t_2 on logarithmic scale.

$$\therefore \quad \log t = \frac{1}{2} [\log t_1 + \log t_2]$$

$$= \frac{1}{2} [\log 10^{-16} + \log 10^{17}]$$

$$= \frac{1}{2} (-16 + 17) = 0{\cdot}5 \simeq 1$$

$$\therefore \quad t = \mathbf{10 \text{ s}}$$

This is the time taken by an athlete to run a 100 metre track.

TYPE IV. MAGNIFICATION

Formulae used.

1. Linear magnification $= \dfrac{\text{size of image}}{\text{size of object}}$
2. Linear magnification $= \sqrt{\text{Areal magnification}}$

Units used. Size of image & size of object must have same units.

Magnification has no units.

Example 16 **A 35 mm wide slide with 24 mm × 36 mm picture is projected on a screen placed 12 cm from the slide. The image of the slide picture on the screen measures 1·0 m × 1·5 m. What is the linear magnification of the arrangement ?**

Solution. Area of object = 24 mm × 36 mm

$$= 24 \times 36 \times 10^{-6} \text{ m}^2$$

Area of image = $1{\cdot}0\text{ m} \times 1{\cdot}5\text{ m} = 1{\cdot}5\text{ m}^2$

$$\therefore \text{ Areal magnificatuion} = \frac{1{\cdot}5\text{ m}^2}{24 \times 36 \times 10^{-6}}$$

$$= 1736$$

Linear magnification $= \sqrt{1736} = 41{\cdot}77$

Example 17 **If the size of a nucleus ($\approx 10^{-15}$ m) is scaled up to the tip of a sharp pin ($\approx 10^{-5}$ m), what roughly is the size of an atom ?**

(NCERT Solved Example)

Solution. Magnification $= \dfrac{\text{size of tip of pin}}{\text{size of nucleus}}$

$$= \frac{10^{-5}}{10^{-15}} = 10^{10}$$

As actual size of atom $\approx 10^{-10}$ m

and it is magnified 10^{10} times,

$\therefore$ Apparent size of atom $= 10^{-10} \times 10^{10}$

$$= 10^0 = 1 \text{ m}$$

TYPE V. CONVERSION OF ONE SYSTEM OF UNITS INTO ANOTHER

Formulae used.

$$n_2 = \frac{n_1 u_1}{u_2} = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

Units used. Every quantity must be expressed in its absolute units only.

Example 18 **Find the value of 60 J per min on a system that has 100 g, 100 cm and 1 min. as the base units.**

Solution. Here, $P = \dfrac{60\text{ J}}{\text{min}} = \dfrac{60\text{ J}}{60\text{ s}} = 1$ watt

It is the SI unit of power $= [M^1 L^2 T^{-3}]$

$\therefore \quad a = 1, \quad b = 2, \quad c = -3$

For converting SI system to new system, we write

$M_1 = 1\text{ kg}, L_1 = 1\text{ m}, T_1 = 1\text{ s}, n_1 = 1$ watt

$M_2 = 100\text{ g}, L_2 = 100\text{ cm}, T_2 = 1\text{ min}, n_2 = ?$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$$= 1 \left(\frac{1\text{ kg}}{100\text{ g}}\right)^1 \left(\frac{1\text{ m}}{100\text{ cm}}\right)^2 \left(\frac{1\text{ s}}{1\text{ min}}\right)^{-3}$$

$$= 1 \times 10 \times 1 \times \left(\frac{1}{60}\right)^{-3} = 10 \times 60^3$$

$$= 2{\cdot}16 \times 10^6 \text{ units}$$

Example 19 **In a new system of units called star units, 1 kg* = 10 kg ; 1 m* = 1 km and 1 s* = 1 minute, what will be the value of 1 J of energy in the new system ?**

Solution. Joule is a unit of energy, $[M^1 L^2 T^{-2}]$

$$\therefore \quad 1\text{J} = \frac{1\text{kg}\,(1\text{m})^2}{(1\text{s})^2} = \frac{\left(\frac{1}{10}\text{kg}^*\right)\left(\frac{1}{1000}\text{m}^*\right)^2}{\left(\frac{1}{60}\text{s}^*\right)^2}$$

$$= 3{\cdot}6 \times 10^{-4} \text{ kg}^* \text{ m}^{*2} \text{ s}^{*-2}$$

$$= 3{\cdot}6 \times 10^{-4} \text{ J}^*$$

TYPE VI. CHECKING THE CORRECTNESS OF FORMULAE

Formulae used. Principle of homogeneity of dimensions, *i.e.*, checking if the powers of M, L, T on either side of the equation are equal. Formula is wrong when dimensions of even one of M, L and T fail to match.

Example 20 **Out of the formulae $y = a \sin 2\pi t/T$ and $y = a \sin vt$ for the displacement y of particle undergoing a periodic motion, rule out the wrong formula on the basis of dimensions. Symbols have standard meaning.** **(CBSE 2009)**

Solution. *(i)* $y = a \sin \dfrac{2\pi t}{T}$

LHS $= y = [L]$.:

RHS $= a \sin \dfrac{2\pi t}{T} = L \sin \dfrac{T}{T} = [L]$

As dimensions on both sides are same, this formula is correct.

(ii) $y = a \sin v t$

LHS $= y = [L]$;

RHS $= a \sin vt = L \sin (LT^{-1} \cdot T) = L \sin L$

As angle has to be dimensionless, this formula is wrong.

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Example 21 **By the method of dimensions, test the accuracy of the equation :** $\delta = \dfrac{mgl^3}{4bd^3Y}$

where δ is depression in the middle of a bar of length l, breadth b, depth d, when it is loaded in the middle with mass m. Y is Young's modulus of material of the bar.

Solution. The given relation is $\delta = \dfrac{mgl^3}{4bd^3Y}$

$$\text{LHS} = \delta = [M^0L^1T^0]$$

$$\text{RHS} = \frac{mgl^3}{4bd^3Y} = \frac{M\,(LT^{-2})\,L^3}{(L)(L^3)(ML^{-1}T^{-2})}$$

$$= [M^0L^1T^0]$$

As dimensions on both sides are same, the given relation is **correct.**

Example 22 **Check the correctness of the relation,** $S_{nth} = u + \dfrac{a}{2}(2n-1)$**, where u is initial velocity, a is acceleration and S_{nth} is the distance travellled by the body in nth second.**

Solution. The given relation is

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

Writing the dimensions on either side, we get

$$\text{L.H.S.} = S_{nth} = \frac{\text{distance}}{\text{time}} = \frac{L}{T} = LT^{-1}$$

$$= [M^0\,L^1\,T^{-1}]$$

$$\text{R.H.S.} = u + \frac{a}{2}(2n-1) = LT^{-1} + LT^{-2}\,(T)$$

$$= LT^{-1} + LT^{-1} = LT^{-1}$$

$$= [M^0\,L^1\,T^{-1}].$$

As LHS = RHS, dimensionally

$\therefore$ **Formula is correct.**

Example 23 **The SI unit of energy is J = kg m^2 s^{-2}, that of speed v is ms^{-1} and of acceleration a is ms^{-2}. Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body).**

(a) $K = m^2 v^3$ **(b)** $K = \dfrac{1}{2}mv^2$ **(c)** $K = ma$

(d) $K = \dfrac{3}{16}mv^2$ (e) $K = \dfrac{1}{2}mv^2 + ma$

(NCERT Solved Example)

Solution. (a) $K = m^2v^3 = (\text{kg})^2\,(\text{ms}^{-1})^3$
$= \textbf{kg}^2\,\textbf{m}^3\,\textbf{s}^{-3}$

(b) $K = \dfrac{1}{2}\,mv^2 = \text{kg}\,(\text{ms}^{-1})^2 = \textbf{kg m}^2\,\textbf{s}^{-2}$

(c) $K = ma = \text{kg}\,(\text{ms}^{-2}) = \textbf{kg ms}^{-2}$

(d) $K = \dfrac{3}{16}\,mv^2 = \text{kg}\,(\text{ms}^{-1})^2 = \textbf{kg m}^2\,\textbf{s}^{-2}$

(e) $K = \dfrac{1}{2}mv^2 + ma = \text{kg}\,(\text{ms}^{-1})^2 + \text{kg}\,(\text{ms}^{-2})$
$= \textbf{kg m}^2\,\textbf{s}^{-2} + \textbf{kg ms}^{-2}$

The two parts having different dimensions cannot be added.

Clearly, on the basis of dimensional arguments, (a), (c) and (e) are ruled out. Choices (b) and (d) are correct. The dimensional argument does not distinguish between (b) and (d). From actual derivation in unit 4, we shall find that **choice (b) is correct.**

Example 24 **Find the dimensions of the quantity q from the expression :** $T = 2\pi\sqrt{\dfrac{ml^3\,q}{5Y}}$**, where T is time period of a bar of length l, mass m and Young's modulus Y.**

Solution. The given expression is

$$T = 2\pi\sqrt{\frac{ml^3\,q}{5Y}}$$

Squaring both sides, we get

$$T^2 = 4\pi^2\,\frac{ml^3\,q}{5Y}$$

$$q = \frac{5YT^2}{4\pi^2\,ml^3}$$

$$q = \frac{ML^{-1}\,T^{-2}\,T^2}{ML^3}$$

$$q = [M^0\,L^{-4}]$$

TYPE VII. THE FACT THAT QUANTITIES HAVING DIFFERENT DIMENSIONS CANNOT BE ADDED OR SUBTRACTED

Formulae used. The dimensions of M, L, T on either side of any relation must be equal individually ; *i.e.*, the principle of homogeneity of dimensions must be obeyed.

Further, the quantities added together must have the same dimensions. And the quantities subtracted from one another must also have the same dimensions.

Example 25 **Find the dimensions of a/b in the relation**

$P = ax + bt^2$, where P is pressure, x is distance and t is time.

Solution. As $ax = P$,

$$a = \frac{P}{x} = \frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$$

Again, as $bt^2 = P$,

$$b = \frac{P}{t^2} = \frac{ML^{-1}T^{-2}}{T^2} = [ML^{-1}T^{-4}]$$

$$\frac{a}{b} = \frac{ML^{-2}T^{-2}}{ML^{-1}T^{-4}} = [M^0L^{-1}T^2]$$

Example 26 **In Vander Wall's equation**

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

what are the dimensions of a and b ? Here, P is pressure, V is volume, T is temperature and R is gas constant. (CBSE 2014)

Solution. The given equation is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

As pressure can be added only to pressure therefore, a/V^2 represents pressure P

i.e., $$\frac{a}{V^2} = P$$

or $a = PV^2 = (ML^{-1}T^{-2})(L^3)^2$
$= [M^1L^5T^{-2}]$

Again, from volume V, we can subtract only the volume. Therefore, b must be representing volume only *i.e.*, $b = V = [L^3] = [M^0L^3T^0]$

Example 27 **Find the dimensions of a/b in the relation $P = \dfrac{a - x^2}{bt}$, where x is distance, t is time and P is pressure.**

Solution. $P = \dfrac{a - x^2}{bt}$

$$a = x^2 = [L^2]$$

Again, $$b = \frac{a - x^2}{P \times t} = \frac{L^2}{(ML^{-1}T^{-2}) \times T}$$
$$= [M^{-1}L^3T^1]$$

$$\frac{a}{b} = \frac{L^2}{(M^{-1}L^3T^1)} = [M^1L^{-1}T^{-1}]$$

Example 28 **Find the dimensions of $\dfrac{a \times b}{c}$ in the relation $y = 4 \sin at + 3 \cos bt - ct$, where t is time and y is distance.**

Solution. The given relation is

$$y = 4 \sin at + 3 \cos bt - ct$$

Here, $at = \theta$, $a = \dfrac{\theta}{t} = \dfrac{1}{T} = [T^{-1}]$

$bt = \theta$, $b = \dfrac{\theta}{t} = \dfrac{1}{T} = [T^{-1}]$

$c \times t = y$, $c = \dfrac{y}{t} = \dfrac{L}{T} = [LT^{-1}]$

Now, $$\frac{a \times b}{c} = \frac{[T^{-1}][T^{-1}]}{[LT^{-1}]} = [L^{-1}T^{-1}]$$

Example 29 **Find the dimensions of $\dfrac{c}{a \times b}$ in the relation : $y = a \cos \omega t + b \times t - c\sqrt{t}$, where y is displacement, t is time and ω is angular velocity.**

Solution. Here, $y = a \cos \omega t + b \times t - c\sqrt{t}$

As $\cos \omega t$ is a number, $a = y = [L]$

$$b \times t = y,\ b = \frac{y}{t} = \frac{L}{T} = [LT^{-1}]$$

$$c\sqrt{t} = y,\ c = \frac{y}{\sqrt{t}} = \frac{L}{T^{1/2}} = LT^{-1/2}$$

$$\frac{c}{a \times b} = \frac{LT^{-1/2}}{L(LT^{-1})} = L^{-1}T^{1/2}$$

TYPE VIII. DERIVATION OF FORMULAE

Formula used.
Principle of homogeneity of dimensions.

Example 30 **The period of revolution (T) of a planet around the sun depends upon (*i*) radius (r) of obit (*ii*) mass M of the sun and (*iii*) gravitational constant G. Prove that $T^2 \propto r^3$**

Solution. Let $T = K\, r^a M^b G^c$ ...(*i*)

where K is dimensionless constant of proportionality.

Writing the dimensions in (*i*), we get

$$M^0L^0T^1 = L^a M^b (M^{-1}L^3T^{-2})^c$$
$$= M^{b-c} L^{a+3c} T^{-2c}$$

Applying the principle of homogeneity of dimensions, we get

$$b - c = 0\,;\; b = c = -1/2$$
$$a + 3c = 0\,;\; a = -3c = 3/2$$
$$-2c = 1\,;\; c = -\frac{1}{2}$$

From (*i*) $T = K\, r^{3/2} M^{-1/2} G^{-1/2}$

$\therefore\; T \propto r^{3/2}$ or $T^2 \propto r^3$

Example 31 **The period of vibration of a tuning fork depends on length (l) of its prongs, density (d) and Young's modulus (Y) of its material. Determine an expression for period of vibration using the method of dimensions.**

Solution. Let $T = K\, l^a d^b Y^c$ ...(*i*)

where K is dimensionless constant.

$$[M^0 L^0 T^1] = L^a [ML^{-3}]^b [ML^{-1} T^{-2}]^c$$
$$= M^{b+c} . L^{a-3b-c}\, T^{-2c}$$

Applying the principle of homogeneity of dimensions, we get

$$b + c = 0 \quad ...(ii)$$
$$a - 3b - c = 0 \quad ...(iii)$$
$$-2c = 1, \quad c = -\frac{1}{2}$$

From (*ii*), $b = -c = \dfrac{1}{2}$

From (*iii*), $a = 3b + c = \dfrac{3}{2} - \dfrac{1}{2} = 1$

Putting these values in (*i*), we get

$$T = K\, l^1 d^{1/2} Y^{-1/2} = \boldsymbol{K\, l\sqrt{d/Y}}$$

Example 32 **The depth x to which a bullet penetrates a human body depends on (*i*) coefficient of elasticity, η and (*ii*) KE (E_k) of the bullet. By the method of dimensions, show that**

$$x \propto \left(\frac{E_k}{\eta}\right)^{1/3}$$

Solution. Let $x \propto E_k^a\, \eta^b$,

where a, b are the dimensions

$$x = K\, E_k^a\, \eta^b \quad ...(i)$$

where K is dimensionless constant of proportionality.

Writing the dimensions in (*i*), we get

$$[M^0 L^1 T^0] = (ML^2 T^{-2})^a\, (ML^{-1} T^{-2})^b$$
$$= M^{a+b} L^{2a-b} T^{-2a-2b}$$

Applying the principle of homogeneity of dimensions, we get

$$a + b = 0 \quad ...(ii)$$
$$2a - b = 1 \quad ...(iii)$$
$$-2a - 2b = 0 \quad ...(iv)$$

Add (*ii*) and (*iii*) : $3a = 1$, $a = 1/3$

From (*ii*), $b = -a = -\dfrac{1}{3}$

From (*i*), $x = K\left(\dfrac{E_k}{\eta}\right)^{1/3}$

Hence, $x \propto \left(\dfrac{E_k}{\eta}\right)^{1/3}$

TYPE IX. ERRORS AND PROPAGATION OF ERRORS IN SUM AND DIFFERENCE

Formulae used.

1. Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{1}{n} \times \sum_{i=1}^{i=n} |\Delta a_i|$$

2. Fractional error or Relative error,

$$\delta a = \frac{\text{mean absolute error}}{\text{true value}} = \pm \frac{\Delta a_{\text{mean}}}{a_m}$$

3. Percentage error, $\delta a = \pm \dfrac{\Delta a_{\text{mean}}}{a_m} \times 100\%$

4. When $x = a + b$, $\Delta x = \pm(\Delta a + \Delta b)$
5. When $x = a - b$, $\Delta x = \pm(\Delta a + \Delta b)$

Units used. Mean absolute error has the units of the quantity itself. Fractional error or relative error has no units.

Example 33 **We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2·63 s, 2·56 s, 2·42 s, 2·71 s and 2·80 s. Calculate the absolute errors, relative error and percentage error.** (NCERT Solved Example)

Solution. The observed/measured values of time period are :

$$t_1 = 2{\cdot}63\text{ s},\ t_2 = 2{\cdot}56\text{ s},\ t_3 = 2{\cdot}42\text{ s},$$
$$t_4 = 2{\cdot}71\text{ s},\ t_5 = 2{\cdot}80\text{ s}$$

The mean period of oscillation of the pendulum

$$t_m = \frac{2{\cdot}63 + 2{\cdot}56 + 2{\cdot}42 + 2{\cdot}71 + 2{\cdot}80}{5}\text{ sec}$$

$$= \frac{13{\cdot}12}{5} = 2{\cdot}624\text{ s}$$

As the periods are measured to a resolution of 0·01 s, all time periods are given to the second decimal place. Therefore, it is proper to put this mean time period also to the second decimal place.

$$\therefore \quad t_m = 2{\cdot}62\text{ s}$$

The absolute errors in the measurement are :

$$t_m - t_1 = 2{\cdot}62 - 2{\cdot}63 = -\ \mathbf{0{\cdot}01\ s}$$
$$t_m - t_2 = 2{\cdot}62 - 2{\cdot}56 = \mathbf{0{\cdot}06\ s}$$
$$t_m - t_3 = 2{\cdot}62 - 2{\cdot}42 = \mathbf{0{\cdot}20\ s}$$
$$t_m - t_4 = 2{\cdot}62 - 2{\cdot}71 = -\ \mathbf{0{\cdot}09\ s}$$
$$t_m - t_5 = 2{\cdot}62 - 2{\cdot}80 = -\ \mathbf{0{\cdot}18\ s}$$

Mean absolute error

$$\Delta t_{\text{mean}} = \frac{0{\cdot}01 + 0{\cdot}06 + 0{\cdot}20 + 0{\cdot}09 + 0{\cdot}18}{5}\text{ sec}$$

$$= \frac{0{\cdot}54}{5}\text{ s} = 0{\cdot}11\text{ s}$$

$\therefore$ The period of simple pendulum is

$$t = (2{\cdot}62 \pm 0{\cdot}11)\text{ s}$$

i.e., t lies between $(2{\cdot}62 + 0{\cdot}11)$ s and $(2{\cdot}62 - 0{\cdot}11)$ s or between 2·73 s and 2·51 s.

The relative error, $\delta a = \pm\dfrac{0{\cdot}11}{2{\cdot}62} = \pm\mathbf{0{\cdot}041}$

The percentage error, $\delta a \times 100$

$$= \pm\, 0{\cdot}041 \times 100 = \pm\,\mathbf{4{\cdot}1\%}$$

Example 34 **The temperatures of two bodies measured by a thermometer are : $t_1 = 20°\text{C} \pm 0{\cdot}5°\text{C}$ and $t_2 = 50°\text{C} \pm 0{\cdot}5°\text{C}$. Calculate the temperature difference and the error there in.** (NCERT Solved Example)

Solution. Here, $t_1 = 20°\text{C} \pm 0{\cdot}5°\text{C}$,
$t_2 = 50°\text{C} \pm 0{\cdot}5°\text{C}$

Temperature difference,

$$t = t_2 - t_1 = 50° - 20° = 30°\text{C}$$

Max. error in difference, $\Delta t = \pm(\Delta t_1 + \Delta t_2)$

$$= \pm(0{\cdot}5 + 0{\cdot}5) = \pm\, 1°\text{C}$$

$\therefore$ Temperature difference $= \mathbf{(30 \pm 1)°C}$

Example 35 **Suppose we use a physical balance to measure the mass of an object and find the mean value of our observation to be 156·28 g. Represent this result correctly.**

Solution. If least count of physical balance is 0·1 g, the mass measured will be correctly represented as : $m = \mathbf{(156{\cdot}3 \pm 0{\cdot}1)\ gram}$

Example 36 **In an experiment to measure focal length of a concave mirror, the values of focal length in successive observations are 17·3 cm, 17·8 cm, 18·3 cm, 18·2 cm, 17·9 cm and 18·0 cm. Calculate mean absolute error and percentage error. Express the result in a proper way.**

Solution. Mean value of focal length

$$= \frac{17{\cdot}3 + 17{\cdot}8 + 18{\cdot}3 + 18{\cdot}2 + 17{\cdot}9 + 18{\cdot}0}{6}$$

$$f = \frac{107{\cdot}5}{6} = 17{\cdot}91$$

$= 17{\cdot}9$ cm (rounding off to one place)

Absolute errors in different measurements

$$\Delta f_1 = 17{\cdot}9 - 17{\cdot}3 = 0{\cdot}6\text{ cm}$$
$$\Delta f_2 = 17{\cdot}9 - 17{\cdot}8 = 0{\cdot}1\text{ cm}$$
$$\Delta f_3 = 17{\cdot}9 - 18{\cdot}3 = -\,0{\cdot}4\text{ cm}$$
$$\Delta f_4 = 17{\cdot}9 - 18{\cdot}2 = -\,0{\cdot}3\text{ cm}$$
$$\Delta f_5 = 17{\cdot}9 - 17{\cdot}9 = 0{\cdot}0\text{ cm}$$
$$\Delta f_6 = 17{\cdot}9 - 18{\cdot}0 = -\,0{\cdot}1\text{ cm}$$

$$\text{Mean absolute error} = \frac{1}{6}\,\Sigma(\Delta f)$$

$$= \frac{0{\cdot}6 + 0{\cdot}1 + 0{\cdot}4 + 0{\cdot}3 + 0{\cdot}0 + 0{\cdot}1}{6} = 0{\cdot}25\text{ cm.}$$

$$\text{Percentage error} = \frac{0{\cdot}25}{17{\cdot}9} \times 100 = \mathbf{1{\cdot}4\%}$$

Result : $f = 17{\cdot}9$ cm ± 1·4%

Example 37 **The temperatures of two bodies measured by a thermometer are : $t_1 = 40°C \pm 0{\cdot}2°C$ and $t_2 = 50°C \pm 0{\cdot}5°C$. Calculate the temperature difference and the error there in.** **(CBSE 2012)**

Solution. Here, $t_1 = 40°C \pm 0{\cdot}2°C$;

$$t_2 = 50°C \pm 0{\cdot}5°C$$

$$t_2 - t_1 = 50°C - 40°C = 10°C$$

$$\text{Error} = \pm(\Delta t_1 + \Delta t_2) = \pm(0{\cdot}2°C + 0{\cdot}5°C) = 0{\cdot}7°C$$

$\therefore$ Result = Temp. diff. = **10°C ± 0·7°C**

Example 38 **The refractive index of water as measured by the relation μ = real depth/ apparent depth was found to have values 1·29, 1·33, 1·34, 1·35, 1·32, 1·36, 1·30, 1·33. Calculate (*i*) mean value of μ (*ii*) mean value of absolute error (*iii*) relative error (*iv*) percentage error.**

Solution. Mean value of μ

$$= \frac{1{\cdot}29 + 1{\cdot}33 + 1{\cdot}34 + 1{\cdot}35 + 1{\cdot}32 + 1{\cdot}36 + 1{\cdot}30 + 1{\cdot}33}{8}$$

$$= \frac{10{\cdot}62}{8} = 1{\cdot}327 \simeq \mathbf{1{\cdot}33}$$

$$\Delta\mu_1 = 1{\cdot}33 - 1{\cdot}29 = 0{\cdot}04$$
$$\Delta\mu_2 = 1{\cdot}33 - 1{\cdot}33 = 0{\cdot}00$$
$$\Delta\mu_3 = 1{\cdot}33 - 1{\cdot}34 = -0{\cdot}01$$
$$\Delta\mu_4 = 1{\cdot}33 - 1{\cdot}35 = -0{\cdot}02$$
$$\Delta\mu_5 = 1{\cdot}33 - 1{\cdot}32 = 0{\cdot}01$$
$$\Delta\mu_6 = 1{\cdot}33 - 1{\cdot}36 = -0{\cdot}03$$
$$\Delta\mu_7 = 1{\cdot}33 - 1{\cdot}30 = 0{\cdot}03$$
$$\Delta\mu_8 = 1{\cdot}33 - 1{\cdot}33 = 0{\cdot}00$$

$$\Delta\mu_{\text{mean}} = \frac{0{\cdot}04 + 0{\cdot}01 + 0{\cdot}02 + 0{\cdot}01 + 0{\cdot}03 + 0{\cdot}03}{8}$$

$$= \frac{0{\cdot}19}{8} = 0{\cdot}023 = \mathbf{0{\cdot}02}$$

$$\text{Relative error} = \frac{\Delta\mu_{\text{mean}}}{\mu_{\text{mean}}} = \frac{0{\cdot}02}{1{\cdot}33} = \mathbf{0{\cdot}015}$$

Percentage error = 0·015 × 100% = **1·5%**

TYPE X. PROPAGATION OF ERRORS IN MULTIPLICATION, DIVISION AND RAISING TO SOME POWER

Formulae used.

1. If $x = a \times b$, $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

2. If $x = \dfrac{a}{b}$, $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

3. If $x = a^n$, $\dfrac{\Delta x}{x} = \pm n\left(\dfrac{\Delta a}{a}\right)$

Units used. Fractional errors have no units

Example 39 **The length and breadth of a rectangle are (5·7 ± 0·1) cm and (3·4 ± 0·2) cm. Calculate area of the rectangle with error limits.**

Solution. Here, $l = (5{\cdot}7 \pm 0{\cdot}1)$ cm,

$$b = (3{\cdot}4 \pm 0{\cdot}2) \text{ cm}$$

$$\text{Area } A = l \times b = 5{\cdot}7 \times 3{\cdot}4 = 19.38 \text{ cm}^2 = 19{\cdot}0 \text{ cm}^2$$

(rounding off to two significant figures)

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0{\cdot}1}{5{\cdot}7} + \frac{0{\cdot}2}{3{\cdot}4}\right)$$

$$= \pm\left(\frac{0{\cdot}34 + 1{\cdot}14}{5{\cdot}7 \times 3{\cdot}4}\right)$$

$$\frac{\Delta A}{A} = \pm\frac{1{\cdot}48}{19{\cdot}38}$$

$$\Delta A = \pm\frac{1{\cdot}48}{19{\cdot}38} \times A = \pm\frac{1{\cdot}48}{19{\cdot}38} \times 19{\cdot}38 = \pm 1{\cdot}48$$

$$\Delta A = \pm 1{\cdot}5$$

(rounding off to two significant figures)

$\therefore$ Area = **(19·0 ± 1·5) sq.cm**

Example 40 **The resistance $R = V/I$, where $V = (100 \pm 5)$ V and $I = (10 \pm 0{\cdot}2)$ A. Find the percentage error in R.**

Solution. Here, $V = (100 \pm 5)$ V,
$I = (10 \pm 0{\cdot}2)$ A

$$R = \frac{V}{I} = \frac{100}{10} = 10 \text{ ohm}$$

Now $$\frac{\Delta R}{R} = \pm\left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)$$

$$= \pm\left(\frac{5}{100} + \frac{0{\cdot}2}{10}\right) = \pm\frac{7}{100}$$

$$= \pm\frac{7}{100} \times 100 = \pm 7\%$$

Example 41 **The radius of curvature of a concave mirror, measured by a spherometer is given by $R = \frac{l^2}{6h} + \frac{h}{2}$ The values of l and h are 4·0 cm and 0·065 cm respectively, where l is measured by a metre scale and h by a spherometer. Find the relative error in the measurement of R.**

Solution. Here, $l = (4{\cdot}0 \pm 0{\cdot}1)$ cm

and $h = (0{\cdot}065 \pm 0{\cdot}001)$ cm, $R = ?$

$$R = \frac{l^2}{6h} + \frac{h}{2} = \frac{(4{\cdot}0)^2}{6 \times 0{\cdot}065} + \frac{0{\cdot}065}{2}$$

$$= 41{\cdot}0256 + 0{\cdot}0325 = \mathbf{41{\cdot}0581\ cm}$$

$$\frac{\Delta R}{R} = \pm\left(\frac{2\,\Delta l}{l} + \frac{\Delta h}{h}\right) = \pm\left(\frac{2 \times 0{\cdot}1}{4{\cdot}0} + \frac{0{\cdot}001}{0{\cdot}065}\right)$$

$$= 0{\cdot}05 + 0{\cdot}015 = 0{\cdot}065 = \mathbf{0{\cdot}06}$$

(*rounding off to 2nd place of decimal*)

Example 42 **A small error in the measurement of the quantity having the highest power (in a given formula) will contribute maximum percentage error in the value of the physical quantity to whom it is related. Explain why.**

Solution. We have already established that when

$$x = \frac{l^a\, m^b}{t^c},$$

then $\frac{\Delta x}{x} \times 100 = \pm\left(a\frac{\Delta l}{l} + b\frac{\Delta m}{m} + c\frac{\Delta t}{t}\right) \times 100$

This is the percentage error in x. We find that maximum contribution to percentage error comes from the highest power (a or b or c) in the given formula.

Example 43 **The two specific heat capacities of a gas are measured as**

$C_p = (12{\cdot}28 \pm 0{\cdot}2)$ units and $C_v = (3{\cdot}97 \pm 0{\cdot}3)$ units.

Find the value of gas constant R.

Solution. Here, $C_p = (12{\cdot}28 \pm 0{\cdot}2)$ units

$C_v = (3{\cdot}97 \pm 0{\cdot}3)$ units, $R = ?$

As $R = C_p - C_v$

$\therefore\ R = 12{\cdot}28 - 3{\cdot}97 = 8{\cdot}31$ units

$$\Delta R = \pm(\Delta C_p + \Delta C_v) = \pm(0{\cdot}2 + 0{\cdot}3) = \pm 0{\cdot}5 \text{ units}$$

Hence, $R = \mathbf{(8{\cdot}31 \pm 0{\cdot}5)}$ **units**

Example 44 **Find the relative error in Z if $Z = \frac{A^4\, B^{1/3}}{CD^{3/2}}$** (NCERT Solved Example)

Solution. Here, $Z = \frac{A^4\, B^{1/3}}{CD^{3/2}}$

As per rule,

$$\frac{\Delta Z}{Z} = \pm\left[4\left(\frac{\Delta A}{A}\right) + \frac{1}{3}\left(\frac{\Delta B}{B}\right) + \left(\frac{\Delta C}{C}\right) + \frac{3}{2}\left(\frac{\Delta D}{D}\right)\right]$$

Example 45 **The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20·0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g ?** (NCERT Solved Example)

Solution. Here, $T = 2\pi\sqrt{\frac{L}{g}}$

Square both sides,

$$T^2 = 4\pi^2\frac{L}{g} \quad\text{or}\quad g = 4\pi^2\frac{L}{T^2}$$

As per rule, $\frac{\Delta g}{g} = \pm\left[\frac{\Delta L}{L} + 2\left(\frac{\Delta T}{T}\right)\right]$ ...(*i*)

Here, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$ $\therefore$ $\frac{\Delta T}{T} = \frac{\Delta t}{t}$

As errors in both, L and t are least count errors, therefore, from (*i*)

$$\frac{\Delta g}{g} = \pm\left[\frac{0{\cdot}1}{20{\cdot}0} + 2\left(\frac{1}{90}\right)\right] = \pm 0{\cdot}027$$

$$= \pm\, 0{\cdot}027 \times 100\% = \pm\, 2{\cdot}7\%$$

Example 46 **A physical quantity X is given by $X = \frac{a^2 b^3}{c\sqrt{d}}$. If the percentage error in measurement of a, b, c and d are 4%, 3%, 2% and 1% respectively. Calculate the percentage error in X.**

Solution. Here, $X = \dfrac{a^2 b^3}{c\sqrt{d}}$

$$\therefore \frac{\Delta X}{X}\times 100 = \pm\left(2\frac{\Delta a}{a}+3\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{1}{2}\frac{\Delta d}{d}\right)$$

$$= \pm\left(2\times 4\% + 3\times 3\% + 2\% + \frac{1}{2}\times 1\%\right)$$

$$= \pm\, 19{\cdot}5\%$$

Example 47 **A physical quantity Q is given by $Q = \dfrac{A^2\, B^{3/2}}{C^4\, D^{1/2}}$. The percentage error in A, B, C and D are 1%, 2%, 3% and 4% respectively, what is the percentage error in X ?**

(Central Schools 2005)

Solution. From the given relation, maximum error in Q is

$$\frac{\Delta Q}{Q}\times 100 = \pm\left(2\frac{\Delta A}{A}+\frac{3}{2}\frac{\Delta B}{B}+4\frac{\Delta C}{C}+\frac{1}{2}\frac{\Delta D}{D}\right)$$

$$= \pm\left(2\times 1\% + \frac{3}{2}\times 2\% + 4\times 3\% + \frac{1}{2}\times 4\%\right)$$

$$= \pm\, 19\%$$

Example 48 **The radius of a sphere is measured as (2·1 ± 0·5) cm. Calculate its surface area with error limits.**

Solution. Here, $r = (2{\cdot}1 \pm 0{\cdot}5)$ cm, $A = ?$

$$A = 4\pi r^2 = 4\times\frac{22}{7}(2{\cdot}1)^2 = 55{\cdot}44\,\text{cm}^2$$

$$\frac{\Delta A}{A} = \pm\frac{2\,\Delta r}{r} = \pm\frac{2\times 0{\cdot}5}{2{\cdot}1} = \pm\, 0{\cdot}476$$

$$\Delta A = \pm\, 0{\cdot}476\, A = \pm\, 0{\cdot}476 \times 55{\cdot}44 = 26{\cdot}4\ \text{cm}^2$$

Hence, surface area of sphere = **(55·4 ± 26·4) cm²**

(rounding off to one place of decimal)

Example 49 **The voltage across a lamp is (6·0 ± 0·1) V and the current passing through it is (4·0 ± 0·2) ampere. Find the power consumed by the lamp.**

Solution. Here, $V = (6{\cdot}0 \pm 0{\cdot}1)$ volt,

$I = (4{\cdot}0 \pm 0{\cdot}2)\ A$

Power consumed, $P = VI = (6{\cdot}0)\,(4{\cdot}0)$

$= 24{\cdot}0$ watt

Now, $\dfrac{\Delta P}{P} = \dfrac{\Delta V}{V} + \dfrac{\Delta I}{I} = \dfrac{0{\cdot}1}{6{\cdot}0} + \dfrac{0{\cdot}2}{4{\cdot}0} = \dfrac{0{\cdot}8}{12{\cdot}0}$

$$\Delta P = \frac{0{\cdot}8}{12{\cdot}0}P = \frac{0{\cdot}8\times 24{\cdot}0}{12{\cdot}0} = 1{\cdot}6\ \text{watt}$$

Hence, Power consumed = **(24·0 ± 1·6) watt**

Example 50 **The length and breadth of a rectangular block are 25·2 cm and 16·8 cm, which have both been measured to an accuracy of 0·1 cm. Find the area of the rectangular block.**

Solution. Here, $l = (25{\cdot}2 \pm 0{\cdot}1)$ cm,

$b = (16{\cdot}8 \pm 0{\cdot}1)$ cm, $A = ?$

Now, $A = l \times b = 25{\cdot}2 \times 16{\cdot}8 = 423{\cdot}36\ \text{cm}^2$

$= 423{\cdot}4\ \text{cm}^2$

(rounding off to one decimal place)

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l}+\frac{\Delta b}{b}\right) = \pm\left(\frac{0{\cdot}1}{25{\cdot}2}+\frac{0{\cdot}1}{16{\cdot}8}\right) = \frac{4{\cdot}2}{423{\cdot}36}$$

$$\Delta A = \pm\frac{4{\cdot}2\,A}{423{\cdot}36} = \pm\, 4{\cdot}2\,\text{cm}^2$$

Hence, area of block = **(423·4 ± 4·2) cm²**

Example 51 **To find the value of 'g' by using a simple pendulum, the following observations were made :**

Length of the thread, l = (100 ± 0·1) cm

Time period of oscillation, T = (2 ± 0·1) s

Calculate the maximum permissible error in measurement of 'g'. Which quantity should be measured more accurately and why ?

Solution. Here, $l = (100 \pm 0{\cdot}1)$ cm,

$T = (2 \pm 0{\cdot}1)\ s\,;\quad g = ?$

From $\quad T = 2\pi\sqrt{\dfrac{l}{g}},\quad g = \dfrac{4\pi^2 l}{T^2}$

$$\frac{\Delta g}{g} = \pm\left(\frac{\Delta l}{l}+\frac{2\Delta T}{T}\right) = \pm\left(\frac{0{\cdot}1}{100}+\frac{2\times 0{\cdot}1}{2}\right)$$

$$= \pm\, 0{\cdot}001 + 0{\cdot}1 = \pm\, 0{\cdot}101$$

$$\Delta g = \pm\, 0{\cdot}101\, g = \pm\, 0{\cdot}101 \times 9{\cdot}8\ \text{m/s}^2$$

$$= \pm\, 0{\cdot}9898\ \text{m/s}^2 = \pm\, \mathbf{1{\cdot}0\ m/s^2}$$

In the calculation of $\dfrac{\Delta g}{g}$, $\left(\dfrac{\Delta T}{T}\right)$ is taken twice.

Therefore, ***time period of pendulum must be measured more accurately.***

TYPE XI. SIGNIFICANT FIGURES AND ROUNDING OFF

Formulae used

Refer to Arts 2.36, 2.37 and 2.38 for common rules.

Example 52 **Write down the number of significant figures in the following :**

(i) 5238 N **(ii) 4200 kg**

(iii) 34·000 m **(iv) 0·02340 Nm^{-1}**

Solution. *(i)* 5238 N has **four significant digits**

(ii) 4200 kg = 4·200 × 10^3 kg has **four significant figures.**

(iii) 34·000 m has **five significant digits.**

(iv) 0·02340 Nm^{-1} has **four significant figures.**

Example 53 **Subtract 2·5 × 10^4 from 3·9 × 10^5 with due regard to significant figures.**

Solution. Let $x = 2{\cdot}5 \times 10^4$ $y = 3{\cdot}9 \times 10^5$

$$y - x = 3{\cdot}9 \times 10^5 - 2{\cdot}5 \times 10^4 = 10^5\,(3{\cdot}9 - 0{\cdot}25)$$

$$= 3{\cdot}65 \times 10^5 = \mathbf{3{\cdot}6 \times 10^5}$$

(rounded to one place of decimal)

Example 54 **Each side of a cube is measured to be 7·203 m. What are the total surface area and the volume of the cube to appropriate significant figures ?**

Solution. Here, $l = 7{\cdot}203$ m

Surface area of the cube = $6\,l^2$

$$A = 6\,(7{\cdot}203)^2 = 311{\cdot}299 \text{ m}^2$$

As l has four significant figures, as per rule, surface area and volume both should have four significant figures. Rounding off, we get

$$\mathbf{A = 311{\cdot}3 \text{ m}^2}$$

Volume of cube = $l^3 = (7{\cdot}203)^3$

$$= 373{\cdot}714 \text{ m}^3 = \mathbf{373{\cdot}7 \text{ m}^3}$$

(rounded off to four significant figures)

Example 55 **5·74 g of a substance occupies 1·2 cm^3. Express its density by keeping the significant figures in view.**

Solution. Here, mass, $m = 5{\cdot}74$ g ,

volume, $V = 1{\cdot}2$ cm^3

$$\text{As density, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{5{\cdot}74 \text{ g}}{1{\cdot}2 \text{ cm}^3}$$

$$= 4{\cdot}783 \text{ g cm}^{-3}$$

As mass has 3 significant digits and volume has 2 significant digits, therefore, as per rule, density will have only two significant digits. Rounding off, we get ρ = **4·8 g cm^{-3}**

Example 56 **It is required to find the volume of a rectangular block. A vernier calliper is used to measure the length, width and height of the block. The measured values are found to be 1·37 cm, 4·11 cm and 2·56 cm respectively.**

Calculate correctly, the volume of the block.

Solution. Here, $l = 1{\cdot}37$ cm, $w = 4{\cdot}11$ cm

and $h = 2{\cdot}56$ cm

$$\text{volume, } V = l \times w \times h = 1{\cdot}37 \times 4{\cdot}11 \times 2{\cdot}56$$

$$= 14{\cdot}414592 = 14{\cdot}4$$

(rounding off to 3 significant digits)

$$\frac{\Delta V}{V} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta w}{w} + \frac{\Delta h}{h}\right)$$

$$= \pm\left(\frac{0{\cdot}01}{1{\cdot}37} + \frac{0{\cdot}01}{4{\cdot}11} + \frac{0{\cdot}01}{2{\cdot}56}\right)$$

$$= \pm\,(0{\cdot}00730 + 0{\cdot}00243 + 0{\cdot}00391)$$

$$= \pm\,0{\cdot}0136$$

$$\Delta V = \pm\,0{\cdot}0136 \times 14{\cdot}4 = \pm\,0{\cdot}196 = \pm\,0{\cdot}2$$

(rounding off to one decimal place)

Hence, volume of block = **(14·4 ± 0·2) cm^3**

Example 57 **Solve the following and express the result to appropriate number of significant figures.**

$$\textit{(i)}\ \ \mathbf{\frac{5{\cdot}7 \times 213 \times 0{\cdot}9256}{47{\cdot}8}}$$

(ii) **3·84 + 0·0239 – 0·568**

Solution. *(i)* $\dfrac{5{\cdot}7 \times 213 \times 0{\cdot}9256}{47{\cdot}8} = 23{\cdot}5098$

Rounding off to two significant digits,

we get, result = **24**

(ii) 3·84 + 0·0239 – 0·568 = 3·2959

Rounding off to two places of decimal,

result = **3·30**

Example 58 **Solve the following and express the result to appropriate number of significant figures.**

$$x = \mathbf{\frac{4{\cdot}69 + 0{\cdot}0537 - 0{\cdot}6872}{1{\cdot}9}}$$

Solution. Here

$$\begin{array}{r} 4{\cdot}69 \\ +\ 0{\cdot}0537 \\ \hline 4{\cdot}7437 \\ -\ 0{\cdot}6872 \\ \hline 4{\cdot}0565 \end{array}$$

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Rounding off to two places of decimal, we get

Numerator = 4·0565 ⇒ 4·06

Now, $\frac{4\cdot06}{1\cdot9} = 2\cdot1368$

Rounding off to two significant digits, we get

$$x = \mathbf{2\cdot1}$$

TYPE XII. TYPICAL EXAMPLES

Example 59 **Express the average distance of earth from the sun in (*i*) light year (*ii*) par sec.**

Solution. Average distance of earth from the sun, $r = 1\text{ AU} = 1\cdot496 \times 10^{11}$ m

As 1 ly = $9\cdot46 \times 10^{15}$ m and

1 par sec = $3\cdot1 \times 10^{16}$ m

$$\therefore \quad r = \frac{1\cdot496\times10^{11}}{9\cdot46\times10^{15}}\text{ ly} = \mathbf{1\cdot58 \times 10^{-5}\ ly}$$

$$\text{Again, } r = \frac{1\cdot496\times10^{11}}{3\cdot1\times10^{16}}\text{ par sec} = \mathbf{4\cdot8 \times 10^{-6}\ par\ sec}$$

Example 60 **Fill in the blanks :**

(i) The volume of a cube of side 10 cm. is.... m^3.

(ii) A vehicle moving with a speed of 36 km h^{-1} covers .... m in 1 sec.

(iii) The density of water at 4°C is .... g/cc or .... kg/m^3.

Solution. (*i*) As $V = L^3$

and $L = 10\text{ cm} = 10^{-1}$ m

$\therefore \quad V = (10^{-1}\text{ m})^3 = \mathbf{10^{-3}\ m^3}$

(*ii*) $v = \frac{36\text{ km}}{\text{hr}} = \frac{36\times1000\text{ m}}{60\times60\text{ s}} = \mathbf{10\ m/s.}$

(*iii*) density $= \frac{\text{mass}}{\text{volume}} = \frac{1\text{ g}}{(1\text{ cm})^3} = \mathbf{1\ g/cc}$

$$= \frac{10^{-3}\text{ kg}}{(10^{-2}\text{ m})^3} = \mathbf{10^3\ kg/m^3}$$

Example 61 **It is estimated that per minute, each cm^2 of earth receives about 2 calories of heat energy from the sun. This constant is called solar constant *S*. Express solar constant in SI units.**

Solution. $S = \frac{2\text{ cals.}}{\text{min. cm}^2} = \frac{2\times4\cdot2\text{ joule}}{60\text{ s}\times10^{-4}\text{ m}^2}$

$$= \mathbf{1\cdot4 \times 10^3\ W/m^2}$$

Example 62 **Ten drops of olive oil of radius 0·20 mm spread into a circular film of radius 14·6 cm. on the surface of water. Estimate the size of an oil molecule.**

Solution. Here, $n = 10$,

$r = 0\cdot20\text{ mm} = 2 \times 10^{-4}$ m,

$R = 14\cdot6\text{ cm} = 14\cdot6 \times 10^{-2}$ m.

Let d be the diameter of oil molecule

= thickness of layer

Volume of olive oil = area × thickness of layer

$$n \times \frac{4}{3}\pi r^3 = \pi R^2 \times d$$

$$d = \frac{4}{3} n \frac{r^3}{R^2} = \frac{4}{3} \times \frac{10(2\times10^{-4})^3}{(14\cdot6\times10^{-2})^2}$$

$$= \mathbf{5 \times 10^{-9}\ m}$$

Example 63 **In case of venus, the angle of maximum elongation is found to be approximately 47°. Determine the distance between venus and sun (r_{vs}) and the distance between venus and earth (r_{ve}).**

Solution. Here, angle of maximum elongation

$\in = 47°, \quad r_{vs} = ?, r_{ve} = ?$

We know, $r_{se} = 1\text{ A.U} = 1\cdot496 \times 10^{11}$ m.

$$r_{vs} = r_{es} \times \sin\in$$
$$= 1\cdot496 \times 10^{11} \sin 47°$$
$$= 1\cdot496 \times 10^{11} \times 0\cdot73$$
$$= \mathbf{1\cdot09 \times 10^{11}\ m}$$

$$r_{ve} = r_{se} \times \cos\in$$
$$= 1\cdot496 \times 10^{11} \cos 47°$$
$$= 1\cdot496 \times 10^{11} \times 0\cdot68\text{ m}$$
$$= \mathbf{1\cdot02 \times 10^{11}\ m}$$

Example 64 **Consider a simple pendulum having a bob attached to a string that oscillates under the action of a force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (*l*), mass of the bob (*m*) and acc. due to gravity (*g*). Derive the expression for its time period using method of dimensions.**

Solution. Let $\qquad T = k l^a g^b m^c \qquad$ ...(*i*)

where k is dimensionless constant and a, b, c are the respective dimensions. Writing the dimensions on both sides of (*i*), we get

$$[M^0 L^0 T^1] = L^a (LT^{-2})^b (M^c)$$
$$= M^c L^{a+b} T^{-2b}$$

Equating the dimensions of M, L and T on both sides, we get

$$c = 0,\ a + b = 0 \quad \text{and} \quad -2b = 1 \text{ or } b = -\frac{1}{2}$$

$$\therefore \quad a = -b = \frac{1}{2}$$

From (*i*), $\quad T = kl^{1/2} . g^{-1/2} . m^0 = k\sqrt{\frac{l}{g}}$

By other methods, we find $k = 2\pi$

$$\therefore \qquad T = 2\pi\sqrt{l/g}$$

Example 65 **The heat produced in a wire carrying an electric current depends on the current, the resistance and the time. Assuming that the dependance is of the product of powers type, guess an eqn. between these quantities using dimensional analysis. The dimensional formula of resistance is $ML^2 A^{-2} T^{-3}$ and heat is a form of energy.**

Solution. Let $\qquad H = I^a R^b t^c \qquad$ ...(*i*)

$$[ML^2 T^{-2}] = A^a [ML^2 T^{-3} A^{-2}]^b [T]^c$$
$$= M^b L^{2b} T^{-3b+c} A^{-2b+a}$$

Applying the principle of homogeneity of dimensions, we get

$$b = 1 ; 2b = 2 ; \ -3b + c = -2 ;$$
$$c = -2 + 3b = -2 + 3 = 1, \text{ and}$$
$$-2b + a = 0,\ a = 2b = 2 \times 1$$

Putting these values in (*i*), we get $H = I^2 R^1 t^1$

Example 66 **A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis, find the expression for period of oscillation (T) in terms of radius of star (R), mean density of fluid (ρ) and universal gravitational constant (G).**

Solution. Let $T = K R^a \rho^b G^c \qquad$ ...(*i*)

where a, b, c are the dimensions and K is dimensionless constant of proportionality. Writing the dimensions in (*i*), we get

$$[M^0 L^0 T^1] = L^a (ML^{-3})^b (M^{-1} L^3 T^{-2})^c$$
$$= M^{b-c} L^{a-3b+3c} T^{-2c} \qquad \text{...(ii)}$$

Applying the principle of homogeneity of dimensions, we get

$$b - c = 0 \qquad \text{...(ii)}$$
$$a - 3b + 3c = 0 \qquad \text{...(iii)}$$
$$-2c = 1,\ c = -\frac{1}{2}$$

From (*ii*), $\quad b = c = -\frac{1}{2}$

From (*iii*), $a = 3b - 3c = 3\left(-\frac{1}{2}\right) - 3\left(-\frac{1}{2}\right) = 0$

Putting in (*i*), we get

$$T = K R^{\circ} \rho^{-1/2} G^{-1/2} = \frac{K}{\sqrt{\rho G}}$$

Example 67 **Two clocks are being tested against a standard clock located in a national laboratory. At 12 : 00 : 00 noon by the standard clock, the readings of the two clocks are :**

	Clock I	Clock II
Monday	12 : 00 : 05	10 : 15 : 06
Tuesday	12 : 01 : 15	10 : 14 : 59
Wednesday	11 : 59 : 08	10 : 15 : 18
Thursday	12 : 01 : 50	10 : 15 : 07
Friday	11 : 59 : 15	10 : 14 : 53
Saturday	12 : 01 : 30	10 : 15 : 24
Sunday	12 : 01 : 19	10 : 15 : 11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer ?

Solution. A simple look at the data shows that average reading of clock I is much closer to the standard time than average reading of clock II. Therefore, zero error in clock II is much larger than the zero error in clock I. But zero error can always be corrected by applying necessary correction. Now, over the seven days, range of variation of clock I is 12 : 01 : 50 – 11 : 59 : 08 = 162 *s* and range of variation of clock II is 10 : 15 : 24 – 10 : 14 : 53 = 31 *s*. Therefore, precision of clock II is much better than the precision of clock I. **We will prefer clock II.**

Example 68 **In an experiment in determining the density of a rectangular block, the dimensions of the block are measured with a vernier calliper with a least count of 0·01 cm and its mass is measured with a beam balance of least count 0·1 g, l = 5·12 cm, b = 2·56 cm, t = 0·37 cm and m = 39·3 g. Report correctly the density of the block.**

Solution. The density of block is given by

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{l \times b \times t} = \frac{39{\cdot}3}{5{\cdot}12 \times 2{\cdot}56 \times 0{\cdot}37}$$
$$= 8{\cdot}1037 \text{ g cm}^{-3} = 8{\cdot}1 \text{ g cm}^{-3}$$

(rounding off to two significant digits)

Maximum relative error in density is

$$\frac{\Delta\rho}{\rho} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t}\right)$$

$$= \pm\left(\frac{0{\cdot}1}{39{\cdot}3} + \frac{0{\cdot}01}{5{\cdot}12} + \frac{0{\cdot}01}{2{\cdot}56} + \frac{0{\cdot}01}{0{\cdot}37}\right)$$

$$= \pm\,(0{\cdot}0025 + 0{\cdot}0019 + 0{\cdot}0039 + 0{\cdot}027)$$

$$= \pm\,0{\cdot}0353$$

$$\Delta\rho = \pm\,0{\cdot}0353\,\rho$$

$$= \pm\,0{\cdot}0353 \times 8{\cdot}1037$$

$$= \pm\,0{\cdot}286$$

$$= \pm\,0{\cdot}3 \text{ g cm}^{-3}$$

(rounding off to one decimal place)

Hence, density of block = **(8·1 ± 0·3) g cm^{-3}**

IMPORTANT NOTE

From the above calculation, we find that in ($\Delta\rho / \rho$), maximum contribution is from ($\Delta t/t$). Therefore, to increase accuracy of measurement, thickness t must be measured more precisely.

Example 69 **The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, linear scale reads 1 mm and 39th division on circular scale coincides with the reference line. If zero error is – 0·05 mm. Calculate area of cross section of the wire with correct number of significant figures.**

Solution. Here, pitch = 1 mm, $n = 100$

$$\text{Least count} = \frac{\text{pitch}}{n} = \frac{1}{100} = 0{\cdot}01 \text{ mm.}$$

Observed diameter of wire

$$= 1 \text{ mm} + 39 \times 0{\cdot}01 \text{ mm} = 1{\cdot}39 \text{ mm}$$

$$\text{Corrected diameter of wire} = (1{\cdot}39 + 0{\cdot}05) \text{ mm}$$

$$= 1{\cdot}44 \text{ mm}$$

$$D = 0{\cdot}144 \text{ cm}$$

$$\text{Area of cross section} = \frac{\pi D^2}{4} = \frac{3{\cdot}14}{4}(0{\cdot}144)^2$$

$$A = 0{\cdot}01627 \text{ sq. cm.}$$

Rounding off to three significant figures, we get

$$A = \mathbf{0{\cdot}0163 \text{ sq. cm}}$$

REVISION AT A GLANCE

1. **The SI units** of measurement of seven physical quantities are called **fundamental units.** They are : *metre* (m) for length ; *kilogram* (kg) for mass ; *second* (s) for time ; *ampere* (A) for electric current ; *kelvin* (K) for temperature, *candela* (cd) for luminous intensity and *mole* (mol) for amount of substance.
2. The SI units of measurement of two more quantities are called **supplementary units.** They are : *radian* (rad) for plane angle and *steradian* (sr) for solid angle.
3. **One metre** = 1,650,763·73 wavelengths in vacuum, of radiation corresponding to orange red light of Krypton-86.
4. **One second** = 9,192,631,770 periods of a particular radiation from Cesium-133 atom.
5. **The magnitudes** of quantities, we deal with in Physics, vary from very small (*micro*) to very large (*macro*). To express such small/large magnitudes, we use some **metric prefixes :**

   The most commonly used metric prefixes are deci (10^{-1}), centi (10^{-2}), milli (10^{-3}), micro (10^{-6}), kilo (10^{3}) and mega (10^{6}).
6. **The symbol** of unit combined with prefix is considered as the new symbol of the unit, which can be raised to any power (positive or negative) without any bracket. For example, km^2 means $(10^3 \text{ m})^2 = 10^6 \text{ m}^2$.
7. **Some of the Non SI units in common use are**

   (*a*) For length/distance

1. Astronomical unit,	1 AU = $1{\cdot}496 \times 10^{11}$ m
2. Light year,	1 ly = $9{\cdot}46 \times 10^{15}$ m
3. Parallactic second,	1 par sec = $3{\cdot}084 \times 10^{16}$ m = 3·26 ly.
4. Micron or micrometer,	1 μm = 10^{-6} m
5. Nanometer,	1 nm = 10^{-9} m
6. Angstrom unit,	1 Å = 10^{-10} m

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(*b*) For mass

1.	Pound,	1 lb = 0·4536 kg
2.	Slug,	1 slug = 14·59 kg
3.	Quintal,	1 q = 100 kg
4.	Metric tonne,	1 *t* = 1000 kg
5.	Atomic mass unit,	1 amu = 1 u = $1{\cdot}66 \times 10^{-27}$ kg

(*c*) For time

1.	Year,	1 yr = 365·25 days = $3{\cdot}156 \times 10^{7}$ s
2.	Shake,	1 shake = 10^{-8} s

(*d*) For other quantities

1.	Barn (for area) :	1 barn = 10^{-28} m^2
2.	Litre (for volume),	1 *l* = 10^3 cc = 10^{-3} m^3 where cc represents cubic centimetre, *i.e.*, cm^3.
3.	Gallon (for volume),	In U.S.A., 1 gallon = 3·7854 *l* In U.K., 1 gallon = 4·546 *l*
4.	Pascal (for pressure),	1 Pa = 1 Nm^{-2}
	Pressure exerted by earth's atmosphere,	1 atm. = $1{\cdot}01 \times 10^5$ Pa
5.	Bar (for pressure),	1 bar = 1 atm. = $1{\cdot}01 \times 10^5$ Pa
6.	Torr (for pressure),	1 torr = 1 mm of Hg. col. = 133·3 Pa
7.	Electron volt (for energy/work),	1 eV = $1{\cdot}6 \times 10^{-19}$ J
8.	Erg (for energy/work),	1 erg = 10^{-7} J
9.	Kilowatt hour (for energy),	1 kWh = $3{\cdot}6 \times 10^{6}$ J
10.	Horse Power (for power),	1 hp = 746 W
11.	Dioptre (for power of a lens),	1 D = 1 m^{-1}
12.	Degree (for angle),	$1^\circ = \frac{\pi}{180}$ rad.

8. ***Order of magnitude*** of a quantity gives us a value nearest to the actual value of the quantity, in terms of suitable powers of 10.

9. The distances to be measured vary from radius of proton (10^{-15} m) to size of universe (10^{26} m). Direct methods for the measurement of length involve the use of (*i*) a metre scale for distances from 10^{-3} m to 10^2 m (*ii*) a vernier callipers for distances upto 10^{-4} m (*iii*) a screw gauge and a spherometer for distances upto 10^{-5} m.

Indirect methods for large distances are Echo method, LASER method, RADAR method, SONAR method, Triangulation method, Parallex method, Spectroscopic method etc. For small distances, Avogadro's method and Rutherford method are used.

10. The masses to be measured vary from mass of electron (10^{-30} kg) to mass of universe (10^{55} kg).

An inertial balance is used for measuring inertial mass of a body, and a physical balance is used for measuring gravitational mass of a body. For measuring weight of a body, we use a spring balance.

Different indirect methods are employed for measuring different ranges of masses.

11. The time intervals of events vary from time taken by light to cross a distance of nuclear size (10^{-22} sec) to age of universe (10^{17} sec).

Any phenomenon that repeats itself serves as a clock. Some of the clocks developed are electric oscillators, electronic oscillators, solar clock, quartz crystal clock (with an accuracy of 1 sec in every 10^9 sec) ; atomic clock (with an accuracy of 1 sec in every 10^{11} sec). Decay of elementary particles using photographic emulsions have led to measurement of time intervals ranging from 10^{-16} s to 10^{-24} s.

Radioactive dating technique has been used for measuring long time intervals of the order of 10^{17} sec.

12. **In mechanics,** dimensions are given in terms of powers of mass (M), length (L) and time (T). In **heat and thermodynamics,** power of K (Kelvin for temperature) comes in addition to powers of M, L and T. In **electricity and magnetism,** dimensions are given in terms of M, L, T and A (ampere for current).

13. Note that **constants** *e.g.*, 1/2, π, e, trigonometrical functions like $\sin \omega t$, $\cos \omega t$ etc. have no units and no dimensions.

14. **Quantities having same dimensions** can be added or subtracted from each other, *i.e.,* length can be added to length only ; area can be subtracted from area only and so on. Thus quantities of different dimensions can not be added or subtracted.

15. *More than one physical quantity may have the same dimensions.* Thus a physical quantity cannot be uniquely defined from its dimensions only.

16. The **three main uses** of dimensional analysis are:

    (*i*) **Conversion of one system of units into another, for which we use**

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

where M_1, L_1, T_1 are fundamental units on one system; M_2, L_2, T_2 are fundamental units on the other system; a, b, c are the dimensions of the quantity in mass, length and time; n_1 is numerical value of the quantity in one system and n_2 is its numerical value in the other system.

Note. This formula is valid only in absolute units and not in gravitational units.

(*ii*) **checking the correctness of a given physical relation,**

(*iii*) **derivation of formulae.**

The last two uses are based on the *principle of homogeneity of dimensions*, according to which dimensions of mass, length and time on one side of the dimensional physical relation must be equal to their respective dimensions on the other side of the relation. The relation would be wrong even if dimensions of any one of M, L, T on the two sides were different.

17. Note that mere dimensional correctness of a relation does not ensure its physical correctness. For example, work and torque have the same dimensions, but the two are not physically same.

18. Difference between measured value and true value of a quantity respresents **error of measurement.** The errors are communicated in different mathematical operations as detailed below :

If $\pm \Delta a$, $\pm \Delta b$ and $\pm \Delta x$ are absolute errors in a, b and x respectively, then

(*i*) If $x = (a + b)$, then $\Delta x = \pm [\Delta a + \Delta b]$

(*ii*) If $x = (a - b)$, then $\Delta x = \pm [\Delta a + \Delta b]$

(*iii*) If $x = a \times b$, then $\dfrac{\Delta x}{x} = \pm \left[\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right]$

(*iv*) If $x = \dfrac{a}{b}$, then $\dfrac{\Delta x}{x} = \pm \left[\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right]$

(*v*) If $x = \dfrac{a^n b^m}{c^p}$, then $\dfrac{\Delta x}{x} = \pm \left[n \dfrac{\Delta a}{a} + m \dfrac{\Delta b}{b} + p \dfrac{\Delta c}{c}\right]$

19. **Significant Figures**

    In the measured value of a physical quantity, the digits about the correctness of which we are sure plus the last digit which is doubtful, are called the significant figures.

    For counting significant figures, refer to rules in the text.

20. In any mathematical operation involving addition, subtraction, decimal places in the result will correspond to lowest number of decimal places in any of the numbers involved.

21. In a mathematical operation like multiplication and division, number of significant figures in the product or in the quotient will correspond to the smallest number of significant figures in any of the numbers involved.

22. Problems with *accuracy* are due to errors. The *precision* describes the limitation of the measuring instrument.

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. A standard metre is equal to k wavelengths in vacuum, of the radiation from Krypton 86, where k is

(*a*) 165076·37 (*b*) 16507637·3

(*c*) 1650763·73 (*d*) none of these

2. The prefix femto stands for

(*a*) 10^{15} (*b*) 10^{-15}

(*c*) 10^{5} (*d*) 10^{-5}

3. How many disintegrations per second make up 1 curie ?

(*a*) $3·7 \times 10^{10}$ (*b*) $3·7 \times 10^{13}$

(*c*) $3·7 \times 10^{7}$ (*d*) none of these

4. The mass of sun is of the order of

(*a*) 10^{55} kg (*b*) 10^{42} kg

(*c*) 10^{-30} kg (*d*) 10^{30} kg

5. Time interval between two successive heart beats is of the order of

(*a*) 10 s (*b*) 10^{-2} s

(*c*) 10^{0} s (*d*) 10^{-1} s

6. Accuracy of cesium clock is

(*a*) 1 part in 10^{7} (*b*) 1 part in 10^{13}

(*c*) 1 part in 10^{-7} (*d*) 1 part in 10^{-13}

7. Quartz crystal clocks have an accuracy of 1 sec in every

(*a*) 10^{-9} s (*b*) 10^{9} s

(*c*) 10^{-13} s (*d*) 10^{13} s

8. The dimensional formula of universal gravitational constant is

(*a*) $[M^1 L^3 T^{-2}]$ (*b*) $[M^1 L^{-3} T^2]$

(*c*) $[M^{-1} L^3 T^{-2}]$ (*d*) $[M^{-1} L^3 T^2]$

9. $[M^1 L^2 T^{-1}]$ is dimensional formula of

(*a*) Reynold number (*b*) intensity of wave

(*c*) angular impulse (*d*) torque

10. Which one of the following has the same dimension in length as Planck's constant ?

(*a*) coefficient of viscosity

(*b*) rate of flow

(*c*) pressure gradient

(*d*) torque

11. The dimensions of $\frac{a}{b}$ in the relation $F = ax + bt$ are

(*a*) LT^{-1} (*b*) $L^{-1}T$

(*c*) LT (*d*) $L^{-1}T^{-1}$

12. Name the quantity represented by the dimensional formula $[M^1 L^{-3} T^0]$.

(*a*) specific gravity (*b*) linear mass density

(*c*) impulse (*d*) density

13. Momentum per unit volume has the dimensions :

(*a*) MLT^{-1} (*b*) MLT^{-2}

(*c*) $ML^{-2}T^{-1}$ (*d*) ML^2T^{-1}

14. The dimensional formula for conductance is

(*a*) $[M^{-1} L^{-2} T^3 A^{-2}]$ (*b*) $[M^1 L^2 T^{-3} A^2]$

(*c*) $[M^{-1} L^{-2} T^{-3} A^2]$ (*d*) $[M^{-1} L^{-2} T^3 A^2]$

15. If random error in the arithmetic mean of 100 observations is x, then the random error in the arithmetic mean of 500 observations would be

(*a*) $5x$ (*b*) $x/5$

(*c*) $25x$ (*d*) $x/25$

16. The percentage error in determination of $g = 4\pi^2 \frac{l}{t^2}$, when l and t are measured with ± 1% and ± 2% errors is

(*a*) ± 1% (*b*) ± 2%

(*c*) ± 5% (*d*) ± 9%

17. Precision of measurement depends on

(*a*) least count of measuring instrument

(*b*) temperature of the surroundings

(*c*) carefulness of observer

(*d*) all of the above

18. The number of significant figures in 0·008403 is

(*a*) 6 (*b*) 4

(*c*) 3 (*d*) 2

19. In the measured length x = 7·304 cm = 73·04 mm = 0·07304 m = 0·00007304 km, number of significant figures is

(*a*) 7 (*b*) 3

(*c*) 4 (*d*) 8

20. When we add 0·9825 to 3·04, the correct result with regard to significant figures is

(*a*) 4·0225 (*b*) 4·022

(*c*) 4·02 (*d*) 4·0

21. Each side of a cube is measured to be 3·784 m. Its total surface area with appropriate significant figures is
(*a*) 85·911936 m^2 (*b*) 85·9119 m^2
(*c*) 85·911 m^2 (*d*) 89·91 m^2

22. 7·893 gram of a substance occupies a volume of 1·1 cm^3. The density of substance with appropriate significant figures is
(*a*) 7·175 g cm^{-3} (*b*) 7·2 g cm^{-3}
(*c*) 7·18 g cm^{-3} (*d*) 7·1754 g cm^{-3}

23. A student measured the diameter of a wire using a screw gauge with least count 0·001 and listed some measurements. The most correct measurement is
(*a*) 5·3 cm (*b*) 5·32 cm
(*c*) 5·320 cm (*d*) 5·3200 cm

24. A simple pendulum completes 20 oscillations in 25 s. The time is measured with a stop watch of least count 0·2 s. The error in the measurement of times is
(*a*) 3·2% (*b*) 2·4%
(*c*) 1·6% (*d*) 0·8%

25. A spherometer has a least count of 0·005 mm and its head scale is divided into 200 equal divisions. Distance between any two consecutive threads on the spherometer screw is
(*a*) 0·25 mm (*b*) 0·50 mm
(*c*) 0·75 mm (*d*) 1·0 mm

26. Two resistances are expressed as $R_1 = (4 \pm 0{\cdot}5)\ \Omega$ and $R_2 = (12 \pm 0{\cdot}5)\ \Omega$. What is the absolute error in the net resistance, when they are connected in series ?
(*a*) 1 Ω (*b*) 5 Ω
(*c*) 10 Ω (*d*) 15 Ω

27. The errors in the measurement of mass and velocity of a moving body are 2% and 3% respectively. Error, in kinetic energy obtained by measuring mass and speed, will be
(*a*) 12% (*b*) 10%
(*c*) 8% (*d*) 2%

28. In a circuit, the resistance is 10·845 Ω and current is 3·23 A. Its potential difference with proper significant figures will be
(*a*) 35·0 V (*b*) 35·00 V
(*c*) 35·02 V (*d*) 35·03 V

29. If radius of a circle is 2·14 m, then area of the circle, with due regards for significant figures, will be
(*a*) 14·389 m^2 (*b*) 14·39 m^2
(*c*) 14·4 m^2 (*d*) 14·0 m^2

30. Which of the following physical quantity has the dimensions of $[ML^2T^{-3}]$?
(*a*) work (*b*) power
(*c*) pressure (*d*) impulse

31. Dimensions $ML^{-1}T^{-1}$ are related to
(*a*) work (*b*) torque
(*c*) energy
(*d*) coefficient of viscosity

32. The dimensional formula for pressure gradient is
(*a*) $[ML^2T^{-1}]$ (*b*) $[M^0L^2T^{-2}]$
(*c*) $[M^{-2}L^{-2}T^0]$ (*d*) $[ML^{-2}T^{-2}]$

33. A force is given by : $F = at + bt^2$ (where t is time). What are the dimensions of a and b ?
(*a*) $[MLT^{-3}]$ and $[MLT^{-4}]$
(*b*) $[MLT^{-3}]$ and $[MLT^{-2}]$
(*c*) $[MLT^{-1}]$ and $[MLT^0]$
(*d*) $[MLT^{-4}]$ and $[MLT]$

34. The potential energy of a particle varies with distance x from a fixed origin as $V = \left(\frac{A\sqrt{x}}{x+B}\right)$; where A and B are constants. The dimensions of $A \times B$ are
(*a*) $[ML^{5/2}T^{-2}]$ (*b*) $[ML^2T^{-2}]$
(*c*) $[ML^{3/2}L^{3/2}T^{-2}]$ (*d*) $[ML^{7/2}T^{-2}]$

35. Given that time – period (t) of a soap bubble depends upon its radius (r), density (σ) and surface tension (T). The dimensionally correct relation is
(*a*) $t = k\, r^{-1/2}\sigma^{1/2}T^{1/2}$
(*b*) $t = k\, r^{3/2}\sigma^{1/2}T^{1/2}$
(*c*) $t = k\, r^{-1/2}\sigma^{1/2}T^{3/2}$
(*d*) $t = k\, r^{1/2}\sigma^{1/2}T^{1/2}$

36. 1° (degree) in radian is given as :
(*a*) $1{\cdot}746 \times 10^{-2}$ rad (*b*) $1{\cdot}236 \times 10^{-3}$ rad
(*c*) $0{\cdot}768 \times 10^{-2}$ rad (*d*) $3{\cdot}14 \times 10^{-3}$ rad

37. If length of the square is given by $L = (2{\cdot}0 \pm 0{\cdot}4)$ m, then fractional error is given by
(*a*) 0·3 (*b*) 0·02
(*c*) 0·2 (*d*) 0·32

38. If the % error in the measurement of a side of the cube is 2%, then percentage errors respectively in its volume and area are
(*a*) 6%, 4% (*b*) 4%, 6·5%
(*c*) 3%, 2% (*d*) 9%, 2%

39. The time period measured by a simple pendulum is 4 sec. Four students measured the value separately. Which student is most accurate and precise. The values measured by students are

Student A 4·02 sec
Student B 4·001 sec
Student C 3·982 sec
Student D 4.12 sec

(*a*) Student *A* (*b*) Student *B*
(*c*) Student *C* (*d*) Student *D*

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

40. Assertion. The dimensions of velocity gradient are not the same as dimensions of velocity.

Reason. Velocity gradient $= \dfrac{\text{velocity}}{\text{distance}}$

41. Assertion. $[ML^{-1}T^{-2}]$ is the dimensional formula of pressure gradient.

Reason. Pressure gradient is same as pressure.

42. Assertion. one micron is a practical unit of mass.

Reason. 1 micron = 10^{-6} kg.

43. Assertion. Physics deals with the study of nature and natural phenomenon.

Reason. Physics is a biological science.

44. Assertion. Forces of friction are electromagnetic forces.

Reason. These forces always oppose the relative motion.

45. Assertion. Mechanical equivalent of heat (J) is dimensionless.

Reason. $J = \dfrac{W}{H} = \dfrac{ML^2T^{-2}}{ML^2T^{-2}} = [M^0L^0T^0]$

46. Assertion. International system of units (SI) has seven fundamental units and two supplementary units.

Reason. SI covers the measurement of all quantities.

47. Assertion. In microcosm measurement, the smallest unit of distance is angstrom unit, where 1 Å = 10^{-10} m.

Reason. Size of atomic nucleus ~ 10^{-13} m.

48. Assertion. In macrocosm measurement, largest unit of distance is one parsec, where 1 parsec = $3{\cdot}1 \times 10^{16}$ m.

Reason. One light year is bigger than one parsecond.

49. Assertion. One nautical mile is same as one mile.

Reason. Both represent the same distance.

ANSWERS

I. Multiple Choice Questions

1. (*c*) **2.** (*b*) **3.** (*a*) **4.** (*d*) **5.** (*c*) **6.** (*b*) **7.** (*b*) **8.** (*c*) **9.** (*c*) **10.** (*d*)
11. (*b*) **12.** (*d*) **13.** (*c*) **14.** (*d*) **15.** (*b*) **16.** (*c*) **17.** (*a*) **18.** (*b*) **19.** (*c*) **20.** (*c*)
21. (*d*) **22.** (*b*) **23.** (*c*) **24.** (*d*) **25.** (*d*) **26.** (*a*) **27.** (*c*) **28.** (*a*) **29.** (*c*) **30.** (*b*)
31. (*d*) **32.** (*d*) **33.** (*a*) **34.** (*d*) **35.** (*b*) **36.** (*a*) **37.** (*c*) **38.** (*a*) **39.** (*b*)

II. Assertion-Reason Type Questions

40. (*a*) **41.** (*d*) **42.** (*d*) **43.** (*c*) **44.** (*b*) **45.** (*a*) **46.** (*a*) **47.** (*d*) **48.** (*c*) **49.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. 1 standard metre = 1650763·73 wavelengths

3. 1 curie = $3{\cdot}7 \times 10^{10}$ disintegration/sec.

5. Time interval between two successive heart beats = 1 sec = 10^0 s.

6. Accuracy of cesium clock is 10^{-13} sec.

7. Accuracy of quartz crystal clock is 10^{-9} sec.

8. From $F = \dfrac{G m_1 m_2}{r^2}$

$$G = \frac{F \times r^2}{m_1 \times m_2} = \frac{[MLT^{-2}][L^2]}{MM}$$

$$= [M^{-1} L^3 T^{-2}]$$

9. Angular impulse = Torque × time

$$= M L^2 T^{-2} \times T$$

$$= [ML^2 T^{-1}]$$

10. Planck's constant, $h = \dfrac{E}{\nu} = \dfrac{ML^2 T^{-2}}{T^{-1}}$

$$= [ML^2 T^{-1}]$$

Torque = $[ML^2 T^{-2}]$

Both have same dimension in length.

12. $[M^1 L^{-3} T^0] = \dfrac{M^1}{L^3} = \dfrac{\text{mass}}{\text{volume}}$ = density

13. $\dfrac{\text{Momentum}}{\text{Volume}} = \dfrac{MLT^{-1}}{L^3} = [ML^{-2} T^{-1}]$

14. Conductance $= \dfrac{1}{\text{resistance}} = \dfrac{1}{V/I} = \dfrac{I}{V} = \dfrac{I}{W/q}$

$$= \frac{A}{ML^2 T^{-2}/AT}$$

$$= [M^{-1} L^{-2} T^3 A^2]$$

15. Where number of observations becomes 5 times, random error becomes $x/5$.

16. From $g = 4\pi^2 \dfrac{l}{t^2}$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta t}{t}$$

$$\frac{\Delta g}{g} \times 100 = \pm 1\% + (\pm 2 \times 2\%) = \pm \mathbf{5\%}$$

20. 0·9825 + 3·04 = 4·02

as result is correct upto two places of decimal.

22. Density $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{7{\cdot}893\text{ g}}{1{\cdot}1\text{ cm}^2} = 7{\cdot}175\text{ g cm}^3$.

As the result is correct upto one place of decimal

∴ Density = $7{\cdot}2\text{ g cm}^{-3}$.

23. The result is correct upto third place of decimal. Therefore, d = 5·320 cm.

24. Percentage error $= \dfrac{0{\cdot}2}{25} \times 100 = \mathbf{0{\cdot}8\%}$.

25. Distance between any two consecutive threads on the spherometer screw

= 0·005 × 200 mm = **1·0 mm**

26. $R_s = R_1 + R_2$

$\Delta R_s = \Delta R_1 + \Delta R_2 = 0{\cdot}5 + 0{\cdot}5 = 1\ \Omega$

27. KE. $E = \dfrac{1}{2} mv^2$

$$\frac{\Delta E}{E} = \frac{\Delta m}{m} + 2\left(\frac{\Delta v}{v}\right)$$

$$\frac{\Delta E}{E} \times 100 = 2\% + 2\,(3\%) = 8\%$$

28. $V = RI$ = 10·845 × 3·23 = 35·03258

Rounding off to three significant figures,

V = **35·0 V**.

29. Area $= \pi r^2 = \dfrac{22}{7} \times (2{\cdot}14)^2 = 14{\cdot}379944$

Rounding off to three significant digits,

Area = $14{\cdot}4\text{ m}^2$

32. Pressure gradient $= \dfrac{P}{l} = \dfrac{ML^{-1}T^{-2}}{L} = [ML^{-2} T^{-2}]$

34. P.E. $(V) = \dfrac{A\sqrt{x}}{x + B}$

B has dimensions of $x = [L]$

Now, $V = \dfrac{A\sqrt{x}}{x} = \dfrac{A}{\sqrt{x}}$

$$A = V\sqrt{x} = ML^2 T^{-2}\,[L^{1/2}]$$

$$= ML^{5/2} T^{-2}$$

$$A \times B = [ML^{5/2} T^{-2}] \times [L]$$

$$A \times B = [ML^{7/2} T^{-2}]$$

35. Let $t \propto r^a \sigma^k T^c$

$$t = k\, r^a \sigma^b T^c$$

We can show that $a = 3/2$, $b = 1/2$, $c = 1/2$

$\therefore\ t = k\, r^{3/2} \sigma^{1/2} T^{1/2}$.

36. As $180° = \pi$ radian

$$1° = \frac{\pi}{180} \text{ rad} = \mathbf{1{\cdot}74 \times 10^{-2} \text{ rad}}$$

37. Here, $L = (2{\cdot}0 \pm 0{\cdot}4)$ m

$$\frac{\Delta L}{L} = \frac{0{\cdot}4}{2{\cdot}0} = \mathbf{0{\cdot}2}$$

38. As $A = L^2$, therefore,

$$\frac{\Delta A}{A} = 2\frac{\Delta L}{L} = 2 \times 2\% = \mathbf{4\%}$$

Again, $V = L^3$

$$\therefore\ \frac{\Delta V}{V} = 3\frac{\Delta L}{L} = 3 \times 2\% = \mathbf{6\%}$$

39. Errors in measurement by

student $A = 4{\cdot}02 - 4 = 0{\cdot}02$ s

student $B = 4{\cdot}001 - 4 = 0{\cdot}001$ s

student $C = 4 - 3{\cdot}982 = 0{\cdot}018$ s

student $D = 4{\cdot}12 - 4 = 0{\cdot}12$ s

Clearly, measurement by student B is most accurate and most precise.

II. Assertion-Reason Type Questions

40. Both, Assertion and Reason are true and Reason is correct explanation of Assertion.

41. Both, the Assertion and Reason are false.

42. Both, the Assertion and Reason are false.

43. Assertion is true, but the Reason is false.

44. Both, the Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

45. Both, the Assertion and Reason are true, and Reason is correct explanation of the Assertion.

46. Both, the Assertion and Reason are true, and Reason is correct explanation of the Assertion.

47. Both, the Assertion and Reason are false.

48. Assertion is true, but the Reason is false.

49. Both, the Assertion and Reason are false.

CONCEPTUAL PROBLEMS

1. In defining the standard of length, we have to specify the temperature at which the measurement should be made. Are we justified in calling length a fundamental quantity, if another physical quantity (temperature) has to be specified in choosing a standard ?

Sol. The length of an object changes with temperature, but the unit for measuring length (*i.e.*, metre) has now been defined in terms of wavelength of light. This is not affected by temperature. Hence, the choice of length as a fundamental unit is justified.

2. Which type of phenomenon can be used as a measure of time ? Give three examples.

Sol. A phenomenon which repeats itself at regular intervals can be used as a measure of time. For example : (*i*) oscillations of a pendulum,

(*ii*) rotation of earth around its axis,

(*iii*) revolution of earth around the sun.

3. Why has 'second' been defined in terms of periods of radiation from Cesium–133 ?

Sol. Second has been defined in terms of periods of radiation, because

(*i*) this period is accurately defined,

(*ii*) this period is not affected by changes of place, time and physical conditions like temperature.

(*iii*) The unit is easily reproducible in any good laboratory.

4. What is a coherent system of units ?

Sol. A coherent system is based on a certain set of fundamental units, from which derived units are obtained by multiplication or division without introducing numerical factors.

5. Do AU and A° represent the same unit of length ? (HP Board 2008C)

Sol. No, AU and A° represent two different units of length.

1 AU = 1 astronomical unit = 1.496×10^{11} m

1 A° = 1 angstrom = 10^{-10} m

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6. Will five litres of benzene weigh more in summer or winter ? Comment.

Sol. Weight = mg = $(V\rho)\,g$ where ρ is density. Now, V and g are constants. Therefore, weight $\propto \rho$ In summer, temperature rises. Therefore, due to thermal expansion, density ρ decreases, and hence, the weight decreases. Thus, 5 litres of benzene will weigh more in winter than in summer.

7. Is the time variation of position shown in Fig. 2(Q).1 observed in nature ?

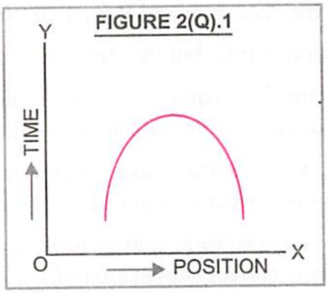
FIGURE 2(Q).1

Sol. No, such a time variation of position is not observed in nature. This is because

(*i*) At a particular instant of time, an object cannot be present at two different positions,

(*ii*) Time increases first and then decreases indicating backward flow of time, which is not possible.

8. Is the measure of an angle dependent on the unit of length ?

Sol. No. This is because

$$\theta\,(\text{radian}) = \frac{\text{length of arc } (l)}{\text{radius } (r)}$$

As θ is the ratio of two lengths, it will not depend upon unit of length.

9. What is meant by angular diameter of moon ? What is its value ?

Sol. Angular diameter of moon is the angle subtended at a point on earth, by two diametrically opposite ends of the moon. Its value is about 0·5°.

I. Measurement of distances

10. Suggest a distance corresponding to each of the following order of length :

(i) 10^7 m **(ii) 10^4 m**

(iii) 10^3 m **(iv) 10^2 m**

(v) 10^{-3} m **(vi) 10^{-6} m**

(vii) 10^{-14} m

Sol. (*i*) 10^7 m — is the order of radius of earth,

(*ii*) 10^4 m — is the order of height of mount everest,

(*iii*) 10^3 m — is the order of distance travelled by sound in air in 3 sec.

(*iv*) 10^2 m — is the order of length of a college playground,

(*v*) 10^{-3} m — is the order of thickness of a cardboard,

(*vi*) 10^{-6} m — mean free path of air molecules,

(*vii*) 10^{-14} m — is the order of size of an atomic nucleus.

11. For a given base line, which will show a greater parallax – a distant star or a nearby star ?

Sol. Parallactic angle, $\theta = \dfrac{\text{base line}}{\text{distance of star}} = \dfrac{b}{s}$

For a given base line, *b* ; θ will be large, when *s* is small. Hence the nearby star will show greater parallax.

12. The screw of a spherometer moves by 4 mm. when its circular scale is given four complete rotations. If circular scale has 200 divisions, calculate pitch and least count of the spherometer.

Sol. $$\text{Pitch} = \frac{\text{distance moved}}{\text{number of complete rotations}}$$

$$= \frac{4\text{ mm}}{4} = 1\text{ mm}$$

$$\text{Least count} = \frac{\text{Pitch}}{\text{no. of divisions on circular scale}}$$

$$= \frac{1\text{ mm}}{200} = 0{\cdot}005\text{ mm}$$

$$= 0{\cdot}0005\text{ cm}$$

II. Measurement of mass and time

13. Mention some repetitive phenomena in nature which could serve as time standards. Which one is most suitable ?

Sol. Some of the repetitive phenomena occurring in nature are :

(*i*) Regular heart beat of a person,

(*ii*) Oscillations of a simple pendulum,

(*iii*) Vibration of atoms,

(*iv*) Rotation of earth about its axis, Earlier, rotation of earth about its axis could be taken as most appropriate. Now, we are shifting to vibration of atoms as the most suitable repetitive phenomenon.

14. Which is the world's most accurate clock ? What is its accuracy ?

Sol. The atomic clock NIST-F1 is considered one of the world's most accurate clocks. It keeps time with an accuracy of one second in twenty million years.

III. Dimensions

15. Do all physical quantities have dimensions ? If no, name three physical quantities which are dimensionless.

Sol. No, all physical quantities do not have dimensions. For example, angle, strain, relative density are dimensionless.

16. Can a quantity having dimensions may have no units ?

Sol. No, a quantity having dimensions must have some units.

17. Can a quantity have units, but still be dimensionless ?

Sol. Yes. For example, angle has units (radian), but is dimensionless.

18. In different systems of units, can a quantity have different dimensions ?

Sol. No, a quantity has same dimensions in all systems of units.

19. Can there be a physical quantity, which has no units and no dimensions ?

Sol. Yes, strain is a physical quantity which has neither units nor dimensions.

20. In the formula $X = 3\,YZ^2$, X and Y have dimensions of capacitance and magnetic induction respectively. The dimensions of Y should be....... . Fill in the blanks.

Sol. From $X = 3\,YZ^2$

$$Y = \frac{X}{3Z^2} = \frac{\text{capacitance}}{(\text{mag. induction})^2}$$

$$= \frac{M^{-1}L^{-2}T^2Q^2}{[MT^{-1}Q^{-1}]^2} = [M^{-3}L^{-2}T^4Q^4]$$

21. The units of Planck's constant are the same as those of......., which is equal to moment of...... Fill in the blanks.

Sol. Angular momentum ; linear momentum.

22. Identify the physical quantity x defined as $x = \frac{I\,F\,v^2}{W\,l^3}$, where I is moment of inertia, F is force, v is velocity, W is work and l is length.

Sol. As $I = [ML^2]$, $F = [ML\,T^{-2}]$, $v = [LT^{-1}]$ and $W = [ML^2T^{-2}]$

$$\therefore \quad x = \frac{I\,F\,v^2}{W\,l^3} = \frac{[ML^2][MLT^{-2}][LT^{-1}]^2}{[ML^2T^{-2}][L^3]}$$

$$= [ML^0\,T^{-2}]$$

These are the dimensions of surface tension; force constant, energy/area.

23. Identify the quantity represented by $CV/\rho\in_0$, where C is capacity, V is potential, ρ is specific resistance and $\in_0$ is electric permittivity of free space.

Sol. As $C = \frac{\in_0 A}{d}$ and $\rho = \frac{RA}{l}$,

where the symbols have their usual meaning, therefore,

$$\frac{CV}{\rho\in_0} = \left(\frac{\in_0 A}{d}\right)\frac{V}{(RA/l)\in_0} = \frac{V}{R} = I \text{, the current}$$

($\because$ l, d both represent distance)

Therefore, the quantity represented is *current*.

IV. Dimensional Analysis

24. Justify $L + L = L$ and $L - L = L$

Sol. When distance is added to distance, we get distance only. This justifies $L + L = L$

Again, when distance is subtracted from distance, we again obtain some distance. This justifies $L - L = L$.

25. Let x and a stand for distance.

Is $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$ dimensionally correct ?

Sol. Integral on LHS has value of the form *log x* which is dimensionless. As RHS is not dimensionless, therefore, the formula is not correct.

26. Use principle of homogeneity of dimensions to find which one of the following relations is correct :

(i) $T^2 = 4\,\pi^2 r^2$ *(ii)* $T^2 = \frac{4\pi^2 r^3}{G}$,

(iii) $T^2 = \frac{4\pi^2 r^3}{GM}$. **(Central Schools 2008)**

Sol. Only relation (*iii*) is correct as

$$\frac{4\pi^2 r^3}{GM} = \frac{L^3}{M^{-1}L^3T^{-2}M} = T^2$$

V. Errors and their propagation

27. Which quantity, in a given formula should be measured most accurately ?

Sol. The quantity which has maximum power (n) should be measured most accurately because any error in the measurement of this quantity is multiplied n times in the final result.

28. 'Problems with accuracy are due to errors'. Is the statement true ?

Sol. Yes, it is true.

29. 'Precision describes the limitation of the measuring instrument'. Is the statement false ?

Sol. No, the statement is true.

30. 'Poor accuracy involves errors that can often be corrected'. Do you agree ?

Sol. Yes, we agree.

31. 'A lack of precision is due to limitation of measuring instrument.' Is it true ?

Sol. Yes, it is true.

32. A physical quantity $x = \frac{a^2 b^{-3/2}}{c^4}$ has a relative error $\frac{\Delta x}{x} = 2\frac{\Delta a}{a} - \frac{3}{2}\frac{\Delta b}{b} - 4\frac{\Delta c}{c}$. Is the statement correct ?

Sol. No, as combined error is additive only without algebraic sign. The correct statement is

$$\frac{\Delta x}{x} = 2\frac{\Delta a}{a} + \frac{3}{2}\frac{\Delta b}{b} + 4\frac{\Delta c}{c}$$

33. Can an instrument be called precise without being accurate ? Can it be accurate without being precise ?

Sol. Yes, an instrument can be precise without being accurate. But the measurements cannot be accurate without being precise.

34. Which of the following length measurement is most accurate and why ?

(*i*) 500·0 cm (*ii*) 0·005 cm (*iii*) 6·00 cm

(CBSE 2014)

Sol. Error in (*i*) is 0·1 cm $= \frac{1}{10}$ cm

Error in (*ii*) is 0·0001 $= \frac{1}{10000}$ cm

Error in (*iii*) is 0·01 cm $= \frac{1}{100}$ cm

∴ Measurement (*ii*) is most accurate, upto fourth place of decimal.

35. Which of the following length measurement is (*i*) most precise and (*ii*) least precise ?

(*a*) l = 5 cm (*b*) l = 5·00 cm

(*c*) l = 5·000 cm (*d*) l = 5·0000 cm.

Sol. l = 5·0000 is most precise as it has been taken with an instrument having least count = 0·0001 cm.

Similarly, l = 5 cm is least precise, as this measurement is taken with an instrument having least count = 1 cm.

36. Of the following, which measurement is most accurate and which one is most precise ?

(*i*) 5·00 mm (*ii*) 5·00 cm

(*iii*) 5·00 m (*iv*) 50·00 m.

Sol. Relative error in measurement of 50·00 m is 0·01 m.

Therefore, it is **most accurate.**

The measurement 5·00 mm is **most precise** as this value is measured using a device with least count of 0·01 mm.

VI. Significant figures and Rounding off

37. In a number without decimal, what is the significance of zeros on the right of non-zero digits ?

Sol. All such zeros are not significant. For example, x = 678000 has only three significant figures.

38. What is the difference between 5·0 and 5·00 ?

Sol. 5·0 indicates that measurement is correct upto first place of decimal, whereas 5·00 indicates that the measurement is correct upto second place of decimal. Thus, for a given quantity, 5·00 is more accurate a value than 5·0.

39. In the expression, surface area = 4 π r^2, the factor 4 is an exact number. How many number of significant figures are there in the factor 4 ?

Sol. In the expression, surface area = 4 π r^2, the factor 4 is an exact number–which has infinite number of significant figures. It can be written as 4 or 4·0 or 4·00 or 4·000 and so on.

40. If all measurements in an experiment are taken upto same number of significant figures, then which mesurement is responsible for maximum error ?

Sol. The maximum error will be due to

(*i*) measurement which is least accurate

(*ii*) measurement of the quantity which has maximum power in the formula.

41. The mean value of period of oscillation of a simple pendulum in an experiment is 2·825 s. The arithmetic mean of all the absolute errors is 0·11 s. Round off the period to appropriate no. of significant figures.

Sol. As mean absolute error is 0·11 s – has two significant digits, therefore mean value of time period should also have two significant digits *i.e.*, $T = \mathbf{2{\cdot}8\ s}$

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

1. Does magnitude of a quantity change with change in the system of units ?

Ans. No, magnitude of a quantity does not change.

2. Can a body have zero mass and zero weight ?

Ans. No, mass of a body can never be zero, but its weight may be zero.

3. Name two types of mass.

Ans. Inertial mass and gravitational mass.

4. Human heart is an inbuilt clock. Comment.

Ans. True, because human heart beats at a regular rate.

5. How many times is a millisecond larger than a microsecond ?

Ans. 10^3 times $\because \quad \frac{1\,\text{millisec}}{1\,\text{microsec}} = \frac{10^{-3}\,\text{s}}{10^{-6}\,\text{s}} = 10^3$

6. How many light years make 1 par sec ?

Ans. 3.26 light years make 1 par sec.

7. What is the accuracy of metre defined in terms of wavelength of light radiation ?

Ans. The accuracy of standard metre so defined is 1 part in 10^9.

8. Name two commonly used units for wavelength of light.

Ans. (*i*) Nanometre (nm) : $1\text{ nm} = 10^{-9}\text{ m}$

(*ii*) Angstrom (Å) : $1\text{ Å} = 10^{-10}\text{ m}$

9. Express 1 micron in metre.

Ans. 1 micron = 10^{-6} metre.

10. Which unit is used to measure size of a nucleus ?

Ans. The size of nucleus is measured in *fermi*

1 fermi = 10^{-15} m

11. How many Angstrom are there in one metre ? **(HP Board 2008C)**

Ans. As $1\text{ Å} = 10^{-10}$ m, therefore,

$$1\,\text{m} = \frac{1}{10^{-10}}\text{ Å} = 10^{10}\text{ Å}.$$

12. How many times larger is a kg than an mg ?

Ans. $\frac{1\,\text{kg}}{1\,\text{mg}} = \frac{10^3\,\text{g}}{10^{-3}\,\text{g}} = 10^6 \quad \therefore\ 1\text{ kg} = 10^6\text{ mg}$

13. Which is the smallest practical unit of time ?

Ans. The smallest practical unit of time is

$$1\text{ shake} = 10^{-8}\text{ s}.$$

14. How many quintals are there in one metric ton ?

Ans. $\frac{1\,\text{metric ton}}{1\,\text{quintal}} = \frac{1000\,\text{kg}}{100\,\text{kg}} = 10$

$\therefore$ 10 quintals = 1 metric ton.

15. What is one carat ?

Ans. Carat is a unit of small mass.

1 carat = 200 mg

16. How many degrees are there in one radian ?

Ans. π radian = 180°

$$1\text{ radian} = \frac{180^\circ}{\pi} = \frac{180\times7}{22} = \frac{630}{11} = \mathbf{57{\cdot}3^\circ}$$

17. Write in ascending order : light year, astronomical unit, par sec.

Ans. Astronomical unit ; light year ; par sec.

18. What is represented by 1 bar ? What is its value in SI units ?

Ans. 1 bar represents one standard atmospheric pressure.

$$1\text{ bar} = 10^5\text{ N/m}^2$$

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19. Is light year a unit of time ?

Ans. No, light year is a unit of distance.

20. Name the unit for measuring nuclear cross sections.

Ans. 1 barn = 10^{-28} m^2

I. Measurement of distances

21. What is the order of magnitude of 499 and 0·050 ?

Ans. $499 = 4{\cdot}99 \times 10^2 = 1 \times 10^2$;
order of magnitude is 2
$0{\cdot}050 = 5{\cdot}0 \times 10^{-2} \rightarrow 10 \times 10^{-2} = 10^{-1}$;
order of magnitude is – 1.

22. What is the order of magnitude of radius of earth ?

Ans.
$$R = 6400 \text{ km} = 6{\cdot}4 \times 10^6 \text{ m} \simeq 10 \times 10^6 \text{ m} = 10^7 \text{ m}$$
Order of magnitude of R is 7.

23. What is the estimated size of observable universe ?

Ans. The estimated size of observable universe is 10^{26} metre.

24. What is the average distance of moon from earth ?

Ans. The distance of moon from earth is of the order of 10^8 m.

25. Which instrument is used for measuring distances upto 10^{-4} m ?

Ans. A vernier callipers is used for this purpose.

26. How far away is the nearest star alpha centuri from earth ?

Ans. The distance of nearest star alpha centuri from earth is $\simeq 10^{16}$ metre.

27. Name the device that can be used to measure the number of wavelengths of light in a given distance.

Ans. Optical interferometer.

28. What does the word LASER stand for ?

Ans. LASER stands for Light Amplification by Stimulated Emission of Radiation.

29. What is the order of size of our galaxy ?

Ans. Size of our galaxy is of the order of 10^{20} metre.

30. What is the order of mean free path of an air molecule ?

Ans. Mean free path of an air molecule is of the order of 10^{-6} m.

31. How are the pitch and least count of a spherometer related ?

Ans. Least count
$$= \frac{\text{pitch}}{\text{total no. of divisions on the circular scale}}$$

32. What is meant by angular diameter of moon ?

Ans. Angular diameter of moon is the angle subtended at a point on earth, by two diameterically opposite ends of the moon. Its value is about 0·5°.

II. Measurement of mass and time

33. What is the order of mass of universe.

Ans. Mass of universe is of the order of 10^{55} kg.

34. Which technique is used for measuring age of rocks, fossils etc. ?

Ans. Age of rocks, fossils etc. is measured using the technique, called Radioactive Dating.

35. Are there more microseconds in a second than the number of seconds in a year ?

Ans. No. Number of microseconds in a second = 10^6 and Number of seconds in a year
$$= 365 \times 24 \times 60 \times 60 = 3{\cdot}1536 \times 10^7.$$

36. Express in scientific notation :
(*i*) 13780 kg (*ii*) 0.00000523 s

Ans. (*i*) $1{\cdot}3780 \times 10^4$ kg
(*ii*) $5{\cdot}23 \times 10^{-6}$ s

37. What is the efficiency of time realisation in cesium atomic clocks ?

Ans. The uncertainty in time realisation of a cesium atomic clock is $\pm\, 1 \times 10^{-13}$. It implies that such clocks gain or loss no more than 3 μs in one year.

38. Which is the most accurate clock ?

Ans. A cesium clock is most accurate. Two cesium clocks may differ only by 1 second after running for 5000 years.

39. What is the order of age of earth ?

Ans. Age of earth is of the order of 10^{17} s.

40. What is human life expectancy ?

Ans. About 100 years which is of the order of 10^9 sec.

41. What is the order of mass of universe ?

Ans. Mass of universe is of the order of 10^{55} kg.

42. What is the smallest mass measured indirectly so far ?

Ans. The smallest mass measured indirectly so far is that of an electron ($\approx 10^{-30}$ kg).

43. What is the shortest time interval measured indirectly so far ?

Ans. The shortest time interval is the time taken by light to cross a distance of nuclear size ($\approx 10^{-22}$ s).

III. Dimensions

44. Name the physical quantities having dimensions $[M^1 L^2 T^{-2}]$.

Ans. Work, energy and torque.

45. Name two physical quantities which have dimensions $[M^1 L^{-1} T^{-2}]$.

Ans. The dimensions of pressure and stress are $[M^1 L^{-1} T^{-2}]$.

46. How many times the unit of energy is affected when units of force and length are doubled ?

Ans. The unit of energy becomes 4 times. This is because energy = work = force × distance.

47. What are the dimensions of rate of flow ?

Ans. $\text{Rate of flow} = \dfrac{\text{Volume}}{\text{time}} = \dfrac{L^3}{T} = [M^0 L^3 T^{-1}]$

48. Give two examples of non dimensional variables.

Ans. Strain and specific gravity.

49. Name any three dimensional constants.

Ans. Velocity of light in vacuum ; Gravitational constant and Planck's constant.

50. Name any two non dimensional constants.

Ans. All trigonometrical ratios, π, e etc. are non dimensional constants.

51. The dimensional formula of Hubble constant is same as that of frequency. Comment.

Ans. Yes, true. Hubble constant

$$= \frac{\text{velocity}}{\text{distance}} = \frac{LT^{-1}}{L} = [T^{-1}]$$

which represents frequency.

52. Can there be a physical quantity which has no units and no dimensions.

Ans. Yes, for example, strain has no units and no dimensions.

53. All constants are dimensionless. Comment.

Ans. No, all constants are not dimensionless. For example, universal gravitational constant, Planck's constant etc. do have dimensions.

54. Can a quantity have units, but still be dimensionless ?

Ans. Yes, for example, angle is dimensionless, but it has units.

55. What are the dimensions of rate of flow ?

Ans. Rate of flow represents volume flowing per second. Its dimensional formula is $[L^3 T^{-1}]$

56. What are the dimensions of linear mass density ?

Ans. linear mass density = mass/length = $[M^1 L^{-1}]$

57. What type of quantity is Avogadro's number?

Ans. Avogadro's number is a dimensionless constant measured in mole^{-1}.

IV. Dimensional Analysis

58. What are the dimensions of a and b in the relation $F = at + bx$, where F is force and x is distance ?

Ans. $a = \dfrac{F}{t} = \dfrac{MLT^{-2}}{T} = [M^1 L^1 T^{-3}]$,

$$b = \frac{F}{x} = \frac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$$

59. The dimensions of Boltzmann constant are the same as that of

(i) pressure density (ii) Stefan's constant
(iii) Planck's constant (iv) entropy

Ans. Boltzmann constant

$$k = \frac{R}{N} = \frac{PV}{TN} = \frac{W}{TN} = \frac{ML^2 T^{-2}}{K}$$

$$= [M^1 L^2 T^{-2} K^{-1}]$$

$$\text{Entropy} = \frac{Q}{T} = \frac{ML^2 T^{-2}}{K}$$

$$= [M^1 L^2 T^{-2} K^{-1}]$$

Choice (*iv*) is correct.

V. Errors and their propagation

60. Measuring process is essentially ....

Fill in the blanks

Ans. a process of comparison.

61. What is error of measurement ?

Ans. It is the difference in the true value and measured value of a quantity.

62. What are personal errors ?

Ans. These are the errors which arise due to the inexperience of the observer.

63. What is the reliability of measurement of length using a metre scale ?

Ans. It is upto $\dfrac{1}{10}$ cm or $\dfrac{1}{1000}$ metre.

64. Maximum absolute error in difference of two quantities is equal to ...... of the absolute errors in the individual quantities. Fill in the blanks.

Ans. sum.

65. Fill in the blanks :

(*i*) Maximum .... error in product of quantities is .... of .... errors in the individual quantities.

(*ii*) Maximum .... error in a quantity raised to power (*n*) is .... the .... error in the individual quantity.

Ans. (*i*) fractional ; sum ; fractional.

(*ii*) fractional ; *n* times ; fractional.

66. Which of the following length measurement is most accurate and why ?

(*i*) 2·0 cm (*ii*) 2·00 cm (*iii*) 2·000 cm.

Ans. (*iii*), because it is correct upto third place of decimal.

67. What is error in density of a cube when its mass is uncertain by ± 2 % and length of its edge is uncertain by ± 1% ?

Ans. As $\rho = \frac{M}{V} = \frac{M}{L^3}$

$$\therefore \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L} = \pm(2\% + 3 \times 1\%)$$

$$= \pm 5\ \%$$

VI. Significant figures and Rounding off

68. Find the number of significant figures in 0·005.

Ans. one.

69. Round off to four significant figures

(*i*) 36·879 (*ii*) 1·0084

Ans. 36·88 ; 1·008.

70. Round off the following numbers as indicated :

(*i*) 25·653 to 3 digits

(*ii*) 4·996 × 10⁵ to 3 digits

(*iii*) 0·6995 to 1 digit

(*iv*) 3·350 to 2 digits

(*v*) 3·450 to 2 digits.

Ans. (*i*) 25·7 (*ii*) $5{\cdot}00 \times 10^5$

(*iii*) 0·7 (*iv*) 3·4(*v*) 3·4.

71. Solve with due regard to significant figures :

$$\sqrt{6{\cdot}5 - 6{\cdot}32}$$

Ans. $\sqrt{6{\cdot}5 - 6{\cdot}32} = \sqrt{0{\cdot}18} = \sqrt{0{\cdot}2}$, upto one decimal place = 0·44 (having 2 significant figures)

72. Change of units does not change the number of significant figures in a measurement. Is it true ?

Ans. Yes, it is true. For example $x = 5{\cdot}608$ cm = 56·08 mm = 0·05608 m ; all have four significant figures.

73. Round off 3·250 and 3·750 to one place of decimal.

Ans. 3·250 → 3·2 and 3·750 → 3·8, as per prescribed rules.

74. Subtract 10·5 from 10·587 and express the result with correct number of significant figures.

Ans. 10·587 – 10·5 = 0·087 → 0·1, rounding off to one place of decimal.

75. Add 0·4382 to 876 and write the result with correct number of significant figures.

Ans. 876 + 0·4382 = 876·4382 → 876, rounding off to no decimal point.

SHORT ANSWER QUESTIONS

Carrying 2 marks

1. Which of these is largest : astronomical unit, light year and par sec ?

Ans. The unit par sec is larger than light year which in turn is larger than an astronomical unit.

2. Name three units which can be used for measuring large masses.

Ans. 1 slug = 14·57 kg ; 1 quintal = 100 kg and 1metric ton = 1000 kg.

3. Which unit can be used for measurement of very small masses ?

Ans. 1 atomic mass unit (a.m.u) = $1{\cdot}66 \times 10^{-27}$ kg.

4. How many a.m.u. make 1 kg ?

Ans. As $1{\cdot}66 \times 10^{-27}$ kg = 1 a.m.u

$$\therefore 1\text{kg} = \frac{1}{1{\cdot}66 \times 10^{-27}}\ \text{a.m.u.}$$

$$= 0{\cdot}6 \times 10^{27}\ \text{a.m.u.}$$

5. What is common between bar and torr ?

Ans. Both, bar and torr are the units of pressure.

1 bar = 1 atmospheric pressure = 760 mm of Hg. col. = 10^5 N/m^2

1 torr = 1 mm. of Hg col. ∴ 1 bar = 760 torr.

6. Why are length, mass and time chosen as fundamental quantities in mechanics ?

Ans. Length, mass and time are chosen as the fundamental quantities in mechanics because :

(*i*) they represent our basic scientific notations,

(*ii*) there is nothing simpler to them,

(*iii*) they cannot be obtained from one another,

(*iv*) all other quantities in mechanics can be obtained from them.

7. SI is rational system of units while MKS system is not rational. Why ?

Ans. SI is a rational system because it assigns only one unit to a particular physical quantity. For example, SI unit of all types of energy is **joule.** On the contrary, on MKS system, mechanical energy is measured in joule, heat energy is measured in calorie, electric energy is measured in watt-hour. So MKS system is not rational.

8. Why is platinum iridium alloy used in making prototype metre and kilogram ?

Ans. This is because this alloy

(*i*) is least affected by temperature variations

(*ii*) is non corrosive, does not wear out easily

(*iii*) does not change with time.

I. Measurement of distances

9. What is meant by order of magnitude of a quantity ?

Ans. Order of magnitude of a quantity is defined as the power of 10, which gives us a value nearest to the actual value of the quantity.

10. Suggest a distance corresponding to each of the following order of length.

(*i*) 10^7 m (*ii*) 10^{-6} m (*iii*) 10^4 m

Ans. (*i*) 10^7 m is the order of radius of earth.

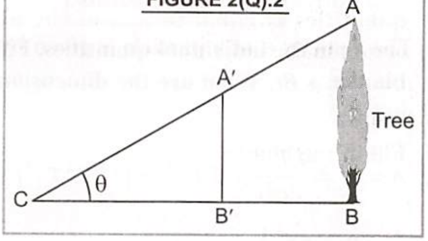
(*ii*) 10^{-6} m is the order of mean free path of air molecule.

(*iii*) 10^4 m is the order of height of mount everest.

11. Suggest some indirect method for measuring the height of a tree on a sunny day.

Ans. In Fig. 2(Q).2, AB is height of a tree. On a sunny day, let the shadow of tree be CB, where $\angle ACB = \theta$.

FIGURE 2(Q).2



Take a rod $A'B'$ and fix it at such a position that the tip of shadow of rod coincides with the point C.

From Fig. 2(Q).2,

$$\tan\theta = \frac{A'B'}{CB'} = \frac{AB}{CB}$$

$$\therefore \quad AB = \frac{A'B' \times CB}{CB'}$$

Measuring $A'B'$, CB' and CB, we can calculate height AB of the tree.

12. Write the full name of the technique used in locating (*a*) position of an aeroplane in space, (*b*) position of an object under water.

Ans. (*a*) RADAR, *i.e.*, radio detection and ranging,

(*b*) SONAR, *i.e.*, sound navigation and ranging.

II. Measurement of mass and time

13. Who maintains Indian Standard Time ?

Ans. The Indian Standard Time (IST) is maintained by the National Physical Laboratory (NPL) in New Delhi ; using a cesium atomic clock.

14. What is the basic difference between inertial mass, gravitational mass and weight of a body ?

Ans. Inertial mass of a body is measure of inertia of the body. It is equal to force required to produce unit acceleration in the body.

Gravitational mass of a body is the force experienced by the body in a gravitational field of unit intensity.

Weight of a body is the force with which the body is attracted towards the centre of earth.

15. What is the technique used for measuring large time intervals ?

Ans. For measuring large time intervals, we use the technique of radioactive dating. Large time intervals are measured by studying the ratio of number of radioactive atoms decayed to the number of surviving atoms in the specimen.

III. Dimensions

16. The velocity v of a particle is given by $v = At^2 + Bt$. What are the dimensions of A and B ?

Ans. $A = \frac{v}{t^2} = \frac{LT^{-1}}{T^2} = LT^{-3} = [M^0L^1T^{-3}]$

$B = \frac{v}{t} = \frac{LT^{-1}}{T} = LT^{-2} = [M^0L^1T^{-2}]$

17. Which of the following has the same dimension as Planck's constant : Torque, gravitational constant, angular momentum ?

Ans. Angular momentum = moment of momentum

$= MLT^{-1} \times L$

$= [ML^2\,T^{-1}]$

Free $E = h\nu$, $h = \frac{E}{\nu} = \frac{ML^2T^{-2}}{T^{-1}} = [ML^2\,T^{-1}]$

18. Give names of a scalar quantity and a vector quantity which have same dimensions.

Ans. Work is scalar and torque is vector. Both have the same dimensions $[ML^2T^{-2}]$.

19. Give two examples each of dimensionless constants and dimensional variables.

Ans. Dimensionless constants are pure numbers and mathematical constants like π, e etc. Dimensional variables have dimensions and variable value. For example, velocity, force etc.

20. Can a quantity have constant value and be dimensionless ?

Ans. Yes, Joule's mechanical equivalent of heat has a constant value, but is dimensionless.

21. Give three examples of dimensionless variables.

Ans. Three examples of dimensionless variables are specific gravity, angle, strain.

IV. Dimensional Analysis

22. Momentum per unit volume ; divided by pressure represents reciprocal of velocity. Comment.

Ans. True. $\frac{\text{Momentum/Volume}}{\text{pressure}} = \frac{MLT^{-1}/L^3}{MLT^{-2}/L^2}$

$= TL^{-1} = (LT^{-1})^{-1} = v^{-1} = \frac{1}{v}.$

23. Which physical quantity is represented by $\sqrt{\lambda g}$, where λ is wavelength and g is acceleration due to gravity ?

Ans. $\sqrt{\lambda g} = \sqrt{(L)(LT^{-2})} = [LT^{-1}]$.

This represents speed or velocity.

24. In the relation $C = \nu\lambda$ true dimensionally ?

Ans. Yes, $C = LT^{-1}$

and $\nu\lambda = \left(\frac{1}{T}\right)(L) = LT^{-1}$.

25. In the equation $y = A \sin(\omega t - kx)$, obtain the dimensional formula of ω and k. Given x is distance and t is time.

Ans. The given equation is $y = A \sin(\omega t - kx)$

The argument of a trigonometrical function, *i.e.*, angle is dimensionless

i.e., $\omega t = \theta$

$\omega = \frac{\theta}{t} = \frac{1}{T} = T^{-1} = [M^0\,L^0\,T^{-1}]$

Also, $kx = \theta$

$k = \frac{\theta}{x} = \frac{1}{L} = L^{-1} = [M^0\,L^{-1}\,T^0]$

V. Errors and their propagation

26. The mass of a body is measured by two persons is 10·2 kg and 10·23 kg. Which is more accurate and why ?

Ans. The value $m = 10{\cdot}23$ kg is more accurate, being correct upto 2nd place of decimal.

27. When $y = x^4$, what is the relative error in y ?

Ans. $\frac{\Delta y}{y} = 4\left(\frac{\Delta x}{x}\right)$, *i.e.*, it is four times the relative error in x.

28. When you take 500 observations instead of 100 observations of a measurement, by what factor is probable error reduced ?

Ans. The probable error reduces to 1/5th.

29. What is a systematic error ? How can it be removed ?

Ans. Systematic error is that which is always positive or always negative. Such an error can be removed by detecting the source of error and applying the necessary correction.

30. Which of the following lengths measured is most accurate and why ?

(a) 500·0 cm **(b) 0·0005 cm**

(c) 6·00 cm **(Central Schools 2014)**

Ans. The measurement (*b*) 0·0005 cm is most accurate as it is correct upto fourth place of decimal.

VI. Significant figures and Rounding off

31. Add $6{\cdot}75 \times 10^3$ cm to $4{\cdot}52 \times 10^2$ cm.

Ans. $x = 6{\cdot}75 \times 10^3$, $y = 4{\cdot}52 \times 10^2$ cm

$= 0{\cdot}452 \times 10^3$ cm

$x + y = (6{\cdot}75 \times 10^3 + 0{\cdot}452 \times 10^3)$ cm

$= 7{\cdot}202 \times 10^3$ cm $= \mathbf{7{\cdot}20 \times 10^3}$ **cm**

(upto two places of decimal).

32. A substance weighing 5·74 g occupies a volume of 1·2 cm³. Calculate its density with due regard to significant digits.

Ans. $\rho = \frac{M}{V} = \frac{5{\cdot}74}{1{\cdot}2} = 4{\cdot}78$ g/cc = **4·8 g/cc**, rounded off to have two significant digits.

33. What importance is attached to the final zeros in a number without any decimal point ?

Ans. Such zeros are treated as non significant.

For example, in $x = 86400$, number of significant figures is **3**.

34. State the number of significant figures in (*i*) 0·007 m (*ii*) $2{\cdot}67 \times 10^{-24}$ kg

(CBSE 2014)

Ans. (*i*) 0·007 m has **one** significant digit, 7.

(*ii*) $2{\cdot}67 \times 10^{-24}$ kg has **three** significant digits, 2, 6, 7.

35. Add 8·2 and 10·163 and round off the sum to proper number of significant figures.

Ans. 8·2 + 10·163 = 18·363

Rounding off to one decimal place, sum = **18·4**

36. The mean value of period of oscillation of a simple pendulum in an experiment is 2·825 s. The arithmetic mean of all the absolute errors is 0·11 s. Round off the period of simple pendulum to the approximate number of significant figures. Give reason.

(Central Schools 2008)

Ans. The absolute error = 0·11 s has two significant figures. Therefore, period of oscillation of simple pendulum, rounded off to two significant digits = **2·8 s**

SHORT ANSWER QUESTIONS Carrying 3 marks

1. What are the essential requirements for the choice of standard unit ? Who decides the units ? [Art. 2.7]

2. Why *mks* system had to be rationalised to obtain SI ? Define the unit of temperature on SI. [Art. 2.8]

3. Name the fundamental units and supplementary units on SI and their symbol of representation. [Art. 2.9]

4. What are the advantages of SI ?[Art. 2.10]

5. Define Astronomical unit, light year and parsecond. Establish relation between them. [Art 2.11]

6. What is meant by giga, micro and femto ? Establish the relation between them. [Art. 2.12]

7. What is meant by order of magnitude ? Illustrate with atleast three examples ? [Art. 2.14]

8. What are the instruments used for the measurement of length from 10^{-5} m to 10^2 m. Give the least count of each instrument. [Art. 2.16]

9. How do you measure the size of the molecule of oleic used ? [Art. 2.18]

10. Name the quantities represented by the dimensional formula

$[M^1 L^2 T^{-1}]$; $[M^1 L^2 T^{-2}]$ $[M^1 L^{-3} T^0]$.

Ans. $[M^1 L^2 T^{-1}] = [M^1 L^2 T^{-2}]\,(T) \rightarrow$ joule–sec which are the units of **Planck's constant.**

$[M^1 L^2 T^{-2}]$ is the dimensional formula of work or **energy.**

$[M^1 L^{-3} T^0]$ is the dimensional formula of **density.**

11. Choose the pairs of quantities which have same dimensions : Impulse, force, work, momentum, moment of force, tension.

Ans. (*i*) Impulse and momentum have the same dimensions : $[M^1 L^1 T^{-1}]$

(*ii*) Force and tension have the same dimensions : $[M^1 L^1 T^{-2}]$.

(*iii*) Work and moment of force have the same dimensions : $[M^1 L^2 T^{-2}]$.

12. The dimensions of quantities in one or more of the following pairs are the same. Identify the pair (s).

(i) Torque and work

(ii) Angular momentum and work

(iii) Energy and Young's modulus

(iv) Light year and wavelength.

Ans. (*i*) Torque and work, the dimensional formula of each is $[M^1 L^2 T^{-2}]$

(*ii*) Light year and wavelength, both represent distance $[L]$.

13. Find the dimensions of Planck's constant. If its value in cgs system is 6·62 × 10^{-27} erg-sec, what will be its value on mks system ?

Ans. $h = \frac{E}{\nu} = \frac{[ML^2 T^{-2}]}{[T^{-1}]} = [M^1 L^2 T^{-1}]$

$h = 6{\cdot}62 \times 10^{-27}$ erg-sec

$= 6{\cdot}62 \times 10^{-27} \times 10^{-7}$ joule-sec

$= \mathbf{6{\cdot}62 \times 10^{-34}}$ **Js**

14. Calculate the dimensions of universal gravitational constant. If its value in SI units is 6·67 × 10^{-11}, what will be its value in cgs system ?

Ans. From $F = \frac{G m_1 m_2}{r^2}$

$$G = \frac{F r^2}{m_1 m_2} = \frac{(MLT^{-2})(L^2)}{M^2} = [M^{-1} L^3 T^{-2}]$$

$G = 6{\cdot}67 \times 10^{-11}\ \text{kg}^{-1}\ \text{m}^3\ \text{s}^{-2}$

$= 6{\cdot}67 \times 10^{-11}\ (10^3\ \text{g})^{-1}\ (10^2\ \text{cm})^3\ \text{s}^{-2}$

$= \mathbf{6{\cdot}67 \times 10^{-8}\ g\ cm^3\ s^{-2}}$.

15. What are the limitations of dimensional analysis ?

Ans. Art. 2.31.

16. A capacitor of capacitance $C = (2{\cdot}0 \pm 0{\cdot}1)\ \mu$ F is charged to a voltage $V = (20 \pm 0{\cdot}5)$ volt. Calculate the charge Q with error limits.

Ans. $Q = CV = 2{\cdot}0 \times 20 = 40$ micro coulomb

$= 40 \times 10^{-6}$ C

$$\frac{\Delta Q}{Q} = \frac{\Delta C}{C} + \frac{\Delta V}{V} = \frac{0{\cdot}1}{2{\cdot}0} + \frac{0{\cdot}5}{20} = \frac{3}{40}$$

$$\Delta Q = \frac{3}{40} \times Q = 3{\cdot}0\,\mu\text{C}$$

Hence, $\mathbf{Q = (40 \pm 3{\cdot}0) \times 10^{-6}}$ **C**

17. Which of the following length measurment is most accurate and why ?

(*i*) 4·00 cm (*ii*) 0·004 mm (*iii*) 40·00 cm.

Ans. (*i*) $\frac{\Delta x}{x} = \frac{0{\cdot}01}{4{\cdot}00} = 0{\cdot}0025$

(*ii*) $\frac{\Delta x}{x} = \frac{0{\cdot}001}{0{\cdot}004} = 0{\cdot}25$

(*iii*) $\frac{\Delta x}{x} = \frac{0{\cdot}01}{40{\cdot}00} = 0{\cdot}00025$

The last observation has the least fractional error and hence it is most accurate.

18. What is meant by accuracy and precision of measuring instruments ?

Ans. Art. 2.35.

19. What is meant by significant figures ? Give any four rules for counting significant figures.

Ans. Arts. 2.36 and 37.

20. What is meant by permissible error in a result ? Give examples.

Ans. Art. 2.40.

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Explain the need for the measurements in Physics and briefly explain the measuring process of any physical quantity. **[Arts. 2.1 and 2.2]**
2. Explain the concept of mass, length and time. Why mass, length and time are called fundamental quantities. **[Arts. 2.3, 2.4, 2.5 and 2.6]**
3. What do you know about SI ? Define the fundamental and supplementary units on SI ? **[Art. 2.9]**
4. Explain how you will measure (*i*) the size of astronomical object and (*ii*) the distance of a nearby star. **[Art. 2.17]**
5. Discuss briefly the methods used for the measurement of small and large time intervals. **[Art. 2.22]**
6. Explain what is meant by dimensions of a physical quantity, its dimensional formula and dimensional equation. **[Arts. 2.23 and 2.24]**

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7. Derive the dimensional formulae for acc. due to gravity, constant of gravitation, surface tension, coefficient of viscosity, coefficient of elasticity, Planck's constant, gas constant, torque, specific gravity and impulse. **[Art. 2.25]**

8. Explain the principle of homogeneity of dimensions. What are its uses ? Illustrate by giving one example of each. **(J & K Board 2011)** **[Arts. 2.29 & 2.30]**

9. Explain the uses of dimensional equations giving atleast one example in each case. **[Arts. 2.28, 2.29 & 2.30]**

10. What do you understand by errors of measurement ? Discuss briefly the various types of errors. **[Art. 2.32]**

11. Discuss how errors propagate in sum, difference, product and division of quantities. **[Art. 2.34]**

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. The advantages of SI are the following :

(*i*) It is a coherent system of units,

(*ii*) It is a rational system of units,

(*iii*) It is an absolute system of units,

(*iv*) It is a metric system

(*v*) In current electricity, absolute units on SI happen to be the practical units.

Note that SI applies to all branches of science whereas *mks* system is confined to mechanics only.

(*a*) In macro-cosm measurements, we use

(*i*) **Astronomical unit** (AU)

$$1 \text{ AU} = 1{\cdot}496 \times 10^{11} \text{ m.}$$

(*ii*) **Light year** is distance travelled by light in vacuum in one year.

$$1 \text{ ly} = 9{\cdot}46 \times 10^{15} \text{ m}$$

(*iii*) **Par sec** is the radius of a circle, at the centre of which an arc of circle 1 AU long subtends an angle of 1″.

$$1 \text{ par sec} = 3{\cdot}1 \times 10^{16} \text{ m.}$$

(*b*) In micro-cosm measurement, we use

(*i*) 1 micron = 10^{-6} m

(*ii*) 1 nanometer = 10^{-9} m

(*iii*) 1 angstrom (1 Å) = 10^{-10} m

(*iv*) 1 fermi (fm) = 10^{-15} m

Some of the other commonly used units are :

(*i*) 1 barn = 10^{-28} m^2

(*ii*) 1 hactare = 10^4 m^2

(*iii*) 1 atomic mass unit (amu) = $1{\cdot}66 \times 10^{-27}$ kg

(*iv*) 1 bar = 1 atmospheric pressure = 10^5 N/m^2

(*v*) 1 torr = 1 mm of Hg col. Therefore, 1 bar = 760 torr.

Commonly used metric prefixes for powers of 10 are :

deci (*d*) 10^{-1} ; *centi* (*c*) 10^{-2} ; milli (*m*) 10^{-3}
micro (μ) 10^{-6} ; nano (*n*) 10^{-9} ; pico (*p*) 10^{-12}
deca (da) 10^1 ; hecto (*h*) 10^2 ; kilo (*k*) 10^3,
mega (M) 10^6.

Order of magnitude of a quantity is the power of 10 which gives us a value nearest to the actual value of the quantity. For this, a number less than 5 is treated as 1 and a number between 5 and 10 is treated as 10.

Read the above passage carefully and answer the following very short answer and short answer questions :

1. Out of astronomical unit, light year and parsec, which unit is the largest ?

2. If 1 torr represents pressure of 1 mm of Hg column and one bar represents atmospheric pressure, then how are bar and torr related ?

3. Out of fermi, angstrom, nanometer and micron, which is the shortest unit of distance.

4. What is meant by order of magnitude ? Radius of earth is 6400 km. What is its order of magnitude ?

CASE 2. *The dimensions* of a physical quantity are the powers (or exponents) to which the units of base quantities are raised to represent a derived unit of that quantity.

The expression which shows how and which of the base quantities represent a physical quantity is called the *dimensional formula* of the quantity.

When a physical quantity is equated to its dimensional formula, we get the *dimensional equation* of the quantity.

Four types of quantities are :

(*i*) Dimensional constants,

(*ii*) Dimensional variables,

(*iii*) Dimensionless constants,

(*iv*) Dimensionless variables.

Dimensional equations are used for

(*i*) Conversion of one system of units into another

(*ii*) Checking the correctness of various formulae

(*iii*) Derivation of formulae.

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For conversion of one system of units into another, we use

$$Q = n_1 u_1 = n_2 u_2 = n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$$

$$\therefore\ n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

This equation is to be applied only after expressing the quantity in its absolute units.

For checking the correctness of formulae and also for derivation of formulae, we use the principle of homogeneity of dimensions. According to this principle, only that formula is correct, in which dimensions of various fundamental units on one side of the relation are equal to their respective dimensions on the other side of the relation.

Read the above passage carefully and answer the following very short answer and short answer questions :

5. The dimensional formula of which quantity is $[ML^{-1}T^{-1}]$?
6. What is the correct dimensional formula of universal gravitational constant (G) ?
7. What is the correct dimensional formula of specific heat ?
8. In the Van der waal's equation

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT,$$ where p is pressure, V is volume, T is temp. R is gas constant. What are the dimensions of $(a \times b)$?

CASE 3. The expression which shows how and which of the base quantities represent a physical quantity is called the **dimensional formula** of the quantity.

When a physical quantity is equated to its dimensional formula, we obtain dimensional equation of the quantity. For example

(*i*) Pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$

(*ii*) Moment of inertia = mass × (radius of gyration)^2

$$= [ML^2T^0]$$

(*iii*) Planck's constant,

$$h = \frac{E}{\nu} = \frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$$

In thermal quantities, we represent temperature in degree Kelvin (K). In electrical quantities, ampere (A) is fundamental unit of current and so on. For example,

(*iv*) Stefan's constant, $\sigma = \dfrac{E}{T^4} = \dfrac{\text{energy/sec/area}}{T^4}$

$$= \frac{ML^2T^{-2}}{T\,L^2\,K^4} = [M^1L^0T^{-3}K^{-4}]$$

(*v*) Electric permittivity ($\in_0$)

From $F = \dfrac{q_1 q_2}{4\pi \in_0 r^2}$, $\in_0 = \dfrac{q_1 q_2}{4\pi F r^2}$

$$= \frac{[AT]^2}{[MLT^{-2}]\,L^2}$$

$$\in_0 = [M^{-1}L^{-3}T^4A^2]$$

and so on.

Read the above passage carefully and answer the following very short answer and short answer questions :

9. What are the dimensions of Avogadro's number (N) ?
10. $[ML^{-2}T^{-2}]$ is the dimensional formula of which quantity ?
11. What is the dimensional formula of relative electric permittivity ($\in_r$) ?
12. What is entropy ? What is its dimensional formula ?

ANSWERS

1. Out of the three given units, **Parsec** is the largest.
2. Atmospheric pressure = 1 bar = 760 mm of Hg col. = 760 torr
3. Out of given units, shortest unit of distance is 1 fermi = 10^{-15} m.
4. Order of magnitude of a quantity = power of 10 which gives us a value nearest to the actual value of the quantity.

$R = 6{\cdot}4 \times 10^6$ m $\simeq 10^7$ m. The order of magnitude is **7**.

5. $[ML^{-1}T^{-1}]$ is the dimensional formula of coefficient of viscosity.
6. From $F = \dfrac{G\,m_1 m_2}{r^2}$

$$G = \frac{F r^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{M^2} = [M^{-1}L^3T^{-2}]$$

7. From $Q = sm\,\theta$

$$s = \frac{Q}{m\theta} = \frac{ML^2 T^{-2}}{MK} = [M^0 L^2 T^{-2} K^{-1}]$$

8. Here, $p = \frac{a}{V^2}$ or $a = p\,V^2 = ML^{-1}\,T^{-2} \times [L^3]^2$

$$= ML^5\,T^{-2}$$

and $b = V = L^3$

$\therefore\ a \times b = [ML^5\,T^{-2}] \times L^3 = [M\,L^8\,T^{-2}]$

9. Avogadro's number (N) = number of atoms in one gram atom/mole.

$$= [M^0\,L^0\,T^0]$$

10. $ML^{-2}\,T^{-2}$ is the dimensional formula of

$$\text{pressure gradient} = \frac{\text{pressure}}{\text{distance}} = \frac{ML^{-1}T^{-2}}{L} = [ML^{-2}\,T^{-2}]$$

11. Relative electric permittivity $(\in_r)$

$$\in_r = \frac{\in}{\in_0} = [M^0\,L^0\,T^0] \rightarrow \text{dimensionless}$$

12. Entropy of a system is heat capacity of the system.

It is measured in joule/kelvin $= \frac{ML^2\,T^{-2}}{K}$

$$= [ML^2\,T^{-2}\,K^{-1}]$$

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. Measurement is the foundation of all experimental science and technology. **The measuring process is essentially a process of comparison.** The difference in the true value and the measured value of a quantity is called error of measurement, *i.e.,* error in a quantity = (true value – measured value) of the quantity.

The errors in measurement can be broadly classified as :

(*i*) **Systematic errors,** the causes of which are known. Such errors can be minimised.

(*ii*) **Random errors,** which occur irregularly due to unpredictable variations in experimental conditions.

(*iii*) **Gross errors** which occur on account of shear carelessness of the observer.

Random errors often follow the well known Gaussian Law of normal distribution. So these errors can be minimised by repeating the observation a large number of times and taking arithmetic mean of all the observations.

Absolute error in the measurement of a physical quantity is the magnitude of difference between the true value and individual measured value of the quantity. The absolute errors may be positive in certain cases and negative in other cases. **Mean absolute error** is the arithmetic mean of the absolute errors in all the measurements. While taking arithmetic mean, we ignore the (±) sign of absolute errors.

$$\Delta a_{\text{mean}} = \frac{1}{n}\sum_{i=1}^{i=n} |\,\Delta a_i\,|$$

Relative error or fractional error, $\delta_a = \frac{\Delta a_{\text{mean}}}{a_m}$

Percentage error, $\delta_a = \frac{\Delta a_{\text{mean}}}{a_m} \times 100\%$

Based on the above paragraph, answer questions no. 1 to 4 :

1. Measuring process is

(*a*) process of comparison

(*b*) process of finding error

(*c*) process of minimising error

(*d*) none of these

2. Which of the following follow the well known Gaussian Law of normal distribution ?

(*a*) Systematic errors

(*b*) Random errors

(*c*) Gross errors

(*d*) all the above

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, A and R are true and R is correct explanation of A.

(*b*) Both, A and R are true, but R is **not** a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) Both, A and R are false.

3. Assertion. The magnitude of difference between the true value and measured value of the quantity represents absolute error in the quantity.

Reason. The absolute error may be positive or negative.

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4. **Assertion.** Mean absolute error in a quantity is the arithmetic mean of absolute errors in all the measurements.

Reason. While taking arithmetic mean, we ignore the (±) sign of absolute errors.

CASE 2. As is known, the result of an experiment is calculated by performing mathematical operations, like addition, subtraction, multiplications, division, raising to some power etc. on several measurements. The propagation of errors in five different mathematical operations are :

(*i*) If $x = (a + b)$, $\Delta x = \pm (\Delta a + \Delta b)$

(*ii*) If $x = (a - b)$, $\Delta x = \pm (\Delta a + \Delta b)$

(*iii*) If $x = a \times b$, $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$

(*iv*) If $x = a/b$, $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$

(*v*) If $x = \frac{a^n}{b^m}$, $\frac{\Delta x}{x} = \pm \left(n \frac{\Delta a}{a} + m \frac{\Delta b}{b} \right)$

As fractional error in a quantity raised to power n is n times the fractional error in the individual quantity, therefore, the quantity with maximum power should be measured with highest degree of accuracy or with least error.

If a quantity appears with a power less than one in an expression, then its contribution to error in the final result is reduced.

Based on the above paragraph, answer questions no. 5 to 8 :

5. The maximum absolute error in difference of two quantities is equal to

(*a*) sum of absolute errors in the two quantities

(*b*) difference of absolute errors in the two quantities

(*c*) product of absolute errors in the two quantities

(*d*) none of the above

6. The fractional error or relative error in a quantity raised to power n is

(*a*) $\frac{1}{n}$ times the fractional error in the quantity

(*b*) $\frac{1}{n^2}$ times the fractional error in the quantity

(*c*) n times the absolute error in the quantity

(*d*) n times the fractional error in the quantity

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, the Assertion and Reason are true, and Reason is correct explanation of Assertion.

(*b*) Both, the Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

(*c*) Assertion is true, but Reason is false.

(*d*) Both Assertion and Reason are false.

7. **Assertion.** In an expression, if a quantity appears with a power less than one, its contribution to error in the final result is reduced.

Reason. When $x = a^n$, $\frac{\Delta x}{x} = n \left(\frac{\Delta a}{a} \right)$

When n is less than one, $\frac{\Delta x}{x}$ reduces.

8. **Assertion.** When length l and time period t are measured with ± 2% and ± 3% errors respectively, the percentage error in g is ± 8%.

Reason. From $g = 4\pi^2 \frac{l}{t^2}$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} + 2 \frac{\Delta t}{t} \right) \times 100\%$$

$$= (\pm 2 \pm 2 \times 3)\%$$

$$= \mathbf{8\%}$$

ANSWERS

1. (*a*) **2.** (*b*) **3.** (*b*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*a*) **8.** (*a*)

HINTS/EXPLANATIONS For Difficult Questions

1. Measuring process is a process of comparison.
2. Random errors follow the Gaussian Law of normal distribution.
3. Both, A and R are true, but R is not a true explanation of A.
4. Both, A and R are true, but R is not a correct explanation of A.
5. The maximum absolute error in difference of two quantities = sum of absolute errors in two quantities.
6. The fractional error in a quantity raised to power *n* is *n* times the fractional error in the quantity.
7. Both, A and R are true and R is correct explanation of A.
8. Both, the Assertion and Reason are true, and Reason is correct explanation of the Assertion.

PROBLEMS FOR PRACTICE

TYPE I. SIMPLE CONVERSION OF UNITS

1. How many light years are there in 1 metre ? **[Ans. $1·057 \times 10^{-16}$ ly]**

2. The radius of gold nucleus is 41·3 fermi. Express its volume in m^3. **[Ans. $2·95 \times 10^{-40}$ m^3]**

3. How many metric tons are there in a teragram ? **[Ans. 10^6]**

4. How much longer than a microsecond is a millisecond ? **[Ans. 10^3]**

5. Calculate surface area of a solid cylinder of diameter 4 cm and height 20 cm in mm^2. **[Ans. 27657·1 mm^2]**

6. Express an acceleration of 10 ms^{-2} in km h^{-2}. **[Ans. 1.296×10^5 km h^{-2}]**

7. Find the value of one light year in giga metre. **[Ans. $9·46 \times 10^6$ Gm]**

TYPE II. ORDER OF MAGNITUDE AND INDIRECT METHODS FOR MEASURING DISTANCES

8. Estimate the order of magnitude of surface area of earth. **[Ans. 15]**

9. Estimate the order of number of seconds in a century. **[Ans. 9]**

10. When planet Jupiter is at a distance of 824·7 million km from earth, its angular diameter is measured to be 35·72″ of arc. Calculate the diameter of Jupiter. **[Ans. $1·428 \times 10^5$ km]**

11. A star is located 9 ly away from us. What is its distance in par sec ? What is the parallax shown by this star when viewed from two locations 3×10^{11} m apart. Given 1 par sec $= 3·084 \times 10^{16}$ m, and 1 ly $= 9·46 \times 10^{15}$ m. **[Ans. 2·76 par sec ; 0·73″]**

12. When the observations are taken at an interval of 6 months, the angle of parallax for a star is 0·4″. Find the distance of star in par sec. **[Ans. 5·02 par sec]**

13. A drop of olive oil of radius 0·25 mm spreads into a circular film of diameter 20 cm on the water surface. Estimate the size of the oil molecule. **[Ans. $2·08 \times 10^{-9}$ m]**

14. The shadow of a pole standing on a level ground is found to be 45 m longer when the sun's altitude is 30° than when it was 60°. Determine the height of the pole. **[Ans. 38·92 m]**

15. A drop of olive oil of radius 0.3 mm spreads into a rectangular film of 30 cm × 15 cm on the water surface. Calculate the size of the oil molecule. **[Ans. $2·51 \times 10^{-9}$ m]**

TYPE III. MEASUREMENT OF TIME

16. An air molecule spins once in 10^{-12} s. How many times (order of magnitude) would an air molecule spin around its axis, while the earth revolves once around the sun ? **[Ans. 10^{19}]**

17. If two atomic clocks, allowed to run for 60 years differ from eachother by 0·2 s only, calculate the accuracy of standard atomic clock measuring a time interval of 1 sec. **[Ans. 1 part in 10^{10}]**

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TYPE IV. MAGNIFICATION

18. If the size of an atom (= 1 Å) were enlarged to the tip of a sharp pin ($\simeq 10^{-5}$ m), how large would the height of mount everest ($\simeq 10^4$ m) be ? **[Ans. 10^9 m]**

19. If an atom of size 10^{-10} m were enlarged to the size of the earth ($\approx 10^7$ m), how large would its nucleus be ? Take size of nucleus = 10^{-14} m. **[Ans. 10^3 m]**

20. If size of an atom ($\approx 10^{-10}$ m) is scaled up to 1 m, what would be the size of nucleus ($\approx 10^{-14}$ m) ? **[Ans. 10^{-4} m]**

TYPE V. CONVERSION OF ONE SYSTEM OF UNITS INTO ANOTHER

21. Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time. **[Ans. 9×10^6 new units]**

22. If the value of universal gravitational constant is $6{\cdot}67 \times 10^{-11}$ Nm^2 kg^{-2}, then find its value in CGS system. (HP Board 2009) **[Ans. $6{\cdot}67 \times 10^{-8}$ dyne cm^2 g^{-2}]**

23. Convert a power of one mega watt on a system whose fundamental units are 10 kg, 1 dm and 1 minute. **[Ans. $2{\cdot}16 \times 10^{12}$ new units of power]**

24. When one metre, one kg and one minute are taken as fundamental units, the magnitude of a force is 36 units. What is the value of this force on cgs system ? **[Ans. 10^3 dyne]**

25. If the unit of force were kilonewton, that of time millisecond and that of power kilowatt, what would be the units of mass and length ? **[Ans. 1 kg ; 10^{-3} m]**

TYPE VI. CHECKING THE CORRECTNESS OF FORMULAE

26. On the basis of dimensional arguments, rule out the wrong relation for Kinetic Energy.

(*i*) $\frac{3}{16} mv^2$ (*ii*) $\frac{1}{2} mv^2 + ma$ (CBSE 2008)

[Ans. Relation (*ii*) is wrong]

27. Check the correctness of the equation

$$FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

(CBSE 2008) **[Ans. Correct]**

28. The rate of flow (V) of a liquid flowing through a pipe of radius r and pressure gradient (P/l) is given by Poiseuille's equation $V = \frac{\pi}{8}\frac{P r^4}{\eta l}$.

Check the dimensional correctness of this relation. **[Ans. Correct]**

29. Using dimensional analysis, check the correctness of the following relations :

(*i*) $S_{nth} = u + \frac{a}{2}(2n - 1)$ (*ii*) $\lambda = h/mv$

(*iii*) $E = mc^2$

where the symbols have their usual meanings.

[Ans. All the three relations are correct]

30. The critical velocity (v) of flow of a liquid through a pipe of radius (r) is given by $v = \frac{\eta}{\rho r}$ where ρ is density of liquid and η is coefficient of viscosity of the liquid. Check if the relation is correct dimensionally. **[Ans. Correct]**

TYPE VII. THE FACT THAT QUANTITIES HAVING DIFFERENT DIMENSIONS CANNOT BE ADDED OR SUBTRACTED

31. Find the dimensions of $a \times b$ in the relation $P = \frac{a - t^2}{b\sqrt{x}}$, where x is distance, t is time and P is power. **[Ans. $M^{-1}L^{-5/2}T^7$]**

32. The position of a particle moving along x-axis depends on time according to the equation $x = at^2 + bt^3$, where x is in metre and t is in sec. What are the units and dimensions of a and b ? What do they represent ?

[Ans. ms^{-2}, ms^{-3}, LT^{-2} ; LT^{-3}; acc. and rate of change of acc.]

33. The velocity v of a particle depends upon time t, according to the equation $v = a + bt + \frac{c}{d+t}$

Write the dimensions of a, b, c and d.

[Ans. $[LT^{-1}]$, $[LT^{-2}]$, $[L]$, $[T]$]

34. Write the dimensions of a/b in the relation

$$P = \frac{a - t^2}{bx}$$

where P is pressure, x is distance and t is time. **[Ans. $[M^1 T^{-2}]$]**

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35. Find the dimensions of $a \times b$ in the relation $P = a\sqrt{t} - bx^2$, where x is distance, t is time and P is power. **[Ans. $M^2 L^2 T^{-13/2}$]**

36. Find the dimensions of $\frac{a \times b}{c}$ in the relation $F = \frac{a}{\sqrt{x}} + bx = cx^2$ where F is force and x is distance. **[Ans. $ML^{5/2} T^{-2}$]**

TYPE VIII. DERIVATION OF FORMULAE

37. A small spherical ball of radius r falls with velocity v through a liquid having coefficient of viscosity η. Find viscous drag F on the wall if it depends on r, v, η. Take $K = 6\pi$.

[Ans. $F = 6\pi\eta r v$]

38. The critical angular velocity ω_c of a cylinder inside another cylinder containing a liquid at which its turbulance occurs depends on viscosity η, density ρ and distance d between walls of the cylinder. Obtain an expression for ω_c using method of dimensions. $\left[\textbf{Ans. } \omega_c = \frac{K\eta}{\rho d^2}\right]$

39. Experiments show that frequency (n) of a tuning fork depends on length (l) of the prong, density (d) and the Young's modulus (Y) of its material. On the basis of dimensional analysis, derive an expression for frequency of tuning fork.

$$\left[\textbf{Ans. } n = \frac{K}{l}\sqrt{\frac{Y}{d}}\right]$$

40. Calculate the dimensions of linear momentum and surface tension in terms of velocity (v), density (ρ) and frequency (ν) as fundamental units. **[Ans. $p = k\rho v^4 \nu^{-3}$; $\sigma = k\rho v^3 \nu^{-1}$]**

41. The velocity (v) of transverse waves on a string may depend upon (*i*) length (l) of string, (*ii*) tension T in the string and (*iii*) mass per unit length (m) of the string. Derive the formula dimensionally. **[Ans. $v = k\sqrt{T/m}$]**

42. The frequency (ν) of an oscillating drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (S) of the liquid. Deduce the formula dimensionally.

$$\left[\textbf{Ans. } \nu = k\sqrt{\frac{S}{\rho r^3}}\right]$$

43. Using the method of dimensions, derive an expression for rate of flow (V) of a liquid through a pipe of radius (r) under a pressure gradient (P/l). Given that V also depends on coefficient of viscosity (η) of the liquid.

(J & K Board, 2011) $\left[\textbf{Ans. } V = k\frac{P r^4}{l\eta}\right]$

44. A small spherical ball of radius r falls with a velocity v through a liquid of coefficient of viscosity η. Obtain an expression for viscous drag F on the ball assuming that it depends upon r, v and η. **[Ans. $F = K r v \eta$]**

TYPE IX. ERRORS AND PROPAGATION OF ERRORS IN SUM AND DIFFERENCE

45. The length of a rod as measured in an experiment was found to be 2·48 m, 2·46 m, 2·49 m, 2·50 m and 2·48 m. Find the average length, absolute error in each observation and the percentage error.

[Ans. 2·48 m ; 0·00 m, 0·02 m, – 0·01 m, – 0·02 m, 0·00 m ; 0·01 m ; 0·40%]

46. If $l_1 = (12{\cdot}0 \pm 0{\cdot}1)$ cm and $l_2 = (8{\cdot}5 \pm 0{\cdot}5)$ cm find $(l_1 + l_2)$ and $(l_1 - l_2)$ with proper error limits.

[Ans. (20·5 ± 0·6) cm ; (3·5 ± 0·6) cm]

47. In an experiment, refractive index of glass was observed to be 1·45, 1·56, 1·54, 1·44, 1·54 & 1·53. Calculate (*i*) mean value of refractive index (*ii*) mean absolute error (*iii*) fractional error (*iv*) percentage error. Express the result in terms of absolute error and percentage error.

[Ans. (*i*) 1·51 (*ii*) ± 0·04 (*iii*) ± 0·03 (*iv*) ± 3% ; μ = 1·51 ± 0·04 ; μ = 1·51 ± 3%].

48. The initial and final temperatures of a liquid are measured to be (67·7 ± 0·2)°C and (76·3 ± 0·3)°C. Calculate the rise in temperature.

[Ans. (8·6 ± 0·5)°C]

49. The diameter of a wire as measured by a screw gauge was found to be 0·026 cm, 0·028 cm, 0·029 cm, 0·027 cm, 0·024 cm and 0·027 cm. Calculate (*i*) mean value of diameter

(*ii*) mean absolute error

(*iii*) relative error (*iv*) percentage error.

Also, express the result in terms of absolute error and percentage error.

[Ans. (*i*) 0·027 cm (*ii*) 0·001 cm (*iii*) 0·037 (*iv*) 3·7% (0·027 ± 0·001) cm 0·027 cm ± 3·7%]

50. In the above question, what is the difference in lengths of the two pieces of wire ?

[Ans. (12·2 ± 0·3) cm]

51. The specific heats of a gas are measured as C_p = (12·28 ± 0·2) units and C_v = (3·97 ± 0·3) units. Find the value of gas constant R and percentage error in R.

[Ans. (8·31 ± 0·5) units ; ± 6·016%]

TYPE X. PROPAGATION OF ERRORS IN MULTIPLICATION, DIVISION AND RAISING TO SOME POWER

52. The length and breadth of a rectangle are (5·7 ± 0·1) cm and (2·4 ± 0·2) cm. Calculate area of the rectangle with error limits.

[Ans. (13·7 ± 1·4) sq. cm]

53. The voltage across a lamp is (6·0 ± 0·1) volt and the current passing through it is (4·0 ± 0·2) A. Find the power consumed with error limits.

[Ans. (24·0 ± 1·6) watt]

54. If percentage errors in a, b, c, d are 1%, 2%, 3% and 4% respectively, what will be the percentage error in $X = \dfrac{a^{1/3} b^4}{c\, d^{2/3}}$. **(CBSE 2008)**

[Ans. ± 14%]

55. The specific resistance ρ of a thin wire of radius r cm, resistance R ohm and length L is given by

$\rho = \dfrac{\pi r^2 R}{L}$. If $L = 78 \pm 0{\cdot}01$ cm,

$r = 0{\cdot}26 \pm 0{\cdot}02$ cm and $R = 32 \pm 1\ \Omega$, what is the percentage error in ρ ? **[Ans. 18·51%]**

56. The radius of a sphere is measured to be (2·1 ± 0·5) cm. Calculate its surface area with error limits. **[Ans. (55·4 ± 26·4) cm^2]**

57. A physical quantity x is calculated from

$x = \dfrac{a b^2}{\sqrt{c}}$. Calculate % error in x, when % error in measuring a, b, c are 4, 2 and 3 respectively.

[Ans. ± 9·5%]

TYPE XI. SIGNIFICANT FIGURES AND ROUNDING OFF

58. State the number of significant figures in the following : (*i*) 0·0070300 m (*ii*) 2·73 × 10^{-4} kg (*iii*) 1·0850 m (*iv*) 5·097 × 10^3 s

[Ans. 5 ; 3, 5 ; 4]

59. Add 2·384 × 10^{-4} to 1·7 × 10^{-5} and express the result to correct number of significant figures.

[Ans. 1·9 × 10^{-5}]

60. Subtract 2·5 × 10^4 from 3·81 × 10^5 with due regard to significant figures. **[Ans. 3·6 × 10^5]**

61. Write down the number of significant figures in the following :

(*i*) 0·039 (*ii*) 2·000 (*iii*) 0·050 (*iv*) 3·08 × 10^6

[Ans. (*i*) two (*ii*) four (*iii*) two (*iv*) three]

62. Round off to three significant digits

(*i*) 0·03927 kg (*ii*) 4·085 × 10^8 s

[Ans. 0·0393 kg ; (*ii*) 4·08 × 10^8 s]

63. A jeweller puts a diamond weighing 5·42 g in a box weighing 1·2 kg. Find the total weight of the box and the diamond to correct number of significant figures. **[Ans. 1·2 kg]**

64. Subtract 3·2 × 10^{-6} from 4·7 × 10^{-4} with due regard to significant figures. **[Ans. 4·7 × 10^{-4}]**

65. Add 3·8 × 10^{-6} to 4·2 × 10^{-5} with due regard to significant figures. **[Ans. 4·6 × 10^{-5}]**

66. Solve with due regard to significant figures :

$$\frac{2{\cdot}91 \times 0{\cdot}3842}{0{\cdot}080}$$ **[Ans. 14]**

67. Solve the following and express the result to appropriate number of significant figures :

(*i*) $\dfrac{2{\cdot}51 \times 10^{-4} \times 1{\cdot}81 \times 10^7}{0{\cdot}4463}$

(*ii*) 1·567 + 0·958 − 0·27

[Ans. (*i*) 10·2 × 10^3 (*ii*) 2·26]

TYPE XII. TYPICAL EXAMPLES

68. Convert an acceleration of 2 km h^{-2} into cm s^{-2}. **[Ans. 0·0154 cm s^{-2}]**

69. The relative density of lead is 11·3. Its density = ... g cm^{-3} = ... kg m^{-3}. Fill in the blanks.

[Ans. 11·3 g cm^{-3} = 11·3 × 10^3 kg m^{-3}]

70. Fill in the blanks by suitable conversion of units :

(*i*) 5 ms^{-2} = ... km h^{-2}

(*ii*) G = 6·67 × 10^{-11} Nm^2 kg^{-2}
= .... cm^3 s^{-2} g^{-1}

[Ans. 6·480 × 10^4 km h^{-2} ; 6·67 × 10^{-8} cm^3 s^{-2} g^{-1}]

71. How many amu would make up 1 kg ?

[Ans. 6·024 × 10^{26}]

72. How many astronomical units make up one light year ? **[Ans. $6{\cdot}3 \times 10^4$]**

73. The value of universal gravitational constant is $6{\cdot}67 \times 10^{-8}$ dyne g^{-2} cm^2. What is its value in MkS system ? **[Ans. $6{\cdot}67 \times 10^{-11}$ Nm^2 kg^{-2}]**

74. The acceleration of a body is 2 km/h^2. Express it in cm/s^2 **[Ans. $0{\cdot}0154$ cm/s^2]**

75. Red corpuscles of human blood stream are known to be flattened discs. Blood count shows RBCs of the order of 5×10^6 in each cubic millimeter of blood. If the adult body contains 5 litres of blood, what is the order of total number of red corpuscles it contains ? **[Ans. 10^{13}]**

76. Find the percentage empty space in one mole of nitrogen gas at STP. Given, radius of nitrogen molecule is 2 Å. **[Ans. 99·9%]**

77. The orbital radius of mercury around the sun is 0·38 A U. Determine the angle of maximum elongation for mercury and its distance from earth, where elongation is maximum. **[Ans. 22·3° ; $1{\cdot}384 \times 10^8$ km]**

78. Find the dimensional formula of $\frac{1}{4\pi\epsilon_0}\frac{e^2}{hc}$, where symbols have their usual meaning. **[Ans. $[M^0 L^0 T^0]$]**

79. The number of particles given by

$$n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$$

are crossing a unit area perpendicular to x-axis in unit time. n_1, n_2 are number of particles per unit volume, for the values of x meant to be x_1 and x_2. What is the dimensional formula of diffusion constant D ? **[Ans. $[L^2 T^{-1}]$]**

80. Assuming that critical velocity (v_c) of a viscous liquid flowing through a capillary tube depends upon the radius (r) of the tube, density (ρ) and coefficient of viscosity (η) of the liquid. Use method of dimensions to obtain a relation between these quantities. **[Ans. $v_c \propto \frac{\eta}{\rho r}$]**

81. A U-tube of uniform cross section contains mercury upto a height h in either limb. The mercury in one limb is depressed a little and then released. Obtain an expression for time period (T) of oscillation, assuming that T depends on h, ρ and g, where ρ is density of mercury. $\left[\textbf{Ans. } T = K\sqrt{\frac{h}{g}}\right]$

82. The nth division of main scale coincides with (n + 1)th division of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

(IIT Mains 2003) $\left[\textbf{Ans. } \frac{a}{n+1} \textbf{unit}\right]$

83. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in proper significant figures. **[Ans. 2·6 cm^2]**

84. In a vernier callipers, 10 vernier scale divisions coincide with 9 MSD each of value 1 mm. While measuring length of a cylinder using this calliper, main scale reading is 5.1 cm and 7th vernier division coincides with any main scale division. When diameter of cylinder is measured, main scale reading is 1.7 cm and 3rd vernier division coincides with any main scale division. Calculate curved surface area of the cylinder with correct number of significant figures. **[Ans. 28·1 cm^2]**

85. How many parsec make up 1 metre ? What is the order of magnitude ? **[Ans. $3{\cdot}225 \times 10^{-17}$; – 17]**

HINTS

For Difficult Problems

1. As 1 ly = $9{\cdot}46 \times 10^{15}$ m

$$\therefore\ 1\,\text{m} = \frac{1}{9{\cdot}46\times10^{15}}\ \text{ly}$$

$$= \mathbf{1{\cdot}057 \times 10^{-16}\ ly}$$

2. Here, $r = 41{\cdot}3$ fermi $= 41{\cdot}3 \times 10^{-15}$ m, $V = ?$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3{\cdot}14\,(41{\cdot}3 \times 10^{-15})^3$$

$$= \mathbf{2{\cdot}95 \times 10^{-40}\ m^3}$$

3. 1 Teragram = 10^{12} g :

1 metric ton = 10^3 kg = 10^6 g

$$\frac{1\ \text{Teragram}}{1\ \text{metric ton}} = \frac{10^{12}\ \text{g}}{10^6\ \text{g}} = 10^6$$

∴ 1 Teragram = **10^6 metric ton**

4. $$\frac{1\,\text{millisecond}}{1\,\text{microsecond}} = \frac{10^{-3}\ \text{s}}{10^{-6}\ \text{s}} = 10^3$$

∴ **1 millisecond = 10^3 microsecond**

5. Surface area of a cylinder of radius r and height h is $= 2\pi r(r+h)$

6. $$a = \frac{10\,\text{m}}{(1\,\text{s})^2} = \frac{10\times10^{-3}\ \text{km}}{\left(\frac{1}{60\times60}h\right)^2}$$

$$= (3600)^2 \times 10^{-2}\ \text{km h}^{-2}$$

$$= \mathbf{1{\cdot}296 \times 10^5\ km\ h^{-2}}$$

7. We know that $1\ ly = 9{\cdot}46 \times 10^{15}$ m

As $1\ \text{Gm} = 10^9$ m

$$\therefore\ 1\text{ly} = \frac{9{\cdot}46\times10^{15}}{10^9}\text{Gm} = \mathbf{9{\cdot}46 \times 10^6\ Gm}$$

8. As is known, radius of earth,

$R = 6{\cdot}4 \times 10^6$ m

surface area of earth $= 4\pi R^2$

$= 4 \times 3{\cdot}14\,(6{\cdot}4 \times 10^6)^2\ \text{m}^2$

$A = 5{\cdot}14 \times 10^{14}\ \text{m}^2$

As $5{\cdot}14 > 5$, therefore order of magnitude of surface area = **15**

9. 1 century = 100 years

$= 100 \times 365 \times 24 \times 60 \times 60$ s

$= 3{\cdot}65 \times 2{\cdot}4 \times 3{\cdot}6 \times 10^8$ s

$= 3{\cdot}1536 \times 10^9$ s

As $3{\cdot}1536 < 5$, therefore order of number of seconds in a century = **9**

10. Here, $r = 824{\cdot}7$ million km

$= 824{\cdot}7 \times 10^6$ km

$$\theta = 35{\cdot}72'' = \frac{35{\cdot}72}{60\times60}\times\frac{\pi}{180}\ \text{radian}$$

$l = ?$

From $l = r\,\theta$

$$l = 824{\cdot}7\times10^6 \times \frac{35{\cdot}72}{60\times60}\times\frac{\pi}{180}\ \text{km}$$

$= \mathbf{1{\cdot}428 \times 10^5\ km}$

11. $r = 9\ \text{ly} = 9 \times 9{\cdot}46 \times 10^{15}$ m

$$= \frac{9\times9{\cdot}46\times10^{15}}{3{\cdot}084\times10^{16}}\ \text{par sec}$$

= 2·76 par sec

$l = 3 \times 10^{11}$ m

$$\theta = \frac{l}{r} = \frac{3\times10^{11}}{9\times9{\cdot}46\times10^{15}}\ \text{radian}$$

$$= \frac{3\times180\times60\times60\times10^{-4}}{9\times9{\cdot}46\pi}\ \text{sec}$$

θ = 0·73 sec

12. Here, for observations at an interval of six months, l = diameter of earth's orbit

$= 2 \times 1{\cdot}5 \times 10^{11}$ m

$= 3 \times 10^{11}$ m

$$\theta = 0{\cdot}4'' = \frac{0{\cdot}4}{60\times60}\times\frac{\pi}{180}\ \text{rad}$$

distance of star, $r = ?$

From $l = r\,\theta$

$$r = \frac{l}{\theta} = \frac{3\times10^{11}\times60\times60\times180}{0{\cdot}4\times\pi}$$

$= 1{\cdot}547 \times 10^{17}$ m

As 1 par sec $= 3{\cdot}08 \times 10^{16}$ m

$$\therefore\ r = \frac{1{\cdot}547\times10^{17}}{3{\cdot}08\times10^{16}}\ \text{par sec} = \mathbf{5{\cdot}02\ par\ sec}$$

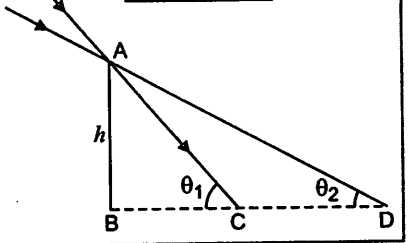
13. Suppose the film is one molecule thick. If d is diameter of an oil molecule, then

$$d = \frac{\text{volume of oil drop}}{\text{area of the film}} = \frac{\frac{4}{3}\pi r^3}{\pi R^2} = \frac{4}{3}\frac{r^3}{R^2}$$

$$d = \frac{4}{3}\frac{(0{\cdot}25\times10^{-3})^3}{(0{\cdot}1)^2} = \mathbf{2{\cdot}08 \times 10^{-9}\ m}$$

14. Let h be the height of pole AB. As is clear from Fig. 2(Q).3.

FIGURE 2(Q).3



$\theta_1 = \angle ACB = 60^\circ$; $\theta_2 = \angle ADB = 30^\circ$

1st shadow $= BC = x_1$

2nd shadow $= BD = x_2$ and $x_2 - x_1 = 45$ m.

Now, $x_1 = h \cot \theta_1$ and $x_2 = h \cot \theta_2$

$\therefore x_2 - x_1 = h(\cot \theta_2 - \cot \theta_1) = 45$

$$h = \frac{45}{\cot\theta_2 - \cot\theta_1} = \frac{45}{\cot 30^\circ - \cot 60^\circ}$$

$$= \frac{45}{\sqrt{3} - 1/\sqrt{3}}$$

$$= \frac{45 \times \sqrt{3}}{2} = \mathbf{38{\cdot}92\ m}$$

15. $d = \dfrac{\text{Volume of drop}}{\text{Area of film}} = \dfrac{\frac{4}{3}\pi r^3}{l \times b}$

$$= \frac{4}{3} \times \frac{22}{7}\,\frac{(0{\cdot}3 \times 10^{-3})^3}{30 \times 15 \times 10^{-4}} = \mathbf{2{\cdot}51 \times 10^{-9}\ m}$$

16. Time taken by earth to revolve once around the sun = 365 days = $365{\cdot}25 \times 24 \times 60 \times 60$ s

Time of spin = 10^{-12} s.

$$\text{No. of spins} = \frac{365{\cdot}25 \times 24 \times 60 \times 60}{10^{-12}}$$

$$= 3{\cdot}16 \times 10^{19} \approx \mathbf{10^{19}}$$

17. As per question,

error in measuring 60 years ($60 \times 365{\cdot}25 \times 24 \times 60 \times 60$ s) = $0{\cdot}2$ s

error in measuring 1 sec

$$= \frac{0{\cdot}2}{60 \times 365{\cdot}25 \times 24 \times 60 \times 60}$$

$$= 1{\cdot}056 \times 10^{-10}\ \text{s}$$

$\therefore$ Accuracy is **1 part in 10^{10}**

18. Magnification $= \dfrac{10^{-5}\text{m}}{1\,\text{Å}} = \dfrac{x}{10^4\text{m}}$

or $\dfrac{10^{-5}\text{m}}{10^{-10}\text{m}} = \dfrac{x}{10^4\text{m}}$

$$x = 10^5 \times 10^4\ \text{m} = \mathbf{10^9\ m}$$

19. $\dfrac{\text{Size of nucleus}}{\text{Size of atom}} = \dfrac{x}{10^7} = \dfrac{10^{-14}}{10^{-10}} = 10^{-4}$

$$x = 10^{-4} \times 10^7 = \mathbf{10^3\ m}$$

21. Joule is SI unit of work = $[ML^2T^{-2}]$

$\therefore\quad a = 1, b = 2, c = -2$

To convert SI system to new system, we write

$M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ s, $n_1 = 100$ J

$M_2 = 250$ g ; $L_2 = 20$ cm ;

$T_2 = \frac{1}{2}$ min $= 30$ s, $n_2 = ?$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$$= 100 \left(\frac{1\,\text{kg}}{250\,\text{g}}\right)^1 \left(\frac{1\,\text{m}}{20\,\text{cm}}\right)^2 \left(\frac{1\,\text{sec}}{30\,\text{sec}}\right)^{-2}$$

$$= 100 \times \left(\frac{1000\,\text{g}}{250\,\text{g}}\right)\left(\frac{100\,\text{cm}}{20\,\text{cm}}\right)^2 \times 30^2$$

$= 100 \times 4 \times 5^2 \times 30^2 = \mathbf{9 \times 10^6}$ **new units**

22. The dimensional formula of G is $[M^{-1}L^3T^{-2}]$

To convert *MKS* system to *CGS* system, we write

$M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ s,

$n_1 = 6{\cdot}67 \times 10^{-11}$

$M_2 = 1$ g, $L_2 = 1$ cm, $T_2 = 1$ s, $n_2 = ?$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^{-1} \left(\frac{L_1}{L_2}\right)^3 \left(\frac{T_1}{T_2}\right)^{-2}$$

$$= 6{\cdot}67 \times 10^{-11} \left(\frac{1\,\text{kg}}{1\,\text{g}}\right)^{-1} \left(\frac{1\,\text{m}}{1\,\text{cm}}\right)^3 \left(\frac{1\,\text{s}}{1\,\text{s}}\right)^{-2}$$

$= 6{\cdot}67 \times 10^{-11} \times (10^3)^{-1} \times (10^2)^3 \times 1$

$= \mathbf{6{\cdot}67 \times 10^{-8}}$ **dyne cm² g⁻²**

23. Here, $F = M^1 L^1 T^{-2} = 1$ kN $= 1000$ N ...(*i*)

$T = T^1 = 1$ ms $= 10^{-3}$ s ...(*ii*)

$P = ML^2T^{-3} = 1$ kW $= 1000$ W ...(*iii*)

$\dfrac{F}{T} = M^1 L^1 T^{-3} = \dfrac{1000}{10^{-3}} = 10^6\ \text{Ns}^{-1}$ ...(*iv*)

Divide (*iii*) by (*iv*)

$\dfrac{ML^2T^{-3}}{MLT^{-3}} = \dfrac{1000}{10^6} = 10^{-3}$ or $L = \mathbf{10^{-3}\ m}$

Put values of L & T in (*i*)

$M(10^{-3})(10^{-3})^{-2} = 10^3$

$\therefore\qquad M = \mathbf{1\ kg}$

26. $K.E. = \dfrac{1}{2}mv^2 = M(LT^{-1})^2 = [ML^2T^{-2}]$

Now, $\dfrac{3}{16}mv^2 = M(LT^{-1})^2 = [ML^2T^{-2}]$

$\therefore$ This formula is dimensionally correct for K.E.

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(*ii*) $\frac{1}{2}mv^2 + ma = M\,(LT^{-1})^2 + MLT^{-2}$

This is not permissible as quantities with different dimensions cannot be added.

Hence Relation (*ii*) is wrong.

27. LHS $= FS = [MLT^{-2}]\,[L] = [M^1L^2T^{-2}]$

$$\text{RHS} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= M\,(LT^{-1})^2 - M\,(LT^{-1})^2$$
$$= M\,(LT^{-1})^2 = [M^1L^2T^{-2}]$$

As dimensions on LHS = dimensions on RHS, therefore, formula is **correct dimensionally.**

28. LHS $= V = \frac{L^3}{T} = [M^0\,L^3\,T^{-1}]$

$$\text{RHS} = \frac{\pi}{8}\,\frac{P\,r^4}{\eta\,l} = \frac{(ML^{-1}T^{-2})(L^4)}{(ML^{-1}T^{-1})(L)} = [M^0\,L^3\,T^{-1}]$$

As LHS = RHS, dimensionally, therefore the **relation is correct.**

29. (*i*) Refer to Solved Example 24, Page 2/54.

(*ii*) $\lambda = h/mv$

LHS $= \lambda = [L]$

$$\text{RHS} = \frac{h}{mv} = \frac{ML^2\,T^{-1}}{MLT^{-1}} = [L]$$

The relation is **dimensionally correct.**

(*iii*) $E = mc^2$

LHS $= E = [M^1\,L^2\,T^{-2}]$

RHS $= mc^2 = M\,(LT^{-1})^2 = [M^1\,L^2\,T^{-2}]$

The relation is **dimensionally correct.**

31. $P = \frac{a - t^2}{b\sqrt{x}}$

Dimension of $a = [T^2]$

$$\text{Again, } \frac{T^2}{b\sqrt{x}} = P,\quad b = \frac{T^2}{P\sqrt{x}} = \frac{T^2}{[ML^2T^{-3}]\,L^{1/2}}$$

$$b = M^{-1}\,L^{-5/2}\,T^5$$

$$\therefore\ a \times b = T^2 \times M^{-1}\,L^{-5/2}\,T^5$$
$$= [M^{-1}\,L^{-5/2}\,T^7]$$

32. $x = at^2 + bt^3$

Each term on RHS must have dimensions of x.

Therefore, $a = x/t^2 = \text{m/s}^2 = [LT^{-2}]$

'*a*' represents acceleration.

Also, $b = x/t^3 = \text{m/s}^3 = [LT^{-3}]$

∴ **'*b*' represents rate of change of acceleration with time.**

35. $P = a\sqrt{t} - bx^2$

$$a\sqrt{t} = P = ML^2\,T^{-3}$$

$$a = \frac{P}{\sqrt{t}} = ML^2\,T^{-7/2}$$

$$\text{Again, } bx^2 = P,\ b = \frac{P}{x^2} = \frac{ML^2\,T^{-3}}{L^2} = MT^{-3}$$

$$a \times b = (ML^2\,T^{-7/2}) \times (MT^{-3})$$
$$= M^2\,L^2\,T^{-13/2}$$

36. The given relation is $F = \frac{a}{\sqrt{x}} + bx + cx^2$

$$\text{As } \frac{a}{\sqrt{x}} = F,\ a = \sqrt{x}\,F = L^{1/2}\,(MLT^{-2})$$
$$= [ML^{3/2}\,T^{-2}]$$

$$\text{Again, } bx = F,\ b = \frac{F}{x} = \frac{MLT^{-2}}{L} = [M^1\,T^{-2}]$$

$$\text{Also, } cx^2 = F,\ c = \frac{F}{x^2} = \frac{MLT^{-2}}{L^2} = [M\,L^{-1}T^{-2}]$$

$$\text{Now, } \frac{a \times b}{c} = \frac{(ML^{3/2}T^{-2}) \times (M^1\,T^{-2})}{(M\,L^{-1}T^{-2})}$$
$$= [ML^{5/2}\,T^{-2}]$$

37. Let $F = K\,r^a\,v^b\,\eta^c$ ...(*i*)

where K = dimensionless constant = $6\,\pi$

Writing dimensions in (*i*),

$$[MLT^{-2}] = L^a\,(LT^{-1})^b\,(ML^{-1}T^{-1})^c$$
$$= M^c\,L^{a+b-c}\,T^{-b-c}$$

Applying principle of homogeneity of dimensions, we get

$c = 1$; $a + b - c = 1$; $-b - c = -2$,

$b = 2 - c = 2 - 1 = 1$

From $a + b - c = 1$,

$a = 1 - b + c = 1 - 1 + 1 = 1$

Put in (*i*) $F = K\,r^1\,v^1\,\eta^1 = \mathbf{6\,\pi\,r\,v\,\eta}$

38. Let $\omega_c = K\,\eta^a\,\rho^b\,d^c$ ...(*i*)

where K is dimensionless constant. Writing the dimensions in (*i*), we get

$$[M^0\,L^0\,T^{-1}] = (ML^{-1}T^{-1})^a\,(ML^{-3})^b\,L^c$$
$$= M^{a+b}\,.L^{-a-3b+c}\,.T^{-a}$$

Applying principle of homogeneity of dimensions, we get

$a + b = 0,\ -a - 3b + c = 0,\ -a = -1,\ a = 1$

$\therefore\ b = -a - 1,\ c = a + 3b = 1 - 3 = -2$

Put in (i), $\omega_c = K\eta^1\rho^{-1}.d^{-2} = \dfrac{K\eta}{\rho d^2}$

42. Let $v = k\, r^a \rho^b s^c$ ...(i)

$[M^0 L^0 T^{-1}] = L^a (ML^{-3})^b (MT^{-2})^c$,

$[M^0 L^0 T^{-1}] = M^{b+c} L^{a-3b} T^{-2c}$

$\therefore b + c = 0$ ...(ii)

$a - 3b = 0$ ...(iii)

$-2c = -1, \therefore c = \dfrac{1}{2}$

From (ii), $b = -c = -\dfrac{1}{2}$

From (iii), $a = 3b = -\dfrac{3}{2}$.

Put in (i) to get the required formula.

44. Let $F = K\, r^a v^b \eta^c$ ...(i)

where K is dimensionless constant of proportionality.

$$[M^1 L^1 T^{-2}] = L^a (LT^{-1})^b (ML^{-1} T^{-1})^c = M^c L^{a+b-c} T^{-b-c}$$

Applying the principle of homogeneity of dimensions, we get

$c = 1, \quad a + b - c = 1$ ...(ii)

$-b - c = -2$ ...(iii)

From (iii), $-b - 1 = -2$, or $-b = +1 - 2 = -1$, $b = 1$

From (ii), $a + 1 - 1 = 1$, or $a = 1$

Putting these values in (i), we get

$$\boldsymbol{F = K r^1 v^1 \eta^1}$$

45. Average length

$$= \frac{2\cdot48 + 2\cdot46 + 2\cdot49 + 2\cdot50 + 2\cdot48}{5} = \frac{12\cdot41}{5}\text{ m}$$

= 2·482 m = 2·48 m (rounded off to 2 places)

Absolute errors in different measurements are :

$\Delta l_1 = 2\cdot48 - 2\cdot48 = 0\cdot00$ m

$\Delta l_2 = 2\cdot48 - 2\cdot46 = 0\cdot02$ m

$\Delta l_3 = 2\cdot48 - 2\cdot49 = -0\cdot01$ m

$\Delta l_4 = 2\cdot48 - 2\cdot50 = -0\cdot02$ m

$\Delta l_5 = 2\cdot48 - 2\cdot48 = 0\cdot00$ m

Mean absolute error $= \dfrac{\Sigma|\Delta L|}{5}$

$$= \frac{0\cdot00 + 0\cdot02 + 0\cdot01 + 0\cdot02 + 0\cdot00}{5} = 0\cdot01\text{ m}$$

∴ Correct length = (2·48 ± 0·01) m

Percentage error $= \dfrac{0\cdot01}{2\cdot48} \times 100 = \mathbf{0\cdot40\%}$

46. $l_1 + l_2 = 12\cdot0 + 8\cdot5 = 20\cdot5$ cm

Error $= \pm(\Delta l_1 + \Delta l_2) = \pm(0\cdot1 + 0\cdot5) = \pm 0\cdot6$ cm

$l_1 - l_2 = 12\cdot0 - 8\cdot5 = 3\cdot5$ cm

Error $= \pm(\Delta l_1 + \Delta l_2) = \pm(0\cdot1 + 0\cdot5) = \pm 0\cdot6$ cm

Hence $l_1 + l_2 = (20\cdot5 \pm 0\cdot6)$ cm, and

$l_1 - l_2 = (3\cdot5 \pm 0\cdot6)$ cm

47. Mean value of

$$\mu = \frac{1\cdot45 + 1\cdot56 + 1\cdot54 + 1\cdot44 + 1\cdot54 + 1\cdot53}{6} = \mathbf{1\cdot51}$$

Absolute errors are :

$1\cdot51 - 1\cdot45 = 0\cdot06, \quad 1\cdot51 - 1\cdot56 = -0\cdot05$

$1\cdot51 - 1\cdot54 = -0\cdot03, \quad 1\cdot51 - 1\cdot44 = 0\cdot07$;

$1\cdot51 - 1\cdot54 = -0\cdot03$; $\quad 1\cdot51 - 1\cdot53 = -0\cdot02$

Mean absolute error

$$= \pm\frac{0\cdot06 + 0\cdot05 + 0\cdot03 + 0\cdot07 + 0\cdot03 + 0\cdot02}{6}$$

$= \pm 0\cdot26/6 = \pm\, \mathbf{0\cdot04}$

Fractional error $= \dfrac{\pm 0\cdot04}{1\cdot51} = \pm\mathbf{0\cdot03}$

Percentage error = ± 0·03 × 100 = **± 3%**

$\mu = 1\cdot51 \pm 0\cdot04$ or $\mu = \mathbf{1\cdot51 \pm 3\%}$

49. (i) Mean value of diameter,

$$D = \frac{0\cdot026 + 0\cdot028 + 0\cdot029 + 0\cdot027 + 0\cdot024 + 0\cdot027}{6}$$

$$D = \frac{0\cdot161}{6} = 0\cdot268 = \mathbf{0\cdot027\ cm}$$

Now, $\Delta D_1 = 0\cdot027 - 0\cdot026 = 0\cdot001$

$\Delta D_2 = 0\cdot027 - 0\cdot028 = -0\cdot001$

$\Delta D_3 = 0\cdot027 - 0\cdot029 = -0\cdot002$

$\Delta D_4 = 0\cdot027 - 0\cdot027 = 0\cdot000$

$\Delta D_5 = 0\cdot027 - 0\cdot024 = 0\cdot003$

$\Delta D_6 = 0\cdot027 - 0\cdot027 = 0\cdot000$

(ii) Mean absolute error

$$= \frac{|\Delta D_1| + |\Delta D_2| + |\Delta D_3| + |\Delta D_4| + |\Delta D_5| + |\Delta D_6|}{6}$$

$$= \frac{0\cdot001 + 0\cdot001 + 0\cdot002 + 0\cdot000 + 0\cdot003 + 0\cdot000}{6}$$

$$= \frac{0\cdot007}{6} = \mathbf{0\cdot001\ cm}$$

(iii) Relative error $= \dfrac{0\cdot001}{0\cdot027} = \mathbf{0\cdot037}$

(iv) Percentage error = 0·037 × 100% = **3·7%**

(v) Diameter of wire = **(0·027 ± 0·001) cm**

= 0·027 cm ± 3·7%

50. Here, $l_1 = (35\cdot2 \pm 0\cdot1)$ cm

$l_2 = (47\cdot4 \pm 0\cdot2)$ cm

$l_2 - l_1 = 47\cdot4 - 35\cdot2 = 12\cdot2$ cm

Error in difference of lengths

$$= \pm (0.1 + 0.2)\text{ cm} = \pm 0.3\text{ cm}$$

$\therefore\ l_2 - l_1 =$ **(12·2 ± 0·3) cm**

51. Here, $C_p = (12.28 \pm 0.2)$ units

$C_v = (3.97 \pm 0.3)$ units

$R = C_p - C_v$

$= (12.28 - 3.97) \pm (0.2 + 0.3)$

$= (8.31 \pm 0.5)$ units

Percentage error in R

$$= \frac{\Delta R}{R} \times 100 = \frac{(\Delta C_p + \Delta C_v)}{C_p - C_v} \times 100$$

$$= \frac{(0.2 + 0.3)}{8.31} \times 100 = \pm \mathbf{6.016\%}$$

52. Here, $l = (5.7 \pm 0.1)$ cm, $b = (2.4 \pm 0.2)$ cm

$A = l \times b = 5.7 \times 2.4 = 13.68$ sq. cm.

$= 13.7$ sq. cm.

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5.7} + \frac{0.2}{2.4}\right)$$

$= \pm (0.0175 + 0.0833)$

$\Delta A = \pm (0.1008) \times 13.7 = \pm 1.38$ sq. cm.

± 1.4 sq. cm.

Hence area = **(13·7 ± 1·4) sq. cm**

53. Here, $V = (6.0 \pm 0.1)$ volt, $I = (4.0 \pm 0.2)$ A, $P = ?$

$P = V \times I = 6.0 \times 4.0 = 24.0$ watt

$$\frac{\Delta P}{P} = \pm\left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) = \pm\left(\frac{0.1}{6.0} + \frac{0.2}{4.0}\right)$$

$= \pm (0.0166 + 0.0500) = \pm 0.066$

$\Delta P = \pm 0.066 \times P = \pm 0.066 \times 24.0$

$= \pm 1.584 = \pm 1.6$ watt

Hence, power = **(24·0 ± 1·6) watt**

54. Here, $X = \dfrac{a^{1/3} b^4}{c\, d^{2/3}}$

Max. percentage error in X is given by

$$\frac{\Delta X \times 100}{X} = \pm\left[\frac{1}{3}\frac{\Delta a}{a} + 4\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{2}{3}\frac{\Delta d}{d}\right]$$

$$= \pm\left[\frac{1}{3} \times 1\% + 4 \times 2\% + 3\% + \frac{2}{3} \times 4\%\right]$$

$$= \pm\left[\frac{1}{3} + 8 + 3 + \frac{8}{3}\right]\% = \pm \mathbf{14\%}$$

55. Max. percentage error in $\rho = \dfrac{\pi r^2 R}{L}$ is

$$\frac{\Delta\rho}{\rho} \times 100 = \pm\left(2\frac{\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta L}{L}\right) \times 100$$

$$= \pm\left(\frac{2 \times 0.02}{0.26} + \frac{1}{32} + \frac{0.01}{78}\right) \times 100$$

$= \pm (15.38 + 3.12 + 0.012) = \pm$ **18·51%**

56. Surface area of a sphere = $4\,\pi\, r^2$

58. (*i*) 0·0070300 m : **Five :** 7, 0, 3, 0, 0

(*ii*) 2.73×10^{-4} kg : **Three :** 2,7,3

(*iii*) 1·0850 m : **Five :** 1,0,8,5,0

(*iv*) 5.097×10^3 s : **Four :** 5,0,9,7

59. $2.384 \times 10^{-4} + 1.7 \times 10^{-5}$

$= 10^{-5}\,[0.2384 + 1.7] = 1.9384 \times 10^{-5}$

Rounding off to one place of decimal :

sum = $\mathbf{1.9 \times 10^{-5}}$

60. $3.81 \times 10^5 - 2.5 \times 10^4 = 10^5\,(3.81 - 0.25)$

$= 3.56 \times 10^5$

Rounding off to one place of decimal,

difference = $\mathbf{3.6 \times 10^5}$

62. (*i*) **0·0393 kg** ($\because$ the last digit 7 > 5)

(*ii*) $\mathbf{4.08 \times 10^8}$ **s**

($\because$ the digit to be rounded off is 5 and preceding digit is even)

63. Weight of diamond = 5·42 g = 0·00542 kg

Total weight = 1·2 + 0·00542

= 1·20542 kg = **1·2 kg**

(The result is correct upto one place of decimal)

65. $3.8 \times 10^{-6} + 4.2 \times 10^{-5} = 10^{-5}\,[0.38 + 4.2]$

$= 10^{-5} \times 4.58 = \mathbf{4.6 \times 10^{-5}}$

(The result is correct upto one place of decimal)

67. (*i*) $\dfrac{2.51 \times 10^{-4} \times 1.81 \times 10^7}{0.4463} = 10.1795 \times 10^3$

$= \mathbf{10.2 \times 10^3}$,

rounded off to three significant digits.

(*ii*) $1.567 + 0.958 - 0.27 = 2.255 =$ **2·26**,

rounded off to two places of decimal.

68. Here, $a = 2\text{ km h}^{-2}$

$$= \frac{2\,\text{km}}{(1\,\text{h})^2} = \frac{2 \times 10^3 \times 10^2\,\text{cm}}{(60 \times 60\,\text{s})^2}$$

$= \mathbf{0.0154\ cm/s^2}$

69. Density = Relative density × density of water

$= 11{\cdot}3\ \text{g cm}^{-3} \times 1\ \text{g cm}^{-3} = \mathbf{11{\cdot}3\ g\ cm^{3}}$

$= \mathbf{11{\cdot}3 \times 10^{3}\ kg\ m^{-3}}$

70. (*i*) $5\ \text{ms}^{-2} = 5\ (10^{-3}\ \text{km}) \left(\dfrac{1}{60\times 60}\ \text{h}\right)^{-2}$

$= 5 \times 10^{-3} \times 3600 \times 3600\ \text{km h}^{-2}$

$= \mathbf{6{\cdot}480 \times 10^{4}\ km\ h^{-2}}$

(*ii*) $G = 6{\cdot}67 \times 10^{-11}\ \text{Nm}^{2}\ \text{kg}^{-2}$

$= 6{\cdot}67 \times 10^{-11}\ (\text{kg ms}^{-2})\ \text{m}^{2}\ \text{kg}^{-2}$

$= 6{\cdot}67 \times 10^{-11}\ \text{m}^{3}\ \text{s}^{-2}\ \text{kg}^{-1}$

$= 6{\cdot}67 \times 10^{-11}\ (10^{2}\ \text{cm})^{3}\ \text{s}^{-2}\ (10^{3}\ \text{g})^{-1}$

$\mathbf{G = 6{\cdot}67 \times 10^{-8}\ cm^{3}\ s^{-2}\ g^{-1}}$

71. As $1\ \text{amu} = 1{\cdot}66 \times 10^{-27}\ \text{kg}$

$\therefore \quad 1\,\text{kg} = \dfrac{1}{1{\cdot}66\times 10^{-27}}\ \text{amu}$

$= \mathbf{6{\cdot}024 \times 10^{26}\ amu}$

72. As is known, $1\ \text{ly} = 9{\cdot}46 \times 10^{5}\ \text{m}$

and $\quad 1\ \text{Au} = 1{\cdot}5 \times 10^{11}\ \text{m}$

$\therefore \quad \dfrac{10\ \text{ly}}{1\,\text{Au}} = \dfrac{9{\cdot}46\times 10^{15}}{1{\cdot}5\times 10^{11}} = 6{\cdot}3\times 10^{4}$

Hence $\quad \mathbf{1\ ly = 6{\cdot}3 \times 10^{4}\ Au}$

73. $G = 6{\cdot}67\times 10^{-8}\ \dfrac{\text{dyne cm}^{2}}{(\text{gram})^{2}}$

$= 6{\cdot}67\times 10^{-8}\ \dfrac{(10^{-5}\ \text{N})\,(10^{-2}\ \text{m})^{2}}{(10^{-3}\ \text{kg})^{2}}$

$\mathbf{G = 6{\cdot}67\times 10^{-11}\ Nm^{2}\ kg^{-2}}$

74. $a = \dfrac{2\,\text{km}}{h^{2}} = \dfrac{2\times 10^{3}\times 10^{2}\ \text{cm}}{(60\times 60\,s)^{2}}$

$= \dfrac{2\times 10^{5}}{3600\times 3600}\ \text{cm/s}^{2}$

$= \mathbf{0{\cdot}0154\ cm/s^{2}}$

75. Number of RBCs in $1\ \text{mm}^{3} = 5 \times 10^{6}$

Total volume of blood = 5 litres

$= 5 \times 10^{3} \times 10^{3}\ \text{mm}^{3}$

Total number of RBCs $= 5 \times 10^{6} \times 5 \times 10^{6}$

$= 2{\cdot}5 \times 10^{13} \approx \mathbf{10^{13}}$

76. Here, $r = 2\ \text{Å} = 2 \times 10^{-10}\ \text{m}$

Volume of one molecule of nitrogen $= \dfrac{4}{3}\pi r^{3}$

$= \dfrac{4}{3}\times\dfrac{22}{7}(2\times 10^{-10})^{3}\ \text{m}^{3}$

Number of molecules in one mole

$= 6{\cdot}023 \times 10^{23}$

∴ Total volume occupied by all molecules in one mole

$V = 6{\cdot}023\times 10^{23}\times\dfrac{4}{3}\times\dfrac{22}{7}(2\times 10^{-10})^{3}\ \text{m}^{3}$

$V = 2{\cdot}02 \times 10^{-5}\ \text{m}^{3}$

Volume occupied by one mole of gas at NTP

$V' = 22{\cdot}4\ \text{litre} = 22{\cdot}4 \times 10^{-3}\ \text{m}^{3}.$

∴ Space occupied $= \dfrac{V}{V'} = \dfrac{2{\cdot}02\times 10^{-5}}{22{\cdot}4\times 10^{-3}} \approx 10^{-3}$

And empty space $= 1 - 10^{-3} = 0{\cdot}999$

$= \mathbf{99{\cdot}9\%}$

77. Here, $r_{ps} = 0{\cdot}38$ A U ; $r_{es} = 1$ A U

The angle of maximum elongation $\in$ is given by

$\sin \in = \dfrac{r_{ps}}{r_{es}} = \dfrac{0{\cdot}38}{1}$

$\in = \sin^{-1}(0{\cdot}38) = \mathbf{22{\cdot}3^{\circ}}$

Now, $r_{pe} = r_{es} \times \cos \in$

$= 1{\cdot}496 \times 10^{11} \cos(22{\cdot}3^{\circ})$

$= 1{\cdot}496 \times 10^{11} \times 0{\cdot}9252 = 1{\cdot}384 \times 10^{11}\ \text{m}$

$= \mathbf{1{\cdot}384 \times 10^{8}\ km}$

78. $\dfrac{e^{2}}{4\pi\epsilon_{0}\,hc} = \dfrac{F\,r^{2}}{hc}$, where $F = \dfrac{e^{2}}{4\pi\epsilon_{0}\,r^{2}}$

$= \dfrac{[MLT^{-2}]\,[L^{2}]}{[ML^{2}\,T^{-1}]\,[LT^{-1}]} = 1 = [M^{0}\,L^{0}\,T^{0}]$

79. From $n = -D\left(\dfrac{n_{2}-n_{1}}{x_{2}-x_{1}}\right)$

$D = \dfrac{n\,(x_{2}-x_{1})}{-(n_{2}-n_{1})}$

Dimensions of n = number/area/time $= L^{-2}\,T^{-1}$

Dimensions of n_{1}, n_{2} = number/volume $= L^{-3}$

Dimensions of $x_{2} - x_{1}$ = diff. between two positions $= L$

$\therefore \quad D = \dfrac{(L^{-2}\,T^{-1})(L)}{L^{-3}} = [L^{2}\,T^{-1}]$

80. Let $v_c = k\, r^a \rho^b \eta^c$

$[M^0 L^1 T^{-1}] = L^a [ML^{-3}]^b [ML^{-1} T^{-1}]^c$

$= M^{b+c} L^{a-3b-c} T^{-c}$

$\therefore\ b + c = 0,\ a - 3b - c = 1,\ -c = -1.$

Simplify to get the required relation.

81. Let $T = K h^a \rho^b g^c$ ...(*i*)

where *a*, *b*, *c* are the dimensions and *K* is dimensionless constant.

Writing the dimensions in (*i*), we get

$[M^0 L^0 T^1] = L^a (ML^{-3})^b (LT^{-2})^c$

$= M^b L^{a-3b+c} T^{-2c}$

Applying the principle of homogeneity of dimensions, we get

$b = 0\,;\ a - 3b + c = 0,\ -2c = 1,\ c = -\frac{1}{2}.$

From $a - 3b + c = 0,\ a - 3 \times 0 - \frac{1}{2} = 0,\ a = \frac{1}{2}$

Putting these values in (*i*), we get

$T = K h^{1/2} \rho^0 g^{-1/2}$

$$\boldsymbol{T = K\sqrt{\frac{h}{g}}}$$

82. $(n + 1)$ VSD $= n$ MSD

$\therefore\ 1\,\text{VSD} = \left(\frac{n}{n+1}\right)\text{MSD}$

Least count of Vernier Callipers

$= 1\,\text{MSD} - 1\,\text{VSD} = \left[1 - \left(\frac{n}{n+1}\right)\right]\text{MSD}$

$= \frac{1}{n+1}\,\text{MSD} = \boldsymbol{\frac{a}{n+1}}$ **unit**

83. Value of 1 MSD = 1 mm = 0·1 cm.

Value of 1 CSD $= \frac{1}{100}$ mm = 0·001 cm.

Here, diameter, *d* = 1·47 mm = 0·147 cm.

l = 5·6 cm

Curved surface area of wire

$A = 2\pi r \times l = \pi d \times l$

$= \frac{22}{7} \times (0{\cdot}147)\,(5{\cdot}6)\ \text{cm}^2$

$= 2{\cdot}5872\ \text{cm}^2$

Rounding off to two significant figures,

$\boldsymbol{A = 2{\cdot}6\ \text{cm}^2}$

84. V.C. = 1 MSD − 1 VSD $= \left(1 - \frac{9}{10}\right)$MSD

$= \frac{1}{10}\,\text{MSD} = \frac{1}{10}\,\text{mm} = 0{\cdot}1\ \text{mm} = 0{\cdot}01\ \text{cm}$

$l = (5{\cdot}1 + 7 \times 0{\cdot}01)\ \text{cm} = 5{\cdot}17\ \text{cm}$

$D = (1{\cdot}7 + 3 \times 0{\cdot}01)\ \text{cm} = 1{\cdot}73\ \text{cm}$

curved surface area $= 2\pi r \times l = \pi D l$

$= 3{\cdot}14\ (1{\cdot}73)\ 5{\cdot}17\ \text{cm}^2$

$A = 28{\cdot}084\ \text{cm}^2$

Rounding off to three significant digits,

$\boldsymbol{A = 28{\cdot}1\ \text{cm}^2}$

85. As is known,

1 par sec = $3{\cdot}1 \times 10^{16}$ metre

$\therefore$ 1 metre $= \frac{1}{3{\cdot}1 \times 10^{16}}$ par sec

$= \boldsymbol{3{\cdot}225 \times 10^{-17}}$ **par sec**

As 3·225 < 5, therefore, order of magnitude = **− 17**

NCERT EXERCISES WITH SOLUTIONS

Q. 1. Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to ......m^3

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ...$(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers ...m in 1 s

(d) The relative density of lead is 11.3. Its density is ...g cm^{-3} or ....kg m^{-3}

Sol. (*a*) Here, length of side, $L = 1\text{ cm} = 10^{-2}\text{ m}$

$$\text{Volume of cube} = L^3 = (10^{-2}\text{ m})^3 = \mathbf{10^{-6}\ m^3}$$

(*b*) Here, $r = 2{\cdot}0\text{ cm} = 20\text{ mm}$, $h = 10{\cdot}0\text{ cm} = 100\text{ mm}$

$$\text{Surface area of solid cylinder} = (2\pi r)\,h = 2 \times \frac{22}{7} \times 20 \times 100\text{ mm}^2 = \mathbf{1{\cdot}26 \times 10^4\ mm^2}$$

(*c*) Here, speed, $v = 18\text{ km h}^{-1} = \dfrac{18 \times 1000\text{ m}}{60 \times 60\text{ s}} = 5\text{ ms}^{-1}$ $\therefore$ Distance covered in 1 second = **5 m**

(*d*) Here, relative density = 11·3

$$\therefore\ \text{density} = \mathbf{11{\cdot}3\ g/cc} = \frac{11{\cdot}3 \times 10^{-3}\text{ kg}}{(10^{-2}\text{ m})^3} = \mathbf{11{\cdot}3 \times 10^3\ kg\ m^{-3}}$$

Q. 2. Fill in the blanks by suitable conversion of units :

(a) 1 kg m^2 s^{-2} = ... g cm^2 s^{-2} (b) 1 m = ... light year (c) 3 ms^{-2} = ... km h^{-2}

(d) $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2} = ... cm^3 s^{-2} g^{-1}

Sol. (*a*) $1\text{ kg m}^2\text{ s}^{-2} = 1 \times 10^3\text{ g }(10^2\text{ cm})^2\text{ s}^{-2} = 10^7\text{ g cm}^2\text{ s}^{-2}$

(*b*) We know, 1 light year = 9.46×10^{15} m $\therefore\ 1\text{m} = \dfrac{1}{9.46 \times 10^{15}}$ light year $= 1.053 \times 10^{-16}$ light year

(*c*) $3\text{ ms}^{-2} = 3 \times 10^{-3}\text{ km}\left(\dfrac{1}{60 \times 60}\text{h}\right)^{-2} = 3 \times 10^{-3} \times 3600 \times 3600\text{ km h}^{-2} = 3.888 \times 10^4\text{ km h}^{-2}$

(*d*) $G = 6.67 \times 10^{-11}\text{ Nm}^2\text{ kg}^{-2} = 6.67 \times 10^{-11}\text{ (kg ms}^{-2})\text{ m}^2\text{ kg}^{-2}$

$= 6.67 \times 10^{-11}\text{ m}^3\text{ s}^{-2}\text{ kg}^{-1} = 6.67 \times 10^{-11}\ (100\text{ cm})^3\text{ s}^{-2}\ (1000\text{ g})^{-1} = \mathbf{6.67 \times 10^{-8}\ cm^3\ s^{-2}\ g^{-1}}$

Q. 3. A calorie is a unit of heat energy and it equals about 4.2 J, where 1 J = 1 kg m^2 s^{-2}. Suppose, we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m and the unit of time is γ s. Show that a calorie has a magnitude of 4.2 α^{-1} β^{-2} γ^2 in terms of the new units.

Sol. Here, 1 calorie = $4.2\text{ J} = 4.2\text{ kg m}^2\text{ s}^{-2}$ ...(*i*)

As new unit of mass = α kg $\therefore$ $1\text{ kg} = \dfrac{1}{\alpha}$ new unit of mass = α^{-1} new unit of mass

Similarly, 1 m = β^{-1} new unit of length, 1 s = γ^{-1} new unit of time

Putting these values in (*i*), we obtain

1 calorie = 4.2 (α^{-1} new unit of mass) (β^{-1} new unit of length)^2 (γ^{-1} new unit of time)$^{-2}$

= **4.2 α^{-1} β^{-2} γ^2 new unit of energy,** which was to be proved.

Q. 4. Explain this statement clearly : (i) To call a dimensionless quantity 'large' or 'small' is meaningless without specifying a standard for comparison. (ii) In view of this, reword the following statements, wherever necessary.

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(a) Atoms are very small objects (b) A jet plane moves with great speed
(c) The mass of jupiter is very large (d) The air inside this room contains a large number of molecules
(e) A proton is much more massive than an electron
(f) The speed of sound is much smaller than the speed of light.

Sol. *(i)* The statement is true. This is because a dimensionless quantity can be large or small only in comparison to some standard. For example, angle is dimensionless. $\angle\theta = 60°$ is larger than $\angle\theta = 30°$, but smaller than $\angle\theta = 90°$.

(ii) *(a)* The size of an atom is smaller than the sharp tip of a pin.

(b) A jet plane moves faster than a superfast train.

(c) The mass of jupiter is very large compared to the mass of earth.

(d) The air inside this room contains more number of molecules than in one mole of air.

(e) The statement is already correct *(f)* The statement is already correct.

Q. 5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit, if light takes 8 min and 20 sec. to cover the distance ?

Sol. We are given that velocity of light in vacuum, $c = 1$ new unit of length s^{-1}. Time taken by light of sun to reach the earth, $t = 8$ min 20 s $= 8 \times 60 + 20 = 500$ s.

∴ Distance between the sun and earth

$$x = c \times t = 1 \text{ new unit of length s}^{-1} \times 500 \text{ s} = \mathbf{500\ new\ units\ of\ length}$$

Q. 6. Which of the following is the most precise device for measuring length ? (a) a Vernier callipers with 20 divisions on the sliding scale, coinciding with 19 main scale divisions (b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale (c) an optical instrument that can measure length to within a wave length of light.

Sol. The most precise device is that whose least count is minimum. Now

(a) Least count of this vernier callipers $= 1\text{ SD} - 1\text{ VD} = 1\text{SD} - \frac{19}{20}\text{SD} = \frac{1}{20}\text{SD}$

$$= \frac{1}{20}\text{mm} = \frac{1}{200}\text{cm} = 0{\cdot}005\,\text{cm}$$

(b) Least count of screw gauge $= \dfrac{\text{pitch}}{\text{no.of divisions on circular scale}} = \dfrac{1}{100}\text{mm} = \dfrac{1}{1000}\text{cm} = 0{\cdot}001\text{cm}$

(c) Wavelength of light, $\lambda \approx 10^{-5}$ cm. $= 0{\cdot}00001$ cm.

Obviously, **the most precise measurement is with optical instrument.**

Q. 7. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3·5 mm. What is his estimate on the thickness of hair ?

Sol. Magnification, $m = \dfrac{\text{observed width } (y)}{\text{real width } (x)}$

$$x = \frac{y}{m} = \frac{3{\cdot}5\,\text{mm}}{100} = \mathbf{0{\cdot}035\,mm}$$

Q. 8. (a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?

(b) A screw gauge has pitch of 1·0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the gauge arbitrarily by increasing the number of divisions on the circular scale ?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

Sol. *(a)* The diameter of a thread is so small that it cannot be measured using a metre scale. We wind a number of turns of the thread on the metre scale so that the turns are closely touching one another.

Measure the length (l) of the windings on the scale which contains n number of turns

$$\therefore \text{ Diameter of thread } = \frac{l}{n}$$

(b) As least count $= \dfrac{\text{pitch}}{\text{number of divisions on circular scale}}$

∴ theoretically speaking, least count decreases on increasing the number of divisions on the circular scale.

Hence, accuracy would increase. Practically, it may not be possible to take the reading precisely due to low resolution of human eye.

(c) A large number of observations (say, 100) will give more reliable result than smaller number of observations (say, 5). This is because of probability of making a positive random error of certain magnitude is equal to that of making a negative random error of the same magnitude. Therefore, when number of observations is large, **random errors are likely to cancel** and the result may be more reliable.

Q. 9. The photograph of a house occupies an area of $1{\cdot}75\ \text{cm}^2$ on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is $1{\cdot}55\ \text{m}^2$. What is the linear magnification of the projector screen arrangement ?

Sol. Here, area of object $= 1{\cdot}75\ \text{cm}^2$ and area of image $= 1{\cdot}55\ \text{m}^2 = 1{\cdot}55 \times 10^4\ \text{cm}^2$

$$\therefore \text{ Areal magnification } = \frac{\text{area of image}}{\text{area of object}} = \frac{1{\cdot}55 \times 10^4}{1{\cdot}75} = 8857$$

Linear magnification $= \sqrt{8857} = \mathbf{94{\cdot}1}$

Q.10. State the number of significant figures in the following :

(a) $0{\cdot}007\ \text{m}^2$ (b) $2{\cdot}64 \times 10^{24}$ kg (c) $0{\cdot}2370\ \text{g cm}^{-3}$ (d) $6{\cdot}320$ J (e) $6{\cdot}032\ \text{N m}^{-2}$ (f) $0{\cdot}0006032\ \text{m}^2$

Sol. The number of significant figures is as follows : (a) one (b) three (c) four (d) four (e) four (f) four

Q. 11. The length, breadth and thickness of a metal sheet are 4·234 m, 1·005 m and 2·01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. Here, length, $l = 4{\cdot}234$ m, breadth, $b = 1{\cdot}005$ m, thickness, $t = 2{\cdot}01\ \text{cm} = 2{\cdot}01 \times 10^{-2}$ m

$$\begin{aligned}\text{Area of the sheet} &= 2\,(l \times b + b \times t + t \times l) = 2\,(4{\cdot}234 \times 1{\cdot}005 + 1{\cdot}005 \times 0{\cdot}0201 + 0{\cdot}0201 \times 4{\cdot}234)\\ &= 2\,(4{\cdot}3604739) = 8{\cdot}7209478\ \text{m}^2\end{aligned}$$

As area can contain a maximum of three significant digits, therefore, rounding off, we get

$$\textbf{Area} = \mathbf{8{\cdot}72\ m^2}$$

Also, volume $= l \times b \times t$

$V = 4{\cdot}234 \times 1{\cdot}005 \times 0{\cdot}0201 = 0{\cdot}0855289 = \mathbf{0{\cdot}0855\ m^3}$ (containing three significant figures)

Q. 12. The mass of a box measured by a grocer's balance is 2·3 kg. Two gold pieces of masses 20·15 g and 20·17 g are added to the box. What is (a) total mass of the box (b) the difference in masses of gold pieces to correct significant figures.

Sol. Here, mass of the box, $m = 2{\cdot}3$ kg

Mass of one gold piece, $m_1 = 20{\cdot}15\ \text{g} = 0{\cdot}02015$ kg

Mass of other gold piece, $m_2 = 20{\cdot}17\ \text{g} = 0{\cdot}02017$ kg

(a) Total mass $= m + m_1 + m_2 = 2{\cdot}3 + 0{\cdot}02015 + 0{\cdot}02017 = 2{\cdot}34032$ kg.

As the result is correct only upto one place of decimal, therefore, on rounding off

total mass = **2·3 kg**

(b) Difference in mases $= m_2 - m_1 = 20{\cdot}17 - 20{\cdot}15 = \mathbf{0{\cdot}02}$ g (correct upto two places of decimal).

Q. 13. A famous relation in Physics relates moving mass m to the rest mass m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special theory of relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes $m = \dfrac{m_0}{(1 - v^2)^{1/2}}$

Guess where to put the missing c ?

Sol. According to the principle of homogeneity of dimensions, powers of M, L, T on either side of the formula must be equal. For this, on RHS, the denominator $(1 - v^2)^{1/2}$ should be dimensionless. Therefore, instead of $(1 - v^2)^{1/2}$, we should write $(1 - v^2/c^2)^{1/2}$

Hence, the correct formula would be $$m = \frac{m_0}{(1 - v^2/c^2)^{1/2}}$$

Q. 14. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å : 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0·5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms ?

Sol. Here, $r = 0{\cdot}5$ Å $= 0{\cdot}5 \times 10^{-10}$ m.

Volume of each atom of hydrogen $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3{\cdot}14\,(0{\cdot}5 \times 10^{-10})^3 = 5{\cdot}236 \times 10^{-31}\ \text{m}^3$

Number of hydrogen atoms in one gram mole of hydrogen = Avogadro's number $= 6{\cdot}023 \times 10^{23}$

$\therefore$ Atomic volume of one gram mole of hydrogen atom $= 5{\cdot}236 \times 10^{-31} \times 6{\cdot}023 \times 10^{23} = \mathbf{3{\cdot}154 \times 10^{-7}\ m^3}$

Q. 15. One mole of an ideal gas at NTP occupies 22·4 litres (molar volume). What is the ratio of molar volume to atomic volume of a mole of hydrogen ? Take size of hydrogen molecule to be 1 Å. Why is this ratio so large ?

Sol. Atomic volume $= \frac{4}{3}\pi R^3 \times N = \frac{4}{3}\pi\ (0{\cdot}5 \times 10^{-10})^3 \times 6{\cdot}023 \times 10^{23} = 3{\cdot}154 \times 10^{-7}\ \text{m}^3$.

Molar volume = 22·4 litre $= 22{\cdot}4 \times 10^{-3}\ \text{m}^3$.

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22{\cdot}4 \times 10^{-3}}{3{\cdot}154 \times 10^{-7}} = \mathbf{7{\cdot}1 \times 10^4}$$

This ratio is large due to large intermolecular separations.

Q. 16. Explain this common observation clearly. If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the moon, the stars etc.) seem to be stationary.

Sol. The line joining the object to the eye is called *the line of sight.* When a train moves rapidly, the line of sight of a nearby tree changes its direction of motion rapidly. Therefore, the trees appear to run in opposite direction.

On the contrary, the line of sight of far off objects (hill tops, the moon, the stars etc.) does not change its direction so much, due to extremely large distance from the eye. Hence, distant hill tops, moon, the stars etc. appear stationary.

Q. 17. The sun is a hot plasma (ionised matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At such high temps, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the sun to be ? In the range of densities of solids, liquids or gases ? Check if your guess is correct from the following data : mass of sun = $2{\cdot}0 \times 10^{30}$ kg ; radius of the sun = $7{\cdot}0 \times 10^8$ m

Sol. Here, $M = 2{\cdot}0 \times 10^{30}$ kg ; $R = 7{\cdot}0 \times 10^8$ m ; Density, $\rho = ?$

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3} = \frac{3 \times 2{\cdot}0 \times 10^{30}}{4 \times 3{\cdot}14 (7 \times 10^8)^3} = \mathbf{1{\cdot}392 \times 10^3\ kg/m^3}$$

This is the order of density of solids and liquids ; and not gases.

The high density of sun is due to inward gravitational attraction on outer layers, due to the inner layers of the sun.

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NCERT EXEMPLAR PROBLEMS

WITH ANSWERS,
HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. The number of significant figures in 0·06900 is

(*a*) 5 (*b*) 4
(*c*) 2 (*d*) 3

2. The sum of the numbers 436·32, 227·2 and 0·301 in appropriate significant figures is

(*a*) 663·821 (*b*) 664
(*c*) 663·8 (*d*) 663·82

3. The mass and volume of a body are 4·237 g and 2·5 cm^3, respectively. The density of the material of the body in correct significant figures is

(*a*) 1·6048 g cm^3 (*b*) 1·69 g cm^{-3}
(*c*) 1·7 g cm^{-3} (*d*) 1·695 g cm^{-3}

4. The numbers 2·745 and 2·735 on rounding off to 3 significant figures will give

(*a*) 2·75 and 2·74 (*b*) 2·74 and 2·73
(*c*) 2·75 and 2·73 (*d*) 2·74 and 2·74

5. The length and breadth of a rectangular sheet are 16·2 cm and 10·1 cm respectively. The area of the sheet in appropriate significant figures and error is

(*a*) 164 ± 3 cm^2 (*b*) 163·62 ± 2·6 cm^2
(*c*) 163·6 ± 2·6 cm^2 (*d*) 163·62 ± 3 cm^2

6. Which of the following pairs of physical quantities does not have same dimensional formula ?

(*a*) Work and torque
(*b*) Angular momentum and Planck's constant
(*c*) Tension and surface tension
(*d*) Impulse and linear momentum

7. Measure of two quantities along with the precision of respective measuring instrument is

$$A = 2{\cdot}5\ \text{ms}^{-1} \pm 0{\cdot}5\ \text{ms}^{-1}$$
$$B = 0{\cdot}10\ \text{s} \pm 0{\cdot}01\ \text{s}$$

The value of AB will be

(*a*) (0·25 ± 0·08) m (*b*) (0·25 ± 0·5) m
(*c*) (0·25 ± 0·05) m (*d*) (0·25 ± 0·135) m

8. You measure two quantities as $A = 1{\cdot}0$ m ± 0·2 m, $B = 2{\cdot}0$ m ± 0·2 m. We should report correct value for $\sqrt{AB}$ as :

(*a*) 1·4 m ± 0·4 m (*b*) 1·41 m ± 0·15 m
(*c*) 1·4 m ± 0·3 m (*d*) 1·4 m ± 0·2 m

9. Which of the following measurements is most precise ?

(*a*) 5·00 mm (*b*) 5·00 cm
(*c*) 5·00 m (*d*) 5·00 km

10. The mean length of an object is 5 cm. Which of the following measurements is most accurate ?

(*a*) 4·9 cm (*b*) 4·805 cm
(*c*) 5·25 cm (*d*) 5·4 cm

11. Young's modulus of steel is 1·9 × 10^{11} N/m^2. When expressed in CGS units of dynes/cm^2, it will be equal to (1N = 10^5 dyne, 1 m^2 = 10^4 cm^2)

(*a*) 1·9 × 10^{10} (*b*) 1·9 × 10^{11}
(*c*) 1·9 × 10^{12} (*d*) 1·9 × 10^{13}

12. If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula

(*a*) $(P^1 A^{-1} T^1)$ (*b*) $(P^2 A^1 T^1)$
(*c*) $(P^1 A^{-1/2} T^1)$ (*d*) $(P^1 A^{1/2} T^{-1})$

MULTIPLE CHOICE QUESTIONS-II

13. On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is *not* correct :

(*a*) $y = a \sin 2\pi t/T$ (*b*) $y = a \sin vt$

(*c*) $y = \dfrac{a}{T}\sin\left(\dfrac{t}{a}\right)$

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(*d*) $y = a\sqrt{2}\left(\sin\frac{2\pi t}{T} - \cos\frac{2\pi t}{T}\right)$

14. If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity ?

(*a*) $(P - Q)/R$ (*b*) $PQ - R$
(*c*) PQ/R (*d*) $(PR - Q^2)/R$
(*e*) $(R + Q)/P$

15. Photon is quantum of radiation with energy $E = h\nu$ where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of

(*a*) Linear impulse
(*b*) Angular impulse
(*c*) Linear momentum
(*d*) Angular momentum

16. If Planck's constant (h) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities ?

(*a*) Mass of electron (m_e)
(*b*) Universal gravitational constant (G)
(*c*) Charge of electron (e)
(*d*) Mass of proton (m_p)

17. Which of the following ratios express pressure ?

(*a*) Force/Area (*b*) Energy/Volume
(*c*) Energy/Area (*d*) Force/Volume

18. Which of the following are *not* a unit of time ?

(*a*) Second (*b*) Parsec
(*c*) Year (*d*) Light year

ANSWERS

1. (*b*) **2.** (*c*) **3.** (*c*) **4.** (*d*) **5.** (*a*) **6.** (*c*) **7.** (*a*) **8.** (*d*) **9.** (*a*) **10.** (*a*)
11. (*c*) **12.** (*d*) **13.** (*b,c*) **14.** (*a, e*) **15.** (*b, d*) **16.** (*a,b,d*) **17.** (*a,b*) **18.** (*b,d*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. In $0{\cdot}0\underline{6900}$, the number of significant figures is 4, as marked.

2. $$\begin{array}{r} 436{\cdot}32 \\ 227{\cdot}2 \\ 0{\cdot}301 \\ \hline 663{\cdot}821 \end{array}$$

The result of summation is valid upto one place of decimal, as per rules. $\therefore$ Given sum $= 663{\cdot}8$

3. Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{4{\cdot}237}{2{\cdot}5} = 1{\cdot}6948$ g/c.c.

As per rules, the result of division will have two significant figures.

$\therefore$ Density $= 1{\cdot}7$ g/c.c.

4. On rounding off to 3 significant figures, $2{\cdot}745 \to 2{\cdot}74$, as per rules ; $2{\cdot}735 \to 2{\cdot}74$, as per rules.

5. Here, $l = (16{\cdot}2 \pm 0{\cdot}1)$ cm ; $b = (10{\cdot}1 \pm 0{\cdot}1)$ cm ; $A = l \times b = 16{\cdot}2 \times 10{\cdot}1 = 163{\cdot}62$

Rounding off to three significant digits, $A = 164$ cm^2.

$$\frac{\Delta A}{A} = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) = \frac{0{\cdot}1}{16{\cdot}2} + \frac{0{\cdot}1}{10{\cdot}1} = \frac{1{\cdot}01 + 1{\cdot}62}{16{\cdot}2 \times 10{\cdot}1}$$

$$\Delta A = 2{\cdot}63$$

Rounding off to one significant figure, $\Delta A = 3$ cm^2 $\therefore A = (164 \pm 3)$ cm^2

6. Tension = force = $[MLT^{-2}]$

$$\text{Surface Tension} = \frac{\text{force}}{\text{length}} = \frac{MLT^{-2}}{L} = [ML^0\,T^{-2}]$$

7. Here, $A = 2{\cdot}5 \text{ ms}^{-1} \pm 0{\cdot}5 \text{ ms}^{-1}$, $B = 0{\cdot}10 \text{ s} \pm 0{\cdot}01 \text{ s}$

$x = AB = (2{\cdot}5)(0{\cdot}10) = 0{\cdot}25 \text{ m}$

$$\frac{\Delta x}{x} = \frac{\Delta A}{A} + \frac{\Delta B}{B} = \frac{0{\cdot}5}{2{\cdot}5} + \frac{0{\cdot}01}{0{\cdot}10} = \frac{0{\cdot}05 + 0{\cdot}025}{0{\cdot}25} = \frac{0{\cdot}075}{0{\cdot}25}$$

$\Delta x = 0{\cdot}075 = 0{\cdot}08$ m, rounding off to two significant figures.

$\therefore$ $AB = (0{\cdot}25 \pm 0{\cdot}08)$ m.

8. Here, $A = 1{\cdot}0 \text{ m} \pm 0{\cdot}2 \text{ m}$, $B = 2{\cdot}0 \text{ m} \pm 0{\cdot}2 \text{ m}$

$$x = \sqrt{AB} = \sqrt{(1{\cdot}0)(2{\cdot}0)} = 1{\cdot}414 \text{ m}$$

Rounding off to two significant digits, $x = \sqrt{AB} = 1{\cdot}4 \text{ m}$.

Now, $$\frac{\Delta x}{x} = \frac{1}{2}\left[\frac{\Delta A}{A} + \frac{\Delta B}{B}\right] = \frac{1}{2}\left[\frac{0{\cdot}2}{1{\cdot}0} + \frac{0{\cdot}2}{2{\cdot}0}\right] = \frac{0{\cdot}6}{2 \times 2{\cdot}0}$$

$$\Delta x = \frac{0{\cdot}6\,x}{2 \times 2{\cdot}0} = 0{\cdot}15 \times 1{\cdot}414 = 0{\cdot}2121$$

Rounding off to one significant digit, $\Delta x = 0{\cdot}2$ m

Hence, $\sqrt{AB} = 1{\cdot}4 \text{ m} \pm 0{\cdot}2 \text{ m}$.

9. All measurements are correct upto two places of decimal. However, the absolute error in (*a*) is 0·01 mm, which is least of all the four. So, it is most precise.

10. Here, $x = 5$ cm

$\Delta x_1 = 5 - 4{\cdot}9 = 0{\cdot}1$ cm, $\quad \Delta x_2 = 5 - 4{\cdot}805 = 0{\cdot}195$ cm

$\Delta x_3 = 5 - 5{\cdot}25 = -0{\cdot}25$ cm, $\quad \Delta x_4 = 5 - 5{\cdot}4 = -0{\cdot}4$ cm

Choice (*a*) is most accurate.

11. Here, $Y = 1{\cdot}9 \times 10^{11} \text{ N/m}^2 = 1{\cdot}9 \times 10^{11} \times \frac{10^5 \text{ dyne}}{(10^2 \text{ cm})^2} = 1{\cdot}9 \times 10^{12} \text{ dyne/cm}^2$.

12. Let energy, $U \propto P^a A^b T^c$

$$U = k\,P^a A^b T^c \qquad \text{...}(i)$$

where k is dimensionless constant of proportionality.

From (*i*), $[ML^2T^{-2}] = (MLT^{-1})^a (L^2)^b T^c = M^a L^{a+2b} T^{-a+c}$

Applying the principle of homogeneity of dimensions, we get

$$a = 1, \; a + 2b = 2\,; \; b = \frac{2-a}{2} = \frac{2-1}{2} = \frac{1}{2}$$

$$-a + c = -2, \; c = -2 + a = -2 + 1 = -1$$

$\therefore$ from (*i*), $U = k\,[P^1 A^{1/2} T^{-1}]$

Choice (*d*) is correct.

Multiple Choice Questions – II

13. As angle is dimensionless, therefore, choices (*b*) and (*c*) are not correct.

14. As *P*, *Q*, *R* are physical quantities having different dimensions, therefore, they can neither be added nor subtracted. Thus (*a*) and (*e*) can never be meaningful. (*b*) is meaningful only if *PQ* and *R* have same dimensions. Similarly, (*d*) is meaningful only when *PR* have the dimensions of Q^2.

15. From $E = h\nu$

$$h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{T^{-1}} = [M^1 L^2 T^{-1}]$$

Angular impulse $= \tau \times t = [ML^2T^{-2} \times T = [M^1 L^2 T^{-1}]$

Angular momentum $= m\,v\,r = M\,[LT^{-1}]\,L = [M^1 L^2 T^{-1}]$.

16. Here, $h = [ML^2\,T^{-1}]$; $c = [LT^{-1}]$

$m_e = M,\ G = [M^{-1}\,L^3\,T^{-2}]$

$e = AT$; $m_p = M$.

Now, $$\frac{hc}{G} = \frac{(M^1 L^2 T^{-1})(LT^{-1})}{M^{-1} L^3 T^{-2}} = M^2$$

$$M = \sqrt{\frac{hc}{G}}$$

Again, $$\frac{h}{c} = \frac{ML^2 T^{-1}}{LT^{-1}} = ML$$

$$L = \frac{h}{cM} = \frac{h}{c}\sqrt{\frac{G}{hc}} = \frac{\sqrt{Gh}}{c^{3/2}}$$

From $c = LT^{-1}$, $$T = \frac{L}{c} = \frac{\sqrt{Gh}}{c^{3/2}.c} = \frac{\sqrt{Gh}}{c^{5/2}}$$

Hence, out of (*a*), (*b*), (*d*), any one can be taken to express *L*, *M*, *T* in terms of three chosen fundamental quantities.

17. Pressure $= \dfrac{\text{force}}{\text{area}}$, by definition

Pressure $= \dfrac{\text{force} \times \text{distance}}{\text{area} \times \text{distance}} = \dfrac{\text{energy}}{\text{volume}}$.

18. Parsec and light year, both are units of distance ; and not of time.

VERY SHORT ANSWER QUESTIONS

19. Why do we have different units for same physical quantity ?

Ans. The value of a given physical quantity may vary over a wide range. To express the quantity in proper format, we may need different units.

Further, a given quantity may be expressed in terms of different quantities, will have same dimensions. This would lead to different equivalent units of the same quantity.

20. The radius of atom is of the order of 1 Å and radius of nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus ?

Ans. Here, $R_A \simeq 1\ \text{Å} = 10^{-10}$ m, $R_N \simeq 1$ fermi $= 10^{-15}$ m

$$\therefore \quad \frac{V_A}{V_N} = \frac{\frac{4}{3}\pi R_A^3}{\frac{4}{3}\pi R_N^3} = \left(\frac{R_A}{R_N}\right)^3 = \left(\frac{10^{-10}}{10^{-15}}\right)^3 = 10^{15}.$$

21. Name the device used for measuring the mass of atoms and molecules.

Ans. A mass spectrograph is used for measuring the mass of atoms and molecules.

22. Express unified atomic mass unit in kg.

Ans. Unified atomic mass unit, *i.e.*, 1 a.m.u. $= 1{\cdot}67 \times 10^{-27}$ kg

Infact, by definition, $$1 \text{ a.m.u.} = \frac{1}{12} \times \text{mass of an atom of } C^{12}$$

$$= \frac{1}{12} \times \frac{12\,\text{g}}{6{\cdot}023 \times 10^{23}} = 1{\cdot}67 \times 10^{-24}\ \text{g} = 1{\cdot}67 \times 10^{-27}\ \text{kg}$$

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23. A function $f(\theta)$ is defined as : $f(\theta) = 1 - \theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} ...$

Why is it necessary for θ to be a dimensionless quantity ?

Ans. Here, $f(\theta) = 1 - \theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} - ....$.

As $f(\theta)$ is sum of different powers of θ, therefore θ must be dimensionless. This is because quantities having different dimensions cannot be added.

24. Why length, mass and time are chosen as base quantities in mechanics ?

Ans. In mechanics, length, mass and time are chosen as the base quantities because

(*i*) there is nothing simpler to length, mass and time.

(*ii*) all other quantities in mechanics can be expressed in terms of length, mass and time.

(*iii*) length, mass and time cannot be derived from one another.

SHORT ANSWER QUESTIONS

25. (*a*) The earth-moon distance is about 60 earth radius. What will be the diameter of the earth (approximately in degrees) as seen from the moon ?

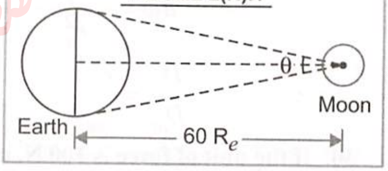
(*b*) Moon is seen to be of (1/2)° diameter from the earth. What must be the relative size compared to the earth ?

(*c*) From parallax measurement, the sun is found to be at a distance of about 400 times the earth-moon distance. Estimate the ratio of sun-earth diameters.

Ans. (*a*) As is clear from Fig. 2(N).1,

$$\theta = \frac{l}{r} = \frac{2R_e}{60R_e} = \frac{1}{30}\text{ radian} = \frac{1}{30} \times \frac{180^\circ}{\pi} = \frac{6^\circ}{\pi} = \frac{42^\circ}{22} = 1{\cdot}9^\circ$$

FIGURE 2(N).1

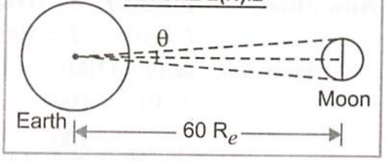


(*b*) As is clear from Fig. 2(N).2,

$$\theta = \frac{l}{r} = \frac{2R_m}{60R_e} = \frac{1^\circ}{2} = \frac{1}{2} \times \frac{\pi}{180}\text{ rad}$$

$$\therefore \quad \frac{R_m}{R_e} = \frac{30\pi}{2 \times 180} = \frac{\pi}{12} = \frac{11}{42} \simeq \frac{1}{4}$$

FIGURE 2(N).2



(*c*) We are given $\frac{\text{distance of sun from earth}}{\text{distance of moon from earth}} = \frac{r_{\text{sun}}}{r_{\text{moon}}} = 400$

We have to calculate $= \frac{\text{diameter of sun}}{\text{diameter of earth}} = \frac{D_{\text{sun}}}{D_{\text{earth}}} = ?$

As seen from the earth, both sun and moon appear to be of same angular diameter.

$$\therefore \qquad \theta = \frac{D_{\text{sun}}}{r_{\text{sun}}} = \frac{D_{\text{moon}}}{r_{\text{moon}}}$$

$$\frac{D_{\text{sun}}}{D_{\text{moon}}} = \frac{r_{\text{sun}}}{r_{\text{moon}}} = 400 \qquad ...(i)$$

From (*b*) above, $\frac{D_{\text{earth}}}{D_{\text{moon}}} = 4$ ...(*ii*)

$\therefore$ Dividing (*i*) by (*ii*), we get $\frac{D_{\text{sun}}}{D_{\text{earth}}} = \frac{400}{4} = 100$

26. Which of the following time measuring devices is most precise ?
(*a*) A wall clock. (*b*) A stop watch. (*c*) A digital watch. (*d*) An atomic clock.
Give reason for your answer.

Ans. A wall clock may measure correctly upto one second. A stop watch may measure fraction of a second. A digital watch may measure some fraction of a second.

An atomic clock is the most precise time measuring device as atomic oscillators are repeated with a precision of 1 s in 10^{13} s.

27. The distance of a galaxy is of the order of 10^{25} m. Calculate the order of magnitude of time taken by light to reach us from the galaxy.

Ans. Time taken by light to reach us from galaxy, $t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{10^{25}}{3\times10^{8}} \simeq 3 \times 10^{16}$ second

28. The vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0·5 mm, calculate the minimum inaccuracy in the measurement of distance.

Ans. Minimum inaccuracy in the measurement of distance by Vernier callipers

$$= \text{Vernier constant} = 1\text{ MSD} - 1\text{ VSD} = 1\text{MSD} - \frac{49}{50}\text{MSD} = \frac{1}{50}\text{MSD} = \frac{1}{50}\times 0{\cdot}5\text{ mm} = 0{\cdot}01\text{ mm}$$

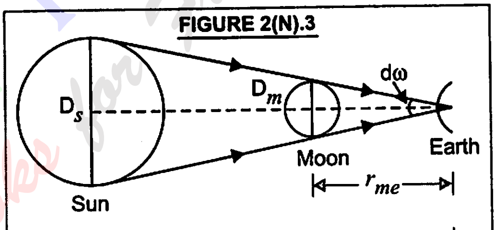
29. During a total solar eclipse the moon almost entirely covers the sphere of the sun. Write the relation between the distances and sizes of the sun and moon.

Ans. As is clear from Fig. 2(N).3.

$$d\omega = \frac{\pi D_s^2/4}{r_{se}^2} = \frac{\pi D_m^2/4}{r_{me}^2}$$

$$\therefore \quad \frac{D_s}{D_m} = \frac{r_{se}}{r_{me}} \quad \text{or} \quad \frac{R_s}{R_m} = \frac{r_{se}}{r_{me}}$$

FIGURE 2(N).3



30. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s, what is the unit of mass in this system of units ?

Ans. Here, $F = MLT^{-2} = 100$ N ...(*i*)

$L = 10$ m ; $T = 100$ s

From (*i*) ; $M\,(10)\,(100)^{-2} = 100$

$M\,10^{-3} = 100$

$$M = \frac{100}{10^{-3}} = 10^5\text{ kg}$$

31. Give an example of (*a*) a physical quantity which has a unit but no dimensions.
(*b*) a physical quantity which has neither unit nor dimensions.
(*c*) a constant which has a unit.
(*d*) a constant which has no unit.

Ans. (*a*) Angle in a plane is measured in radian, though it has no dimensions.
(*b*) Strain has neither units nor dimensions.
(*c*) Universal gravitational constant (*G*) has a constant value : $6{\cdot}67 \times 10^{-11}$ Nm^2 kg^{-2}
(*d*) Reynold number is a constant which has no units.

32. Calculate the length of the arc of a circle of radius 31·0 cm which subtends an angle of $\dfrac{\pi}{6}$ at the centre.

Ans. Here, $l = ?$, $r = 31{\cdot}0$ cm, $\theta = \dfrac{\pi}{6}$

From $l = r\,\theta$

$$l = 31{\cdot}0 \times \frac{\pi}{6} = 16{\cdot}2\text{ cm}$$

33. Calculate the solid angle subtended by the periphery of an area of 1 cm^2 at a point situated symmetrically at a distance of 5 cm from the area.

Ans. $d\omega = \dfrac{ds}{r^2} = \dfrac{1\,\text{cm}^2}{(5\,\text{cm})^2} = \mathbf{4 \times 10^{-2}}$ **steradian**

34. The displacement of a progressive wave is represented by $y = A\sin(\omega t - kx)$, where x is distance and t is time. Write the dimensional formula of (*i*) ω and (*ii*) k.

Ans. Here, $y = A\sin(\omega t - kx)$

As angle is dimensionless, therefore $\omega t = 1,\ \omega = \dfrac{1}{t} \rightarrow [T^{-1}]\ ;\ kx = 1,\ k = \dfrac{1}{x} \rightarrow [L^{-1}]$

35. Time for 20 oscillations of a pendulum is measured as $t_1 = 39\cdot 6$ s ; $t_2 = 39\cdot 9$ s ; $t_3 = 39\cdot 5$ s. What is the precision in the measurements ? What is the accuracy of the measurement ?

Ans. Here, $t_1 = 39\cdot 6$ s, $t_2 = 39\cdot 9$ s and $t_3 = 39\cdot 5$ s.

Precision in the measurement = least count of measuring instrument = 0·1 s.

Mean value of time for 20 oscillations $t = \dfrac{t_1 + t_2 + t_3}{3} = \dfrac{39\cdot 6 + 39\cdot 9 + 39\cdot 5}{3} = 39\cdot 7\,\text{s}$

$$\Delta t_1 = t - t_1 = 39\cdot 7 - 39\cdot 6 = 0\cdot 1 \text{ s}$$
$$\Delta t_2 = t - t_2 = 39\cdot 7 - 39\cdot 9 = -\,0\cdot 2 \text{ s}$$
$$\Delta t_3 = t - t_3 = 39\cdot 7 - 39\cdot 5 = 0\cdot 2 \text{ s}$$

Mean absolute error $= \dfrac{|\Delta t_1| + |\Delta t_2| + |\Delta t_3|}{3} = \dfrac{0\cdot 1 + 0\cdot 2 + 0\cdot 2}{3} = 0\cdot 17$

$\therefore$ Accuracy of measurement = **± 0·2 s,** rounded off to one decimal place.

Infact, accuracy in measurement depends upon many factors like personal errors, instrumental errors, random errors, errors due to external causes and so on.

LONG ANSWER QUESTIONS

36. A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5 J measure in this new system ?

Ans. Here, $M_1 = 1$ kg, $L_1 = 1$ m, $T_1 = 1$ s, $M_2 = \alpha$ kg, $L_2 = \beta\,m$ and $T_2 = \gamma$ s, $n_1 = 5$ J, $n_2 = ?$

For energy, $U = [M^1 L^2 T^{-2}]$

As $n_2 u_2 = n_1 u_1$

$$n_2 = n_1\left(\frac{u_1}{u_2}\right) = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2} = n_1\left(\frac{1}{\alpha}\right)^1\left(\frac{1}{\beta}\right)^2\left(\frac{1}{\gamma}\right)^{-2} = \frac{n_1\gamma^2}{\alpha\beta^2} = \frac{5\gamma^2}{\alpha\beta^2} \text{ new units of energy.}$$

37. The volume of a liquid flowing out per second of a pipe of length *l* and radius *r* is written by a student as

$$v = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

where *P* is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $ML^{-1}T^{-1}$.

Check whether the equation is dimensionally correct.

Ans. The given relation is $v = \dfrac{\pi}{8}\dfrac{P\,r^4}{\eta l}$

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$$\text{LHS} = v = \frac{L^3}{T} = L^3\,T^{-1} = [M^0\,L^3\,T^{-1}]$$

$$\text{RHS} = \frac{\pi}{8}\,\frac{P\,r^4}{\eta\,l} = \frac{(ML^{-1}T^{-2})(L^4)}{(ML^{-1}T^{-1})(L)} = [M^0\,L^3\,T^{-1}]$$

As LHS = RHS, dimensionally. $\therefore$ Formula is dimensionally correct.

38. A physical quantity X is related to four measurable quantities a, b, c and d as follows :

$$X = a^2\,b^3\,c^{5/2}\,d^{-2}$$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity X ? If the value of X calculated on the basis of the above relation is 2·763, to what value should you round off the result ?

Ans. Here, $X = a^2\,b^3\,c^{5/2}\,d^{-2}$

$$\frac{\Delta X}{X} = \pm\left[2\left(\frac{\Delta a}{a}\right) + 3\left(\frac{\Delta b}{b}\right) + \frac{5}{2}\left(\frac{\Delta c}{c}\right) + 2\left(\frac{\Delta d}{d}\right)\right] = \pm\left[2(1\%) + 3(2\%) + \frac{5}{2}(3\%) + 2(4\%)\right] = \pm\,23{\cdot}5\%$$

Mean absolute error in X = ± 0·235 = ± 0·24, rounding off to two digits.

The value of X calculated on the basis of the above relation has to be rounded off so as to contain two significant digits.

$\therefore$ $X = 2{\cdot}8$

39. In the expression $P = E\,l^2\,m^{-5}\,G^{-2}$, E, m, l and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

Ans. Here, $P = E\,l^2\,m^{-5}\,G^{-2} = [ML^2\,T^{-2}]\,[ML^2\,T^{-1}]^2\,[M]^{-5}\,[M^{-1}\,L^3\,T^{-2}]^{-2} = M^{1+2-5+2}\,L^{2+4-6}\,T^{-2-2+4}$

$P = [M^0\,L^0\,T^0]$

Hence, P is a dimensionless quantity.

40. If velocity of light c, Planck's constant h and gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of dimensions of these quantities.

Ans. Here, $c = [LT^{-1}]$, $h = [ML^2\,T^{-1}]$

$G = [M^{-1}\,L^3\,T^{-2}]$

Let $m = c^x\,h^y\,G^z$ ...(*i*)

$$[M^1\,L^0\,T^0] = (LT^{-1})^x\,(ML^2\,T^{-1})^y\,(M^{-1}\,L^3\,T^{-2})^z = M^{y-z}\,L^{x+2y+3z}\,T^{-x-y-2z}$$

Applying principle of homogeneity of dimensions, we get

$y - z = 1$ ...(*ii*)

$x + 2y + 3z = 0$ ...(*iii*)

$-x - y - 2z = 0$ ...(*iv*)

Add (*ii*), (*iii*) and (*iv*),

$$2y = 1,\; y = \frac{1}{2}$$

From (*ii*), $z = y - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

From (*iv*), $x = -y - 2z = -\frac{1}{2} + 1 = \frac{1}{2}$

Putting in (*i*), we get $m = c^{1/2}\,h^{1/2}\,G^{-1/2}$

$$m = \sqrt{\frac{ch}{G}}$$

Proceeding as above, we can show that $L = \sqrt{\dfrac{hG}{c^3}}$ and $T = \sqrt{\dfrac{hG}{c^5}}$

41. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's Third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r. Show using dimensional analysis, that $T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$,

where k is a dimensionless constant and g is acceleration due to gravity.

Ans. Here, we given that $T^2 \propto r^3$ or $T \propto r^{3/2}$

Also, T is a function of g and R.

Let $T \propto r^{3/2}\, g^a\, R^b$. where a, b are the dimensions of g and R.

or $T = k\, r^{3/2}\, g^a\, R^b$ ...(*i*)

where k is dimensionless constant of proportionality.

From (*i*), $[M^0 L^0 T^1] = L^{3/2}\, (LT^{-2})^a\, (L)^b = M^0\, L^{a+b+3/2}\, T^{-2a}$

Applying the principle of homogeneity of dimensions, we get

$$a + b + 3/2 = 0 \quad \text{...(ii)}$$

$$-2a = 1,\; a = -\frac{1}{2}$$

From (*ii*), $$-\frac{1}{2} + b + \frac{3}{2} = 0$$

$$b = -1$$

Putting these values in (*i*), we get

$$T = k\, r^{3/2}\, g^{-1/2}\, R^{-1}$$

$$T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$$

This is the required relation.

42. In an experiment to estimate the size of a molecule of oleic acid 1 mL of oleic acid is dissolved in 19 mL of alcohol. Then 1 mL of this solution is diluted to 20 mL by adding alcohol. Now 1 drop of this diluted solution is placed on water in a shallow trough. The solution spreads over the surface of water forming one molecule thick layer. Now, lycopodium powder is sprinkled evenly over the film and its diameter is measured. Knowing the volume of the drop and area of the film we can calculate the thickness of the film which will give us the size of oleic acid molecule.

Read the passage carefully and answer the following questions :

(*a*) Why do we dissolve oleic acid in alcohol ?

(*b*) What is the role of lycopodium powder ?

(*c*) What would be the volume of oleic acid in each mL of solution prepared ?

(*d*) How will you calculate the volume of *n* drops of this solution of oleic acid ?

(*e*) What will be the volume of oleic acid in one drop of this solution ?

Ans. (*a*) Oleic acid is dissolved in alcohol, because it does not dissolve in water.

(*b*) When lycopodium powder is sprinkled evenly, it spreads over the entire surface of water.

When a drop of prepared solution is dropped on water, oleic acid does not dissolve in water. Instead, it spreads on the water surface pushing the lycopodium powder away to clear a circular area where the drop falls. We can therefore, measure the area over which oleic acid spreads.

(*c*) Volume of oleic acid in each *mL* of solution prepared $= \frac{1}{20} mL \times \frac{1}{20} = \frac{1}{400} mL$

(*d*) Volume of *n* drops of this solution of oleic acid can be calculated by using a burette and measuring cylinder.

(*e*) If 1 *mL* of solution contains *n* drops, then volume of oleic acid in one drop $= \frac{1}{(400n)} mL$

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43. (*a*) How many astronomical units (A.U.) make 1 parsec ?

(*b*) Consider a sunlike star at a distance of 1 parsec. When it is seen through a telescope with 100 magnification, what should be the angular size of the star ? Sun appears to be (1/2)° from the earth. Due to atmospheric fluctuations, eye can't resolve objects smaller than 1 arc minute.

(*c*) Mars has approximately half of the earth's diameter. When it is closest to the earth it is at about 1/2 A.U. from the earth. Calculate what size it will appear when seen through the same telescope.

Ans. (*a*) By definition, 1 par sec = distance at which 1 A.U. long arc subtends an angle of 1 sec.

$$1 \text{ par sec} = \frac{1 \text{A.U.}}{1 \text{arc sec}}$$

As $$1 \text{ arc sec} = \frac{1}{60 \times 60} \times \frac{\pi}{180} \text{radian}$$

$$\therefore \quad 1 \text{par sec} = \frac{1 \text{A.U.}}{\pi / 60 \times 60 \times 180} = \frac{3600 \times 180}{\pi} \text{A.U.} = 206181 \text{A.U.}$$

$$1 \text{ par sec} = 2 \times 10^5 \text{ A.U.}$$

(*b*) At 1 A.U., sun's diameter is $\frac{1^\circ}{2}$

$\therefore$ At 1 par sec = 2×10^5 A.U., star's diameter will be $= \frac{1/2^\circ}{2 \times 10^5} = \frac{60'}{4 \times 10^5} = 15 \times 10^{-5}$ min

With magnification = 100, star should look of diameter = $15 \times 10^{-5} \times 100 = 15 \times 10^{-3}$ min

However, due to atmospheric flutuations, it will still look of diameter about 1 minute.

Stars can't be magnified using telescope.

(*c*) We are given $\frac{D_{\text{mars}}}{D_{\text{earth}}} = \frac{1}{2}$, where D is for diameter.

From Table 2.5 on page 2/13, we know $\frac{D_{\text{earth}}}{D_{\text{sun}}} = \frac{1}{100} \quad \therefore \quad \frac{D_{\text{mars}}}{D_{\text{sun}}} = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$

At 1 A.U. sun's diameter $= \frac{1^\circ}{2} \quad \therefore \quad$ mar's diameter $= \frac{1}{2} \times \frac{1}{200} = \frac{1^\circ}{400}$

At $\frac{1}{2}$ A.U., mar's diameter $= \frac{1}{400} \times 2^\circ = \frac{1^\circ}{200}$.

with magnification = 100, mar's diameter $= \frac{1}{200} \times 100^\circ = \frac{1^\circ}{2} = 30'$

This is larger than resolution limit due to atmospheric fluctuations. Hence, we conclude that planet looks magnified when seen through telescope.

44. Einstein's mass - energy relation emerging out of his famous theory of relativity relates mass (*m*) to energy (*E*) as $E = mc^2$, where *c* is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear level is usually measured in MeV, where 1 MeV = 1·6 × 10^{-13} J ; the masses are measured in unified atomic mass unit (*u*) where 1 *u* = 1·67 × 10^{-27} kg.

(*a*) Show that the energy equivalent of 1 *u* is 931·5 MeV.

(*b*) A student writes the relation as 1 *u* = 931·5 MeV. The teacher points out that the relation is dimensionally incorrect. Write the correct relation.

Ans. (*a*) When $m = 1 \ u = 1{\cdot}67 \times 10^{-27}$ kg

$$E = mc^2 = 1{\cdot}67 \times 10^{-27} (3 \times 10^8)^2 \text{ J} = 1{\cdot}67 \times 9 \times 10^{-11} \text{ J}$$

$$E = \frac{1{\cdot}67 \times 9 \times 10^{-11}}{1{\cdot}6 \times 10^{-13}} \text{MeV} = 939{\cdot}4 \text{MeV}$$

(*b*) The dimensionally correct relation is 1 $u \times c^2$ = **939·4 MeV**

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I. Multiple Choice Questions (with One Correct Answer)

I. Units and Dimensions

1. The dimensions $[MLT^{-2} A^{-2}]$ belong to the
   (a) Magnetic flux
   (b) Self inductance
   (c) Magnetic permeability
   (d) Electric permittivity **(NEET 2022)**

2. Plane angle and solid angle have
   (a) Units but no dimensions
   (b) Dimensions but no units
   (c) No units and no dimensions
   (d) Both units and dimensions **(NEET 2022)**

3. The area of a rectangular field (in m^2) of length 55·3 m and breadth 25 m after rounding off the value for correct significant figures is
   (a) 138×10^1 m^2 (b) 1382 m^2
   (c) 1382·5 m^2 (d) 14×10^2 m^2
   (NEET 2022)

4. If E and G respectively denote energy and gravitational constant, then $\frac{E}{G}$ has the dimensions
   (a) $[M^2 L^{-2} T^{-1}]$ (b) $[M^2 L^{-1} T^0]$
   (c) $[M L^{-1} T^{-1}]$ (d) $[M L^0 T^0]$
   (NEET 2021)

5. If force (F) according (A) and time (T) are chosen as the fundamental physical quantities, find the dimensions of energy.
   (a) $[FA^{-1} T]$ (b) $[FAT]$
   (c) $[FAT^2]$ (d) $[FAT^{-1}]$
   (NEET 2021)

6. A screw gauge gives the following readings when used to measure the diameter of a wire : Main scale reading = 0 mm. Circular scale reading = 52 divisions. If 1 mm on main scale = 100 divisions on circular scale, diameter of wire is
   (a) 0·052 cm (b) 0·52 cm
   (c) 0·026 cm (d) 0·26 cm
   (NEET 2021)

7. If E and G respectively denote energy and gravitational constant, then E/G has the dimensions of
   (a) $[M^2 L^{-2} T^{-1}]$ (b) $[M^2 L^{-1} T^0]$
   (c) $[M L^{-1} T^{-1}]$ (d) $[M L^0 T^0]$
   (NEET 2021)

8. If force F, acceleration A and time T are chosen as the fundamental physical quantities, find the dimensions of energy.
   (a) $[F A^{-1} T]$ (b) $[F A T]$
   (c) $[F A T^2]$ (d) $[F A T^{-1}]$
   (NEET 2021)

ANSWERS

1. (c) 2. (a) 3. (d) 4. (b) 5. (c) 6. (a) 7. (b) 8. (c)

9. A screw gauge gives the following readings, when used to measure the diameter of a wire

Main scale reading : 0 mm

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions on the circular scale. The diameter of the wire from the above data is

(*a*) 0·052 cm (*b*) 0·52 cm
(*c*) 0·026 cm (*d*) 0·26 cm

(NEET 2021)

10. Dimensions of stress are

(*a*) $[ML^2T^{-2}]$ (*b*) $[ML^0T^{-2}]$
(*c*) $[ML^{-1}T^{-2}]$ (*d*) $[MLT^{-2}]$

(NEET 2020)

11. Taking into account the significant figures, what is the value of 9·99 m – 0·0099 m ?

(*a*) 9·98 m (*b*) 9·980 m
(*c*) 9·9 m (*d*) 9·9801 m

(NEET 2020)

12. A screw gauge has least count of 0·01 mm and there are 50 divisions in its circular scale. The pitch of the screw gauge is

(*a*) 0·25 mm (*b*) 0·50 mm
(*c*) 1·0 mm (*d*) 0·01 mm

(NEET 2020)

13. Time intervals measured by a clock give the following readings

1·28 s, 1·24 s, 1·27 s, 1·21 s and 1·28 s

What is the percentage relative error of the observations ?

(*a*) 1·6% (*b*) 2%
(*c*) 4% (*d*) 16% **(NEET 2020)**

14. The angle of 1 minute of arc in radian is nearly equal to

(*a*) $1{\cdot}75 \times 10^{-2}$ rad (*b*) $2{\cdot}91 \times 10^{-4}$ rad
(*c*) $4{\cdot}85 \times 10^{-4}$ rad (*d*) $4{\cdot}80 \times 10^{-6}$ rad

(NEET 2020)

15. Dimensional formula for thermal conductivity is here (*K* denotes the temperature)

(*a*) $[MLT^{-3}]K$ (*b*) $[MLT^{-2}K]$
(*c*) $[MLT^{-2}K^{-2}]$ (*d*) $[MLT^{-3}K^{-1}]$

(JEE Main 2020)

16. The density of a solid metal sphere of diameter *d* is ρ. The maximum error in the density of the sphere $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring mass and diameter are 6·0% and 1·5% respectively, the value of *x* is

(*a*) 1050 (*b*) 105
(*c*) 850 (*d*) 950

(JEE Main 2020)

17. If speed *V*, area *A* and force *F* are chosen as fundamental units, the dimensions of Young's modulus *Y* will be

(*a*) $F^1A^{-1}V^0$ (*b*) FA^2V^{-1}
(*c*) FA^2V^{-3} (*d*) FA^2V^{-2}

(JEE Main 2020)

18. A quantity *f* is given by $f = \sqrt{\frac{hc^5}{G}}$, where *c* is velocity of light, *G* is gravitational constant and *h* is Planck's constant. Dimension of *f* are that of

(*a*) area (*b*) momentum
(*c*) volume (*d*) energy

(JEE Main 2020)

19. If the screw on a screw gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale, the least count of the screw gauge is

(*a*) 0·001 mm (*b*) 0·001 cm
(*c*) 0·02 mm (*d*) 0·01 cm

(JEE Main 2020)

20. The density of a material in SI units is 128 kg/m^3. In certain units, where the unit of length is 25 cm and the unit of mass is 50 gram, the numerical value of the density of the material is

(*a*) 640 (*b*) 410
(*c*) 40 (*d*) 16

(JEE Main 2020)

21. Using screw gauge of pitch 0·1 cm and 50 divisions on circular scale, thickness of an object is measured. It should be correctly recorded as

(*a*) 2·123 cm (*b*) 2·125 cm
(*c*) 2·121 cm (*d*) 2·124 cm

(JEE Main 2020)

ANSWERS

9. (*a*) **10.** (*c*) **11.** (*a*) **12.** (*b*) **13.** (*a*) **14.** (*b*) **15.** (*d*) **16.** (*a*) **17.** (*a*) **18.** (*d*) **19.** (*b*)
20. (*c*) **21.** (*d*)

II. Dimensional Analysis

22. If energy (E), velocity (v) and time (T) are chosen as the fundamental quantities, the dimensional formula of surface tension will be :

(a) $[E v^{-1} T^{-2}]$ (b) $[E v^{-2} T^{-2}]$

(c) $[E^{-2} v^{-1} T^{-3}]$ (d) $[E v^{-2} T^{-1}]$

(AIPMT 2015)

23. E, m, L and G denote energy, mass, angular momentum and gravitational constant respectively ; $EL^2/(m^5 G^2)$ has the dimensions of

(a) Length (b) Mass

(c) Time (d) Angle

24. A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\epsilon_0}$ is [c is velocity of light, G is universal constant of gravitation and e is charge] :

(a) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\epsilon_0}\right]^{1/2}$ (b) $c^2\left[G\frac{e^2}{4\pi\epsilon_0}\right]^{1/2}$

(c) $\frac{1}{c^2}\left[\frac{e^2}{G\,4\pi\epsilon_0}\right]^{1/2}$ (d) $\frac{1}{c}G\frac{e^2}{4\pi\epsilon_0}$

(NEET 2017)

25. If force (F), velocity (V) and time (T) are taken as fundamental units, then the dimensions of mass are :

(a) $[FVT^{-1}]$ (b) $[FVT^{-2}]$

(c) $[FV^{-1} T^{-1}]$ (d) $[FV^{-1} T]$

(AIPMT 2014)

26. Assuming that the mass m of the largest stone that can be moved by a flowing river depends upon the velocity v of water, its density ρ and acceleration due to gravity g, then m is directly proportional to

(a) v^4 (b) v^6

(c) v^5 (d) v

27. The position x of a particle at time t is given by

$$x = \frac{V_0}{a}(1 - e^{-at})$$

where V_0 is a constant and $a > 0$.

The dimensions of V_0 and a are

(a) $M^0 L T^{-1}$ and T^{-1}

(b) $M^0 L T^0$ and T^{-1}

(c) $M^0 L T^{-1}$ and LT^{-2}

(d) $M^0 L T^{-1}$ and T

28. In terms of basic units of mass (M), length (L), time (T) and charge (Q), the dimensions of magnetic permeability of vacuum (μ_0) would be

(a) (MLQ^{-2}) (b) $(LT^{-1} Q^{-1})$

(c) $(ML^2 T^{-1} Q^{-2})$ (d) (LTQ^{-1})

(AIPMT 2015)

29. If speed of light (c), acceleration due to gravity (g) and pressure (p) are taken as fundamental units, the dimensions of gravitational constant (G) are

(a) $c^0 g p^{-3}$ (b) $c^2 g^3 p^{-2}$

(c) $c^0 g^2 p^{-1}$ (d) $c^2 g^2 p^{-2}$

30. If the energy, $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light, then the values of p, q and r are, respectively

(a) –1/2, 1/2 and 5/2 (b) 1/2, –1/2 and –5/2

(c) –1/2, 1/2 and 3/2 (d) 1/2, –1/2 and –3/2

31. Let $[\epsilon_0]$ denote the permittivity of vaccum. If M = mass, L = length, T = time and A = electric current, then :

(a) $[\epsilon_0] = M^{-1} L^{-3} T^2 A$

(b) $[\epsilon_0] = M^{-1} L^{-3} T^4 A^2$

(c) $[\epsilon_0] = MLT^{-2} A^{-2}$

(d) $[\epsilon_0] = ML^2 T^{-1} A$ **(JEE Main 2013)**

32. If $[\epsilon_0]$ and $[\mu_0]$ represent dimensions of permittivity and permeability of free space respectively, what is the relation between $[\epsilon_0]$ and $[\mu_0]$?

(a) $[\mu_0] = [\epsilon_0] L^2 T^{-2}$ (b) $[\mu_0] = [\epsilon_0] L^{-2} T^2$

(c) $[\mu_0] = [\epsilon_0]^{-1} L^2 T^{-2}$ (d) $[\mu_0] = [\epsilon_0]^{-1} L^{-2} T^2$

(JEE Advanced 2018)

III. Errors & Significant Figures

33. Time period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Measured value of l is 20·0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s.

ANSWERS

22. (b) **23.** (d) **24.** (a) **25.** (d) **26.** (b) **27.** (a) **28.** (a) **29.** (c) **30.** (a) **31.** (b) **32.** (d)

using a wrist watch of 1 s resolution. The accuracy in the determination of g is

(*a*) 2% (*b*) 3%
(*c*) 1% (*d*) 5%

(JEE Main 2015)

34. The diameter of a circle is 2·486 m. Calculate the area with due regard to significant figures. Take $\pi = 3{\cdot}142$

(*a*) 4·85 m^3 (*b*) 4·85454 m^3
(*c*) 4·854 m^3 (*d*) 4·8545 m^3

35. The length, breadth and thickness of a rectangular object are 4·576 m, 1·243 m and 1·22 cm respectively. Find the area of its face and its volume to correct significant figures.

(*a*) 5·688 m^3, $6{\cdot}94 \times 10^{-3}$ m^3
(*b*) 5·062 m^2, $6{\cdot}94 \times 10^{-2}$ m^3
(*c*) 5·688 m^2, $6{\cdot}94 \times 10^{-2}$ m^3
(*d*) 5·6 m^2, $6{\cdot}9 \times 10^{-1}$ m^3

36. The length of a cylinder is measured with a meter rod having least count 0·1 cm. Its diameter is measured with a vernier callipers with vernier constant 0·01 cm. Given that the length of cylinder is 5·0 cm and its radius is 2·00 cm. The percentage error in the calculated volume is :

(*a*) 4% (*b*) 3%
(*c*) 2% (*d*) 1%

37. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long. In a particular reading, the student measures the extension in the length of the wire to be 0·8 mm with an uncertainty of ± 0·05 mm at a load of exactly 1·0 kg. The student also measures the diameter of the wire to be 0·4 mm with an uncertainty of ± 0·01 mm. Take $g = 9{\cdot}8$ m/s^2 (exact). The Young's modulus obtained from the readings is :

(*a*) $(2{\cdot}0 \pm 0{\cdot}3) \times 10^{11}$ N/m^2
(*b*) $(2{\cdot}0 \pm 0{\cdot}2) \times 10^{11}$ N/m^2
(*c*) $(2{\cdot}0 \pm 0{\cdot}1) \times 10^{11}$ N/m^2
(*d*) $(2{\cdot}0 \pm 0{\cdot}05) \times 10^{11}$ N/m^2

38. In an experiment to measure the height of a bridge by dropping stone into water underneath, if the error in measurement of time is 0·1 s at the end of 2 s, then the error in estimation of height of bridge will be

(*a*) 0·49 m (*b*) 0·98 m
(*c*) 1·96 m (*d*) 2·12 m

39. The following observations were taken for determining surface tension T of water by capillary method :

Diameter of capillary, $D = 1{\cdot}25 \times 10^{-2}$ m
rise of water, $h = 1{\cdot}45 \times 10^{-2}$ m

Using $g = 9{\cdot}80$ m/s^2 and the simplified relation $T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to

(*a*) 2·4% (*b*) 10%
(*c*) 0·15% (*d*) 1·5%

(JEE Main 2017)

40. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of – 0.004 cm, the correct diameter of the ball is

(*a*) 0.053 cm (*b*) 0.525 cm
(*c*) 0.521 cm (*d*) 0.529 cm

(NEET 2018)

41. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0·03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(*a*) 3·32 mm (*b*) 3·73 mm
(*c*) 3·67 mm (*d*) 3·38 mm

(AIEEE 2008)

42. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) (are) true ?

(*a*) Error ΔT in measuring T, the time period, is 0·05 seconds
(*b*) Error ΔT in measuring T, the time period, is 1 second
(*c*) Percentage error in the determination of g is 5%
(*d*) Percentage error in the determination of g is 2.5%

(IIT 2010)

ANSWERS

33. (*b*) **34.** (*c*) **35.** (*c*) **36.** (*b*) **37.** (*b*) **38.** (*c*) **39.** (*d*) **40.** (*d*) **41.** (*d*) **42.** (*c*)

43. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(*a*) 0.9% (*b*) 2.4%

(*c*) 3.11% (*d*) 4.2% **(IIT 2011)**

44. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58·5 degree

Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0·5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data

(*a*) 58·59° (*b*) 58·77°

(*c*) 58·65° (*d*) 59° **(AIEEE 2012)**

45. A student measured the length of a rod and wrote it as 3·50 cm. Which instrument did he use to measure it ?

(*a*) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm

(*b*) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

(*c*) A meter scale.

(*d*) A vernier calliper where 10 divisions in vernier scale match with 9 divisions in main scale and main scale has 10 divisions in 1 cm. **(JEE Main 2014)**

46. Time period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Measured value of l is 20·0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s. using a wrist watch of 1 s resolution. The accuracy in the determination of g is

(*a*) 2% (*b*) 3%

(*c*) 1% (*d*) 5%

(JEE Main 2015)

47. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0{\cdot}2\ \text{s}^{-1}$. The measurement of A has an error of 1·25%. If the error in the measurement of time is 1·50%, the percentage error is the value of $E(t)$ at $t = 5$ sec is

(*a*) 1% (*b*) 2%

(*c*) 3% (*d*) 4%

(JEE Advanced 2015)

48. A student measures the time period of 100 oscillations of a simple pendulum 4 times. The data set is 90 s, 91 s, 92 s and 95 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be

(*a*) (92 ± 2) s (*b*) (92 ± 5) s

(*c*) (92 ± 1·8) s (*d*) (92 ± 3) s

(JEE Main 2016)

49. The following observations were taken for determining surface tension T of water by capillary method :

Diameter of capillary, $D = 1{\cdot}25 \times 10^{-2}$ m

rise of water, $h = 1{\cdot}45 \times 10^{-2}$ m

Using $g = 9{\cdot}80\ \text{m/s}^2$ and the simplified relation $T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to

(*a*) 2·4% (*b*) 10%

(*c*) 0·15% (*d*) 1·5%

(JEE Main 2017)

50. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(*a*) 2.5% (*b*) 3.5%

(*c*) 4.5% (*d*) 6%

(JEE Main 2018)

51. A screw gauge with a pitch of 0·5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of aluminium. Before starting the measurement, it is found that when the two jaws of the screw are brought in contact, 45th division coincides with the main scale line and that the zero of main scale is barely visible. What is the thickness of the sheet, if the main scale reading is 0·5 mm and the 25th division coincides with the main scale line ?

(*a*) 0·75 mm (*b*) 0·80 mm

(*c*) 0·70 mm (*d*) 0·50 mm

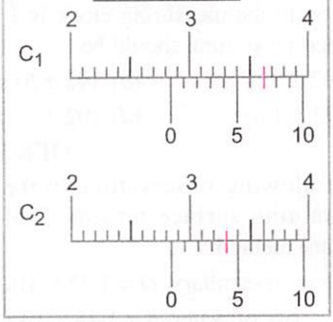
(JEE Main 2016)

ANSWERS

43. (*c*) **44.** (*c*) **45.** (*d*) **46.** (*b*) **47.** (*d*) **48.** (*a*) **49.** (*d*) **50.** (*c*) **51.** (*b*)

52. There are two vernier callipers, both of which have 1 cm divided into 10 equal divisions on the main scale. The vernier scale of calliper C_1 has 10 equal divisions that correspond to 9 main scale divisions. The vernier scale of the other calliper C_2 has 10 equal divisions, that correspond to 11 main scale divisions. The readings of the two callipers are shown in Fig. 2(CF).1. The measured values (in cm) by C_1 and C_2 respectively are

FIGURE 2(CF).1



(a) 2·87 and 2·87 (b) 2·87 and 2·83

(c) 2·85 and 2·82 (d) 2·87 and 2·86

(JEE Advanced 2016)

53. In an experiment, the percentages of error occurred in the measurement of physical quantities A, B, C, D are 1%, 2%, 3% and 4% respectively. Then the maximum percentage error in $X = \dfrac{A^2 B^{1/2}}{C^{1/3} D^3}$ will be

(a) (3/13)% (b) 16%

(c) – 10% (d) 100% **(NEET 2019)**

II. Multiple Choice Questions (with One or More than One Correct Answers)

54. Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option (s) is/(are)

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$

(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

(JEE Advanced 2015)

55. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0, and speed of light c, the dimensionally correct equation(s) is (are) :

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$

(c) $I = \varepsilon_0 cV$ (d) $\mu_0 cI = \varepsilon_0 V$

(JEE Advanced 2015)

56. Let $\in_0$ denote the permittivity of vacuum and μ_0 denote permeability of vacuum. If M = mass, L = length, T = time, I = electric current, then

(a) $[\varepsilon_0] = [M^{-1}L^{-3}T^2 I]$ (b) $[\varepsilon_0] = [M^{-1}L^{-3}T^4 I^2]$

(c) $[\mu_0] = [MLT^{-2}I^{-2}]$ (d) $[\mu_0] = [ML^2T^{-2}I]$

57. Consider a vernier callipers in which each 1 cm of main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the vernier callipers, 5 divisions of the vernier scale coincide with 4 divisions of the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then :

(a) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·01 mm.

(b) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·005 mm.

(c) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·01 mm.

(d) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·005 mm. **(JEE Advanced 2015)**

58. A length scale (l) depends on the permittivity ($\in$) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of a certain charged particle, and charge (q) carried by each of the particles. Which of the following expression (s) for l is (are) dimensionally correct ?

(a) $l = \sqrt{\dfrac{nq^2}{\in k_B T}}$ (b) $l = \sqrt{\dfrac{\in k_B T}{nq^2}}$

(c) $l = \sqrt{\dfrac{q^2}{\in n^{2/3} k_B T}}$

(d) $l = \sqrt{q^2/\in n^{1/3} k_B T}$ **(JEE Advanced 2016)**

ANSWERS

52. (b)	53. (b)	54. (a,c,d)	55. (a,c)	56. (b,c)	57. (b,c)	58. (b,d)

III Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1

Whether a given relation/formula is correct or not can be checked on the basis of the *principle of homogeneity of dimensions.* According to this principle, only that formula is correct, in which the dimensions of the various terms on one side of the relation are equal to the respective dimensions of these terms on the other side of the relation.

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions :

59. The distance travelled by a body in nth second is given by $S_{\text{nth}} = u + \frac{a}{2}(2n-1)$ where u is initial velocity and a is acceleration. The dimensions of S_{nth} are

(*a*) L (*b*) LT^{-1} (*c*) LT^{-2} (*d*) $L^{-1}T$

60. Power P is related to distance x and time t as

$$P = \frac{b - x^2}{at}$$

The dimensional formula of b is

(*a*) $[M^0 LT^{-2}]$ (*b*) $[M^0 L^2 T^2]$

(*c*) $[M^0 L^2 T^{-2}]$ (*d*) $[M^0 L^2 T^0]$

Comprehension 2

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of measurement and vice-versa.

In addition or subtraction, the number of decimal places in the result should equal the smallest number of decimal places in any term in the operation.

In multiplication and division, the number of sigificant figures in the product or in the quotient is the same as the smallest number of significant figures in any of the factors.

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions :

61. The area enclosed by a circle of diameter 1·06 m with correct number of significant figures is

(*a*) 0·88 m^2 (*b*) 0·883 m^2

(*c*) 1·88 m^2 (*d*) 0·882026 m^2.

62. Subtract $2{\cdot}6 \times 10^4$ from $3{\cdot}9 \times 10^5$ with due regard to significant figures

(*a*) $3{\cdot}64 \times 10^5$ (*b*) $3{\cdot}7 \times 10^5$

(*c*) $3{\cdot}6 \times 10^5$ (*d*) $3{\cdot}65 \times 10^6$

Comprehension 3

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$$

The series expansion of $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $(\Delta y/y)$ is $\left(1 \mp \frac{\Delta y}{y}\right)$.

The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$$

The above derivation makes the assumption that $\frac{\Delta x}{x} << 1$ and $\frac{\Delta y}{y} << 1$. Therefore, the higher powers of these quantities are neglected.

63. In an experiment, the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1·0 *s*. For $|x| \le 1$, $\ln(1 + x) = x$, upto first power in x. The error $\Delta\lambda$ in the determination of decay constant λ in s^{-1} is

(*a*) 0·04 (*b*) 0·03

(*c*) 0·02 (*d*) 0·01

(JEE Advanced 2018)

ANSWERS

59. (*b*) **60.** (*d*) **61.** (*b*) **62.** (*c*) **63.** (*c*)

IV. Matching Type Questions

64. Column I gives four physical quantities and column II gives their units. Select the correct match out of the four given choices

Column I	Column II
A. Capacitance	(*p*) Ohm-metre
B. Inductance	(*q*) Columb volt^{-1}
C. Magnetic Induction	(*r*) newton – (amp. m)$^{-1}$
D. Specific resistance	(*s*) volt-sec (amp)$^{-1}$

(*a*) A-*p*, B-*q*, C-*r*, D-*s* (*b*) A-*q*, B-*p*, C-*r*, D-*s*
(*c*) A-*s*, B-*r*, C-*p*, D-*q* (*d*) A-*q*, B-*s*, C-*r*, D-*p*

65. Match the physical quantities given in column I with dimensions expressed in terms of mass *M*, length *L*, time *T* and charge *Q* given in column II.

Column I	Column II
A. Angular momentum	(*p*) $[ML^2T^{-2}]$
B. Heat	(*q*) $[ML^2T^{-2}Q^{-1}]$
C. Potential	(*r*) $[ML^2T^{-1}Q^{-1}]$
D. Magnetic flux	(*s*) $[ML^2T^{-1}]$

(*a*) A-*p*, B-*q*, C-*r*, D-*s* (*b*) A-*s*, B-*q*, C-*p*, D-*r*
(*c*) A-*s*, B-*p*, C-*q*, D-*r* (*d*) A-*r*, B-*s*, C-*p*, D-*q*

V. Matrix-Match Type Questions

Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*r, s* ; B-*p, q* ; C-*p* ; D-*q, s*, then the correctly bubbled matrix will look like the one shown here :

	p	q	r	s
A	○	○	●	●
B	●	●	○	○
C	●	○	○	○
D	○	●	○	●

66.	Column I		Column II
A.	same dimension in mass	*p*.	electric field
B.	same dimension in time	*q*.	magnetic field
C.	(–1) dimension in current	*r*.	inductive reactance
D.	zero dimension in length	*s*.	resistivity

67.	Column I		Column II
A.	dimension in length is – 1	*p*.	Rydberg constant
B.	dimension in time is – 2	*q*.	Power of a lens
C.	dimension in mass is zero	*r*.	Coefficient of elasticity
D.	dimension in mass is 1	*s*.	Binding energy/nucleon

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

A	B	C	D
0	●	0	0
1	1	1	1
2	2	2	●
3	3	3	3
●	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	●	9

68. If units of measurement of two systems are in the ratio 2 : 1, then the ratio of units of angular momentum in the two systems will be :

69. To find the distance *d* over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assures that the distance depends on the mass density ρ of the fog, intensity (power/area) *S* of the light from the signal and its frequency *f*. The engineer finds that *d* is proportional to $S^{1/n}$. The value of *n* is :

[JEE (Advanced) 2014]

ANSWERS

64. (*d*) **65.** (*c*) **66.** A-*p*, *q*, *r*, *s* ; B-*p*, *r*, *s* ; C-*p*, *q* ; D-*q*
67. A-*p*, *q* ; B-*r*, *s* ; C-*p*, *q* ; D-*r*, *s* **68.** (4) **69.** (3)

70. The dimensions of T in the dimensional formula for mobility are :

71. If C is capacitance, V is potential, ρ is specific resistance and $\in_0$ is electric permittivity of free space, then the dimension of ampere (A) in $\dfrac{CV}{\rho \in_0}$ are :

72. If σ is Stefan's constant and b is Wien's constant, then the dimensions of length in $\sigma\, b^4$ are :

73. To find the distance d over which a signal can be seen clearly in foggy conditions, a railway engineer uses dimensional analysis and assures that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is : **(JEE Advanced 2014)**

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. The following questions consist of two statements each, printed as **Assertion** and **Reason.**

While answering these questions, you are required to choose any one of the following four responses.

A. If both, **Assertion** and **Reason** are true and the **Reason** is the correct explanation of the **Assertion.**

B. If both, **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

C. If **Assertion** is true but the **Reason** is false.

D. If both, **Assertion** and **Reason** are false.

74. Assertion. If the units of force and length are doubled, the unit of energy will become 4 times.

Reason. The unit of energy is independent of the units of force and length.

(*a*) A (*b*) B (*c*) C (*d*) D

75. Assertion. The error in the measurement of radius of a sphere is 0·3%. The permissible error in calculation of its surface area is 0·6%.

Reason. The permissible error is calculated by the formula $\dfrac{\Delta A}{A} = \dfrac{4\,\Delta r}{r}$

(*a*) A (*b*) B (*c*) C (*d*) D

76. Assertion. Distance travelled by the particle in the nth second has dimensions of length.

Reason. It is the distance travelled by the particle in the given time.

(*a*) A (*b*) B (*c*) C (*d*) D

77. Assertion. Light year and year, both measure time.

Reason. Because light year is the time light takes to reach the earth from the sun.

(*a*) A (*b*) B (*c*) C (*d*) D

78. Assertion. Force cannot be added to pressure.

Reason. Because their dimensions are different.

(*a*) A (*b*) B (*c*) C (*d*) D

79. Assertion. Rate of flow of a liquid represents velocity of flow.

Reason. The dimensions of rate of flow are $[M^0 L^1 T^{-1}]$.

(*a*) A (*b*) B (*c*) C (*d*) D

80. Assertion. Planck's constant (h) represents angular momentum.

Reason. Because, both have the same dimensions ; $[M^1 L^2 T^{-1}]$.

(*a*) A (*b*) B (*c*) C (*d*) D

81. Assertion. Units of Rydberg constant R are m^{-1}.

Reason. It follows from Bohr's formula $\bar{\nu} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$, where the symbols have their usual meaning.

(*a*) A (*b*) B (*c*) C (*d*) D

82. Assertion. Parallax method cannot be used for measuring distances of stars more than 100 light years away.

Reason. Because parallax angle reduces so much that it cannot be measured accurately.

(*a*) A (*b*) B (*c*) C (*d*) D

ANSWERS

70. (2) **71.** (1) **72.** (4) **73.** (3) **74.** (*c*) **75.** (*c*) **76.** (*d*) **77.** (*d*) **78.** (*a*) **79.** (*d*) **80.** (*a*)
81. (*a*) **82.** (*a*)

FOR ENGINEERING STUDENTS

DIRECTIONS. In the following questions, read the two statements and check if

(A) Statement-1 is true, Statement-2 is true ; and Statement-2 is correct explanation of Statement-1.

(B) Statement-1 is true ; Statement-2 is true, but Statement-2 is not a correct explanation of Statement-1.

(C) Statement-1 is true, but Statement-2 is false.

(D) Statement-1 is false, but Statement-2 is true.

83. Statement-1. Out of three measurements $l = 0{\cdot}7$ m ; $l = 0{\cdot}70$ m and $l = 0{\cdot}700$ m, the last one is most accurate.

Statement-2. In every measurement, only the last significant digit is not accurately known.

(*a*) A (*b*) B (*c*) C (*d*) D

84. Statement-1. nm is not same as m N

Statement-2. 1 nm = 10^{-9} m and 1m N = 10^{-3} N

(*a*) A (*b*) B (*c*) C (*d*) D

85. Statement-1. Distance travelled in *n*th second has the dimensions of velocity.

Statement-2. Because it is the distance travelled in one (particular) second.

(*a*) A (*b*) B (*c*) C (*d*) D

86. Statement-1. Velocity gradient has the dimensions of frequency.

Statement-2. Velocity gradient is rate of change of velocity with distance.

(*a*) A (*b*) B (*c*) C (*d*) D

87. Statement-1. If error in measurement of distance and time are 3% and 2% respectively, error in calculation of velocity is 5%.

Statement-2. Velocity $= \dfrac{\text{distance}}{\text{time}}$

(*a*) A (*b*) B (*c*) C (*d*) D

88. Statement-1. The dimensional formula of electric potential is $[ML^2T^{-3}A^{-1}]$.

Statement-2. Electric potential is equal to work done.

(*a*) A (*b*) B (*c*) C (*d*) D

VIII. Multiple Choice Questions (Based on Experimental Skills)

(Measurements using vernier callipers ; screw gauge and spherometer)

89. Consider a vernier callipers in which each 1 cm of main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the vernier callipers, 5 divisions of the vernier scale coincide with 4 divisions of the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then :

(*a*) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·01 mm.

(*b*) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·005 mm.

(*c*) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·01 mm.

(*d*) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of screw gauge is 0·005 mm.

90. The vernier constant of a vernier callipers is 0·1 mm and it has a zero error of 0·04 cm. While measuring diameter of a rod, the main scale reading is 1·2 cm and 5th vernier division is coinciding with any scale division. The corrected diameter of the rod is

(*a*) 1·21 cm (*b*) 1·21 mm

(*c*) 1·29 mm (*d*) 1·29 cm

91. What is the use of a thin strip at the back of vernier callipers ?

(*a*) for measuring internal diameter of a beaker

(*b*) for measuring depth of a cylinder

(*c*) for measuring diameter of a hollow cylinder

(*d*) none of these

92. When the two jaws of a vernier calliper are in touch, zero of vernier scale lies to the right of zero of main scale and coinciding vernier division is 3. If vernier constant is 0·1 mm, the zero correction is

(*a*) – 0·03 cm (*b*) + 0·03 cm

(*c*) – 0·03 mm (*d*) + 0·03 mm

ANSWERS

83. (*a*) **84.** (*a*) **85.** (*a*) **86.** (*a*) **87.** (*b*) **88.** (*c*) **89.** (*b,c*) **90.** (*a*) **91.** (*b*) **92.** (*a*)

93. The circular scale of a screw gauge has 200 divisions. When it is given 4 complete rotations, it moves through 2 mm. The least count of screw gauge is

(*a*) $0{\cdot}25 \times 10^{-2}$ cm (*b*) $0{\cdot}25 \times 10^{-3}$ cm

(*c*) 0·001 cm (*d*) 0·001 mm

94. While measuring diameter of a wire using a screw gauge, the main scale reading is 7 mm and zero of circular scale is 35 division above the reference line. If screw gauge has a zero error of – 0·003 cm, the correct diameter of wire is (given least count = 0·001 cm)

(*a*) 0·735 cm (*b*) 0·732 cm

(*c*) 0·738 cm (*d*) 7·38 cm

95. When a screw gauge is completely closed, zero of circular scale is 4 divisions below the reference line of graduation. If least count of screw gauge is 0·001 cm, the zero correction is

(*a*) – 0·004 cm (*b*) + 0·004 cm

(*c*) – 0·004 mm (*d*) + 0·004 mm

96. Two spherometers *A* and *B* have the same pitch. *A* has 100 divisions on periphery of its circular disc and *B* has 200 divisions on periphery of its circular disc. Then

(*a*) Both *A* and *B* have same least count

(*b*) L.C. of *A* is twice the L.C. of *B*

(*c*) L.C. of *A* is half the L.C. of *B*

(*d*) Nothing can be said

97. If *h* is the height or depth (sagitta) of a spherical surface and *l* is the mean distance between the legs of spherometer, then radius of curvature *R* of the surface is

(*a*) $R = \frac{l^2}{h} + \frac{h}{2}$ (*b*) $R = \frac{l^2}{6h} + \frac{h}{2}$

(*c*) $R = \frac{l^2}{6h} - \frac{h}{2}$ (*d*) $R = \frac{h^2}{6l} + \frac{l}{2}$

98. A student measured the length of a rod and wrote it as 3·50 cm. Which instrument did he use to measure it ?

(*a*) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(*b*) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

(*c*) A meter scale

(*d*) A vernier calliper where the 10 divisions in vernier scale match with 9 divisions on the main scale and main scale has 10 divisions in 1 cm. **[JEE (Advanced) 2014]**

HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. $[MLT^{-2}\,A^{-2}]$ is the dimensional formula of magnetic permeability.

2. Unit of plane angle is radian

Dimensions of plane angle

$$= \frac{\text{arc}}{\text{radius}} = \frac{L}{L} = [M^0\,L^0\,T^0]$$

Unit of solid angle is steradian

Dimensions of solid angle

$$= \frac{\text{area}}{(\text{radius})^2} = \frac{L^2}{L^2} = [M^0\,L^0\,T^0]$$

Both have units, but no dimension.

3. Here, $L = 55{\cdot}3$ m, $B = 25$ m

$$A = L \times B = 55{\cdot}3 \times 25 \text{ m}^2 = 1382{\cdot}5$$

Rounding off to two significant figures.

$$\mathbf{A = 14 \times 10^2\ m^2}$$

4. E = energy = $M\,L^2\,T^{-2}$

$$G = \text{gravitional constant} = \frac{F\,r^2}{m_1\,m_2}$$

$$= \frac{(MLT^{-2})\,L^2}{M^2} = [M^{-1}\,L^3\,T^{-2}]$$

$$\frac{E}{G} = \frac{M\,L^2\,T^{-2}}{M^{-1}\,L^3\,T^{-2}} = [M^2\,L^{-1}\,T^0]$$

5. Let energy, $E = F^a\,A^b\,T^c$

$$[ML^2\,T^{-2}] = [MLT^{-2}]^a\,[LT^{-2}]^b\,T^c$$
$$= M^a\,L^{a+b}\,T^{-2a-2b+c}$$

ANSWERS

93. (*b*) 94. (*c*) 95. (*a*) 96. (*b*) 97. (*b*) 98. (*d*)

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HINTS/EXPLANATIONS

For Difficult Questions

Applying the principle of homogeneity of dimensions

$a = 1, (a + b) = 2,$

$-2a - 2b + c = -2$

or $-2 \times 2 + c = -2, c = 2 ; b = 2 - a = 2 - 1 = 1.$

Hence $E = F^1 A^1 T^2$

7. Energy, $E = ML^2 T^{-2}$

Gravitational constant,

$$G = \frac{F r^2}{m_1 m_2} = \frac{MLT^{-2} \cdot L^2}{M^2} = M^{-1} L^3 T^{-2}$$

$$\frac{E}{G} = \frac{ML^2 T^{-2}}{M^{-1} L^3 T^{-2}} = [M^2 L^{-1} T^0]$$

8. Energy $(E) \propto F^a A^b T^c$

$$E = kF^a A^b T^c$$

$$ML^2T^{-2} = (MLT^{-2})^a (LT^{-2})^b T^c$$

$$= M^a L^{a+b} T^{-2a-2b+c}$$

Applying the principle of homogeneity of dimensions, we get

$1 = a$

$2 = a + b ; b = 2 - a = 2 - 1 = 1$

$-2 = -2a - 2b + c$

or $c = -2 + 2a + 2b = -2 + 2 + 2 = 6$

$\therefore E = k F^1 A^1 T^2$ or $E \propto F^1 A^1 T^2$

9. Here, 1 CSD $= \frac{1}{100}$ mm.

Diameter of wire = Main scale reading + $\frac{1}{100}$ × CS reading

$$= 0 + \frac{1}{100} \times 52 = 0.52 \text{ mm} = \mathbf{0.052 \text{ cm}}$$

10. Stress $= \frac{\text{force}}{\text{area}} = \frac{M LT^{-2}}{L^2} = [M L^{-1} T^{-2}]$

11. $9.99 - 0.0099 = 9.9801$

$= 9.98$ m ; rounding off.

12. Least count = 0.01 mm

Number of divisions on circular scale = 50

$\therefore$ Pitch = 0.01 × 50 mm = **0.50 mm**

13. Mean value of observations

$$t = \frac{1.28 + 1.24 + 1.27 + 1.21 + 1.28}{5} = \frac{6.28}{5} = 1.26 \text{ s.}$$

$\Delta t_1 = 1.28 - 1.26 = 0.02$

$\Delta t_2 = 1.24 - 1.26 = -0.02$

$\Delta t_3 = 1.27 - 1.26 = 0.01$

$\Delta t_4 = 1.21 - 1.26 = -0.05$

$\Delta t_5 = 1.28 - 1.26 = 0.02$

$$\text{Mean } \Delta t = \frac{\Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_4 + \Delta t_5}{5}$$

$$= \frac{0.02 + 0.02 + 0.01 + 0.05 + 0.02}{5}$$

$$= 0.024$$

$$\text{Percent relative error } = \frac{0.02}{1.26} \times 100 = 1.587$$

$$= \mathbf{1.6\%}$$

14. $\theta = 1 \text{ minute} = \frac{1}{60}^{\circ} = \frac{\pi}{180} \times \frac{1}{60}$ radian

$\theta = 0.000291 \text{ rad} = \mathbf{2.91 \times 10^{-4} \text{ rad.}}$

15. From the relation $\frac{\Delta Q}{\Delta t} = K'A \left(\frac{\Delta T}{\Delta x}\right)$, where K' is thermal conductivity and K is temperature.

$$K' = \frac{\Delta Q}{\Delta t} \times \frac{\Delta x}{A \Delta T}$$

$$= \frac{ML^2T^{-2}}{T} \times \frac{L}{L^2 K} = [MLT^{-3}K^{-1}]$$

16. Density $(\rho) = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{\frac{4}{3}\pi r^3}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\left|\frac{\Delta r}{r}\right|$$

$$\frac{x}{100} = 6\% + 3 \times 1.5\% = 10.5\%$$

$$x = 10.5 \times 100 = \mathbf{1050}$$

17. Let $Y = KV^a A^b F^c$ ...(*i*)

Here K is dimensionless constant.

$$ML^{-1} T^{-2} = (LT^{-1})^a (L^2)^b (MLT^{-2})^c$$

$$= M^c L^{a+2b+c} T^{-a-2c} \quad \text{...(i)}$$

Applying the principle of homogeneity of dimensions, we get

$c = 1, a + 2b + c = -1, -a - 2c = -2 ;$

$a = -2c + 2 = -2 + 2 = 0$

From $a + 2b + c = -1$

$a + 2b + 1 = -1$, or $2b = -2, b = -1$

From (*i*), $Y = KV^0 A^{-1} F^1$. Here $\mathbf{Y = F^1 A^{-1} V^0}$

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18. Here, $f = \sqrt{\dfrac{hc^5}{G}} = \sqrt{\dfrac{(ML^2T^{-1})(LT^{-1})^5}{M^{-1}L^3T^{-2}}}$

$= \sqrt{M^2L^4T^{-4}} = [ML^2T^{-2}] =$ **energy**

19. Pitch $= \dfrac{3}{6}\text{ mm} = 0{\cdot}5\text{ mm} = 0{\cdot}05\text{ cm}$

Least count $= \dfrac{\text{pitch}}{50} = \dfrac{0{\cdot}05}{50}\text{ cm} = \mathbf{0{\cdot}001\,cm}$

20. Density $= \dfrac{\text{mass}}{\text{volume}} = 128\ \dfrac{\text{kg}}{\text{m}^3}$

$= 128 \times \left(\dfrac{1000}{50}\right) \times \dfrac{1}{(100/25)^3}$

= 40 new units

21. Here, pitch = 0·1 cm

Number of divisions on circular scale = 50

Least count $= \dfrac{0{\cdot}1}{50}\text{ cm} = 0{\cdot}002$

Thickness of an object is correct upto third place of decimal and a correct multiple of 2.

Hence the correct answer is 2·124 cm *i.e.* **choice (*d*).**

22. Let $S \propto E^a v^b T^c\ ;\ S = kE^a v^b T^c$ ...(*i*)

$\therefore\ M^1L^0T^{-2} = k\,[M^1L^2T^{-2}]^a\,[L^1T^{-1}]^b\,[T]^c$

or $M^1L^0T^{-2} = k\,M^aL^{2a}T^{-2a}L^bT^{-b}T^c$

or $M^1L^0T^{-2} = k\,M^aL^{-2a+b}T^{-2a-b+c}$

Equating powers of *M*, *L* and *T*, we get

$\therefore\quad \mathbf{a = 1}$

$2a + b = 0,\ b = -2a = -2$ or $\mathbf{b = -2}$

or $-2a - b + c = -2\ ;\ -2(1) - (-2) + c = -2$

$\mathbf{c = -2}$

Putting values of *a*, *b* and *c* in (*i*)

$\mathbf{S = k\,E^1 v^{-2} T^{-2}}$

23. Here, E = Energy $(M^1L^2T^{-2})$, m = mass (M^1)

L = angular momentum $= mvr = (ML^2T^{-1})$

G = gravitational constant $= (M^{-1}L^3T^{-2})$

$$\therefore\quad \frac{EL^2}{m^5G^2} = \frac{M^1L^2T^{-2}\times(ML^2T^{-1})^2}{M^5(M^{-1}L^3T^{-2})^2}$$

$$= \frac{M^3L^6T^{-4}}{M^3L^6T^{-4}} = M^0L^0T^0$$

which are the dimensions of Angle

$$= \frac{\textbf{arc}}{\textbf{radius}} = \mathbf{M^0L^0T^0}$$

24. Let $L \propto c^x G^y \left(\dfrac{e^2}{4\pi\epsilon_0}\right)^z$

$$L = K\,c^x G^y \left(\frac{e^2}{4\pi\epsilon_0}\right)^z \quad ...(i)$$

where K = dimensionless constant.

The dimensions of $c = LT^{-1}$,

$$G = \frac{Fr^2}{m_1m_2} = [M^{-1}L^3T^{-2}]$$

and $\dfrac{e^2}{4\pi\epsilon_0} = Fr^2 = [ML^3T^{-2}]$

From (*i*),

$M^0L^1T^0 = (LT^{-1})^x\,[M^{-1}L^3T^{-2}]^y\,[ML^3T^{-2}]^z$

$= M^{-y+z}\,L^{x+3y+3z}\,T^{-x-2y-2z}$

Applying the principle of homogeneity of dimensions,

$-y + z = 0$ ...(*ii*)

$x + 3y + 3z = 1$ ...(*iii*)

$-x - 2y - 2z = 0$ ...(*iv*)

Add (*iii*) and (*iv*) : $y + z = 1$

Add (*ii*) $\quad 2z = 1,\ \ z = \dfrac{1}{2}$

From (*ii*), $\ y = z = \dfrac{1}{2}$

From (*iv*), $-x = 2y + 2z = 2\times\dfrac{1}{2} + 2\times\dfrac{1}{2} = 2$,

$x = -2$

Put in (*i*), $\ L = K\,c^{-2}G^{1/2}\left(\dfrac{e^2}{4\pi\epsilon_0}\right)^{1/2}$

$$= \frac{\mathbf{1}}{\mathbf{c^2}}\left(\frac{\mathbf{Ge^2}}{\mathbf{4\pi\epsilon_0}}\right)^{\mathbf{1/2}} \quad \text{(omitting } K\text{)}$$

25. Dimensional formula of force $F = [MLT^{-2}]$

or $F = [M\,(LT^{-1})\,T^{-1}] = [MVT^{-1}]$

or $\mathbf{M = [FV^{-1}T]}$

26. Let $m = k\,v^a\,\rho^b\,g^c$; where k is dimensionless constant.

$[M^1L^0T^0] = (LT^{-1})^a\,(ML^{-3})^b\,(LT^{-2})^c$

$= M^b\,L^{a-3b+c}\,T^{-a-2c}$

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Applying the principle of homogeneity of dimensions, we get

$$b = 1\ ;\ -a - 2\,c = 0 \quad\text{or}\quad c = \frac{-a}{2}$$

$$a - 3\,b + c = 0$$

$$a - 3 - a/2 = 0 \qquad\text{or}\quad a = 6$$

Hence, $\boldsymbol{m \propto v^6}$

27. As $a \times t$ is dimensionless.

$$\therefore \quad a = \frac{1}{t} = \frac{1}{T} = \mathbf{[T^{-1}]}$$

Also, $x = \dfrac{V_0}{a}$, $V_0 = x\,a = \mathbf{[LT^{-1}]}$

28. Force acting per unit length of each of two parallel wires carrying currents in the same direction placed certain distance apart in vacuum is

$$\frac{F}{l} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$

$$\frac{MLT^{-2}}{L} = \mu_0\frac{(Q/T)^2}{L} \quad\text{or}\quad \mu_0 = \mathbf{[MLQ^{-2}]}$$

29. Let $G = c^x\,g^y\,p^z$

$$[M^{-1}\,L^3\,T^{-2}] = (LT^{-1})^x\,(LT^{-2})^y\,(ML^{-1}\,T^{-2})^z$$
$$= M^z\,L^{x+y-z}\,T^{-x-2y-2z}$$

Applying principle of homogeneity of dimensions, we get

$$z = -1,\ x + y - z = 3\ ;$$
$$-x - 2\,y - 2\,z = -2$$

On solving, we get, $y = 2$, $z = 0$

$$\therefore \quad \boldsymbol{G = c^0\,g^2\,p^{-1}}$$

30. $E = G^p\,h^q\,c^r$ ...(*i*)

$$[M^1\,L^2\,T^{-2}] = (M^{-1}\,L^3\,T^{-2})^p\,(ML^2\,T^{-1})^q\,(LT^{-1})^r$$
$$= M^{-p+q}\,L^{3p+2q+r}\,T^{-2p-q-r}$$

Applying principle of homogeneity of dimensions, we get

$-p + q = 1$ ...(*ii*)

$3\,p + 2\,q + r = 2$ ...(*iii*)

$-2\,p - q - r = -2$ ...(*iv*)

Add (*iii*) and (*iv*), $p + q = 0$ ...(*v*)

Add (*ii*) and (*v*), $2q = 1,\ q = \dfrac{1}{2}$

From (*ii*), $p = q - 1 = \dfrac{1}{2} - 1 = -\dfrac{1}{2}$

Put in (*iii*), $-\dfrac{3}{2} + 1 + r = 2,\ r = 5/2$

Choice (*a*) is correct.

31. From Coulomb's law, $F = \dfrac{q_1\,q_2}{4\,\pi\epsilon_0\,r^2}$

$$\epsilon_0 = \frac{q_1\,q_2}{4\,\pi\,F\,r^2} = \frac{(AT)\ (AT)}{(MLT^{-2})\,L^2} = [M^{-1}\,L^{-3}\,T^4\,A^2]$$

Choice (*b*) is correct.

32. As is know, $c = \dfrac{1}{\sqrt{\mu_0\epsilon_0}}$

$$\text{or}\ c^2 = \frac{1}{\mu_0\epsilon_0} \quad\text{or}\quad \mu_0 = \frac{1}{c^2\,\epsilon_0} = \frac{1}{(LT^{-1})^2\,\epsilon_0}$$

$$[\mu_0] = [\epsilon_0]^{-1}\,[L]^{-2}\,[T]^2$$

33. Here, $l = 20$ cm, $\Delta l = 1$ mm $= 0{\cdot}1$ cm, $t = 90$ sec, $\Delta t = 1$ sec

$T = 2\pi\sqrt{\dfrac{l}{g}}$, Squaring both sides, we get

$$T^2 = 4\,\pi^2 \times \frac{l}{g} \quad\text{or}\quad g = 4\,\pi^2\,\frac{l}{T^2}$$

$$\therefore \quad \frac{\Delta g}{g}\times 100 = \frac{\Delta l}{l}\times 100 + \frac{2\,\Delta T}{T}\times 100$$

As $\dfrac{\Delta T}{T} = \dfrac{\Delta t}{t}$, therefore,

$$\frac{\Delta g}{g}\times 100 = \frac{0{\cdot}1}{20}\times 100 + 2\times\frac{1}{90}\times 100$$

$$\frac{\Delta g}{g}\times 100 = \frac{100}{200} + \frac{200}{90} = \frac{1}{2} + \frac{20}{9} \approx \mathbf{3\%}$$

34. Here, $D = 2{\cdot}486$ m, $\pi = 3{\cdot}142$

$$\text{As Area} = \frac{\pi\,D^2}{4} = \frac{3{\cdot}142\times(2{\cdot}486)^2}{4}$$

$$= \frac{3{\cdot}142\times 6{\cdot}180196}{4} = \frac{19{\cdot}418175832}{4}$$

$$= 4{\cdot}85454\ \text{m}^3$$

Since, the diameter has been expressed in four significant figures, final result of area should also be expressed in four significant figures

$\boldsymbol{A = 4{\cdot}854\ \text{m}^3}$

35. Here, $l = 4{\cdot}576$ m, $b = 1{\cdot}243$ m,

$t = 1{\cdot}22$ cm $= 0{\cdot}0122$ m

As, Area $= l \times b = 4{\cdot}576 \times 1{\cdot}243 = 5{\cdot}687968\ \text{m}^3$

As l and b have 4 significant figures, area shold also be expressed in 4 significant figures.

$\therefore$ **A = 5·688 m²** [rounding off to 4 significant figures]

As volume $= l \times b \times t$

$V = 4\cdot576 \times 1\cdot243 \times 0\cdot0122 = 0\cdot0693932096\ \text{m}^3$

As thickness has 3 significant figures, So rounding off volume to 3 significant figures

$$\boldsymbol{V = 6\cdot94 \times 10^{-2}\ \text{m}^3}$$

36. Here $l = 5\cdot0$ cm, $r = 2\cdot00$ cm

Also $\Delta l \rightarrow$ least count of meter rod $= 0\cdot1$ cm

$\Delta r \rightarrow$ Vernier constant $= 0\cdot01$ cm

V (Volume) $= \pi r^2 l$

$$\therefore \quad \frac{\Delta V}{V} = 2\left(\frac{\Delta r}{r}\right) + \frac{\Delta l}{l} = 2 \times \left(\frac{0\cdot01}{2\cdot00}\right) + \frac{0\cdot1}{5\cdot0}$$

$$= 0\cdot03$$

$$\%\text{error} = \frac{\Delta V}{V} \times 100 = 0\cdot03 \times 100 = \mathbf{3\%}$$

37. Here, $F = 1$ kg wt $= 9\cdot8$ N, $L = 2$ m,

$\Delta d = 0\cdot01$ mm, $\Delta l = 0\cdot05$ mm

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F\,l}{A \times \Delta l}$$

$$Y = \frac{F\,L}{\pi r^2 l} = \frac{4\,F\,L}{\pi d^2 l}$$

$$Y = \frac{4 \times 9\cdot8 \times 2}{\frac{22}{7} \times (0\cdot4 \times 10^{-3})^2 \times 0\cdot8 \times 10^{-3}}$$

$$= \frac{548\cdot8}{2\cdot816 \times 10^{-9}}$$

$$\approx \mathbf{2 \times 10^{11}\ N/m^2}$$

$$\therefore \quad \frac{\Delta Y}{Y} = \frac{2\,\Delta d}{d} + \frac{\Delta l}{l}$$

[l is the extension in wire. As F, *i.e.*, load, and L are exactly known, $\therefore$ ΔF and ΔL are zero]

$$\frac{\Delta Y}{Y} = \frac{2 \times 0\cdot01}{0\cdot4} + \frac{0\cdot05}{0\cdot8} = 0\cdot1125 = 0\cdot1$$

$\Delta Y = 0\cdot1 \times Y = 0\cdot1 \times 2 \times 10^{11} = 0\cdot2 \times 10^{11}\ \text{N/m}^2$

Young modulus obtained from readings

$= Y \pm \Delta Y$

$= \mathbf{(2\cdot0 \pm 0\cdot2) \times 10^{11}\ N/m^2}$

38. From $s = ut + \frac{1}{2}at^2$

$$h = 0 + \frac{1}{2} \times 9\cdot8\,(2)^2 = 19\cdot6\,\text{m}$$

$$\frac{\Delta h}{h} = \pm 2\left(\frac{\Delta t}{t}\right) \qquad (\because\ a = g = \text{const})$$

$$= \pm 2\left(\frac{0\cdot1}{2}\right) = \pm\frac{1}{10}$$

$$\Delta h = \pm\frac{h}{10} = \pm\frac{19\cdot6}{10} = \mathbf{\pm\, 1\cdot96\ m}$$

39. Given surface tension, $T = \frac{r\,h\,g}{2} \times 10^3$

$$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$$

% error in surface tension

$$\frac{\Delta T}{T} \times 100 = \left(\frac{\Delta r}{r} + \frac{\Delta h}{h}\right) \times 100 = \left(\frac{\Delta D}{D} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0\cdot01}{1\cdot25} + \frac{0\cdot01}{1\cdot45}\right) \times 100 = (0\cdot8 + 0\cdot689) \times 100$$

$$= 1\cdot489\% \approx \mathbf{1\cdot5\%}$$

40. Diameter of the ball $= MSR + CSR \times$ least count $-$ zero error

$= 0\cdot5 + 25 \times 0\cdot001 - (-\,0\cdot004)$

$= 0\cdot5 + 0\cdot025 + 0\cdot004$

$=$ **0·529 cm**

41. Pitch of screw gauge $= \frac{1}{2}$ mm $= 0\cdot5$ mm

Least count of screw gauge

$$= \frac{0\cdot5}{50}\,\text{mm} = 0\cdot01\,\text{mm}$$

Zero error $= -\,0\cdot03$ mm

Zero correction $= +\,0\cdot03$ mm

Observed diameter of wire

$= 3$ mm $+ 35 \times (0\cdot01)$ mm $= 3\cdot35$ mm

Corrected diameter of wire $= (3\cdot35 + 0\cdot03)$ mm

= 3·38 mm

42. Let t be the time for n oscillations and T be the time period of oscillations of the simple pendulum. Then $\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{40} = 0\cdot05$

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 l\, n^2}{t^2}$$

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$$\therefore \quad \frac{\Delta g}{g} = \frac{2\Delta t}{t} = 2 \times \frac{1}{40} = \frac{1}{20}$$

% error in determination of g

$$= \frac{\Delta g}{g} \times 100 = \frac{1}{20} \times 100 = \mathbf{5\%}$$

43. Least count of screw gauge

$$= \frac{\text{pitch}}{\text{no. of divisions on circular scale}}$$

$$= \frac{0.5\,\text{mm}}{50} = 0{\cdot}01 \text{ mm.}$$

Diameter of ball, $D = \text{MSR} + \text{CSR} \times \text{LC}$

$= 2{\cdot}5 \text{ mm} + (20) \times 0{\cdot}01 \text{ mm} = 2.7 \text{ mm}$

As density, $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{M}{\dfrac{4\pi}{3}\left(\dfrac{D}{2}\right)^3}$

Relative error in the density, $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta M}{M} + \dfrac{3\Delta D}{D}$

Relative percentage error in the density is

$$= \frac{\Delta\rho}{\rho} \times 100 = \left(\frac{\Delta M}{M} + \frac{3\Delta D}{D}\right) \times 100$$

$$= \frac{\Delta M}{M} \times 100 + \frac{3\Delta D}{D} \times 100$$

$$= 2\% + 3 \times \frac{0{\cdot}01}{2{\cdot}7} \times 100$$

$$= 2\% + 1{\cdot}11\% = \mathbf{3{\cdot}11\%}$$

44. Here, main scale reading, $x = 58{\cdot}5^\circ$

Vernier scale reading $n = 09$ divisions

V.C. = 1 MSD – 1 VSD

$$= 1\,\text{MSD} - \frac{29}{30}\,\text{MSD} = \frac{1}{30}\,\text{MSD}$$

$$= \frac{1}{30} \times 0{\cdot}5^\circ = \frac{1^\circ}{60}$$

$\therefore$ Angle of prism = $x + n$ (V.C.)

$$= 58{\cdot}5^\circ + \frac{9^\circ}{60} = \mathbf{58{\cdot}65^\circ}$$

45. As the measured value is 3·50 cm, the least count must be = 0·01 cm = 0·1 mm

For given screw gauge in option (*a*),

$$\text{least count} = \frac{1\,\text{mm}}{100} = 0{\cdot}01 \text{ mm}$$

$$\text{Least count in option } (b) = \frac{1\,\text{mm}}{50} = 0{\cdot}02 \text{ mm}$$

Least count in option (*c*) = 1 mm

Least count in option (*d*) = 1 MSD – 1 VSD

$$= 1\,\text{MSD} - \frac{9}{10}\,\text{MSD}$$

$$= \frac{1}{10}\,\text{MSD} = \frac{1}{10} \times 1\,\text{mm} = \mathbf{0{\cdot}1\ mm}$$

Thus vernier calliper is used for the measurement. Hence **option (*d*) is true.**

46. Here, $l = 20$ cm, $\Delta l = 1$ mm $= 0{\cdot}1$ cm,

$t = 90$ sec, $\Delta t = 1$ sec

$T = 2\pi\sqrt{\dfrac{l}{g}}$, Squaring both sides, we get

$$T^2 = 4\pi^2 \times \frac{l}{g} \text{ or } g = 4\pi^2 \frac{l}{T^2}$$

$$\therefore \quad \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + \frac{2\,\Delta T}{T} \times 100$$

As $\dfrac{\Delta T}{T} = \dfrac{\Delta t}{t}$, therefore,

$$\frac{\Delta g}{g} \times 100 = \frac{0{\cdot}1}{20} \times 100 + 2 \times \frac{1}{90} \times 100$$

$$\frac{\Delta g}{g} \times 100 = \frac{100}{200} + \frac{200}{90} = \frac{1}{2} + \frac{20}{9} \approx \mathbf{3\%}$$

47. Given $E(t) = A^2 \exp(-\alpha t)$, $\alpha = 0{\cdot}2 \text{ s}^{-1}$,

$$\frac{dA}{A} = 1{\cdot}25\%, \quad \frac{dt}{t} = 1{\cdot}50\%, \quad t = 5 \text{ sec.}, \quad \frac{dE}{E} = ?$$

Taking log of both sides, we get

$\log E = \log (A^2 e^{-\alpha t})$

$\log E = \log A^2 + \log e^{-\alpha t}$

or $\log E = 2 \log A - \alpha t$

Differentiating both sides, of given relation, we get

$$\frac{dE}{E} = \pm \frac{2\,dA}{A} \pm \alpha\,dt \qquad \text{...(i)}$$

$$\because \quad \frac{dt}{t} = 1{\cdot}50\%, \ t = 5\,\text{sec}$$

$$\therefore \quad \frac{dt}{5} = 1{\cdot}50\%, \ dt = 7{\cdot}50\%$$

From (*i*), $\dfrac{dE}{E} = \pm\, 2\,(1{\cdot}25) \pm 0{\cdot}2 \times 7{\cdot}5$

$$= \pm\, 2{\cdot}5 \pm 1{\cdot}5 = \mathbf{\pm\, 4\%}$$

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48. Mean value of time period

$$= \frac{90+91+92+95}{4} = 92\,\text{s}$$

Mean absolute error

$$= \frac{|92-90|+|92-91|+|92-92|+|92-95|}{4}$$

$$= \frac{2+1+0+3}{4} = 1{\cdot}5\,\text{s}$$

$\therefore$ Value of time period = $(92 \pm 1{\cdot}5)$ s

As least count = 1 s

$\therefore$ Value of time period = $\mathbf{(92 \pm 2)}$ **s**

49. Given surface tension, $T = \dfrac{rhg}{2} \times 10^3$

$$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$$

% error in surface tension

$$\frac{\Delta T}{T}\times 100 = \left(\frac{\Delta r}{r}+\frac{\Delta h}{h}\right)\times 100 = \left(\frac{\Delta D}{D}+\frac{\Delta h}{h}\right)\times 100$$

$$= \left(\frac{0{\cdot}01}{1{\cdot}25}+\frac{0{\cdot}01}{1{\cdot}45}\right)\times 100 = (0{\cdot}8 + 0{\cdot}689)\times 100$$

$$= 1{\cdot}489\% \approx \mathbf{1{\cdot}5\%}$$

50. Here, $\dfrac{\Delta m}{m} = 1{\cdot}5\%$; $\dfrac{\Delta l}{l} = 1\%$;

$$\frac{\Delta\rho}{\rho} = ?$$

As $\rho = \dfrac{m}{V} = \dfrac{m}{l^3}$

$\therefore$ Error in density $= \dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + 3\left(\dfrac{\Delta l}{l}\right)$

$= 1{\cdot}5\% + 3 \times 1\% = \mathbf{4{\cdot}5\%}$

51. Here, pitch = 0·5 mm

Least count $= \dfrac{0{\cdot}5\,\text{mm}}{50} = 0{\cdot}01$ mm

Zero error = – (50 – 45) × 0·01 = – 0·05 mm

Zero correction = + 0·05 mm

Thickness of sheet = 0·5 mm + 25 × 0·01 mm + 0·05 mm

= **0·80 mm**

52. For vernier C_1,

10 VSD = 9 MSD = 9 mm

$\therefore$ 1 VSD = 0·9 mm

LC = 1 MSD – 1 VSD

= (1 – 0·9) mm = 0·1 mm

Reading of C_1 = MSR + (VSR) × LC

= 28 mm + 7 (0·1) = 28·7 mm

= **2·87 cm**

For vernier C_2 (which is abnormal),

We can show that Reading of C_2 = 28·3 mm

= **2·83 cm**

53. Here, $\dfrac{\Delta A}{A} = 1\%$, $\dfrac{\Delta B}{B} = 2\%$

$\dfrac{\Delta C}{C} = 3\%$; $\dfrac{\Delta D}{D} = 4\%$

As $X = \dfrac{A^2 B^{1/2}}{C^{1/3} D^3}$

$\therefore$ $\dfrac{\Delta X}{X}\times 100$

$$= 2\left(\frac{\Delta A}{A}\right)+\frac{1}{2}\left(\frac{\Delta B}{B}\right)+\frac{1}{3}\left(\frac{\Delta C}{C}\right)+3\left(\frac{\Delta D}{D}\right)$$

$$= 2\times 1\% + \frac{1}{2}\times 2\% + \frac{1}{3}\times 3\% + 3\times 4\%$$

= 2% + 1% + 1% + 12%

= **16%**

II. Multiple Choice Questions (with One or More than One Correct Answers)

54. Here, Dimensional formula for (*i*) Length L = ?

(*ii*) planck's constant $h = ML^2T^{-1}$

(*iii*) gravitational constant $G = M^{-1}L^3T^{-2}$

(*iv*) speed of light $C = LT^{-1}$

for length L

let $L \propto h^x c^y G^z$ or $L = kh^x c^y G^z$

$$M^0L^1T^0 = k\,[ML^2T^{-1}]^x\,(LT^{-1})^y\,[M^{-1}L^3T^{-2}]^z$$

$$M^0L^1T^0 = kM^{x-z}\,L^{2x+y+3z}\,T^{-x-y-2z}$$

Equating the powers of M, L, T, we get,

$x - z = 0$, $x = z$

$2x + y + 3z = 1$

$-x - y - 2z = 0$

HINTS/EXPLANATIONS....

For Difficult Questions

On solving, we get $x = \frac{1}{2}$, $y = -\frac{3}{2}$, $z = \frac{1}{2}$

$$\therefore \quad L = k\,h^{1/2}\,c^{-3/2}\,G^{1/2}$$

$$L = k\sqrt{\frac{hG}{c^3}}\,,\ L \propto \sqrt{h},\ L \propto \sqrt{G}$$

option (*c*) and (*d*) are correct.

Similarly, for mass M

Let $M \propto h^x c^y G^z$ or $M = kh^x c^y G^z$

or $M^1L^0T^0 = k\,[ML^2\,T^{-1}]^x\,(LT^{-1})^y\,[M^{-1}L^3T^{-2}]^z$

or $M^1L^0T^0 = kM^{x-z}\,L^{2x+y+3z}\,T^{-x-y-2z}$

Equating the powers of M, L, T, we get

$$x - z = 1 \qquad \text{...}(i)$$

$$2x + y + 3z = 0 \qquad \text{...}(ii)$$

$$-x - y - 2z = 0 \qquad \text{...}(iii)$$

On solving, we get $x = \frac{1}{2}$, $y = \frac{1}{2}$, $z = -\frac{1}{2}$

$$M = k\,h^{1/2}\,c^{1/2}\,G^{-1/2} = k\sqrt{\frac{hc}{G}}$$

$$M \propto \sqrt{h},\ M \propto \sqrt{c},\ M \propto \frac{1}{\sqrt{G}}$$

Hence option (*a*) is correct.

55. Here, Dimensional formula for

(*i*) $V = ML^2T^{-3}A^{-1}$

(*ii*) $I = [A^1]$

(*iii*) $c = [LT^{-1}]$

(*iv*) $\varepsilon_0 = [M^{-1}L^{-3}T^4A^2]$

(*v*) $\mu_0 = [M^1L^1T^{-2}A^{-2}]$

(*a*) $\mu_0 I^2 = \varepsilon_0 V^2$

$$MLT^{-2}\,A^{-2} \times A^2 = M^{-1}L^{-3}T^4A^2 \times (ML^2T^{-3}A^{-1})^2$$

$$MLT^{-2} = MLT^{-2}$$

Hence option (*a*) is correct.

(*b*) $\varepsilon_0 I = \mu_0 V$

$$M^{-1}L^{-3}T^4\,A^2 \times A = M^1L^1T^{-2}A^{-2} \times ML^2T^{-3}A^{-1}$$

$$= M^2L^3T^{-5}A^{-3}$$

$\therefore$ LHS ≠ RHS, dimensionally.

Hence option (*b*) is incorrect.

(*c*) $I = \varepsilon_0\,cV$

$$A = M^{-1}L^{-3}T^4A^2 \times LT^{-1} \times ML^2T^{-3}A^{-1} = A$$

L.H.S. = R.H.S., dimensionally.

Hence option (*c*) is correct.

(*d*) $\mu_0\,cI = \varepsilon_0\,V$

$$M^1L^1T^{-2}\,A^{-2} \times LT^{-1} \times A$$

$$= M^{-1}L^{-3}T^4A^2 \times ML^2T^{-3}A^{-1}$$

$$M^1L^2T^{-3}A^{-1} \neq M^0L^{-1}T^1A^1$$

L.H.S. ≠ R.H.S.

Hence option (*d*) is incorrect.

56. As $F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$

or $\varepsilon_0 = \frac{1}{4\pi}\frac{q^2}{r^2\,F}$

$$\varepsilon_0 = \frac{I^2\,T^2}{L^2\,MLT^{-2}} = \mathbf{M^{-1}L^{-3}T^4I^2}$$

As $B = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

$$\frac{F}{qv} = \frac{\mu_0}{4\pi} \times \frac{I\,dl\sin\theta}{r^2} \qquad [\because F = B\,q\,v\sin\theta]$$

$$\mu_0 = \frac{F\,r^2}{q\,v\,I\,dl} = \frac{MLT^{-2}.L^2}{IT.LT^{-1}.I.L} = \mathbf{MLT^{-2}I^{-2}}$$

Choices (*b*) and (*c*) are correct.

57. Smallest division on main scale of vernier callipers

$$= \frac{1}{8}\text{ cm} = 0{\cdot}125\text{ cm}$$

5 division of the vernier scale

= 4 div. of main scale

= 4 × 0·125 = 0·5 cm

1 division of vernier scale $= \frac{0{\cdot}5}{5} = 0{\cdot}1$ cm

L.C of vernier callipers = 1 MSD – 1 VSD

= 0·125 – 0·1 = 0·025 cm

L.C. of screw gauge $= \frac{\text{pitch}}{100}$

If pitch of screw gauge is twice the L.C of vernier callipers, then L.C of screw gauge

$$= \frac{0{\cdot}025 \times 2}{100} = \frac{0{\cdot}05}{100}\text{ cm} = \frac{0{\cdot}5}{100}\text{ mm}$$

$$= \mathbf{0{\cdot}005\ mm}$$

Hence option (*b*) is correct.

HINTS/EXPLANATIONS

For Difficult Questions

Also, If L.C of linear scale of screw gauge is twice the L.C. of vernier callipers,

then L.C of linear scale of screw gauge

$$= 2 \times 0{\cdot}025 = 0{\cdot}05 \text{ cm}$$

$$\text{pitch} = 2 \times 0{\cdot}05 = 0{\cdot}1 \text{ cm} = 1 \text{ mm}$$

$$\text{L.C of screw gauge} = \frac{1\,\text{mm}}{100} = \mathbf{0{\cdot}01\ mm}$$

Hence option (*c*) is correct.

58. Here, $l = [L]$, $q = [AT]$, $T = [K]$

$n = [L^{-3}]$, $\in = [M^{-1} L^{-3} A^2 T^4]$,

$k_B = [ML^2 T^{-2} K^{-1}]$

(*b*) $\sqrt{\dfrac{\in k_B T}{nq^2}}$

$$= \sqrt{\frac{(M^{-1} L^{-3} A^2 T^4)(ML^2 T^{-2} K^{-1}).K}{L^{-3}(AT)^2}}$$

$$= [M^0 LT^0 A^0] = [L]$$

(*d*) $\sqrt{\dfrac{q^2}{\in n^{1/3} k_B T}}$

$$= \sqrt{\frac{A^2 T^2}{(M^{-1} L^{-3} A^2 T^4) L^{-1} (ML^2 T^{-2} K^{-1}) K}}$$

$$= [L]$$

Hence **choices (*b*) and (*d*) are correct.**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

59. S_{nth} = distance travelled by a body in one particular second $= \dfrac{L}{T} = LT^{-1}$

60. As x^2 is subtracted from b, therefore, dimensional formula of $b = x^2 = L^2 = [M^0 L^2 T^0]$

61. Here, $d = 2r = 1{\cdot}06$ m, $r = \dfrac{1{\cdot}06}{2} = 0{\cdot}530$ m

$$\text{Area enclosed} = \pi r^2 = \frac{22}{7}(0{\cdot}530)^2$$

$$= 0{\cdot}88282 \text{ m}^2 = \mathbf{0{\cdot}883\ m^2}$$

(with 3 significant figures)

62. $3{\cdot}9 \times 10^5 - 2{\cdot}6 \times 10^4 = 10^5(3{\cdot}9 - 0{\cdot}26)$

$= 10^5 \times 3{\cdot}64 = \mathbf{3{\cdot}6 \times 10^5}$

(correct upto one place of decimal)

63. As is know, $N = N_0 e^{-\lambda t}$

$\therefore \quad \ln N = \ln N_0 - \lambda t$

$$\frac{dN}{N} = -d\lambda t$$

In terms of errors, $\dfrac{\Delta N}{N} = -(\Delta\lambda)\, t$

or $\Delta\lambda = \dfrac{\Delta N}{N(t)} = \dfrac{40}{(3000 - 1000) \times 1}$

$$\Delta\lambda = \frac{40}{2000} = \mathbf{0{\cdot}02}$$

IV. Matching Type Questions

64. Capacitance = Q/V = columb (volt)$^{-1}$

Inductance, $L = \dfrac{e}{dI/dt}$ = volt-sec (amp)$^{-1}$

Magnetic induction, $B = \dfrac{F}{I\,l} = \dfrac{\text{newton}}{\text{amp.metre}}$

Specific resistance,

$$\rho = \frac{R \times a}{l} = \frac{\text{ohm m}^2}{\text{m}}$$

ohm-m

Choice (*d*) is correct.

65. Angular momentum $= I\omega = (ML^2)(T^{-1})$

Heat (energy) $= [ML^2 T^{-2}]$

$$\text{Potential} = \frac{\text{work}}{\text{charge}} = \frac{ML^2 T^{-2}}{Q} = [ML^2 T^{-2} Q^{-1}]$$

$$\text{Magnetic flux } \phi = B \times A = \frac{FA}{I\,l} = \frac{MLT^{-2}.L^2}{AL}$$

$$= ML^2 T^{-2} A^{-1} = [ML^2 T^{-1} Q^{-1}]$$

Choice (*c*) is correct.

HINTS/EXPLANATIONS

For Difficult Questions

V. Matrix-Match Type Questions

66. Electric field = $[M^1 L^1 T^{-3} A^{-1}]$

Magnetic field = $[M^1 L^0 T^{-2} A^{-1}]$

Inductive reactance = $[M^1 L^2 T^{-3} A^{-2}]$

Resistivity = $[M^1 L^3 T^{-3} A^{-2}]$

For all the four, dimension in mass = 1.

Electric field, inductive reactance and resistivity have the same dimension in time.

Electric field and magnetic field have (– 1) dimension in current. And magnetic field has zero dimension in length.

67. Rydberg constant = $[M^0 L^{-1} T^0]$

Power of lens = $[M^0 L^{-1} T^0]$

Coefficient of elasticity = $[M^1 L^{-1} T^{-2}]$

Binding energy/nucleon = $[M^1 L^2 T^{-2}]$

The dimension in length is – 1, in case of Rydberg constant and power of lens.

Dimension of time is (– 2) in coefficient of elasticity and binding energy/nucleon. The dimension in mass is zero in case of Rydberg constant and power of lens. And, the dimension of mass is 1 in case of coefficient of elasticity and B.E./nucleon.

VI. Integer Type Questions

68. Here, $\dfrac{M_1}{M_2} = \dfrac{L_1}{L_2} = \dfrac{T_1}{T_2} = \dfrac{2}{1}$

Angular momentum = $[ML^2 T^{-1}]$

$$\therefore \quad \frac{M_1 L_1^2 T_1^{-1}}{M_2 L_2^2 T_2^{-1}} = \left(\frac{M_1}{M_2}\right) \times \left(\frac{L_1}{L_2}\right)^2 \left(\frac{T_2}{T_1}\right)$$

$$= \frac{2}{1}\left(\frac{2}{1}\right)^2 \times \frac{1}{2} = 4$$

69. Let a, b and c be the dimensions of ρ, S and f to represent d. So,

$$d = k\, \rho^a S^b f^c \qquad \text{...(i)}$$

where k is a dimensionless constant

Writing the dimensions in (*i*),

$$M^0 L^1 T^0 = (ML^{-3})^a (MT^{-3})^b (T^{-1})^c$$
$$= M^{a+b} L^{-3a} T^{-3b-c}$$

Equating the powers of M, L and T, we have

$a + b = 0$ or $a = -b$

$-3a = 1$ or $a = -\dfrac{1}{3}$ or $b = +\dfrac{1}{3}$

From (*i*), $d \propto S^b$ so $d \propto S^{1/3}$

Given $d \propto S^{1/n}$, hence $n = 3$

70. Mobility $= \dfrac{\text{drift velocity}}{\text{electric field}} = \dfrac{v}{F/q} = \dfrac{qv}{F}$

$$= \frac{(AT)(LT^{-1})}{(MLT^{-2})} = [M^{-1} L^0 T^2 A^1]$$

The dimension of $T = 2$

71. Here, $C = \dfrac{q}{V} = \dfrac{q}{W/q} = \dfrac{q^2}{W} = \dfrac{(AT)^2}{ML^2 T^{-2}}$

$$= [M^{-1} L^{-2} T^4 A^2]$$

$$V = [ML^2 T^{-3} A^{-1}]$$

$$\rho = \frac{Ra}{l} = \frac{V}{I}\frac{a}{l} = \frac{(ML^2 T^{-3} A^{-1})(L^2)}{AL}$$
$$= [ML^3 T^{-3} A^{-2}]$$

From $\quad F = \dfrac{q_1 q_2}{4\pi\epsilon_0 r^2}$

$$\epsilon_0 = \frac{q_1 q_2}{4\pi F r^2} = \frac{(AT)^2}{(MLT^{-2})(L^2)} = [M^{-1} L^{-3} T^4 A^2]$$

Now, $\dfrac{CV}{\rho\epsilon_0} = \dfrac{q}{\rho\epsilon_0}$

$$= \frac{AT}{(ML^3 T^{-3} A^{-2})(M^{-1} L^{-3} T^4 A^2)}$$
$$= [A]$$

∴ dimension of $A = 1$

72. From $E = \sigma T^4$, $\sigma = \dfrac{E}{T^4}$

where E is energy/sec/area and T is temperature.

$$\therefore \quad \sigma = \frac{ML^2 T^{-2}}{T.L^2} \times \frac{1}{K^4} = [M^1 T^{-3} K^{-4}]$$

HINTS/EXPLANATIONS

For Difficult Questions

From $b = \lambda_m T = LK = [LK]$

Now $\sigma\, b^4 = (M^1\, T^{-3}\, K^{-4})\,(LK)^4$
$= [M^1\, L^4\, T^{-3}]$

$\therefore$ Dimension of length = **4**

73. Let a, b and c be the dimensions of ρ, S and f to represent d. So,

$$d = k\, \rho^a\, S^b\, f^c \qquad \text{...}(i)$$

where k is a dimensionless constant

Writing the dimensions in (i),

$$M^0 L^1 T^0 = (ML^{-3})^a\,(MT^{-3})^b\,(T^{-1})^c$$
$$= M^{a+b}\, L^{-3a}\, T^{-3b-c}$$

Equating the powers of M, L and T, we have

$a + b = 0$ or $a = -b$

$-3a = 1$ or $a = -\dfrac{1}{3}$ or $b = +\dfrac{1}{3}$

From (i), $d \propto S^b$ so $d \propto S^{1/3}$

Given $d \propto S^{1/n}$, hence $n = \mathbf{3}$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

74. Here, assertion is true, but reason is false.

As Energy = Work = $F \times S$

$\therefore$ When the unit of force and length are doubled the unit of energy will become 4 times. Also, unit of energy depends upon force and length.

75. Here, assertion is true, but reason is false.

as $A = 4\,\pi\, r^2$, $\therefore$

$$\frac{\Delta A}{A} = \frac{2\,\Delta r}{r} = 2 \times 0{\cdot}3\% = \mathbf{0{\cdot}6\%}$$

Also $\dfrac{\Delta A}{A} \neq \dfrac{4\,\Delta r}{r}$, So reason is false.

76. Here, both the assertion and reason are false.

As $S_{nth} = u + \dfrac{a}{2}\,(2n-1)$

$$\therefore \quad S_{nth} = LT^{-1} + LT^{-2}.T = [LT^{-1}]$$

Also, it is the distance travelled by particle in particular second, not in the given time.

77. Light year is unit of distance and not of time. Therefore, both the assertion and reason are false.

78. $F = [M^1\, L^1\, T^{-2}]$;
$P = [M^1\, L^{-1}\, T^{-2}]$

Quantities of different dimensions cannot be added.

79. The dimensions of rate of flow are $[M^0\, L^3\, T^{-1}]$, as it represents volume flowing/sec.

80. Dimensions of $h = \dfrac{E}{\nu} = \dfrac{ML^2\, T^{-2}}{T^{-1}} = M^1\, L^2\, T^{-1}$

which are same as the dimensions of angular momentum.

81. $\bar{\nu} = \dfrac{1}{\lambda} = m^{-1}$ and n_1, n_2 are pure numbers.

82. Parallax angle $\theta = \dfrac{l}{r}$. As distance of star (r) increases, θ reduces. Measurement becomes inaccurate.

FOR ENGINEERING STUDENTS

83. Fractional error in the three measurements respectively are

$$\pm\frac{0{\cdot}1}{0{\cdot}7};\ \pm\frac{0{\cdot}01}{0{\cdot}70};\ \pm\frac{0{\cdot}001}{0{\cdot}700}$$

The last one has the least error, *i.e.*, it is most accurate. Choice (*a*) is correct.

84. nm represents a nanometre (= 10^{-9} m) and mN represents a milli Newton (= 10^{-3} N). Choice (*a*) is correct.

85. Both the statements are true, and statement-2 is correct explanation of the statement-1.

86. Velocity gradient = change of velocity with distance $= \dfrac{LT^{-1}}{L} = [T^{-1}]$ and T^{-1} represents frequency. Therefore, both the statements are true and statement-2 is correct explanation of the statement-1.

87. Both the statements are true. But statement-2 is not a correct explanation of statement-1. Infact

$$\upsilon = \frac{L}{T}$$

$$\therefore \quad \frac{\Delta\upsilon}{\upsilon} = \pm\left(\frac{\Delta L}{L} + \frac{\Delta T}{T}\right) = \pm\,(3\,\% + 2\,\%)$$
$$= \pm\,\mathbf{5\,\%}$$

88. The statement-1 is true. But statement-2 is false. Electric potential at a point is work done in carrying unit charge from infinity to that point.

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HINTS/EXPLANATIONS

For Difficult Questions

VIII. Multiple Choice Questions (Based on Experimental Skills)

89. Smallest division on main scale of vernier callipers

$$= \frac{1}{8}\text{ cm} = 0{\cdot}125\text{ cm}$$

5 division of the vernier scale

$$= 4\text{ div. of main scale}$$
$$= 4 \times 0{\cdot}125$$
$$= 0{\cdot}5\text{ cm}$$

1 division of vernier scale $= \frac{0{\cdot}5}{5} = 0{\cdot}1$ cm

L.C of vernier callipers = 1 MSD – 1 VSD

$$= 0{\cdot}125 - 0{\cdot}1 = 0{\cdot}025\text{ cm}$$

L.C. of screw gauge $= \frac{\text{pitch}}{100}$

If pitch of screw gauge is twice the L.C of vernier callipers, then L.C of screw gauge

$$= \frac{0{\cdot}025 \times 2}{100} = \frac{0{\cdot}05}{100}\text{ cm} = \frac{0{\cdot}5}{100}\text{ mm}$$

$$= \mathbf{0{\cdot}005\text{ mm}}$$

Hence option (*b*) is correct.

Also, If L.C of linear scale of screw gauge is twice the L.C. of vernier callipers,

then L.C of linear scale of screw gauge

$$= 2 \times 0{\cdot}025 = 0{\cdot}05\text{ cm}$$

$$\text{pitch} = 2 \times 0{\cdot}05 = 0{\cdot}1\text{ cm} = 1\text{ mm}$$

L.C of screw gauge $= \frac{1\text{mm}}{100} = \mathbf{0{\cdot}01\text{ mm}}$

Hence option (*c*) is correct.

90. V.C. = 0·1 mm = 0·01 cm ;

zero error = 0·04 cm.

Zero correction = – 0·04 cm.

Observed diameter = (1·2 + 5 × 0·01) cm

$$= 1{\cdot}25\text{ cm}$$

Corrected diameter = (1·25 – 0·04) cm

$$= \mathbf{1{\cdot}21\text{ cm}}$$

91. The strip at the back of vernier callipers is used for measuring depth of a cylinder.

92. As zero of vernier scale is to the right of zero of main scale, therefore, zero correction is *negative*. Its magnitude $= 3 \times 0{\cdot}1\text{ mm} = 0{\cdot}3\text{ mm} = \mathbf{0{\cdot}03\text{ cm}}$

93. Pitch $= \frac{2}{4}\text{ mm} = 0{\cdot}5\text{ mm}$

$$\text{L.C.} = \frac{\text{pitch}}{200} = \frac{0{\cdot}5}{200}\text{ mm} = 0{\cdot}25 \times 10^{-2}\text{ mm}$$

$$= \mathbf{0{\cdot}25 \times 10^{-3}\text{ cm}}$$

94. Observed diameter = 0·7 cm + 35 (0·001) cm

$$= 0{\cdot}735\text{ cm}$$

Corrected diameter = (0·735 + 0·003) cm

$$= \mathbf{0{\cdot}738\text{ cm}}$$

95. As zero of circular scale is below the reference line of graduation, zero correction is *negative*. Its magnitude is $4 \times 0{\cdot}001$ cm $= \mathbf{0{\cdot}004\text{ cm}}$

96. As $\text{L.C.} = \frac{\text{pitch}}{\text{no.of divs.on C.S.}}$

and B has twice the number of divisions on *A*,

∴ **L.C. of *A* is twice the L.C. of *B***

97. The correct relation for *R* is $R = \frac{l^2}{6h} + \frac{h}{2}$

98. Since, the measured value is 3·50 cm, the least count of measuring instrument must be 0·01 cm. It is so for the given vernier calliper.

Least count for vernier calliper = Vernier constant

$$= 1\,MSD - 1\,VSD = 1\,MSD - (9/10)\,MSD$$

$$= \frac{1}{10}\,(MSD) = \frac{1}{10} \times 0{\cdot}1\text{ cm} = \mathbf{0{\cdot}01\text{ cm}}$$

UNIT-2

KINEMATICS

CHAPTER 3.

MOTION IN A STRAIGHT LINE

CHAPTER 4.

MOTION IN A PLANE

3

MOTION IN A STRAIGHT LINE

3.1. INTRODUCTION

The concept of motion is too common. We observe people walking around on foot ; on bikes ; in cars, in trains, in aeroplanes etc. A body is said to be in motion, when it changes with time its position with respect to the surroundings. In this unit, we shall study the concepts of distance and displacement ; speed and velocity ; uniform and non uniform motion ; relative velocity. We shall investigate further the applications of these concepts in day to day life.

The branch of Physics which deals with the study of motion of material objects is called Mechanics.

Mechanics can be broadly classified into following branches :—

(i) Statics. ***It is a branch of mechanics which deals with the study of material objects at rest.***

An object can be at rest, even when a number of forces acting on it are in equilibrium. Thus, *statics is the branch of mechanics which deals with the study of motion of objects under the effect of forces in equilibrium.* Here, time factor does not play any role.

(ii) Kinematics. ***It is that branch of mechanics which deals with the study of motion of material objects without taking into account the factors which cause motion.***

Here, time factor plays an essential role. The term Kinematics is derived from a Greek word **'Kinema'** meaning motion.

(iii) Dynamics. ***It is that branch of mechanics which deals with the study of motion of objects taking into account the factors which cause motion.***

Here also time factor plays an essential role. The term dynamics is derived from the Greek word **'Dynamis'** meaning power. Since the force is the cause of motion, therefore dynamics is based on the concept of force.

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3.2. OBJECT IN MOTION

Rest. ***An object is said to be at rest if it does not change its position with time, with respect to its surroundings.***

A book lying on a table, a person sitting in a chair are the examples of rest.

Motion. ***An object is said to be in motion if it changes its position with time, with respect to its surroundings.***

A bird flying in air, a train moving on rails, a ship sailing on water, a man walking on road are some of the examples of motion, visible to the eye. Motion of gas molecules is an example of motion, invisible to the eye.

Rest and motion are relative. It means an object in one situation can be at rest but in another situation, the same object can be in motion. ***For example,*** a person sitting in a moving train is at rest with respect to his fellow passengers but is in motion with respect to the objects outside the train. Similarly a person sitting in his house is at rest *w.r.t.* earth but is in motion *w.r.t.* other planets.

3.3. TYPES OF MOTION OF A BODY

Mainly the motion of a body can be of following three types :

(1) Rectilinear or translatory motion.

Rectilinear motion is that motion in which a particle or point mass body is moving along a straight line.

Translatory motion is that motion in which a body, which is not a point mass body is moving such that all its constituent particles move simultaneously along parallel straight lines and all its constituent particles shift through equal distance in a given interval of time. e.g.,

A body slipping along the inclined plane has translatory motion. Rectilinear or translatory motion can be uniform or non-uniform.

(2) Circular or Rotatory motion

A circular motion is that motion in which a particle or a point mass body is moving on a circle, about a fixed point which is the centre of a circle.

A rotatory motion is that motion in which a body, which is not a point mass body, is moving such that all its constituent particles move simultaneously along concentric circles, whose centres lie on a line, called axis of rotation and shift through equal angle in a given time.

Circular or rotatory motion can be two dimensional or three dimensional motion and can be uniform or non-uniform motion. If the circular or rotatory motion is uniform, it is periodic also.

(3) Oscillatory or Vibratory motion

Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position) in a definite interval of time.

In such a motion the body is confined within well defined limits (called extreme positions) on either side of mean position. It means a periodic and bounded motion of a body about a fixed point is called an oscillatory motion, *e.g.*, the motion of the pendulum of wall clock is oscillatory motion.

If in the oscillatory motion, the amplitude is very small, (*i.e.*, microscopic), the motion of body is said to be a ***vibratory motion.*** In the oscillatory or vibratory motion, if the restoring force (or torque) is linear, the motion of the body is said to be **simple harmonic motion.** This motion has been discussed in detail in chapter 14.

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3.4. CONCEPT OF POINT MASS OBJECT

When an object in motion covers a very large distance as compared to its size or dimensions, then to study the motion of that object, its size or dimensions can be neglected and object can be regarded as a point object.

Thus, an object can be considered as a point object if during motion in a given time, it covers distances much greater than its own size.

For example, while studying the revolution of earth around the sun, the diameter of earth being very small as compared to the length of its orbit around the sun, the earth can be regarded as a point object. Similarly, a car travelling a few hundred kilometre distance, can be taken as a point object. But if a car is travelling a distance which is not very large as compared to the size of the car, then the car can not be taken as a point object.

It is important to note that the point object is a mathematical concept to simplify the problems.

3.5. FRAME OF REFERENCE

When a point object is in motion and we want to specify its position, then we need to use a reference point and a system having a set of axes. The most convenient system is a rectangular coordinate system, consisting of three mutually perpendicular axes, labelled X-, Y- and Z-axis. The point of intersection O of these three axes is called origin which serves as a **reference point** or **the position of the observer.** The observer has a clock with him to record the time. The position of the object at a given instant of time can be described in terms of position coordinates (x, y, z), *i.e.*, the distances of the given position of object along the X-, Y- and Z-axis. *This coordinate system alongwith a clock constitutes a* **frame of reference.**

Thus, the frame of reference is a system of coordinate axes attached to an observer having a clock with him, with respect to which, the observer can describe position, displacement, acceleration etc. of a moving object.

Frames of reference can be of two types :

(*a*) Inertial frame of reference (*b*) Non-inertial frame of reference.

(a) Inertial frame of reference is one in which Newton's first law* of motion holds good. For example, a frame of reference attached to a person in a bus at rest or in a bus moving with a uniform velocity along a straight path.

(b) Non-inertial frame of reference is one in which Newton's first law of motion does not hold good. For example, a frame of reference attached to a person in a bus moving with variable velocity or moving with acceleration along a straight path.

We know that earth is revolving around the sun in almost in a circular orbit and is having spinning motion also about its axis. Due to it, the velocity of earth is changing with time, so the Newton's first law of motion does not hold good for earth. Therefore, the frame of reference attached to a person on earth for observing the things outside the earth is non-inertial frame of reference. However, for the motion of the object on earth, the earth can be taken at rest. Then the frame of reference attached to a person on earth is taken as inertial frame of reference.

3.6. MOTION IN ONE, TWO AND THREE DIMENSIONS

(*i*) One dimensional motion.

The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with respect to time.

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*For Newton's first law of motion refer to chapter 5 of this book.

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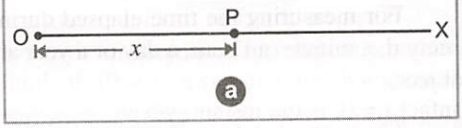
Suppose an object is moving along a straight line OX with origin at O. Fig. 3.1(a). At time t, let the object be at P, where $OP = x$. It means, to know the position of the object at a given time t, we are required to know only one coordinate, *i.e.*, distance x. Such *a motion of an object in which only one coordinate is changing with time is called* **one dimensional motion.**

FIGURE 3.1



In one dimensional motion, the object moves along a straight line or a well defined straight path. Therefore, one dimensional motion is sometimes known as **rectilinear** or **linear motion.**

For example, the motion of a train along a straight railway track, an object dropped from a certain height above the ground, a man walking on a level and narrow road, oscillations of a mass suspended from a vertical spring etc., are one dimensional motion.

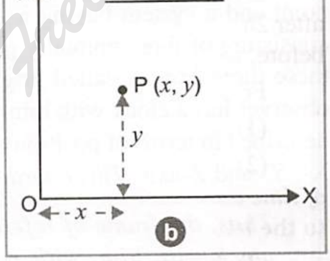
(*ii*) **Two dimensional motion.**

> ***The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with respect to time.***

In such a motion, the object moves in a plane.

Suppose an object is moving in X-Y plane with origin at O. Fig. 3.1(b). At time t, let the object be at P, whose coordinates are (x, y). It means to know the position of the object, moving in X-Y plane, at a given time t, we are required to know two coordinates, *i.e.*, distances along X-axis and Y-axis. *Such a motion of an object in which two coordinates are changing with time is called* **two dimensional motion.**

FIGURE 3.1



For example, an insect crawling over the floor, the earth revolving around the sun, a billard ball moving over the billard table, are two dimensional motion.

(*iii*) **Three dimensional motion.**

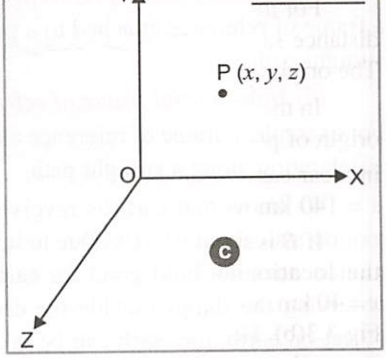
> ***The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with respect to time.***

In such a motion, the object moves in space.

Suppose an object is moving in space, having three mutually perpendicular axes labelled as X- , Y- and Z-axes, with origin at O. Fig. 3.1(c). At time t, let the object be at P, whose coordinates are (x, y, z). It means to know the position of the object in space at a given time, we are required to know three coordinates, *i.e.*, distances along X-, Y- and Z-axes. *Such a motion of an object in which three coordinates are changing with time is called* **three dimensional motion.**

In three dimensional motion, the object moves in space. **For example :** a kite flying on a windy day, the random motion of a gas molecule, a flying aeroplane or bird etc. are three dimensional motions.

FIGURE 3.1



3.7. MOTION IN A STRAIGHT LINE

The motion of a point object in a straight line is one dimensional motion. During such a motion, the point object occupies a definite position on the path at each instant of time. Therefore, the motion of the point object can be described by specifying the distance x of the point object and the corresponding instant of time t. Mathematically, the position of the object in one dimensional motion can be expressed as follows:

$$x = x(t) \quad \text{or} \quad x = f(t)$$

Here, the distance x is the function of time t. Thus, to study one dimensional motion, we have to measure both the position and time at various instants. In order to understand and represent such a motion graphically, the following important concepts are to be understood.

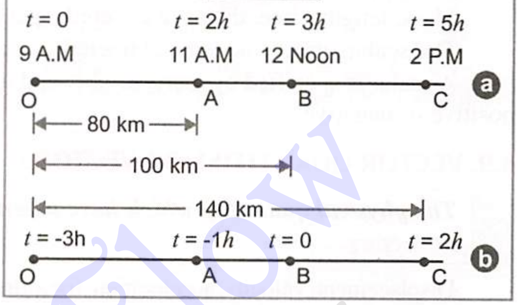
(a) Origin, unit and sense of passage of time

For measuring the time elapsed during the motion of an object, select a convenient unit of time say a second, a minute, an hour, a day or a year and also select some instant of time as the origin. *The origin of time is assigned zero value (i.e., $t = 0$).* It should be clearly understood that $t = 0$, is not the begining of motion. Infact $t = 0$, is the instant, we choose arbitrarily to describe the motion.

Consider a person leaving his home O at 9 A.M. by car, through a stright road. He crosses the city A at a distance of 80 km from his home at 11 A.M., reaches another city B at a distance, 100 km from his home at 12 noon and finally reaches his destination C at a distance 140 km from his home at 2 PM ; Fig 3.3(a).

If we consider the instant of time 9 A.M. (when the person left his home O) as the origin of time axis, *i.e.*, $t = 0$, then the person crosses city A, at $t = 2$ h ; city B at $t = 3$ h and finally reaches his destination C at $t = 5$h.

FIGURE 3.2



If the instant of time 12 noon is chosen as origin of time axis, then the person reaches his destination C after 2h (*i.e.*, $t = 2$h) and crossed the city A, one hour before, *i.e.*, when $t = -1$ h and left his home three hours before, *i.e.*, when $t = -3$ h ; Fig. 3.2(b).

From the above discussion, we conclude the following :

(1) ***The origin of time axis can be shifted to any point on the time axis.***

(2) ***The time measured after the origin of time axis (i.e., the time measured to the right of the origin of time axis) is taken as positive and the time measured before the origin of time axis*** **(*i.e.*, time measured to the left of the origin of time axis)** ***is taken as negative.***

(3) ***The negative time coordinate of a point on time axis indicates that the object had reached that point at a time which was before the origin of the time axis.***

(4) ***The time interval between two points on time axis does not change due to shift in the origin of time axis.***

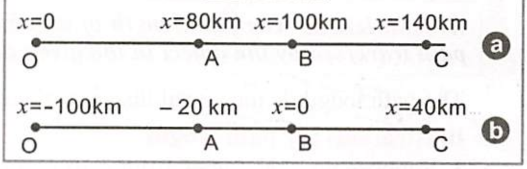
(b) Origin, unit and direction for position measurement

For measuring the position of an object at different locations during motion, select a convenient unit of distance say a centimetre, metre, kilometre etc and also select some location as the origin on position axis. The origin on position axis is assigned zero value (*i.e.*, $x = 0$).

In the above illustration (used in the unit and sense of passage of time), if the home O is selected as the origin of position axis (*i.e.*, $x = 0$), then the location of city A, city B and destination C of the person going in a car can be written as $x = 80$ km, $x = 100$ km and $x = 140$ km respectively. Fig. 3.3(a).

If B is chosen as the origin of position axis, then the locations of C, A and O on position axis are $x = 40$ km, $x = -20$ km and $x = -100$ km respectively ; Fig 3.3(b). Here, positive sign shows that the location is to the right of the origin of position axis and negative sign shows that the location is to the left of the origin of position axis.

FIGURE 3.3



From the above discussion, we conclude the following:

(1) ***The origin of position axis can be shifted to any point on the position axis.***

(2) ***The distances measured to the right of the origin of position axis are taken positive and those measured to the left of the origin of position axis are taken negative.***

(3) ***In case of motion along a vertical line (say along Y axis), the distances measured above the origin of position axis are taken positive and those measured below the origin of position axis are taken negative.***

(4) *The distance between two locations on position axis does not change due to shift in the origin of position axis.*

3.8. SCALAR QUANTITIES OR SCALARS

The physical quantities which have only magnitude but no direction, are called ***scalar quantities*** **or** ***scalars.***

Mass, length, time, distance covered, speed, temperature, work etc. are the few examples of scalars. The scalars can be added, subtracted, multiplied and divided using ordinary laws of algebra.

A scalar is specified by mere number and unit, where number represents its magnitude. A scalar may be positive or negative.

3.9. VECTOR QUANTITIES OR VECTORS

The physical quantities which have magnitude as well as direction are called ***vector quantities*** **or** ***vectors.***

Displacement, velocity, acceleration, force, torque, gravitational intensity etc. are the few examples of vectors.

Vectors cannot be added, subtracted, and multiplied by ordinary laws of algebra. For these operations on vectors we have to use *laws of vectors. It is important to note that the division of a vector by another vector is not a valid operation in vector algebra, because the division of a vector by a direction is not possible.*

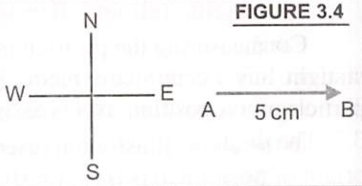
A vector in writing, *can be represented either by a single letter in bold face or by a single letter with an arrow head on it.* Thus, the velocity which is a vector quantity can be represented by υ or $\vec{\upsilon}$. Its positive magnitude is represented by υ or $|\vec{\upsilon}|$, called modulus of velocity vector. The modulus or magnitude of a vector is a scalar quantity.

Geometrically or graphically, *a vector is represented by a straight line with an arrow head, i.e., arrowed line.*

Here, the length of the arrowed line drawn on a suitable scale represents the magnitude and the arrow head represents the direction of the given vector.

For example, a velocity of 50 ms^{-1} due east is represented by line *AB* of length 5 cm, with an arrow head pointing east, on the scale, 1 cm = 10 ms^{-1}, as shown in Fig. 3.4. The initial or starting point *A* of the arrowed line is called the **tail** or **origin of the vector.** The point *B* which is at the end of the arrowed line is called **tip** or the **terminus** or **head of the vector.**

FIGURE 3.4



3.10. PATH LENGTH AND DISPLACEMENT

(i) Path length. *The path length of an object in motion in a given time is the length of actual path traversed by the object in the given time.*

The path length is the actual distance travelled by the object in the given time during motion.

Illustrations for path length

1. Suppose an object goes from *A* to *C* following the path *ABC*, in a certain time *t*, Fig. 3.5. The path length traversed by the object in time $t = AB + BC$.

2. If the object goes from *A* to *B*, *B* to *C* and *C* to *A* in time *t*, Fig. 3.5, then the path length traversed by object in time $t = AB + BC + CA$.

3. If an object goes once around a circle of radius *r* in time *t*, Fig. 3.6, then the path length traversed in time $t = 2\pi r$.

FIGURE 3.5

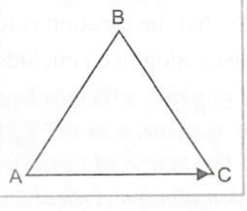
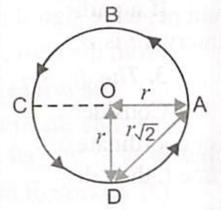


FIGURE 3.6



From above, we note that **the path length is the actual distance travelled.** It has only magnitude and no direction. Hence, **path length (*i.e.,* distance travelled) is a scalar quantity.** It can never be zero or negative.

> ***(ii) Displacement.* The displacement of an object in motion in a given time is defined as the change in position of the object, i.e., the difference between the final and initial positions of the object, in a given time. It is the shortest distance between the two positions of the object and its direction is from initial to final position of the object, during the given interval of time. It is represented by the vector drawn from the initial position to its final position.**

Displacement is a vector quantity as it possesses both, the magnitude and direction.

Illustrations for displacement

1. When an object goes on the path ABC, Fig. 3.5, then the displacement of the object is $\overrightarrow{(AC)}$. The arrow head at (AC) shows that the object is displaced from A to C. In case the object is displaced from C to A, then the displacement of the object is $\overrightarrow{(CA)}$, [*i.e.,* a vector drawn from C to A]. Then $\overrightarrow{(CA)} = -\overrightarrow{(AC)}$.

Here, the displacement $\overrightarrow{(CA)}$ is having the same magnitude as that of $\overrightarrow{(AC)}$ but opposite direction.

2. If an object goes from A to B, B to C and C to A in time t, Fig. 3.5, then the displacement of the object in time t will be $= \overrightarrow{(AA)} = \vec{0}$.

3. Refer to Fig. 3.6, the displacement of the object for the motion along the circular path ABC, $= \overrightarrow{(AC)}$, where $(AC) = 2\,r$.

The displacement of object for the motion of object along the circular path $ABCD$, $= \overrightarrow{(AD)}$ where

$$(AD) = \sqrt{OA^2 + OD^2} = \sqrt{r^2 + r^2} = r\sqrt{2}$$

The ***displacement of an object in motion can be positive, zero or negative.***

Characteristics of Displacement

1. Displacement of a moving object in a given interval of time is a vector quantity as it has both magnitude and direction.

2. The displacement of a moving object in a given interval of time can be positive, negative or zero.

Illustration for positive displacement

Consider a particle starting from point O moving along a straight line OX, with O as an origin of coordinate axis. Let the particle move from A to B in time t, where $OA = x_i$ and $OB = x_f$. Fig. 3.7. The displacement of particle in time interval t is

$\Delta x = OB - OA = x_f - x_i$ which is positive as $x_f > x_i$

FIGURE 3.7

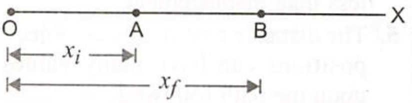


Illustration of Negative displacement

Refer to Fig. 3.8, if the particle moves from A to B in time t, where $OA = x_i$ and $OB = x_f$, then the displacement of particle in time interval t is $\Delta x = x_f - x_i$, which is negative as $x_f < x_i$

FIGURE 3.8

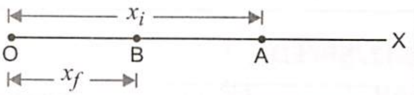


Illustration of zero displacement

If a particle is at rest, then its displacement in a given time interval is zero

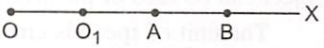
If a particle in motion returns to its initial position after a certain time t, then its displacement in time interval t is zero.

3. *The displacement of a particle in a given interval of time is independent of the choice of the origin.*

Consider a particle moving along a straight line OX, with O as origin for coordinate axis. Fig. 3.9. Let the particle go from A to B in interval of time t, then displacement of particle in time t is

$$= OB - OA = AB$$

FIGURE 3.9

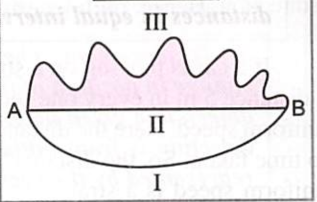


If O_1 is the origin for position measurement, then displacement in time $t = O_1B - O_1A = AB$

4. *The displacement of a particle in a given time does not tell the shape of the path followed.*

Let a particle be moving from A to B in time t, following firstly the path I, then path II and later on path III. Fig. 3.10. In all such cases, the displacement of particle in time interval t is $= AB$, which is independent of the shape of the path followed.

FIGURE 3.10

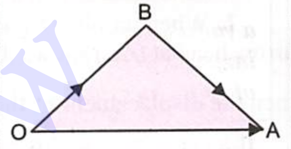


5. *The magnitude of displacement of a particle between two positions is the shortest distance between them.*

6. *The magnitude of displacement of a particle in a given time t is equal to distance travelled by the particle in time t* if the particle is moving along a straight line without changing its direction of motion.

7. The magnitude of displacement of a particle in a given time is less than the distance travelled by particle in that time if the particle does not move along a straight line.

FIGURE 3.11



Let a particle go from O to A in time t following path OBA, then displacement of particle $= (\overrightarrow{OA})$

$$\text{Distance travelled} = (OB + BA)$$

As path $(OB + BA) >$ path OA, so the distance travelled > magnitude of displacement

3.11. DIFFERENCE BETWEEN DISTANCE (PATH LENGTH) AND DISPLACEMENT

DISTANCE	DISPLACEMENT
1. It is the actual path traversed by the object in the given time.	**1.** It is the shortest distance between the initial position and final positions of the object in the given time.
2. It is a scalar quantity.	**2.** It is a vector quantity.
3. The distance travelled by an object in a given time is never negative or zero, but is always positive.	**3.** The displacement of an object in a given time can be positive, zero or negative.
4. The distance travelled in a given time is either equal to or greater than displacement but never less than displacement.	**4.** The displacement of an object in a given time can be equal to or less than the distance travelled but never greater than the distance travelled.
5. The distance covered by an object between two positions can have many values, depending upon the path followed.	**5.** The displacement of an object between two positions has a unique value.
6. The distance travelled by the object between two positions tells the type of path followed.	**6.** The displacement of an object between two positions does not tell the type of path followed.

3.12. SPEED

> ***Speed** of an object in motion is defined as the ratio of total path length (i.e., actual distance covered) and the corresponding time taken by the object, i.e.,*

$$\text{Speed} = \frac{\text{total path length}}{\text{time taken}} = \frac{\text{actual distance travelled}}{\text{time taken}}$$

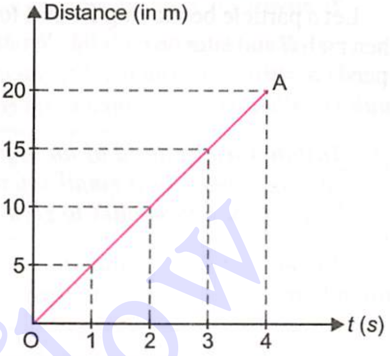
Speed is a scalar quantity. It gives no idea about the direction of motion of the object. The speed of the object can be zero or positive but never negative.

The unit of speed is cm s^{-1} in cgs system and ms^{-1} in mks system or SI. The dimensional formula of speed is $[M^0L^1T^{-1}]$.

(a) *Uniform speed.* ***An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.***

If a car is moving on a straight road from A to B and covers a distance 5 m in every one second, then the car is moving with a uniform speed. Here the distance travelled is directly proportional to time taken. So, the distance-time graph of the car moving with uniform speed is a straight line OA as shown in Fig. 3.12. The speed of this car is 5 m s^{-1}, which is the slope of straight line OA.

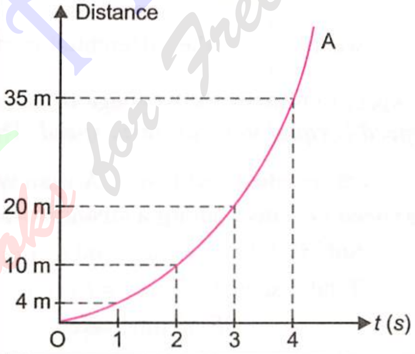
FIGURE 3.12



(b) *Variable speed.* ***An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.***

If a car is moving on a straight road and covers 4 m in first second, 6 m in 2nd second, 10 m in third second and so on, then the distance travelled is not directly proportional to time, so the distance time graph is a curve OA as shown in Fig. 3.13.

FIGURE 3.13



(c) *Average speed.* ***When an object is moving with a variable speed, then the average speed of the object is that constant speed with which the object covers the same distance in a given time as it does while moving with variable speed during the given time. Average speed for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken i.e.,***

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

(*i*) If a particle travels distances S_1, S_2, S_3 etc. with speeds v_1, v_2, v_3 etc. respectively, in same direction then total distance travelled $= S_1 + S_2 + S_3 + \ldots\ldots$

$$\text{Total time taken } = \frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3} + \ldots\ldots\ldots$$

$$\text{Average speed, } v_{av} = \frac{S_1 + S_2 + \ldots\ldots}{\left(\frac{S_1}{v_1} + \frac{S_2}{v_2} + \ldots\ldots\right)} \quad \text{...(1)}$$

If $S_1 = S_2 = S$; *i.e.,* the body covers equal distances with different speeds then,

$$v_{av} = \frac{2S}{S\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2v_1 v_2}{(v_1 + v_2)} \quad \text{...(2)}$$

It means average speed is equal to harmonic mean of individual speeds.

(*ii*) If a particle travels with speeds v_1, v_2, v_3, etc. during time intervals t_1, t_2, t_3, etc. respectively, then total distance travelled $= v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots\ldots$

Total time taken $= t_1 + t_2 + t_3 + \ldots\ldots$

$$\text{So average speed, } v_{av} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \ldots\ldots}{t_1 + t_2 + t_3 + \ldots\ldots} \quad \text{...(3)}$$

If $t_1 = t_2 = t_3 = ... = t_n = t$ (say) then from (3), we have

$$v_{av} = \frac{(v_1 + v_2 + v_3 + ... + v_n)\,t}{nt} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n} \quad ...(4)$$

It means average speed is equal to arithemetic mean of individual speeds.

(d) Instantaneous speed. When the speed of an object is variable, then the object possesses different speeds at different instants. *The speed of an object at a given instant of time or at a particular point of its path is called its instantaneous speed.*

Instantaneous speed of an object at an instant of time t is defined as the limiting value of the average speed in a small internal of time (Δt) taken around that instant, provided this time interval Δt approaches to zero.

Let at an instant of time t, an object while moving covers a distance Δx in a small interval of time Δt around time t, so that $\Delta t \to 0$, then

$$\boxed{\text{Instantaneous speed } = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

where $\frac{dx}{dt}$ is the differential coefficient of x with respect to t, *i.e.*, first derivative of distance with respect to time in the language of calculus. In case of a **uniform motion of an object,** ***the instantaneous speed is equal to its uniform speed.*** The instantaneous speed is measured by **speedometer.**

Sample Problem **A man walks for 1 minute at a speed of 1 ms^{-1} and then runs for 1 min at a speed of 3 ms^{-1} along a straight track. What is the average speed of the man ?**

Sol. Here, $t_1 = 1$ min $= 60$ s ; $v_1 = 1$ ms^{-1} ; $t_2 = 1$ min $= 60$ s ; $v_2 = 3$ ms^{-1}.

Total distance covered $= v_1 t_1 + v_2 t_2 = 1 \times 60 + 3 \times 60 = 4 \times 60$ m

Total time taken $= t_1 + t_2 = 60 + 60 = 120$ s

$$\therefore \quad \text{Average speed } = \frac{4 \times 60}{120} = \mathbf{2\ ms^{-1}}$$

3.13. VELOCITY

Velocity of an object in motion is defined as the ratio of displacement and the corresponding time interval taken by object, i.e.,

$$\text{Velocity } = \frac{\text{displacement}}{\text{time interval}}$$

Velocity is a vector quantity, as it has both, the magnitude (speed) and direction. The velocity of an object can be positive, zero or negative according as its displacement is positive, zero or negative.

The unit of velocity is cm s^{-1} in cgs system and ms^{-1} in mks system or SI. The dimensional formula of velocity is $[M^0L^1T^{-1}]$.

An object is said to be in a uniform motion if its velocity is uniform and constant throughout the motion, *i.e.*, the object is moving with uniform velocity.

Different Types of Velocities

(a) Uniform velocity. *It is that velocity of object with which, it undergoes equal displacements (in magnitude and direction) in equal intervals of time, howsoever small these intervals may be.*

(b) Variable velocity. *An object is said to be moving with a variable velocity if either its speed changes with time or direction of motion changes with time or both change with time.*

(c) Average velocity ($\vec{v}_{av}$). When an object is moving with variable velocity, *then average velocity of the object is defined as the ratio of total displacement to the total time interval in the given displacement,*

i.e.,
$$(\vec{v}_{av}) = \frac{\text{total displacement}}{\text{total time}}$$

If $\vec{x}_1$ and $\vec{x}_2$ are the displacements of an object in timings t_1 and t_2 respectively, then

$$\vec{v}_{av} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1}$$

In uniform motion along a straight line, the average velocity is equal to uniform velocity.

(d) Instantaneous velocity ($\vec{v}_i$). *When an object is moving with a variable velocity, then the velocity of an object at a given instant of time or at a particular point of its path is called its instantaneous velocity. It is equal to the limiting value of the average velocity of the object in a small interval of time around that instant provided this time interval approaches to zero.* Then, $\vec{v}_i = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$

Thus instantaneous velocity of an object is equal to the first derivative of its displacement w.r.t. time at the given instant of time.

In uniform motion along a straight line, **the instantaneous velocity = uniform velocity = average velocity.**

Sample Problem **A cylist moving on a circular track of radius 200 m completes one revolution is 5 minutes. What is his (*a*) average speed (*b*) average velocity in one full revolution ?**

Sol. Here, $r = 200$ m ; $t = 5$ min $= 5 \times 60$ *s*

Distance travelled in one revolution, $S = 2\pi r$

Average speed, $v_{av} = \frac{S}{t} = \frac{2\pi r}{t} = \frac{2\pi \times 200}{5 \times 60} = \mathbf{4{\cdot}19\ ms^{-1}}$

Displacement in one revolution, $\vec{S} = 0$.

$\therefore$ Average velocity $= \frac{\vec{S}}{t} = \frac{0}{5 \times 60} = \mathbf{0}$

3.14. UNIFORM MOTION IN A STRAIGHT LINE

An object is said to be in uniform motion if its velocity is uniform, i.e., it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.

Consider an object in uniform motion along a straight line *OX* with a uniform velocity $\vec{v}$. Let point *O* be the origin for position measurement. Let the time be measured from the instant, the object is at *O*. Let *A* and *B* be the positions of the object at the instants of time t_1 and t_2 respectively, where

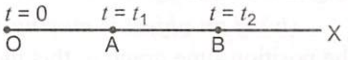
$$\overrightarrow{OA} = \vec{x}_1 \quad \text{and} \quad \overrightarrow{OB} = \vec{x}_2 \text{, Fig. 3.14.}$$

$\therefore$ Displacement of the object in time interval $(t_2 - t_1)$

$$= \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{x}_2 - \vec{x}_1$$

As, velocity $= \frac{\text{displacement}}{\text{time interval}}$ $\therefore$ $\boxed{\vec{v} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1}}$

FIGURE 3.14



Following are the important features of uniform motion :

***(i)* For a uniform motion along a straight line in a given direction, the magnitude of the displacement is equal to the actual distance covered by the object.**

***(ii)* The velocity in uniform motion does not depend upon the time interval ($t_2 - t_1$).**

***(iii)* The velocity in uniform motion is independent of choice of origin.**

***(iv)* The velocity of an object is taken to be positive if the object is moving towards the right of origin and is taken to be negative if the object is moving towards the left of origin.**

***(v)* No force is required for an object to be in uniform motion.**

***(vi)* The average and instantaneous velocities have same value in a uniform motion.**

3.15. FORMULAE FOR UNIFORM MOTION

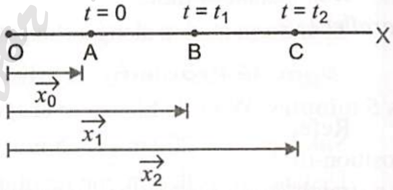
This formula connects the position, velocity and time of an object in uniform motion. Consider an object, moving with uniform velocity $\vec{v}$ along the straight line OX. Let the origin for position measurement be at point O and the origin for time measurement be taken at the instant, when the object is at A, such that $\overrightarrow{OA} = \vec{x}_0$ [Fig. 3.15.] Let the object be at positions B and C at the instants of time t_1 and t_2, where $\overrightarrow{OB} = \vec{x}_1$ and $\overrightarrow{OC} = \vec{x}_2$. Then $\overrightarrow{AB} = \vec{v}\, t_1$ and $\overrightarrow{AC} = \vec{v}\, t_2$.

Here, $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ and $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$$\therefore \qquad \vec{x}_1 = \vec{x}_0 + \vec{v}\, t_1 \qquad \text{...(5)}$$

$$\text{and} \qquad \vec{x}_2 = \vec{x}_0 + \vec{v}\, t_2 \qquad \text{...(6)}$$

FIGURE 3.15



Subtracting (5) from (6), we get

$$\vec{x}_2 - \vec{x}_1 = \vec{v}\,(t_2 - t_1) \qquad \text{or} \qquad \vec{x}_2 = \vec{x}_1 + \vec{v}\,(t_2 - t_1) \qquad \text{...(7)}$$

The relations (5), (6) and (7) represent the kinematics of uniform motion along a straight line.

Special cases. 1. If $\vec{x}_2 - \vec{x}_1 = \vec{s}$ and $t_2 - t_1 = t$ then from (7), we get $\vec{s} = \vec{v}\, t$ ...(8)

2. For a uniform motion in one dimension, displacement is equal to distance travelled and velocity is equal to speed, hence the arrow heads over the displacement and velocity vectors may be dropped. It means we can write, $x_1 = x_0 + v\, t_1$; $x_2 = x_0 + v\, t_2$ and $(x_2 - x_1) = v\,(t_2 - t_1)$.

3.16. POSITION-TIME GRAPH FOR A MOVING OBJECT

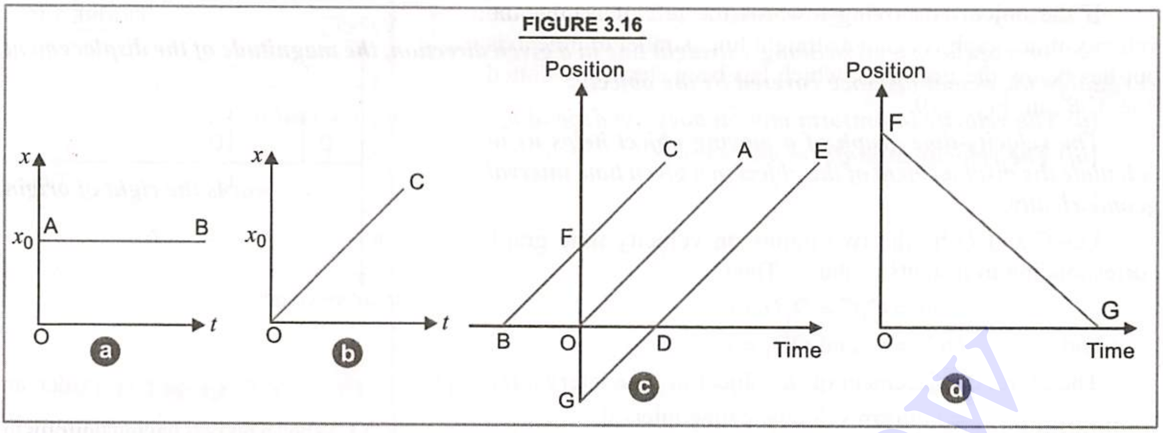
(*i*) ***If an object is at rest or stationary,*** its position will not change with time. Let the object be stationary at position $x(t) = x_0$ from the origin. Then the position (x)-time (t) graph for the stationary object is a straight line AB parallel to time axis. Fig. 3.16(a). Such a graph describes a train standing at the railway line or a bus standing on the road side.

(*ii*) ***If an object is in uniform motion along a straight line, starting from, the origin O,*** the position (x)-time (t) graph is a straight line OC inclined to time axis as shown in Fig. 3.16(b).

(*iii*) Position-time graph of uniform motion in one dimension of an object (*a*) starting from origin is shown by st. line OA ; (*b*) starting from left of origin is shown by st. line BC and (*c*) starting from right of origin, is shown by st. line DE in Fig. 3.16(c).

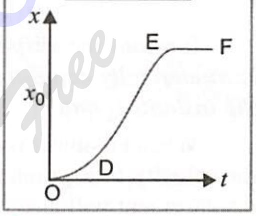
(*iv*) ***If an object is moving with a constant negative velocity starting from the positive position,*** then the position-time graph of this motion in one dimension is a straight line FG inclined to time axis, as shown in Fig. 3.16(d).

FIGURE 3.16



(*v*) ***If an object is in non-uniform motion along a straight line, i.e.,*** a car starting from rest at time $t = 0$ second from the origin, moves along a straight path. It picks up some speed. It then moves with constant speed for some time. After that the brakes are applied and the car comes to rest. The position-time graph for this case is as shown in Fig. 3.17.

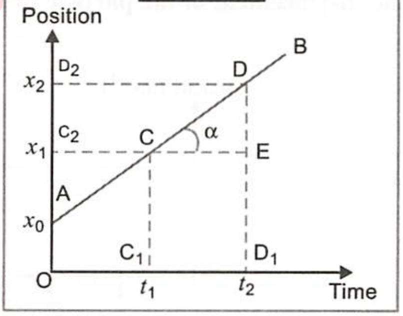
FIGURE 3.17



It is ***important to note that position-time graph cannot be a straight line parallel to position axis, because it will indicate infinite velocity.***

3.17. POSITION-TIME GRAPH OF UNIFORM MOTION IN ONE DIMENSION

FIGURE 3.18



Refer to motion of the object as given in Article 3.15, the position-time graph of an object in a uniform motion in one dimension is a straight line AB, inclined to time axis as shown in Fig. 3.18. Here, the position coordinates of the object at the instants $t = 0$, t_1 and t_2 respectively are x_0, x_1 and x_2. Let C and D be the two points on the position-time graph corresponding to instants t_1 and t_2. Draw CC_1, CC_2, DD_1 and DD_2, perpendiculars on time axis and position axis, as shown in Fig. 3..18.

Then, $t_2 - t_1 = OD_1 - OC_1 = C_1D_1 = CE$

and $x_2 - x_1 = OD_2 - OC_2 = C_2D_2 = ED$

$$\therefore \text{ Velocity of the object, } v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{ED}{CE} = \tan\alpha = \text{slope of position-time graph.}$$

Thus, velocity of an object in uniform motion is equal to the slope of position–time graph with time axis. Hence, the importance of the position-time graph of uniform motion lies in the fact that its slope gives us velocity of the object.

IMPORTANT NOTE

Greater is the slope of position-time graph, larger is the uniform velocity of that body.

3.18. VELOCITY-TIME GRAPH IN UNIFORM MOTION

Consider an object moving with a uniform velocity $\vec{v}$ along a straight line OX. Let the object be moving towards the right of origin O (*i.e.*, positive direction of *x*-axis) for position and time measurements, hence, $\vec{v}$ is positive. The velocity-time graph of the object is a straight line AB parallel to time axis, Fig. 3.19.

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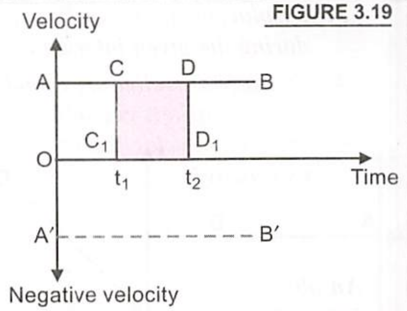
If the object is moving towards the left of origin, the velocity-time graph is again a straight line parallel to time axis but lies below the time axis, which has been shown by dotted line $A' B'$, in. Fig. 3.19.

FIGURE 3.19



The velocity-time graph of a moving object helps us to calculate the displacement of the object in a given time interval geometrically.

Let C and D be the two points on velocity-time graph corresponding to instants t_1 and t_2. Then,

$$OA = C_1C = D_1D = v$$

and $$OC_1 = t_1 \text{ and } OD_1 = t_2.$$

Therefore, displacement of the object in time interval $(t_2 - t_1)$

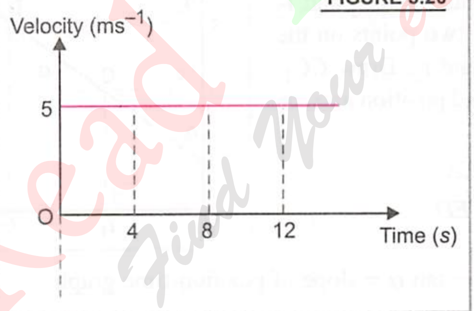
$$= \text{uniform velocity} \times \text{time interval}$$
$$= v\,(t_2 - t_1) = C_1C \times (OD_1 - OC_1)$$
$$= C_1C \times C_1D_1 = \text{area } CC_1\,D_1D$$

It means the ***displacement of an object in time interval $(t_2 - t_1)$ is numerically equal to the area under velocity-time graph between the instants t_1 and t_2.***

When an object is moving with negative velocity, the area below the velocity-time graph is taken negative and corresponding to this, the displacement will also be negative.

Sample Problem **The velocity time graph of a uniform motion of a particle along a st. line is shown in Fig. 3.20. What is the displacement of the particle in time interval 8 s to 12 s ?**

FIGURE 3.20



Sol. Displacement of a particle having uniform motion is equal to the area which velocity-time graph encloses with time axis.

Displacement of particle in time interval 8 s to 12 s (*i.e.*, 4 s) is = velocity × time interval = 5 × 4 = **20 m.**

DO YOU KNOW ?

1. When a body is having uniform motion along a straight line in a given direction, the magnitude of the displacement of body is equal to the actual distance travelled by the body in the given time.

2. The average and instantaneous velocity in a uniform motion are equal in magnitude.

3. In a uniform motion along a st. line, the slope of position-time graph gives the velocity of the body.

4. The position-time graph of a body moving along a st. line can never be a st. line parallel to position axis because it will indicate infinite velocity.

5. The speed of a body can never be negative.

6. The slope of velocity-time graph of uniform motion with time axis is zero.

7. If the two bodies are moving with unequal uniform velocities, then their position-time graphs must intersect each other.

Retain in Memory

1. The **importance** of *velocity-time graph of uniform motion lies in the fact that it helps us to measure the displacement undergone by an object in a given time as the area enclosed by velocity-time graph with time axis.*
2. *In velocity-time graph, for the given interval of time, the positive area enclosed with time axis gives positive displacement and the negative area enclosed with time axis gives negative displacement.*

3. *Displacement of a body in a given interval of time is equal to total area of velocity-time graph, during the given interval of time which is to be added with* **proper sign.**
4. *Distance travelled by a body in a given interval of time is equal to total area of velocity-time graph,* **without considering sign.** It means, even if the body is moving with negative velocity, the *area of velocity-time graph is to be taken* **positive** for the *measurement of distance travelled by a body.*

3.19. NON-UNIFORM MOTION

An object is said to be in non-uniform motion if it undergoes equal displacements in unequal intervals of time, howsoever small these intervals may be.

Clearly, in non-uniform motion, the velocity of the object is different at different instants.

For example, when we run or travel in a vehicle from one position to another, we start from the condition of rest. We gain speed for some time and then may travel with constant velocity for sometime and finally slow down and come to a stop. In scuh a case, the velocity of object is different at different instants and hence this motion is non-uniform motion.

In non-uniform motion, the velocity of object in different forms is explained below.

(i) Variable velocity. ***An object is said to be moving with a variable velocity, if either its speed or its direction of motion or both change with time.***

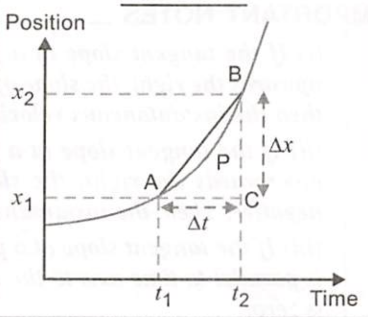
(ii) Average velocity. For a given motion of an object, ***the average velocity is that uniform velocity with which the object will cover the same displacement in a given interval of time as it does with its actual variable velocity during the given interval of time.***

Quantitatively, ***the average velocity of an object is equal to the ratio of change in position or displacement*** $(\Delta \vec{x})$ ***to the time interval*** (Δt)***, in which the displacement occurs, i.e.,***

$$\text{Average velocity, } \bar{v} = \frac{\text{displacement}}{\text{time interval}} = \frac{\Delta \vec{x}}{\Delta t}$$

Consider an object moving with variable velocity. Let the position-time graph of this motion be represented by a curve, as shown in Fig. 3.21. Let us take two points on this graph. The point A corresponds to time t_1 and position x_1. The point B corresponds to time t_2 and position x_2. The change in position of object (or magnitude of displacement) in time interval $(t_2 - t_1) = x_2 - x_1$.

FIGURE3.21



$$\text{Slope of straight line } AB = \tan\theta = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$$= v_{av} \quad \text{or} \quad \bar{v} \text{ (magnitude of average velocity).}$$

Thus, the magnitude of average velocity of the object over a given interval of time denoted by two points on the position-time graph is equal to the slope of straight line joining those two points with time axis on position-time graph.

The average velocity of an object can be positive or negative depending on the sign of displacement. It is zero if the displacement is zero.

(iii) **Instantaneous velocity.** The average velocity of the object in motion tells us how fast an object has been moving over a given interval of time. But it does not tell, how fast the given object is moving at different instants of time during motion. For this, we define **instantaneous velocity.**

The instantaneous velocity of an object at an instant of time t, is defined as the limit of average velocity as time interval Δ t, around time t, becomes infinitesimally small.

Thus, instantaneous velocity at instant of time t is, $v_i = \underset{\Delta t \to 0}{\text{Limit}} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

where $\dfrac{dx}{dt}$ = the differential coefficient of x with respect to t, *i.e.*, first derivative of displacement with respect to time, in the language of calculus. Thus,

the instantaneous velocity of an object at a given instant is the first derivative of displacement with respect to time at that instant.

We can obtain the **value of instantaneous velocity graphically** as explained below :

Suppose an object is moving along a straight line with variable velocity. Let the position-time graph of this motion be represented by a curve as shown in Fig. 3.22. From the graph, we note that the average velocity of the object over the time interval Δ t = 4 s, (*i.e.*, from time 4 s to 8 s) is the slope of straight line AB.

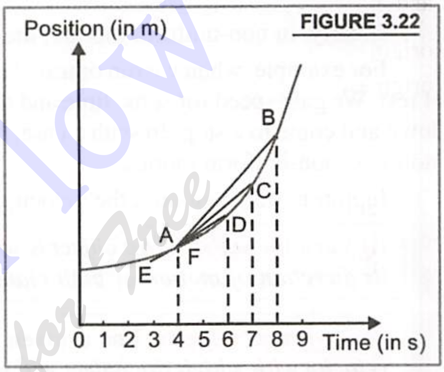


FIGURE 3.22

If we decrease the interval of time Δ t from 4 s to 3 s (*i.e.*, from t = 4 s to t = 7 s), then the line AB becomes AC. On making the time interval Δ t = 2 s (*i.e.*, t = 4 s to 6 s), the line AB becomes AD. As Δt approaches zero, the point B approaches point A. In this situation the line AB would become the tangent EF to the curve at A. The slope of the tangent with time axis would give the value of instantaneous velocity corresponding to point A.

Thus, *the instantaneous velocity is defined as the slope of the tangent to the position-time graph at position corresponding to given instant of time.*

IMPORTANT NOTES

(i) If the tangent slope at a point on position-time graph is upwards the right, the slope of tangent at that point is positive, then the instantaneous velocity is taken positive.

(ii) If the tangent slope at a point on position-time graph is downwards the right, the slope of tangent at that point is negative. Then, the instantaneous velocity is taken negative.

(iii) If the tangent slope at a point on the position-time graph is parallel to time axis to the right, the instantaneous velocity is zero.

DO YOU KNOW ?

1. The magnitude of instantaneous velocity is equal to the instantaneous speed at the given instant.

2. In a uniform motion in one dimension, the average velocity = instantaneous velocity = uniform velocity.

3. A speedometer in an automobile measures the instantaneous speed and odometer measures the distance covered by the automobile.

4. A cheetah is the fastest land animal over distances less than 500 metres and can achieve peak speed of 100 km/h.

Curiosity Question

Q. How can the toll taker determine that you were speeding the vehicle or not ?

Ans. On some toll roads, the ticket is stamped with time, you enter the toll road and the time you exit. The toll taker knows the distance between entry point and exit point of the vehicle and thus can determine the time spent by you to drive between entry point and exit point, which helps to determine the average speed of vehicle. If your average speed is above the posted speed limit, you are speeding the vehicle.

3.20. RELATIVE VELOCITY IN ONE DIMENSIONAL MOTION

When two objects A and B are moving with different velocities, then the velocity of one object A with respect to another object B is called relative velocity of object A *w.r.t.* B. Thus,

> ***The relative velocity of one object w.r.t. another is the velocity with which one object moves w.r.t. another object. Hence relative velocity is defined as the time rate of change of relative position of one object with respect to another.***

The concept of relative velocity in one dimension is analysed as follows :—

(a) Expression for relative velocity. Consider two objects A and B, moving with uniform velocities $\vec{v}_1$ and $\vec{v}_2$ along parallel straight tracks in the same direction. Let $\vec{x}_{01}$ and $\vec{x}_{02}$ be their displacements from the origin at instant $t = 0$. If at any time t, $\vec{x}_1$ and $\vec{x}_2$ are the displacements (distances) of the two objects *w.r.t.* the origin of the position axis, then for the object A,

$$\vec{x}_1 = \vec{x}_{01} + \vec{v}_1\, t \qquad \text{...(9)}$$

and for the object B,

$$\vec{x}_2 = \vec{x}_{02} + \vec{v}_2\, t \qquad \text{...(10)}$$

Subtracting the above equations we have,

$$\vec{x}_2 - \vec{x}_1 = (\vec{x}_{02} - \vec{x}_{01}) + (\vec{v}_2 - \vec{v}_1)\, t \qquad \text{...(11)}$$

where, $\vec{x}_{02} - \vec{x}_{01} = \vec{x}_0$, ***initial displacement of object B w.r.t. to object A at t = 0***

$\vec{x}_2 - \vec{x}_1 = \vec{x}$, relative displacement of object B w.r.t. to object A at time t.

Thus, the relation (11) can be written as

$$\vec{x} = \vec{x}_0 + (\vec{v}_2 - \vec{v}_1)\, t \quad \text{or} \quad \vec{x} - \vec{x}_0 = (\vec{v}_2 - \vec{v}_1)\, t \quad \text{or} \quad \boxed{\frac{\vec{x} - \vec{x}_0}{t} = \vec{v}_2 - \vec{v}_1} \qquad \text{...(12)}$$

Here, L.H.S. of relation (12) gives the time rate of change of relative position of object B w.r.t. object A i.e., the relative velocity of object B w.r.t. object A.

∴ relative velocity of object B w.r.t. object A,

$$\vec{v}_{BA} = \vec{v}_2 - \vec{v}_1 = \text{velocity of object } B - \text{velocity of object } A. \qquad \text{...(13)}$$

If the objects are in one dimensional motion, in the same direction, the magnitude of displacement of object is equal to actual path length (total distance) travelled by object in a given time ; then we can ignore arrowheads from the letters. Now, from (12), we have

$$\frac{x - x_0}{t} = v_2 - v_1 = v_{BA} \qquad \text{...(14)}$$

(b) Position-time graph in relative velocity

From (14), we note that the relative velocity v_{BA} will be zero, negative or positive depending upon the relative magnitudes of v_1 and v_2.

(*i*) **It the two objects A and B are moving with same velocity, i.e.,** $v_1 = v_2$ or $(v_2 - v_1) = 0$ then from (14), $x - x_0 = 0$ or $x = x_0$, *i.e.*, the two objects will remain always constant distance apart, which is equal to the relative distance between them initially (at $t = 0$). Therefore, their position–time graphs are parallel straight lines as shown in Fig. 3.23(a). But the graph for relative displacement $(x - x_0) = x\,(t)$ with time, t will be straight line PQ parallel to time axis as shown in Fig. 3.23(b)

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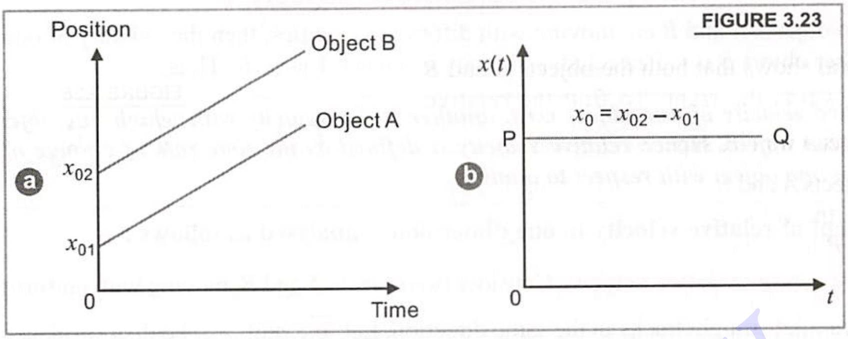
FIGURE 3.23



(*ii*) **If $v_2 > v_1$ *i.e.*, $(v_2 - v_1)$ is positive**, then from (14), we note that $(x - x_0)$ is positive. It means the relative separation between two objects will increase by an amount $(v_2 - v_1)$ after each unit of time. Therefore, their position–time graphs will open out gradually as shown in Fig. 3.24.

FIGURE 3.24

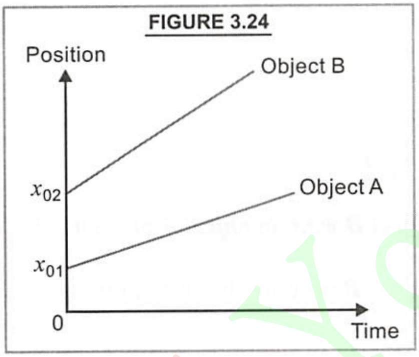
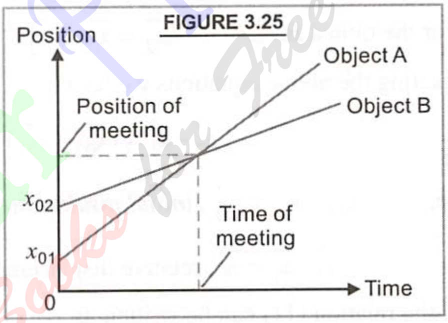


FIGURE 3.25



(*iii*) **If $v_1 > v_2$, *i.e.*, $(v_2 - v_1)$ is negative**, then from (14), we note that $(x - x_0)$ is negative. It means the separation between the two objects will go on decreasing by the amount $v_1 - v_2$ after each unit of time. After some time, the two objects will meet and then the object *B*, which was to the right of *A* will get more and more to the left of *A*. The position-time graphs of this motion will be as shown in Fig. 3.25. The time coordinates corresponding to point of intersection gives their ***time of meeting*** and the corresponding position coordinates gives the ***position of meeting.***

(c) Determination of relative velocity

For the determination of the relative velocity of body *A* with respect to body *B*, we follow the following procedure :

1. Bring the body *B* at rest by imposing equal and opposite velocity of *B* on it.

2. Impose the equal and opposite velocity of *B* on body *A* also. Now the body *A* will have two velocities (*i*) its own velocity $\vec{v}_A$ and (*ii*) equal and opposite velocity of *B*, *i.e.*, $(-\vec{v}_B)$.

3. Find the resultant velocity of $\vec{v}_A$ and $-\vec{v}_B$. It will give us the relative velocity of *A* w.r.t. *B*, *i.e.*,

$$\vec{v}_{AB} = \vec{v}_A + (-\vec{v}_B).$$

To understand it, we consider the following examples. Let $\vec{v}_A$ and $\vec{v}_B$ be the uniform velocities of the objects *A* and *B*, where $v_A > v_B$.

(i) When the two objects are moving along parallel straight lines in the same direction i.e., angle between them is 0°

FIGURE 3.26

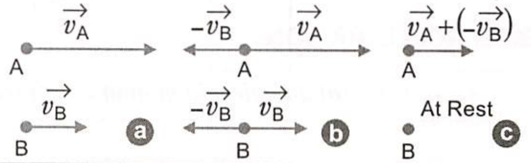


Fig. 3.26(a) shows that both the objects *A* and *B* are moving towards the right. To find the relative velocity of object *A* w.r.t. *B*, superimpose velocity $-\vec{v}_B$ on both the objects *A* and *B*, Fig. 3.26(b). Due to which in Fig. 3.26(c), the velocity of object *B* becomes zero, *i.e.*, the object *B* is brought to rest and the velocity of object *A* becomes $\vec{v}_A+(-\vec{v}_B)=\vec{v}_A-\vec{v}_B$. Hence, relative velocity of object *A* w.r.t. *B* is given by

$$\vec{v}_{AB}=\vec{v}_A-\vec{v}_B$$

Since v_{AB}, v_A and v_B all are in the same direction, we can write

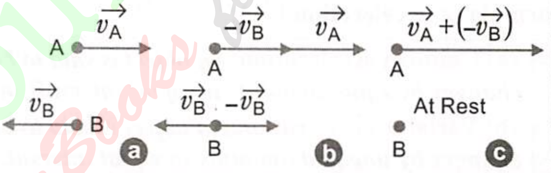
$$v_{AB}=v_A-v_B$$

If object *A* is a car moving with velocity 50 km/h due east and object *B* is a train moving with velocity 30 km/h due east, then relative velocity of car w.r.t. train = 50 – 30 = **20 km/h due east.**

Thus, if two objects are moving in the same direction, the magnitude of relative velocity of one object w.r.t. another is equal to difference in magnitude of two velocities.

(ii) When the two objects are moving along parallel straight lines in opposite directions, *i.e.*, angle between them is 180°. Fig. 3.27(a) shows that the object *A* is moving towards the right and object *B* is moving towards the left. To find the relative velocity of object *A* *w.r.t.* *B*, superimpose velocity $-\vec{v}_B$ on both the objects *A* and *B*, Fig. 3.27(b). Due to which, in Fig. 3.27(c), the velocity of object *B* becomes zero *i.e.*, the object *B* is brought to rest and the velocity of object *A* becomes $=\vec{v}_A+(-\vec{v}_B)=\vec{v}_A-\vec{v}_B$. Therefore, the relative velocity of object *A* w.r.t. object *B* is given by

FIGURE 3.27



$$\vec{v}_{AB}=\vec{v}_A-\vec{v}_B$$

Since the direction of $\vec{v}_B$ is opposite to that of $\vec{v}_A$ then magnitude of $\vec{v}_{AB}$, will be $v_{AB}=v_A+v_B$

If object *A* is a car moving with velocity 50 km/h due east and object *B* is a train moving with velocity 30 km/h due west, then velocity of *B* is,

$$\vec{v}_B = 30 \text{ km/h west} = -30 \text{ km/h due east.}$$

The relative velocity of car w.r.t. train is

$$v_{AB}=50-(-30)=80 \text{ km/h due east.}$$

Thus, if *two objects are moving in opposite directions, the magnitude of relative velocity of one object w.r.t. other is equal to the sum of the magnitude of their velocities.*

3.21. ACCELERATED MOTION

In non-uniform motion of an object, the velocity of the object is different at different instants, *i.e.*, the velocity of the object keeps on changing with time. Such a motion is said to be an **accelerated motion.** For example, *when a vehicle moves from one position to another on a road, with different velocities at different instants, its motion is said to be* **accelerated motion.**

In an accelerated motion, if the change in velocity of an object in each unit of time is constant, the obeject is said to be moving with constant acceleration and such a motion is called **uniformly accelerated motion.** On the other hand, if the change in velocity in each unit of time is not constant, the object is said to be moving with variable acceleration and such a motion is called **non-uniformly accelerated motion.**

3.22. ACCELERATION

Acceleration of an object in motion is defined as the rate of change in velocity of the object, i.e.,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

Acceleration is a vector quantity. It be positive, negative or zero. *e.g.*,

(*i*) If the acceleration is acting in the direction of motion of a body, *i.e.*, the velocity of a body is increasing with time, then the **acceleration of a body is positive.**

(*ii*) If the acceleration is acting opposite to the direction of motion of a body, *i.e.*, the velocity of a body is decreasing with time, then the **acceleration of a body is negative.**

(*iii*) If the body is moving with a constant velocity, then the **acceleration of a body is zero.**

IMPORTANT NOTE

Negative acceleration is also called **retardation** *or* **deceleration.**

The **unit of acceleration** is ms^{-2} in mks system or SI and cm s^{-2} in cgs system. The **dimensional formula of acceleration** is $[M^0L^1T^{-2}]$.

(a) Uniform acceleration. *An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time, howsoever small the intervals may be.*

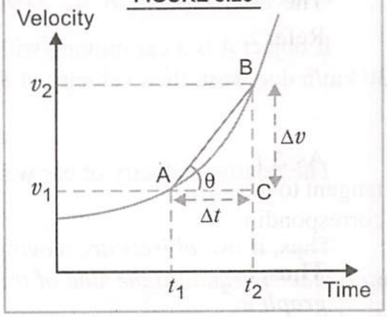
(b) Variable acceleration. *An object is said to be moving with a variable acceleration if its velocity changes by unequal amounts in equal intervals of time, howsoever small the intervals may be.*

(c) Average acceleration. **When an object is moving with a variable acceleration, then** *the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken, i.e.,*

$$\text{Average acceleration, } \vec{a}_{av} = \frac{\text{total change in velocity}}{\text{total time taken}} = \frac{\Delta \vec{v}}{\Delta t}$$

Consider an object moving along a straight line with a variable acceleration. Let the velocity-time graph of this motion be represented by curve as shown in Fig. 3.28. We take two points A and B on this curve. The point A corresponds to time t_1 and velocity v_1. The point B corresponds to time t_2 and velocity v_2. Change in velocity in time interval $(t_2 - t_1) = v_2 - v_1$.

FIGURE 3.28



$$\text{Slope of straight line } AB = \tan\theta = \frac{BC}{AC}$$

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} = a_{av} \text{ or } \bar{a}$$

where a_{av} or $\bar{a}$ is the magnitude of average acceleration.

Thus, the slope of straight line joining two points on velocity-time graph with time axis gives the average acceleration of the object between those two points.

The average acceleration can be positive or negative depending upon the sign of slope of velocity-time graph. It is zero if the change in velocity of the object in the given interval of time is zero.

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(d) Instantaneous acceleration. When an object is moving with variable acceleration, then the object possesses different accelerations at different instants.

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration.

Consider an object moving with non-uniform acceleration along a straight line. Let $\vec{v}$ and $\vec{v} + \Delta \vec{v}$ be the velocities of the object at instants of time t and $t + \Delta t$, where Δt is very small time interval. The acceleration of the object at an instant t, *i.e.*, instantaneous acceleration is nearly equal to average acceleration in small time interval Δt, when $\Delta t \to 0$, because in this time-interval the acceleration of the object can be considered to be uniform, even though the object has non-uniform acceleration. Therefore, instantaneous acceleration 'a_i' will be approximately equal to average acceleration in small time Δt and is, given by

$$\vec{a}_i \simeq \frac{\Delta \vec{v}}{\Delta t}$$

The sense of approximation in the above expression can be removed by making the time interval Δt as small as possible, tending to zero. Thus instantaneous acceleration, $\vec{a}_i = \underset{\Delta t \to 0}{\text{Limit}} \left(\frac{\Delta \vec{v}}{\Delta t} \right)$

Hence, ***the instantaneous acceleration of an object at a given instant is defined as the limiting value of the average acceleration in a small time interval around the given instant, when the time interval tends to zero.***

$\therefore$ Instantaneous acceleration, $\vec{a}_i = \underset{\Delta t \to 0}{\text{Limit}} \left(\frac{\Delta \vec{v}}{\Delta t} \right) = \frac{d\vec{v}}{dt}$

As, $\vec{v} = \frac{d\vec{x}}{dt}$, therefore, $\vec{a}_i = \frac{d}{dt}\left(\frac{d\vec{x}}{dt} \right) = \frac{d^2 \vec{x}}{dt^2}$

Thus, ***the instantaneous acceleration of an object (i) is equal to the first time derivative of velocity at the given instant or (ii) is equal to the second time derivative of position of the object at the given instant.***

The value of instantaneous acceleration can be obtained graphically as explained below.

Refer Fig. 3.1, the average acceleration of the object in time interval $(t_2 - t_1)$ is

$$a_{av} = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta v}{\Delta t} = \text{slope of straight line } AB$$

As Δt approaches zero, the point B approaches point A. In this situation, the line AB would become tangent to the curve at point A. The slope of this tangent would give us the value of instantaneous acceleration corresponding to point A.

Thus, ***instantaneous acceleration is also defined as the slope of the tangent to the velocity-time graph at a position, corresponding to given instant of time.***

IMPORTANT NOTE

If an object is moving with uniform acceleration, then, instantaneous acceleration = uniform acceleration = average acceleration.

Retain in Memory

1. If a body is moving with a uniform acceleration, then its average velocity is
$$v_{av} = \frac{u+v}{2}$$
where u is the initial velocity and v is the final velocity of the body.
2. Average velocity of a body can be positive or negative but average speed is always positive.
3. For a uniform motion along a straight line magnitude of average velocity is equal to its average speed.
4. An object speeds up even if its acceleration and velocity are negative. It is so in case of a falling body, where upward motion is positive and downward motion is negative.

Sample Problem **The velocity of a particle is given by $v = (3\,t^2 + 6)$ cm s^{-1}. Find (*i*) the change in velocity of the particle during the time interval between $t_1 = 2$ s and $t_2 = 4$ s (*ii*) the average acceleration during the same interval and (*iii*) instantaneous acceleration at $t_2 = 4$ s.**

Sol. Given, $v = 3\,t^2 + 6$

(*i*) When $t_2 = 2$ s, $v_1 = 3 \times 2^2 + 6 = 18$ cm s^{-1} When $t_2 = 4$ s, $v_2 = 3 \times 4^2 + 6 = 54$ cm s^{-1}

Change in velocity $= v_2 - v_1 = 54 - 18 = \mathbf{36}$ **cm s^{-1}**

(*ii*) Average acceleration, $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{36}{4-2} = \mathbf{18}$ **cm s^{-2}**

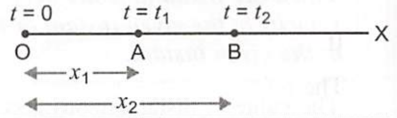
(*iii*) Instantaneous acceleration, $a_i = \dfrac{dv}{dt} = \dfrac{d}{dt}(3t^2+6) = 6t$

At $t = 4$ s, $a_i = 6 \times 4 = \mathbf{24}$ **ms^{-2}**

3.23. FORMULAE FOR UNIFORMLY ACCELERATED MOTION ALONG A STRAIGHT LINE

Consider an object, moving with a uniform acceleration 'a' along a straight line OX, with origin at O. Let the object reach at points A and B at instants t_1 and t_2. Let x_1 and x_2 be the displacements of the object at times t_1 and t_2 respectively and v_1 and v_2 be the velocities of the object at positions A and B respectively, Fig. 3.29.

FIGURE 3.29



(i) Velocity-time relation

Acceleration of the object $= \dfrac{\text{change in velocity}}{\text{time taken}}$

$$\therefore \qquad a = \frac{v_2 - v_1}{t_2 - t_1} \qquad \text{...(15)}$$

or $v_2 - v_1 = a\,(t_2 - t_1)$ or $\boxed{v_2 = v_1 + a\,(t_2 - t_1)}$ **...(16)**

Let the origin of time axis be taken at A and u be the velocity of the object at A. v be the velocity of the object at B after time t. Then $v_1 = u\,;\, t_1 = 0\,;\, v_2 = v$ and $t_2 = t$.

Putting values in (16), we get $\boxed{v = u + a\,t}$ **...(17)**

The relations (16) and (17) are the required velocity-time relations.

(ii) Position-time relation

let v_{av} be average velocity of the object during the motion of object from A to B,

Then, $v_{av} = \dfrac{\text{displacement}}{\text{time taken}} = \dfrac{x_2 - x_1}{t_2 - t_1}$ or $x_2 - x_1 = v_{av}\,(t_2 - t_1)$

But, $v_{av} = \dfrac{\text{initial velocity} + \text{final velocity}}{2} = \dfrac{v_1 + v_2}{2}$

$$\therefore \qquad x_2 - x_1 = \left(\frac{v_1 + v_2}{2}\right)(t_2 - t_1) \qquad \text{...(18)}$$

or
$$x_2 = x_1 + \left[\frac{v_1 + v_1 + a\,(t_2 - t_1)}{2}\right](t_2 - t_1) \qquad \text{[from (16)]}$$

$$= x_1 + v_1\,(t_2 - t_1) + \frac{1}{2}\,a\,(t_2 - t_1)^2 \qquad \text{...(19)}$$

If x_0, u are the displacement and velocity of the object at $t_1 = 0$ and x be the displacement of object at time t, then using $t_1 = 0$, $x_1 = x_0$, $v_1 = u$, $t_2 = t$ and $x_2 = x$, in relation (5) we have,

$$x = x_0 + u\,(t - 0) + \frac{1}{2}\,a\,(t - 0)^2 \quad \text{or} \quad \boxed{x = x_0 + u\,t + \frac{1}{2}\,a\,t^2} \qquad \text{...(20)}$$

or
$$x - x_0 = ut + \frac{1}{2}at^2 \qquad \text{...(21)}$$

Using, $x - x_0 = S$ = distance travelled in time t, we have $\boxed{S = ut + \frac{1}{2}at^2}$ ...(22)

The relations (19), (20), (21) and (22) are the required position-time relations

(iii) Position-velocity relation

From (16), we have $(t_2 - t_1) = \dfrac{v_2 - v_1}{a}$

Putting this value in (18) we get

$$x_2 - x_1 = \left(\frac{v_2 + v_1}{2}\right)\left(\frac{v_2 - v_1}{a}\right) = \frac{v_2^2 - v_1^2}{2a} \quad \text{or} \quad \boxed{v_2^2 - v_1^2 = 2a\,(x_2 - x_1)} \qquad \text{...(23)}$$

If u and v are the velocities of an object at positions x_0 and x respectively, then using $v_1 = u$, $v_2 = v$, $x_1 = x_0$ and $x_2 = x$ in (23), we get,

$$v^2 - u^2 = 2a\,(x_0 - x) \qquad \text{...(24)}$$

If $\quad x_0 - x = S$, then $\boxed{v^2 - u^2 = 2\,\text{aS}}$ ...(25)

The relations (23), (24) and (25) are the required position–velocity relations.

Retain in Memory

If a body is moving with non-uniform acceleration such that its acceleration is $\vec{a}_1$ in time t_1 and $\vec{a}_2$ in time t_2, then the average acceleration is given by

$$\vec{a}_{av} = \frac{\vec{a}_1\,t_1 + \vec{a}_2\,t_2}{t_1 + t_2}$$

Sample Problem **A particle is moving with a uniform acceleration 4 ms^{-2} for time 2 seconds and then 5 ms^{-2} for time 3 seconds. What is the average acceleration of the particle during motion.**

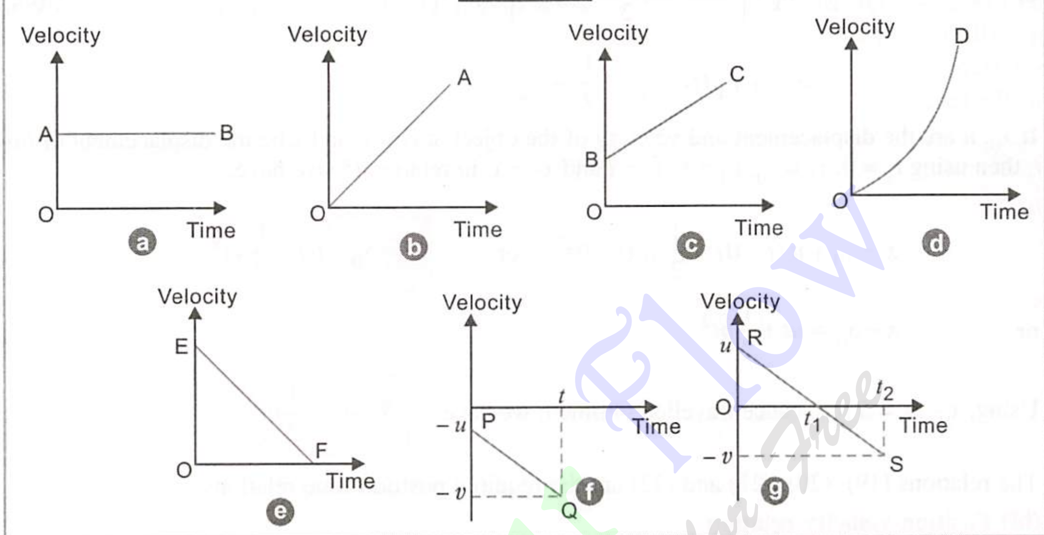
Sol. Here, $a_1 = 4\text{ ms}^{-2}$; $t_1 = 2$ s ; $a_2 = 5\text{ ms}^{-2}$; $t_2 = 3$ s

$$a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2} = \frac{4\times2 + 5\times3}{2+3} = \frac{23}{5} = \mathbf{4{\cdot}6\ ms^{-2}}$$

3.24. VELOCITY-TIME GRAPH OF AN ACCELERATED MOTION

(*i*) **When an object is moving with zero acceleration, *i.e.*, a constant velocity,** the velocity-time graph is a straight line *AB* parallel to time axis. Fig. 3.30(a).

FIGURE 3.30



(*ii*) **When an object is moving with constant positive acceleration, having zero initial velocity,** then the velocity-time graph is a straight line *OA*. Here, slope of graph is less than 90°, Fig. 3.30(b).

(*iii*) **When an object is moving with positive constant acceleration having some initial velocity,** then the velocity-time graph is represented by straight line *BC*, Fig. 3.30(c).

(*iv*) **When an object is moving with increasing acceleration, having zero initial velocity,** the velocity-time graph is a curve *OD* as shown in Fig. 3.30(d). ***The slope of the velocity-time graph at the given instant of time gives the instantaneous acceleration at that time.***

(*v*) **When an object is moving with constant negative acceleration, having positive initial velocity,** then velocity-time graph is a straight line EF as shown in Fig. 3.30(e). Here, the slope of graph is more than 90°.

(*vi*) **When an object is moving with uniform negative acceleration having negative initial velocity,** the velocity-time graph is a straight line, *PQ* Fig. 3.30(f).

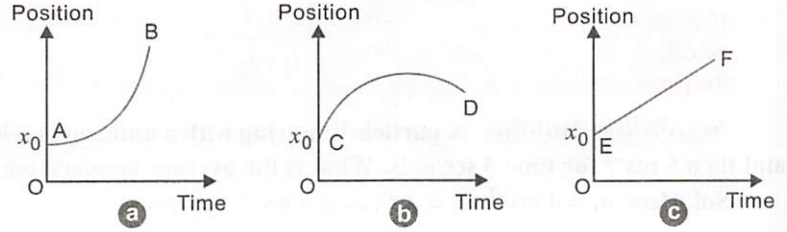
(*vii*) **When an object is moving with uniform negative acceleration having positive initial velocity,** changes direction of motion at time t_1, but continuously moving with uniform acceleration, the velocity-time graph is a straight line, *RS* Fig. 3.30(g).

3.25. POSITION-TIME GRAPH FOR ACCELERATED MOTION

(*i*) **When an object is moving with uniform positive acceleration,** the position-time graph is a curve *AB* as shown in Fig. 3.31(a).

(*ii*) **When an object is moving with negative acceleration,** then position-time graph is a curve *CD* as shown in Fig. 3.31(b).

FIGURE 3.31



It is to be noted that the slope of position-time graph at any instant of time gives the instantaneous velocity of the object at the given instant of time.

(*iii*) **When object is moving with zero acceleration,** the position-time graph is a straight line *EF*, Fig. 3.4(c).

Sample Problem **A particle starts with an initial velocity $3\cdot0\ \text{ms}^{-1}$ along the positive x-direction and it accelerates uniformly at the rate of $0\cdot60\ \text{ms}^{-2}$. (a) Find the distance travelled by it in the first three seconds. (b) How much time does it take to reach the velocity 9·0 m/s ? (c) How much distance will it cover in reaching the velocity $9\cdot0\ \text{ms}^{-1}$?**

Sol. Here, $u = 3\cdot0\ \text{ms}^{-1}$; $a = 0\cdot60\ \text{ms}^{-2}$

(*a*) We have to calculate S when $t = 3$ s

$$S = ut + \frac{1}{2}at^2 = 3\cdot0 \times 3 + \frac{1}{2} \times 0\cdot6 \times 3^2 = 9\cdot0 + 2\cdot7 = \mathbf{11\cdot7\ m}$$

(*b*) We have to calculate t when $v = 9\cdot0$ m/s. As $v = u + at$, so $t = (v - u)/a = (9 - 3)/0\cdot6 = \mathbf{10\ s}$

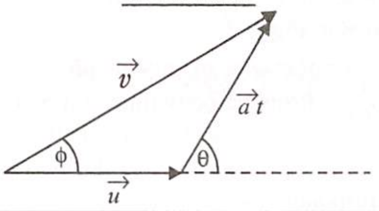
(*c*) Here, $S = ?$, $v = 9\cdot0\ \text{ms}^{-2}$; As $v^2 = u^2 + 2\,aS$

so $$S = \frac{(v^2 - u^2)}{2a} = \frac{(9\cdot0)^2 - (3\cdot0)^2}{2 \times 0\cdot6} = \frac{81 - 9}{1\cdot2} = \frac{72}{1\cdot2} = \mathbf{60\ m}$$

DO YOU KNOW ?

1. The equations of motion, *i.e.*, $v = u + at$; $S = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2\,aS$; can be applied only when acceleration of the body is uniform.
2. The equations of motion given above can not be applied to circular motion or simple harmonic motion.
3. When acceleration of a body is not constant, integration of acceleration will give us velocity and integration of velocity will give us displacement.
4. The first and second equations of motion are vector equations, *i.e.*,
$$\vec{v} = \vec{u} + \vec{a}\,t \qquad \text{...}(i)$$
$$\vec{S} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2 \qquad \text{...}(ii)$$
while third equation of motion is a scalar equation, *i.e.*, $\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\,\vec{a}\cdot\vec{S}$ or $v^2 = u^2 + 2\,\vec{a}\cdot\vec{S}$
5. If the velocity and acceleration are collinear, conventionally we can take, the direction of motion to be positive and so the variables (*i.e.*, the various physical quantities) in the direction of motion are taken to be positive while opposite to it are negative.
For example, If a body is moving with constant acceleration along the direction of motion, then acceleration '*a*' is positive. The equations of motion will be $v = u + at$; $S = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2\,aS$.
If a body is moving with constant acceleration opposite to the direction of motion, the equations of motion will be, $v = u - at$; $S = ut - \frac{1}{2}at^2$; $v^2 = u^2 - 2\,aS$.
6. If acceleration and velocity are not collinear, then v and S can be calculated using the (*i*) and (*ii*) respectively and applying the parallelogram law of vector addition. According to which
$$v = [u^2 + (at)^2 + 2\,u\,at\cos\theta]^{1/2}$$
and $$\tan\phi = \frac{at\sin\theta}{u + at\cos\theta}$$
where θ is angle between $\vec{u}$ and $\vec{a}\,t$, Fig. 3.5.
For detail of parallelogram law of vectors, refer to Unit 2(c).

FIGURE 3.32



3.26. DISTANCE TRAVELLED BY UNIFORMLY ACCELERATED BODY IN *n*th SECOND

Distance travelled by a uniformly accelerated body in *n*th second can be obtained by subtracting the distance travelled by the body in $(n-1)$ second from the distance travelled by the body in *n* seconds.

If, u = initial velocity of the body, a = uniform acceleration of the body,

S_n, S_{n-1} = distances travelled by the body in *n* seconds and $(n-1)$ seconds respectively,

D_n = distance travelled in *n*th second.

Then, $$D_n = S_n - S_{n-1} \qquad \text{...(26)}$$

We know that the distance travelled in *t* seconds is given by ; $S = ut + \frac{1}{2}at^2$

∴ Distance travelled in *n* seconds is, $$S_n = un + \frac{1}{2}an^2$$

Distance travelled in $(n-1)$ seconds is, $$S_{n-1} = u\,(n-1) + \frac{1}{2}a\,(n-1)^2$$

Putting these values in (26), we get

$$D_n = [un + \frac{1}{2}an^2] - [u\,(n-1) + \frac{1}{2}a\,(n-1)^2] = un + \frac{1}{2}a\,n^2 - un + u - \frac{1}{2}a\,n^2 + a\,n - \frac{a}{2}$$

$$= u + a\,n - \frac{a}{2} = u + a\,(n - \frac{1}{2}) \quad \text{or} \quad \boxed{D_n = u + \frac{a}{2}\,(2n-1)} \qquad \text{...(27)}$$

The relation (27) is the required relation.

IMPORTANT NOTE

The dimensions of D_n are the same as that of velocity *i.e.* $[M^0 LT^{-1}]$

Sample Problem **A particle having initial velocity 5 m/s moves with a constant acceleration 2 ms^{-2}, for a time 10 second along a straight line. Find the displacement of the particle in the last one second and the total distance travelled in 10 seconds.**

Sol. Here, $u = 5\ \text{ms}^{-1}$; $a = 2\ \text{ms}^{-2}$; $t = 10$ s ; $D_{10} = ?$; $S = ?$

As, $D_n = u + \frac{a}{2}\,(2n-1)$ so, $D_{10} = 5 + \frac{2}{2}\,(2\times10-1) = 5 + 19 = \mathbf{24\ m}$

Now, $S = ut + \frac{1}{2}at^2 = 5\times10 + \frac{1}{2}\times2\times10^2 = 50 + 100 = \mathbf{150\ m}$

3.27. VELOCITY-TIME GRAPH OF A UNIFORMLY ACCELERATED MOTION

Consider an object moving with a uniform acceleration '*a*' along a straight line. Let *u* be its initial velocity of the object. The velocity-time graph of this motion is a straight line inclined to time axis as shown in Fig. 3.33.

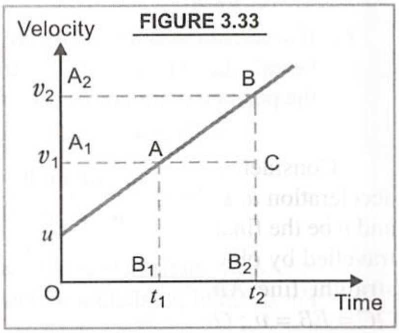


FIGURE 3.33

Importance of this graph :

(*a*) From velocity time graph of an object, we can calculate the velocity of the object at a particular instant of time.

(*b*) **Acceleration :** Take two points *A* and *B* on this graph corresponding to times t_1 and t_2. Let v_1 and v_2 be the corresponding velocities of the object at times t_1 and t_2 respectively. Then,

$v_1 = OA_1$; $v_2 = OA_2$; $t_1 = OB_1$ and $t_2 = OB_2$

Acceleration, $$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{OA_2 - OA_1}{OB_2 - OB_1} = \frac{A_1A_2}{B_1B_2} = \frac{CB}{AC} = \text{slope of st. line } AB.$$

Thus, acceleration of an object in a uniformly accelerated motion in one dimension is equal to the slope of the velocity- time graph with time axis.

(*c*) **Distance travelled in a given time interval**

Distance travelled = average velocity × time interval = $\left(\frac{v_1 + v_2}{2}\right) \times (t_2 - t_1)$

$$= \frac{1}{2}\,(OA_1 + OA_2) \times B_1B_2 = \text{area of trapezium } ABB_2B_1.$$

Thus, the distance travelled by an object in a given time interval of a uniformly accelerated motion is equal to the area under velocity-time graph with time axis for the given interval of time.

Note : The total displacement of the body in the given time is equal to the area which velocity-time graph encloses with time axis. The area to be added with proper sign, i.e., the area above time axis is positive and below the time axis is negative.

Sample Problem **The velocity-time graph for linear motion is shown in Fig. 3.34. Find (a) the distance travelled and (b) displacement, between 5 and 40 s. (c) Acceleration between 15 to 25 seconds.**

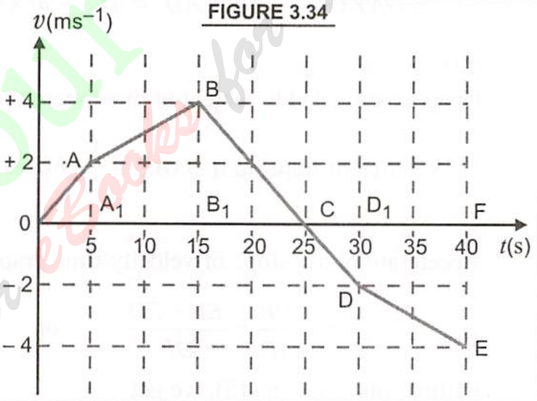
Sol. (*a*) Distance travelled by body in time 5 to 15 seconds = Area $A_1ABB_1 = \frac{(2+4)\times 10}{2} = 30$ m

Distance travelled by body in time 15 to 25 seconds = Area $BB_1C = \frac{4\times 10}{2} = 20$ m

Distance travelled by body in time 25 to 30 seconds = Area $CDD_1 = \frac{5\times 2}{2} = 5$ m

Distance travelled by body in time 30 to 40 seconds = Area $D_1DEF = \frac{(2+4)\times 10}{2} = 30$ m

FIGURE 3.34



∴ Total distance travelled between 5 to 40 seconds = 30 + 20 + 5 + 30 = **85 m**

(*b*) Here, the area CDD_1 and area D_1DEF being below the time axis will show negative displacement. Hence, total displacement between 5 to 40 seconds = 30 + 20 − 5 − 30 = **15 m**

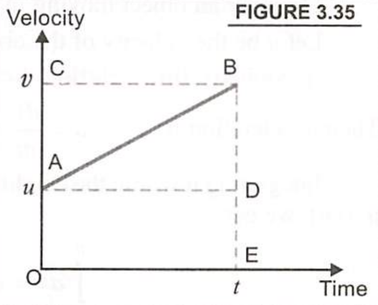
(*c*) Acceleration of the body in time interval 15 s to 25 s is the slope of the line *BC*

$$= \frac{-BB_1}{CB_1} = \frac{-4}{(25-15)} = \frac{-4}{10} = \mathbf{-0{\cdot}4\ m/s^2}$$

3.28. DERIVATION OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION FROM VELOCITY–TIME GRAPH

Consider an object moving along a straight line with uniform acceleration *a*. Let *u* be the initial velocity of the object at time $t = 0$ and v be the final velocity of the object at time *t*. Let *S* be the distance travelled by object in time *t*. Velocity-time graph of this motion is a straight line *AB*, as shown in Fig. 3.35. Where $OA = ED = u$; $OC = EB = v$; $OE = AD = t$.

FIGURE 3.35



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(i) $v = u + at$

We know that ***slope of velocity-time graph of uniformly accelerated motion represents the acceleration of the object.***

i.e., Acceleration = slope of the velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{DB}{OE} = \frac{EB - ED}{OE} = \frac{v - u}{t} \quad \text{or} \quad v - u = at \quad \text{or} \quad \boxed{v = u + at}$$

(ii) $S = ut + \frac{1}{2}at^2$

We know that the ***area under the velocity-time graph for a given time interval represents the distance covered by a uniformly accelerated object in that time interval.***

From graph, acceleration, a = slope of velocity-time graph AB

$$\therefore \quad a = \frac{DB}{AD} = \frac{DB}{t} \quad \text{or} \quad DB = a\,t$$

Now distance travelled by object in time t is

S = area of trapezium $OABE$ = area of rectangle $OADE$ + Area of triangle ADB

$$= OA \times OE + \frac{1}{2} DB \times AD \; = u\,t + \frac{1}{2} at \times t = u\,t + \frac{1}{2} a\,t^2 \text{. Thus,} \qquad \boxed{S = ut + \frac{1}{2}at^2}$$

(iii) $v^2 - u^2 = 2\,a\,S$

Distance travelled by object in time interval t is

$$S = \text{area of trapezium } OABE = \frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + ED) \times OE \qquad \text{...(28)}$$

$$(\because OA = ED)$$

Acceleration, a = slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{EB - ED}{OE} \quad \text{or} \quad OE = \frac{EB - ED}{a}$$

Putting this value in (15), we get

$$S = \frac{1}{2}(EB + ED) \times \frac{(EB - ED)}{a} = \frac{1}{2a}(EB^2 - ED^2) = \frac{1}{2a}(v^2 - u^2)$$

or $$\boxed{v^2 - u^2 = 2\,aS}$$

3.29. EQUATIONS OF UNIFORMLY ACCELERATED MOTION BY CALCULUS METHOD

Consider an object moving in a straight-line with uniform acceleration a.

Let u be the velocity of the object at time $t = 0$, and v be the velocity of the object at time t.

(i) Velocity-time relation. Let at an instant of time t, dv be the change in velocity in time interval dt.

Then acceleration is, $$a = \frac{dv}{dt} \quad \text{or} \quad dv = a\,dt$$

Integrating it within the conditions of motion (*i.e.*, when time changes from 0 to t, velocity changes from u to v), we get

$$\int_u^v dv = a \int_0^t dt \quad \text{or} \quad v - u = at \quad \text{or} \quad \boxed{v = u + at} \qquad \text{...(29)}$$

(ii) Distance-time relation. Consider an object moving in a straight line with uniform acceleration 'a'. Let at an instant t, dx be the displacement of the object in time interval dt. Then, its instantaneous velocity v is given by

$$v = \frac{dx}{dt} \quad \text{or} \quad dx = v\,dt \quad \text{or} \quad dx = (u + at)\,dt \qquad (\because\ v = u + at)$$

Let x_0 displacement of the object at $t = 0$ and x be the displacement of the object at time t. Integrating the above relation within the conditions of motion (*i.e.*, when time changes from 0 to t, displacement changes from x_0 to x), we get

$$\int_{x_0}^{x} dx = \int_{o}^{t} (u + at)\,dt = u\int_0^t dt + a\int_0^t t\,dt \quad \text{or} \quad (x)_{x_0}^{x} = u\,(t)_0^t + a\left(\frac{t^2}{2}\right)_0^t$$

$$\text{or} \quad x - x_0 = ut + \frac{1}{2}at^2 \quad \text{or} \quad \boxed{x = x_0 + ut + \frac{1}{2}at^2} \qquad \text{...(30)}$$

If $x - x_0 = S$ = the distance covered by object in time t, then from (30),

$$\boxed{S = ut + \frac{1}{2}at^2} \qquad \text{...(31)}$$

(iii) Velocity-displacement relation

Consider an object moving in a straight line with initial velocity u and uniform acceleration a.

$$\text{Then,} \quad a = \frac{dv}{dt} = \frac{dv}{dx}\times\frac{dx}{dt} = \frac{dv}{dx}\times v \qquad \text{or} \qquad a\,dx = v\,dv$$

Let u, v be the velocity of the object at positions given by displacements x_0 and x. Integrating the above expression within the conditions of motion, we have,

$$\int_{x_0}^{x} a\,dx = \int_u^v v\,dv \quad \text{or} \quad a\,(x)_{x_0}^{x} = \left(\frac{v^2}{2}\right)_u^v \quad \text{or} \quad a\,(x - x_0) = \frac{v^2}{2} - \frac{u^2}{2} \quad \text{or} \quad v^2 - u^2 = 2a\,(x - x_0)$$

Putting $x - x_0 = S$, we get, $\boxed{v^2 - u^2 = 2\,aS}$ ...(32)

which is the required equation.

(iv) Distance travelled by an object in nth second

Let A and B be two points on the velocity time graph as shown in Fig. 3.36, corresponding to time $(n - 1)$ and n seconds respectively. Let v_{n-1} and v_n be the velocity of the object at A and B respectively.

Distance travelled in nth second is

D_n = distance travelled in n seconds

– distance travelled in $(n - 1)$ seconds

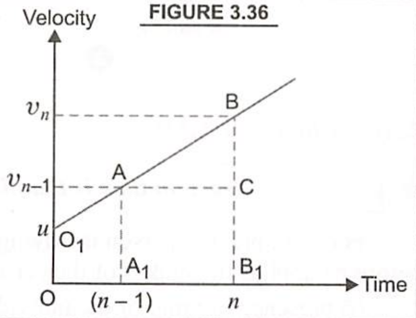
= area OO_1ABB_1 – area OO_1AA_1

= area of trapezium ABB_1A_1

$$= \frac{1}{2}[AA_1 + BB_1]\times A_1B_1 = \frac{1}{2}\,[v_{n-1} + v_n]\times[n - (n-1)]$$

$$= \frac{1}{2}\,[v_{n-1} + v_n]\times 1$$

FIGURE 3.36



The velocity of object at time t is $v = u + at$

$\therefore v_n = u + an$ and $v_{n-1} = u + a\,(n-1)$

$$D_n = \frac{1}{2}\{[u + a\,(n-1) + (u + an)]\} = \frac{1}{2}[2u + 2an - a] = \frac{1}{2}[2u + a\,(2n-1)] = u + \frac{a}{2}[2n-1]$$

i.e., $\boxed{D_n = u + \frac{a}{2}(2n-1)}$

3.30. MOTION OF AN OBJECT UNDER FREE FALL

If an object is released from a height near the surface of earth, it is accelerated downwards under the influence of gravity with acceleration equal to acceleration due to gravity, g. If the air resistance is neglected, the object is said to be in free fall. If the height through which the object falls is very small compared to the radius of the earth, then g can be taken to be constant. The value of g near the surface of earth is generally taken equal to $9{\cdot}8\ \text{ms}^{-2}$. It means, **free fall is a case of motion with uniform acceleration.**

In case of free fall (*i*) if downward direction is taken positive, and the object is released from rest then $u = 0$, $a = g = 9{\cdot}8\ \text{m/s}^2$, $t = t$, $S = s$. Then equations of motion become

$$v = u + at = 0 + gt = gt$$

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2\,aS = 0 + 2\,gs = 2\,gs$$

In case of free fall, if downward direction is taken negative, and the object is released from rest then, $u = 0$; $a = -g = -9{\cdot}8\ \text{m/s}^2$, $t = t$ and $S = s$.

The equations of motion become

$$v = u + at = 0 - gt = -gt = -9{\cdot}8\,t \quad \text{...(33)}$$

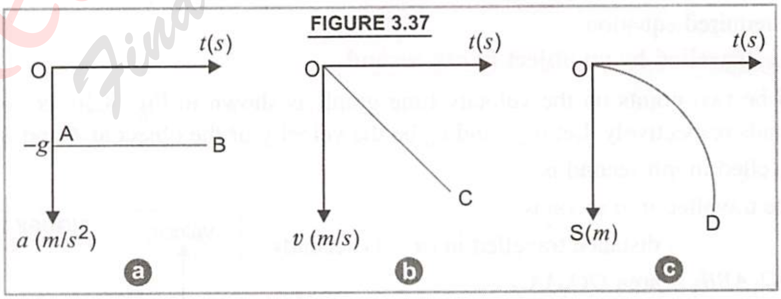
$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(-g)\,t^2 = -\frac{1}{2}gt^2 = -4{\cdot}9\,t^2 \quad \text{...(34)}$$

$$v^2 = u^2 + 2\,aS = 0 + 2\,(-g)\,s = -2\,gs = -2 \times 9{\cdot}8\,s = -19{\cdot}6\,s \quad \text{...(35)}$$

The variation of acceleration (a) with time (t) for free fall is shown in Fig. 3.37(a).

The variation of velocity (v) with time (t) for free fall is shown in Fig. 3.37(b) and the variation of distance (S) with time (t) for free fall is shown in Fig. 3.37(c).

FIGURE 3.37



3.31. REACTION-TIME

Reaction time is that time which a person takes to observe, think and act.

For example, if a person is driving a car and suddenly a boy appears on the road, then the time elapsed before he applies the brakes of the car is the reaction time. Reaction time depends on

(*i*) presence of mind of the individual and (*ii*) the complexity of the situation.

One can measure his reaction time by the following experiment : Your friend is holding a long rod vertically in the gap between the thumb and fore-finger of your right hand. He is to drop that rod and you are to catch hold the falling rod. You will see that the rod is caught by you in a small time later than it is dropped by your friend. After you catch it, note the distance *d* travelled by the rod. Let this distance *d* be 0·21 m. If *t* is the reaction time, then $u = 0$, $a = 9{\cdot}8\ \text{m/s}^2$, $S = 0{\cdot}21$ m, $t = ?$

Using the relation, $S = ut + \frac{1}{2}at^2$

$$\therefore \quad 0{\cdot}21 = 0 + \frac{1}{2} \times 9{\cdot}8\, t^2 \quad \text{or} \quad t = \sqrt{\frac{2 \times 0{\cdot}21}{9{\cdot}8}} \approx \mathbf{0{\cdot}2\ s}$$

3.32. HANG-TIME

It is that time for which a person while jumping hangs in the air in defiance of gravity. The value of hang-time of a jumper is maximum when he jumps vertically upwards with maximum initial velocity. It is always found to be less than one second.

The value of hang-time of a person depends on (*i*) the strength of the muscles of a person (*ii*) the speed he gets while leaving the ground during vertical jump.

Curiosity Questions

Q. 1. Why going in a bus from plane to hill station on a road with slopes and curves, one feels vomitting ?

Ans. The roads leading to hill station are generally sloppy and curved. When a bus is going on a road to a hill station, the tangential and centripetal accelerations of the bus are not constant. This will cause uneven vibrations in the stomach of the passengers sitting in the bus. As a result of it, the stomach of passengers becomes queasy, resulting in vomitting.

Q. 2. How do the safety devices used in the high speed cars protect the driver and occupants ?

Ans. The safety devices used in the high speed cars consist of shoulder straps and air bags with sensors. When collision of a car is going to take place, the sensor fitted in the car triggers the devices. Due to it, the shoulder straps and air bags move into place (*i.e.*, in between body of driver or occupant and solid body of car). If the driver applies the brakes to stop the car, the safety devices allow the car occupants to go down slowly with the car, instead of being thrown into the car body or into the air when the speed of car reduces to zero, very rapidly. As a result, the driver and the occupants of the car remain almost safe.

SOLVED EXAMPLES

TYPE I. DISTANCE COVERED, DISPLACEMENT AVERAGE SPEED AND AVERAGE VELOCITY

Formulae used.

Average speed = total distance travelled/ total time taken

Average velocity = displacement/total time taken

Units used. In SI units, displacement and distance are in *metre* and time is in *second*. In cgs system, displacement or distance is in *cm* and time is in *second*.

Example 1 **A body travels from *A* to *B* at 40 ms^{-1} and from *B* to *A* at 60 ms^{-1}. Calculate the average speed and average velocity.**

(HP Board 2007)

Solution. Total time taken by the body while going from *A* to *B* and then from *B* to *A* is

$$t_1 + t_2 = \frac{AB}{v_1} + \frac{BA}{v_2} = AB\left[\frac{1}{40} + \frac{1}{60}\right] = \frac{AB}{24}\ s$$

Total distance travelled = $AB + BA = 2\,AB$

Average speed,

$$v_{av} = \frac{2\,AB}{t_1 + t_2} = \frac{2\,AB}{(AB/24)} = \mathbf{48\ ms^{-1}}$$

Displacement of body in time $(t_1 + t_2) = 0$

[$\because$ the body returns to its initial position after time $(t_1 + t_2)$]

Average velocity,

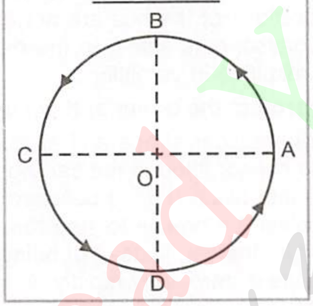
$$\vec{v}_{av} = \frac{\text{displacement}}{t_1 + t_2} = \frac{0}{t_1 + t_2} = \mathbf{0}$$

Example 2 **A particle moves along a circle of radius R. It starts from A and moves in antilockwise direction. Calculate the distance travelled by the particle (a) from A to B (b) from A to C, (c) from A to D (d) in one complete revolution. Also calculate the magnitude of displacement in each case. Fig. 3.38.**

Solution. (*a*) Distance travelled by particle from

A to B $= \dfrac{2\pi R}{4} = \dfrac{\pi R}{2}$

FIGURE 3.38



$$\text{Displacement} = |\overrightarrow{AB}| = \sqrt{OA^2 + OB^2}$$
$$= \sqrt{R^2 + R^2} = \sqrt{2}\,R$$

(*b*) For the motion from A to C, distance travelled $= \dfrac{2\pi R}{2} = \pi R$

$$\text{Displacement} = |\overrightarrow{AC}| = 2R$$

(*c*) For the motion from A to D, distance travelled $= \dfrac{2\pi R \times 3}{4} = \dfrac{3}{2}\pi R$

$$\text{Displacement} = |\overrightarrow{AD}| = \sqrt{R^2 + R^2}$$
$$= \sqrt{2}\,R$$

(*d*) For one complete revolution, *i.e.*, motion from A to A, total distance travelled $= 2\,\pi\text{R}$

Displacement = **zero**, since the final position coincides with the initial position.

Example 3 **A car is moving along a straight line OP. It moves from O to P in 18 seconds and returns from P to Q in 6 seconds, where OP = 360 m and OQ = 240 m. What are the average velocity and average speed of the car in going (*a*) from O to P ? and (*b*) from O to P and back to Q ?** **(NCERT Solved Example)**

Solution. (*a*) For a motion from O to P,

displacement = 360 m

and path length = 360 m

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{360\text{ m}}{18\text{ s}}$$
$$= \mathbf{20\ ms^{-1}\ along}\ \overrightarrow{OP}$$

$$\text{Average speed} = \frac{\text{path length}}{\text{time}}$$
$$= \frac{360\text{ m}}{18\text{ s}} = \mathbf{20\ ms^{-1}}$$

(*b*) For a motion from O to P and back to Q;

displacement = OQ = 240 m

path length = $OP + PQ$ = 360 + (360 – 240)

= 360 + 120 = 480 m

$$\text{Average velocity} = \frac{240}{18+6}$$
$$= \mathbf{10\ ms^{-1}\ along}\ \overrightarrow{OP}$$

$$\text{Average speed} = \frac{480}{18+6} = \mathbf{20\ ms^{-1}}$$

Example 4 **A man walks on a straight road from his home to a market 3 km away with a speed of 6 km/h. Finding the market closed, he instantly turns and walks back with a speed of 9 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min ?**

Solution. Time taken by man to go from his home to market,

$$t_1 = \frac{\text{distance}}{\text{speed}} = \frac{3\text{ km}}{6\text{ km/h}} = \frac{1}{2}\text{ h} = 30\text{ min}$$

Time taken by man to go from market to home,

$$t_2 = \frac{3\text{ km}}{9\text{ km/h}} = \frac{1}{3}\text{ h} = 20\text{ min}$$

Total time taken $= t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\text{ h}$

$= 50\text{ min}$

(i) **0 to 30 min**

(a) Average velocity $= \frac{\text{displacement}}{\text{time}}$

$= \frac{3{\cdot}0\text{ km}}{(1/2)\text{ h}} = \mathbf{6\text{ km/h}}$

(b) Average speed $= \frac{\text{path length}}{\text{time}} = \frac{3{\cdot}0\text{ km}}{(1/2)\text{ h}}$

$= \mathbf{6\text{ km/h}}$

(ii) **0 to 50 min**

Total path length covered = 3·0 + 3·0
= 6·0 km

Total displacement = zero

(a) Average velocity $= \frac{\text{displacement}}{\text{time}}$

$= \frac{0}{(5/6)\text{ h}} = \mathbf{0}$

(b) Average speed $= \frac{\text{total path length}}{\text{time}}$

$= \frac{6\text{ km}}{(5/6)\text{ h}} = \mathbf{7{\cdot}2\text{ km/h}}$

(iii) **0 to 40 min**

Distance moved in 30 min (from home to market) = 3·0 km

Distance moved in 10 min (from market to home) with speed 9 km/h $= 9 \times \frac{10}{60} = 1{\cdot}5$ km.

So, displacement = 3·0 − 1·5 = 1·5 km

Total path length travelled = 3·0 + 1·5
= 4·5 km

(a) Average velocity $= \frac{1{\cdot}5\text{ km}}{(40/60)\text{ h}} = \mathbf{2{\cdot}25\text{ km/h}}$

(b) Average speed $= \frac{4{\cdot}5\text{ km}}{(40/60)\text{ h}} = \mathbf{6{\cdot}75\text{ km/h}}$

Example 5 **A particle travels along a straight line. It covers half the distance with a speed v. The remaining part of the distance was covered with speed v_1 for half the time and with speed v_2 for the other half the time. Find the average speed of the particle over the entire motion.**

Solution. Let S be the total distance to be travelled by the particle.

Time taken by particle to travel half the distance $(S/2)$ is $t_1 = \frac{S/2}{v} = \frac{S}{2v}$

Let $2t_2$ be the time taken by particle to cover the remaining distance $S/2$

Distance travelled by particle in time t_2 while travelling with speed v_1 is

$$S_1 = v_1 t_2$$

Distance travelled by particle in time t_2 while travelling with speed v_2 is

$$S_2 = v_2 t_2$$

$$S_1 + S_2 = v_1 t_2 + v_2 t_2$$

But $S_1 + S_2 = \frac{S}{2} = v_1 t_2 + v_2 t_2 = (v_1 + v_2) t_2$

or $2t_2 = \frac{S}{(v_1 + v_2)}$

Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{S}{t_1 + 2t_2} = \frac{S}{(S/2v) + S/(v_1 + v_2)}$$

$$= \frac{2v(v_1 + v_2)}{v_1 + v_2 + 2v}$$

Example 6 **A man starts from his home at 8.00 am to his office. He walks with a speed of 2 ms^{-1} on a road upto his office 3·0 km away from his home. He stays in the office upto 4.00 pm and returns to his home by bus which moves non-stop with a speed of 10 ms^{-1}. Calculate**

***(i)* the time taken by man to reach his office and**

***(ii)* time taken by man to reach his home. Also plot *x-t* graph of his motion.**

(HP Board 2009)

Solution. Here, distance between home to office, $S = 3{\cdot}0\text{ km} = 3000\text{ m}$

(i) Man going to office from home,

Speed of man, $v = 2\text{ ms}^{-1}$

Time taken by man to reach office,

$$t = \frac{S}{v} = \frac{3000}{2} = 1500\text{ s} = \frac{1500}{60} = 25\text{ min}$$

So the man reaches the office 8:25 am

Stay of the man in the office is from 8:25 am to 4:00 pm, so no distance has been travelled by man during this time.

(*ii*) *Man returning to his home from office, by bus*

speed of bus, $v_1 = 10\ \text{ms}^{-1}$

$$\text{Time taken, } t_1 = \frac{S}{v_1} = \frac{3000}{10} = 300\ \text{s}$$

$$= \frac{300}{60} = 5\ \text{min}$$

It means the man reaches his home at 4:05 pm $x - t$ graph of this motion is shown in Fig. 3.39.

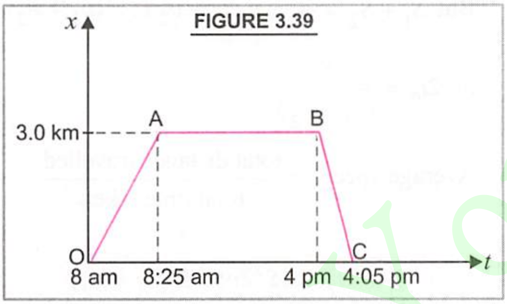


FIGURE 3.39

Example 7 **A man wants to climb to the top of a vertical pole of height 11 m. He climbs 5 m in 2 s and then slips 3 m in 2 s. This process repeats. Plot $x - t$ graph of the motion of the man. Find the time taken by the man to reach the top of the pole and total distance covered by man.**

Solution. At time $t = 0$, the man starts to climb a pole.

After 2 seconds he will climb 5 m high and will slip down to 2 m (*i.e.*, 5 m – 3 m = 2 m) after 4 s. After 6 s, he will be at 7 m height but after 8 s, he will come down to 4 m (*i.e.*, 7 m – 3 m = 4 m). After 10 s, he will be at 4 + 5 = 9 m height and after 12 s, he will come down to height = 9 m – 3 m = 6 m. After 14 s, he will be at height = 6 m + 5 m = 11 m.

Thus time taken by man to reach the top = **14 s.**

The $x - t$ graph is shown in Fig. 3.40.

$$\text{Total distance covered} = OA + AB + BC + CD + DE + EF + FG$$

$$= 5 + 3 + 5 + 3 + 5 + 3 + 5$$

$$= \mathbf{29\ m}$$

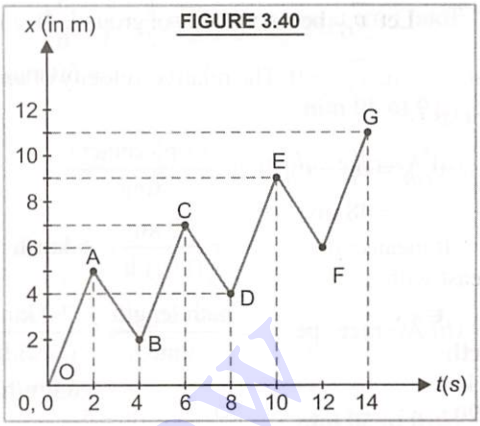


FIGURE 3.40

TYPE II. RELATIVE VELOCITY

Formulae used.

(*i*) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ (*ii*) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

where $\vec{v}_{AB}$ = relative velocity of object A w.r.t. B

$\vec{v}_{BA}$ = relative velocity of object B w.r.t. A

$\vec{v}_A, \vec{v}_B$ = velocity of object A and B respectively.

Units used. The SI unit of v_A, v_B, v_{AB} and v_{BA} is ms^{-1} and cgs unit is cm s^{-1}.

Example 8 **Two railway tracks are parallel to west-east direction. Along one track, train A moves with a speed of 30 ms^{-1} from west to east, while along the second track, train B moves with a speed of 48 ms^{-1} from east to west. Calculate (i) relative speed of B w.r.t. A and (ii) relative speed of ground w.r.t. B.**

Solution. Let the motion from west to east be taken positive. Here,

$\vec{v}_A = 30\ \text{ms}^{-1}$ (west to east)

and $\vec{v}_B = 48\ \text{ms}^{-1}$ (east to west)

or $\vec{v}_B = -48\ \text{ms}^{-1}$ (west to east).

(*i*) The relative velocity of the train B w.r.t. A will be

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = [(-48) - 30]\ \text{(west to east)}$$

$$= -78\ \text{ms}^{-1}\ \text{(west to east)}$$

$$= \mathbf{78\ ms^{-1}\ (east\ to\ west)}$$

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(*ii*) Let $\vec{v}_G$ be the velocity of ground. As ground is at rest, so, $\vec{v}_G = 0$. The relative velocity of ground w.r.t. B is

$$\vec{v}_{GB} = \vec{v}_G - \vec{v}_B = 0 - (-48) \text{ (west to east)}$$
$$= \mathbf{48\ ms^{-1}} \textbf{ (west to east)}$$

It means ground will appear to move from west to east with a speed of 48 ms^{-1} w.r.t. train B.

Example 9 **Two parallel rail tracks run north-south. Train *A* moves north with a speed of 54 km h^{-1} and train *B* moves south with a speed of 90 km h^{-1}. What is the (*a*) relative velocity of *B* with respect to *A* ? (*b*) relative velocity of ground with respect to *B* ? and (*c*) velocity of a monkey running on the roof of the train *A* against the motion with a velocity of 18 km h^{-1} with respect to the train *A* as observed by a man standing on the ground ?** (CBSE 2005)
(NCERT Solved Example)

Solution. Let positive direction of motion be from south to north. Then,

$$v_A = +54 \text{ km h}^{-1} = 15 \text{ ms}^{-1} ;$$
$$v_B = -90 \text{ km h}^{-1} = -25 \text{ ms}^{-1}$$

(*a*) Relative velocity of B w.r.t. A

$$= v_{BA} = v_B - v_A = -25 - 15 = -40 \text{ ms}^{-1}$$

i.e., **train *B* appears to *A* to move with a speed of 40 ms^{-1} from north to south.**

(*b*) Relative velocity of ground w.r.t. B

$$= 0 - v_B = 0 - (-25) = \mathbf{25\ ms^{-1}} \textbf{ due north}$$

(*c*) Let v_m be the velocity of monkey w.r.t. ground. Here, relative velocity of the monkey w.r.t. train A $= v_m - v_A = -18 \text{ km h}^{-1} = -5 \text{ ms}^{-1}$

$$\therefore\ v_m = v_A - 5 = 15 - 5 = \mathbf{10\ ms^{-1}} \textbf{ due north}$$

Example 10 **A motor boat covers the distance between the two spots on the river in 9 *h* and 13 *h* down stream and upstream respectively. Find the time required by the boat to cover this distance in still water.**

Solution. Let u be the velocity of boat in still water and v be the velocity of river. Let x be the distance between two given spots. As per question.

For downstream, $u + v = \dfrac{x}{9}$

For upstream, $u - v = \dfrac{x}{13}$

On adding, $2u = \dfrac{x}{9} + \dfrac{x}{13} = \dfrac{22x}{117}$ or $u = \dfrac{11x}{117}$

Time taken by boat to cover distance x in still water is $t = \dfrac{x}{u} = \dfrac{x}{11x/117} = \mathbf{10{\cdot}63\ h}$

TYPE III. UNIFORMLY ACCELERATED MOTION

Formulae used.

(*i*) $v = u + at$ (*ii*) $S = ut + \dfrac{1}{2}at^2$

(*iii*) $v^2 = u^2 + 2\,aS$ (*iv*) $D_n = u + \dfrac{a}{2}(2n - 1)$

Units used. In SI units, u and v are taken in ms^{-1} ; a in ms^{-2}, S in m ; D_n in m.

Example 11 **A body covers 10 m in 4th second and 15 m in 6th second of its uniformly accelerated motion. How far will it travel in the next 3 seconds.**

Solution. Here, $D_4 = 10$ m ; $D_6 = 15$ m ; $S_9 - S_6 = ?$

Let u and a be the initial velocity and uniform acceleration of the body. As

$$D_n = u + \frac{a}{2}(2n - 1)$$

$$\therefore \quad D_4 = u + \frac{a}{2}(2 \times 4 - 1) = 10 \qquad ...(i)$$

$$\text{and} \quad D_6 = u + \frac{a}{2}(2 \times 6 - 1) = 15 \qquad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$2a = 5 \quad \text{or} \quad a = (5/2) \text{ ms}^{-2}$$

From (*i*),

$$u = 10 - \frac{5/2}{2}(8 - 1) = 10 - \frac{35}{4} = \frac{5}{4} \text{ ms}^{-1}$$

Distance travelled in t seconds is

$$S = ut + \frac{1}{2}at^2$$

When $t = 9$s, then

$$S_9 = \frac{5}{4} \times 9 + \frac{1}{2} \times \frac{5}{2} \times 9^2 = 112{\cdot}5 \text{ m}$$

When $t = 6$ s,

$$S_6 = \frac{5}{4} \times 6 + \frac{1}{2} \times \frac{5}{2} \times 6^2 = 52{\cdot}5 \text{ m}$$

$\therefore$ Distance travelled in the next three seconds

$$= S_9 - S_6 = 112{\cdot}5 - 52{\cdot}5 = \mathbf{60{\cdot}0\ m}$$

Example 12 A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multi-storey building. The height of the point from where the ball is thrown is 25·0 m from the ground. (*a*) How high the ball will rise ? and (*b*) how long will it be before the ball hits the ground ?Take, $g = 10$ ms^{-2}.

(NCERT Solved Example)

Solution. (*a*) *Taking vertical upward motion of ball upto highest point, we have*

$$u = 20\ \text{ms}^{-1},\ a = -10\ \text{m/s}^2,$$
$$v = 0,\ S = ?,\ t = t_1\ \text{(say)}$$

As, $\quad v^2 = u^2 + 2\,a\,S$

$\therefore \quad 0 = 20^2 + 2\,(-10)\,S$

or $\quad S = \dfrac{20 \times 20}{2 \times 10} = \mathbf{20\ m}$

(*b*) As, $\quad v = u + at$

$\therefore \quad 0 = 20 + (-10)\,t_1 \quad$ or $\quad t_1 = 2$ s

Taking vertical downward motion of ball from highest point to ground, we have

$$u = 0\ \text{ms}^{-1}\ ;\ a = 10\ \text{m/s}^2\ ;$$
$$S = 20 + 25 = 45\ \text{m}\ ;\ t = t_2\ \text{(say)}$$

As $\quad S = ut + \dfrac{1}{2}at^2$

$\therefore \quad 45 = 0 + \dfrac{1}{2} \times 10 \times t_2^2$

or $\quad t_2 = \left(\dfrac{2 \times 45}{10}\right)^{1/2} = 3$ s

Total time $= t_1 + t_2 = 2 + 3 = \mathbf{5\ s}$

Example 13 A driver of car takes 0·20 s (which is the reaction time of driver) to apply the brakes after he sees an obstruction at a distance of 25 m from him. If he is driving car at a speed of 54 km h^{-1} and the brakes cause a deceleration of 6·0 ms^{-2}, predict whether he will avert collision or not.

Solution. Here, reaction time = 0·20 s ;

During the time of reaction, the car continuous to move with uniform speed of 54 km h^{-1} = 15 ms^{-1}

Distance covered by car in time 0·20 s

$$= 15 \times 0{\cdot}2 = 3{\cdot}0\ \text{m}$$

Taking motion of car when brake is applied and its velocity becomes zero. Then

$$u = 15\ \text{ms}^{-1},\ a = -6{\cdot}0\ \text{ms}^{-2},\ v = 0,\ S = ?$$

$$v^2 = u^2 + 2\,aS$$

or $\quad S = \dfrac{v^2 - u^2}{2\,a} = \dfrac{0 - (15)^2}{2\,(-6)} = 18{\cdot}75$ m

Total distance travelled by car = 3·0 + 18·75
= 21·75 m

As this distance is less than 25 m, so the driver **will avert the collision.**

Example 14 A body travels a distance of 2 m in 2 seconds and 2·2 m in next 4 seconds. What will be the velocity of the body at the end of 7th second from the start ?

Solution. Here, case (*i*) $S = 2$ m, $t = 2$ s

Case (*ii*) $S = 2 + 2{\cdot}2 = 4{\cdot}2$ m, $t = 2 + 4 = 6$ s

Let u and a be the initial velocity and uniform acceleration of the body.

we know that, $\quad S = ut + \dfrac{1}{2}at^2$

Case (*i*), $\quad 2 = u \times 2 + \dfrac{1}{2}a \times 2^2$

or $\quad 1 = u + a \qquad$ **...(*i*)**

Case (*ii*), $\quad 4{\cdot}2 = u \times 6 + \dfrac{1}{2}a \times 6^2$

or $\quad 0{\cdot}7 = u + 3a \qquad$ **...(*ii*)**

Subtracting (*ii*) from (*i*), we get

$$0{\cdot}3 = 0 - 2a = -2a$$

or $\quad a = -0{\cdot}3/2 = -0{\cdot}15$ ms^{-2}

From (*i*), $\quad u = 1 - a = 1 + 0{\cdot}15 = 1{\cdot}15$ ms^{-1}

For the velocity of body at the end of 7th second, we have $\quad u = 1{\cdot}15$ ms^{-1} ;

$$a = -0{\cdot}15\ \text{ms}^{-2},\ v = ?,\ t = 7\ \text{s}$$

As, $\quad v = u + at$

$\therefore \quad v = 1{\cdot}15 + (-0{\cdot}15) \times 7 = \mathbf{0{\cdot}1\ ms^{-1}}$

Example 15 A motor car starts from rest and accelerates uniformly for 10 s to a velocity of 30 ms^{-1}. It then runs at a constant speed and is finally brought to rest in 80 m with a constant retardation. Total distance covered is 830 m. Find the value of acceleration, retardation and total time taken.

Solution. Let S_1, S_2 and S_3 be the distances covered by motor boat with uniform acceleration, constant velocity and uniform retardation respectively.

Taking motion of motor car while moving with a uniform acceleration, we have

$$u = 0,\ t_1 = 10\ \text{s},\ v = 30\ \text{ms}^{-1},\ a = ?$$

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As, $v = u + at_1$, so $30 = 0 + a \times 10$

or $$a = \mathbf{3\ ms^{-2}}$$

Distance covered, $$S_1 = ut + \frac{1}{2}at_1^2$$

$$= 0 + \frac{1}{2}\times 3 \times 10^2 = 150 \text{ m}$$

Taking motion of motor car while moving with uniform velocity, we have

$$S_1 = 150 \text{ m} ; S_3 = 80 \text{ m and } S = 830 \text{ m}$$

As $S_1 + S_2 + S_3 = 830$

∴ $150 + S_2 + 80 = 830$

or $S_2 = 830 - 230 = 600$ m

This distance is covered with a uniform velocity $30\ ms^{-1}$, so time taken, $t_2 = \dfrac{600}{30} = 20\,s$

Taking motion of motor car with uniform retardation, we have

$$u = 30\ ms^{-1}, v = 0, S = 80 \text{ m}, a = ?$$

As $v^2 = u^2 + 2\,aS$ So, $0 = 30^2 + 2\,a \times 80$

or $a = -\dfrac{900}{160} = -\dfrac{45}{8}\ ms^{-1} = \mathbf{-\,5{\cdot}625\ ms^{-2}}$

Time taken, $t_3 = \dfrac{v-u}{a} = \dfrac{0-(30)}{-45/8} = 5{\cdot}33\,s$

Total time taken $= t_1 + t_2 + t_3$

$= 10 + 20 + 5{\cdot}33$

$= \mathbf{35{\cdot}33\ s}$

Example 16 **A man 16 m behind the door of a train when it starts moving with an acceleration $a = 2\ ms^{-2}$. The man runs with a maximum constant speed to get into the train. How for does he has to run and after what time does he get into the train ? What is his maximum constant speed ?**

Solution. Let v be the maximum constant speed of the man. He will get into the train after time t, when his velocity becomes equal to the velocity of train.

∴ $$v = 0 + 2 \times t = 2\,t \qquad ...(i)$$

Distance moved by man in time t,

$$S = vt = 2t \times t = 2t^2$$

Distance moved by train in time t is

$$S_1 = 0 + \frac{1}{2} \times at^2 = \frac{1}{2} \times 2 \times t^2 = t^2$$

Man will get into the train, when

$$S = S_1 + 16$$

∴ $$2\,t^2 = t^2 + 16 \text{ or } t^2 = 16$$

or $$t = \mathbf{4}s$$

Distance moved by man $= 2 \times (4)^2 = \mathbf{32\ m}$

Maximum constant speed of man,

$$v = 2\,t = 2 \times 4 = \mathbf{8\ ms^{-1}}$$

Example 17 **Two buses A and B are at positions 60 m and 160 m from the origin at time t = 0. They start moving in the same direction simultaneously with uniform speed 54 km h^{-1} and 36 km h^{-1}. Determine the time and position at which A overtaken B.**

Solution. Here, $x_1\,(0) = 60$ m, $x_2\,(0) = 160$ m,

$v_1 = 54 \text{ km h}^1 = 15\ ms^{-1}$,

$v_2 = 36 \text{ km h}^{-1} = 10\ ms^{-1}$

The positions of two buses at any instant t are

$x_1\,(t) = x_1\,(0) + v_1\,t = 60 + 15\,t$

$x_2\,(t) = x_2\,(0) + v_2\,t = 160 + 10\,t$

When bus A overtakes bus B, then

$x_1\,(t) = x_2\,(t)$ or $60 + 15\,t = 160 + 10\,t$

or $5\,t = 100$ or $t = \mathbf{20\ s}$

∴ $x_1\,(20) = 60 + 15 \times 20 = \mathbf{360\ m}$

$x_2\,(20) = 160 + 10 \times 20 = \mathbf{360\ m}$

Example 18 **Prove that the distances traversed during equal intervals of time by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely 1 : 3 : 5 : 7......]**

(NCERT Solved Example)

Solution. Here, $u = 0$, $a = g$,

Distance travelled in nth second is given by

$$D_n = u + \frac{a}{2}\,(2n-1)$$

Distance travelled in 1st second [*i.e.,* when $n = 1$] will be

$$D_1 = 0 + \frac{g}{2}\,(2\times 1 - 1) = \frac{g}{2}$$

Distance travelled in 2nd second [*i.e.,* when $n = 2$] will be

$$D_2 = 0 + \frac{g}{2}\,(2\times 2 - 1) = \frac{3\,g}{2}$$

Distance travelled in 3rd second [*i.e.,* when $n = 3$] will be

$$D_3 = 0 + \frac{g}{2}\,[2\times 3 - 1] = \frac{5\,g}{2}$$

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Distance travelled in 4th second [*i.e.*, when $n = 4$] will be

$$D_4 = 0 + \frac{g}{2}(2\times4-1) = \frac{7\,g}{2}$$

$\therefore$ $D_1 : D_2 : D_3 : D_4 =$ **1 : 3 : 5 : 7 (Proved)**

Example 19 **A tennis ball is dropped on to the floor from a height of 4·00 m. It rebounds to a height of 3·00 m. If the ball was in contact with the floor for 0·010 sec, what was its average acceleration during contact ?**

Solution. Here, $h_1 = 4{\cdot}00$ m, $h_2 = 3{\cdot}00$ m, $\Delta t = 0{\cdot}01$ s. Let v_1 be the velocity of the ball (acting downwards) just before striking the floor and v_2 be the velocity of the ball (acting upwards) just after striking the floor. Then, change in velocity of the ball in time $\Delta t = v_2 - (-v_1) = v_2 + v_1$

$$\therefore \quad \text{acceleration, } a = \frac{v_2 + v_1}{\Delta t} \qquad ...(i)$$

When body falls from height h_1,

then $u = 0, v = v_1, a = g$ and $S = h_1$

As, $v^2 = u^2 + 2a\,S$,

$$\therefore \quad v_1^2 = 0 + 2\,g\,h_1 \quad \text{or} \quad v_1 = \sqrt{2\,g\,h_1}$$

Taking motion of the ball after striking the floor, then $u = v_2, v = 0, a = -g, S = h_2$

As, $v^2 = u^2 + 2a\,S$, we have

$$0 = v_2^2 + 2(-g)\,h_2 \quad \text{or} \quad v_2 = \sqrt{2\,g\,h_2}$$

Putting values in (*i*), we get,

$$a = \frac{\sqrt{2\,g\,h_2} + \sqrt{2\,g\,h_1}}{\Delta t}$$

$$= \frac{\sqrt{2\times9{\cdot}8\times3} + \sqrt{2\times9{\cdot}8\times4}}{0{\cdot}01} = \mathbf{1652\ m/s^2}$$

Example 20 **A parachutist bails out from an aeroplane and after dropping through a distance of 40 m opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long is he in the air ? At what height did he bail out from the plane ?**

Solution. *When the parachutist drops for the first 40 m*

$u = 0$; $S = 40$ m ; $a = g = 9{\cdot}8$ ms^{-2} ;

$v = ?$ $t = t_1$

As, $v^2 - u^2 = 2\,a\,S$

$\therefore$ $v^2 = 2 \times 9{\cdot}8 \times 40 = 784$

or $v = (784)^{1/2} = 28$ ms^{-1}

Also, $v = u + at$

$\therefore$ $28 = 0 + 9{\cdot}8\,t_1$ or $t_1 = 28/9{\cdot}8 = 2{\cdot}8$ s

Now the final velocity after the drop of 40 m is the initial velocity for the rest of the fall.

Therefore, $u = 28$ ms^{-1} ; $a = -2$ ms^{-2} ;

$v = 2$ ms^{-1} ; $t = t_2$

As, $v = u + at$

$\therefore$ $2 = 28 + (-2) \times t_2$ or $t_2 = 13$ seconds

Therefore, the total time the parachutist is in air $T = t_1 + t_2 = 2{\cdot}8 + 13 =$ **15·8 seconds.**

Height through which the parachutist has fallen during the second half of the fall is given by

$$v^2 - u^2 = 2a\,S$$

or $2^2 - (28)^2 = 2(-2)\,S$ or $S = 195$ m

Thus, the height at which the parachutist bailed out $= 40 + 195 =$ **235 m**

Example 21 **A body is dropped from rest at a height of 150 m and simultaneously, another body is dropped from rest from a point 100 m above the ground. What is the difference between heights after they have fallen for (i) 3s (ii) 5s. Consider that the body on reaching ground remains there and acceleration due to gravity be 10 m/s^2.**

Solution. (*i*) *For the first body*, taking downward motion of the body, we have

$u = 0, t = 3$ s, $a = 10$ m/s^2, $S = ?$

$$\text{As,} \quad S = ut + \frac{1}{2}at^2$$

$$\therefore \quad S = 0\times3 + \frac{1}{2}\times10\times3^2 = 45 \text{ m}$$

Height of body above ground $= 150 - 45$
$= 105$ m.

For second body

Downward distance travelled in 3 seconds
$= 45$ m

Height of body above ground $= 100 - 45$
$= 55$ m

Thus, difference in heights $= 105 - 55 =$ **50 m**

(*ii*) Similarly distance covered by each body in 5 second will be given by

$$S = \frac{1}{2}gt^2 = \frac{1}{2}\times10\times5^2 = 125 \text{ m}$$

Now, the height of first body above ground
$= 150 - 125 = 25$ m

For second body, as the distance travelled is more than its height from ground, it means this body will be on ground just after 5 seconds. Therefore, the height of this body from ground is zero.

Thus, difference in height of two bodies

$= 25 - 0 = \mathbf{25\ m}$

Example 22 **A balloon starts rising from the ground with an acceleration 2 ms^{-2}. After 5 second, a stone is released from the balloon. Find the time taken by the stone to reach the ground after its release and total height attained by balloon when stone reaches the ground. Take $g = 10$ ms^{-2}.**

Solution. Taking upward motion of balloon and stone, $u = 0$, $a = 2$ ms^{-2}, $t = 5$ s,

velocity attained by the balloon after 5 s is

$$v = u + at = 0 + 2 \times 5 = 10 \text{ ms}^{-1}$$

Height attained by the balloon after 5 s is

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 5^2 = 25 \text{ m}$$

Now the stone will fall from a height of 25 m with initial velocity 10 ms^{-1} acting upwards.

Taking vertical downward motion of stone from balloon to ground we have

$u = -10$ ms^{-1}, $a = g = 10$ ms^{-2} ; S = 25 m,

$t = ?$

$$S = ut + \frac{1}{2}at^2$$

$$25 = -10 \times t + \frac{1}{2} \times 10 \times t^2 = -10\,t + 5\,t^2$$

or $$t^2 - 2\,t - 5 = 0$$

or $$t = \frac{2 \pm \sqrt{4+20}}{2} = \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$$

$$= 1 \pm 2{\cdot}449 = 3{\cdot}449 \text{ s or } -1{\cdot}449 \text{ s}$$

Since time t is a positive quantity, so time taken by stone to reach the ground = 3·449 = **3·45** s

Total time for upward motion of balloon

$T = 5 + 3{\cdot}45 = 8{\cdot}45$ s

Total height attained by balloon in time 8·45 s

$$h = ut + \frac{1}{2}aT^2 = 0 + \frac{1}{2} \times 2 \times (8{\cdot}45)^2$$

$$= \mathbf{71{\cdot}4\ m}$$

TYPE IV : GRAPHS FOR UNIFORMLY ACCELERATED MOTION

Formulae used. (*i*) Slope of distance-time graph of a uniform motion along a st. line gives the velocity of the body.

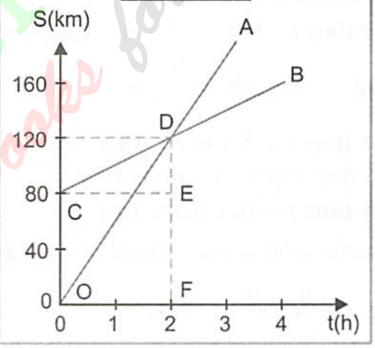
(*ii*) Slope of velocity-time graph of a uniformly accelerated motion at an instant represents the acceleration of the body at that instant.

(*iii*) Area enclosed by velocity-time graph with time axis is equal to the distance travelled by body for the given time.

(*iv*) Change in velocity is equal to the area of acceleration-time graph for the given interval of time.

Example 23 **Fig. 3.41 shows the distance (*S*) – time (*t*) graphs of two trains, which start moving simultaneously in the same direction. From the graphs, find :**

FIGURE 3.41



(*a*) How much *B* is ahead of *A* when motion starts

(*b*) What is the speed of *B* ?

(*c*) When and where *A* will catch *B* ?

(*d*) What is the difference in speeds of *A* and *B* ?

Solution. (*a*) When motion starts, *B* is ahead of *A* by the distance *OC* = **80 km.**

(*b*) Speed of $B = \frac{DE}{CE} = \frac{120-80}{2-0} = \mathbf{20\ km\ h^{-1}}$

(*c*) Since the two graphs meet at point *D*, so *A* will catch *B* after **2 hours** and at a distance of **120 km** from the origin.

(*d*) Speed of $A = \frac{DF}{OF} = \frac{120-0}{2-0} = \mathbf{60\ km\ h^{-1}}$

Difference in speed = 60 – 20 = **40 km h^{-1}**

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Example 24 **Velocity (v) time (t) graph of a car starting from rest is shown in Fig. 3.42. Draw acceleration (a) – time (t) graph for the motion of the car and find total distance travelled by car.**

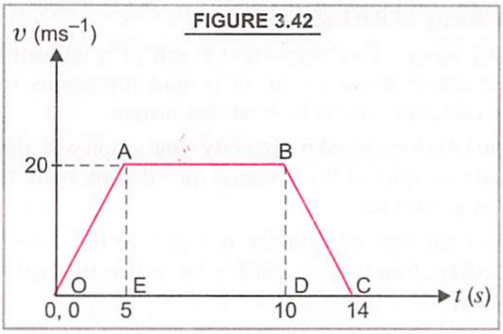
FIGURE 3.42

Solution. From the graph we note that car starting from rest accelerates for time 0 to 5 s and attains a velocity 20 ms^{-1}. It then moves with a constant velocity 20 ms^{-1} from 5 s to 10 s. After that the motion of car is retarded for time 10 s to 14 s and it comes to rest at time t = 14 s.

For time t = 0 to t = 5 s, the acceleration of car

$$= \text{slope of } v - t \text{ graph } = \frac{20-0}{5} = 4 \text{ ms}^{-2}$$

For time t = 5 s to t = 10 s, velocity of car is constant, hence acceleration is zero.

For time t = 10 s to t = 14 s

Acceleration of car = slope of $v - t$ graph

$$\text{or } \quad a = \frac{0-20}{4} = -5 \text{ ms}^{-2}$$

Thus acceleration (a) – time (t) graph is of the type as shown in Fig. 3.43.

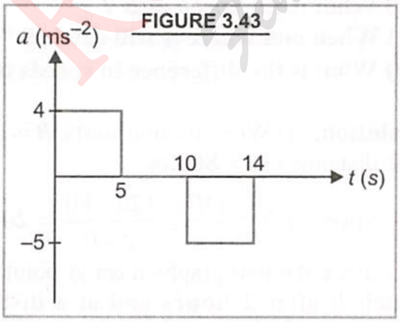
FIGURE 3.43

Total distance travelled by car in 14 seconds

= Area $OABC$

= area of ΔOAE + Area of $\square ABDE$ + Area ΔBCD

$$= \frac{20 \times 5}{2} + 20 \times 5 + \frac{20 \times 4}{2}$$

= 50 + 100 + 40 = **190 m**

Example 25 **A body starting from rest accelerates uniformly along a st. line, at the rate of 10 ms^{-2} for 5 s. It moves for 2 s with uniform velocity of 50 ms^{-1}. Then, it retards uniformly and comes to rest in 3 s. Draw velocity–time graph of the body and find the total distance travelled by body.** **(CBSE 2004)**

Solution. Taking the motion of the body from 0 to 5 seconds we have

$$u = 0;\ v = ?\ ;\ a = 10 \text{ ms}^2\ ;\ t = 5 \text{ s.}$$

As, $v = u + at$

$\therefore \quad v = 0 + 10 \times 5 = 50$ m/s.

The velocity–time graph of the motion of the body is as shown in Fig. 3.44.

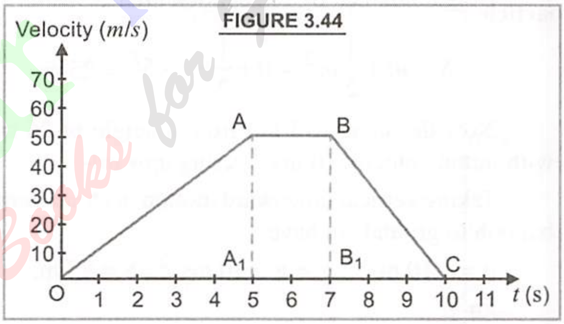
FIGURE 3.44

Total distance travelled by body during motion = area of trapizium $OABC$

$$= \frac{1}{2}(OC + AB) \times AA_1$$

$$= \frac{1}{2}(10 + 2) \times 50 = \textbf{300 m}$$

Example 26 **Fig. 3.45 shows the time - acceleration graph for a particle in rectilinear motion. Find the average acceleration in first twenty seconds.**

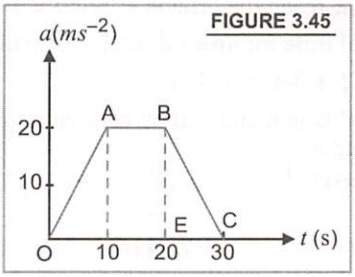
FIGURE 3.45

Solution. Average acceleration (a)

$$= \frac{\text{change in velocity}}{\text{time taken}}$$

$\therefore$ change in velocity

= average acceleration × time taken

= area of acceleration-time graph

$\therefore$ Average acceleration

$$= \frac{\text{area of acceleration-time graph}}{\text{time taken}}$$

$$= \frac{\text{area } OABE}{20} = \frac{300}{20} = \mathbf{15\ ms^{-2}}$$

Example 27 **A particle starts from rest at time $t = 0$ and moves on a straight line with acceleration a (ms^{-2}) as plotted in Fig. 3.46. Find the time at which the speed of the particle is maximum. Also calculate the displacement of the particle from starting point after 4 s.**

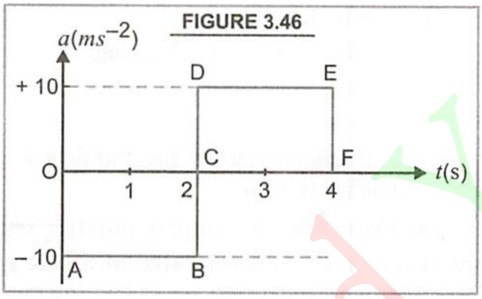
FIGURE 3.46

Solution. Taking motion from 0 to 2 seconds, we have

$u = 0$, $a = -10\ m/s^2$, $t = 2$ s, $v = ?$

$v = u + at = 0 + (-10) \times 2 = -20\ ms^{-1}$

Taking motion from 2 to 4 seconds we have

$u = -20\ ms^{-1}$; $a = 10\ m/s^2$; $t = 2$ s ; $v = ?$

Now, $v = u + at = -20 + 10 \times 2 = 0$.

Therefore, the speed is maximum at time

$t = $ **2 seconds**

Taking motion from 0 to 2 seconds, if S_1 is the distance covered, then

$$S_1 = \frac{v^2 - u^2}{2a} = \frac{(-20)^2 - (0)^2}{2(-10)} = -20\,\text{m}$$

Taking motion from 2 to 4 seconds, if S_2 is the distance covered then

$$S_2 = \frac{v^2 - u^2}{2a} = \frac{0 - (-20)^2}{2 \times 10} = -20\,\text{m}$$

$\therefore$ Total displacement $= S_1 + S_2$

$= -20 + (-20)$

$=$ **− 40 m**

Example 28 **A ball is thrown upward with an initial velocity of 80 ms^{-1}. After how much time will it return to ground ? Draw velocity-time graph for the ball and find from the graph**

(*a*) the maximum height attained by the ball and (*b*) height of the ball after 12 s. Take $g = 10\ ms^{-2}$

Solution. Here, $u = 80\ ms^{-1}$; At the highest point $v = 0$, $a = -10\ ms^{-2}$.

Let the ball reach the highest point in time t. As

$v = u + at$; so $0 = 80 + (-10)\,t$

or $t = 80/10 = 8$ s

The ball will return to ground in time

$= 2\,t = 2 \times 8 =$ **16 s**

Velocity of the ball at different instants of time will be as follows :

At $t = 0$, $v = 80 - 10 \times 0 = 80\ ms^{-1}$

At $t = 4$ s, $v = 80 - 10 \times 4 = 40\ ms^{-1}$

At $t = 8$ s, $v = 80 - 10 \times 8 = 0$

At $t = 12$ s, $v = 80 - 10 \times 12 = -40\ ms^{-1}$

At $t = 16$ s, $v = 80 - 10 \times 16 = -80\ ms^{-1}$

The velocity-time graph will be a st. line ABC as shown in Fig 3.47.

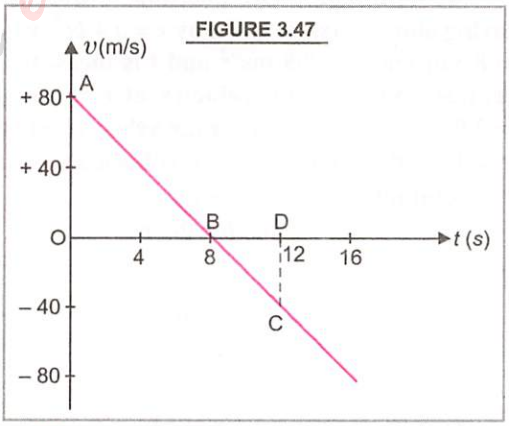
FIGURE 3.47

(*a*) Maximum height attained by the ball

= area OAB

$$= \frac{1}{2} \times 8 \times 80 = \mathbf{320\ m}$$

(*b*) Height attained after 12 *s*.

= area OAB + Area BCD

$$= 320 + \frac{1}{2}(12 - 8) \times (-40) = 320 - 80 = \mathbf{240\ m}$$

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TYPE V. CALCULUS METHOD FOR ACCELERATED MOTION

Example 29 **The velocity of a particle is given by $v = v_0 + gt + \frac{1}{2} ft^2$. If its position is $x = 0$ at $t = 0$, then what is its displacement after $t = 1$s ?**

Solution. Here, $v = \frac{dx}{dt} = v_0 + gt + \frac{1}{2} ft^2$

or $$dx = \left(v_0 + gt + \frac{1}{2} ft^2 \right) dt$$

Integrating it within the conditions of motion, *i.e.,* as t changes 0 to 1 s, x changes 0 to x, we get

$$\int_0^x dx = \int_0^1 \left(v_0 + gt + \frac{1}{2} ft^2 \right) dt$$

$$x = v_0\,(t)_0^1 + g\left(\frac{t^2}{2}\right)_0^1 + \frac{1}{2} f \left(\frac{t^3}{3}\right)_0^1$$

$$= v_0 + \frac{g}{2} + \frac{f}{6}$$

Example 30 **The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8{\cdot}5$ m and $b = 2{\cdot}5$ ms^{-2} and t is measured in seconds. What is the velocity at $t = 0$ s and $t = 2{\cdot}0$ s ? What is the average velocity between $t = 2{\cdot}0$ s and $t = 4{\cdot}0$ s ?** **(NCERT Solved Example)**

Solution. Given, $x = a + bt^2$;

where, $a = 8{\cdot}5$ m and $b = 2{\cdot}5$ m/s^2

$\therefore$ velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2)$

$= 2\,bt = 2 \times 2{\cdot}5 \times t$

$= 5{\cdot}0$ t

At $t = 0$, $v = 5{\cdot}0 \times 0 = \mathbf{0}$ **ms^{-1}**

At $t = 2{\cdot}0$ s, $v = 5{\cdot}0 \times 2 = \mathbf{10}$ **ms^{-1}**

At time $t = 2{\cdot}0$ s,

position, $x_2 = a + b \times 2^2 = a + 4\,b$

At time $t = 4{\cdot}0$ s,

position, $x_4 = a + b \times 4^2 = a + 16\,b$

$\therefore$ Displacement in time interval 2·0 s to 4·0 s

$= x_4 - x_2 = (a + 16\,b) - (a + 4\,b) = 12\,b$

$\therefore$ Average velocity $= \frac{\text{displacement}}{\text{time interval}}$

$$= \frac{12\,b}{(4-2)} = \frac{12 \times 2{\cdot}5}{2} = \mathbf{15\ ms^{-1}}$$

Example 31 **The distance x of a particle moving in one dimension, under the action of a constant force is related to time t by equation**

$$t = \sqrt{x} + 3$$

where x is in metres and t in seconds. Find the displacement of the particle when its velocity is zero.

Solution. Given, $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$

Squaring both the sides, we get

$x = (t-3)^2 = t^2 - 6\,t + 9$

Differentiating it w.r.t time t, we get

velocity, $v = \frac{dx}{dt} = 2t - 6$

when $v = 0$, then

$2t - 6 = 0$ or $t = 3$ second.

At $t = 3$, displacement is given by,

$x = t^2 - 6t + 9 = 3^2 - 6 \times 3 + 9 = \mathbf{0}$

Hence, **displacement of the particle is zero when its velocity is zero.**

Example 32 **An object moving with a speed of 6·25 ms^{-1} is decelerated at a rate given by, $\frac{dv}{dt} = -2{\cdot}5\sqrt{v}$ where v is the instantaneous speed. What is the time taken by the object to come to rest ?**

Solution. Given, $\frac{dv}{dt} = -2{\cdot}5\sqrt{v}$

or $$\frac{dv}{\sqrt{v}} = -2{\cdot}5\,dt$$

Let the object will come to rest after time t, then velocity of object will be changing in time t from 0·625 ms^{-1} to zero.

Integrating the above relation within the conditions of motion, we get

$$\int_{6\cdot25}^{0} \frac{dv}{\sqrt{v}} = \int_0^t -2{\cdot}5\,dt \quad \text{or} \quad [2\sqrt{v}]_{6\cdot25}^{0} = -2{\cdot}5[t]_0^t$$

or $0 - 2\sqrt{6{\cdot}25} = -2{\cdot}5\,t$ or $-2 \times 2{\cdot}5 = -2{\cdot}5\,t$

or $t = \mathbf{2\ s}$

TYPE VI. TYPICAL EXAMPLES

Example 33 **Four particles A, B, C and D are situated at the corners of a square $ABCD$ of side a at $t = 0$. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC, C along CD and D along DA. At what time will these particles meet each other ?**

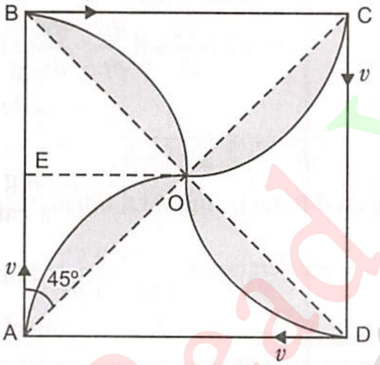
Solution. The motion of four particles A, B, C and D is as shown in Fig. 3.48 by solid curved lines. They will be meeting at the centre O of the square $ABCD$. It means, the particle at A will be covering a displacement AO before meeting the other particles. The velocity of the particle at A along AO

$$= v \cos 45° = \frac{v}{\sqrt{2}}$$

$$\cos 45° = AE/AO$$

or $$AO = \frac{AE}{\cos 45°} = \frac{a/2}{1/\sqrt{2}} = \frac{a}{\sqrt{2}}$$

FIGURE 3.48



The time take by particle at A to go from A to O

$$= \frac{AO}{v \cos 45°} = \frac{a/\sqrt{2}}{v\,(1/\sqrt{2})} = \frac{a}{v}$$

Example 34 **A long belt is moving horizontally with a speed of 5 km h^{-1}. A child runs on this belt to and fro with a speed of 9 km/h (w.r.t. belt) between his father and mother located 50 m apart on the belt. For an observer on a stationary plateform outside, what is t**

(a) speed of the child running in the direction of motion of the belt, (b) speed of the child running opposite to the direction of the belt, and (c) time taken by the child in cases (a) and (b) ?

Which of the answers change, if motion is viewed by one of the parents ?

Solution. Let us consider left to right be the positive direction of x-axis.

(*a*) Here, velocity of belt, $v_B = +5$ km h^{-1}

Speed of child w.r.t. belt, $v_C = +9$ km h^{-1} $= 5/2$ ms^{-1}

Speed of child w.r.t. stationary observer,

$$v_C' = v_C + v_B = 9 + 5 = \mathbf{14\ km\ h^{-1}}$$

(*b*) Here, $v_B = +5$ km h^{-1} ; $v_C = -9$ km h^{-1}

Speed of child w.r.t. stationary observer,

$$v_C' = v_C + v_B = -9 + 5 = \mathbf{-4\ km\ h^{-1}}$$

Here, negative sign shows that the child will appear to run in a direction opposite to the direction of motion of the belt.

(*c*) Distance between parents, $s = 50$ m.

Since parents and child are located on the same belt, the speed of child as observed by stationary observer in either direction (either from mother to father or from father to mother) will be 9 km h^{-1}.

Time taken by child in case (*a*) and (*b*) is,

$$t = 50/(5/2) = \mathbf{20\ s}$$

If the motion is observed by one of the parents, answer to **case (a) or case (b) will get altered.** It is so because speed of child w.r.t. either of mother or father is 9 km/h.

But answer (*c*) remains unaltered due to the fact that parents and child are on the same belt and as such all are equally affected by the motion of belt.

Example 35 **A bird flies for 4s with a velocity $v = (t - 2)$ ms^{-1} in a straight line. Calculate the displacement of the bird and distance travelled by the bird.**

Solution. Velocity, $v = (t - 2)$

$$\therefore \text{ Acceleration } a = \frac{dv}{dt} = \frac{d}{dt}(t-2) = 1\ ms^{-2}$$

When $t = 0$, $v_0 = (0 - 2) = -2$ ms^{-1}

When $t = 2s$, $v = 2 - 2 = 0$

When $t = 4s$, $v = 4 - 2 = 2$ ms^{-1}

Taking motion of bird for time 0 to 2s, we have $u = v_0 = -2$ ms^{-1}, $a = 1$ ms^{-2},

$t = 2s$, $S = S_1$ (say)

$$S = ut + \frac{1}{2}at^2$$

$$\therefore\ S_1 = -2\times 2 + \frac{1}{2}\times 1\times 2^2 = -4 + 2 = -2\text{ m}$$

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Taking motion of bird from 2 to 4 s, we have

$u = 0, a = 1 \text{ ms}^{-2}, t = 2 \text{ s}, S = S_2$ (say)

$$S_2 = 0 \times 2 + \frac{1}{2} \times 1 \times 2^2 = 2 \text{ m}$$

$\therefore$ Displacement in 4 s $= S_1 + S_2 = -2 + 2 = \mathbf{0}$

Distance travelled in 4 s $= |S_1| + S_2$

$= 2 + 2 = \mathbf{4\ m}$

Example 36 **If *a*, *b*, *c* be the distances moved by a particle moving with a constant acceleration during the *l*th, *m*th and *n*th second of its motion respectively, show that**

$$\boldsymbol{a (m - n) + b (n - l) + c (l - m) = 0.}$$

Solution. Let u be the initial velocity of the particle and A be its uniform acceleration. Using the relation ; $D_n = u + \frac{a}{2}(2n - 1)$, we have

$$a = u + \frac{A}{2}(2l - 1) \quad \text{...}(i)$$

$$b = u + \frac{A}{2}(2m - 1) \quad \text{...}(ii)$$

and $$c = u + \frac{A}{2}(2n - 1) \quad \text{...}(iii)$$

Subtracting (*iii*) from (*ii*) ;

$$b - c = \frac{A}{2}(2m - 2n) = A\,(m - n)$$

or $a\,(m - n) = a\,(b - c)/A$

$= (ab - ac)/A$ ...(*iv*)

Subtracting (*i*) from (*iii*) ;

$$c - a = \frac{A}{2}(2n - 2l) = A\,(n - l)$$

or $b\,(n - l) = b\,(c - a)/A$

$= (bc - ab)/A$ ...(*v*)

Subtracting (*ii*) from (*i*) ;

$$a - b = \frac{A}{2}(2l - 2m) = A\,(l - m)$$

or $c\,(l - m) = c\,(a - b)/A = (ac - bc)/A$ ...(*vi*)

Adding (*iv*), (*v*) and (*vi*), we get

$a\,(m - n) + b\,(n - l) + c\,(l - m)$

$$= \frac{ab - ac}{A} + \frac{bc - ab}{A} + \frac{ac - bc}{A} = \mathbf{0}$$

Example 37 **A juggler throws balls into air. He throws one whenever the previous one is at its highest point. How high do the balls rise if he throws *n* balls each second. Acceleration due to gravity = *g*.**

Solution. Since the juggler is throwing n balls each second and he throws second ball when the first ball is at the highest point, so time taken by each ball to reach the highest point is $t = 1/n$.

Taking vertical upward motion of ball up to the highest point, we have,

$v = 0, a = -g, t = 1/n, u = ?$

As, $v = u + at$;

so $0 = u + (-g)\,1/n$

or $u = g/n$ ...(i)

Also, $v^2 = u^2 + 2\,as$,

so $0 = u^2 - 2\,gh$,

i.e., $h = (u^2/2\,g) = \mathbf{g/(2\,n^2)}$

Example 38 **The relation between time *t* and distance *x* is, $t = \alpha x^2 + \beta x$ where α and β are constants. Show that retardation is $2\,\alpha\,v^3$, where *v* is the instantaneous velocity.**

Solution. $t = \alpha x^2 + \beta x$. Differentiating it w.r.t. time t, we have

$$1 = \alpha\,2\,x \frac{dx}{dt} + \beta \frac{dx}{dt} = \frac{dx}{dt}(2\,\alpha\,x + \beta)$$

or $$\frac{dx}{dt} = v = \frac{1}{2\alpha x + \beta}$$

Again differentiating w.r.t. time t, we have

$$\frac{dv}{dt} = \text{acceleration} = \frac{-2\,\alpha\,v}{(2\,\alpha\,x + \beta)^2} = -2\,\alpha\,v^3 \ ;$$

so retardation $= \mathbf{2\,\alpha\,v^3}$

Example 39 **As soon as a car starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity-time graphs are shown in Fig. 3.49. Calculate**

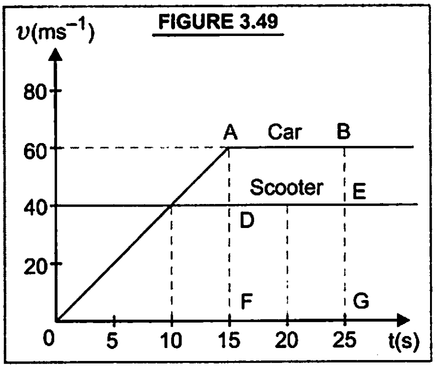


FIGURE 3.49

(*a*) the difference between the distances travelled by the car and the scooter in 25 s.

(*b*) the time when car will catch up the scooter.

(*c*) the distance of the car and scooter from the starting point at the meeting point.

Solution. (*a*) Distance travelled by the car in 25 s

$= \text{Area } OAF + \text{Area } ABGF$

$= \frac{1}{2} \times 15 \times 60 + 60 \times (25 - 15)$

$= 450 + 600 = 1050 \text{ m}$

Distance travelled by the scooter in 25 s

$= 25 \times 40 = 1000 \text{ m}$

Difference in the distances travelled

$= 1050 - 1000 = \mathbf{50 \text{ m}}$

(*b*) The distance travelled by car in 15 s

$= \text{area } OAF = \frac{1}{2} \times 15 \times 60 = 450 \text{ m}$

Distance travelled by scooter in 15 s

$= 40 \times 15 = 600 \text{ m}$

Let the car catch up the scooter in time $(15 + t)$ s

After 15 second, the velocity of car is constant. The distance travelled by car in time $(15 + t)$ s is

$= (450 + 60\,t) \text{ m}$

Distance travelled by scooter in time $(15 + t)$ is

$= 40\,(15 + t) \text{ m}$

The car will catch up the scooter, when

$(450 + 60\,t) = 40\,(15 + t)$

$= 600 + 40\,t$

or $t = 7{\cdot}5 \text{ s}$

$\therefore$ Total time, the car will catch up the scooter

$= 15 + 7{\cdot}5 = \mathbf{22{\cdot}5 \text{ s}}$

(*c*) Distance of car at meeting point distance of scooter from starting point in time 22·5 s

$= 22{\cdot}5 \times 40 = \mathbf{900 \text{ m}}$

REVISION AT A GLANCE

1. **Rest.** An object is said to be at rest if it does not change its position with time w.r.t its surroundings.
2. **Motion.** An object is said to be in motion if it changes its position with time, w.r.t. its surroundings.
3. **Rest and motion are relative.** It means an object in one situation is at rest and in another situation the same object is in motion.
4. **Point mass object.** An object can be considered as a point mass object if during motion in a given time, it covers a distance much greater than its own size.
5. **One dimensional motion.** The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with respect to time. In such a motion, an object moves along a straight line or on a well defined straight path.
6. **Two dimensional motion.** The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a plane.
7. **Three dimensional motion.** The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a space.
8. **Total path length.** It is the total distance travelled by an object during motion. It is defined as the length of the actual path transversed by an object during motion in a given interval of time. Path length is a scalar quantity. Its value can never be zero or negative, during the motion of an object.
9. **Displacement.** The displacement of an object in a given interval of time is defined as the change in position of the object along a particular direction during that time and is given by the vector drawn from the initial position to final position. The displacement of an object can be positive, zero or negative. The displacement of an object between two positions has a unique value, which is the shortest distance between them. The magnitude of the displacement of an object in a given time interval can be equal or less than the actual distance travelled but never greater than actual distance travelled.

10. **Speed.** The speed of an object is defined as the ratio of path length covered and time taken during the change of position of the object in any direction *i.e.* speed = distance travelled/(time taken). Speed is a scalar quantity. It can be zero or positive but never negative.
11. **Uniform Speed.** An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.
12. **Variable Speed.** An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.
13. **Average Speed.** The average speed of an object for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken *i.e.* Average speed = total distance travelled/total time taken.
14. **Instantaneous Speed.** The speed of an object at a given instant of time is called its instantaneous speed. It is also defined as the first derivative of distance w.r.t. time, at the given instant of time.
15. **Velocity.** The velocity of an object is defined as the ratio of displacement of the object and time taken *i.e.* velocity = displacement /time taken. Velocity = speed + direction. The velocity is a vector quantity. The velocity of an object can be positive, zero or negative.
16. **Uniform velocity and variable velocity.** If an object undergoes equal displacements in equal intervals of time, it is said to be moving with a uniform velocity. If an object undergoes unequal displacements in equal intervals of time or equal displacements in unequal intervals of time, it is said to be moving with a variable velocity.
17. **Average velocity.** The average velocity of an object is equal to the ratio of the total displacement, to the total time interval for which the motion takes place.
18. **Instantaneous velocity.** The velocity of an object at a given instant of time is called its instantaneous velocity. When a body is moving with a uniform velocity, its instantaneous velocity = average velocity = uniform velocity. Instantaneous velocity of an object is also defined as the first time derivative of displacement at the given instant.
19. **Uniform motion in a straight line.** An object is said to be in uniform motion if it undergoes equal displacement in equal intervals of time, howsoever small these intervals may be. The position of an object in uniform motion in a straight line is given by $\vec{x} = \vec{x}_0 + \vec{v}\,t$ where $\vec{x}_0$, $\vec{x}$ are the displacements of an object at timings $t = 0$ and $t = t$. And $\vec{v}$ is the uniform velocity of the object.
20. **Velocity-time graph of a uniform motion** in one dimension is a st. line parallel to time axis. The area enclosed by this graph with time axis measures the displacement of an object in a given interval of time.
21. **Position-time graph of a uniform motion in one dimension** is a straight line inclined to time axis. The slope of the position-time graph with time axis tells the velocity of the object.
22. **Relative velocity.** The relative velocity of one object w.r.t. another is the velocity with which one object moves w.r.t. another object. If $\vec{v}_A$ and $\vec{v}_B$ are the velocity of two objects A and B, and θ is the angle between them, then relative velocity of object A w.r.t. B is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

where,
$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180° - \theta)} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

and
$$\tan\beta = \frac{v_B \sin(180° - \theta)}{v_A + v_B \cos(180° - \theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

Here, β is the angle which $\vec{v}_{AB}$ makes with the direction of $\vec{v}_A$

23. Acceleration. The acceleration of an object is defined as the ratio of change in velocity of the object to the time taken *i.e.* Acceleration = change in velocity/time taken. Acceleration is a vector quantity. Acceleration is positive, if the velocity is increasing and is negative if velocity is decreasing. The negative acceleration is called ***retardation*** or ***deceleration.***

24. Uniform acceleration and variable acceleration. If the velocity of an object changes by equal amounts is equal intervals of time, it is said to be moving with a uniform acceleration.

If the velocity of an object changes by unequal amounts in equal intervals of time, it is said to be moving with a variable acceleration.

25. Average acceleration. The average acceleration of an object for a given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken *i.e.* average acceleration = total change in velocity/total time taken.

26. Instantaneous acceleration. The acceleration of an object at a given instant or at a given point of motion is called its instantaneous accelertion. It is defined as the first time derivative of velocity at a given instant of time or it is also equal to the second time derivative of the position of the object at a given instant of time *i.e.*

Instantaneous acceleration, $a_i = \underset{\Delta t \to 0}{\text{Limit}} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$

27. Formulae for uniformly accelerated motion along a straight line.

(*i*) $v = u + a\,t$

(*ii*) $S = ut + \frac{1}{2}at^2$

(*iii*) $v^2 = u^2 + 2\,aS$

(*iv*) $D_n = u + \frac{a}{2}(2n - 1)$

28. Velocity-time graph of a uniformly accelerated motion is a st. line inclined to time axis. The slope of this st. line tells the acceleration of the object and area enclosed by the st. line with time axis tells the distance travelled by object in a given time interval.

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. Two cars 1 and 2, starting from rest are moving with speeds v_1 and v_2 ($v_1 > v_2$). Car 2 is ahead of car 1 by 's', when the driver of car 1 sees car 2. What minimum retardation 'a' should be given to car 1 to avoid collision ?

(*a*) $\dfrac{v_1 - v_2}{s}$ (*b*) $\dfrac{v_1 + v_2}{s}$

(*c*) $\dfrac{(v_1 + v_2)^2}{2\,s}$ (*d*) $\dfrac{(v_1 - v_2)^2}{2\,s}$

2. An automobile travelling with a speed of 60 kmh^{-1}, can brake to stop within a distance of 20 m. If car is going twice as fast *i.e.*, 120 kmh^{-1}, the stopping distance will be

(*a*) 20 m (*b*) 40 m

(*c*) 60 m (*d*) 80 m

3. A bullet moving with a speed of 100 ms^{-1} can just penetrate two planks of equal thickness. Then, the number of such planks penetrated by the same bullet when the speed is doubled, will be

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 10

4. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time 't' and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is 15 s, then

(*a*) $s = ft$ (*b*) $s = 24\,ft$

(*c*) $s = \dfrac{ft^2}{36}$ (*d*) $s = \dfrac{ft^2}{72}$

5. A particle starts from rest and has an acceleration of 2 ms^{-2} for 10 s. After that, it travels for 30 s with constant speed and then undergoes a retardation of 4 ms^{-2} and comes back to rest. The total distance covered by the particle is

(*a*) 650 m (*b*) 700 m
(*c*) 750 m (*d*) 800 m

6. A particle (A) is dropped from a tower of height of 50 m and another particle (B) is thrown in horizontal direction with speed of 5 ms^{-1} from the same height. The correct statement is

(*a*) both particles will reach at ground simultaneously
(*b*) both particles will reach at ground with same speed
(*c*) particle (A) will reach at ground first with respect to particle (B)
(*d*) particle (B) will reach at ground first with respect to particle (A)

7. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from rest and from the same point O along three different frictionless paths. The speeds of the three objects on reaching the ground will be in the ratio of

(*a*) 1 : 1 : 1 (*b*) $m_1 : m_2 : m_3$
(*c*) $m_1 : 2\,m_2 : 3\,m_3$ (*d*) $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

8. A stone is released from the top of a tower. If its velocity at half of the height is 10 ms^{-1}, then height of the tower is (Take $g = 10\ ms^{-2}$)

(*a*) 8 m (*b*) 10 m
(*c*) 12 m (*d*) 16 m

9. A body projected vertically upwards with a velocity u returns to the starting point in 4 s. If $g = 10\ ms^{-2}$, the value of u is

(*a*) 5 ms^{-1} (*b*) 10 ms^{-1}
(*c*) 15 ms^{-1} (*d*) 20 ms^{-1}

10. A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What will be the position of the ball from the ground after $T/3$ seconds of its fall ?

(*a*) $\frac{h}{9}$ metres (*b*) $\frac{7h}{9}$ metres
(*c*) $\frac{8h}{9}$ metres (*d*) $\frac{17h}{18}$ metres

11. A body, thrown upwards with some velocity, reaches the maximum height of 20 m. Another body with double the mass thrown up, with double the initial velocity, will reach a maximum height of

(*a*) 40 m (*b*) 80 m
(*c*) 160 m (*d*) 200 m

12. A body thrown upwards with some velocity, reaches the maximum height of 100 m. Another body is thrown upwards with half the initial velocity will reach a maximum height of

(*a*) 200 m (*b*) 100 m
(*c*) 50 m (*d*) 25 m

13. When two particles each of mass 'm' are dropped from a height 'h' and '$2\,h$' respectively, then ratio of their times to reach the ground is

(*a*) 1 : 2 (*b*) 2 : 1
(*c*) $1 : \sqrt{2}$ (*d*) $\sqrt{2} : 1$

14. A particle covers 150 m in 8th second when starts from rest its acceleration is

(*a*) 5 ms^{-2} (*b*) 10 ms^{-2}
(*c*) 15 ms^{-2} (*d*) 20 ms^{-2}

15. From the top of a tower, a particle is thrown vertically downwards with a velocity of 10 ms^{-1}. The ratio of the distances covered by it in the 3rd and 2nd seconds of its motion is
(Take $g = 10\ ms^{-2}$)

(*a*) 5 : 7 (*b*) 7 : 5
(*c*) 3 : 2 (*d*) 2 : 3

16. A particle travelling with uniform acceleration in a straight line, travels 55 m in the 8th second and 85 m in the 13th second of its motion. The initial velocity and uniform acceleration of the particle is

(*a*) 10 ms^{-1} ; 6 ms^{-2} (*b*) 60 ms^{-1} ; 6 ms^{-2}
(*c*) 6 ms^{-1} ; 10 ms^{-2} (*d*) 10 ms^{-1} ; 60 ms^{-2}

17. The distance covered by a particle varies with time as : $x = at^2 - bt^3$. The time 't' of body, when its acceleration is zero, is

(*a*) $\frac{a}{b}$ (*b*) $\frac{b}{a}$
(*c*) $\frac{a}{3b}$ (*d*) $\frac{2a}{3b}$

18. A body moves along a straight line and its displacement at any time t is given by
$s = (t^3 - 6\,t^2 + 3\,t + 4)\ m$. The velocity of body, when its acceleration is zero, is

(*a*) – 9 ms^{-1} (*b*) – 15 ms^{-1}
(*c*) 3 ms^{-1} (*d*) 42 ms^{-1}

19. A 120 m long train is moving towards west at a speed of 10 ms^{-1}. A bird, flying towards east at a speed of 5 ms^{-1}, crosses the train. What is the time taken by the bird to cross the train ?

(*a*) 8 s (*b*) 10 s

(*c*) 12 s (*d*) 16 s

20. A police van moving on a highway with a speed of 36 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 72 kmh^{-1}. If muzzle speed of the bullet is 150 km h^{-1}, with what speed does the bullet hits the thief's car ?

(*a*) 80 kmh^{-1} (*b*) 120 kmh^{-1}

(*c*) 114 kmh^{-1} (*d*) 160 kmh^{-1}

21. What is the value of acceleration due to gravity at the highest point if a body is projected vertically upward.

(*a*) 9·81 m/s^2 (*b*) 9·81 m/s

(*c*) $0 < a < 9{\cdot}81$ m/s^2 (*d*) $a > 9{\cdot}81$ m/s^2

22. A boy completes a semicircle of radius 14 m in 10 seconds. The distance travelled by the boy is

(*a*) 22 m (*b*) 14 m

(*c*) 44 m (*d*) 88 m

23. A helicopter is raising upward with a velocity of 2 m/s and a food packet is dropped from helicopter. What will be the initial velocity of food packet for the observer on the ground ?

(*a*) – 2 m/s (*b*) – 4 m/s

(*c*) 0 m/s (*d*) + 2 m/s

24. Two trains *A* and *B* are moving with a velocity of 60 km/h and 40 km/h respectively towards north direction. What is the relative velocity of train *B* w.r.t. train *A*

(*a*) Zero

(*b*) 20 km/h towards south

(*c*) 20 km/h towards north

(*d*) 100 km/h towards north

25. An stone is dropped from a tower of height '*H*' m. It take '*T*' *s* to reach the ground. Its speed after *T*/4 s.

(*a*) $\sqrt{2gH}$ m/s (*b*) $2\sqrt{gH}$ m/s

(*c*) $\sqrt{\frac{gH}{2}}$ m/s (*d*) $\frac{1}{2}\sqrt{\frac{gH}{2}}$ m/s

26. A boy throws a stone upwards from a stationary lift with velocity *u* m/s which comes back to lift in t_1 second. If he throws another stone upward with same velocity *u* m/s from a lift moving up with constant velocity *u*/2 m/s and it take a time t_2 to reach back to lift. Which of the following correctly represent the relation between t_1 and t_2 ?

(*a*) $t_1 > t_2$ (*b*) $t_1 < t_2$

(*c*) $t_1 = t_2$ (*d*) $t_2 = \frac{3}{2} t_1$

27. The position of a particle is given by $x = (-2t^3 + 3t^2 + 5)$ *m*. Where '*x*' is in meter and '*t*' is in second. Find the acceleration of particles when its velocity is zero.

(*a*) 6 m/s^2 (*b*) – 6 m/s^2

(*c*) 12 m/s^2 (*d*) – 6 m/s

28. Free fall is a case of motion with uniform acceleration. What is the value of acceleration ?

(*a*) 9·8 m/s^2 (*b*) Zero

(*c*) – 9·8 m/s^2 (*d*) 1

29. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is

(*a*) 10 (*b*) 9

(*c*) 8 (*d*) 7

30. The distance *x* covered by a particle in one dimensional motion varies with time *t* as $x^2 = at^2 + 2bt + c$, where *a*, *b* and *c* are constants. Acceleration of the particle depends on *x* as x^{-n}, the value of *n* is

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) none of the above

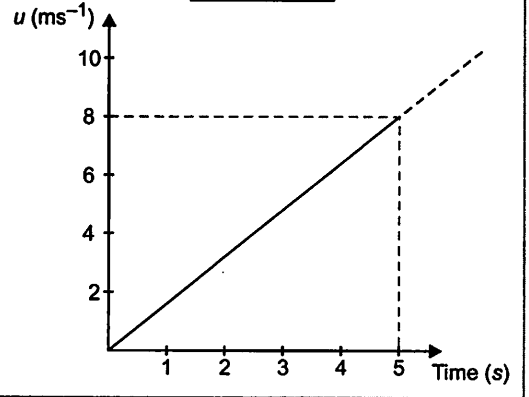
31. A particle starts from the origin at $t = 0$, with an initial velocity of $\vec{u} = 3\hat{i}$ from the origin and moves in the *x*-*y* plane with a constant acceleration, $\vec{a} = (6\hat{i} + 4\hat{j})$ m/s^2. The *x*-coordinate of the particle at the instant when its *y*-coordinate is 32 m is D meters. The value of *D* is

(*a*) 70 m (*b*) 65 m

(*c*) 60 m (*d*) 50 m

32. The speed verses time graph for a particle is shown in the Fig. 3(Q).1, The distance travelled (in *m*) by the particle during the time interval $t = 0$ to $t = 5$ s will be

FIGURE 3(Q).1



(*a*) 40 m (*b*) 20 m

(*c*) 60 m (*d*) 30 m

33. A car moving in a straight line at 30 ms^{-1} slows uniformly to a speed of 10 ms^{-1} in 5 second. Find the acceleration of the car.

(*a*) 2 ms^{-2} (*b*) 4 ms^{-2}

(*c*) – 2 ms^{-2} (*d*) – 4 ms^{-2}

34. In the above question, displacement in the third second is

(*a*) 20 m (*b*) 15 m

(*c*) 30 m (*d*) 18 m

35. The motion of a particle is described by the equation $v = at$, where a is a constant and equals 8 ms^{-2}. The distance travelled by the particle in first 4 seconds is

(*a*) 32 m (*b*) 64 m

(*c*) 36 m (*d*) 72 m

36. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest assuming that is faces constant resistance to motion.

(*a*) 1·5 cm (*b*) 1·0 cm

(*c*) 2·0 cm (*d*) 1·6 cm

37. A person walks a distance of 40 m towards west with a speed of 1·5 ms^{-1} and 30 m towards north with a speed of 2 ms^{-1}. The average speed of the journey is

(*a*) 1·75 ms^{-1} (*b*) 1·68 ms^{-1}

(*c*) 1·85 ms^{-1} (*d*) 1·70 ms^{-1}

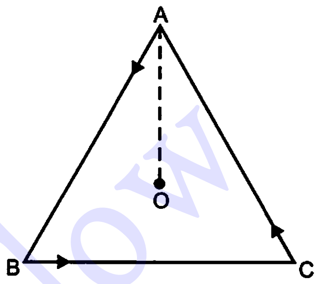
38. In above question, the average velocity for his journey is

(*a*) 1·68 ms^{-1} (*b*) 1·45 ms^{-1}

(*c*) 1·28 ms^{-1} (*d*) 1·20 ms^{-1}

39. Three persons A, B and C are initially at the three corners of an equilateral triangle of side a. Each person now moves with same speed v, such that A moves towards B, B towards C and C towards A always. Those three persons meet after the time

FIGURE 3(Q).2



(*a*) $\dfrac{a}{v}$ (*b*) $\dfrac{3a}{v}$

(*c*) $\dfrac{2a}{3v}$ (*d*) $\dfrac{3a}{2v}$

40. A particle moves along a straight line such that its displacement x varies with time t as $x = a + bt + ct^2$, where a, b and c are constants. The average velocity of particle during 3rd second would be

(*a*) $b + 7c$ (*b*) $b + 5c$

(*c*) $b + 4c$ (*d*) $4b + c$

41. A food packet is released from a helicopter which is rising at 4 ms^{-1}. Then, the velocity of the packet after 2 seconds is (g = 10 ms^{-2})

(*a*) 20 ms^{-1} upwards (*b*) 20 ms^{-1} downwards

(*c*) 16 ms^{-1} upwards (*d*) 16 ms^{-1} downwards

42. A balloon in going vertically upwards with a velocity of 10 ms^{-1}. When it is at 75 m above the ground, a stone is released, the height of balloon above the ground when the stone reaches the ground is (g = 10 ms^{-2})

(*a*) 25 m (*b*) 75 m

(*c*) 125 m (*d*) 150 m

43. A particle is projected vertically upwards is at a certain height at time 4 seconds and again at time 6 seconds, then the velocity of projection is :
(g = 10 ms^{-2})

(*a*) 25 ms^{-1} (*b*) 40 ms^{-1}

(*c*) 50 ms^{-1} (*d*) 80 ms^{-1}

44. In the above question, height of the point from the ground, where the particle is crossing while going up or coming down is

(*a*) 100 m (*b*) 120 m
(*c*) 150 m (*d*) 200 m

45. A body is projected vertically upwards from ground. In the last two seconds of its ascent distance travelled by it will be numerically equal to

(*a*) g (*b*) $g/2$
(*c*) 2 g (*d*) 4 g

46. A freely falling body crossess the points A, B and C with velocities v, $2\,v$ and $3\,v$. Then $AB : AC$ is equal to

(*a*) 1 : 2 (*b*) 1 : 3
(*c*) 1 : 25 (*d*) 3 : 8

47. A body starts from rest and moves with a uniform acceleration of 10 ms^{-2} in the first 10 seconds. During the next 10 seconds it moves with the maximum velocity attained. The total displacement of the body is

(*a*) 1000 m (*b*) 1500 m
(*c*) 2000 m (*d*) 2500 m

48. A train starts from rest with an acceleration of 2 ms^{-2}. A man who is 9 m behind the train runs with uniform speed and just manages to get into the train in 7·5 seconds. The speed of the man is

(*a*) 4·5 ms^{-1} (*b*) 8·7 ms^{-1}
(*c*) 9·4 ms^{-1} (*d*) 12·7 ms^{-1}

49. The displacement of the particle moving in one dimension under the action of a constant force is related to the time t by the equation, $t = \sqrt{x} + 3$, where x is in metres and t is in seconds. The displacement of the particle when the velocity is zero is

(*a*) 1·5 m (*b*) 2·0 m
(*c*) 2·5 m (*d*) zero

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both A and R are true, and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

50. Assertion. Earth revolving around the sun is a three dimensional motion.

Reason. For earth revolving around the sun, x, y and z-axis coordinates are changing with time.

51. Assertion. Displacement can be equal to or less than the distance travelled by object in motion.

Reason. Displacement tells the shape of path followed by the object in motion.

52. Assertion. Displacement is a vector quantity.

Reason. Distance is a scalar quantity.

53. Assertion. A man walks 4 m to the north, 12 m to west and finally climbs up a vertical pole upto a height of 3 m. His displacement will be 13 m.

Reason. Displacement $= \sqrt{x^2 + y^2 + z^2}$.

54. Assertion. A person walks a distance 30 m towards west with a speed of 2 ms^{-1} and 40 m towards north with a speed of 1·5 ms^{-1}. The average velocity of his journey is 1·2 ms^{-1}.

Reason. Average velocity $= \dfrac{\text{distance travelled}}{\text{time taken}}$.

55. Assertion. The acceleration of a body has the direction of velocity.

Reason. If velocity increases, the acceleration also increases.

56. Assertion. The acceleration of a particle in motion is increasing with time.

Reason. The displacement of a particle is proportional to the cube of time.

57. Assertion. A body has a constant speed but it possesses varying velocity.

Reason. Speed is a scalar and velocity is a vector quantity.

58. Assertion. A body moving with a uniform speed may have an acceleration.

Reason. A body moving on a circular path with constant time period has centripetal acceleration.

59. Assertion. A body is projected vertically up from the ground. Distance travelled by it in the last second of its upward journey is independent of the initial velocity.

Reason. Distance travelled in *n*th second of its vertical motion is, $D_n = u - \frac{g}{2}(2n-1)$.

60. Assertion. The shape of displacement – time graph for a body projected vertically upwards is a curve with decreasing slope.

Reason. For upward motion, the displacement is,

$$S = ut - \frac{1}{2}gt^2$$

61. Assertion. Acceleration of a body due to gravity in vacuum is same irrespective of size and mass of the body.

Reason. The acceleration due to gravity depends upon the mass and radius of the earth.

62. Assertion. A stone is falling freely from rest for 5 seconds. The total distance covered by it in the last second of its motion equals the distance covered in the first three seconds.

Reason. Distance travelled in *n*th second is,

$$D_n = u + \frac{a}{2}(2n-1).$$

63. Assertion. A body covers first half of distance with speed v_1 and second half of distance with speed v_2, then the average speed is, $2v_1 v_2/(v_1 + v_2)$.

Reason. Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

64. Assertion. The displacement of a freely falling body in successive seconds is in the ratio 1 : 3 : 5.

Reason. It is so if the body is moving with uniform velocity.

65. Assertion. A hydrogen filled balloon stops rising after it has attained certain height in the sky.

Reason. The atmospheric pressure decreases with height and becomes zero when maximum height is attained.

ANSWERS

I. Multiple Choice Questions

1. (*d*)	**2.** (*d*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*a*)	**7.** (*a*)	**8.** (*b*)	**9.** (*d*)	**10.** (*c*)
11. (*c*)	**12.** (*d*)	**13.** (*c*)	**14.** (*d*)	**15.** (*b*)	**16.** (*a*)	**17.** (*c*)	**18.** (*a*)	**19.** (*a*)	**20.** (*c*)
21. (*a*)	**22.** (*c*)	**23.** (*d*)	**24.** (*b*)	**25.** (*d*)	**26.** (*c*)	**27.** (*b*)	**28.** (*c*)	**29.** (*c*)	**30.** (*b*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*a*)	**35.** (*b*)	**36.** (*b*)	**37.** (*b*)	**38.** (*d*)	**39.** (*c*)	**40.** (*b*)
41. (*d*)	**42.** (*c*)	**43.** (*c*)	**44.** (*b*)	**45.** (*c*)	**46.** (*d*)	**47.** (*b*)	**48.** (*b*)	**49.** (*d*)	

II. Assertion-Reason Type Questions

50. (*d*)	**51.** (*c*)	**52.** (*b*)	**53.** (*a*)	**54.** (*c*)	**55.** (*d*)	**56.** (*a*)	**57.** (*b*)	**58.** (*a*)	**59.** (*b*)
60. (*a*)	**61.** (*b*)	**62.** (*a*)	**63.** (*a*)	**64.** (*c*)	**65.** (*b*)				

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. Initial relative velocity of car 1 w.r.t. car 2 is $(v_1 - v_2)$. Distance between the two cars is *s*. Let *a* be the acceleration of car 1 to avoid collision. Final relative velocity of car 1 is zero.

$$\therefore \quad 0 = (v_1 - v_2)^2 + 2as \text{ or } -a = \frac{(v_1 - v_2)^2}{2s}$$

$$\therefore \text{ Retardation } = -a = \left(\frac{v_1 - v_2^2}{2s}\right)$$

2. Let *F* be the retarding force on automobile, when break is applied. Then loss in K.E. = workdone.

Case I. $\frac{1}{2}mv_1^2 = Fs_1$ ...(*i*)

Case II. $\frac{1}{2}mv_2^2 = Fs_2$ ...(*ii*)

Dividing (*ii*) by (*i*), we get

$$\frac{s_2}{s_1} = \frac{v_2^2}{v_1^2} = \left(\frac{120}{60}\right)^2 = 4$$

or $s_2 = 4s_1 = 4 \times 20 = \mathbf{80\ m}$

3. Let *x* be the thickness of each plank and *n* be the no. of planks penetrated by bullet when its speed is doubled. Then

$$\frac{1}{2}\,\mathrm{m}\,(100)^2 = F \times 2x \qquad ...(i)$$

and $$\frac{1}{2}\,\mathrm{m}\,(200)^2 = F \times nx \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{n}{2} = \frac{(200)^2}{(100)^2} = 4 \quad \text{or} \quad n = \mathbf{8}$$

4. During acceleration of car ; $u = 0,\ a = f,$

$$S = S,\ v = v_1$$

As $v^2 = u^2 + 2\,aS$, so, $v_1^2 = 0 + 2\,f\,S = 2\,f\,S$

or $$v_1 = \sqrt{2\,f\,S}$$

During uniform motion of car ; distance travelled is

$$S_1 = v_1 t = \sqrt{2\,f\,S}\ t$$

During deceleration of car ;

$$u = v_1 = \sqrt{2\,f\,S},\ a = -f/2, v = 0, S = S_2$$

As $v^2 - u^2 = 2\,aS$

or $0^2 - (\sqrt{2\,f\,S})^2 = 2\,(-f/2)\,S_2$

or $$S_2 = 2\,S$$

Given ; $S + S_1 + S_2 = 15\ S$

$\therefore\quad S + (\sqrt{2\,f\,S})\,t + 2\,S = 15\,S$

or $(\sqrt{2\,f\,S})\ t = 12S$ or $2\,f\,S\,t^2 = 144\ S^2$

or $$S = \frac{2\,ft^2}{144} = \frac{\mathbf{ft^2}}{\mathbf{72}}$$

5. Let S, S_1, S_2 be the distances travelled by particle with acceleration, with uniform speed and then with uniform retardation to come to rest.

Velocity of particle moving with uniform acceleration

$$v = at = 2 \times 10 = 20 \text{ m/s}$$

Distance travelled,

$$S = \frac{1}{2}\,a_1 t^2 = \frac{1}{2} \times 2 \times 10^2 = 100 \text{ m}$$

Distance travelled with a uniform speed v is

$$S_1 = v\ t_1 = 20 \times 30 = 600 \text{ m}$$

Distance travelled with a constant retaration is

$$S_2 = \frac{v^2}{2a_2} = \frac{(20)^2}{2 \times 4} = 50 \text{ m}$$

Total distance travelled $= S + S_1 + S_2$

$$= 100 + 600 + 50 = \mathbf{750\ m}$$

6. As both the particles have zero vertical downward velocity and will cover same vertical distance before reaching to ground hence will reach ground simultaneously.

7. Let h be the vertical height of each object while falling from rest at a height h from ground. The speed of the object on reaching ground is $v = \sqrt{2\,gh}$. As h is same for each of the falling object, hence speed of three objects on reaching the ground will be the same. Hence **option (*a*) is true.**

8. Let h be the height of tower. Velocity of stone at height $h/2$ is

$$v = \sqrt{2\,gh/2} = \sqrt{gh}$$

or $$h = \frac{v^2}{g} = \frac{(10)^2}{10} = \mathbf{10\ m}$$

9. As body is going vertically upward and then vertically downwards, before returning to starting point, so displacement, $S = 0$. Here, $u = ?$, $a = -\,10\ \mathrm{ms^{-2}}$, $t = 4\ s$,

As $S = ut + \frac{1}{2}\,at^2$

$\therefore\quad 0 = u \times 4 + \frac{1}{2}\,(-10)\ (4)^2$ or $u = 20\ \mathrm{ms^{-1}}$

10. Here, $h = \frac{1}{2}\,g\,T^2$

Let x be the vertical distance of fall of body in time $T/3$ seconds, then

$$x = \frac{1}{2}\,g\ (T/3)^2 = \left(\frac{1}{2}\,g\,T^2\right)\frac{1}{9} = \frac{h}{9}$$

$\therefore$ Vertical height of ball from ground

$$= h - x = h - h/9 = \mathbf{8\ h/9\ metres}$$

11. As, $$\frac{1}{2}\,mu^2 = F \times S$$

$$\therefore\quad \frac{1}{2}\,mu^2 = F \times 20 \qquad ...(i)$$

and $$\frac{1}{2}\,2m(2u)^2 = F \times S_1 \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we have

$$\frac{S_1}{20} = 8 \quad \text{or} \quad S_1 = 20 \times 8 = \mathbf{160\ m}$$

12. $h = \dfrac{u^2}{2g}$;

$$h_1 = \frac{(u/2)^2}{2g} = \frac{1}{4}\frac{u^2}{2g} = \frac{h}{4} = \frac{100}{4} = \mathbf{25\ m}$$

13. $t_1 = \sqrt{2h/g}$ and $t_2 = \sqrt{2(2h)/g} = \sqrt{2}\, t_1$

$$\therefore \quad \frac{t_1}{t_2} = \frac{1}{\sqrt{2}}$$

14. Here $D_8 = 150$ m ; $u = 0$, $a = ?$

As $\quad D_n = u + \dfrac{a}{2}(2n - 1)$

$$\therefore \quad 150 = 0 + \frac{a}{2}(2 \times 8 - 1)$$

or $\quad a = \dfrac{150 \times 2}{15} = \mathbf{20\ m/s^2}$

15. $$\frac{D_3}{D_2} = \frac{u + \frac{a}{2}(3 \times 2 - 1)}{u + \frac{a}{2}(2 \times 2 - 1)} = \frac{u + 5a/2}{u + 3a/2}$$

$$= \frac{10 + 5 \times 10/2}{10 + 3 \times 10/2} = \frac{35}{25} = \frac{7}{5}$$

16. $D_8 = u + \dfrac{a}{2}(2 \times 8 - 1) = 55 \quad \ldots(i)$

$D_{13} = u + \dfrac{a}{2}(2 \times 13 - 1) = 85 \quad \ldots(ii)$

Solving (*i*) and (*ii*), we get,

$\mathbf{a = 6\ ms^{-2}}$, $\mathbf{u = 10\ ms^{-1}}$

17. $x = at^2 - bt^3$;

Velocity, $v = \dfrac{dx}{dt} = 2at - 3bt^2$

Acceleration, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(2at - 3bt^2)$

$$= 2a - 6bt$$

When, $a = 0$, then $0 = 2a - 6bt$ or $\mathbf{t = a/3b}$

18. $S = t^3 - 6t^2 + 3t + 4$

Velocity, $v = \dfrac{dS}{dt} = 3t^2 - 12t + 3$

Acceleration, $a = \dfrac{dv}{dt} = 6t - 12$

When, $a = 0$, then $6t - 12 = 0$ or $t = 2$ *s*.

$\therefore \quad v = 3 \times 2^2 - 12 \times 2 + 3 = \mathbf{-9\ ms^{-1}}$

19. Relative velocity of bird w.r.t. train $= v_b + v_T$

$$= 5 + 10 = 15\ \text{m s}^{-1}$$

Time taken by the bird to cross the train of length 120 m is

$$t = \frac{120}{15} = \mathbf{8\ s}$$

20. Speed of police van w.r.t. ground,

$$v_{PG} = 36\ \text{km h}^{-1}$$

Speed of thief car w.r.t. ground,

$$v_{TG} = 72\ \text{km h}^{-1}$$

Speed of bullet w.r.t. police van,

$$v_{BP} = 150\ \text{km h}^{-1}$$

Speed with which the bullet will hit the thief's car will be

$$v_{BT} = v_{BG} + v_{GT} = v_{BP} + v_{PG} + v_{GT}$$
$$= v_{BP} + v_{PG} - v_{TG} \quad [\because v_{GT} = -v_{TG}]$$
$$= 150 + 36 - 72$$
$$= \mathbf{114\ km\ h^{-1}}$$

21. As the height attained by a vertically projected body will be very small as compared to radius of earth, so the value of acceleration due to gravity can be taken constant *i.e.*, **9·81 ms^{-2}**.

22. Distance travelled $= \pi r = \dfrac{22}{7} \times 14 = \mathbf{44\ m}$

23. Velocity of helicopter w.r.t. ground,

$$v_{HG} = 2\ \text{ms}^{-1}$$

Initial velocity of food packet dropped from helicopter w.r.t. ground,

$$v_{FG} = v_{FH} + v_{HG} = 0 + v_{HG}$$
$$= v_{HG} = 2\ \text{ms}^{-1}$$

Velocity of food packet w.r.t. observer on ground,

$$v_{FO} = v_{FG} = \mathbf{+\ 2\ ms^{-1}}$$

24. : $\quad v_A = 60$ km h^{-1} towards north ;

$v_B = 40$ km h^{-1} towards north

$v_{BA} = v_B - v_A = 40 - 60$

$= -20$ km h^{-1} towards north

= 20 km h^{-1} towards south

25. $T = \sqrt{2H/g}$;

$v = u + at = 0 + g\ (T/4)$

$$= g \times \frac{1}{4}\sqrt{\frac{2H}{g}} = \frac{\mathbf{1}}{\mathbf{2}} \times \sqrt{\frac{\mathbf{gH}}{\mathbf{2}}}$$

26. In both the cases, the displacement of stone w.r.t. lift is zero. Therefore $\mathbf{t_1 = t_2}$.

27. Here, $x = -2\,t^3 + 3\,t^2 + 5$

Velocity, $v = \dfrac{dx}{dt} = -6t^2 + 6t$; when $v = 0$,

then, $\quad 0 = -6\,t^2 + 6\,t \quad$ or $\quad t = 1$ s.

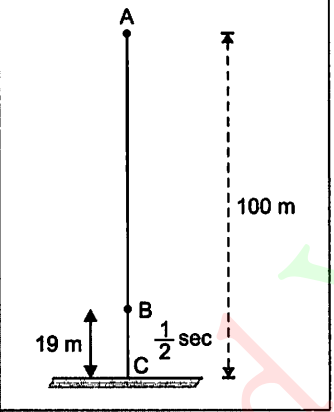
Acceliteration, $a = \dfrac{dv}{dt} = -12\,t + 6$

$$= -12 \times 1 + 6 = \mathbf{-6\ ms^{-2}}$$

28. In free fall a body is under the acceleration of earth, which is the negative direction of y-axis. So value of $g = -9{\cdot}8\ \text{ms}^{-2}$.

29. Let t be the time taken by ball to fall vertical height 100 m (= AC) and g be the acceleration due to gravity on the surface of that planet. Fig. 3(Q).3, then

FIGURE 3(Q).3



$$100 = \frac{1}{2}\,gt^2 \qquad \text{...}(i)$$

The velocity of the ball at B will be

$$v = \sqrt{2 \times g \times (100 - 19)}$$

$$= \sqrt{2 \times g \times 81}$$

Taking vertical downward motion of the ball from A to B, we have,

$$(100 - 19) = \frac{1}{2}\,g\left(t - \frac{1}{2}\right)^2 \qquad \text{...}(ii)$$

Dividing (ii) by (i), we get

$$\frac{81}{100} = \frac{(t - 1/2)^2}{t^2} = \frac{t^2 - t + 1/4}{t^2}$$

or $\quad 81\,t^2 = 100\,t^2 - 100\,t + 25$

or $\quad 19\,t^2 - 100\,t + 25 = 0$

On solving, $t = 5$ s.

From (i), $\quad g = \dfrac{2 \times 100}{t^2} = \dfrac{2 \times 100}{5^2} = \mathbf{8\ ms^{-2}}$

30. Given, $\quad x^2 = at^2 + 2\,bt + c$

Differentiating it w.r.t., time, we have

$$2x\frac{dx}{dt} = 2at + 2b \quad \text{or} \quad xv = at + b \qquad \text{...}(i)$$

$$\left[\because \frac{dx}{dt} = v = \text{velocity}\right]$$

$$\therefore \quad v = \frac{(at + b)}{x} \qquad \text{...}(ii)$$

Differentiating (i) w.r.t. t, we have

$$\frac{dx}{dt}\,v + x\frac{dv}{dt} = a$$

or $\; v\,.\,v + x\,A = a \qquad \left[\dfrac{dv}{dt} = A = \text{acceleration}\right]$

or $\quad x\,A = a - v^2 = a - \left(\dfrac{at + b}{x}\right)^2$

$$= \frac{ax^2 - (at + b)^2}{x^2}$$

$$\therefore \text{ Acceleration, } A = \frac{a\,(at^2 + 2bt + c) - (at + b)^2}{x^3}$$

$$= \frac{ac - b^2}{x^3} = (ac - b^2)\,x^{-3}$$

$\therefore \; A \propto x^{-3}$; Given $\; A \propto x^{-n}$, hence $n = 3$

31. Taking motion of the particle along y-axis, we have

$$y = u_y\,t + \frac{1}{2}\,a_y\,t^2$$

or $\; 32 = 0 \times t + \dfrac{1}{2} \times 4 \times t^2 \;$ or $\; t^2 = 16$ or $t = 4$ s

Taking motion of particle along x-axis, we have

$$x = u_x\,t + \frac{1}{2}\,a_x\,t^2 = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$$

$$= 12 + 48 = \mathbf{60\ m}$$

32. Distance travelled = area under speed-time graph

$$= \frac{1}{2} \times 8 \times 5 = \mathbf{20\ m}$$

33. Here, $u = 30\ \text{ms}^{-1}$, $v = 10\ \text{ms}^{-1}$, $t = 5$ s, $a = ?$

As, $v = u + at \;\therefore\; a = \dfrac{v - u}{t} = \dfrac{10 - 30}{5} = \mathbf{-4\ ms^{-2}}$

34. As, $D_n = u + \frac{a}{2}(2n-1)$

Here, $n = 3$, then $D_3 = 30 + \frac{(-4)}{2}[2\times3-1]$

$= 30 - 10 = \mathbf{20\ m}$

35. For the motion given by $v = at = 8\,t$, the acceleration a (= 8 ms^{-2}) is constant throughout the motion. When $t = 0$, $u = a \times 0 = 0$.

Distance travelled in t (= 4) seconds is

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 8 \times (4)^2 = \mathbf{64\ m}$$

36. Case I, Let u be the initial velocity of the bullet fired from gun. Then, $u = u$, $v = u/2$, $S = 3$ cm,

As, $v^2 = u^2 + 2\,a\,s$

$$\therefore \left(\frac{u}{2}\right)^2 = u^2 + 2a\times 3$$

or $$6a = \frac{u^2}{4} - u^2 = \frac{-3u^2}{4} \quad \text{or} \quad a = -\frac{u^2}{8}$$

Let the bullet further penetrate through distance x before coming to stop. Then

$u = u/2$, $v = 0$, $a = -u^2/8$, $s = x$

As, $v^2 = u^2 + 2\,a\,s$

$$\therefore\ 0 = \left(\frac{u}{2}\right)^2 + 2\left(\frac{-u^2}{8}\right)\times x \quad \text{or} \quad \frac{2u^2}{8}x = \frac{u^2}{4}$$

or $x = \mathbf{1\ cm}$

37. Distance travelled along west = 40 m ;

time taken $= \frac{40}{1{\cdot}5} = \frac{80}{3}$ s

Distance travelled along north = 30 m,

time taken $= \frac{30}{2} = 15$ s.

Total distance travelled = 40 + 30 = 70 m.

Total time taken $= \frac{80}{3} + 15 = \frac{125}{3}$ s

$\therefore$ Average speed $= \frac{70}{125/3} = \frac{42}{25} = \mathbf{1{\cdot}68\ ms^{-1}}$

38. Displacement along west, $x = 40$ cm,

Time taken $= \frac{40}{1{\cdot}5} = \frac{80}{3}$ s

Displacement along, y north = 30 m

Time taken $= \frac{30}{2} = 15$ s

Total displacement $= \sqrt{x^2 + y^2}$

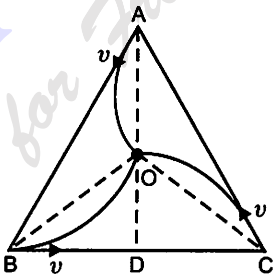
$= \sqrt{40^2 + 30^2} = 50$ m

Total time taken $= \frac{80}{3} + 15 = \frac{125}{3}$ s

Average velocity $= \frac{50}{125/3} = \frac{6}{5} = \mathbf{1{\cdot}20\ ms^{-1}}$

39. Those three persons will meet at the centroid O of the triangle ABC. Here each person would move along covering spiral like path as shown in Fig. 3(Q).4. It means the person A will be covering a distance AO before meeting the other persons. The velocity of the person at A along $AO = v\cos 30° = v\sqrt{3}/2$

FIGURE 3(Q).4



Distance $AO = \frac{2}{3}AD = \frac{2}{3}\sqrt{a^2 - a^2/4} = \frac{a}{\sqrt{3}}$

$\therefore$ time taken by a person at A to go from A to O

$$= \frac{AO}{v\cos 30°} = \frac{a/\sqrt{3}}{v\sqrt{3}/2} = \mathbf{\frac{2a}{3v}}$$

40. Comparing the given equation, $x = a + bt + ct^2$ with the equation, $x = x_0 + ut + \frac{1}{2}At^2$, we have $u = b$, $A = 2\,c$ (which is constant throughout the motion). Average velocity during 3rd second is the distance travelled in 3rd second.

$$D_3 = u + \frac{A}{2}(2\times3-1)$$

$$= b + \frac{2c}{2}(5) = \mathbf{b + 5\,c}$$

41. Taking vertically downward motion of the food packet for time 2 seconds, we have

$u = -4\ ms^{-1}$, $a = g = 10\ ms^{-2}$, $v = ?$, $t = 2$ s

As, $v = u + at$ $\therefore v = -4 + 10\times 2 = \mathbf{16\ ms^{-1}}$

42. Taking vertically downward motion of stone released from balloon at a height of 75 m upto the ground, we have

$u = -10\ \text{ms}^{-1}$, $a = g = 10\ \text{ms}^{-2}$, $S = 75$ m ; $t = ?$

As, $$S = ut + \frac{1}{2}at^2$$

$$\therefore \quad 75 = -10t + \frac{1}{2}\times 10t^2$$

or $5t^2 - 10t - 75 = 0$

or $t^2 - 2t - 15 = 0$

On solving, $t = 5$ s or -2 s. Time can not be negative, hence, $t = 5$ s.

Height gained by balloon in 5 seconds

$= 10 \times 5 = 50$ m

$\therefore$ Total height of balloon from ground $= 75 + 50$

$=$ **125 m**

43. Let S be the height of the particle from ground at timings 4 seconds and 6 seconds when projected vertically upwards with velocity u from ground

Then, $$S = u\times 4 - \frac{1}{2}\times 10\times 4^2$$

$$= u\times 6 - \frac{1}{2}\times 10\times 6^2$$

or $S = 4u - 80 = 6u - 180$ ...(*i*)

or $4u - 80 = 6u - 180$

or $2u = 100$

or $u = \mathbf{50\ ms^{-1}}$

44. From above solution, $u = 50\ \text{ms}^{-1}$.

$\therefore \quad S = 4u - 80 = 4 \times 50 - 80$

$=$ **120 m**

45. We known that the distance covered by body in a given time while going upwards is equal to the distance covered by body while coming downwards. Taking vertically downward motion of body from highest position for 2 seconds, we have

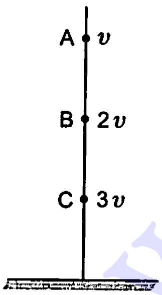
$u = 0$, $a = g$, $S = ?$, $t = 2$ s.

As, $$S = ut + \frac{1}{2}at^2$$

$$\therefore \quad S = 0\times 2 + \frac{1}{2}\times g\times 2^2 = 2\text{ g}$$

46. Refer to Fig. 3(Q).5 for a freely falling body from A to B,

FIGURE 3(Q).5



$$(2v)^2 = v^2 + 2g \times (AB)$$

or $$AB = \frac{3v^2}{2g}$$

For the body falling from A to C,

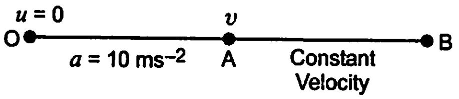
$$(3v)^2 = v^2 + 2g \times (AC)$$

or $$AC = \frac{8v^2}{2g}$$

$$\therefore \quad \frac{AB}{AC} = \frac{3}{8}$$

47. Refer to Fig. 3(Q).6,

FIGURE 3(Q).6



Let $OA = S_1$ and $AB = S_2$

Taking motion of the body from O to A, for 10 s, we have

$u = 0$, $a = 10\ \text{ms}^{-2}$, $S = S_1$, $t = 10$ s, $v = ?$

As, $$S = ut + \frac{1}{2}at^2$$

$$\therefore \quad S_1 = 0\times 10 + \frac{1}{2}\times 10\times 10^2 = 500\text{ m}$$

$v = u + at = 0 + 10 \times 10 = 100\ \text{ms}^{-1}$

Taking motion of body from A to B for 10 s, we have

$S_2 = v \times 10 = 100\times 10 = 1000$ m

$\therefore$ Total displacement $= S_1 + S_2$

$= 500 + 1000 =$ **1500 m**

48. Taking motion of train for 7·5 s, we have

$u = 0,\ a = 2\ \text{ms}^{-2},\ t = 7{\cdot}5\ \text{s},\ S = ?$

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 2\times(7{\cdot}5)^2 = \frac{225}{4}\ \text{m}$$

Total distance to be travelled by man with uniform speed v to catch the train in 7·5 s

$$= 9 + \frac{225}{4} = \frac{261}{4}\ \text{m}$$

$$\therefore \text{ Speed of man } = \frac{261/4}{7{\cdot}5} = \frac{261}{30} = \mathbf{8{\cdot}7\ ms^{-1}}$$

49. Given, $t = \sqrt{x} + 3$

or $\quad x = (t-3)^2 = t^2 - 6\,t + 9 \qquad \ldots(i)$

Differentiating it w.r.t. t, we get

$$\frac{dx}{dt} = 2\,t - 6\times 1 + 0 = 2\,t - 6$$

when velocity $\left(= \dfrac{dx}{dt}\right)$ becomes zero, then

$$0 = 2\,t - 6 \quad \text{or} \quad t = 3\ \text{s}$$

Putting value in (i),

displacement, $x = 3^2 - 6\times 3 + 9 = \mathbf{0\ m}$

II. Assertion-Reason Type Questions

50. **Both Assertion and Reason are wrong.** Earth revolving around the sun is two dimensional motion.

51. Here **Assertion is true but Reason is false** as the distance travelled tells the shape of path followed and not the displacement.

52. **Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.**

53. **Both Assertion and Reason are true and Reason is the correct explanation of Assertion** because displacement $= \sqrt{4^2 + 12^2 + 3^2} = 13\,\text{m}$.

54. Time taken for displacement 30 m along west

$$= \frac{30}{2} = 15\,\text{s}$$

Time taken for displacement 40 m along north

$$= \frac{40}{1{\cdot}5} = \frac{80}{3}\,\text{s}$$

$$\text{Net displacement } = \sqrt{30^2 + 40^2} = 50\,\text{s}$$

$$\text{Total time taken } = 15 + \frac{80}{3} = \frac{125}{3}\,\text{s}$$

$$\text{Average velocity } = \frac{\text{displacement}}{\text{time taken}}$$

$$\text{Average velocity } = \frac{50}{125/3} = \frac{150}{125} = \frac{6}{5} = 1{\cdot}2\ \text{ms}^{-1}$$

Thus **Assertion is true but Reason is false.**

55. Acceleration of a body has the direction of change in velocity. If velocity increases, the acceleration may or may not change. Thus **both Assertion and Reason are false.**

56. From Reason, $S \propto t^3$ or $S = kt^3$

$$\text{Velocity, } v = \frac{dS}{dt} = \frac{d\,(kt^3)}{dt} = 3\,kt^2$$

$$\text{Acceleration, } A = \frac{dv}{dt} = \frac{d\,(3\,kt^2)}{dt} = 6\,kt \quad \therefore A \propto t$$

Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

57. If a body is moving on a circular path with a definite period, its speed is constant but velocity is varying from point to point of its motion. Hence **both Assertion and Reason are true but Reason is not the correct explanation of Assertion.**

58. **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

59. Distance travelled in last second of its vertical upward motion = distance travelled in one second while falling from its highest position.

Here, $u = 0,\ a = g,\ t = 1$ s, so

$$S = ut + \frac{1}{2}at^2 = 0\times 1 + \frac{1}{2}\,g\times 1^2 = \frac{g}{2}$$

Here, **both Assertion and Reason are true but the Reason is not the correct explanation of Assertion.**

60. For a body projected vertically upwards, the displacement at time t is $\quad S = ut - \dfrac{1}{2}\,gt^2$

Slope of displacement – time graph at time t is,

$$\frac{dS}{dt} = \frac{d}{dt}\left(ut - \frac{1}{2}\,gt^2\right) = u - gt$$

This shows that as t increases slope (dS/dt) decreases. **Thus both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

61. Acceleration due to gravity is independent of size and mass of the body but depends upon the mass and radius of earth. **Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.**

62. Here, $u = 0$, $n = 5$, $D_5 = 0 + \frac{g}{2}(2 \times 5 - 1) = \frac{9g}{2}$.

Distance travelled in first second is

$$D_1 = 0 \times 1 + \frac{1}{2} g\,(2 \times 1 - 1) = \frac{g}{2}$$

Distance travelled in 2nd second is

$$D_2 = 0 \times 2 + \frac{1}{2} \times g\,(2 \times 2 - 1) = \frac{3g}{2}$$

Distance travelled in 3rd second is

$$D_3 = 0 \times 2 + \frac{1}{2} \times g\,(2 \times 3 - 1) = \frac{5g}{2}$$

$$\therefore D_1 + D_2 + D_3 = \frac{g}{2} + \frac{3g}{2} + \frac{5g}{2} = \frac{9g}{2} = D_5$$

Thus **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

63. Total time taken to travel a distance S is

$$= \frac{d/2}{v_1} + \frac{d/2}{v_2} = \frac{d}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$$

$$\text{Average speed} = \frac{d}{\frac{d}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]} = \frac{2 v_1 v_2}{v_1 + v_2}$$

Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

64. A freely falling body is moving with a uniform acceleration and its displacement in nth second is $D_n = u + \frac{a}{2}(2n - 1)$

$$\therefore D_1 = 0 + \frac{g}{2}(2 \times 1 - 1) = \frac{g}{2}$$

$$D_2 = 0 + \frac{g}{2}(2 \times 2 - 1) = \frac{3g}{2}$$

$$D_3 = 0 + \frac{g}{2}(2 \times 3 - 1) = \frac{5g}{2}$$

$$\therefore D_1 : D_2 : D_3 = 1 : 3 : 5$$

Thus Assertion is true but Reason is false.

65. Both Assertion and Reason are true but Reason is not the correct explanation of Assertion as the hydrogen filled balloon will stop rising when weight of balloon becomes equal to the weight of air displaced by balloon.

CONCEPTUAL PROBLEMS

I. Displacement and Velocity

1. Can an object be at rest as well as in motion at the same time ? Explain with illustration.

Sol. Yes, an object can be at rest with respect to one object and can be in motion with respect to another object. *e.g.* A body lying on earth is at rest w.r.t. earth but the same body is in motion with respect to a person on another planet because earth and the planet are in relative motion. Thus rest and motion are relative terms.

2. State in the following cases, whether the motion is one, two or three dimensional motion :

(a) a kite flying on a windy day (b) a speeding car on a long straight high way (c) a carrom coin rebounding from the side of the board (d) a planet revolving around its star.

Sol. (*a*) The motion of kite in the sky is **three dimensional motion** because the kite while flying may be changing the direction as well as losing or gaining the height, *i.e.*, the motion is in space.

(*b*) The motion of speeding car on a long straight highway is **one dimensional motion**, since the motion is confined along a st. line.

(*c*) The motion of a carrom coin rebounding from the side of the board is **two dimensional motion**, since the motion is confined in the plane of board.

(*d*) The motion of a planet revolving around its star is **two dimensional motion**, becuase the motion is confined in a fixed plane.

3. An object is in uniform motion along a straight line. What will be position-time graph for the motion of the object if

(a) $x_0 = +$ ve, $v = +$ ve

(b) $x_0 = +$ ve, $v = -$ ve

(c) $x_0 = -$ ve, $v = +$ ve and

(d) both x_0 and v are negative ? The letters x_0 and v represent position of the object at time $t = 0$ and uniform velocity of the object respectively.

Sol. The position of the object at any time t while moving with uniform velocity along a straight line is given by, $x = x_0 + v\,t$

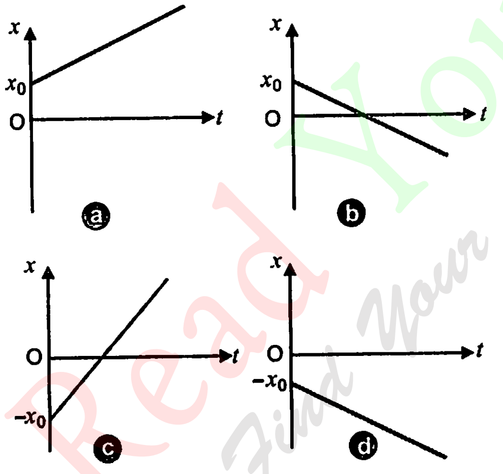
(*a*) If $x_0 = +$ ve and $v = +$ ve, position - time graph will be as shown in Fig. 3(Q).7(a)

(*b*) If $x_0 = +$ ve and $v = -$ ve, the position - time graph will be as shown in Fig. 3(Q).7(b).

(*c*) If $x_0 = -$ ve and $v = +$ ve, the position - time graph will be as shown in Fig. 3(Q).7(c).

(*d*) If both x_0 and v are, negative, the position - time graph will be as shown in Fig. 3(Q).7(d).

FIGURE 3(Q).7

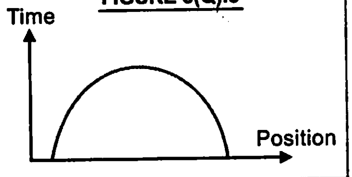


4. A drunkard walking in a narrowlane takes 5 steps forward and 3 steps backward each step of 1 m long, per second and so on. Determine how long the drunkard takes to fall in a pit 15 m away from the start.

Sol. The effective distance travelled by drunkard in 8 steps = 5 – 3 = 2 m. Therefore, he takes 40 steps to move 10 metres. Now he will have to cover 5 metres more to reach the pit, for which he has to take only 5 forward steps. Therefore, he will have to take = 40 + 5 = 45 steps to move 15 metres. Thus he will fall into the pit after taking 45 steps, *i.e.*, **after 45 seconds** from the start.

5. Is the time variation of position, shown in Fig. 3(Q).8 observed in nature ?

FIGURE 3(Q).8



Sol. No, because in the given variation, with the increase of position, the time first increases and then decreases which is impossible.

6. A body covers one-third of its journey with speed v_1, next one-third with speed v_2 and last one-third with speed v_3. Calculate the average speed of the body during the entire journey.

Sol. Let s be the total distance travelled by body. Total time taken,

$$t = \frac{s/3}{v_1} + \frac{s/3}{v_2} + \frac{s/3}{v_3} = \frac{s}{3}\left[\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}\right]$$

$$= \frac{s}{3}\left[\frac{v_2 v_3 + v_1 v_3 + v_1 v_2}{v_1 v_2 v_3}\right]$$

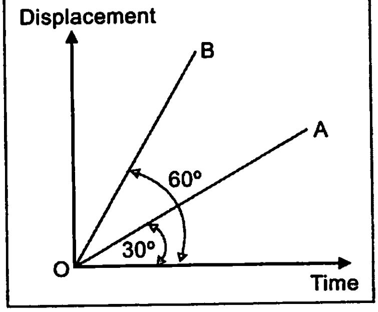
$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{s}{\frac{s}{3}\left[\frac{v_2 v_3 + v_1 v_3 + v_1 v_2}{v_1 v_2 v_3}\right]}$$

$$= \frac{3 v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

7. Two straight lines drawn on the same displacement - time graph make angles 30° and 60° with time-axis respectively Fig. 3(Q).9. Which line represents greater velocity ? What is the ratio of two velocities ?

FIGURE 3(Q).9



Sol. Since the slope of the displacement – time graph of uniform motion in one dimension represents the velocity of the object, hence the line showing greater slope in graph corresponds to greater velocity of the object. Therefore, **the line making angle 60° with time axis represents greater velocity :**

Ratio of two velocities ;

$$\frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

8. A body covered a distance of *l* metre along a semicircular path. Calculate the magnitude of displacement of the body and the ratio of distance to displacement.

Sol. Let r be the radius of semicircular path.

Here, $l = (2\pi r/2)$

or $r = l/\pi$.

Diameter $= 2r = 2l/\pi$;

Magnitude of displacement $=$ diameter

$= \mathbf{2\,l/\pi}$

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{l}{2l/\pi} = \frac{\pi}{2}$$

9. For ordinary terrestrial experiments, which of the observers below are inertial and which are non-inertial ? (a) a child revolving in a giant wheel. (b) a driver in a sports car moving with a constant high speed of 200 km/h on a straight road. (c) the pilot of an aeroplane which is taking off. (d) a cyclist negotiating a sharp turn. (e) the guard of a train which is slowing down to stop at a station.

Sol. An inertial observer is one which is unaccelerated (*i.e.*, $a = 0$). A non inertial observer is an accelerated observer. On this basis, we find that

(*a*) a child revolving in a giant wheel is a **non-inertial observer.**

(*b*) a driver with a constant speed is an **inertial observer.**

(*c*) at take off, aeroplane has an acceleration. Therefore, the pilot is a **non-inertial observer.**

(*d*) a cyclist negotiating a sharp turn has an acceleration. Therefore, he is a **non-inertial observer.**

(*e*) the guard of the train is a **non-inertial observer,** as the train in this case has retardation.

10. A person goes to post-office slowly and purchases post cards. Then he comes back steadily. Draw the time-velocity and time-displacement graphs for the person.

Sol. For a person, the time-velocity graph is as shown in Fig. 3(Q).10(a). Here the areas A and B are equal but of opposite signs because the net displacement of the person in going to post-office and coming back is zero.

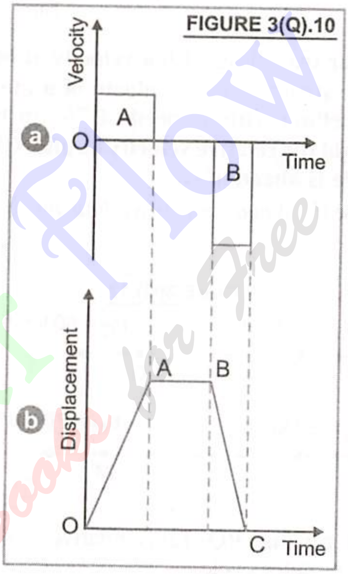


FIGURE 3(Q).10

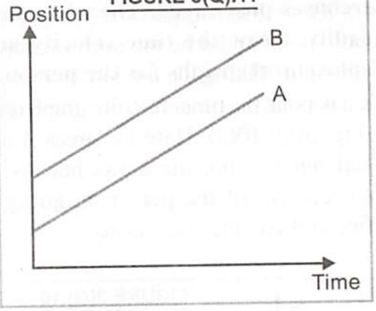
The time-displacement graph is as shown in Fig. 3(Q).10(b). In this graph, as the velocity of person in going is less than that of return journey, hence the slope of portion OA is smaller than that of portion BC because the slope of time - displacement graph gives the velocity.

II. Relative Velocity

11. Draw position–time graphs for two objects having zero relative velocity.

Sol. Let us consider the two objects be at different positions when $t = 0$. As the relative velocity of the two objects is zero, therefore, the two objects must be moving with the same & uniform velocity in the same direction. Since the slope of position–time graph of uniform motion in a st. line represents velocity, hence the position-time graphs for two objects in motion having zero relative velocity are parallel straight lines equally inclined to time axis as shown in Fig. 3(Q).11.

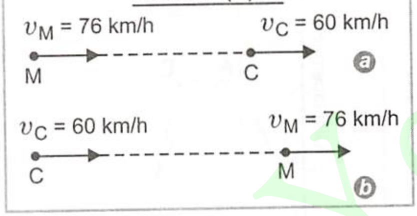
FIGURE 3(Q).11



12. A car travelling with a velocity of 60 km h^{-1} on a straight road is ahead of a motor cycle travelling with a speed of 76 km h^{-1}. How would the relative velocity be altered if motor cycle is ahead of car ?

Sol. **Case (*i*).** *When car is travelling ahead the motor cycle.*

FIGURE 3(Q).12



Refer to Fig. 3(Q).12(*a*), relative velocity of car w.r.t. motor cycle,

$$v_{CM} = v_C - v_M = 60 - 76 = \mathbf{-16\ km\ h^{-1}}$$

Case (*ii*). *When motor cycle is travelling ahead of the car.*

Refer to Fig. 3(Q).12(*b*), relative velocity of car w.r.t. motor cycle,

$$v_{CM} = v_C - v_M = 60 - 76 = -16\ \text{km h}^{-1}$$

It means relative velocity will remain the same as it does not depend on the position of the two bodies. But in case (*i*), the separation between car and motor cycle will be decreasing. But in case (*ii*), the separation between car and motor cycle will be increasing.

13. When a person is standing on earth, the trees and houses appear stationary to him. However, when he is sitting in a running train all these objects appear to move in backward direction. Why ?

Sol. For a person standing on earth, the relative velocity of trees and houses with respect to person is zero. For a person sitting in a running train, the relative velocity of trees and houses with respect to person is negative

$$[\because v_{TP} = v_T - v_P = 0 - v_P = -v_P]$$

That is why, the trees and houses appear to move in backward direction to a person sitting in a running train.

14. Wind is blowing west to east along two parallel tracks. Two trains moving with the same speed in opposite directions on these tracks have the steam tracks. If one stream track is double than the other, what is the speed of each train ?

Sol. Let v and w be the velocity of each train and wind respectively. As per question, $(v + w) = 2(v - w)$ or $v = 3w$, *i.e.*, velocity of each train is **3 times the velocity of wind.**

III. Accelerated motion

15. The direction in which an object moves is given by the direction of velocity of the object and not by the direction of acceleration. Explain this statement with suitable example.

Sol. When an object is thrown up, the direction of motion of the object and hence, its velocity are along vertical in upward direction. As the object moves up, it is always attracted by earth in downward direction, *i.e.*, the acceleration is in vertical downward direction. Hence, the direction of motion of the object is that of velocity and not that of acceleration.

16. From top of a building a ball is dropped while another is projected horizontally at the same time. (a) Which ball will strike the ground first ? (b) Which ball will strike the ground with more speed.

Sol. As the horizontal velocity does not affect the vertical motion and initial vertical velocity of both the ball is zero, so both the balls will be reaching the ground simultaneously, *i.e.*,

$$t_1 = t_2 = \sqrt{2h/g}$$

Velocity of the ball dropped vertically while reaching the ground is

$$v_1 = \sqrt{2gh}$$

For the ball projected horizontally, the horizontal component velocity $v_{H_2} = u$ and vertical component velocity, $v_{V_2} = \sqrt{2gh}$.

So the total velocity of ball,

$$v_2 = \sqrt{v_{H_2}^2 + v_{V_2}^2} = \sqrt{u^2 + 2gh}$$

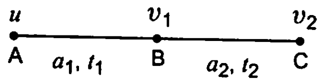
which is greater than v_1.

It means the ball projected horizontally will strike the ground, with more speed than the ball dropped vertically downwards.

17. A body travels along a straight line with uniform acceleration a_1 for time t_1 and with uniform acceleration a_2 for time t_2. What is the average acceleration ?

Sol. Refer to Fig. 3(Q).13, motion from A to B is in time t_1 and from B to C in time t_2.

FIGURE 3(Q).13



As change in velocity = acceleration × time interval

so, $v_1 - u = a_1 t_1$

and $v_2 - v_1 = a_2 t_2$

Adding the above equations, we get

$$v_2 - u = a_1 t_1 + a_2 t_2 \quad \text{...(i)}$$

If a_{av} is the average acceleration for the motion from A to C, then

$$v_2 - u = a_{av}(t_1 + t_2) \quad \text{...(ii)}$$

From (*i*) and (*ii*),

$$a_{av}(t_1 + t_2) = a_1 t_1 + a_2 t_2$$

$$\therefore \quad a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$$

18. The driver of a train moving at a speed v_1, sights another train at a disance d, ahead of him moving in the same direction with a slower speed v_2. He applies the brakes and gives a constant retardation a to his train. Show that there will be no collision if $d > (v_1 - v_2)^2/2a$.

Sol. For no collision of two trains, the relative velocity of faster train w.r.t slower train $(v_1 - v_2)$ should become zero, while travelling a relative displacement d with retardation a.

Here ; $u = v_1 - v_2$; $v = 0$;

$a = -a$; $S = d$.

As, $v^2 = u^2 + 2aS$,

so, $0 = (v_1 - v_2)^2 + 2(-a)d$

or $d = (v_1 - v_2)^2/2a$

There will be no collision if

$$d > (v_1 - v_2)^2/2a$$

19. Two ends of a train moving with a constant acceleration pass a certain point with velocities u and v. Show that the velocity with which the middle point of the train passes the same point is $\sqrt{(u^2 + v^2)/2}$.

Sol. Let x be the total length of the train, V be the velocity of the train while passing a certain middle point and a be the uniform acceleration of the train. Taking the motion of the train when middle point is passing from the given point, we have

$$u = u, v = V, S = x/2 ; a = a$$

Using, $v^2 = u^2 + 2aS$, we have

$$V^2 = u^2 + 2ax/2 = u^2 + ax \quad \text{...(i)}$$

Taking the motion of train when the last end of train is passing from the given point, then

$$u = u, v = v, a = a, S = x$$

Now, we have,

$$v^2 = u^2 + 2ax \quad \text{or} \quad ax = \frac{v^2 - u^2}{2}$$

Putting this value in (*i*), we get

$$V^2 = u^2 + \frac{v^2 - u^2}{2} = \frac{u^2 + v^2}{2}$$

or $V = \sqrt{(u^2 + v^2)/2}$

20. A passenger is standing 'd' metres away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed v towards the bus. What must be the minimum speed of the passenger so that he may catch the bus.

Sol. Let the passenger catches the bus after time t. Distance travelled by the bus in time t,

$$S_1 = 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad \text{...(i)}$$

Distance travelled by the passenger,

$$S_2 = vt + 0 = vt \quad \text{...(ii)}$$

The passenger will catch the bus if,

$$d + S_1 = S_2$$

or $d + \frac{1}{2}at^2 = vt$ or $\frac{1}{2}at^2 - vt + d = 0$

or $$t = \frac{[v \pm \sqrt{v^2 - 2ad}]}{a/2}$$

The passenger will catch the bus if t is real,

i.e., $v^2 \geq 2ad$ or $v \geq \sqrt{2ad}$

Thus, minimum speed of passenger for catching the bus is $\sqrt{2ad}$

21. In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes that point. Assuming that the cars start from rest and travel with constant acceleration a_1 and a_2. Find the relation between v, t, a_1 and a_2.

Sol. Let S be the distance to be covered by each car. Let t_1, t_2 be the times taken by cars A and B to complete their journey and v_1, v_2 be their velocities at the finishing point. According to given problem

$$v_1 - v_2 = v \quad \text{and} \quad t_2 - t_1 = t$$

As, distance travelled = average velocity × time interval

when $u = 0$, so

$$S = \frac{0+v_1}{2}t_1 = \frac{0+v_2}{2}t_2 \quad \text{or} \quad S = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

or $v_1 = 2\,S/t_1$ and $v_2 = 2\,S/t_2$

Also $S = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$

or $t_1 = \sqrt{\frac{2S}{a_1}}$ and $t_2 = \sqrt{\frac{2S}{a_2}}$

Now

$$\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{(2\,S/t_1) - (2\,S/t_2)}{t_2 - t_1} = \frac{2\,S}{t_1 t_2}$$

$$= \frac{2\,S}{\sqrt{\frac{2\,S}{a_1}} \times \sqrt{\frac{2\,S}{a_2}}} = \sqrt{a_1 a_2}$$

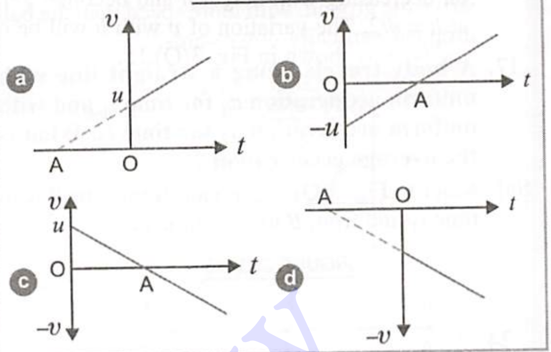
or $v = t\sqrt{a_1 a_2}$

IV. Graphs for uniformly accelerated motion

22. An object has a uniformly accelerated motion. The object always slows down before the time, when its velocity becomes zero. Establish this statement graphically when (*i*) both initial velocity (u) and acceleration (a) are positive (*ii*) u is negative and a is positive (*iii*) u is positive and a is negative and (*iv*) both u and a are negative.

Sol. (*i*) **When both u and a are positive.** In this case, the $v-t$ graph is a straight line as shown in Fig. 3(Q).14(a). In this graph, at the time corresponding to point A, velocity of object becomes zero. From the graph, it is clear that before the time corresponding to point A, the velocity is negative but its negative magnitude decreases with time till it becomes zero at point A.

FIGURE 3(Q).14



(*ii*) **When u is negative and a is positive.** In this case, the v–t graph is a straight line as shown in Fig. 3(Q).14(b). At the time, corresponding to point A, the velocity becomes zero. From the graph, it is clear that before this time, the velocity is negative but its negative magnitude decreases with time till it becomes zero at point A.

(*iii*) **When u is positive and a is negative.** In this case, the v–t graph is a straight line shown in Fig. 3(Q).14(c). At the time corresponding to point A, velocity is zero. From graph, it is clear that before this time, corresponding to point A, the velocity decreases.

(*iv*) **When both u and a are negative.** In this case, the v–t graph is a straight line shown in Fig. 3(Q).14(d). If we produce this straight line graph backwards, it meets the time axis at point A for which velocity is zero. Before this time, velocity is positive and decreases till it becomes zero at point A.

23. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Show how its velocity v varies with height above the ground before and after the bounce, neglecting subsequent motion and air resistance.

Sol. If a ball falls from a height d and reaches at height h then using the relation,

$$v^2 = u^2 + 2\,a\,S, \text{ we have } u = 0,\ S = (d-h),$$

then $v^2 = 2\,g\,(d-h)$

This relation shows that the variation of v and h is a parabolic one.

As the ball falls down, its velocity goes on increasing and becomes maximum at $h = 0$.

At $h = 0$, the direction of velocity is reversed.

Then the ball moves upwards, its velocity goes on decreasing with height h and becomes zero at $h = d/2$. The variation of v with h will be of the type as shown in Fig. 3(Q).15.

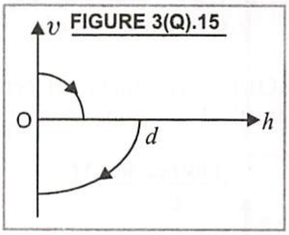


FIGURE 3(Q).15

24. Two boys are standing at the ends A and B of a ground, where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in time t. Find the value of t.

Sol. Let the boy at A catches the boy B at point C after time t, Fig. 3(Q).16.

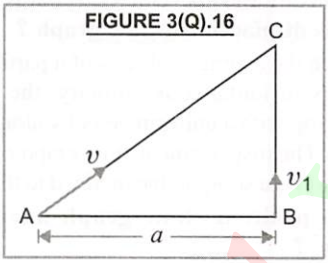


FIGURE 3(Q).16

Here, $AC = vt,\ BC = v_1 t$

As $AC^2 = AB^2 + BC^2$

$\therefore$ $(vt)^2 = a^2 + (v_1 t)^2$

or $t^2 (v^2 - v_1^2) = a^2$ or $t = \dfrac{a}{\sqrt{v^2 - v_1^2}}$

V. Calculus method for accelerated motion

25. If the initial velocity of a particle is u and collinear acceleraion at any time t is at, calculate the velocity of the particle after time t.

Sol. By definition, acceleration $= \dfrac{dv}{dt} = at$

or $dv = at\,dt$

Integrating it, we have $\displaystyle\int_u^v dv = \int_0^t at\,dt$

or $v - u = \dfrac{1}{2}at^2$ or $v = u + \dfrac{1}{2}at^2$

26. The driver of a car travelling at a velocity v suddenly sees a broad wall in front of him at a distance r. Is it better to brake or to turn sharply ?

Sol. *It is better to apply brake than to turn sharply for the reason discussed below.*

Let m be the mass of the car. When the driver applies the brakes, let the car stop at distance x. Then retardation, $a = v^2/2\,x$

Retarding force, $F = ma = \dfrac{mv^2}{2x}$ or $x = \dfrac{m\,v^2}{2\,F}$

There will be no collision if $x \le r$

or $\dfrac{m\,v^2}{2\,F} \le r$ or $F \ge \dfrac{m\,v^2}{2\,r}$ ...(*i*)

If the driver takes a sharp turn of radius x, then centripetal force on car is,

$F' = \dfrac{m\,v^2}{x}$ or $x = \dfrac{m\,v^2}{F'}$

To avoid collision, $x \le r$ so

$\dfrac{m\,v^2}{F'} \le r$ or $F' \ge \dfrac{m\,v^2}{r}$ ...(*ii*)

From (*i*) and (*ii*) we note that $F = F'/2$

It means to avoid collision braking force required is half the centripetal force. Therefore, **braking is better.**

27. The acceleration experienced by a boat after the engine is cut off, is given by $\dfrac{dv}{dt} = -k\,v^3$, where k is a constant. If v_0 is the magnitude of the velocity at cut off, find the magnitude of the velocity at time t after the cut off.

Sol. Given, $\dfrac{dv}{dt} = -k\,v^3$ or $v^{-3}\,dv = -k\,dt$

Integrating it within the conditions of motion *i.e.* as time changes from 0 to t, velocity changes from v_0 to v, we have

$$\int_{v_0}^{v} v^{-3}\,dv = -k\int_0^t dt \quad \text{or} \quad \left[\frac{v^{-2}}{-2}\right]_{v_0}^{v} = -k\,(t)_0^t$$

or $-\dfrac{1}{2}\left[\dfrac{1}{v^2} - \dfrac{1}{v_0^2}\right] = -kt$

or $\dfrac{1}{v^2} = 2\,kt + \dfrac{1}{v_0^2} = \dfrac{2\,kt\,v_0^2 + 1}{v_0^2}$

or $v = \dfrac{v_0}{\sqrt{1 + 2\,kt\,v_0^2}}$

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Displacement and Velocity

1. Mention the condition when an object in motion (*a*) can be considered point object (*b*) can not be considered point object.

Ans. An object can be considered as a point object if (*a*) its size is negligble in comparison to the distance travelled by object (*b*) its size is not negligible in comparison to the distance travelled by object.

2. Can a body have a constant speed and still have a varying velocity ? (HP Board 2006 C)

Ans. Yes ; If a body is in a uniform circular motion, it has a constant speed but varying velocity as the direction of motion of body at different points on circular motion is different.

3. Which speed is measured by the speedometer of your scooter ?

Ans. The speedometer measures the instantaneous speed of the scooter at a given instant of time.

4. Why is the speed, in general, greater than the magnitude of the velocity ?

Ans. If there is a change in the direction of motion of a body, then the actual path length in a given time is greater than the displacement in that time. Therefore speed (= path length/time taken) is greater than the magnitude of velocity (= displacement/time taken).

5. Can a particle in one-dimensional motion have zero speed and a non-zero velocity ?

Ans. No ; If the speed of a particle is zero, then its velocity will always be zero, since velocity = speed + direction.

6. What does the tangent at a point to the position–time graph for an object in non-uniform motion along a straight line represent ?

Ans. Instantaneous speed or instantaneous velocity at the given instant.

7. What will be the nature of position - time graph for a uniform motion ?

Ans. A straight line inclined to time axis.

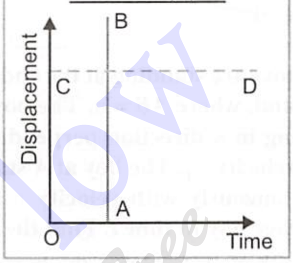
8. If the displacement - time graph of a particle is parallel to

(*a*) displacement axis (*b*) the time axis,

what will be the velocity of the particle ?

Ans. Fig. 3(Q).17. (*a*) Infinity (*b*) Zero. Infact it is not a practical situation.

FIGURE 3(Q).17



9. The average velocity of a particle is equal to its instantaneous velocity. What is the nature of its displacement-time graph ?

Ans. When the average velocity of a particle is equal to its instantaneous velocity, the particle is moving with a uniform velocity along a straight line. The displacement-time graph of a uniform motion is a straight line inclined to the time axis.

10. Can position - time graph have negative slope ?

Ans. Yes, when the velocity of the object is negative.

11. What do you understand by positive and negative time.

Ans. The instant of time which is taken after the origin of time (*i.e.,* zero time) is called positive time and the instant of time which is taken before the origin of time is called negative time.

12. Can a body have a constant velocity but a varying speed ?

Ans. No, since velocity = speed + direction ; therefore a body can not possess a constant velocity but a varying speed.

13. The body travels a distance s_1 with velocity v_1 and s_2 with velocity v_2 in the same direction. Calculate the average velocity of the body.

Ans. Time taken for distance (s_1) is given by,

$$t_1 = s_1/v_1$$

Time taken for distance (s_2) is given by,

$$t_2 = s_2/v_2$$

Total distance covered = $s_1 + s_2$;

Total time taken = $t_1 + t_2$

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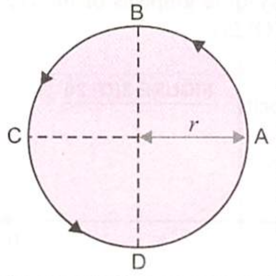
$$\therefore \quad \text{Average velocity} = \frac{\text{total distance}}{\text{total time taken}}$$

$$= \frac{s_1 + s_2}{t_1 + t_2} = \frac{s_1 + s_2}{(s_1 / v_1) + (s_2 / v_2)}$$

$$= \frac{(s_1 + s_2)\, v_1\, v_2}{s_1\, v_2 + s_2\, v_1}$$

14. A cyclist moves on a circular track from *A* to *D* in time *t* as shown in Fig. 3(Q).18. What is the average speed and average velocity of the cyclist.

FIGURE 3(Q).18



Ans. Distance travelled in time $t = ABCD = \frac{3\pi r}{2}$

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{3\pi r/2}{t}$$

$$= \frac{3\pi r}{2t}$$

Displacement in time $t = AD = \sqrt{r^2 + r^2} = r\sqrt{2}$

$$\text{Average velocity} = \frac{r\sqrt{2}}{t}$$

15. A boy walks to his school at a distance of 6 km with constant speed of 2·5 km h^{-1} and walks back with a constant speed of 4 kmh^{-1} What is average speed for the round trip in kmh^{-1} ?

Ans. Total time in going and coming back,

$$T = \frac{6}{2{\cdot}5} + \frac{6}{4} \; ;$$

total distance travelled,

$$S = 6 + 6 = 12 \text{ km} \; ;$$

$$\text{Average speed} = \frac{S}{T} = \frac{12}{(6/2{\cdot}5) + (6/4)}$$

$$= \frac{40}{13} \text{ km h}^{-1}$$

II. Relative Velocity

16. Can the relative velocity of two bodies be greater than the absolute velocity of either body.

Ans. Yes, when two bodies are moving in opposite directions along a straight path, then the relative velocity of each body is greater than the absolute velocity of either body.

17. Though the rain is falling vertically downwards, the front screen of a moving car gets wet while the back screen remains dry. Why ?

Ans. It is due to the fact that the rain strikes the car in the direction of relative velocity of rain with respect to car.

III. Accelerated motion

18. Can the direction of velocity of a body change, when acceleration is constant ?

Ans. Yes, when the body moves vertically upwards, after reaching the highest point, the body starts falling down, *i.e.,* the direction of velocity is reversed, whereas the acceleration is constant.

19. Light travels in a straight line at a constant speed of 3 × 10^8 m/s. What is the acceleration of light ?

Ans. As speed of light is constant along a st. line. Hence, its acceleration is zero.

20. Why does time occur twice in a unit of accleration ?

Ans. $$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{(\text{change in position/time taken})}{\text{time taken}}$$

$$= \frac{\text{change in position}}{(\text{time taken})^2}$$

21. Is the acceleration of a car greater when the accelerator is pushed to the floor or when brake pedal is pushed hard ?

Ans. Acceleration of a car is greater when brake pedal is pushed hard, because car suddenly comes to rest, *i.e.,* the rate of change of velocity of car is large.

22. Which of the two ; velocity and acceleration, gives the direction of motion of the body. Explain it with the help of an illustration.

Ans. It is the velocity of body and not the acceleration which gives the direction of motion of the body. When a body is projected vertically upwards, its velocity and direction of motion are upwards but its acceleration is vertically downwards.

23. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with same initial speed. Which one will rise to the greater height ?

Ans. When a ball is projected vertically upward with speed u, the maximum height attained, $h_{max} = u^2/2g$, which is independent of mass of ball. Thus both the balls (of different masses) will attain same height when projected vertically upward with the same velocity.

24. Is it possible to have a constant rate of change of velocity when velocity changes both in magnitude and direction ?

Ans. Yes, in projectile motion.

25. Is it possible that the brakes of a car are so perfect that the car stops instantaneously ? Comment.

Ans. It is not possible to have perfect brakes which can stop the car instantaneously because if the car is to stop instantaneously, its velocity is to be zero in an extremely small interval of time. As a result of it, the car needs infinite retardation, which is not possible.

26. A train is moving on horizontal rails with a uniform acceleration *a*. A passenger sitting in a boggie drops a stone inside the boggie. What will be the acceleration of stone (*i*) w.r.t. boggie and (*ii*) w.r.t. the rails ?

Ans. (*i*) g w.r.t. boggie and

(*ii*) $\sqrt{a^2 + g^2}$ w.r.t. rails.

27. If the acceleration of the particle is constant in magnitude but not in direction, what type of path does the body follow ?

Ans. Circular path.

IV. Graphs for uniformly accelerated motion

28. How can the distance travelled be calculated from velocity-time graph in a uniform one dimensional motion ?

Ans. Distance travelled by an object in a given time is equal to the area which the velocity-time graph encloses with the time axis for the given interval of time. Finding that area under velocity-time graph, distance travelled will be determined.

29. What is wrong with the speed time graph as shown in Fig. 3(Q).19.

Ans. Speed being scalar can not have negative value.

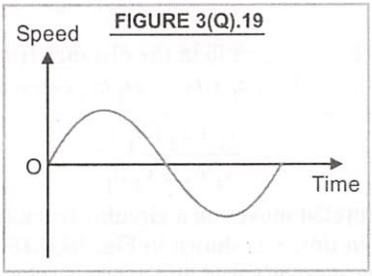


FIGURE 3(Q).19

30. A ball is thrown vertically upwards. Draw its velocity-time curve.

Ans. Velocity-time graph is of the type as shown in Fig. 3(Q).20.

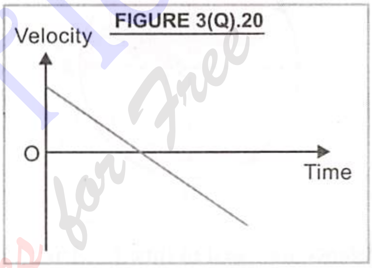


FIGURE 3(Q).20

31. What does the area under acceleration-time graph for any interval of time represents when the acceleration of the moving body is varying with time ?

Ans. The area under acceleration-time graph for a given interval of time represents the change in speed of the body during that interval of time.

32. What does the slope of velocity-time graph represent ?

(HP Board 2007, CBSE Delhi 2010)

Ans. The slope of velocity-time graph represents acceleration.

33. What is the nature of the displacement time curve of a body moving with constant acceleration ?

Ans. If a body starts from rest and moves along a straight line with a uniform acceleration a, then the displacement (x) of the body in time t is

$$x = \frac{1}{2}at^2$$

which is an equation of a parabola. Thus the displacement-time graph for a body moving with constant acceleration is a parabola.

V. Calculus method for accelerated motion

34. What is the ratio of the distance travelled by a body falling freely from rest in the first, second, and third seconds of its fall.

Ans. Here, $u = 0$, $a = g$;

Using the relation $D_n = u + \frac{a}{2}(2n-1)$

we have $D_1 : D_2 : D_3 = \frac{g}{2}(2 \times 1 - 1)$

$: \frac{g}{2}(2 \times 2 - 1) : \frac{g}{2}(2 \times 3 - 1) = \mathbf{1 : 3 : 5}$

35. Can we use the equations of kinematics to find the height attained by a body projected upwards with any velocity ?

Ans. No, the equations of kinematics can be used only if the body is moving with a uniform acceleration. The acceleration due to gravity is uniform only near the surface of earth but decreases with height. Therefore if a body is projected upwards with any velocity, we **cannot** find its height using equations of kinematics.

36. The displacement of a body is given to be proportional to the cube of time elapsed. What is the nature of the acceleration of the body ?

Ans. Given, $S \propto t^3$ or $S = kt^3$

velocity, $v = \frac{ds}{dt} = 3kt^2$

Acceleration, $a = \frac{dv}{dt} = 6kt$ *i.e.* $a \propto t$

Thus **acceleration increases with time.**

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Displacement and Velocity

1. When can an object be considered as a point object ? Explain in brief.

Ans. An object can be considered a point object if during motion in a given time it covers a distance much greater than its own size. For example, if a car of size 3 metres is moving a few hundred kilometre distance, then to study this motion, the car can be taken as a point object, since 3 m is very much less than few hundred kilometre distance.

2. Given below are some examples of motion. State in each case, if the motion is one, two or three dimensions : (*i*) A flying bird in the sky (*ii*) A football kicked by a player (*iii*) Earth revolving around the sun (*iv*) The motion of the bob of a simple pendulum.

Ans. (*i*) A flying bird in the sky is a **three dimensional motion.** (*ii*) A football kicked by a player is a **two dimensional motion.** (*iii*) Earth revolving around the sun is **a two dimensional motion.** (*iv*) The motion of the bob of simple pendulum is **one dimensional motion.**

3. What are the characteristics of uniform motion ? **(HP Board 2009)**

Ans. Some of the characteristics of uniforn motion are as follows :

(*i*) In uniform motion, the velocity of object neither depends on the choice of origin nor on the choice of time interval.

(*ii*) In uniform motion along a straight line, the magnitude of displacement of object in a given time is equal to the distance travelled.

(*iii*) In uniform motion along a straight line, the instantaneous velocity = average velocity = uniform velocity.

(*iv*) In uniform motion, no force is required to maintain the motion.

4. Is magnitude of the displacement of an object and total distance covered by it in certain time interval same ? Explain.

Ans. Not necessarily. If an object covers a complete circular track of radius r, then its displacement is zero but distance travelled is $2\pi r$.

5. Is it true that in one dimensional motion, a particle with zero speed may have non-zero velocity ? Explain.

Ans. It is not possible because, velocity = speed + direction. If speed is zero, the velocity is also zero.

6. Explain that a body can have zero average velocity but not zero average speed.

Ans. Average velocity = displacement/(total time taken) and average speed = total distance travelled/(total time taken). If an object completes a circular path of radius r in time t, then its displacement is zero but distance travelled by body is $2\pi r$. Therefore, the average velocity of body = zero but average speed of body = $2\pi r/t$.

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7. Draw velocity-time graph of a uniform motion of an object along a straight line. What do you study from this graph ?

Ans. Velocity-time graph of a uniform motion along a straight line is a straight line *AB* parallel to time axis. Fig. 3(Q).21.

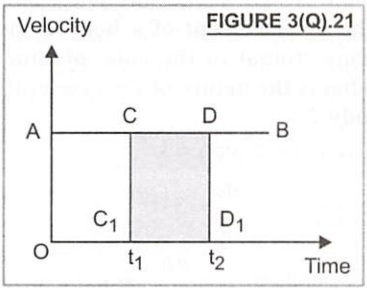


FIGURE 3(Q).21

This graph helps us to calculate the displacement of the object in a given interval of time, which is equal to the area under velocity-time graph for the given interval of time. The displacement of the object in time interval $(t_2 - t_1)$

$= $ uniform velocity × time interval

$= v\,(t_2 - t_1) = C_1\,C \times (OD_1 - OC_1)$

$= C_1\,C \times C_1\,D_1 = $ area $CC_1\,D_1\,D$

8. Show that average velocity of the object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.

Ans. Since, average velocity $= \dfrac{\text{displacement}}{\text{total time taken}}$

and average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

Since, displacement of an object can be equal to or less than the total distance travelled in a given time, hence, average velocity of an object over an interval of time is either equal to or smaller than the average speed of the object over the same interval of time.

9. Two particles are moving with constant speed *v* such that they are always at a constant distance *d* apart and their velocities are always equal and opposite. After what time, they return to their initial positions ?

Ans. Since, the particles while moving with constant speed *v*, are always at a constant distance *d*, they must be at the two ends of the diameter of a circular path. The diameter of this circular path $= d$. Each particle will return to its initial position after describing a circular path $= 2\,\pi\,r$ $= \pi\,d$ $(\because\ 2\,r = d)$

Time interval after which each particle will return to its initial position

$= $ distance travelled/speed $= \pi\,d/v$

10. Show that the slope of displacement-time graph is equal to the velocity of uniform motion. **(HP Board 2004, 2007 C)**

Ans. Refer to Art. 3.17.

11. A body travels with velocity v_1 for time t_1 second and with velocity v_2 for time t_2 second in the same direction, find the average velocity of the body.

Ans. Displacement travelled in time $(t_1 + t_2)$

$$= v_1\,t_1 + v_2\,t_2$$

$\therefore$ Average velocity $= \dfrac{\text{displacement}}{\text{total time taken}}$

$$= \frac{v_1\,t_1 + v_2\,t_2}{(t_1 + t_2)}$$

12. The displacement *x* of the body in motion is given by $x = A \sin(\omega t + \theta)$; Determine the time at which the displacement is maximum.

Ans. The value of displacement *x* will be maximum, when the value of $\sin(\omega t + \theta)$ is maximum. It will be so if $\sin(\omega t + \theta) = 1 = \sin \pi/2$

or $\omega t + \theta = \dfrac{\pi}{2}$ or $\omega t = \dfrac{\pi}{2} - \theta$

$$\therefore \quad t = \left(\frac{\pi}{2\omega} - \frac{\theta}{\omega}\right)$$

13. An athelete completes one round of a circular track of radius *R* in 40 seconds. What will be the displacement at the end of 2 min. 20 second ?

Ans. In 2 minutes 20 seconds, an athelete will complete $= \dfrac{2 \times 60 + 20}{40} = 3{\cdot}5$ rounds, *i.e.*, three complete rotations and one half rotation, *i.e.*, finally it will be at the opposite end of the diameter from the starting point. So displacement $= 2\,R$.

14. A person travels along a straight road for the first half length with a velocity v_1 and the second half length with velocity v_2. What is the mean velocity of the person ?

Ans. Time taken by person to travel first half length,

$$t_1 = \frac{(d/2)}{v_1} = \frac{d}{2v_1}$$

Time taken by person to travel second half length, $t_2 = \dfrac{d/2}{v_2} = \dfrac{d}{2v_2}$

$$\therefore \text{ Total time } = t_1 + t_2 = \frac{d}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$$

$$= \frac{d\,(v_1 + v_2)}{2 v_1 v_2}$$

∴ Mean velocity or average velocity

$$= \frac{d}{t_1 + t_2} = \frac{2 v_1 v_2}{(v_1 + v_2)}$$

II. Relative Velocity

15. The speedometer of a car moving eastward reads 50 km/h. It passes another car which travels westward at 50 km/h. (i) Do both the cars have same speed ? (ii) Do they have the same velocity ? (iii) What is the relative velocity of car A w.r.t. car B.

Ans. (*i*) Since, speed is a scalar quantity and speedometer reads instantaneous speed of the car, hence **both the cars have same speed.**

(*ii*) Since, velocity is a vector quantity, hence **both the cars are having opposite velocities** as they are moving in opposite directions.

(*iii*) Relative velocity of car *A* w.r.t. car *B*

$= v_A + v_B$

= (50 km/h + 50 km/h)

= 100 km/h due east.

16. When two bodies move uniformly towards each other, the distance between them decreases by 6 metres/second. If both the bodies move in the same direction with their same speeds, the distance between them increases by 4 metres/second. What are the speeds of the two bodies.

Ans. Let u and v be the velocities of two bodies. As per question, $u + v = 6$

and $u - v = 4$ or $2u = 10$

On solving we get

$u = 5$ m/s and $v = 1$ m/s

III. Accelerated motion

17. An object can accelerate while travelling at constant speed, but not at constant velocity. Is it true ? Explain

Ans. Yes ; the direction of an object can change at constant speed. It is so when an object is moving on a circular path with constant speed. Now it has centripetal acceleration. The constant velocity shows zero acceleration.

18. If a speedometer is attached to a freely falling body, then how much would its speed readings increase with each second of fall ?

Ans. A speedometer measures the instantaneous speed of a moving body. The speed of a falling body increases by g (= 9·8 ms^{-2}) for each second of fall. The speed reading of speedometer will increase by 9·8 ms^{-1} for each second of fall.

19. A man standing on the edge of a cliff throws a stone straight up with initial speed *u* and then throws another stone straight down with same initial speed and from the same position. Find the ratio of the speeds, the stones would have attained when they hit ground at the base of the cliff.

Ans. Since a stone thrown vertically up with speed u returns back the same point with the same speed, hence, both the stones will strike the ground with the same speed. Hence, the ratio of the speeds of the two stones will be **1 : 1.**

20. Acceleration is defined as the rate of change of velocity. Suppose we call the rate of change of acceleration as 'SLAP'.

(*i*) What is the unit of SLAP ?

(*ii*) How can we calculate instantaneous SLAP ?

Ans. Given, SLAP $= \dfrac{\text{change in acceleration}}{\text{time taken}}$

(*i*) The SI unit of SLAP is $\dfrac{m/s^2}{s} = ms^{-3}$

(*ii*) Instantaneous SLAP is the value of SLAP at the given instant, which is the first derivative of acceleration at the given instant and can be written as

$$\text{Instantaneous SLAP} = \underset{\Delta t \to 0}{\text{Limit}}\left(\frac{\Delta \vec{a}}{\Delta t}\right) = \frac{d\vec{a}}{dt}$$

21. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward direction with the greater speed ?

Ans. Let u be the initial velocity of projection of body and v be the velocity of the same body while passing downwards through point of projection. The displacement of body $S = 0$. Using the relation $v^2 = u^2 + 2\,aS$,

and $u = u$, $v = ?$; $a = -g$, $S = 0$, we have

$$v^2 = u^2 + 2(-g) \times 0 = u^2 \quad \text{or} \quad v = u$$

It means the final speed is independent of mass of the body. Hence, both the bodies will acquire the same speed while passing through point of projection.

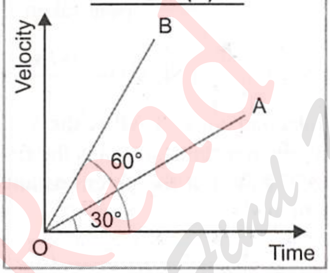
22. For a particle in one dimensional motion, the instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

Ans. We know that instantaneous speed, $v_i = \frac{ds}{dt}$. Where $\frac{ds}{dt}$ = rate of change of distance. The instantaneous velocity, $\vec{v}_i = \frac{d\vec{s}}{dt}$ = rate of change of displacement. In one dimensional motion having uniform acceleration, the direction of velocity does not change with time, only the magnitude of velocity changes with time. Therefore,

$$|d\vec{s}| = ds \quad \text{or} \quad v_i = \frac{ds}{dt} = \frac{|d\vec{s}|}{dt} = |\vec{v}_i|.$$

23. Two straight lines drawn on the same velocity-time graph make angles 30° and 60° with the time axis respectively, as shown in Fig. 3(Q).22. Which line represents greater acceleration ? What is the ratio of the two accelerations ?

FIGURE 3(Q).22



Ans. Slope of velocity-time graph = acceleration of the object.

As slope of line OB > slope of line OA. Therefore, the line OB making an angle 60° with time axis represents greater velocity than the line OA making an angle 30° with time axis.

Ratio of the two accelerations

$$= \frac{a_A}{a_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = \mathbf{1:3}$$

24. Points P, Q and R are in vertical line such that $PQ = QR$. A ball at P is allowed to fall freely. What is the ratio of the times of descent through PQ and QR ?

Ans. Let t_1, t_2 be the times of descent through PQ and QR respectively. Let $PQ = QR = y$. Then

$$y = \frac{1}{2} g t_1^2 \quad \text{and} \quad 2y = \frac{1}{2} g (t_1 + t_2)^2$$

$$\text{or} \quad \frac{2y}{y} = \frac{(t_1+t_2)^2}{t_1^2} \quad \text{or} \quad \sqrt{2} = \frac{t_1+t_2}{t_1} = 1 + \frac{t_2}{t_1}$$

$$\text{or} \quad \frac{t_2}{t_1} = \sqrt{2} - 1 \quad \text{or} \quad \frac{t_1}{t_2} = \frac{1}{\sqrt{2}-1} = \mathbf{1 : (\sqrt{2} - 1)}$$

25. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in $T/3$ seconds from ground ?

Ans. Taking vertically downward motion of ball from top of tower to ground, we have

$$u = 0, a = g, S = h, t = T$$

$$\text{As} \quad S = ut + \frac{1}{2} a t^2$$

$$\therefore \quad h = 0 + \frac{1}{2} g T^2 \quad \text{or} \quad T = \sqrt{\frac{2h}{g}}$$

Distance covered in time $(T/3)$s

$$h' = \frac{1}{2} g (T/3)^2 = \frac{h}{9}$$

Height of the ball from ground after $T/3$ seconds

$= h - h/9 = \mathbf{8\,h/9}$

IV. Graphs for uniformly accelerated motion

26. What do the slopes of 'distance-time' and 'velocity-time' graphs represent ? What do positive and negative values of these slopes imply ?

Ans. The slope of tangent at a point on distance-time graph of a uniformly accelerated motion in one dimension represents the instantaneous velocity at that instant of time.

The slope of velocity-time graph of uniformly accelerated motion at a given instant represents the acceleration of the moving body at that given instant.

For the distance time graph of an accelerated motion, the positive value of slope represents that the velocity of the moving body is increasing with time and the negative value of slope represents that the velocity of the moving body is decreasing with time.

Similarly for the velocity-time graph of a uniformly accelerated motion, the positive value of slope represents that the acceleration is positive, *i.e.,* the velocity of body is increasing with time and negative value of slope represents that the acceleration of body is negative, *i.e.,* the velocity of body is decreasing with time.

27. How can one determine (*i*) the distance (*ii*) the displacement covered by a uniformly accelerated body from its velocity-time graph ?

Ans. (*i*) The distance travelled by a body in a given interval of time is equal to total area enclosed by velocity time graph, without considering sign. It means, even if the body is moving with negative velocity, the area of velocity-time graph is to be taken positive for the measurement of distance travelled by the body.

(*ii*) Displacement of a body in a given interval of time is equal to total area enclosed by velocity-time graph, during the given interval of time, which is to be added with proper sign.

V. Calculus method for accelerated motion

28. An electron starting from rest has a velocity that increases linearly with time that is $v = kt$, where $k = 2$ m/s^2. What will be the distance covered in first 3 seconds of its motion ?

Ans. $v = dx/dt = kt$ or $dx = k\,t\,dt$. Integrating it,

we have $x = \frac{1}{2}k\,t^2$ when $t = 3$ s, then

$$x = \frac{1}{2}\times 2\times 3^2 = \mathbf{9\ m}$$

29. In a case of a motion, displacement is directly proportional to the square of the time elapsed. What do you think about its acceleration *i.e.,* constant or variable ? Explain why ?

(Pb. Board 2003)

Ans. Given $x \propto t^2$ or $x = kt^2$ where k is a constant

Velocity, $v = \dfrac{dx}{dt} = 2kt$

Acceleration, $a = \dfrac{dv}{dt} = 2k = a$ constant

Hence, the **acceleration of moving object is constant.**

30. If the velocity of a particle is given by $v = \sqrt{180 - 16x}$ m/s, what will be its acceleration ?

Ans. Acceleration, $a = \dfrac{dv}{dt} = \dfrac{dv}{dx}\times\dfrac{dx}{dt} = v\dfrac{dv}{dx}$

Given, $v^2 = (180 - 16\,x)$;

Differentiating it w.r.t. x, we have

$$2v\frac{dv}{dx} = -16$$

So, acceleration, $a = v\dfrac{dv}{dx} = -\dfrac{16}{2} = \mathbf{-8\ m/s^2}$

31. The distance traversed by a moving particle at any instant is half of the product of its velocity and the time of traverse. Show that the acceleration of particle is constant.

Ans. Let at an instant t, v be the velocity of the moving particle and S be the distance travelled by the particle. As per question.

$$S = v\,t/2 \qquad \text{...}(i)$$

Differentiating it w.r.t. time t, we have

$$\frac{dS}{dt} = \frac{1}{2}\frac{dv}{dt}\times t + \frac{v}{2}$$

or $$v = \frac{1}{2}a\times t + \frac{v}{2}$$

$$\left(\because \frac{dv}{dt} = \text{acceleration} = a\right)$$

or $$at = v$$

Differentiating it again w.r.t. time t, we have

$$\frac{da}{dt}\times t + a = \frac{dv}{dt} = a \quad \text{or} \quad \frac{da}{dt}\times t = 0$$

or $$\frac{da}{dt} = 0$$

Therefore ; $\qquad$ **a = a constant.**

32. The acceleration of a particle, starting from rest, varies with time according to relation ; $a = -r\omega^2 \sin \omega t$. Find the displacement of this particle at a time t.

Ans. Acceleration, $a = \dfrac{dv}{dt} = -r\,\omega^2 \sin \omega\, t$

or $\quad dv = -r\,\omega^2 \sin \omega t\, dt$;

Integrating it we have

$$v = -r\,\omega^2\left(-\frac{\cos \omega t}{\omega}\right) = r\,\omega \cos \omega t.$$

Now, velocity $v = \dfrac{dx}{dt} = r\,\omega \cos \omega t$

or $$dx = r\,\omega \cos \omega t\, dt$$

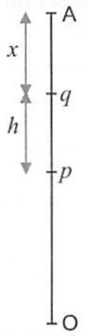
Integrating it, we have,

$$x = r\,\omega\left(\frac{\sin \omega t}{\omega}\right) = \mathbf{r \sin \omega t}$$

33. An object is thrown vertically upward with some speed. It crosses 2 points p, q which are separated by h metre. If t_p is the time between p and highest point and coming back and t_q is the time between q and highest point and coming back, relate acceleration due to gravity, t_p, t_q and h. **(AIPMT Mains 2007)**

Ans. Let u' be the velocity of the object while crossing point p and v' be its velocity while crossing point q. Fig. 3(Q).23. A is the highest point of vertical motion of object. As per question, the time taken by the object in going from p to $A = t_p/2$. The time taken by the object in going from q to $A = t_q/2$.

FIGURE 3(Q).23



Taking vertical upward motion of object from p to A, we have

$$u = u', v = 0, a = -g, t = t_p/2$$

As, $v = v + at$

$\therefore$ $0 = u' + (-g)\, t_p/2$

or $u' = g t_p/2$ ...(*i*)

Taking vertical upward motion of object from q to A, we have,

$$u = v', v = 0, a = -g, t = t_q/2$$

As $v = u + at$

$\therefore$ $0 = v' + (-g)\, t_q/2$

or $v' = g\, t_q/2$

Taking vertical upward motion of object from p to q, we have,

$$u = u', v = v', a = -g, S = h$$

As, $v^2 = u^2 + 2\,aS$ $\therefore$ $v'^2 = u'^2 + 2\,(-g)\,h$

or $$2\,gh = u'^2 - v'^2 = \frac{g^2 t_p^2}{4} - \frac{g^2 t_q^2}{4}$$

or $$g = \frac{\mathbf{8h}}{(t_p^2 - t_q^2)}$$

34. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for some time t and then decelerates at the rate $f/2$ to come to rest. If the total distance is $5\,S$, then prove that

$$S = \frac{1}{2} f t^2.$$

Ans. *During acceleration of object,*

$$u = 0, a = f, S = S, v = v_1$$

As, $v^2 = u^2 + 2\,a\,S$ so $v_1^2 = 0^2 + 2\,f\,S = 2\,f\,S$

or $v_1 = \sqrt{2\,f\,S}$

During uniform motion of object,

$$u = v_1 = \sqrt{2\,f\,S}\,, t = t,$$

Distance travelled, $S_1 = ut = \sqrt{2\,f\,S}\,.t$

During deceleration of object,

$$u = \sqrt{2\,f\,S}\,, a = -f/2, v = 0, S = S_2$$

As, $v^2 - u^2 = 2\,a\,S$

so $0 - (\sqrt{2\,f\,S})^2 = 2(-f/2)\,S_2$ or $S_2 = 2\,S$

Given $S + S_1 + S_2 = 5\,S$

or $S + \sqrt{2\,f\,S}\,t + 2\,S = 5\,S$

or $\sqrt{2\,f\,S}\,t = 2\,S$ or $S = \frac{1}{2} f t^2$

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. Explain the basic concepts of statics, kinematics and dynamics. **[Art. 3.1]**
2. Differentiate between one, two and three dimensional motion. **[Art. 3.5]**
3. Define uniform velocity of an object moving along a straight line. What will be the shapes of position-time and velocity - time graphs of such a motion ? **[Arts. 3.13, 16 & 18]**
4. Discuss the importance of graphical study of the uniform motion of an object in one dimension. **[Arts. 3.17 & 18]**
5. What do you understand by non uniform motion ? Explain variable velocity and instantaneous velocity of an object in one dimensional motion. **[Art. 3.19]**

6. Define relative velocity of an object w.r.t. another. Draw position - time graphs of two objects moving along a straight line, when their relative velocity is (*i*) zero and (*ii*) non-zero. **[Art. 3.20]**
7. Define and explain the term acceleration. Derive the velocity-time relation of a body moving under constant acceleration. **[Arts. 3.22 & 3.23]**
8. Find the distance travelled by the uniformly accelerated object moving in one dimension in *n*th second. **[Art. 3.26]**
9. Draw velocity-time graph of a uniformly accelerated motion in one dimension and explain that the distance travelled is equal to the area under velocity-time graph. **[Art. 3.27]**
10. From the velocity-time graph of uniform accelerated motion deduce the equations of motion in (*i*) velocity and time (*ii*) distance and time (*iii*) distance and velocity. **[Art. 3.28]**
11. Discuss the motion of an object under free fall and draw (*a*) acceleration-time, (*b*) velocity-time and (*c*) position-time graph for this motion. **[Art. 3.30]**
12. What are the reaction time and hang time ? Explain them properly. **[Arts. 3.31 and 3.32]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Explain path length and displacement with illustrations and distinguish them. **[Art. 3.10 & 11]**
2. What do you understand by relative velocity of an object w.r.t. another ? Obtain an expression for the relative position of the two objects at time *t* in terms of their velocities and positions, when motion takes place along a straight line. **[Art. 3.20]**
3. Explain clearly (*i*) uniform acceleration (*ii*) variable acceleration (*iii*) Average acceleration (*iv*) instantaneous acceleration and show that instantaneous acceleration is the limiting value of average acceleration. **[Art. 3.22]**
4. Deduce the following relations analytically for a uniformly accelerated motion along a st. line, where the terms have their usual meanings

   (*i*) $v = u + at$ (*ii*) $s = ut + \frac{1}{2}at^2$

   (*iii*) $v^2 = u^2 + 2\,as$ **[Art. 3.23]**
5. Deduce the equations of uniformly accelerated motion in one dimension by following calculus method. **[Art. 3.29]**

CASE-BASED VERY SHORT/SHORT QUESTIONS

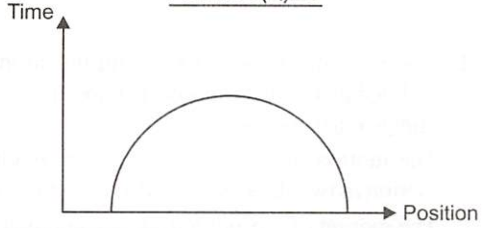
CASE 1. An object is said to be in motion if its position changes with time with respect to its surroundings. Motion of an object can be rectilinear or translatory motion, circular or rotatory motion and oscillatory or vibratory motion.

Rectilinear or translatory motion can be uniform or non-uniform and is one dimensional motion. Circular or rotatory motion can be uniform or non-uniform and can be two or three dimensional motion. Oscillatory or vibratory motion can be one or two dimensional motion.

Read the above paragraph carefully and answer the following very short and short answer questions :

1. What is the type of motion of an object slipping along the inclined plane ?
2. What is the type of motion of earth around the sun ?
3. What is the type of motion of a pendulum in pendulum clock ?
4. (*i*) Is the time variation of position for a motion shown in Fig. 3(Q).24 observed in nature ?

FIGURE 3(Q).24



(*ii*) Is it correct to say that in a uniform motion in one dimension, the average velocity = instantaneous velocity = uniform velocity ?

CASE 2. When two objects A and B are moving with different velocities along a straight line, then the velocity of object A with respect to object B is called relative velocity of A, with respect to B.

If $\vec{v}_A$ and $\vec{v}_B$ are the velocities of object A and B respectively, then the relative velocity of A w.r.t. the body B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

The relative velocity of object B w.r.t. object A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

If two objects A and B are moving in the same direction along a straight path, then

$$|\vec{v}_{AB}| = |\vec{v}_A| - |\vec{v}_B| \quad \text{or} \quad v_{AB} = v_A - v_B$$

If two objects A and B are moving in the opposite directions along a straight path, then

$$|\vec{v}_{AB}| = |\vec{v}_A| - |-\vec{v}_B| = v_A + v_B$$

If two objects A and B are sitting in a moving train, then relative velocity of object A w.r.t. object B is zero and the relative velocity of object A or B w.r.t. ground is equal to the velocity of train because ground is assumed to be at rest.

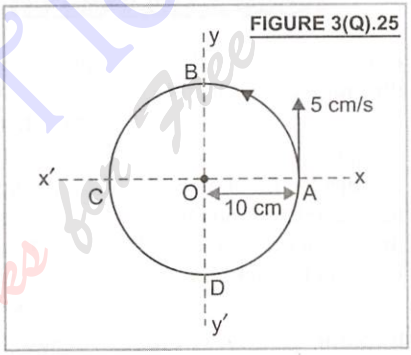
Read the above paragraph carefully and answer the following very short and short answer questions :

5. Can the relative velocity of two bodies be greater than the absolute velocity of either body ?
6. Though the rain is falling vertically downwards, the front screen of a moving car gets wet while the back screen remains dry. Why ?
7. Draw position-time graph for two objects having zero relative velocity.
8. Wind is blowing west to east along parallel tracks. Two trains moving with the same speed in opposite directions on these tracks have the stream tracks. If one stream track is double than the other, what is the speed of each train.

CASE 3. When an object is in motion, then the total length of the actual path covered in a given time is called the distance travelled. But the shortest distance between the initial and final positions of the objects, is called displacement of the object in the given time and its direction is from initial position to final position. The distance travelled by an object in motion can not be zero but displacement of object can be zero if the object reaches its initial position of motion after the given time. The distance travelled by the object in motion, between two positions tells the type of the path followed but displacement of the object between the two positions does not tell the type of path followed.

A body moves along a circle of radius 10cm. It starts from A and moves in anticlockwise direction with a constant speed 5 cm/s.

FIGURE 3(Q).25



Read the above paragraph carefully and answer the following very short and short answer questions :

9. Calculate the distance treavelled by the body while going from A to B.
10. Calculate the time taken by body in going from A to B.
11. What is the displacement of the body in going from A to B ?
12. Explain the followings : (*i*) Can a body possess displacement without distance covered. (*ii*) Displacement can be positive, negative or zero.

ANSWERS

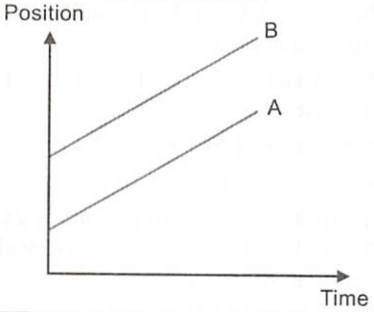
1. The motion of the object slipping along the inclined plane can be uniform or non-uniform one dimensional motion.
2. The motion of earth around the sun is circular motion in two dimensions with a constant period.
3. The motion of a pendulum in pendulum clock is one dimensional translatory motion.
4. (*i*) **No,** because in the given variation, with the increase in position, the time first increase and then decreases, which is impossible.

   (*ii*) **Yes.** In a uniform motion in one dimension, the average velocity = instantaneous velocity = uniform velocity, because in each second of the motion, the displacement is same.
5. **Yes,** when two bodies are moving in opposite directions along a straight path, then the relative velocity of each body is greater than the absolute velocity of either body.
6. It is due to the fact that the rain strikes the car in the direction of relative velocity of rain with respect to car, which is towards the front screen of car and away from the back screen of car.

7. As the relative velocity of two objects is zero, so the two objects must be moving with the same and uniform velocity in the same direction.

Since the slope of position-time graph of uniform motion in a straight line represent velocity, hence the position-time graph for two objects in motion having zero relative velocity, are parallel straight lines equally inclined to time axis as shown in Fig. 3(Q).26.

FIGURE 3(Q).26



8. Let v be the velocity of each train and w be the velocity of wind. As per question, the relative velocity of one stream track train is double than the relative velocity of other stream track train. So, $v + w = 2(v - w)$ or $v = 3w$; *i.e.*, the velocity of each train is 3 times the velocity of wind.

9. Here, $r = 10$ cm. Distance travelled by body from

$$A \text{ to } B = \frac{2\pi r}{4} = \frac{\pi r}{2} = \frac{22}{7} \times \frac{10}{2} = \mathbf{15{\cdot}7\ cm}$$

10. $$\text{Time taken} = \frac{\text{distance}}{\text{speed}} = \frac{15{\cdot}7\,\text{cm}}{5\,\text{cm/s}} = \mathbf{3{\cdot}14\ s}$$

11. $$\text{Displacement} = AB = \sqrt{r^2 + r^2}$$
$$= \sqrt{10^2 + 10^2} = 10\sqrt{2}$$
$$= 10 \times 1{\cdot}414 = \mathbf{14{\cdot}14\ cm}$$

12. (*i*) A body can not have a displacement without distance covered because displacement occurs when body is in motion.

(*ii*) Displacement of a body in motion can be positive if the body goes to the forward position, negative if the body goes to the backward position and is zero if it comes back to the initial position.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. When a body is in motion, then the rate of change of distance travelled by a body is called its speed, which is uniform, if the body covers equal distances in equal intervals of time. The speed of the moving body can never be negative or zero.

FIGURE 3(Q).27

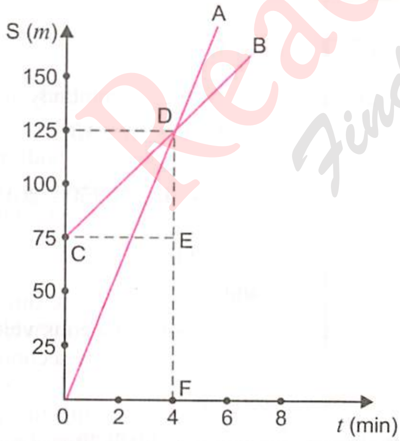


Fig. 3(Q).27 shows, the distance (S) – time (t) graph of two trains A and B which start moving simultaneously in the same direction along the straight paths.

Based on the above paragraph, answer question no. 1 to 4 :

1. What is the speed of body B ?
   (*a*) 25 m/min (*b*) 12·5 m/min
   (*c*) 6·25 m/min (*d*) 9·5 m/min
2. When and where train A will catch train B ?
   (*a*) 2 min, 100 m (*b*) 4 min, 100 m
   (*c*) 4 min, 125 m (*d*) 6 min, 125 m

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

3. **Assertion.** The slope of distance – time graph with time axis tells the speed of the body.

   Reason. Greater the slope of distance – time graph lower is the speed of the body.

4. **Assertion.** Speed of B is greater than the speed of A.

   Reason. The body B reaches the meeting point D earlier than body A.

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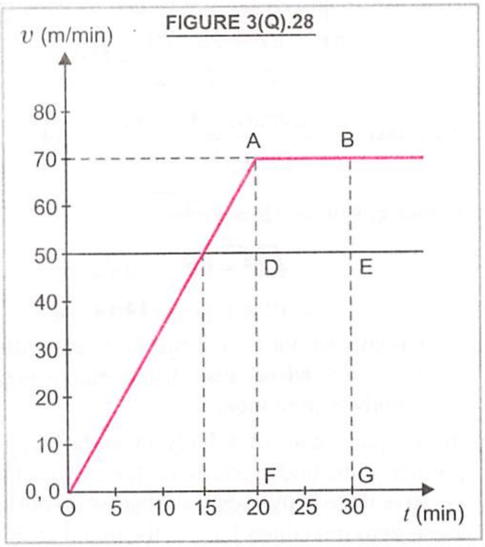
CASE 2. As soon as a car starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity-time graphs are shown in Fig. 3(Q).28.

FIGURE 3(Q).28



Based on the above paragraph, answer question no. 5 to 8 :

5. What is the distance travelled by car in 30 minutes ?
   (*a*) 2100 m (*b*) 1400m
   (*c*) 1150 m (*d*) 700 m

6. What is the difference between distance travelled by car and scooter in 30 minutes ?
   (*a*) 600 m (*b*) 100 m
   (*c*) 350 m (*d*) 800 m

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

7. **Assertion.** The velocity-time graph of a uniformly accelerated motion of a body is straight line inclined with time axis.

   Reason. Acceleration $= \dfrac{\text{change in velocity}}{\text{time taken}}$

8. **Assertion.** The velocity-time graph of a body moving with infinite acceleration can not be shown in graph.

   Reason. For a given time, the area enclosed by the velocity-time graph will be the acceleration.

ANSWERS

1. (*b*) **2.** (*c*) **3.** (*c*) **4.** (*d*) **5.** (*b*) **6.** (*b*) **7.** (*a*) **8.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

1. Speed of $B = \dfrac{DE}{CE} = \dfrac{125-75}{4-0} = \mathbf{12{\cdot}5\ m/min}$

2. A will catch B at meeting point *D* after **4 minutes** at distance of **125 m.**

3. Speed $= \dfrac{\text{distance}}{\text{time}} = \left(\dfrac{DF}{OF}\right) =$ slope of distance time graph for train *A*

   Thus **Assertion is true**. Here, **Reason is false** as larger is the slope of distance-time graph, the greater is its speed.

4. **Both Assertion and Reason are false** as slope of *A* is greater than that of *B*. So the speed of *A* is more than that of *B*. But both the body *A* and *B* meet at *D* at the same time.

5. Distance travelled by car in 30 minutes

$$= \text{area } OAF + \text{area } ABGF$$

$$= \frac{1}{2} \times 20 \times 70 + 70\,(30-20)$$

$$= 700 + 700$$

$$= \mathbf{1400\ m}$$

6. Distance travelled by scooter in 30 minutes

$$= 50 \times 30$$

$$= 1500 \text{ m}$$

   Difference in the distance travelled

$$= 1500 - 1400$$

$$= \mathbf{100\ m}$$

7. **Here both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

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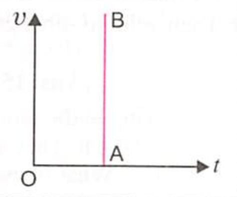
8. Here both Assertion and Reason are false, as velocity-time graph of a body moving with infinite acceleration is a straight line parallel to velocity axis. Fig. 3(Q).29.

The area unclosed by the velocity-time graph for the given time of motion is equal to the distance travelled by body and not the acceleration of the moving body.

FIGURE 3(Q).29



PROBLEMS FOR PRACTICE

TYPE I. DISTANCE COVERED, DISPLACEMENT AVERAGE SPEED AND AVERAGE VELOCITY

1. A body travels from A to B at 40 ms^{-1} and from B to A at 60 ms^{-1}. Calculate the average speed and average velocity.

(HP Board 2007) **[Ans. 48 ms^{-1}, 0]**

2. A cyclist moving on a circular track of radius 50 m completes one revolution in 2 minutes. What is his (*i*) distance covered (*ii*) displacement (*iii*) average speed (*iv*) average velocity, in one full revolution.

[Ans. (*i*) 100 πm ; (*ii*) 0 (*iii*) 50 π/min (*iv*) 0]

3. A car is travelling along a straight line. It covers one-half of the total distance with a velocity 10 ms^{-1}. The remaining part of the distance was covered with a velocity 12 ms^{-1}, for half the time and with velocity 16 ms^{-1} for the other half the time. Find the average speed over the whole time of motion. **[Ans. 11·67 ms^{-1}]**

4. A table clock has its minute hand 5·0 cm long. Find the average velocity of the tip of the minute hand (*a*) between 6.00 a.m. to 6.15 a.m. and (*b*) between 6.00 a.m. to 6.30. p.m.

[Ans. (*a*) $7{\cdot}86 \times 10^{-3}$ cm s^{-1} (*b*) $2{\cdot}2 \times 10^{-4}$ cm s^{-1}]

5. The science lecture theatre of a college is 10 m wide and has a door at a corner. A science teacher enters at 12.00 noon through the door and makes 8 rounds along the 10 m wall back and forth during the period and finally leaves the class room at 12.40 p.m. through the same door. Compute the average speed and average velocity. **[Ans. (*a*) 0·067 ms^{-1} (*b*) zero]**

6. A body travelling along a straight line traversed one third of the total distance with a velocity 4 ms^{-1}. The remaining part of the distance was covered with a velocity 2 ms^{-1} for half the time and with velocity 6 ms^{-1} for the other half of time. What is the mean velocity averaged over the whole time of motion ? **[Ans. 4 ms^{-1}]**

7. A drunkard walking in a narrow lane takes 6 steps forward and 4 steps backward, following again 6 steps forward and 4 steps backward and so on. Each step is 1 m long and requires 1 s. Determine how long the drunkard takes to fall in a pit 15 m away from the start. **[Ans. 55 s]**

TYPE II. RELATIVE VELOCITY

8. Two cars are moving in the same direction with the same speed of 30 km h^{-1} at a distance of 5 km from each other. A third car moving in the opposite direction meets these two cars at an interval of 4 minutes. Find the speed of third car. **[Ans. 45 km h^{-1}]**

9. A jet airplane travelling at the speed of 500 km h^{-1} ejects the burnt gases at the speed of 1100 km h^{-1}, relative to the jet airplane. Find the speed of the burnt gases w.r.t. a stationary observer on earth. **[Ans. 600 km h^{-1}]**

10. If a man's speed with and against the water current in a river be 15 km h^{-1}, and 5 km h^{-1}, then find out the man's speed in still water and the speed of the river. **[Ans. 10 km h^{-1} ; 5 km h^{-1}]**

11. Two buses started simultaneously towards each other from towns A and B which are 480 km apart. It took the first bus travelling from A to B eight hours to cover the distance and the second bus travelling from B to A, ten hours. Determine, when the buses will meet after starting and at what distance from A.

[Ans. 4 h 26·7 min ; 266·7 km]

12. A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 kmh^{-1}. The jeep follows it at a speed of 108 kmh^{-1}, crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike.

[Ans. 600 m]

13. Two trains, each of length 100 m, are running on parallel tracks. One overtakes the other in 20 second and one crosses the other in 10 second. Calculate the velocities of two trains. **[Ans. 15 ms^{-1} ; 5 ms^{-1}]**

14. Two cars are moving in the same direction with the same speed 30 km/h. They are separated by a distance of 4 km. What is the speed of a car moving in the opposite driection if it meets these two cars at an interval of 5 minutes. **[Ans. 18 km/h]**

TYPE III. UNIFORMLY ACCELERATED MOTION

15. A train moves with a constant speed of 36 km h^{-1} in the first 10 minutes, with another constant speed of 45 km h^{-1} in the next 10 minutes and then with an acceleration of 5 ms^{-2} in the last 10 minutes. Calculate the average speed of the train for this journey and the total distance travelled. **[Ans. 1842 km/h ; 921 km]**

16. An object is moving with a uniform acceleration. Its velocity after 5 s is 25 ms^{-1} and after 8 s is 34 ms^{-1}. Find the distance travelled by the object in 10th second. **[Ans. 38·5 m]**

17. A body covers 12 m in 2nd second and 20 m in 4th second. Find what distance the body will cover in 4 seconds after the 5th second. **(Pb. Board 2003) [Ans. 136 m]**

18. An automobile starts from rest and accelerates uniformly for 30 second to a speed of 72 km h^{-1}. It then moves with a uniform velocity and it is finally brought to rest in 50 m with a constant retardation. If the total distance travelled is 950 m, find the acceleration, the retardation and total time taken. **[Ans. 2/3 ms^{-2}, 4 m s^{-2}, 65 sec]**

19. Two bodies are released from the same height at an interval of 1 s. How long after the first body begins to fall will the two bodies be 10 m apart ? (g = 10 m/s^2) **[Ans. 1·5 s]**

20. A stone is dropped from the top of a cliff and is found to travel 44·1 m in the last second before it reaches the ground. Find the height of the cliff. **[Ans. 122·5 m]**

21. A ball thrown up is caught by the thrower after 4 second. How high did it go and with what velocity was it thrown ? How far below its highest point was in 3 second after start ? Acceleration due to gravity is 9·8 ms^{-2}. **[Ans. 19·6 m, 19·6 ms^{-1}, 4·9 m]**

22. From the top of a tower 100 m in height a ball is dropped and at the same instant another ball is projecred vertically upwards from the ground so that it just reaches the top of tower. At what height do the two balls pass one another ? **[Ans. 75 m from ground]**

23. A body starting from rest, was observed to cover 20 m in 1 second and 40 m during the next second. How far had it travelled before the first observation was taken ? **[Ans. 2·5 m]**

24. A ball is dropped from the roof of a tower of height h. The total distance covered by it in the last second of its motion is equal to the distance covered by it in first three seconds. What is the value of h ? (g = 10 m/s^2) **[Ans. 125 m]**

25. A stone is dropped from a balloon at an altitude of 300 m. How long will the stone take to reach the ground if (*a*) the balloon is ascending with a velocity of 5 ms^{-1}. (*b*) the ballon is descending with a velocity of 5 ms^{-1} (*c*) the balloon is stationary ? **[Ans. (*a*) 8·36s (*b*) 7·33s (*c*) 7·82 s.]**

26. A food packet is released from a helicopter which is rising steadily at 3 ms^{-1}. After 3 seconds, (*i*) what is the velocity of the packet ? (*ii*) how far is it below the helicopter ? Take g = 9·8 m/s^2. **[Ans. (*i*) 26·4 ms^{-1} (*ii*) 44·1 m]**

TYPE IV. GRAPHS FOR UNIFORMLY ACCELERATED MOTION

27. The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3(Q).30. Find

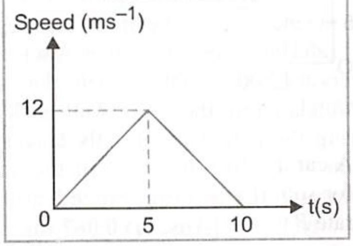
(*i*) distance travelled by the particle between 0 sec to 10 sec

(*ii*) average speed between this interval

(*iii*) the time when the speed was minimum

(*iv*) the time when the speed was maximum.

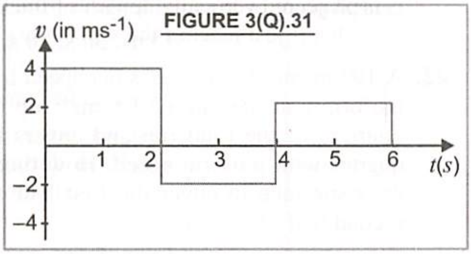
FIGURE 3(Q).30



(CBSE Delhi 2006)

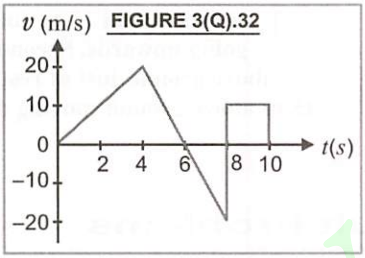
[Ans. (*i*) 60 m ; (*ii*) 6 ms^{-1} (*iii*) 0 or 10 s (*iv*) 15 s]

28. The velocity-time graph of a body moving in a straight line is shown in Fig. 3(Q).31. Find the displacement and the distance travelled by the body in 6 seconds.



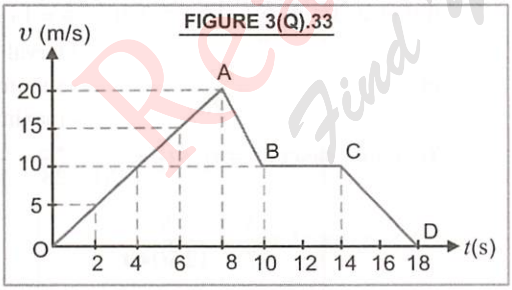
[Ans. 8 m, 16 m]

29. The velocity-time graph of a particle moving along a straight line is shown in Fig. 3(Q).32. Calculate the distance covered between $t = 0$ to $t = 10$ seconds. Also find displacement in time 0 to 10 seconds.



[Ans. 100 m, 60 m]

30. The velocity-time graph of a particle moving along a straight line is shown in the Fig. 3(Q).33 by curve *OABCD*. Calculate the distance covered by the particle between (*i*) t = zero to $t = 18$ seconds (*ii*) $t = 2$ s to $t = 12$ s. and the maximum value of acceleration during this interval.



[Ans. (*i*) 170 m, (*ii*) 125 m, –5 m/s^2]

31. A car accelerates from rest at a constant rate *A* for some time after which it retards at constant rate *B* to come to rest. If the total time lapse is *T* seconds, evaluate the maximum velocity reached and total distance travelled, in terms of *A*, *B* and *T*. $\left[\textbf{Ans. } \left(\frac{AB}{A+B}\right)T, \left(\frac{AB}{A+B}\right)\frac{T^2}{2}\right]$

32. A car accelerates from rest at a constant rate 36 km/h^2 for some time after which it retards at a constant rate 48 km h^{-2} to come to rest. If the total time elapsed is 1 hour, calculate

(*i*) the maximum velocity attained by the car and

(*ii*) the total distance travelled by the car.

[Ans. 20·57 km h^{-1}; 10·29 km]

TYPE V. CALCULUS METHOD FOR ACCELERATED MOTION

33. The acceleration of the particle in ms^{-2} is given by $a = 3t^2 + 2t + 2$, where time t is in second. If the particle starts with a velocity 2 ms^{-1} at time $t = 0$, find the velocity at the end of 3 second. **[Ans. 44 ms^{-1}]**

34. The displacement of particle is zero at $t = 0$ and it is x, at $t = t$. It starts moving in the positive x-direction with a velocity which varies as $v = k\sqrt{x}$, where k is a constant. Show that velocity is directly proportional to time.

35. The relation between time t and distance x is given by $t = ax^2 + bx$, where a and b are constants. What is the acceleration in terms of velocity v of the body ? **[Ans. – 2 av^3]**

TYPE VI. TYPICAL PROBLEMS

36. Three particles *A*, *B* and *C* are situated at the vertices of an equilateral triangle *ABC* of side a at $t = 0$. Each of the particles moves with constant speed v. *A* always has its velocity along *AB*, *B* along *BC* and *C* along *CA*. At what time will these particles meet each other.

[Ans. 2 a/3 v]

37. A car travelling at 60 km h^{-1} overtakes another car travelling at 42 km h^{-1}. Assuming each car to be 5·0 m long, find the time taken during the overtake and the total road distance used for the overtake. **[Ans. 2 s ; 38·3 m]**

38. The speed of a motor launch with respect to still water is 8 ms^{-1} and the speed of stream is 4 ms^{-1}. When the launch begins travelling upstream a float was dropped from it. The launch travelled 4·8 km upstream, turned about and caught up with the float. How long is it before the launch reaches the float. **[Ans. 40 min]**

39. Two persons *P* and *Q* are standing 54 m apart on a long moving belt. Person *P* rolls a round stone towards person *Q* with a speed of 9 ms^{-1} with respect to belt. If the belt is moving with a speed of 4 ms^{-1} in the direction from *P* to *Q*,

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(*a*) What is the speed of the stone w.r.t. an observer on a stationary platform ? (*b*) What is the time taken by stone to travel from P to Q ? (*c*) What will be the speed of the stone with respect to an observer on a stationary platform if person Q rolls the stone with a velocity of 9 ms^{-1} with respect to the belt towards person P and the time taken by the stone to travel from Q to P ?

[Ans. (*a*) 13 ms^{-1} (*b*) 6 s (*c*) – 5 ms^{-1} ; 6 s]

40. The acceleration 'a' in ms^{-2} of a particle is given by $a = 3t^2 + 2t + 2$, where t is the time. If the particle starts out with a velocity $v = 2\ ms^{-1}$ at $t = 0$, then find the velocity at the end of 2 s.

[Ans. 18 ms^{-1}]

41. An athlete runs a distance of 1500 m as follows : (*i*) He starts from rest and accelerates himself uniformly with acceleration 2 ms^{-2} till he covers a distance of 900 m. (*ii*) He then runs the remaining distance of 600 m with a uniform speed developed. Calculate the time taken by the athlete to cover the two parts of the distance covered. Also find the time, when he is at the central point of the total length of track.

[Ans. 30 s, 10 s, 27·4 s]

42. A 100 m sprinter increases her speed from rest uniformly at the rate of 1·5 ms^{-2} up to three quarters of the total run and covers the last quarter with uniform speed. How much time does she take to cover the first half and the second half of the run ?

[Ans. 8·165 s ; 3·502 s]

43. A jugglar maintains four balls in motion, making each in turn rise to a height of 20.0 m from his hand. Find the velocity with which the jugglar project these balls and the position of other three balls at the instant when the fourth ball is just leaving the hand of jugglar. Take $g = 10\ m/s^2$.

[Ans. Third ball 15 m above ground going upwards, Second ball 20 m above ground just at rest ; first ball 15 m above ground moving downward]

HINTS
For Difficult Problems

1. Let x be the distance between A and B.

Total time taken by a body in going from A to B and then from B to A is

$$t_1 + t_2 = \frac{AB}{40} + \frac{BA}{60} = AB\left[\frac{1}{40} + \frac{1}{60}\right] = \frac{AB}{24} = \frac{x}{24}\,\text{s}$$

Total distance covered, $S = x + x = 2x$

$$\text{Average speed} = \frac{S}{t_1 + t_2} = \frac{2x}{x/24} = \mathbf{48\ ms^{-1}}$$

Displacement of the body in going from A to B and then from B to A is zero, since the final location of body coincides with the initial location.

$$\therefore \text{Average velocity} = \frac{\text{displacement}}{t_1 + t_2} = \frac{0}{t_1 + t_2} = \mathbf{0}$$

2. Here $r = 50$ m, $t = 2$ min

(*i*) Distance covered in one revolution (*i.e.* in 2 minutes)

$$= 2\pi r = 2\pi \times 50 = \mathbf{100\ \pi m}$$

(*ii*) Displacement in one revolution = **0** (*i.e.*, in 2 minutes)

(*iii*) Average speed in one revolution $= \dfrac{100\pi}{2}$

$$= \mathbf{50\ \pi\ m/min}$$

(*iv*) Average velocity in one revolution $= \dfrac{0}{2} = \mathbf{0}$

3. Let S be total distance to be travelled by car. Time taken by car to cover the distance $S/2$ with velocity 10 ms^{-1} is $t_1 = \dfrac{S/2}{10} = \dfrac{S}{20}$.

Let t_2 be the time taken to cover the next $S/2$ distance. Then $\dfrac{S}{2} = 12 \times \dfrac{t_2}{2} + 16 \times \dfrac{t_2}{2} = 14\,t_2$

or $$t_2 = \frac{S}{28}.$$

$$\text{Total time taken, } T = t_1 + t_2 = \frac{S}{20} + \frac{S}{28} = \frac{12S}{140}$$

$$\text{Average speed} = \frac{S}{T} = \frac{S}{12S/140}$$

$$= \frac{140}{12} = \mathbf{11{\cdot}67\ ms^{-1}}$$

4. (*a*) At 6.00 a.m., the tip of the minute hand is at 12 mark and at 6·15 a.m. it is 90° away. Thus, displacement

$$= \sqrt{R^2 + R^2} = \sqrt{2}\,R = \sqrt{2} \times 5\ \text{cm}$$

Time taken from 6.00 a.m. to 6.15 a.m. is 15 minutes = 15 × 60 s. The average velocity

$$= \frac{\text{displacement}}{\text{time taken}} = \frac{\sqrt{2}\times 5\,\text{cm}}{15\times 60\,\text{s}}$$

$$= \mathbf{7{\cdot}86 \times 10^{-3}\ cm\ s^{-1}}$$

Direction of average velocity is 45° with vertical.

(*b*) At 6.00 a.m. the minute hand is at 12 mark and at 6.30 p.m. is 180° away. Thus, displacement = $R + R = 2R = 2 \times 5 = 10$ cm.

Time taken from 6.00 a.m. to 6.30 p.m. is 12 hours and 30 minutes = 45000 s.

The average velocity,

$$v_{av} = \frac{\text{displacement}}{\text{time}} = \frac{10}{45000}$$

$$= \mathbf{2{\cdot}2 \times 10^{-4}\ cm/s}$$

Direction of average velocity is 12 mark to the 6 mark on clock panel *i.e.* at 0° angle with vertical.

5. Total distance travelled in 40 minutes

$$= 20 \times 8 = 160\ \text{m}$$

Average speed = $160/(40 \times 60)$ = $\mathbf{0{\cdot}067\ ms^{-1}}$

At 12.00 noon he is at the door and at 12.40 p.m. he is again at the same door, the displacement during the 40 minute time interval is **zero.** So average velocity = **0**

6. Let s be the total distance travelled by body. If t_1 is the time taken to cover first one third distance, then $t_1 = \frac{s/3}{4} = \frac{s}{12}$

Let t_2 be the time for each of the remaining two journeys. Then

$$\frac{2s}{3} = 2t_2 + 6t_2 = 8t_2 \quad \text{or} \quad t_2 = \frac{2s}{3\times 8} = \frac{s}{12}$$

$$\text{Average velocity} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{s}{t_1 + 2t_2} = \frac{s}{(s/12) + 2\,(s/12)} = \mathbf{4\ ms^{-1}}$$

7. Effective distance travelled by drunkard in 10 steps = 6 – 4 = 2 m.

Therefore he takes 50 steps to move 10 m. Now he will have to cover 5 m more to reach the pit. For this he has to take only 5 forward steps. Thus total steps taken to fall in pit = 50 + 5 = 55

As each step is taken in 1s, so the time taken to fall in pit = **55 s.**

8. Let v km h^{-1} be the speed of third car.

Relative speed of third car w.r.t. other two cars

$$= (v + 30)\ \text{km h}^{-1}$$

Separation between two cars, $S = 5$ km

Time taken by third car to cover 5 km distance is t = 4 minutes $= \frac{4}{60}\text{h} = \frac{1}{15}\text{h}$

As distance = relative speed × time

$$\therefore \quad 5 = (v + 30)\times\frac{1}{15}$$

or $\quad v + 30 = 75$

or $\quad v = 75 - 30 = \mathbf{45\ km\ h^{-1}}$

9. Let downward motion be taken positive and upward motion negative

Speed of jet air plane $v_j = -500$ km h^{-1}

Relative velocity of burnt gases w.r.t. jet airplane is $v_{bj} = 1100$ km h^{-1}

Let v_b be the velocity of the burnt gases w.r.t. observer on ground. Then

$$v_{bj} = v_b - v_j \quad \text{or} \quad 1100 = v_b - (-500)$$

or $\quad v_b = 1100 - 500 = \mathbf{600\ km\ h^{-1}}$

10. If v_m, v_w are the speeds of man and river water w.r.t. ground. Then $v_m + v_w = 15$ and $v_m - v_w = 5$. On solving, we get, $v_m = \mathbf{10\ km\ h^{-1}}$ and $v_w = \mathbf{5\ km\ h^{-1}}$

11. Here, distance between two towns A and B, $S = 480$ km ; $v_1 = S/8$ and $v_2 = S/10$. If these buses meet after time t, then

$$\frac{S}{8}\times t + \frac{S}{10}\times t = S$$

On solving we get, t = **(40/9) hours**

$$\therefore \text{ Distance from } A = v_1 \times t = \frac{480}{8}\times\frac{40}{9} = \frac{800}{3}$$

$$= \mathbf{266{\cdot}7\ km}$$

12. Speed of motorbike = 72 km h^{-1} = 20 ms^{-1} ; Speed of police jeep = 108 kmh^{-1} = 30 ms^{-1}

Let the jeep catch up the bike after t seconds. Therefore, $30\,t = 20\,(t + 10)$ or $t = 20$ s ; Distance travelled by jeep in 20 s

$$= 30 \times 20 = \mathbf{600\ m}$$

13. Let u and v be the velocity of two trains A and B. While overtaking, the relative velocity of train A w.r.t. $B = u - v$. While crossing, the relative velocity of train A w.r.t. $B = u + v$. Total distance to be travelled by train A while crossing

$$= 100 + 100 = 200\ \text{m}$$

$$\therefore \quad 20 = \frac{200}{u - v} \quad \text{or} \quad u - v = 10$$

$$\text{and} \quad 10 = \frac{200}{u + v} \quad \text{or} \quad u + v = 20$$

On solving we get,

$$u = 15 \text{ ms}^{-1} \quad \text{and} \quad v = 5 \text{ ms}^{-1}$$

14. Relative velocity of car B w.r.t. car A,

$$|\vec{v}_{BA}| = |\vec{v}_B - \vec{v}_A| = v - v = 0.$$

So relative separation between two cars A and B remains constant at all instants.

If $\vec{v}_C$ is the velocity of car C w.r.t. ground in the opposite direction to car A and B, then relative velocity of car C relative to car A or B will be $v_{rel} = v_C - (-30) = (v_C + 30)$ km/h.
Time taken by car C to cross cars A and B is ;

$$t = \frac{d}{v_{\text{rel.}}}$$

or $$\frac{5}{60} = \frac{4}{v_C + 30} \qquad \left[\text{as } 5 \text{ min.} = \frac{5}{60} \text{h}\right]$$

On solving, $v_C = \mathbf{18}$ **km/h**

15. *For first 10 minute,*

speed = 36 km h^{-1} = 10 m/s

Distance travelled,

$S_1 = 10 \times (10 \times 60) = 6000$ m

For next 10 minutes,

speed = 45 km h^{-1} = (25/2) m/s

Distance travelled, $S_2 = \frac{25}{2} \times (10 \times 60)$

$= 7500$ m

For last 10 minutes, u = (25/2) m/s ;

$a = 5$ ms^{-2}, $t = 10 \times 60$ s

Distance travelled, $S_3 = ut + \frac{1}{2}at^2$

$$= \frac{25}{2} \times (10 \times 60) + \frac{1}{2}(10 \times 60)^2 = 907500 \text{ m}$$

Total distance travelled by train

$= S_1 + S_2 + S_3 = 6000 + 7500 + 907500$

$= 921000$ m = **921 km**

Total time of journey = 10 + 10 + 10

$= 30 \text{ min} = \frac{1}{2} h$

Average speed,

$$v_{av} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{921}{1/2}$$

$= \mathbf{1842}$ **km/h**

16. Case (*i*), $t = 5$ s, $v = 25$ ms^{-1} ;

Case (*ii*), $t = 8$ s, $v = 34$ ms^{-1}.

As $v = u + at$

Case (*i*), $25 = u + a \times 5$ ...(*i*)

Case (*ii*), $34 = u + a \times 8$ ...(*ii*)

Subtracting (*i*) from (*ii*), we have

$$9 = 3a \quad \text{or} \quad a = 3 \text{ ms}^{-2}$$

From (*i*), $25 = u + 3 \times 5$

or $u = 25 - 15 = 10$ ms^{-1}

Distance travelled in 10th second

$$D_{10} = u + \frac{a}{2}(2 \times 10 - 1) = 10 + \frac{3}{2} \times 19$$

$= 10 + 28{\cdot}5 = \mathbf{38{\cdot}5}$ **m**

17. Distance covered in 4 seconds after the fifth second $= S_9 - S_5$. The distance covered in t seconds is given by, $S = ut + \frac{1}{2}at^2$ and distance covered in nth second is given by,

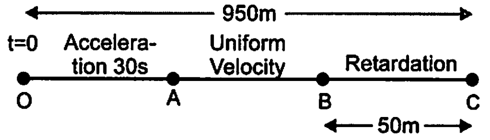
$$D_n = u + \frac{a}{2}(2n - 1)$$

18. Let the automobile be accelerated from O to A. It moves with constant velocity from A to B and then moves with constant retardation from B to C Fig. 3(Q).34.

Taking motion of automobile from O to A, we have

$u = 0$, $a = ?$; $t = 30$ s ; $v = 72$ km/h = 20 m/s ; $S = ?$

FIGURE 3(Q).34



As, $a = \frac{v - u}{t} = \frac{20 - 0}{30} = \frac{\mathbf{2}}{\mathbf{3}}$ **m/s²**

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{2}{3} \times 30^2 = 300 \text{ m}$$

i.e., $OA = 300$ m ; so, $AB = OC - (OA + BC)$

$= 950 - (300 + 50) = 600$ m.

Taking motion of automobile from A to B,

time taken = 600/20 = 30 s

Taking motion of automobile from B to C, we have

$u = 20$ m/s, $v = 0$, $a = ?$; $S = 50$ m ; $t = ?$

$$a = \frac{v^2 - u^2}{2S} = \frac{0 - 20^2}{2 \times 50} = \mathbf{-4 m/s^2} ;$$

$$t = \frac{v-u}{a} = \frac{0-20^2}{-4} = 5\text{ s}$$

$\therefore$ Total time = 30 + 30 + 5 = **65 s**

19. Let the first body fall for t sec and second body for time $(t-1)$ second when their separation becomes 10 m. Then

$$10 = \frac{1}{2}gt^2 - \frac{1}{2}g\,(t-1)^2$$

On solving, $t = \mathbf{1{\cdot}5\ s}$

20. Let S be the height of cliff and t be the total time taken by stone while falling from top to bottom of the cliff. Here, initial velocity, $u = 0$, acceleration $a = 9{\cdot}8\text{ ms}^{-2}$; $D_t = 44.1$ m.

As, $D_t = u + \frac{a}{2}(2t-1)$;

$$\therefore 44{\cdot}1 = 0 + \frac{9{\cdot}8}{2}(2t-1) = 4{\cdot}9\,(2t-1)$$

On solving, $t = 5$ s

$$\therefore\quad S = ut + \frac{1}{2}at^2 = 0\times5 + \frac{1}{2}\times9{\cdot}8\times5^2$$

$$= 4{\cdot}9\times5^2 = \mathbf{122{\cdot}5\ m}$$

21. The position of ball below the highest point in 3 second can be calculated by taking the motion of ball vertically downwards from its highest point for 1 second.

22. Taking vertical upward motion of ball from ground to height 100 m, we have

$u = ?$, $a = -9{\cdot}8\text{ m/s}^2$, $v = 0$, $S = 100$ m.

As, $v^2 = u^2 + 2aS$;

so $\quad 0 = u^2 + 2(-9{\cdot}8)\times100$

or $\quad u = \sqrt{2\times9{\cdot}8\times100} = 14\sqrt{10}$ m/s

Let the two balls meet after time t and at a height x metres from ground. The vertical distance travelled by ball dropped from 100 m in time t = $(100-x)$ metres.

Taking vertical downward motion of one ball for time t, we have

$u = 0$, $a = 9{\cdot}8\text{ m/s}^2$, $S = (100-x)$, $t = t$

As, $S = ut + \frac{1}{2}at^2$

$$\therefore (100-x) = 0 + \frac{1}{2}\times9{\cdot}8\times t^2 = 4{\cdot}9\,t^2 \quad ...(i)$$

Taking vertical upward motion of another ball for time t, we have

$u = 14\sqrt{10}$ m/s ; $a = -9{\cdot}8\text{ m/s}^2$; $t = t$; $S = x$.

As, $S = ut + \frac{1}{2}at^2$; so

$$x = 14\sqrt{10}\,t + \frac{1}{2}(-9{\cdot}8)\,t^2$$

$$= 14\sqrt{10}\,t - 4{\cdot}9\,t^2 \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$100 = 14\sqrt{10}\,t \quad\text{or}\quad t = \frac{100}{14\sqrt{10}} = \frac{10\sqrt{10}}{14}$$

From (*ii*),

$$x = 14\sqrt{10}\times\frac{10\sqrt{10}}{14} - 4{\cdot}9\left(\frac{10}{14}\times\sqrt{10}\right)^2$$

$$= 100 - 25 = \mathbf{75\ m}$$

23. Let the distance travelled in $(t+1)$th second be 20 m and $(t+2)$th second be 40 m. Then using the relation, $D_n = u + \frac{a}{2}(2n-1)$ we get

$$20 = 0 + \frac{a}{2}[2\times(t+1)-1] = \frac{a}{2}(2t+1) \quad ...(i)$$

and $\quad 40 = 0 + \frac{a}{2}[2\times(t+2)-1] = \frac{a}{2}(2t+3) \quad ...(ii)$

On solving (*i*) & (*ii*) we get, $a = 20\text{ ms}^{-2}$ and $t = 1/2$ second. Distance travelled in t seconds

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times20\times\left(\frac{1}{2}\right)^2 = \mathbf{2{\cdot}5\ m}$$

24. Let the ball remain in air for t seconds. Then,

$$D_t = u + \frac{a}{2}(2t-1) = 0 + \frac{10}{2}(2t-1)$$

$$= 10\,t - 5 \quad ...(i)$$

Distance covered in first three seconds.

$$S = \frac{1}{2}gt^2 = \frac{1}{2}\times10\times3^2 = 45 \quad ...(ii)$$

As per question, $10\,t - 5 = 45$

or $\quad t = \mathbf{5\ s}$

$$\therefore\quad h = \frac{1}{2}gt^2 = \frac{1}{2}\times10\times5^2 = \mathbf{125\,m}$$

25. (*a*) Here, $u = -5\text{ ms}^{-1}$, $a = 9{\cdot}8\text{ ms}^{-2}$, $S = 300$ m, $t = ?$

As, $\quad S = ut + \frac{1}{2}at^2$,

$$\therefore\quad 300 = -5\times t + \frac{1}{2}\times9{\cdot}8\times t^2$$

Solve it for t.

(*b*) Here, $u = 5\ \text{ms}^{-1}$, $a = 9{\cdot}8\ \text{ms}^{-2}$, $S = 300$ m,

$$t = ?\ \text{Use, } S = ut + \frac{1}{2}at^2$$

and solve it for t.

(*c*) Here, $u = 0$, $a = 9{\cdot}8\ \text{ms}^{-2}$, $S = 300$ m, $t = ?$

Use, $S = ut + \frac{1}{2}at^2$ and solve it for t.

26. Taking downward motion of food packet dropped from helicopter.

Here, $u = -3\ \text{ms}^{-1}$, $t = 3$ s, $a = +9{\cdot}8\ \text{m/s}^2$

(*i*) $v = u + at = -3 + (+9{\cdot}8)\,3 = \mathbf{+26{\cdot}4\ ms^{-1}}$

This velocity is directed downwards.

(*ii*) Downward distance covered by packet in 3 seconds

$$S = ut + \frac{1}{2}at^2$$

$$= -3\times 3 + \frac{1}{2}\times 9{\cdot}8\times 3^2 = 35{\cdot}1\ \text{m}$$

The helicopter rises up in 3 seconds

$$= 3\ \text{ms}^{-1}\times 3\ \text{s} = 9\ \text{m}$$

Therefore, the distance of the food packet from the helicopter $= 35{\cdot}1 + 9 = \mathbf{44{\cdot}1\ m}$

27. (*i*) Distance travelled by particle in time interval 0 to 10 seconds is

S = area under velocity-tme graph

$$= \frac{1}{2}\times(10-0)\times(12-0) = \mathbf{60\ m}$$

(*ii*) Average speed,

$$v_{av} = \frac{\text{Total distance travelled}}{\text{total time taken}}$$

$$= \frac{60}{10} = \mathbf{6\ ms^{-1}}$$

(*iii*) Speed of particle is minimum at $t = \mathbf{0}$ s and $t = \mathbf{10}$ **s**

(*iv*) Speed of particle is maximum at $t = \mathbf{5}$ **s**

28. Displacement in 6 seconds

$$= 4\times 2 - 2\times 2 + 2\times 2 = \mathbf{8\ m}$$

Distance travelled in 6 seconds

$$= 4\times 2 + 2\times 2 + 2\times 2 = \mathbf{16\ m}$$

29. The area enclosed by velocity-time graph with time axis measures the distance travelled in a given time. Displacement covered from 0 to 6 sec is positive ; from 6 to 8 sec is negative and from 8 to 10 sec is positive. Whereas distance covered is always positive.

∴ Total distance covered in 10 s

$$= \frac{1}{2}\times 6\times 20 + \frac{1}{2}\times 2\times 20 + 2\times 10 = \mathbf{100\ m}$$

Total displacement in 10 s

$$= \frac{1}{2}\times 6\times 20 - \frac{1}{2}\times 2\times 20 + 2\times 10 = \mathbf{60\ m}$$

30. (*i*) Total distance travelled from 0 to 18 s is the area which the velocity-time graph encloses with time axis, *i.e.*, total distance covered = distance travelled from 0 to 8 s + distance travelled from 8 to 10 s + distance travelled from 10 to 14 s + distance travelled from 14 to 18 s

$$= \frac{1}{2}\times 8\times 20 + \frac{1}{2}(10+20)\times 2 + 4\times 10 + \frac{1}{2}\times 4\times 10$$

$= \mathbf{170\ m}$

Distance travelled between 2 to 12 second = distance travelled from 2 to 8 s + distance travelled from 8 to 10 s + distance travelled 10 to 12 s.

$$= \frac{1}{2}\times 6\times(5+20) + \frac{1}{2}\times 2\times(20+10) + 2\times 10$$

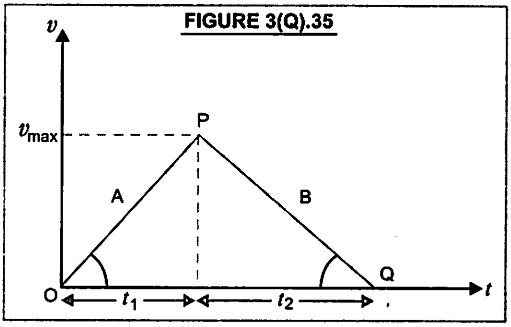
$= \mathbf{125\ m}$

Max. value of acceleration is related with the steepest slope of velocity-time graph. It is from 8 to 10 s for the part AB of graph.

$$\therefore\ \text{Slope of } AB = -\frac{20-10}{10-8} = \mathbf{-5\ ms^{-2}}$$

31. If the car has uniform accelertation for time t_1 and uniform retardation for time t_2, then total time is $T = t_1 + t_2$ ...(*i*)

FIGURE 3(Q).35



If v_{max} be the maximum velocity of the car. As slope of $v-t$ graph gives acceleration, so

$$A = \frac{v_{\text{max}}}{t_1} \quad \text{and} \quad B = \frac{v_{\text{max}}}{t_2}$$

Thus, $\dfrac{1}{A}+\dfrac{1}{B}=\dfrac{t_1+t_2}{v_{\max}}=\dfrac{T}{v_{\max.}}$ [from (*i*)]

or $$v_{\max}=\frac{ABT}{(A+B)} \quad ...(ii)$$

we know that area under $v-t$ graph gives the distance travelled in a given time. So

$$S=\frac{1}{2}(v_{\max.})\times T=\frac{ABT^2}{2(A+B)}$$

32. Here, $A = 36$ km h^{-2} ; $B = 48$ km h^{-2}; $T = 1$ h.
Refer the hint of above question

(*i*) $$v_{\max}=\frac{ABT}{A+B}=\frac{(36\times 48)\times 1}{36+48}$$
$$= \mathbf{20{\cdot}57\ km/h}$$

(*ii*) Total distance, $$S_{\text{Total}}=\left(\frac{AB}{A+B}\right)\frac{T^2}{2}$$

$$=\frac{(36\times 48)\times 1^2}{(36+48)\times 2}=\mathbf{10{\cdot}29\ km}$$

33. Given, $a=\dfrac{dv}{dt}=3t^2+2t+2$

or $dv=(3t^2+2t+2)\,dt$

When time changes from 0 to 3 s, velocity changes from 2 ms^{-1} to v (say)

Integrating the above relation within the conditions of motion, we get

$$\int_2^v dv=\int_0^3(3t^2+2t+2)\,dt$$

or $(v-2)=(t^3+t^2+2t)_0^3=(27+9+2\times 3)$
$=42$

or $v = 2 + 42 = \mathbf{44\ ms^{-1}}$

34. Here, $v=k\sqrt{x}$;

$$\frac{dx}{dt}=k\,x^{1/2} \quad \text{or} \quad \frac{dx}{x^{1/2}}=k\,dt$$

Integrating both sides within the limits [$x = 0$ at $t = 0$ and $x = x$ at $t = t$]

$$\int_0^x\frac{dx}{x^{1/2}}=\int_0^t k\,dt.$$

On solving, $$\sqrt{x}=\frac{kt}{2}$$

As $v=k\sqrt{x}=k\left(\dfrac{kt}{2}\right)=\dfrac{k^2t}{2}$ $\therefore$ $v\propto t$

35. Here, $t = ax^2 + bx$

Differentiating both the sides w.r.t., time t, we get

$$1=2ax\frac{dx}{dt}+b\frac{dx}{dt}=(2ax+b)\frac{dx}{dt}$$

or $$\frac{dx}{dt}=\frac{1}{(2ax+b)}$$

velocity, $$v=\frac{dx}{dt}=\frac{1}{(2ax+b)}=(2ax+b)^{-1}$$

Acceleration $$A=\frac{dv}{dt}=\frac{d}{dt}[(2ax+b)^{-1}]$$

$$=-(2ax+b)^{-2}\times 2a\frac{dx}{dt}$$

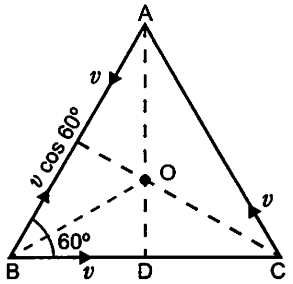
$$=\frac{-2a}{(2ax+b)^2}\times\frac{1}{(2ax+b)}$$

$$=\frac{-2a}{(2ax+b)^3}$$

$$= -\mathbf{2\,a\,v^3} \qquad \left[\because \frac{1}{(2ax+b)}=v\right]$$

36. 1st method. In Fig. 3(Q).36, the velocity of A is along AB. The velocity of B is along BC. The component velocity of B along $BA = v$ cos 60° $= v/2$. Thus, the separation of particles at A and B decreases with the speed $= v + v/2 = 3\,v/2$.

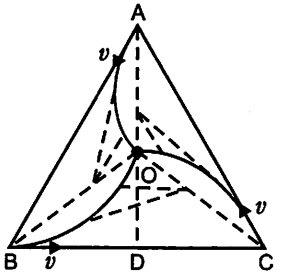
FIGURE 3(Q).36



Since, this speed is constant, the time taken in reducing the separation AB from a to zero is, $t = a/(3\,v/2) = \mathbf{2\,a/3\,v}$

2nd method. The motion of three particles at A, B and C is shown in Fig. 3(Q).37 by solid curved lines. They will be meeting at the centroid O of the triangle ABC. It means a particle at A will be covering a distance AO before meeting the other particle. The velocity of particle at A along $AO = v$ cos 30° $= v\sqrt{3}/2$.

FIGURE 3(Q).37



Distance, $AO = \frac{2}{3} AD = \frac{2}{3}\sqrt{a^2 - a^2/4} = \frac{a}{\sqrt{3}}$

Therefore, the time taken by particle at A to go from A to O

$$= \frac{AO}{v\cos 30°} = \frac{a/\sqrt{3}}{v\sqrt{3}/2} = \frac{2a}{3v}$$

37. Total distance to be travelled for overtaking the car = 5·0 + 5·0 = 10·0 m. Relative velocity of first car w.r.t second car = 60 – 42 = 18 km/h = 8 × 1000/(60 × 60) = 5 ms^{-1}.

Time taken = 10/5 = **2 s.** Road distance used for overtaking = distance travelled by fast car in 2s + length of fast car

= [60 × 1000/(60 × 60)] × 2 + 5

= **38·33 m**

38. Here, speed of motor launch, $v_m = 8\ \text{ms}^{-1}$

speed of stream of water, , $v_w = 4\ \text{ms}^{-1}$

For upstream motion of motor launch, the relative velocity of motor launch w.r.t. water

$$= v_m - v_w = 8 - 4 = 4\ \text{ms}^{-1}$$

Time taken by launch to travel a distance 4·8 km upstream

$$t_1 = \frac{4{\cdot}8\ \text{km}}{4\ \text{ms}^{-1}} = \frac{4800\ \text{m}}{4\ \text{ms}^{-1}} = 1200\,\text{s}$$

For downstream motion of motor launch

Distance moved by float with downstream in 1200 s

$$= 4 \times 1200 = 4800\ \text{m}$$

Before turning about, the distance between launch and float = 4800 + 4800 = 9600 m

This distance is to be covered by launch with its own velocity (= 8 ms^{-1}) because the stream velocity is shared both by launch as well as by float.

Time taken to reach the float,

$$t_2 = \frac{9600}{8} = 1200\,\text{s}$$

Total time taken, to reach the float is

$$t = t_1 + t_2 = 1200 + 1200 = 2400\ \text{s} = \mathbf{40\ min}$$

39. (*a*) Let the direction from P to Q be positive. Given ; speed of the belt, $v_b = +\,4\ \text{ms}^{-1}$; speed of the stone w.r.t. belt, $v_s = +\,9\ \text{ms}^{-1}$ speed of stone w.r.t. a stationary observe

$$= v_s + v_b = 9 + 4 = \mathbf{13\ ms^{-1}}$$

(*b*) Since, persons P and Q are located on belt, the speed of stone w.r.t. P or Q will be 9 ms^{-1}. As distance between P and Q = 54 m, so time taken = 54/9 = **6 s.**

(*c*) Here ; $v_b = +\,4\ \text{ms}^{-1}$; $v_s = -\,9\ \text{ms}^{-1}$

∴ Speed of the stone w.r.t. observer

$$= v_s + v_b = -\,9 + 4 = \mathbf{-\,5\ ms^{-1}}$$

Here, negative sign shows that the stone is moving in a direction opposite to the direction in which the belt is moving.

In this case also, the speed of stone w.r.t. P or Q is still 9 ms^{-1}.

So, time taken $= \frac{54}{9} = \mathbf{6\ s}$

40. $$a = \frac{dv}{dt} = 3\,t^2 + 2\,t + 2$$

or $dv = (3\,t^2 + 2\,t + 2\,)\,dt$

Integrating it within the conditions of motion

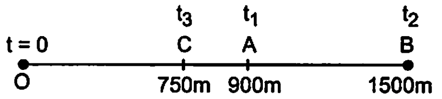
we have $$\int_2^v dv = \int_0^2 (3t^2 + 2t + 2)\,dt$$

or $$(v)_2^v = (t^3 + t^2 + 2t)_0^2$$

or $v - 2 = 2^3 + 2^2 + 2 \times 2$ or $v = \mathbf{18\ m/s}$

41. Refer to Fig. 3(Q).38,

FIGURE 3(Q).38



Taking motion of athelete from 0 to A, we have $u = 0$, $a = 2\ \text{ms}^{-2}$, $S = 900$, $t = t_1$, $v = v_1$ (say)

$v^2 = u^2 + 2\,aS$ or $v_1^2 = 0 + 2 \times 2 \times 900$

or $$v_1 = 60\ \text{ms}^{-1}$$

Time $$t_1 = \frac{v-u}{a} = \frac{60-0}{2} = \mathbf{30\ s}$$

Taking motion of athelete from A to B, this motion is at a constant speed of 60 ms^{-1}.

Time taken, $$(t_2 - t_1) = \frac{1500 - 900}{60} = \mathbf{10\ s}$$

Taking motion of athelete from 0 to C, we have

$u = 0$, $a = 2\ \text{ms}^{-2}$, $t = t_3$, $S = 750$ m

As, $S = ut + \frac{1}{2}at^2$ so, $750 = 0 + \frac{1}{2} \times 2 \times t_3^2$

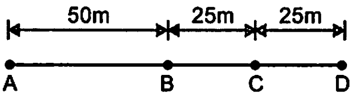
or $t_3 = \sqrt{750} = \mathbf{27{\cdot}4\ s}$

42. In Fig. 3(Q).39,

total distance $AD = 100$ m.

Three quarter distance $= AC = 75$ m and half of the total distance $= AB = 50$ m.

FIGURE 3(Q).39



Therefore, distance

$CD = AD - AC = 100 - 75 = 25$ m

Taking motion of sprinter from A to B, we have ;

$u = 0$; $a = 1{\cdot}5\ \text{ms}^{-2}$; $t = ?$; $S = 50$ m

As, $S = ut + \frac{1}{2}at^2$;

$\therefore\ 50 = 0 \times t + \frac{1}{2}1{\cdot}5 \times t^2$

or $t = (2 \times 50/1{\cdot}5)^{1/2} = \mathbf{8{\cdot}165\ s}$

Taking motion of sprinter from A to C ; $u = 0$;

$a = 1{\cdot}5\ \text{ms}^{-2}$; $t = ?$ $S = 75$ m ; $v = ?$

As $S = ut + \frac{1}{2}at^2$;

$\therefore\ 75 = 0 \times t + \frac{1}{2} \times 1{\cdot}5 \times t^2$

or $t = (75 \times 2/1{\cdot}5)^{1/2} = 10$ s

Also, $v = u + at = 0 + 1{\cdot}5 \times 10 = 15\ \text{ms}^{-1}$

Therefore, time taken from B to $C = 10 - 8{\cdot}165$

$= 1{\cdot}835$ s

Taking motion of sprinter from C to D ;

$u = 15\ \text{ms}^{-1}$; $t = ?$

$S = 25$ m ; $a = 0$

As, $S = ut + \frac{1}{2}at^2$;

$\therefore\ 25 = 15 \times t + \frac{1}{2}(0) \times t^2 = 15t$

or $t = 25/15 = 1{\cdot}667$ s

$\therefore$ time from B to $D = 1{\cdot}835 + 1{\cdot}667 = \mathbf{3{\cdot}502\ s}$

43. Taking vertical upward motion of a ball, we have

$S = 20$ m ; $a = -10\ \text{m/s}^2$;

$u = ?, v = 0, t = ?$

As $v^2 = u^2 + 2aS$ so, $0 = u^2 + 2(-10) \times 20$

or $\mathbf{u = 20\ ms^{-1}}$

Also $v = u + at$

or $t = \frac{v - u}{a} = \frac{0 - 20}{(-10)} = 2\,\text{s}$

So each ball will return to the hand of jugglar after 4 s.

To maintain proper distance, the balls must be projected upwards with an interval of time $= \frac{4}{4} = 1$ s. When the fourth ball is in hand, the third ball has travelled for 1 s, second ball for 2 s and first ball for 3 s.

(*i*) **For Third ball,**

$S = ut + \frac{1}{2}at^2 = 20 \times 1 + \frac{1}{2}(-10)\,1^2 = 15$ m

Thus third ball will be **15 m above the ground** going upwards.

(*ii*) **For second ball,**

$S = 20 \times 2 + \frac{1}{2}(-10) \times 2^2 = 20$ m

The second ball will be **20 m above the ground** just at rest.

(*iii*) **For first ball,**

$S = 20 \times 3 + \frac{1}{2} + (-10) \times 3^2 = 15$ m

The first ball will be **15 m above the ground** moving downwards.

NCERT EXERCISES WITH SOLUTIONS

Q. 1. In which of the following examples of motion, can the body be considered approximately a point object :

(a) a railway carriage moving without jerks between two stations.

(b) a monkey sitting on top of a man cycling smoothly on a circular track.

(c) a spinning cricket ball that turns sharply on hitting the ground.

(d) a tumbling beaker that has slipped off the edge of a table ?

Sol. (*a*) *The carriage can be considered a point object* because the distance between two stations is very large as compared to the size of the railway carriage.

(*b*) *The monkey can be considered as a point object* if the cyclist describes a circular track of very large radius because in that case the distance covered by the cyclist is quite large as compared to the size of monkey. *The monkey can not be considered as a point object* if the cyclist describes a circular track of small radius because in that case the distance covered by the cyclist is not very large as compared to the size of the monkey.

(*c*) *The spinning cricket ball can not be considered as a point object* because the size of the spinning cricket ball is quite appreciable as compared to the distance through which the ball may turn on hitting the ground.

(*d*) *A beaker slipping off the edge of a table can not be considered as a point object* because the size of the beaker is not negligible as compared to the height of the table.

Q. 2. The position-time ($x - t$) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3(N).1. Choose the correct entries in the brackets below :

(a) (A/B) lives closer to the school than (B/A).

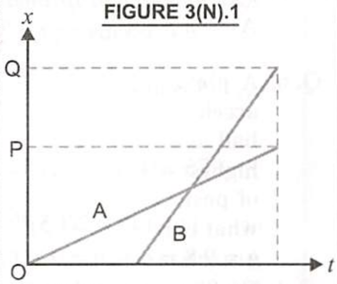
(b) (A/B) starts from the school earlier than (B/A).

(c) (A/B) walks faster than (B/A).

(d) A and B reach home at the (same/different) time.

(e) (A/B) overtakes on the road (once/twice).

FIGURE 3(N).1



Sol. (*a*) As $OP < OQ$, **A lives closer to the school than B.**

(*b*) For $x = 0$, $t = 0$ for A ; while t has some finite value for B. Therefore, **A starts from the school earlier than B.**

(*c*) Since the velocity is equal to slope of x–t graph in case of uniform motion and slope of x–t graph for B is greater than that for A, hence **B walks faster than A.**

(*d*) Corresponding to points P and Q, the value of t from $x - t$ graphs for A and B is same, which can be checked by drawing lines through P and Q parallel to t axis. Thus, both A and B **reach home at the same time.**

(*e*) The $x - t$ graph for A and B intersect each other only once. Since B starts from the school after wards, therefore **B overtakes A on the road once.**

Q. 3. A woman starts from her home at 9·00 a.m., walks with a speed of 5 km/h on straight road up to her office 2·5 km away, stays at the office up to 5·00 p.m. and returns home by an auto with a speed of 25 km/h. Choose suitable scales and plot the $x - t$ graph of her motion.

Sol. Time taken in reaching office

$$= \frac{\text{distance}}{\text{speed}} = \frac{2\cdot5}{5} = 0\cdot5 \text{ hr.}$$

Time taken in returning from office

$$= \frac{2\cdot5}{25} = 0\cdot1 \text{ hr.} = 6 \textit{ minutes}$$

It means the woman reaches the office at 9·30 a.m. and returns home at 5·06 p.m.The $x - t$ graph of this motion will be as shown in Fig. 3(N).2.

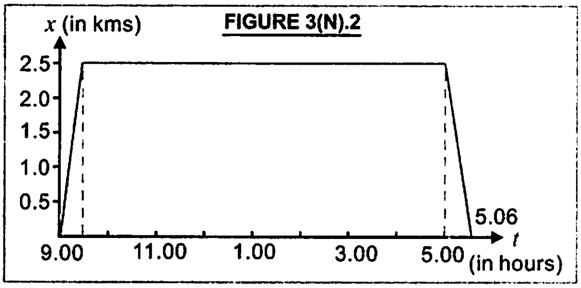


FIGURE 3(N).2

Q. 4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Determine how long the drunkard takes to fall in a pit 13 m away from the start.

Sol. The effective distance travelled by drunkard in 8 steps = 5 – 3 = 2 m

Therefore, he takes 32 steps to move 8 metres.

Now he will have to cover 5 meters more to reach the pit, for which he has to take only 5 forward steps.

Therefore he will have to take = 32 + 5 = 37 steps to move 13 metres. Thus he will fall into the pit after taking 37 steps *i.e.,* after **37 seconds** from the start.

Q. 5. A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?

Sol. Here, $u = 126 \text{ km h}^{-1} = \frac{126\times1000}{60\times60} \text{ ms}^{-1} = 35 \text{ ms}^{-1}$; $v = 0$, $S = 200$ m, a = ? and t = ?

We know, $v^2 = u^2 + 2\,a\,S$ $\therefore$ $0 = (35)^2 + 2 \times a \times 200$ or $a = -\frac{(35)^2}{2\times200} = -\frac{49}{16} = \mathbf{-3\cdot06\ ms^{-2}}$

As, $v = u + at$ $\therefore$ $0 = 35 - \left(\frac{49}{16}\right)t$ or $t = \frac{35\times16}{49} = \frac{80}{7} = \mathbf{11\cdot43\ s}$

Q. 6. A player throws a ball upwards with an initial speed of 29·4 ms^{-1} (a) What is the direction of acceleration during the upward motion of the ball ? (b) What are the velocity and acceleration of the ball at the highest point of its motion. (c) Choose the $x = 0$ and $t = 0$ be the location and time at its highest point, vertically downward direction to be the positive direction of X–axis and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion. (d) To what height does the ball rise and after how long does the ball returns to the player's hands. (Take $g = 9\cdot8$ ms^{-2} and neglect air resistance)

Sol. (*a*) Since the ball is moving under the effect of gravity, the direction of *acceleration due to gravity is always vertically downwards.*

(*b*) At the highest point, the velocity of the ball becomes **zero** and acceleration is equal to the acceleration due to gravity = *9·8 ms^{-2} in vertically downward direction.*

(*c*) When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of X-axis and upward direction as negative direction of X-axis.

During upward motion, sign of position is **negative,** sign of velocity is **negative** and sign of acceleration is **positive.**

During downward motion, sign of position is **positive,** sign of velocity is **positive** and sign of acceleration is also **positive.**

(*d*) Let t be the time taken by the ball to reach the highest point where height from ground be S.

Taking vertical upward motion of the ball, we have, $u = -29\cdot4$ ms^{-1}, $a = 9\cdot8$ ms^{-2}, $v = 0$, $S = S$, t = ?

As, $v^2 - u^2 = 2\,aS$ $\therefore$ $0 - (-29\cdot4)^2 = 2 \times 9\cdot8 \times S$ or $S = \frac{(29\cdot4)^2}{2\times9\cdot8} = \mathbf{-44\cdot1\ m}$

Here –ve sign shows that the distance is covered in upward direction.

As, $v = u + at$ $\therefore$ $0 = -29{\cdot}4 + 9{\cdot}8 \times t,$ or $t = \dfrac{29{\cdot}4}{9{\cdot}8} = 3\text{s}$

It means time of ascent = 3 s

When an object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

Therefore total time after which the ball returns to the player's hand = 3 + 3 = **6 s**

Q. 7. Read each statement below carefully and state with reasons and expamples if it is true or false ;

(a) with zero speed at an instant may have non-zero acceleration at that instant

(b) with zero speed may have non-zero velocity

(c) with constant speed must have zero acceleration

(d) with positive value of acceleration must be speeding up.

Sol. (*a*) **True**, when a body is thrown vertically upwards in the space, then at the highest point, the body has zero speed but has downward acceleration equal to the acceleration due to gravity.

(*b*) **False**, because velocity is the speed of body in a given direction. When speed is zero, the magnitude of velocity of body is zero, hence velocity is zero.

(*c*) **True**, when a particle is moving along a straight line with a constant speed, its velocity remains constant with time. Therefore, acceleration (= change in velocity/time) is zero.

(*d*) The statement depends upon the choice of the instant of time taken as origin, when the body is moving along a straight line with positive acceleration. The velocity of the body at an instant of time t is

$$v = u + at$$

The given statement **is not correct** if a is positive and u is negative, at the instant of time taken as origin. Then for all the times before the time for which v vanishes, there is slowing down of the particle *i.e.* the speed of the particle keeps on decreasing with time. It happens when body is projected vertically upwards. However the **given statement is true** if u is positive and a is positive, at the instant of time taken as origin. It is so when the body is falling vertically downwards.

Q. 8. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s. (g = 10 ms^{-2})

Sol. Taking vertical downward motion of ball from a height 90 m, we have

$$u = 0,\ a = 10\ \text{m/s}^2,\ S = 90\ \text{m},\ t = ?\ ;\ v = ?$$

$$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2\times 90}{10}} = 3\sqrt{2}\ \text{s} = \mathbf{4{\cdot}24\ s}\ ;\ \ v = \sqrt{2aS} = \sqrt{2\times 10\times 90} = 30\sqrt{2}\ \text{m/s}$$

Rebound velocity of ball, $u' = \dfrac{9}{10}v = \dfrac{9}{10}\times 30\sqrt{2} = 27\sqrt{2}$ m/s

Time to reach the highest point is, $t' = \dfrac{u'}{a} = \dfrac{27\sqrt{2}}{10} = 2{\cdot}7\sqrt{2} = 3{\cdot}81$ s

Total time = $t + t' = 4{\cdot}24 + 3{\cdot}81 = 8{\cdot}05$ s

The ball will take further 3·81 s to fall back to floor, where its velocity before striking the floor

$$= 27\sqrt{2}\ \text{m/s}.$$

Velocity of ball after striking the floor

$$= \frac{9}{10}\times 27\sqrt{2} = 24{\cdot}3\sqrt{2}\ \text{m/s}$$

Total time elapsed before upward motion of ball

$$= 8{\cdot}05 + 3{\cdot}81 = 11{\cdot}86\ \text{s}$$

Thus the speed-time graph of this motion will be as shown in Fig. 3(N).3.

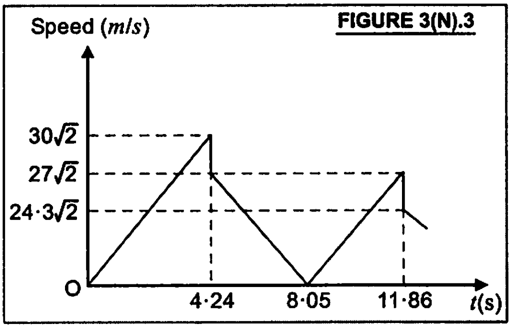


FIGURE 3(N).3

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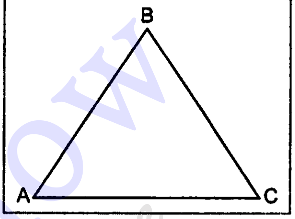
Q. 9. Explain clearly, with examples, the distinction between :

(a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of the path covered by a particle over the same interval.

(b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only]

Sol. (*a*) Magnitude of displacement of a particle in motion for a given time is the shortest distance between the initial and final position of the particle in that time, whereas the total length of the path covered by particle is the actual path traversed by the particle in the given time. If a particle goes from A to B and B to C in time t as shown in Fig 3(N).4, then

FIGURE 3(N).4



Magnitude of displacement = distance AC.

Total path length = distance AB + distance BC

From above we note that total path length ($AB + BC$) is greater than magnitude of displacement (AC).

If there is a motion of the particle in one dimension *i.e.* along a straight line, then the magnitude of displacement becomes equal to total path length traversed by the particle in the given time.

(*b*) Magnitude of average velocity $= \dfrac{\text{magnitude of displacement}}{\text{time interval}} = \dfrac{AC}{t}$

and Average speed $= \dfrac{\text{total path length}}{\text{time interval}} = \dfrac{(AB+BC)}{t}$

As, $(AB + BC) > AC$, so average speed is greater than the magnitude of average velocity. If the particle is moving along a straight line, then in a given time the magnitude of displacement is equal to total path length traversed by particle in that time, so average speed is equal to magnitude of average velocity.

Q. 10. A man walks on a straight road from his home to a market 2·5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back with a speed of 7·5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min ?

Sol. Time taken by man to go from his home to market, $t_1 = \dfrac{\text{distance}}{\text{speed}} = \dfrac{2\cdot5}{5} = \dfrac{1}{2}\text{ h}$

Time taken by man to go from market to his home, $t_2 = \dfrac{2\cdot5}{7\cdot5} = \dfrac{1}{3}\text{ h}$

$\therefore$ Total time taken $t_1 + t_2 = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}\text{ h} = 50\text{ min}.$

(i) 0 to 30 min

(*a*) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{2\cdot5}{1/2} = \mathbf{5\ km/h}$

(*b*) Avereage speed $= \dfrac{\text{distance}}{\text{time}} = \dfrac{2\cdot5}{1/2} = \mathbf{5\ km/h}$

(ii) 0 to 50 min

Total distance travelled = 2·5 + 2·5 = 5 km

Total displacement = zero

(*a*) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = \mathbf{0}$

(*b*) Average speed $= \dfrac{\text{distance}}{\text{time}} = \dfrac{5}{5/6} = \mathbf{6\ km/h}$

(iii) 0 to 40 min.

Distance moved in 30 min (from home to market) = 2·5 km.

Distance moved in 10 min (from market to home) with speed 7·5 km/h $= 7{\cdot}5 \times \dfrac{10}{60} = 1{\cdot}25$ km

So displacement = 2·5 – 1·25 = 1·25 km

Distance travelled = 2·5 + 1·25 = 3·75 km

(*a*) Average velocity $= \dfrac{1{\cdot}25}{(40/60)} = \mathbf{1{\cdot}875\ km/h}$

(*b*) Average speed $= \dfrac{3{\cdot}75}{(40/60)} = \mathbf{5{\cdot}625\ km/h}$

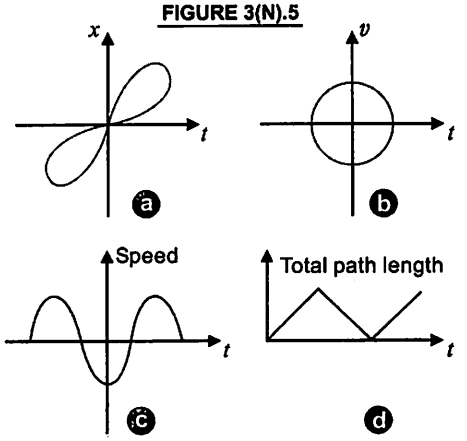
Q. 11. In above questions 9 and 10, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

Sol. Instantaneous speed (v_{ins}) of the particle at an instant is the first derivative of the distance with respect to time at that instant of time *i.e.* $v_{ins} = \dfrac{dx}{dt}$.

Since in instantaneous speed we take only a small interval of time (*dt*) during which direction of motion of a body is not supposed to change, hence there is no difference between total path length and magnitude of displacement for small interval of time *dt*. Hence, instantaneous speed is always equal to magnitude of instantaneous velocity.

Q. 12. Look at the graphs Fig. 3(N).5.(a) to (d) carefully and state, with reasons, which of these cannot possibly represent one dimensional motion of a particle.

FIGURE 3(N).5



Sol. (*a*) *This graph does not represent one dimensional motion* because, at the given instant of time, the particle will have two positions, which is not possible in one dimensional motion.

(*b*) *This graph does not represent one dimensional motion* because, at the given instant of time, particle will have velocity in positive as well as in negative direction, which is not possible in one dimensional motion.

(*c*) *It also does not represent one dimensional motion*, because this graph tells that the particle can have the negative speed but the speed of the particle can never be negative.

(*d*) *It also does not represent one dimensional motion*, because this graph tells that the total path length decreases after certain time but total path length of a moving particle can never decrease with time.

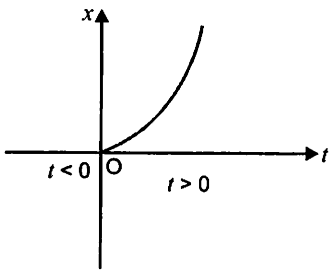


FIGURE 3(N).6

Q. 13. Fig. 3(N).6 shows $x-t$ plot of one dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.

Sol. No, because the x-t graph does not represent the trajectory of the path followed by a particle. From the graph, it is noted that at $t = 0$, $x = 0$.

Context. The above graph can represent the motion of a body falling freely from a tower under gravity.

Q. 14. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car ? (Note, obtain that speed which is relevant for damaging the thief's car ?)

Sol. Muzzle speed of bullet, $v_B = 150 \text{ ms}^{-1} = 540 \text{ km h}^{-1}$

Speed of police van, $v_P = 30$ km/h, Speed of thief car, $v_T = 192$ km/h

Since the bullet is sharing the velocity of the police van, its effective velocity is

$$V_B = v_B + v_P = 540 + 30 = 570 \text{ km/h}$$

The speed of the bullet w.r.t. the thief's car moving in the same direction

$$V_{BT} = V_B - v_T = 570 - 192 = 378 \text{ km/h } = \frac{378 \times 1000}{60 \times 60} = \mathbf{105 \text{ ms}^{-1}}$$

Q. 15. Suggest a suitable physical situation for each of the following graph. Fig. 3(N).7.

Sol. In Fig. 3(N).7(a), the $x-t$ graph shows that initially x is zero *i.e.* at rest, then it increases with time, attains a constant value and again reduces to zero with time, then it increases in opposite direction till it again attains a constant value *i.e.* comes to rest. **The similar physical situation arises when a ball resting on a smooth floor is kicked which rebounds from a wall with reduced speed. It then moves to the opposite wall which stops it.**

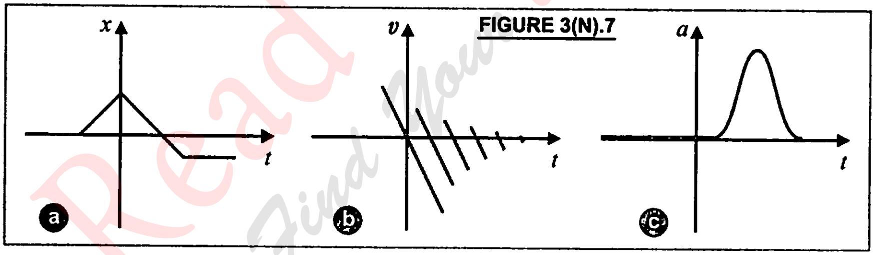


FIGURE 3(N).7

In Fig. 3(N).7(b), the velocity changes sign again and again with passage of time and every time some speed is lost. **The similar physical situation arises when a ball is thrown up with some velocity, returns back and falls freely. On striking the floor, it rebounds with reduced speed each time it strikes against the floor.**

In Fig. 3(N).7(c), initially the body moves with uniform velocity. Its acceleration increases for a short duration and then falls to zero and there after the body moves with a constant velocity. The similar physical situation arises **when a cricket ball moving with a uniform speed is hit with a bat for a very short interval of time.**

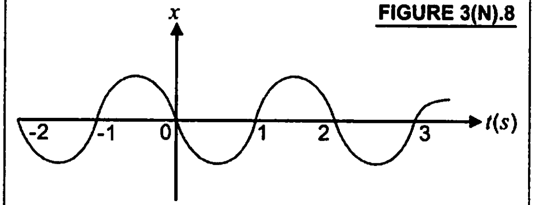


FIGURE 3(N).8

Q. 16. Fig. 3(N).8 gives the $x-t$ plot of a particle executing one dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particles at $t = 0{\cdot}3$ s, $1{\cdot}2$ s, $-1{\cdot}2$ s.

Sol. In the S.H.M. acceleration $a = -\omega^2 x$, where ω (*i.e.*, angular frequency) is constant.

(i) At time $t = 0{\cdot}3$ s, x is negative, the slope of $x - t$ plot is negative, hence **position and velocity are negative**. Since $a = -\omega^2 x$, hence, **acceleration is positive**.

(ii) At time $t = 1{\cdot}2$ s, x is positive, the slope of $x - t$ plot is also positive, hence **position and velocity are positive**. Since $a = -\omega^2 x$, hence **acceleration is negative**.

(iii) At $t = -1{\cdot}2$ s, x is negative, the slope of $x - t$ plot is also negative. But since both x and t are negative here, hence **velocity is positive**. Finally acceleration **'a' is also positive.**

Q. 17. Fig. 3(N).9 shows the $x - t$ plot of a particle in one dimensional motion. Three different equal intervals of time are shown. In which interval the average speed is greatest and in which it is the least ? Give the sign of average speed for each interval.

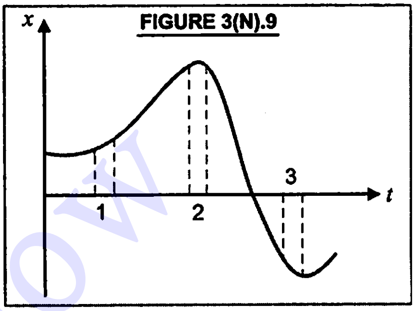


FIGURE 3(N).9

Sol. We know that average speed in a small interval of time is equal to the slope of $x - t$ graph in that inerval of time. The **average speed is the greatest in the interval 3 because** slope is greatest and the **average speed is least in inerval 2** because slope is least there.

The average speed is positive in intervals 1 and 2 because slope of $x - t$ is positive there and **average speed is negative in interval 3** because the slope of $x - t$ is negative.

Q. 18. Fig 3(N).10. gives a speed-time graph of a particle in one dimensional motion. Three different equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ? In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motion, give the signs of 'u' and 'a' in the three intervals. What are the accelerations at the points A, B, C and D ?

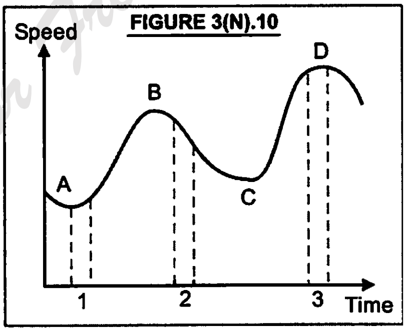


FIGURE 3(N).10

Sol. We know that average acceleration in a small interval of time is equal to slope of velocity-time graph in that interval. As the slope of velocity-time graph is maximum in interval 2 as compared to other intervals 1 and 3, hence the **magnitude of average acceleration is greatest in interval 2.**

The **average speed is greatest in interval 3** for obvious reasons.

In interval 1, the slope of velocity- time graph is positive, hence **acceleration a is positive**. The **speed u is positive in this interval due to** obvious reasons.

In interval 2, the slope of velocity-time graph is negative, hence **acceleration a is negative**. The **speed u is positive** in this interval due to obvious reasons.

In interval 3, the velocity-time graph is parallel to time axis, therefore **acceleration a is zero** in this interval but **v is positive** due to obvious reasons.

At points A, B, C and D, the velocity-time graph is parallel to time axis. Therefore acceleration **a is zero at all the four points.**

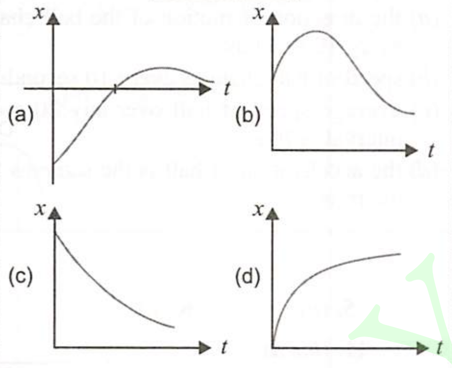
NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. Among the four graphs (Fig. 3(N).11), there is only one graph for which average velocity over the time intervel (0, T) can vanish for a suitably chosen T. Which one is it ?

FIGURE 3(N).11



2. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct ?

(*a*) $x < 0, v < 0, a > 0$ (*b*) $x > 0, v < 0, a < 0$
(*c*) $x > 0, v < 0, a > 0$ (*d*) $x > 0, v > 0, a < 0$

3. In one dimensional motion, instantaneous speed v satisfies $0 \leq v < v_0$.

(*a*) The displacement in time T must always take non-negative values.

(*b*) The displacement x in time T satisfies $-v_0 T < x < v_0 T$.

(*c*) The acceleration is always a non-negative number.

(*d*) The motion has no turning points.

4. A vehicle travels half the distance L with speed V_1 and the other half with speed V_2, then its average speed is

(*a*) $\frac{V_1 + V_2}{2}$ (*b*) $\frac{2V_1 + V_2}{V_1 + V_2}$

(*c*) $\frac{2V_1 V_2}{V_1 + V_2}$ (*d*) $\frac{L(V_1 + V_2)}{V_1 V_2}$

5. The displacement of a particle is given by $x = (t - 2)^2$ where x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is

(*a*) 4 m (*b*) 8 m (*c*) 12 m (*d*) 16 m

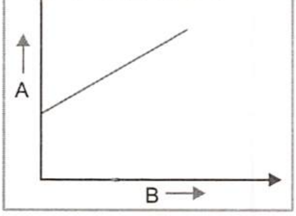
6. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be

(*a*) $(t_1 + t_2)/2$ (*b*) $t_1 t_2/(t_2 - t_1)$
(*c*) $t_1 t_2/(t_2 + t_1)$ (*d*) $t_1 - t_2$

MULTIPLE CHOICE QUESTIONS-II

7. The variation of quantity A with quantity B, plotted in Fig. 3(N).12 describes the motion of a particle in a straight line.

FIGURE 3(N).12



(*a*) Quantity B may represent time.

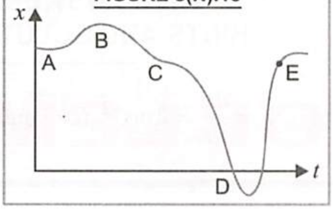
(*b*) Quantity A is velocity if motion is uniform.

(*c*) Quantity A is displacement if motion is uniform.

(*d*) Quantity A is velocity if motion is uniformly accelerated.

8. A graph of x versus t is shown in Fig. 3(N).13. Choose correct alternatives from below.

FIGURE 3(N).13



(*a*) The particle was released from rest at $t = 0$

(*b*) At *B*, the acceleration $a > 0$

(*c*) At *C*, the velocity and the acceleration vanish

(*d*) Average velocity for the motion between *A* and *D* is positive

(*e*) The speed at *D* exceeds that at *E*.

9. For the one-dimensional motion, described by

$$x = t - \sin t$$

(*a*) $x(t) > 0$ for all $t > 0$

(*b*) $v(t) > 0$ for all $t > 0$

(*c*) $a(t) > 0$ for all $t > 0$

(*d*) $v(t)$ lies between 0 and 2

10. A spring with one end attached to a mass and the other to a rigid support is stretched and released.

(*a*) Magnitude of acceleration, when just released is maximum.

(*b*) Magnitude of acceleration, when at equilibrium position, is maximum.

(*c*) Speed is maximum when mass is at equilibrium position.

(*d*) Magnitude of displacement is always maximum whenever speed is minimum.

11. A ball is bouncing elastically with a speed 1 m/s between walls of a railway compartment of size 10 m in a direction perpendicular to walls. The train is moving at a constant velocity of 10 m/s parallel to the direction of motion of the ball. As seen from the ground.

(*a*) the direction of motion of the ball changes every 10 seconds.

(*b*) speed of ball changes every 10 seconds.

(*c*) average speed of ball over any 20 second interval is fixed.

(*d*) the acceleration of ball is the same as from the train.

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*b*) **6.** (*c*)
7. (*a,c,d*) **8.** (*a,c,e*) **9.** (*a,d*) **10.** (*a,c*) **11.** (*b,c,d*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. In graph (*b*), for one value of displacement, there are two timings. As a result of it, for one time, the average velocity is positive and for other time is equivalent negative. Due to it, the average velocity for the two timings (equal to time period) can vanish

2. As lift is coming from 8th to 4th floor, the value of *x* becomes less hence negative, *i.e.*, $x < 0$. Velocity is downwards (*i.e.*, negative). So, $v < 0$. Before reaching 4th floor lift is retarded, *i.e.*, acceleration is upwards. Hence, $a > 0$.

3. The maximum distance covered in time $T = v_0 T$. Therefore, for the object having one dimensional motion, the displacement *x* in time *T* satisfies $-v_0 T < x < v_0 T$.

4. Time taken to travel first half distance, $t_1 = \dfrac{L/2}{v_1} = \dfrac{L}{2v_1}$

Time taken to travel second half distance, $t_2 = \dfrac{L/2}{v_2} = \dfrac{L}{2v_2}$

$$\text{Total time } = t_1 + t_2 = \frac{L}{2v_1} + \frac{L}{2v_2} = \frac{L}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$$

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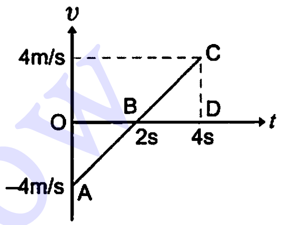
$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{L}{\frac{L}{2}\left[\frac{1}{v_1}+\frac{1}{v_2}\right]} = \frac{2v_1 v_2}{v_1+v_2}$$

5. Here, $x = (t-2)^2$; velocity, $v = \frac{dx}{dt} = 2(t-2)$ m/s ; acceleration, $a = \frac{dv}{dt} = 2\,\text{ms}^{-2}$ (*i.e.*, uniform)

When $t = 0, v = -4$ m/s ; $t = 2$ s, $v = 0$
$t = 4$ s, $v = 4$ m/s.

Velocity (v) – time (t) graph of this motion is as shown in Fig. 3(N).14.

FIGURE 3(N).14



$$\text{Distance travelled} = \text{area } AOB + \text{area } BCD = \frac{4\times2}{2}+\frac{4\times2}{2} = \mathbf{8\,m}$$

6. Velocity of girl, $v_g = \frac{L}{t_1}$; .velocity of escalator, $v_e = \frac{L}{t_2}$

Effective velocity of girl on escalator $= v_g + v_e = \frac{L}{t_1}+\frac{L}{t_2}$

If t is the time taken, then $\frac{L}{t} = \frac{L}{t_1}+\frac{L}{t_2}$ or $t = \frac{t_1 t_2}{t_1+t_2}$

Multiple Choice Questions – II

7. Quantity B may represent time. Thus, option (*a*) is true. If motion is uniform, the velocity-time graph should be a straight line parallel to time axis. Thus, quantity A cannot be a velocity. Thus, option (*b*) is wrong. If motion is uniform, the velocity is constant. The displacement-time graph is a straight line with positive slope. Thus, quantity A is displacement. If motion is uniformly accelerated, the acceleration is constant. The velocity-time graph is a straight line with positive slope. Thus, quantity A is velocity.

8. From graph, when $t = 0$, the particle is released from rest at A, hence $v = 0$.

At B, the graph is parallel to time axis, hence velocity is constant there. Thus acceleration a is zero.

At C, the graph changes slope, where velocity and acceleration vanish.

Average velocity for motion between A and D is negative because the value of x is decreasing with time t. The slope of graph (which represents speed) is more at D than at E. Therefore, speed at D is more than that at E.

9. $x = t - \sin t$; velocity, $v = \frac{dx}{dt} = 1 - \cos t$; acc. $a = \frac{dv}{dt} = \sin t$

$\therefore$ $x(t) > 0$ for all values $t > 0$ and $v(t)$ can be zero for one value of t. $a(t)$ can be zero for one value of t.
If $t = 0$, $v(t) = 1 - 1 = 0$.
If $t = \pi$, $v(t) = 1 - \cos\pi = 1 - (-1) = 2$.

10. Maximum restoring force set up in the spring when stretched by distance r is $F = -kr$ and P.E. of stretched spring $= \frac{1}{2}kx^2$. As $F \propto r$ and this force is directed towards equilibrium position, hence if mass is left free, it will execute damped SHM due to gravity pull.

Magnitude of acceleration in the mass attached to one end of spring when just released is

$$a = \frac{F}{m} = -\frac{k}{m}r = \text{(Maximum)}$$

At equilibrium position, $r = 0$ $\therefore$ $a = -\frac{k}{m}\times 0 = 0$

At equilibrium position, the whole PE of the stretched spring is converted into whole KE of the mass. Hence, speed of mass is maximum at equilibrium position.

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In damped SHM, the speed of mass is minimum at the extreme position, where displacement is maximum. But the magnitude of displacement is not always maximum whenever speed is minimum.

11. Since the ball is moving with a very small speed in the moving train, the direction of motion of the ball is the same as that of the train. The direction of motion of ball does not change with respect to observer on ground.

The speed of the ball as observed by observer on ground before collision with side of train is 10 + 1 = 11 m/s and after elastic collision is 10 – 1 = 9 m/s.

Since the collision of the ball with side of train is perfectly elastic ; the total momentum and kinetic energy are conserved, so average speed of the ball over any 20 seconds interval is constant as observed by observer on ground.

Since train is moving with constant velocity, it is an intertial frame, so acceleration of ball is same as from the train.

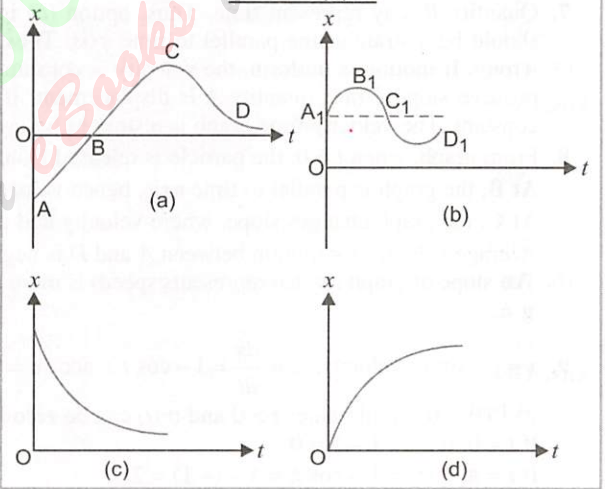
VERY SHORT ANSWER QUESTIONS

12. Refer to the graphs in Fig. 3(N).15. Match the following.

Graph		**Characteristic**
(*a*)	(*i*)	**has $v > 0$ and $a < 0$ throughout.**
(*b*)	(*ii*)	**has $x > 0$ throughout and has a point with $v = 0$ and a point with $a = 0$.**
(*c*)	(*iii*)	**has a point with zero displacement for $t > 0$.**
(*d*)	(*iv*)	**has $v < 0$ and $a > 0$.**

Ans. (*a-iii*), (*b-ii*), (*c-iv*), (*d-i*).

FIGURE 3(N).15



For graph (a), there is a point B for which displacement is zero for $t > 0$, which is option (*iii*).

For graph (b), the condition $x > 0$ throughout. There is a point B_1 at the top of the curve, where slope of the curve, *i.e.*, $dx/dt = v = 0$ and there is a point C_1 on curve, where the curvature of curve changes, which shows, acceleration = 0. This corresponds to option (*ii*).

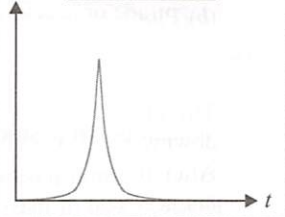
For graph (c), the slope (dx/dt) is negative, hence $v < 0$. The curvature of x-t graph indicates that the negative velocity ($= dx/dt$) is decreasing with time. Therefore, $a > 0$. This corresponds to option (*iv*).

For graph (d), the slope ($= dx/dt$) is positive, hence $v > 0$. The curvature of x-t graph indicates that positive velocity is decreasing with time. Therefore, $a < 0$. This corresponds to option (*i*).

13. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in the backward direction as positive).

Ans. Acceleration of the ball is zero just before it strikes the bat. Neglecting the gravity effect, the ball moving uniformly turned back with same speed by hitting with a bat. When ball is hit by bat it has acceleration due to impulsive force. The variation of acceleration-time graph will be as shown in Fig. 3(N).16.

FIGURE 3(N).16



14. Give examples of a one-dimensional motion where

(*a*) the particle moving along positive *x*-direction comes to rest periodically and moves forward.

(*b*) the particle moving along positive *x*-direction comes to rest periodically and moves backward.

Ans. (*a*) In one dimensional motion, the displacement of the particle in periodic motion must be linked with sine or cosine function. The particle will be moving along +ive *x*-direction only if $t > \sin t$. Hence,

$$x(t) = t - \sin t.$$

$$\text{Velocity, } v(t) = \frac{dx(t)}{dt} = 1 - \cos t \quad \text{and} \quad \text{acceleration, } a(t) = \frac{dv}{dt} = \sin t$$

$v(t) = 0$, when $t = 0$ or 2π and $a(t) = 0$.

When $t = \pi$, displacement, $x(t) = \pi - \sin \pi = \pi$ (positive)

When $t = 2\pi$, displacement, $x(t) = 2\pi - \sin 2\pi = 2\pi$ (positive).

(*b*) In one dimensional motion, where a particle moving along positive *x*-direction comes to rest periodically and moves backward can be represented by $x(t) = \sin t$.

Here, $$v = \frac{dx}{dt} = \cos t$$

$$\text{acceleration, } a = \frac{dv}{dt} = -\sin t$$

When $t = 0$,	$x = 0$,	$v =$ positive,	$a = 0$
When $t = \pi/2$,	$x = +$ive,	$v = 0$,	$a = -$ve
When $t = \pi$,	$x = 0$,	$v = -$ve,	$a = 0$
When $t = 3\pi/2$,	$x = -$ive,	$v = 0$,	$a = +$ive
When $t = 2\pi$,	$x = 0$,	$v = +$ve,	$a = 0$.

15. Give example of a motion where $x > 0$, $v < 0$, $a > 0$ at a particular instant.

Ans. As per given condition, the motion can be represented by $x(t) = A + B e^{-\gamma t}$ where $A > B$ and $\gamma > 0$ positive constant.

Now, velocity, $v(t) = -B\gamma e^{-\gamma t}$.

Acceleration, $a(t) = B\gamma^2 e^{-\gamma t}$

Therefore, for this motion $x > 0$, $v < 0$ and $a > 0$ at a particular instant of time.

16. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where g = gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed ?

Ans. Given, $a = g - bv$

A body will fall with a constant speed v_c, when $a = 0$. Therefore, $0 = g - b v_c$ or $v_c = g/b$

SHORT ANSWER QUESTIONS

17. A ball is dropped and its displacement vs time graph is as shown in Fig. 3(N).17 (displacement *x* is from ground and all quantities are +ve upwards).

(*a*) Plot qualitatively velocity vs time graph.

(*b*) Plot qualitatively acceleration vs time graph.

Ans. As *x* is positive upwards and ball is dropped from a height, so velocity of ball increases downwards due to gravity pull. Therefore, v is negative. The acceleration of the ball is downwards and is equal to acceleration due to gravity ($= -g$). After rebound the ball goes upwards, its velocity is positive, but acceleration is $-g$. Therefore, slope of $v - t$ graph is same at every rebounding of ball.

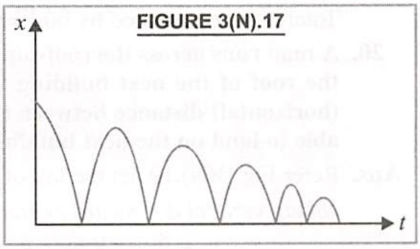


FIGURE 3(N).17

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(*a*) The velocity vs time graph is shown in Fig. 3(N).18(*a*) and
(*b*) acceleration vs time graph is shown in Fig. 3(N).18(*b*).

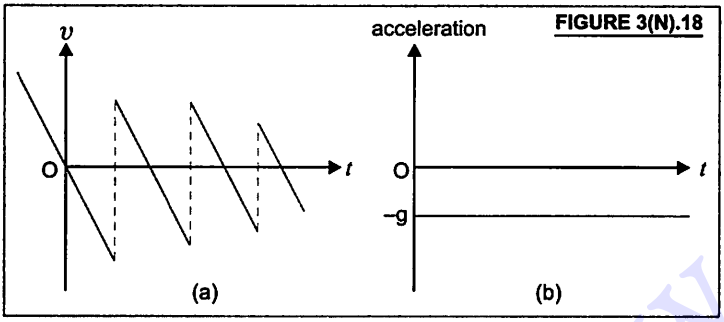

FIGURE 3(N).18

18. A particle executes the motion described by $x(t) = x_0(1 - e^{-\gamma t})$; $t \geq 0, x_0 > 0$.
(*a*) Where does the particle start and with what velocity ?
(*b*) Find maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increase with time and $v(t)$ decreases with time.

Ans. $x(t) = x_0[1 - e^{-\gamma t}]$

$$v(t) = \frac{dx(t)}{dt} = x_0 \gamma e^{-\gamma t}$$

$$a(t) = \frac{dv(t)}{dt} = -x_0 \gamma^2 e^{-\gamma t}$$

(*a*) When $t = 0$, $x(t) = x_0(1 - e^{-0}) = 0$

$v(t) = x_0 \gamma e^{-0} = \gamma x_0$

Thus, particle starts with velocity γx_0 from origin at time $t = 0$.

(*b*) $x(t)$ is maximum, when $t = \infty$. Then $x(t) = x_0$.

$x(t)$ is minimum, when $t = 0$ $x(t) = 0$ $v(t)$ is maximum, when $t = 0$; $v(0) = x_0\gamma$

$v(t)$ is minimum, when $t = \infty$; $v(\infty) = 0$ $a(t)$ is maximum, when $t = \infty$; $a(\infty) = 0$

$a(t)$ is minimum, when $t = 0$ $a(0) = -x_0\gamma^2$

From above we note that $x(t)$ and $a(t)$ increases with time but $v(t)$ decreases with time.

19. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 km/h while the other has the speed of 27 km/h. The bird starts moving from first car towards the other and is moving with the speed of 36 km/h and when the two cars were separated by 36 km. What is the total distance covered by the bird ? What is the total displacement of the bird ?

Ans. Relative velocity of each car w.r.t. another car = 27 + 27 = 54 km/h

Relative velocity of bird w.r.t. a car approaching bird = 18 + 27 = 45 km/h

Time to meet the bird with approaching car, $t = \dfrac{36\,\text{km}}{45\,\text{km/h}} = \dfrac{4}{5} = 0{\cdot}8\,\text{h}$

Total distance covered by bird = $v_b\, t = 36 \times 0{\cdot}8 =$ **28·8 km**

20. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is at a lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building ? (take $g = 10$ m/s^2)

Ans. Refer Fig. 3(N).19, let the fall of man from *O* to *A* takes *t* seconds. The vertical distance covered is 9 m.

Taking vertical downward motion from O to A, we have

$y_0 = 0, y = 9$ m ; $u_y = 0$, $a_y = 10$ m/s^2, $t = ?$

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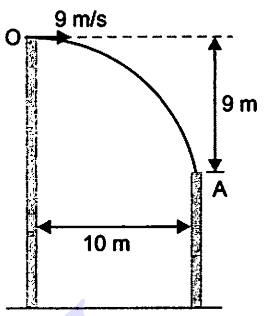
FIGURE 3(N).19



As $$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$\therefore$ $$9 = 0 + 0 + \frac{1}{2} \times 10 \times t^2 = 5t^2$$

or $$t = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} \text{ s}$$

Horizontal distance travelled in t seconds is

$$= 9 \times \frac{3}{\sqrt{5}} = \frac{27\sqrt{5}}{5} \approx 12 \text{ m} > 10 \text{ m}$$

Therefore, he will **land on the next building**

21. A ball A is dropped from a building of height 45 m. Simultaneously another ball B is thrown up with a speed 40 m/s. Calculate the relative speed of the balls as a function of time.

Ans. Speed of falling ball A after time t, $v_1 = gt$ (downwards)

Speed of projected upward ball B after time t, $v_2 = (40 - gt)$ (upwards) $= -(40 - gt)$ (downwards)

Relative velocity of ball A w.r.t. another ball B $= v_1 - v_2 = gt - [-(40 - gt)] =$ **40 m/s**

At time $t = 0$, the relative velocity of ball A w.r.t. ball B, $u_{AB} = 0 - (-40) = 40 \text{ ms}^{-1}$. Since $v_{AB} = u_{AB}$, therefore, there is no acceleration of ball A w.r.t. ball B. Hence the relative speed of ball A at any instant of time remains constant.

22. The velocity-displacement graph of a particle is shown in Fig. 3(N).20.

(*a*) Write the relation between v and x.

(*b*) Obtain the relation between acceleration and displacement and plot it.

FIGURE 3(N).20

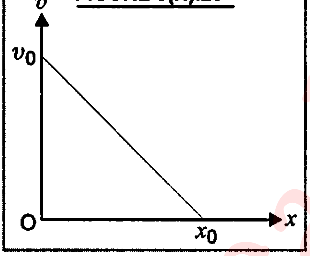


FIGURE 3(N).21

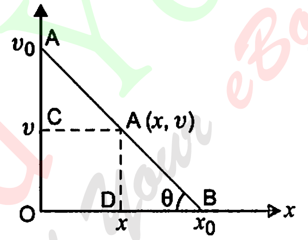
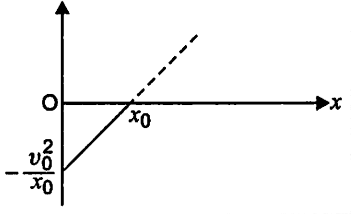


FIGURE 3(N).22



Ans. Refer Fig. 3(N).21, initial velocity $= v_0$, when $t = 0$. Distance travelled in time $t = x_0$. Let at an instant of time, the particle has velocity v and its displacement is x, which is located by position A on the graph.

From graph, $$\tan\theta = \frac{v_0}{x_0} = \frac{v_0 - v}{x}$$

or $$\frac{v_0 x}{x_0} = v_0 - v \quad \text{or} \quad v = -\frac{v_0}{x_0} x + v_0 \qquad \text{...}(i)$$

Acceleration, $$a = \frac{dv}{dt} = -\frac{v_0}{x_0}\frac{dx}{dt} + 0 = -\frac{v_0}{x_0}\left[-\frac{v_0}{x_0} x + v_0\right] \qquad \text{[from } (i)]$$

or $$a = \frac{v_0^2}{x_0^2} x - \frac{v_0^2}{x_0}$$

Graph between **a and x is shown in Fig. 3(N).22.**

LONG ANSWER QUESTIONS

23. It is a common observation that rain clouds can be at about a kilometre altitude above the ground.

(*a*) If a rain drop falls from such a height freely under gravity, what will be its speed ? Also calculate in km/h. ($g = 10$ m/s^2)

(*b*) A typical rain drop is about 4 mm diameter. Momentum is mass × speed in magnitude. Estimate its momentum when it hits ground.

(*c*) Estimate the time required to flatten the drop.

(*d*) Rate of change of momentum is force. Estimate how much force such a drop would exert on you.

(*e*) Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.

(Assume that umbrella is circular and has a diameter of 1 m and cloth is not pierced through it)

Ans. Here, $h = 1000$ m, $g = 10$ m/s^2

(*a*) Velocity of rain drop after falling height h is

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1000} = 100\sqrt{2}\ \text{m/s} = \frac{100\sqrt{2} \times 60 \times 60}{1000}\ \text{km/h} = 360\sqrt{2} \approx \mathbf{510\ km/h}$$

(*b*) Diameter of drop, $d = 2r = 4$ mm or $r = 2$ mm $= 2 \times 10^{-3}$ m

$$\text{Mass of rain drop, } m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3} \times \frac{22}{7} \times (2 \times 10^{-3})^3 \times 10^3 \approx 3{\cdot}4 \times 10^{-5}\ \text{kg}$$

Momentum of rain drop when it hits the ground, $P = mv = (3{\cdot}4 \times 10^{-5}) \times 100\sqrt{2} \approx \mathbf{4{\cdot}7 \times 10^{-3}\ kg\ m/s}$

(*c*) Time required to flatten the drop is

$$\Delta t = \frac{d}{v} = \frac{4 \times 10^{-3}}{100\sqrt{2}} = 0{\cdot}028 \times 10^{-3} = \mathbf{28 \times 10^{-6}\ s}$$

$$(d)\ \text{Force, } F = \frac{\text{change in momentum}}{\text{time}} = \frac{P - 0}{\Delta t} = \frac{4{\cdot}7 \times 10^{-3}}{28 \times 10^{-6}} \approx \mathbf{168\ N}$$

(*e*) Given, radius of umbrella, $R = 1/2$ m

$$\text{Area of umbrella} = \pi R^2 = \frac{22}{7}\left(\frac{1}{2}\right)^2 = \frac{22}{28} = \frac{11}{14} \approx 0{\cdot}8\ \text{m}^2$$

with average separation of 5 cm ($= 5 \times 10^{-2}$ m), number of drops those will hit umbrella simultaneously

$$= \frac{0{\cdot}8}{(5 \times 10^{-2})^2} \approx 320$$

Net force on umbrella = 320 × 168 = 53760 N ≈ **54000 N**

24. A motor car moving at a speed of 72 km/h can not come to a stop in less than 3·0 s while for a truck this time interval is 5·0 s. On a highway the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump onto (collide with) the truck. Human response time is 0·5 s.

Ans. Here, $u = 72$ km/h $= 20$ m/s ; Given, the car is behind the truck.

$$\text{Retardation of truck} = \frac{20}{5} = 4\ \text{m/s}^2$$

$$\text{Retardation of car} = \frac{20}{3}\ \text{m/s}^2$$

Let car be at a distance x from truck and t be the time taken to cover this distance.

As car decelerates only after 0·5 s, so using relation, $v = u + at$, we have

$$V_{\text{car}} = 20 - \left(\frac{20}{3}\right)(t - 0{\cdot}5) \qquad \text{and} \qquad V_{\text{truck}} = 20 - 4t$$

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The car will just touch to the truck if

$$20-4t=20-\frac{20}{3}(t-0\cdot5) \quad \text{or} \quad 4t=\frac{20}{3}(t-0\cdot5)$$

or $$t=\frac{5}{3}(t-0\cdot5) \quad \text{or} \quad 3t=5t-2\cdot5 \quad \text{or} \quad t=2\cdot5/2=(5/4)\text{ s}$$

Distance travelled by truck in this time t.

$$S_{\text{truck}}=50\left(\frac{5}{4}\right)-\frac{1}{2}(4)\left(\frac{5}{4}\right)^2=21\cdot875\text{ m}$$

Distance travelled by car in time t

$$S_{\text{car}}=(20\times0\cdot5)+20\left(\frac{5}{4}-0\cdot5\right)-\frac{1}{2}\left(\frac{20}{3}\right)\left(\frac{5}{4}-0\cdot5\right)^2=23\cdot125\text{ m}$$

$\therefore$ $$S_{\text{car}}-S_{\text{truck}}=23\cdot125-21\cdot875=1\cdot250\text{ m}$$

Therefore, to avert the collision, the car must maintain the distance from truck **more than 1·250 m.**

25. A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time t is given by $v(t)=2t(3-t)$; $0<t<3$ s and $v(t)=-(t-3)(6-t)$ for $3<t<6$ s in m/s. It repeats this cycle till it reaches the height of 20 m.

(*a*) At what time is its velocity maximum ? (*b*) At what time is its average velocity maximum ?

(*c*) At what time is its acceleration maximum in magnitude ?

(*d*) How many cycles (counting fractions) are required to reach the top ?

Ans. $v(t)=2t(3-t)=6t-2t^2$ ...(*i*)

$$a=\frac{dv(t)}{dt}=6-4t$$

(*a*) When velocity $v(t)$ is maximum, $\frac{dv(t)}{dt}=0$ so $6-4t=0$ or $t=(3/2)$ s

(*b*) From (*i*), $\frac{ds}{dt}=6t-2t^2$ or $ds=(6t-2t^2)\,dt$

Distance travelled in time interval 0 to 3 seconds will be

$$S_1=\int_0^3(6t-2t^2)dt=\left(\frac{6t^2}{2}-\frac{2t^3}{3}\right)_0^3=\left(\frac{6\times3^2}{2}-\frac{2\times3^3}{3}\right)=9\text{ m}$$

$$v_{av}=\frac{S_1}{t}=\frac{9}{3}=3\text{ m/s}$$

From (*i*), $3=6t-2t^2$ or $2t^2-6t+3=0.$

$\therefore$ $$t=\frac{6\pm\sqrt{36-24}}{4}=\frac{6\pm\sqrt{12}}{4}=\frac{9\cdot464}{4}\approx\frac{9}{4}\text{s}$$

(*c*) In periodic motion, acceleration is maximum where the velocity is zero. It is so when $t=0$ or $t=3$ s

(*d*) $$S_1=\int_0^3(6t-2t^2)dt=\left(\frac{6t^2}{2}-\frac{2t^3}{3}\right)_0^3=3\times3^2-\frac{2}{3}\times3^3=27-18=9\text{ m}$$

$$S_2 = \int_3^6 -(t-3)(6-t)\,dt = \int_3^6 (18-9t+t^2)\,dt = \left(18t - \frac{9t^2}{2} + \frac{t^3}{3}\right)_3^6$$

$$= \left(18\times6 - \frac{9\times6^2}{2} + \frac{6^3}{3}\right) - \left(18\times3 - \frac{9\times3^2}{2} + \frac{3^3}{3}\right)$$

$$= 18 - 22{\cdot}5 = -4{\cdot}5 \text{ m}$$

Total distance travelled in one cycle = $S_1 + S_2 = 9 - 4{\cdot}5 = 4{\cdot}5$ m.

No. of cycles covered in total distance to be covered $= \dfrac{20}{4{\cdot}5} \approx 4{\cdot}44 = 5$

26. A man is standing on top of a building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is + 15 m at $t = 2$ s. The gap is found to remain constant. Calculate the velocity with which the balls were thrown and the exact time interval between their throw.

Ans. Let the first ball be thrown with velocity u and second ball with a time interval x. Taking vertical motion of ball first for time $(t + x)$, we have

$$S_1 = u(t+x) - \frac{1}{2}g(t+x)^2 \qquad \text{...(}i\text{)}$$

Taking upward motion of second ball for time t, we have

$$S_2 = \frac{u}{2}t - \frac{1}{2}gt^2 \qquad \text{...(}ii\text{)}$$

$$\therefore \quad S_1 - S_2 = u\left(t + x - \frac{t}{2}\right) - \frac{1}{2}g(t^2 + 2xt + x^2 - t^2)$$

$$= u\left(\frac{t}{2} + x\right) - \frac{1}{2}g(2xt + x^2)$$

As per question, $S_1 - S_2 = 15$ m ; $t = 2$ s and $x = 1$ s [as $x < 2$ s]

Then, $\quad 15 = u\left(\dfrac{2}{2} + 1\right) - \dfrac{1}{2}\times10(2\times1\times2 + 1^2)$

or $\quad 15 = u \times 2 - 25$

or $\quad u = 40/2 = $ **20 m/s**

The velocity of other ball $= \dfrac{u}{2} = \dfrac{20}{2} = $ **10 m/s**. Time interval $x =$ **1 s**

Competition FOCUS

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I. Multiple Choice Questions (with One Correct Answer)

I. Displacement and velocity

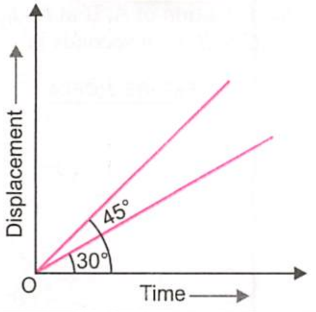
1. A student goes from his house to his friend's house with speed v_1. Finding the door of his friend's home closed, he returns back to his own house with the speed v_2. Then the average speed and net displacement of student is [consider distance between two houses be S]

(*a*) $\dfrac{v_1+v_2}{2}, 0$ (*b*) $\sqrt{v_1 v_2}, 2S$

(*c*) $\dfrac{2v_1 v_2}{v_1+v_2}, 0$ (*d*) $v_1 v_2, S$

2. The displacement-time graphs of two moving particles make angles of 30° and 45° with the x-axis as shown in Fig. 3(CF).1. The ratio of their respective velocity is

FIGURE 3(CF).1



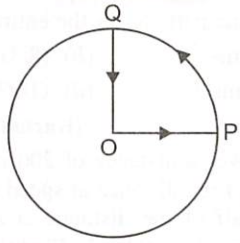
(*a*) $\sqrt{b} : 1$ (*b*) $1 : 1$

(*c*) $1 : 2$ (*d*) $1 : \sqrt{3}$

(NEET 2022)

3. A cyclist starts from the centre O of a circular park of radius one kilometre, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the Fig. 3(CF).2. If the round trip takes ten minutes, the net displacement and average speed of the cyclist (in metre and kilometre per hour) is

FIGURE 3(CF).2



(*a*) 0, 1   (*b*) 21·4, $\dfrac{\pi+4}{2}$

(*c*) $\dfrac{\pi+4}{2}, 0$ (*d*) 0, 21·4

ANSWERS

1. (*c*) **2.** (*d*) **3.** (*d*)

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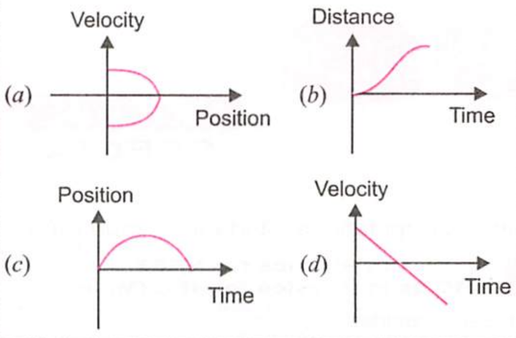
4. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

FIGURE 3(CF).3



(JEE Main 2018)

5. Train A and Train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1·8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (Take the distance between the tracks as negligable)

(*a*) 29·5 ms^{-1} (*b*) 30·5 ms^{-1}

(*c*) 31·5 ms^{-1} (*d*) 28·5 ms^{-1}

(JEE Main 2020)

6. A body of mass m moving along a straight line covers half the distance with a speed of 2 ms^{-1}. The remaining half of the distance is covered in two equal time intervals with a speed of 3 ms^{-1} and 5 ms^{-1} respectively. The average speed of the particle for the entire journey is

(*a*) (3/8) ms^{-1} (*b*) (8/3) ms^{-1}

(*c*) (4/3) ms^{-1} (*d*) (16/3) ms^{-1}

(Karnataka CET 2009)

7. A car moves a distance of 200 m. It covers the first half of the distance at speed of 40 km/h and second half of the distance at a speed v. The average speed is 48 km/h. Find the value of v.

(*a*) 56 km/h (*b*) 60 km/h

(*c*) 50 km/h (*d*) 58 km/h

8. A taxi leaves the station A for station B every 10 minutes. Simultaneously, a taxi also leaves the station B for station A every 10 minutes. The taxies move at the same constant speed and go from A to B or vice versa in 2 hours. How many taxies coming from other side will meet each taxi enroute from B to A ?

(*a*) 11 (*b*) 12

(*c*) 23 (*d*) 24.

II. Accelerated Motion

9. A man throws ball into the air one after the other. Throwing one when other is at the highest point. How high the balls rise if he throws twice a second.

(*a*) 2·45 m (*b*) 1·225 m

(*c*) 19·6 m (*d*) 4·9 m

10. A bus begins to move with an acceleration of 1 ms^{-2}. A man who is 48 m behind the bus starts running at 10 ms^{-1} to catch the bus. The man will be able to catch the bus after

(*a*) 6 s (*b*) 5 s

(*c*) 3 s (*d*) 7 s

(*e*) 8 s **(Kerala PET 2011)**

11. A particle is moving such that its position coordinates (x, y) are (2 m, 3 m) at time $t = 0$, (6 m, 7 m) at time $t = 2$ s and (13 m, 14 m) at time $t = 5$ s. Average velocity vector $(\vec{V}_{av})$ from $t = 0$ to $t = 5$ s is :

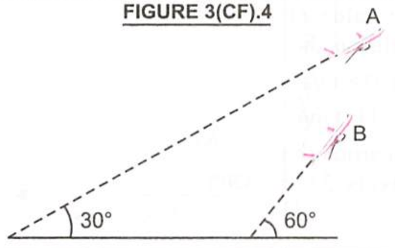
(*a*) $\frac{1}{5}(13\hat{i}+14\hat{j})$ (*b*) $\frac{7}{3}(\hat{i}+\hat{j})$

(*c*) $2(\hat{i}+\hat{j})$ (*d*) $\frac{11}{5}(\hat{i}+\hat{j})$

(AIPMT 2014)

12. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in Fig. 3(CF).4. The speed of A is $100\sqrt{3}$ m/s. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. The observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is

FIGURE 3(CF).4



ANSWERS

4. (*b*) **5.** (*a*) **6.** (*b*) **7.** (*b*) **8.** (*c*) **9.** (*b*) **10.** (*e*) **11.** (*d*) (*a*) 3·5 s (*b*) 5 s

(*c*) 6·5 s (*d*) 7 s

(JEE Advanced 2014)

13. Two balls of equal masses are thrown upwards along the same vertical direction at an interval of 2 seconds, with the same initial velocity of 39·2 m/s. Then these collide at a height of

(*a*) 44·1 m (*b*) 73·5 m

(*c*) 117·6 m (*d*) 196·0 m

14. Two balls of equal masses are thrown upwards along the same vertical direction at an interval of 2 seconds, with the same initial velocity of 45 ms^{-1}. Then these balls collide at a height of (use $g = 10\ ms^{-2}$)

(*a*) 54·2 m (*b*) 96·25 m

(*c*) 162·7 m (*d*) 217·0 m

15. In a car race on straight road, car *A* takes a time *t* less than car *B* at the finish and passing fishing point with a speed *v* more than that of car *B*. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then *v* is equal to :

(*a*) $\dfrac{2a_1a_2}{a_1+a_2}t$ (*b*) $\sqrt{2a_1a_2}\,t$

(*c*) $\sqrt{a_1\,a_2}\,t$ (*d*) $\left(\dfrac{a_1+a_2}{2}\right)t$

(JEE Main 2019)

16. A body is projected vertically upwards with a velocity of 10 m/s. It reaches the maximum height *h* in time *t*. In time *t*/2 the height covered is ($g = 10\ m/s^2$)

(*a*) *h*/2 (*b*) (2/5) *h*

(*c*) (3/4) *h* (*d*) (5/8) *h*

17. Two trains travelling on the same track are approaching each other with equal speeds of 40 m/s. The drivers of the trains begin to decelerate simultaneously when just 2·0 km apart. Assuming decelerations to be uniform and equal, the value of the deceleration to barely avoid collision should be

(*a*) 0·8 m/s^2 (*b*) 2·1 m/s^2

(*c*) 11·0 m/s^2 (*d*) 11·8 m/s^2

18. A particle moving with a uniform acceleration travels 24 metre and 64 metre in first two successive intervals of 4 seconds each. Its initial velocity is

(*a*) 1 m/s (*b*) 2 m/s

(*c*) 5 m/s (*d*) 10 m/s.

19. Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant ? ($g = 10\ m/s^2$).

(*a*) 1·25 m (*b*) 2·50 m

(*c*) 3·75 m (*d*) 4·00 m

20. A body released from the top of a tower falls through half the height of tower in 3 seconds. It will reach the ground after nearly

(*a*) 3·5 sec (*b*) 4·24 sec

(*c*) 4·71 sec (*d*) 6 sec

21. A ball is thrown up, it reaches a maximum height and then comes down. If t_1 and t_2 ($t_2 > t_1$) are the times that the ball takes to be at a particular height then the time taken by the ball to reach the highest point is

(*a*) $(t_1 + t_2)$ (*b*) $(t_2 - t_1)$

(*c*) $(t_2 - t_1)/2$ (*d*) $(t_2 + t_1)/2$

22. A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distances of one metre each will then be

(*a*) all equal being equal to $\sqrt{(2/g)}$ second

(*b*) in the ratio of square roots of the integers 1, 2, 3, 4

(*c*) in the ratio of the differences in the square roots of integers *i.e.* $(\sqrt{1}-\sqrt{0}),\ (\sqrt{2}-\sqrt{1}),\ (\sqrt{3}-\sqrt{2}),\ (\sqrt{4}-\sqrt{3}),....$

(*d*) in the ratio $\dfrac{1}{\sqrt{1}}:\dfrac{1}{\sqrt{2}}:\dfrac{1}{\sqrt{3}}:\dfrac{1}{\sqrt{4}}$.

23. A man in a balloon rising vertically with an acceleration of 4·9 ms^{-2} releases a ball 2 seconds after the balloon is let go from the ground. The greatest height above the ground reached by the ball is

(*a*) 9·8 m (*b*) 14·7 m

(*c*) 19·6 m (*d*) 24·5 m.

ANSWERS

12. (*b*) **13.** (*b*) **14.** (*b*) **15.** (*c*) **16.** (*c*) **17.** (*a*) **18.** (*a*) **19.** (*c*) **20.** (*b*) **21.** (*d*)
22. (*c*) **23.** (*b*)

24. A bullet loses 1/20 of its velocity in passing through a plank. What is the least number of planks required to stop the bullet

(*a*) 8 (*b*) 7

(*c*) 11 (*d*) 14

25. A balloon rises from rest with a constant acceleration $g/8$. A stone is released from it when it has risen to height h. The time taken by the stone to reach the ground is

(*a*) $4\sqrt{\frac{h}{g}}$ (*b*) $2\sqrt{\frac{h}{g}}$

(*c*) $\sqrt{\frac{2h}{g}}$ (*d*) $\sqrt{\frac{g}{h}}$ **(AIIMS 2010)**

26. A ball released from the top of a tower travels $\frac{11}{36}$ of the height of the tower in the last second of its journey. The height of the tower is ($g = 10\ \text{ms}^{-2}$)

(*a*) 11 m (*b*) 36 m

(*c*) 180 m (*d*) 110 m

27. If a particle is thrown vertically upwards, then its velocity so that it covers same distance in 5th and 6th seconds would be

(*a*) 48 m/s (*b*) 14 m/s

(*c*) 49 m/s (*d*) 7 m/s

28. A stone is dropped from a certain height which can reach the ground in 5 seconds. If the stone is stopped after 3 seconds of its fall and then allowed to fall again, then the time taken by the stone to reach the ground for the remaining distance is

(*a*) 2 s (*b*) 3 s

(*c*) 4 s (*d*) none of these.

29. A ball is dropped from a bridge 122·5 m above a river. After the ball has been falling for 2*s*, a second ball is thrown straight down after it. What must the initial velocity of the second ball be so that both hit the water at the same time ?

(*a*) 40 m/s (*b*) 55·5 m/s

(*c*) 26·1 m/s (*d*) 9·6 m/s

(AIIMS 2015)

30. A toy car with charge q moves on a frictionless horizontal plane surface under the influence of a uniform electric field $\vec{E}$. Due to the force $q\vec{E}$, its velocity increases from 0 to 6 m/s in one second duration. At that instant the direction of the field is reversed. The car continues to move for two more seconds under the influence of this field. The average velocity and the average speed of the toy car between 0 to 3 seconds are respectively

(*a*) 1 m/s, 3.5 m/s (*b*) 1 m/s, 3 m/s

(*c*) 2 m/s, 4 m/s (*d*) 1.5 m/s, 3 m/s

(NEET 2018)

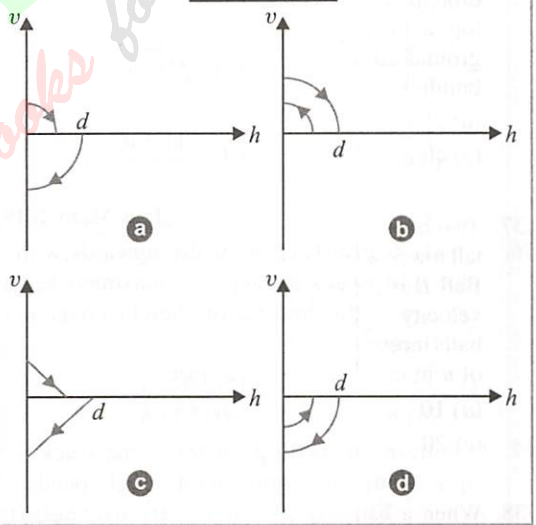
31. A particle moving in a straight line with initial velocity u and uniform acceleration f. If the sum of the distances travelled in tth and $(t + 1)$th seconds is 100 cm, then its velocity after t seconds in cm/s is

(*a*) 20 (*b*) 30

(*c*) 50 (*d*) 80

32. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

FIGURE 3(CF).5



(JEE Main 2020)

33. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at $2\ \text{ms}^{-2}$. He reaches the ground with a speed of $3\ \text{ms}^{-1}$. At what height, did he bail out ?

(*a*) 111 m (*b*) 293 m

(*c*) 182 m (*d*) 243 m

ANSWERS

24. (*c*) 25. (*b*) 26. (*c*) 27. (*c*) 28. (*c*) 29. (*c*) 30. (*b*) 31. (*c*) 32. (*a*) 33. (*b*)

34. A balloon is ascending vertically with an acceleration of $0{\cdot}2\ \text{ms}^{-2}$. Two stones are dropped from it at an interval of 2 s. The distance between them when the second stone dropped is (take $g = 9{\cdot}8\ \text{ms}^{-2}$)

(*a*) 0·4 m (*b*) 4·9 m
(*c*) 19·6 m (*d*) 20·0 m.

35. If a train engine crosses a signal with a velocity u and has constant acceleration and the last bogey of the train crosses the signal with velocity v, then middle point of the train crosses the signal with velocity :

(*a*) $\dfrac{u+v}{2}$ (*b*) $\sqrt{\dfrac{v^2+u^2}{2}}$
(*c*) $\sqrt{\dfrac{v^2-u^2}{2}}$ (*d*) $\dfrac{v-u}{2}$

(JEE Main 2021)

36. A stone is released from top of a building of height h. When it goes by 5 m from top, another stone is dropped from the height which is 25 m below the top of the building. Both the stones reached the ground simultaneously. Find the height of the building. ($g = 10\ \text{ms}^{-2}$)

(*a*) 25 m (*b*) 35 m
(*c*) 40 m (*d*) 45 m

(JEE Main 2021)

37. Two balls A and B are placed at the top of 180 m tall tower. Ball A is released from the top at $t = 0$ s. Ball B is thrown vertically down with an initial velocity u at $t = 2$ s. After a certain time, both balls meet 100 m above the ground. Find the value of u in ms^{-1}. [Use $g = 10\ \text{ms}^{-2}$]

(*a*) 10 (*b*) 15
(*c*) 20 (*d*) 30

(JEE Main 2022)

38. When a ball dropped into a lake from a height 4·9 m above the water level, it hits the water with a velocity u and then sinks to the bottom with constant velocity u. It reaches the bottom of the lake 4·0 s after it is dropped. The approximate depth of the lake is :

(Use $g = 9{\cdot}8\ \text{ms}^{-2}$)

(*a*) 19·6 m (*b*) 29·4 m
(*c*) 39·2 m (*d*) 73·5 m

(JEE Main 2022)

39. A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10 m in t s, the distance travelled by the toy in the next t s will be :

(*a*) 10 m (*b*) 20 m
(*c*) 30 m (*d*) 40 m

(JEE Main 2022)

40. A person sitting in the ground floor of a building notices through the window, of height 1·5 m, a ball dropped from the roof of the building crosses the window in 0·1 s. What is the velocity of the ball when it is at the topmost point of the window ? ($g = 10\ \text{ms}^{-2}$)

(*a*) $20\ \text{ms}^{-1}$ (*b*) $15{\cdot}5\ \text{ms}^{-1}$
(*c*) $14{\cdot}5\ \text{ms}^{-1}$ (*d*) $4{\cdot}5\ \text{ms}^{-1}$

(NEET 2020)

41. A small block slides down a smooth inclined plane, starting from rest at $t = 0$. Let S_n be the distance travelled by the block in the time interval $t = (n-1)$ to $t = n$. Then the ratio $\dfrac{S_n}{S_{n+1}}$ is

(*a*) $\dfrac{2n}{2n-1}$ (*b*) $\dfrac{2n-1}{2n}$
(*c*) $\dfrac{2n-1}{2n+1}$ (*d*) $\dfrac{2n+1}{2n-1}$

(NEET 2021)

III. Accelerated Motion using Calculus Method

42. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$, where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by

(*a*) $-2n\beta^2 x^{-4n-1}$ (*b*) $-2\beta^2 x^{-2n+1}$
(*c*) $-2n\beta^2 x^{-4n+1}$ (*d*) $-2n\beta^2 x^{-2n-1}$

(AIPMT 2015)

43. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$, where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be

(*a*) $a + \dfrac{b^2}{4c}$ (*b*) $a + \dfrac{b^2}{c}$

ANSWERS

34. (*d*) 35. (*b*) 36. (*d*) 37. (*d*) 38. (*b*) 39. (*c*) 40. (*c*) 41. (*c*) 42. (*a*) 43. (*d*)

(c) $a+\dfrac{b^2}{2c}$ (d) $a+\dfrac{b^2}{3c}$

(JEE Main 2019)

44. The position vector of a particle changes with time according to the relation

$$\vec{r}\,(t)=15t^2\,\hat{i}+(4-20t^2)\,\hat{j}.$$

What is the magnitude of acceleration at $t = 1$?

(a) 40 (b) 100

(c) 25 (d) 50

(JEE Main 2019)

45. The displacement x of a particle varies with time t as $x = a\,e^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) be independent of β

(b) drop to zero when $\alpha = \beta$

(c) go on decreasing with time

(d) go on increasing with time

46. The acceleration of a particle (a) is related to its velocity (v) by $a = -2\,v$. What is the nature of velocity-time curve ?

(a) Linearly increasing

(b) Exponentially decreasing

(c) Exponentially increasing

(d) Linearly decreasing **(IAS 2010)**

47. An object, moving with a speed of 6·25 m/s, is decelerated at a rate given by :

$$\frac{dv}{dt}=-2{\cdot}5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(a) 2 s (b) 4 s

(c) 8 s (d) 1 s **(AIEEE 2011)**

48. The acceleration experienced by a moving boat after its engine is cut off, is given by $\dfrac{dv}{dt}=-k\,v^3$, where k is a constant. If v_0 is the magnitude of the velocity at cut off. Then the magnitude of the velocity at time t after the cut off is

(a) $\dfrac{v_0}{\sqrt{1-2k\,v_0^2\,t}}$ (b) $\dfrac{v_0}{\sqrt{1+2k\,v_0^2\,t}}$

(c) $\dfrac{v_0}{1-2k\,v_0\,t}$ (d) $\dfrac{v_0}{1+2k\,v_0\,t}$

(WB JEE 2006)

49. The displacement of the particle is zero at $t = 0$ and it is x at $t = t$. It starts moving in the positive x-direction with a velocity which varies, $v = k\sqrt{x}$, where k is a constant. Then the relation for variation of velocity with time is

(a) $\dfrac{2k^2}{t}$ (b) $\dfrac{2t^2}{k}$

(c) $\dfrac{k^2\,t}{2}$ (d) $\dfrac{2}{kt^2}$

50. The retardation experienced by a moving motor boat after its engine is cut-off, at the instant t is given by, $a = -k\,v^4$, where k is a constant. If v_0 is the magnitude of velocity at the cut-off, the magnitude of velocity at time t after the cut-off is

(a) $\dfrac{v_0}{\sqrt{3kt\,v_0^3}}$ (b) $\dfrac{v_0}{(3kt\,v_0^3+1)^{1/3}}$

(c) $\sqrt{3kt\,v_0^3}$ (d) $(3kt\,v_0^3+1)^{1/3}$

51. The position co-ordinates of a particle moving in a 3-D co-ordiantes system is given by

$$x = a\cos\omega t\ ;\ \ y = a\sin\omega t\ \text{ and }\ z = a\,\omega t$$

The speed of the particle is :

(a) $\sqrt{2}\,a\omega$ (b) $a\omega$

(c) $\sqrt{3}\,a\omega$ (d) $2\,a\omega$

(JEE Main 2019)

52. A particle moving along X-axis has acceleration f, at time t, given by $f = f_0\left(1-\dfrac{t}{T}\right)$, here f_0 and T are constants. The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle velocity (v_x) is

(a) $\dfrac{1}{2}f_0\,T^2$ (b) $f_0\,T^2$

(c) $\dfrac{1}{2}f_0\,T$ (d) $f_0\,T$ **(AIPMT 2007)**

53. The x and y coordinates of the particle at any time are $x = 5\,t - 2\,t^2$ and $y = 10\,t$ respectively, where x and y are in meters and t in seconds. The acceleration of the particle at $t = 2\ s$ is

(a) 0 (b) 5 m/s^2

(c) – 4 m/s^2 (d) – 8 m/s^2

(NEET 2017)

ANSWERS

44. (d) **45.** (d) **46.** (b) **47.** (a) **48.** (b) **49.** (c) **50.** (b) **51.** (a) **52.** (c) **53.** (c)

54. A particle is moving with velocity $\vec{v} = K(y\,\hat{i} + x\,\hat{j})$, where K is a constant. The general equation for its path is

(*a*) $y^2 = x +$ constant (*b*) $xy =$ constant

(*c*) $y^2 = x^2 +$ constant (*d*) $y = x^2 +$ constant

(JEE Main 2019, AIEEE 2010)

55. A particle crossing the origin of coordinates at time $t = 0$, moves in the x-y plane with a constant acceleration a in the y-direction. If the equation of motion is $y = bx^2$ (b is a constant), its velocity component in the x-direction is

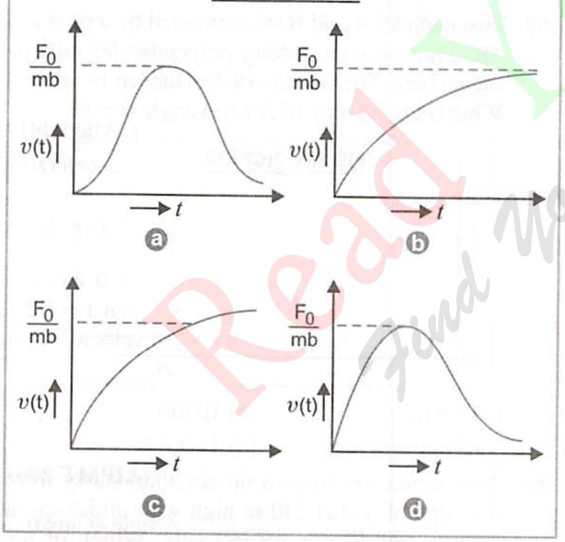
(*a*) $\sqrt{\dfrac{2b}{a}}$ (*b*) $\sqrt{\dfrac{a}{2b}}$

(*c*) $\sqrt{\dfrac{a}{b}}$ (*d*) $\sqrt{\dfrac{b}{a}}$

(*e*) $\sqrt{ba}$ **(Kerala PET 2011)**

56. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0\, e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves ?

FIGURE 3(CF).6



(AIEEE 2012)

57. The position vector of a particle $\vec{R}$ as a function of time is given by

$$\vec{R} = 4\sin(2\pi t)\,\hat{i} + 4\cos(2\pi t)\,\hat{j},$$

where R is in metres, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x- and y-directions respectively. Which one of the following statements is **wrong** for the motion of particle ?

(*a*) Path of particle is a circle of radius 4 metre

(*b*) Acceleration vector is along $-\vec{R}$

(*c*) Magnitude of acceleration vector is v^2/R, where v is the velocity of the particle

(*d*) Magnitude of the velocity of particle is 8 metre/second **(RE-AIPMT 2015)**

58. A particle is moving with speed $v = b\sqrt{x}$ along x-axis. Calculate the speed of the particle at time $t = \tau$ (assuming that the particle is at origin at $t = 0$).

(*a*) $b^2\tau$ (*b*) $\dfrac{b^2\tau}{4}$

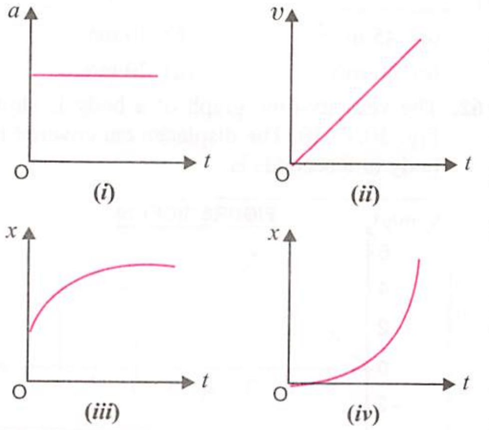
(*c*) $\dfrac{b^2\tau}{2}$ (*d*) $\dfrac{b^2\tau}{\sqrt{2}}$

(JEE Main 2019)

IV. Graphical study of one dimensional motion

59. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration), v = velocity ; x = displacement, t = time).

FIGURE 3(CF).7



(*a*) (*i*), (*ii*), (*iii*) (*b*) (*i*)

(*c*) (*ii*), (*iii*) (*d*) (*i*), (*ii*), (*iv*)

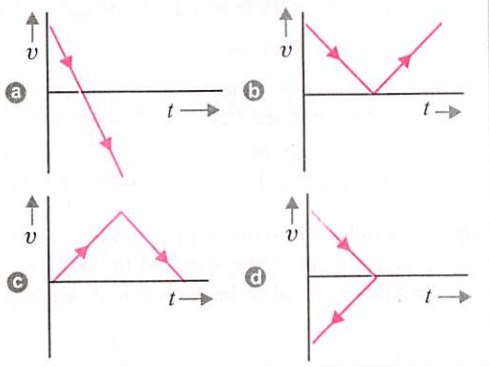
(JEE Main 2019)

ANSWERS

54. (*c*) 55. (*b*) 56. (*b*) 57. (*d*) 58. (*c*) 59. (*d*)

60. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time ?

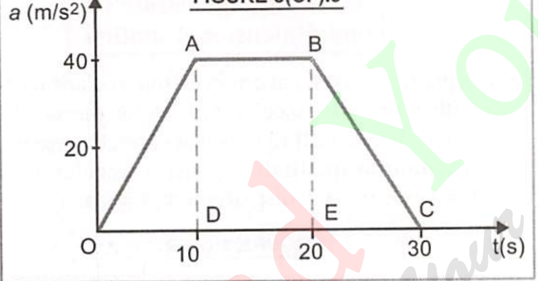
FIGURE 3(CF).8



(JEE Main 2017)

61. Fig. 3(CF).9 shows the time acceleration graph for a particle in rectilinear motion. The average acceleration in first twenty seconds is

FIGURE 3(CF).9

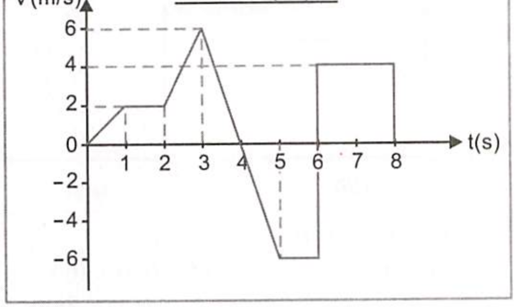


(*a*) 45 m/s^2 (*b*) 40 m/s^2

(*c*) 30 m/s^2 (*d*) 20 m/s^2

62. The velocity-time graph of a body is shown in Fig. 3(CF).10. The displacement covered by the body in 8 seconds is

FIGURE 3(CF).10



(*a*) 9 m (*b*) 12 m

(*c*) 18 m (*d*) 27 m.

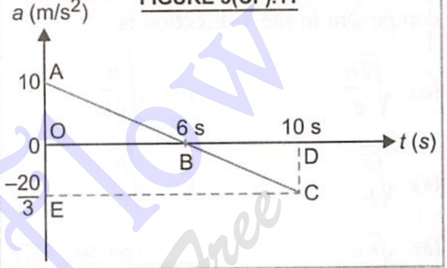
63. In the above question, the total distance covered by the body in 8 seconds is

(*a*) 9 m (*b*) 12 m

(*c*) 18 m (*d*) 27 m.

64. The acceleration time graph of a particle is shown in the Fig. 3(CF).11. At time $t = 0$, the velocity of the particle is 8 ms^{-1}. Its velocity after $t = 10$ s is

FIGURE 3(CF).11

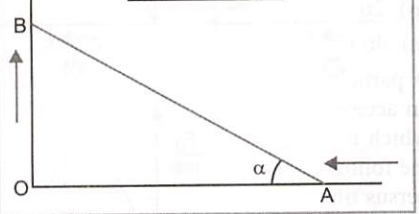


(*a*) $\frac{50}{3}$ ms^{-1} (*b*) $\frac{70}{3}$ ms^{-1}

(*c*) $\frac{74}{3}$ ms^{-1} (*d*) $\frac{144}{3}$ ms^{-1}

65. Two particles *A* and *B* are connected by a rigid rod *AB*. The rod slides along perpendicular rails as shown here. The velocity of *A* to the left is 10 m/s. What is the velocity of *B* when angle α = 30°.

FIGURE 3(CF).12



(*a*) 9·8 m/s (*b*) 10 m/s

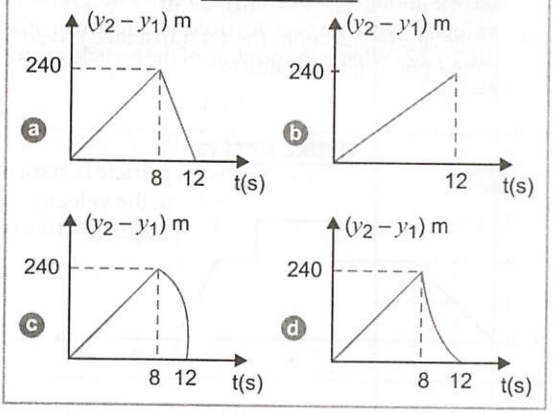
(*c*) 5·8 m/s (*d*) 17·3 m/s

66. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ? [Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2] (The figures are schematic and not drawn to scale)

ANSWERS

60. (*a*) **61.** (*c*) **62.** (*a*) **63.** (*d*) **64.** (*c*) **65.** (*d*)

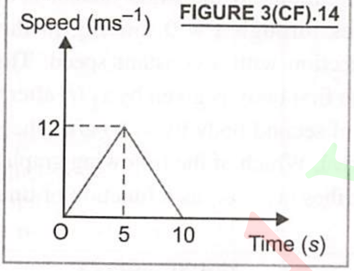
FIGURE 3(CF).13



(JEE Main 2015)

67. The speed-time graph of a particle moving along a fixed direction is shown in the Fig. 3(CF).14. The distance traversed by the particle between $t = 2$ s to 6 s is

FIGURE 3(CF).14



(*a*) 26 m (*b*) 36 m

(*c*) 46 m (*d*) 56 m

68. A particle starts from rest at $t = 0$ and undergoes an acceleration a in ms^{-2} with time t in seconds which is shown in Fig. 3(CF).15. Which one of the following plot represents velocity v (in ms^{-1}) versus time (in seconds) ?

FIGURE 3(CF).15

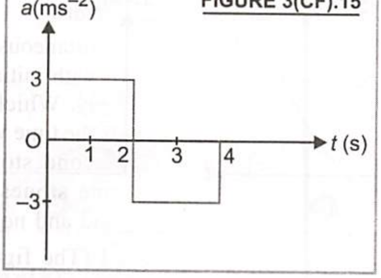
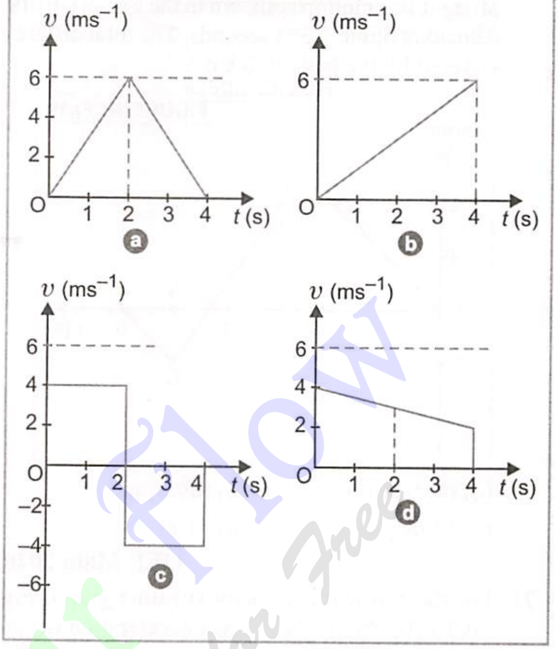


FIGURE 3(CF).16



69. What will be the a vs x graph for the graph shown in Fig. 3(CF).17

FIGURE 3(CF).17

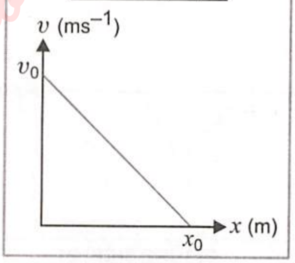
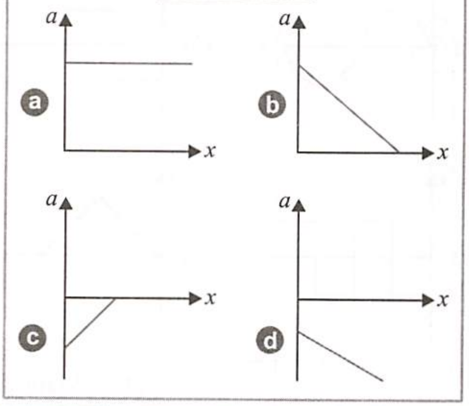


FIGURE 3(CF).18



ANSWERS

66. (*c*) 67. (*b*) 68. (*a*) 69. (*c*)

70. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the Fig. 3(CF).19. The point S is at 4·333 seconds. The total distance covered by the body in 6 s is :

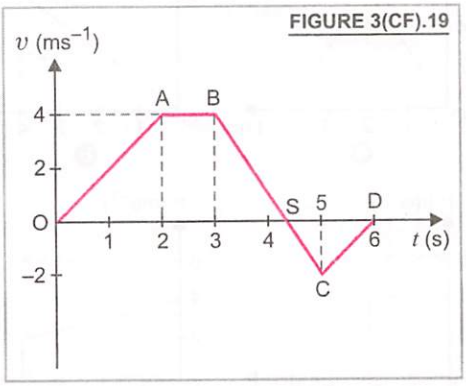


FIGURE 3(CF).19

(*a*) (37/3) m (*b*) (49/4) m

(*c*) 12 m (*d*) 11 m

(JEE Main 2020)

71. For the following velocity (v)-time graph, Fig. 3(CF).20, choose the correct acceleration (a) v/s time (t) graph

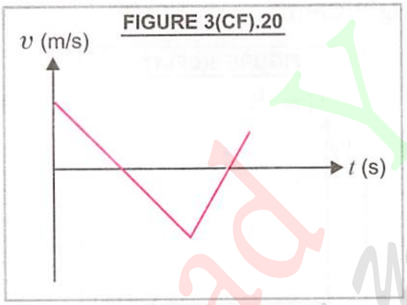


FIGURE 3(CF).20

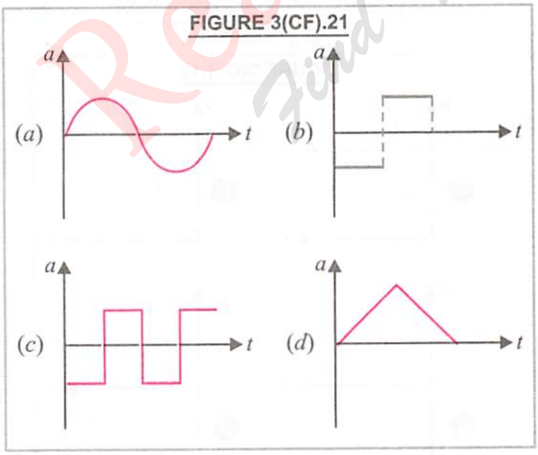


FIGURE 3(CF).21

(JEE Main 2021)

72. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in Fig. 3(CF).22. What is the position of the particle in time $t = 5$ s ?

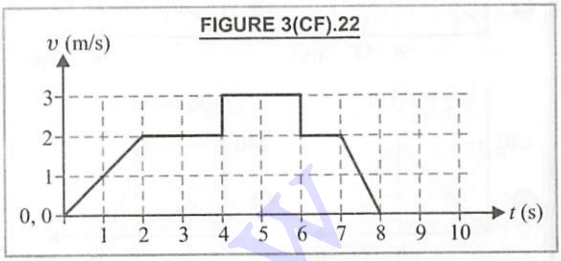


FIGURE 3(CF).22

(*a*) 9 m (*b*) 6 m

(*c*) 10 m (*d*) 3 m

(JEE Main 2019)

73. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t ?

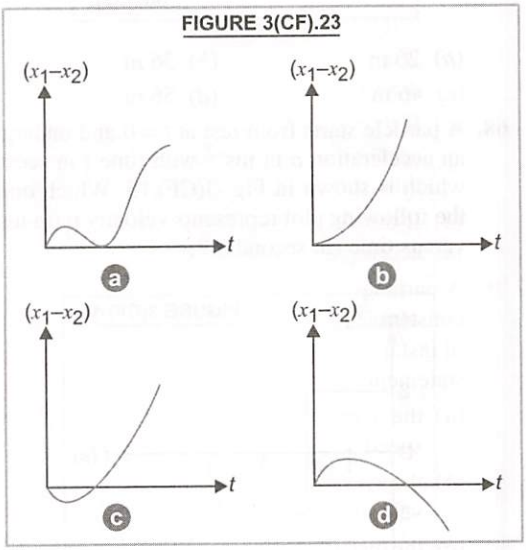


FIGURE 3(CF).23

(AIEEE 2008)

ANSWERS

70. (*a*) **71.** (*b*) **72.** (*a*) **73.** (*c*)

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II. Multiple Choice Questions (with One or More than One Correct Answers)

74. Which of the following statements are true for motion with uniform velocity

(*a*) the motion is always along a straight line path

(*b*) the motion is always in the same direction

(*c*) magnitude of displacement is less than the distance covered

(*d*) average velocity is equal to instantaneous velocity

75. An object while moving may not have

(*a*) variable speed but constant velocity

(*b*) variable velocity but constant speed

(*c*) non-zero acceleration but constant speed

(*d*) non-zero acceleration but constant velocity

76. A particle moves along a straight line as

$$S = u\,(t-2) + a\,(t-2)^2$$

(*a*) the acceleration of particle is 'a'

(*b*) the initial velocity of particle is 'u'

(*c*) at $t = 2$ s, the particle is at origin

(*d*) the acceleration of particle is '$2\,a$'

77. Choose the correct statement for a particle going on a straight line :

(*a*) If the position and velocity are in opposite direction, the particle is moving towards the origin

(*b*) If the acceleration and velocity are in opposite direction, the particle is slowing down

(*c*) If the velocity is zero for a time interval, the acceleration is zero at any moment within that time interval

(*d*) If the velocity is zero at any instant, then the acceleration must be zero at that instant

78. A particle is moving on a straight line path with constant acceleration directed along the direction of instantaneous velocity. Which of the following statements are false about the motion of particle ?

(*a*) the average velocity is less than the average speed

(*b*) the average velocity is equal to the instaneous velocity

(*c*) the distance covered is equal to the magnitude of displacement

(*d*) the particle may reverse the direction of motion.

79. A particle is forced to move on a straight line path. It returns to the starting point after 10 seconds. The total distance covered by the particle during this time is 20 m. Which of the following statements are true regarding the motion of the particles.

(*a*) the average velocity of the particle is zero

(*b*) the displacement of the particle is zero

(*c*) the average speed of the particle is $2{\cdot}0$ ms^{-1}

(*d*) the displacement of the particle is 20 m.

80. A body is thrown vertically upward in air. When air resistance is taken into account, the time of Ascent is t_1 and time of descent is t_2. Which of the following is/are true ?

(*a*) $t_1 = t_2$ (*b*) $t_1 < t_2$

(*c*) $t_1 > t_2$ (*d*) $t_2 \leq t_1$

81. Acceleration of a particle which is at rest at $x = 0$ is $\vec{a} = (4 - 2x)\,\hat{i}$. Select the correct alternative(s).

(*a*) Maximum speed of the particle is 4 units

(*b*) Particle further comes to rest at $x = 4$

(*c*) Particle oscillates about $x = 2$

(*d*) Particle will continuously accelerate along the x-axis.

82. The motion of a body is given by the equation $\dfrac{dv(t)}{dt} = 6{\cdot}0 - 3\,v(t)$ where $v\,(t)$ is the speed in ms^{-1} and t in second. If the body was at rest at $t = 0$

(*a*) the terminal speed is $2{\cdot}0$ ms^{-1}

(*b*) the magnitude of initial acceleration is $6{\cdot}0$ ms^{-2}

(*c*) the speed varies with time as $v(t) = 2\,(1 - e^{-3\,t})$ m/s

(*d*) the speed is $1{\cdot}0$ ms^{-1} when the acceleration is half the initial value.

ANSWERS

74. (*a,b,d*) **75.** (*a,d*) **76.** (*b, c,d*) **77.** (*a,b,c*) **78.** (*a,b,d*) **79.** (*a,b,c*)
80. (*b*) **81.** (*b,c*) **82.** (*b,c,d*)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension **Two cars A and B travel in straight line. The distance of A from the starting point is given as a function of time by $x_A(t) = pt + qt^2$, with $p = 2{\cdot}60\ ms^{-1}$ and $q = 1{\cdot}20\ ms^{-2}$. The distance of B from the starting pint is $x_B(t) = rt^2 - st^3$ where $r = 2{\cdot}80\ ms^{-2}$ and $s = 0{\cdot}20\ ms^{-3}$.**
Answer the following questions :

83. Which car is ahead just after they have the starting point ?

(*a*) Car A moves ahead (*b*) Car B moves ahead
(*c*) Cars A and B move simultaneously
(*d*) Data is insufficient to decide.

84. At what time(s) are the cars at the same point ?

(*a*) 2·60
(*b*) 2·27 *s*
(*c*) 5·73 s
(*d*) both 2·27 and 5·73 s

85. At what time (s) do the cars A and B have the same acceleration ?

(*a*) 2·67 s
(*b*) 6·27 s
(*c*) 4·33 s
(*d*) both 6·27 and 4·33 s

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices.

86. A ball thrown up is caught by the thrower after 4 seconds. Use $g = 9{\cdot}8\ ms^{-1}$

	Column I		Column II
(A)	The height of ball after 2 seconds	(*p*)	14·7 m
(B)	The height of ball after 3 seconds	(*q*)	19·6 m
(C)	The speed of the ball after 3 seconds	(*r*)	9·8 ms^{-1}
(D)	The speed of the ball after 4 seconds	(*s*)	19·6 ms^{-1}

(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*p* ; C-*r* ; D-*s*
(*c*) A-*p*, B-*q*, C-*s* ; D-*r* (*d*) A-*q* ; B-*p* ; C-*s* ; D-*r*

87. The velocity (v) – time (t) graph of a body moving in a straight line is shown in Fig. 3(CF).24. Match the quantities of column I and column II

	Column I		Column II
(A)	Distance travelled in 8 seconds	(*p*)	40 m
(B)	Distance travelled in 10 seconds	(*q*)	60 m
(C)	Displacement in 8 seconds	(*r*)	80 m
(D)	Displacement in 10 seconds	(*s*)	100 m

(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*r* ; C-*s* ; D-*p*
(*c*) A-*r*, B-*s*, C-*p* ; D-*q* (*d*) A-*r* ; B-*s* ; C-*q* ; D-*p*

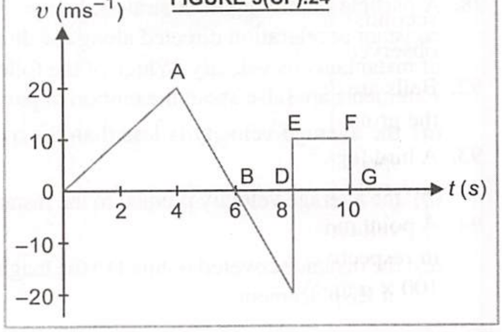


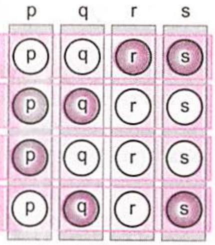
FIGURE 3(CF).24

ANSWERS

83. (*a*) **84.** (*d*) **85.** (*a*) **86.** (*b*) **87.** (*c*)

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answer to these questions have to be appropriately bubbled. If the correct matches are A-*r* ; A-*s* ; B-*p* ; B-*q* ; C-*p*, and D-*q* ; D-*s*, then the correctly bubbled matrix will look like the one shown here.



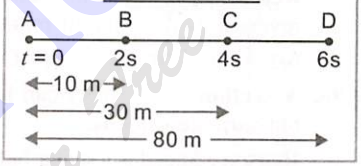
88. **Column I** | **Column II**

Column I	Column II
(A) Linear motion	(*p*) Motion of a bullet fired directly on a target from a distance.
(B) Translatory motion	(*q*) Motion of a particle on a straight line confined in a well defined limits.
(C) Oscillatory motion	(*r*) A body sliding down on an inclined plane.
(D) Projectile motion	(*s*) Motion of a bus on a straight path.

89. A point mass body starts from rest and moves from *A* to *D* on a straight path *ABCD* as shown in Fig. 3(CF).25.

Column I	Column II
(A) Constant velocity	(*p*) Motion from *A* to *B*
(B) Variable velocity	(*q*) Motion from *B* to *C*
(C) Uniform acceleration	(*r*) Motion from *A* to *D*
(D) Variable acceleration	(*s*) Motion from *A* to *C*

FIGURE 3(CF).25



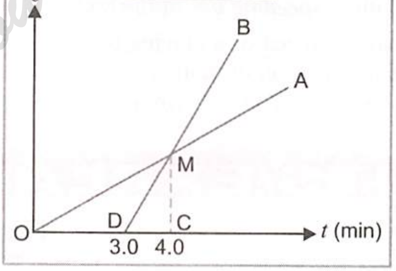
VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

A B C D
0 1 2 3 4 5 6 7 8 9

90. The velocity (v) versus time (t) graphs for two cyclists A and B moving along the same straight path of a high-way from the same point are shown in Fig. 3(CF).26. The cyclist B starts moving at $t = 3{\cdot}0$ min. At what time (in minutes) the two cyclist meet, when time is noted from the start of cyclist A ?

FIGURE 3(CF).26



91. A body falling from rest was observed to fall through 78·4 m in 2 seconds. How long (in seconds) had it been falling before it was observed.

92. Balls are dropped from the roof of a tower at a fixed interval of time. At the moment when 9th ball reaches the ground the nth ball is (3/4)th height of the tower. What is the value of n ? $g = 10\ \text{m/s}^2$.

93. A bird flies for 4 s with a velocity of $|t - 2|$ m/s in a straight line, where t = time in second. The total distance covered by the bird in metres is.

94. A point moving in a straight line travels in its second, fifth and eleven seconds of motion 16 m, 28 m and 52 m respectively. The point is moving with constant acceleration. The total distance moved in 10 seconds is $100 \times a$ m. What is the value of a ?

ANSWERS

88. A-*q*, *r*, *s*, B-*q*, *r*, *s*, C-*q*, D-*p* **89.** A-*q* ; B-*p*, *r*, *s* ; C-*p* ; D-*r*, *s*
90. (6) **91.** (3) **92.** (5) **93.** (4) **94.** (3)

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

(A) both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) both Assertion and Reason are true but the Reason is not a correct explanation of the Assertion.

(C) Assertion is true but the Reason is false.

(D) both Assertion and Reason are false.

95. Assertion : A body can have acceleration even if its velocity is zero at a given instant of time.

Reason : A body is momentarily at rest when it reverses its direction of motion.

(*a*) A (*b*) B (*c*) C (*d*) D

96. Assertion : An object can have constant speed but variable velocity.

Reason : Speed is a scalar but velocity is a vector quantity.

(*a*) A (*b*) B (*c*) C (*d*) D

97. Assertion : The speed of a body can be negative.

Reason : If the body is moving in the opposite direction of positive motion, then its speed is negative.

(*a*) A (*b*) B (*c*) C (*d*) D

98. Assertion : A negative acceleration of a body can be associated with a '*speeding up*' of the body.

Reason : Increase in speed of a moving body is independent of its direction of motion.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

(A) Statement-1 is true ; Statement-2 is true ; Statement-2 is correct explanation of Statement-1.

(B) Statement-1 is true ; Statement-2 is true ; Statement-2 is not correct explanation of Statement-1.

(C) Statement-1 is true ; Statement-2 is false.

(D) Statement-1 is false ; Statement-2 is true.

99. Statement-1 : The position-time graph of a uniform motion in one dimension of a body can have negative slope.

Statement-2 : When the speed of body decreases with time, the position-time graph of the moving body has negative slope.

(*a*) A (*b*) B (*c*) C (*d*) D

100. Statement-1 : A positive acceleration of a body can be associated with a '*slowing down*' of the body.

Statement-2 : Acceleration is a vector quantity.

(*a*) A (*b*) B (*c*) C (*d*) D

101. Statement-1 : When a body is subjected to a uniform acceleration, it always moves in a straight line.

Statement-2 : Straight line motion is the natural tendency of the body.

(*a*) A (*b*) B (*c*) C (*d*) D

HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. As S is the distance between two houses. Time taken by a student in going in one direction,

$$t_1 = \frac{S}{v_1}$$

Time taken by student in coming back, $t_2 = \dfrac{S}{v_2}$

Total time taken, $= t_1 + t_2$

$$= \frac{S}{v_1} + \frac{S}{v_2} = S\left[\frac{1}{v_1} + \frac{1}{v_2}\right] \text{ or } t = S\left[\frac{v_1 + v_2}{v_1 v_2}\right]$$

Total distance travelled $= S + S = 2\,S$

ANSWERS

95. (*a*) 96. (*b*) 97. (*d*) 98. (*c*) 99. (*c*) 100. (*b*) 101. (*d*)

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HINTS/EXPLANATIONS

For Difficult Questions

$$\therefore \text{ Average speed } = \frac{2S(v_1 v_2)}{S(v_1+v_2)} = \frac{2v_1 v_2}{v_1+v_2}$$

As he returns back to his initial position, so net displacement is **zero.**

2. We know that velocity = displacement/time, which is equal to the slope of displacement-time graph.

Therefore, ratio of respective velocity

$$= \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1/\sqrt{3}}{1} = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$$

3. In one round, net displacement of cyclist = **zero**

$$v_{av} = \frac{\text{total distance travelled}}{\text{time taken}}$$

$$= \frac{OP+PQ+QO}{10}\text{ km/min}$$

$$= \frac{1+\frac{\pi}{2}\times 1+1}{10}\text{ km/min}$$

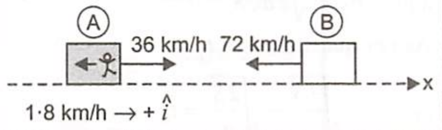
$$= \frac{\pi+4}{20}\text{ km/min} = \frac{\pi+4}{20}\times 60\text{ km/h}$$

$$= \mathbf{21{\cdot}4\ km/h}$$

4. Options (*a*), (*c*) and (*d*) show a uniformly accelerated motion in a straight line with positive initial velocity and constant negative acceleration, whereas **option (*b*) does not correspond to this motion.**

5. Refer to Fig. 3(CF).27, effective velocity of man in train *A*

FIGURE 3(CF).27



$$\vec{v}_m = (-1{\cdot}8\,\hat{i} + 36\,\hat{i})\text{ km/h}$$

$$= \left(-1{\cdot}8\times\frac{5}{18} + 36\times\frac{5}{18}\right)\hat{i}\text{ ms}^{-1}$$

$$= (-0{\cdot}5+10)\,\hat{i}\text{ ms}^{-1} = 9{\cdot}5\,\hat{i}\text{ ms}^{-1}$$

Velocity of man w.r.t. train *B* is

$$\vec{v}_{mB} = \vec{v}_m - \vec{v}_B = 9{\cdot}5\,\hat{i} - (-20\,\hat{i})\text{ ms}^{-1}$$

$$= 29{\cdot}5\text{ ms}^{-1}\,\hat{i}$$

$$\therefore \quad v_{mB} = \mathbf{29{\cdot}5\ ms^{-1}}$$

6. Let *S* be the total distance travelled by the body. Let t_1 be the time taken by the body to travel first half of the distance (*i.e.* *S*/2). Then

$$t_1 = \frac{(S/2)}{2} = \frac{S}{4}$$

Let t_2 be the time taken by body for each time interval for the remaining half journey. Therefore,

$\therefore\ (S/2) = 3\,t_2 + 5\,t_2 = 8\,t_2$ or $t_2 = (S/16)$

Average speed,

$$v_{av} = \frac{S}{t_1 + 2t_2}$$

$$= \frac{S}{(S/4)+2(S/16)} = \frac{\mathbf{8}}{\mathbf{3}}\mathbf{ms^{-1}}$$

7. Here $S = 200\text{ m} = 0{\cdot}2\text{ km}$, $v_1 = 40\text{ km/h}$, $v_2 = v$

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

or

$$48 = \frac{0{\cdot}2}{t}$$

$$t = \frac{0{\cdot}2}{48},$$

For first half distance $S_1 = 100\text{ m} = 0{\cdot}1\text{ km}$

$$\therefore \quad t_1 = \frac{0{\cdot}1}{40} \text{ similarly } t_2 = \frac{0{\cdot}1}{v}$$

$$t = t_1 + t_2 \text{ or } \frac{0{\cdot}2}{48} = \frac{0{\cdot}1}{40} + \frac{0{\cdot}1}{v}$$

$\frac{0{\cdot}2}{48} - \frac{0{\cdot}1}{40} = \frac{0{\cdot}1}{v}$ on solving we get, $\mathbf{v = 60\ km/h}$

8. No. of taxies leaving a station *A* in 2 hours = 2 × 60/10 = 12. Therefore a taxi standing at a location *B*, will encounter 12 taxies in 2 hours. If taxi leaves station *B* and moves towards *A* will be meeting 11 more taxies enroute and it will be meeting the twelfth taxi at the location of *A* which is just going to start. Therefore this taxi is not counted in the total no. of taxies enroute. Thus the total no. of taxies encountered enroute

= 12 + 11 = **23**

HINTS/EXPLANATIONS

For Difficult Questions

9. The time taken by each ball to go from starting point to highest point, $t = 1/2$ sec, which is equal to time taken by each ball to fall back to starting point (= 1/2 sec)

$$\therefore \quad S = \frac{1}{2} \times 9{\cdot}8 \times \left(\frac{1}{2}\right)^2 = \frac{9{\cdot}8}{8}\,\text{m} = \mathbf{1{\cdot}225\,m}$$

10. Let the man be able to catch the bus after time t s.

Then, $10t = 48 + \frac{1}{2} \times 1 \times t^2$ or $t^2 - 20\,t + 96 = 0$

On solving, $t = 8$ s and 12 s

Thus the man will catch the bus for the first time after **8 s.**

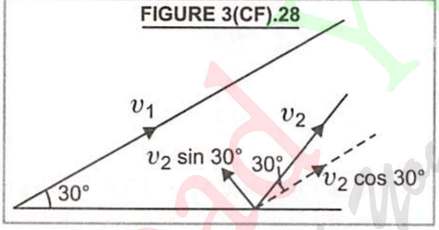
11. $$\vec{V}_{av} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}{t_2 - t_1}$$

$$= \frac{(13-2)\hat{i} + (14-3)\hat{j}}{5-0}$$

$$= \frac{11\hat{i} + 11\hat{j}}{5} = \frac{\mathbf{11}}{\mathbf{5}}(\hat{i} + \hat{j})$$

12. Refer to Fig. 3(CF).28.

FIGURE 3(CF).28



$$v_1 = v_2 \cos 30^\circ$$

or $100\sqrt{3} = v_2 \times \frac{\sqrt{3}}{2}$ or $v_2 = 200$ m/s

$$\therefore \quad t_0 = \frac{d}{v_2 \sin 30^\circ} = \frac{500}{200 \times \frac{1}{2}} = \mathbf{5s}$$

13. Here $u = 39{\cdot}2$ m/s

Let two balls collide at a height S from the ground after t seconds when second ball is thrown upwards.

$\therefore$ the time taken by first ball to reach the point of collision = $(t + 2)$ sec.

Taking motion of the first ball, we have

$$S = 39{\cdot}2\,(t+2) + \frac{1}{2}\,(-9{\cdot}8)\,(t+2)^2$$
$$= 39{\cdot}2\,(t+2) - 4{\cdot}9\,(t+2)^2 \quad \text{...(i)}$$

Taking motion of second ball, we have

$$S = 39{\cdot}2\,t + \frac{1}{2}\,(-9{\cdot}8)\,t^2 = 39{\cdot}2\,t - 4{\cdot}9\,t^2 \quad \text{...(ii)}$$

From (*i*) and (*ii*), we have

$$39{\cdot}2\,(t+2) - 4{\cdot}9\,(t+2)^2 = 39{\cdot}2\,t - 4{\cdot}9\,t^2$$

On solving, we get $t = 3$ sec

From (*ii*), $S = 39{\cdot}2 \times 3 - 4{\cdot}9\,(3)^2 = 117{\cdot}6 - 44{\cdot}1$
$= \mathbf{73{\cdot}5\,m}$

14. Let the two balls collide at a height S from the ground after t second when second ball is thrown upwards. Therefore, the time taken by first ball to reach the point of collision = $(t + 2)$ s.

Taking motion of the first ball, we have

$$S = 45(t+2) + \frac{1}{2}(-10)(t+2)^2$$
$$= 45\,(t+2) - 5\,(t+2)^2 \quad \text{...(i)}$$

Taking motion of the second ball, we have

$$S = 45\,t + \frac{1}{2}\,(-10)\,t^2 = 45\,t - 5\,t^2 \quad \text{...(ii)}$$

From (*i*) and (*ii*),

$45\,(t+2) - 5\,(t+2)^2 = 45\,t - 5\,t^2$

On solving, we get $t = 7/2$ s

From (*ii*), $S = 45 \times 7/2 - 5 \times (7/2)^2$
$= 157{\cdot}5 - 61{\cdot}25 = \mathbf{96{\cdot}25\,m}$

15. Let S be the distance between starting point and finishing point of car race. Let v_1, v_2 be the velocities of car A and B at the finishing point. Let t_1, t_2 be the time taken by cars A and B respectively while reaching the breaking point. Then

$$t_1 = \sqrt{\frac{2S}{a_1}}\ ; \quad t_2 = \sqrt{\frac{2S}{a_2}}\ ;$$

$$v_1 = \sqrt{2a_1 S}$$

and $$v_2 = \sqrt{2a_2 S}$$

As per question, $t_2 - t_1 = t$

or $$\sqrt{\frac{2S}{a_2}} - \sqrt{\frac{2S}{a_1}} = t$$

or $$\frac{t}{\sqrt{2S}} = \frac{1}{\sqrt{a_2}} - \frac{1}{\sqrt{a_1}} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\sqrt{a_1 a_2}}$$

$$\therefore \quad \frac{\sqrt{2S}}{t} = \frac{\sqrt{a_1 a_2}}{\sqrt{a_1} - \sqrt{a_2}} \quad \text{...(i)}$$

HINTS/EXPLANATIONS

For Difficult Questions

And $v_1 - v_2 = v$ or $\sqrt{2a_1 S} - \sqrt{2a_2 S} = v$

or $$\frac{\sqrt{2S}}{v} = \frac{1}{\sqrt{a_1} - \sqrt{a_2}} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{v}{t} = \sqrt{a_1 a_2} \quad \text{or} \quad v = \sqrt{a_1 a_2}\, t$$

16. Here $u = 10$ m/s, $v = 0$

As $v^2 - u^2 = 2(-g)h$ or $(0)^2 - (10)^2 = 2(-10)h$

$h = 5$ m

Also, $v = u + (-g)t$ or $0 = 10 + (-10)t$

∴ $t = 1$ sec

height covered in $t/2$, *i.e.* (1/2 *S*) is

$$h' = ut + \frac{1}{2}(-g)t^2 = 10 \times \frac{1}{2} - \frac{1}{2} \times 10 \times \left(\frac{1}{2}\right)^2$$

$$= 5 - 1.25 = 3.75\,\text{m} = \frac{3.75}{5.0}\text{h} = \mathbf{\frac{3}{4}h}$$

17. Here, $u = 40$ m/s ; Distance to be covered by each train before rest $S = 1000$ m ; $v = 0$; $a = ?$ Using the relation $v^2 = u^2 + 2aS$, we have

$$a = \frac{(v^2 - u^2)}{2S} = \frac{0 - (40)^2}{2 \times 1000} = \mathbf{0.8\,m/s^2}$$

18. $24 = u \times 4 + \frac{1}{2}\, a \times 4^2 = 4u + 8a$

or $6 = u + 2a$ **...(*i*)**

$(24 + 64) = u \times 8 + \frac{1}{2}\, a \times 8^2 = 8u + 32a$

or $11 = u + 4a$ **...(*ii*)**

On solving (*i*) and (*ii*), we get $u =$ **1 m/s**

19. Time taken by one drop to fall through 5 m is

$$t = \sqrt{\frac{2S}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1\,\text{sec.}$$

∴ Time interval for successive two drops is 1/2 second. The distance travelled by falling drop in 1/2 second is ; $S = \frac{1}{2} \times 10 \times \left(\frac{1}{2}\right)^2 = 1.25\,\text{m}$

∴ Height of second drop from ground

$= 5 - 1.25 =$ **3.75 m**

20. Let *h* be the total height of tower ; taking fall of the body in 3 s, $u = 0$,

$$S = \frac{h}{2},\ a = g = 9.8\ \text{m/s}^2,\ t = 3\ \text{sec}$$

or $S = ut + \frac{1}{2}at^2$ ∴ $\frac{h}{2} = 0 \times 3 + \frac{1}{2} \times (9.8)(3)^2$

or $h = 9.8 \times 9$ m

Now, if body takes total time *t* to fall through height *h*, then

$$h = \frac{1}{2} \times 9.8 \times t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{9.8}}$$

or $t = \sqrt{\frac{2 \times 9.8 \times 9}{9.8}} = 3\sqrt{2}$ sec $=$ **4.24 secs**

21. Let *S* be the height of a particular point where the ball crosses in time t_1 and t_2 seconds while going upwards and coming downwards. If *u* is the initial velocity of projection of ball, then

$$S = ut_1 - \frac{1}{2}gt_1^2 = ut_2 - \frac{1}{2}gt_2^2$$

or $u(t_2 - t_1) = \frac{1}{2}g(t_2^2 - t_1^2)$

or $u = \frac{1}{2}g(t_2 + t_1)$. If *T* is the time taken by ball to reach to its highest point then using the relation $v = u + at$, we have $0 = u + (-g)T$

or $T = \frac{u}{g} = \frac{1}{2}g(t_2 + t_1)/g = \mathbf{\frac{1}{2}(t_2 + t_1)}$

22. Using the relation, $S = ut + \frac{1}{2}at^2$; time to fall distance *S* when $u = 0$, $a = g$ will be $t = \sqrt{\frac{2S}{g}}$;

Let t_1, t_2, t_3 .... be the time taken to fall 1 m, 2 m, 3 m..... respectively. Then

$$t_1 = \sqrt{\frac{2 \times 1}{g}}\ ;\quad t_2 = \sqrt{\frac{2 \times 2}{g}}\ ;\quad t_3 = \sqrt{\frac{2 \times 3}{g}}$$

So the time taken to fall 1 m $= t_1 - 0$ $= \sqrt{2/g}\,(\sqrt{1} - 0)$; time taken to fall 2nd metre $= t_2 - t_1 = \sqrt{2/g}\,(\sqrt{2} - \sqrt{1})$; time taken to fall 3rd metre $= (t_3 - t_2) = \sqrt{2/g}\,(\sqrt{3} - \sqrt{2})$ and so on.

HINTS/EXPLANATIONS

For Difficult Questions

23. Velocity of balloon after 2 seconds ; $v = 0 + 4{\cdot}9 \times 2 = 9{\cdot}8\ \text{ms}^{-1}$. Vertical distance travelled by ball, $S_1 = \dfrac{v^2}{2g} = \dfrac{(9{\cdot}8)^2}{2 \times 9{\cdot}8} = 4{\cdot}9\,\text{m}$

Height travelled in 2 sec, $S_2 = \dfrac{1}{2}at^2$

$$= \frac{1}{2} \times 4{\cdot}9 \times 2^2 = 9{\cdot}8\,\text{m}$$

Total height of ball $= S_1 + S_2 = 4{\cdot}9 + 9{\cdot}8$
$= \mathbf{14{\cdot}7\ m}$

24. Let u be the initial velocity of bullet and a be the retardation of bullet while passing through a plank. Velocity of bullet after passing through plank

$$v = u - \frac{u}{20} = \frac{19u}{20}$$

Taking motion of bullet through one plank and using the relation, $v^2 = u^2 + 2as$, we have

$$\left(\frac{19}{20}u\right)^2 = u^2 - 2aS \text{ or } 2aS = u^2 - \left(\frac{19}{20}u\right)^2$$

[$\because$ motion is retarted]

$$2aS = u^2 - \frac{361}{400}u^2 = \frac{39u^2}{400}$$

Taking motion of bullet through n planks

$v = 0,\ S = nS$

$0 = u^2 - 2\,a\,nS$

$$\text{or} \quad n = \frac{u^2}{2aS} = \frac{u^2}{39u^2/400} = \frac{400}{39} = 10{\cdot}25 \approx \mathbf{11}$$

25. Velocity acquired by balloon at height h,

$$v = \sqrt{2ah} = \sqrt{2 \times \frac{g}{8}h} = \frac{\sqrt{gh}}{2}$$

Taking downward motion of stone released from balloon at height h upto surface of ground, we have

$$u = -\frac{\sqrt{gh}}{2}\ ;\ a = g\ ;\ S = h,\ t = t$$

As $\quad S = ut + \dfrac{1}{2}at^2$

$$\therefore \quad h = -\frac{\sqrt{gh}}{2}t + \frac{1}{2}gt^2 \quad \text{or} \quad t^2 - \sqrt{\frac{h}{g}}\,t - \frac{2h}{g} = 0\,.$$

On solving, $t = 2\sqrt{h/g}$ or $-\sqrt{h/g}$

As negative time is not accounted, hence

$$t = 2\sqrt{h/g}$$

26. Let the ball released from the top of tower of height h takes t seconds to reach the ground. As per question, using the relation,

$$D_n = u + \frac{a}{2}(2n - 1)\text{, we have,}$$

$$\frac{11h}{36} = \frac{10}{2}(2t - 1) = 5(2t - 1)$$

$$\text{and} \quad h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10t^2 = 5t^2$$

$$\therefore \quad \frac{11}{36} \times 5t^2 = 5(2t - 1) \quad \text{or} \quad \frac{11}{36} \times t^2 = 2t - 1$$

or $\quad 11\,t^2 - 72\,t + 36 = 0$

On solving, we get $t = 6$ or $(6/11)$ s

Rejecting fractional value of $t\left(= \dfrac{6}{11}\text{s}\right)$

we have, $t = 6$ s $\therefore$ $h = 5 \times 6^2 = \mathbf{180\ m}$

27. The distance covered by particle will be same in 5th and 6th second if it reaches at the highest position in 5 seconds. Using $t = 5$ s,

$a = -9{\cdot}8\ \text{m/s}^2,\ v = 0,\ u = ?$

Now, $v = u + at$; so $0 = u + (-9{\cdot}8) \times 5$

or $\quad u = 9{\cdot}8 \times 5 = \mathbf{49{\cdot}0\ m/s}$

28. Height from where the stone is dropped $= \dfrac{1}{2}gt^2$

$$= \frac{1}{2} \times 9{\cdot}8 \times 5^2 = 4{\cdot}9 \times 25\,\text{m}$$

Vertical downward distance covered by stone in 3 seconds $= \dfrac{1}{2} \times 9{\cdot}8 \times 3^2 = 4{\cdot}9 \times 9$ m

Velocity attained $= 9{\cdot}8 \times 3\ \text{ms}^{-1}$

Remaining distance $= 4{\cdot}9 \times 25 - 4{\cdot}9 \times 9$
$= 4{\cdot}9 \times 16$ m

If t_1 is the time taken to cover the remaining distance when stone is dropped again ($u = 0$)

$$4{\cdot}9 \times 16 = \frac{1}{2} \times 9{\cdot}8 \times t^2$$

$$t = \sqrt{16} = \mathbf{4s}$$

29. Let the first ball hit the water in t seconds

Using ; $\quad S = ut + \dfrac{1}{2}at^2$

HINTS/EXPLANATIONS

For Difficult Questions

For first ball,

$$122{\cdot}5 = 0 + \frac{1}{2} \times 9{\cdot}8 \times t^2 = 4{\cdot}9\, t^2$$

or $t = \sqrt{\dfrac{122{\cdot}5}{4{\cdot}9}} = 5\, s$

For second ball,

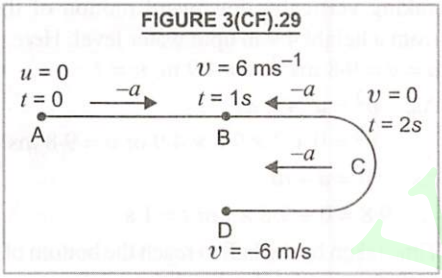
$$122{\cdot}5 = u\,(5-2) + \frac{1}{2} \times 9{\cdot}8 \times (5-2)^2$$

$$= 3\,u + 44{\cdot}1$$

$3\,u = 122{\cdot}5 - 44{\cdot}1 = 78{\cdot}4$ or $u = \mathbf{26{\cdot}1\ ms^{-1}}$

30. The motion of a toy car is shown in Fig. 3(CF).29. Taking motion of the toy car from A to B,

FIGURE 3(CF).29



Acceleration, $a = \dfrac{v-u}{t} = \dfrac{6-0}{1} = 6\ ms^{-2}$

Taking motion of toy car for $t = 0$ to $t = 1$ s

$$S_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 6 \times 1^2 = 3\ \text{m} \quad ...(i)$$

Taking motion of toy car for $t = 1$ s to $t = 2$ s

$$S_2 = 6 \times 1 + \frac{1}{2}(-6) \times 1^2 = 3\ \text{m} \quad ...(ii)$$

Taking motion of toy car for $t = 2$ s to $t = 3$ s

$$S_3 = 0 \times 1 + \frac{1}{2}(-6) \times 1^2 = -3\ \text{m} \quad ...(iii)$$

Total displacement, $|\vec{S}| = S_1 + S_2 + S_3$

$$= 3 + 3 + (-3) = 3\ \text{m}$$

Average velocity, $|\vec{v}_{av}| = \dfrac{3}{3} = \mathbf{1\ ms^{-1}}$

Total distance travelled = 3 + 3 + 3 = 9 m

Average speed, $v_{av} = \dfrac{9}{3} = \mathbf{3\ ms^{-1}}$

31. Given, $100 = D_t + D_{(t+1)}$

$$= \left[u + \frac{f}{2}(2t-1)\right] + \left[u + \frac{f}{2}\{2(t+1)-1\}\right]$$

or $100 = 2\,u + \dfrac{f}{2}\,(4\,t) = 2\,u + 2\,ft$ or $50 = u + ft$

Velocity after t seconds, $v = u + ft = \mathbf{50\ cm/s}$

32. When a ball is falling down from height d and it reaches at height h from ground, then the velocity acquired by the ball is

$$v = \sqrt{2\,g\,(d-h)} \quad \text{or} \quad v^2 = 2\,g\,(d-h) \quad ...(i)$$

This downward velocity of ball is negative.

When $h = d$, $v = 0$ and when $h = 0$, $v = \sqrt{2\,gd}$

From (*i*), v and h variation is a parabola below h- axis.

After collision with ground, the ball can go vertically upward upto a height $d/2$, so its initial upward velocity is

$u' = \sqrt{2\,g\,(d/2)}$. At height h, its velocity is

$$v' = \sqrt{u'^2 - 2\,gh} = \sqrt{2\,g\,(d/2) - 2\,gh} \quad ...(ii)$$

When, $h = d/2$, $v' = 0$. The upward velocity of ball is positive.

From (*ii*), the variation of v' and h is a parabola above h-axis. Thus **graph (*a*) satisfies all these conditions.**

33. First 50 m fall is under the effect of gravity only. The velocity acquired

$$u = \sqrt{2\,gh} = \sqrt{2 \times 9{\cdot}8 \times 50} = 10\sqrt{9{\cdot}8}\ ms^{-1}$$

Taking onward motion of prachutist with retardation 2 m/s^2, we have ; $u = 10\sqrt{9{\cdot}8}$ m/s ;

$a = -2\ ms^{-2}$; $v = 3$ m/s ; $S = ?$

$$S = \frac{v^2 - u^2}{2a} = \frac{3^2 - (100 \times 9{\cdot}8)}{2 \times (-2)} = 243\,\text{m}$$

Total height = 50 + 243 = **293 m**

34. The initial relative velocity of the first stone w.r.t. second stone, when first stone is just dropped is zero *i.e.* $u = 0$.

The relative acceleration of the first stone w.r.t. second, $a = 9{\cdot}8 + 0{\cdot}2 = 10\ ms^{-2}$.

The relative distance between the two stones in 2 seconds is

$$S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 10 \times 2^2 = \mathbf{20{\cdot}0\ m}$$

HINTS/EXPLANATIONS For Difficult Questions

35. Let l be the length of the train. a be the constant acceleration of the train. Taking motion of the whole train we have

$$u = u, v = v, s = l, a = a$$

As, $v^2 - u^2 = 2\,as \quad \therefore v^2 = u^2 + 2\,al$

or $\quad a = \dfrac{v^2 - u^2}{2l}$

For mid point of train, $l' = l/2$

$$\therefore\ v_{\text{last}}^2 = u^2 + 2\,al' = u^2 + 2\,a \times l/2 = u^2 + al$$

$$= u^2 + \left(\frac{v^2 - u^2}{2l}\right) l = \frac{2u^2 + v^2 - u^2}{2}$$

$$v_{\text{last}} = \sqrt{\frac{\mathbf{u^2 + v^2}}{\mathbf{2}}}$$

36. Taking vertical downward motion of first ball for height 5 m, we have

$$u = 0, a = 10 \text{ m/s}^2, s = 5 \text{ m}, t = ?$$

As, $\quad s = ut + \dfrac{1}{2}at^2 \quad \therefore\ 5 = 0 + \dfrac{1}{2} \times 10 \times t^2$

or $\quad t^2 = \dfrac{2 \times 5}{10} = 1 \quad$ or $\quad t = \mathbf{1\ sec.}$

Let the first ball take n seconds to cover height h, then second ball will take $(n - 1)$ second to cover $(h - 25)$ m.

So for first ball, $h = \dfrac{1}{2} \times 10 \times n^2 \quad$ ...(*i*)

For second ball, $(h - 25) = \dfrac{1}{2} \times 10 \times (n - 1)^2 \quad$ ...(*ii*)

or $\quad \dfrac{1}{2} \times 10 \times n^2 - 25 = \dfrac{1}{2} \times 10 \times (n - 1)^2$

or $\quad 5\,n^2 - 25 = 5\,(n - 1)^2$

or $\quad n^2 - 5 = (n - 1)^2 = n^2 - 2\,n + 1$

or $\quad 2\,n = 6 \quad$ or $\quad n = 3$ sec.

From (*i*), $\quad h = \dfrac{1}{2} \times 10 \times 3^2 = \mathbf{45\ m}$

37. Let the two balls meet after 2 seconds at a height h (= 180 – 100= 80 m).

Taking vertically downward motion of ball A, we have, $u = 0$, $a = 10$ ms^{-2}, $t = ?$, $s = h = 80$ m

As $\quad s = ut + \dfrac{1}{2}at^2$

$\therefore\ \quad 80 = 0 + \dfrac{1}{2} \times 10 \times t^2 \quad$ or $\quad t = \sqrt{\dfrac{80 \times 2}{10}} = 4$ s.

Time taken by ball B to meet ball $A = 4 - 2 = 2$ s

Taking vertically downward motion of ball B for time 2 s, we have, $u = u$, $a = 10$ ms^{-2}, $t = 2$ s, $s = 80$ m.

As, $\quad s = ut + \dfrac{1}{2}at^2$

$\therefore\ \quad 80 = u \times 2 + \dfrac{1}{2} \times 10 \times 2^2$

or $\quad 2\,u = 80 - 20 = 60$

or $\quad u = \mathbf{30\ ms^{-1}}$

38. Taking vertically downward motion of the ball from a height 4·9 m upto water level. Here, $u = 0$, $a = g = 9{\cdot}8$ ms^{-2}, $s = 4{\cdot}9$ m, $v = ?$

As, $\quad v^2 = u^2 + 2\,as$

$\therefore\ \quad v^2 = 0 + 2 \times 9{\cdot}8 \times 4{\cdot}9$ or $v = 9{\cdot}8$ ms^{-1}

As, $\quad v = u + at$

$\therefore\ \quad 9{\cdot}8 = 0 + 9{\cdot}8 \times t$ or $t = 1$ s

Time taken by the ball to reach the bottom of water surface $\quad t' = 4 - 1 = 3$ s.

Depth of water level = distance travelled in 3 seconds $\quad = 9{\cdot}8 \times 3 = \mathbf{29{\cdot}4\ m}$

39. Distance travelled by toy in first t seconds is,

$$10 = \frac{1}{2} \times a \times t^2 \qquad \textbf{...(1)}$$

Let x be the distance travelled by toy in the next t seconds. Then total distance travelled by toy in time $(t + t)$ seconds is

$$10 + x = \frac{1}{2}\,a\,(2t^2) \qquad \textbf{...(2)}$$

Dividing (2) by (1), we get

$$\frac{10 + x}{10} = 4 \quad \text{or} \quad 10 + x = 40 \quad \text{or} \quad x = \mathbf{30\ m}$$

40. Let v be the velocity acquired by the ball while reaching at the top of window, during its fall from a height, *i.e.*, the roof of the building.

Taking vertical downward motion of the ball while crossing the window, we have

$$u = v, S = 1{\cdot}5 \text{ m}, t = 0{\cdot}1 \text{ s}, a = 10 \text{ ms}^{-2}$$

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As, $S = ut + \frac{1}{2} at^2$

$\therefore \quad 1{\cdot}5 = v \times 0{\cdot}1 + \frac{1}{2} \times 10 \times (0{\cdot}1)^2$

On solving, $v = \mathbf{14{\cdot}5\ ms^{-1}}$

41. Let θ be the inclination of the inclined plane with the horizontal ground. Acceleration of the block while sliding down the inclined plane, $a = g \sin\theta$

Here, distance travelled in nth second, $D_n = S_n$ and initial speed, $u = 0$.

As, $D_n = u + \frac{a}{2}(2n-1)$

$\therefore \quad S_n = 0 + \frac{g \sin\theta}{2}(2n-1)$

$= \frac{g \sin\theta}{2}(2n-1)$ ...(*i*)

Distance travelled in $(n+1)$th second,

$S_{n+1} = 0 + \frac{g \sin\theta}{2}[2(n+1)-1]$

$= \frac{g \sin\theta}{2}(2n+1)$ ...(*ii*)

Dividing (*i*) by (*ii*), we get

$$\frac{S_n}{S_{n+1}} = \mathbf{\frac{2n-1}{2n+1}}$$

42. Here $v = \beta x^{-2n}$

$\therefore \quad \frac{dv}{dx} = -2n\beta x^{-2n-1}$,

As $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$

or $a = \frac{v\,dv}{dx} = \beta x^{-2n} \times -2n\beta x^{-2n-1}$

$a = \mathbf{-2n\beta^2 x^{-4n-1}}$

43. Here, $x = at + bt^2 - ct^3$

Velocity, $v = \frac{dx}{dt} = a + 2bt - 3ct^2$

Acceleration, $a = \frac{dv}{dt} = 2b - 6ct$

when $a = 0$, then $0 = 2b - 6ct$ or $t = \frac{b}{3c}$

$\therefore \quad v = a + 2b \times \frac{b}{3c} - 3c \times \left(\frac{b}{3c}\right)^2$

$= a + \frac{2}{3}\frac{b^2}{c} - \frac{b^2}{3c} = \mathbf{a + \frac{b^2}{3c}}$

44. $\vec{r} = (15t^2)\hat{i} + (4 - 20t^2)\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = (30t)\hat{i} - (40t)\hat{j}$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = (30)\hat{i} - (40)\hat{j}$

Magnitude of $\vec{a} = \sqrt{30^2 + (-40)^2} = \mathbf{50}$

45. $x = a e^{-\alpha t} + b e^{\beta t}$

$v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$

As t increases, the value of $a\alpha e^{-\alpha t}$ decreases slowly and the value of $b\beta e^{\beta t}$ increases largely. Thus with increase in t, v **will increase.**

46. $a = \frac{dv}{dt} = -2v$ or $\frac{dv}{v} = -2\,dt$

Integrating it, we have,

$\log_e v = -2t + C$ or $v = e^{-2t+C}$

It means the nature of v–t curve is **exponentially decreasing.**

47. $\frac{dv}{dt} = -2{\cdot}5\sqrt{v}$ or $\frac{dv}{\sqrt{v}} = -2{\cdot}5\,dt$

Integrating it with in the conditions as time changes 0 to t, velocity changes 6·25 m/s to zero.

$\therefore \quad \int_{6\cdot25}^{0} \frac{dv}{\sqrt{v}} = -\int_0^t 2{\cdot}5\,dt$

$2(\sqrt{v})_{6\cdot25}^{0} = -2{\cdot}5t$ or $-2\sqrt{6{\cdot}25} = -2{\cdot}5t$

or $-2 \times 2{\cdot}5 = -2{\cdot}5t$ or $t = \mathbf{2\ s}$

48. Here, $\frac{dv}{dt} = -kv^3$ or $\frac{dv}{v^3} = -k\,dt$

Integrating within the conditions of motion, we have

$\int_{v_0}^{v} \frac{dv}{v^3} = -\int_0^t k\,dt$ or $\left[-\frac{1}{2v^2}\right]_{v_0}^{v} = -k[t]_0^t$

$\frac{1}{v^2} - \frac{1}{v_0^2} = 2kt$ or $v = \mathbf{\frac{v_0}{\sqrt{1 + 2kv_0^2 t}}}$

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49. Here, $v = k\sqrt{x}$ or $\frac{dx}{dt} = k\sqrt{x}$

or $x^{-1/2}\,dx = k\,dt$, Integrating it within the conditions of motion (*i.e.*, $t = 0$, $x = 0$ and at $t = t$, $x = x$), we have

$$\int_0^x x^{-1/2}\,dx = \int_0^t k\,dt$$

$$\text{or}\quad \left[\frac{x^{-\frac{1}{2}+1}}{-1/2+1}\right]_0^x = k\,(t)_0^t \quad\text{or}\quad x^{1/2} = \frac{kt}{2}$$

Putting this value in the given relation, we get

$$v = k\left[\frac{kt}{2}\right] = \frac{k^2 t}{2}$$

50. $$a = \frac{dv}{dt} = -k\,v^4 \quad\text{or}\quad -\frac{dv}{k\,v^4} = dt$$

Integrating both the sides, we have

$$-\frac{1}{k}\int_{v_0}^{v} v^{-4}\,dv = \int_0^t dt$$

$$\text{or}\quad -\frac{1}{k}\left[\frac{v^{-3}}{-3}\right]_{v_0}^{v} = t \quad\text{or}\quad \frac{1}{3k}\left[\frac{1}{v^3} - \frac{1}{v_0^3}\right] = t$$

$$\text{or}\quad \frac{1}{v^3} = 3kt + \frac{1}{v_0^3} = \frac{3kt\,v_0^3 + 1}{v_0^3}$$

$$\text{or}\quad v = \frac{v_0}{(3kt\,v_0^3 + 1)^{1/3}}$$

51. $$v_x = \frac{dx}{dt} = \frac{d}{dt}(a\cos\omega t) = -\,a\omega\sin\omega t$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t) = a\omega\cos\omega t$$

$$v_z = \frac{dz}{dt} = \frac{d}{dt}(a\,\omega t) = a\omega$$

$$\therefore\quad v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$= \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + a^2\omega^2}$$

$$= \sqrt{a^2\omega^2(\sin^2\omega t + \cos^2\omega t) + a^2\omega^2}$$

$$= \sqrt{2}\,a\omega$$

52. Given, at time $t = 0$, velocity $v = 0$

Acceleration, $f = f_0\left(1 - \frac{t}{T}\right)$

When $f = 0$, $0 = f_0\left(1 - \frac{t}{T}\right)$

Since f_0 is constant, therefore, $1 - \frac{t}{T} = 0$ or $t = T$

Also, acceleration, $f = \frac{dv}{dt}$

$$\text{or}\quad \int_0^{v_x} dv = \int_0^T f\,dt = \int_0^T f_0\left(1 - \frac{t}{T}\right)dt$$

$$\text{or}\quad v_x = \left(f_0\,t - \frac{f_0\,t^2}{2T}\right)_0^T = f_0\,T - \frac{f_0\,T^2}{2T} = \frac{f_0 T}{2}$$

53. $x = 5\,t - 2\,t^2$;

velocity, $v_x = \frac{dx}{dt} = 5 - 4t$

Acceleration, $a_x = \frac{dv_x}{dt} = -4\ \text{m/s}^2$

$y = 10\,t$

velocity, $v_y = \frac{dy}{dt} = 10$

Acceleration, $a_y = \frac{dv_y}{dt} = 0$

$\therefore$ Acceleration of particle at $t = 2$ s

$$= \sqrt{(-4)^2 + 0^2} = -\,4\ \text{m/s}^2$$

54. $\vec{v} = K(y\,\hat{i} + x\,\hat{j})$

or $v_x\,\hat{i} + v_y\,\hat{j} = K(y\,\hat{i} + x\,\hat{j})$

$\therefore$ $v_x = Ky$

and $v_y = Kx$

$\frac{dx}{dt} = Ky$

and $\frac{dy}{dt} = Kx$

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = Kx \times \frac{1}{Ky} = \frac{x}{y}$$

or $y\,dy = x\,dx$

Integrating it, $y^2 = x^2 + C$

HINTS/EXPLANATIONS

For Difficult Questions

55. Here, $y = bx^2$. Differentiating it w.r.t. t, we get

$$\frac{dy}{dt} = b\,2x\frac{dx}{dt} \quad \text{or} \quad v_y = 2\,bx\,v_x$$

Again differentiating it w.r.t. t, we get

$$\frac{dv_y}{dt} = 2bv_x\frac{dx}{dt} + 2bx\frac{dv_x}{dt} = 2bv_x^2 + 0$$

$[\frac{dv_x}{dt} = 0$, because the particle has constant acceleration along y-direction only]

As per question, $\frac{dv_y}{dt} = a = 2bv_x^2$

or $v_x^2 = \frac{a}{2b}$ or $v_x = \sqrt{\frac{a}{2b}}$

56. Given, $F = F_0\,e^{-bt}$

Acceleration, $a = \frac{F}{m} = \frac{F_0}{m}e^{-bt}$

or $\frac{dv}{dt} = \frac{F_0}{m}e^{-bt}$ or $dv = \frac{F_0}{m}e^{-bt}\,dt$

Integrating it within the conditions of motion, we get

$$\int_0^v dv = \int_0^t \frac{F_0}{m}e^{-bt}\,dt$$

or $$v = \frac{F_0}{m}\left[-\frac{1}{b}e^{-bt}\right]_0^t = -\frac{F_0}{mb}[e^{-bt} - 1]$$

$$= \frac{F_0}{mb}[1 - e^{-bt}]$$

When $t = 0$, $v = 0$ and when $t = \infty$, $v = \frac{F_0}{mb}$

Thus velocity increases continuously and attains maximum value, $v = \frac{F_0}{mb}$ at $t = \infty$. Thus, **option (*b*) is true**

57. Here, $x = 4 \sin 2\,\pi t$ and $y = 4 \cos 2\,\pi t$

Squaring and adding we have

$x^2 + y^2 = 4^2\,(\sin^2 2\,\pi t + \cos^2 2\,\pi t) = 4^2$

It is an equation of a circle of radius 4 metre.

For circular motion, the accelerated is along $-\vec{R}$ and magnitude of acceleration is v^2/R.

Given $\vec{R} = 4\sin(2\pi t)\,\hat{i} + 4\cos(2\pi t)\,\hat{j}$

velocity, $\vec{v} = \frac{d\vec{R}}{dt}$

$$= 4 \times 2\pi\cos(2\pi t)\,\hat{i} - 4 \times 2\pi\sin(2\pi t)\,\hat{j}$$

$$= 8\pi[\cos(2\pi t)\,\hat{i} - \sin(2\pi t)\,\hat{j}]$$

Magnitude of velocity, $= |\vec{v}| = \mathbf{8\,\pi\ ms^{-1}}$

Thus, **option (*d*) is wrong.**

58. $v = \frac{dx}{dt} = b\sqrt{x}$ or $\int_0^x \frac{dx}{\sqrt{x}} = \int_0^\tau b\,dt$

or $2\sqrt{x} = b\,\tau$ or $\sqrt{x} = \frac{b\,\tau}{2}$

$\therefore \quad v = b \times \frac{b\,\tau}{2} = \frac{b^2\,\tau}{2}$

59. Here, $u = 0$, $a = a$ constant. So graph (*i*) is true. At time t, $v = 0 + at = at$ or $v \propto t$. Thus graph (*ii*) is correct.

Also, $x = 0\,(t) + \frac{1}{2}at^2 = \frac{1}{2}at^2$

It is an equation of a parabola. Thus graph (*iv*) is correct. Hence **option (*d*) is true.**

60. When a body is thrown vertically upwards with velocity u, then its velocity v after time t is

$$v = u - g\,t = -g\,t + u$$

It is a straight line for $v - t$ graph, with negative slope. Thus **option (*a*) is true.**

61. Average acceleration $(a) = \frac{\text{change in velocity}}{\text{time taken}}$

$\therefore$ Change in velocity = average acc. × time taken = area of acc – time graph.

Average acceleration $= \frac{\text{area OABE}}{20\,\text{s}} = \frac{600}{20}$

$= \mathbf{30\,m/s^2}$

62. The displacement will be the area which velocity-time graph encloses with time axis. The area enclosed by graph above the time axis is positive and below the time axis is negative.

63. The total distance covered by the body is equal to the sum of all the area of velocity-time graph above and below the time axis.

HINTS/EXPLANATIONS ....

For Difficult Questions

64. As, acceleration $= \dfrac{\text{change in velocity}}{\text{time taken}}$

$\therefore$ change in velocity = acceleration × time taken

Hence, change in velocity in 10 s = area *OAB* + area *BCD*

$$= \frac{1}{2}\times 6\times 10+\frac{1}{2}\times(10-6)\times\left(-\frac{20}{3}\right)$$

$$= 30-\frac{40}{3}=\frac{50}{3}\ \text{ms}^{-1}$$

$\therefore$ Final velocity at 10 s = initial velocity + change in velocity $= 8+\dfrac{50}{3}=\mathbf{\dfrac{74}{3}\ ms^{-1}}$

65. Let particle *B* move upwards with velocity v. At an instant, let $OA = x$ and $OB = y$, $AB = l$. Then

$$x^2+y^2=l^2$$

Differentiating it with time t, we have

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}=-\frac{dx/dt}{\tan 30^\circ}=-10\times\sqrt{3}$$

$$= \mathbf{-17{\cdot}32\ m/s}$$

Since the velocity of *B* is away from origin, hence negative.

66. Taking vertical upward motion of first stone for time t, we have

$y_0 = 240$ m, $u = 10$ m/s, $a = -10$ m/s^2

$t = t$, $y = y$

As $\quad y = y_0 + ut + \dfrac{1}{2}at^2$

$$\therefore\ y_1 = 240+10t+\frac{1}{2}(-10)t^2 = 240+10\,t-5\,t^2$$

Taking vertical upward motion of second stone for time t, we have $y_0 = 240$ m, $u = 40$ m/s, $a = -10$ m/s^2, $t = t$, $y = y_2$

$$y_2 = 240+40t-\frac{1}{2}\times 10\times t^2 = 240+40\,t-5\,t^2$$

When first stone hits the ground, $y_1 = 0$,

$\therefore\ 0 = 240 + 10\,t - 5\,t^2$ or $t^2 - 2\,t - 48 = 0$

or $\quad t = \dfrac{2\pm\sqrt{(2)^2+4\times 48}}{2} = 8\,\text{s}$ or -6 sec

-6 s is meaningless so time $t = 8$ sec

When second stone hits the ground, $y_2 = 0$

$\therefore\ 0 = 240 + 40\,t - 5\,t^2$ or $t^2 - 8\,t - 48 = 0$

on solving we get, $t = 12$ s or -4 s

as -4 s is meaningless so, $t = 12$ s

Relative position of 2nd stone w.r.t. first is

$y_2 - y_1 = 30\,t \quad$ ...(*i*) $[0 < t \le 8\ \text{s}]$

Since $y_2 - y_1$ and t are linearly related.

$\therefore$ graph will be straight line from $t = 0$ to $t = 8$ s.

After 8 sec only second stone would be in motion for further 4 s. Now $y_1 = 240$ m and

$y_2 = 240 + 40\,t - 5\,t^2$

$\therefore\ y_2 - y_1 = 40\,t - 5\,t^2 \quad$ ...(*ii*) $(8 \le t \le 12\ \text{s})$

From (*i*) and (*ii*), we note that

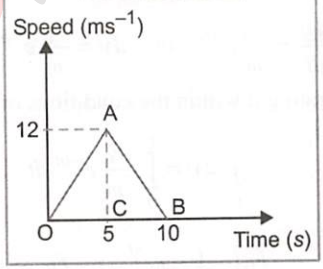
$y_2 - y_1 \propto t$ and $(y_2 - y_1) \propto (40\,t - 5\,t^2)$.

Thus **option (*c*) is true.**

67. Let s_1 be the distance travelled by particle in time 2 to 5 s and s_2 be the distance travelled by particle in time 5 to 6 s.

$\therefore$ Total distance travelled, $s = s_1 + s_2$.

FIGURE 3(CF).30



During the time interval 0 to 5 s, the acceleration of particle is equal to the slope of *OA* *i.e.*

$$a = \frac{12}{5} = 2{\cdot}4\ \text{ms}^{-2}.$$

Velocity at the end of 2 s will be,

$$v = 0 + 2{\cdot}4\times 2 = 4{\cdot}8\ \text{ms}^{-1}$$

Taking motion of particle for time interval 2 s to 5 s, we have

$u = 4{\cdot}8$ ms^{-1}, $a = 2{\cdot}4$ ms^{-2}, $s = s_1$, $t = 5 - 2 = 3$ s.

Then, $\quad s_1 = 4{\cdot}8\times 3 + \dfrac{1}{2}\times 2{\cdot}4\times 3^2 = 25{\cdot}2$ m

Acceleration of the particle during the motion $t = 5$ s to $t = 10$ s is

$$a = \text{slope of line } AB = -\frac{12}{5} = -2{\cdot}4\ \text{ms}^{-2}$$

HINTS/EXPLANATIONS

For Difficult Questions

Taking motion of the particle for the time 1 sec (*i.e.* 5 s to 6 s), we have

$$u = 12\ \text{ms}^{-1},\ a = -2{\cdot}4\ \text{ms}^{-2},\ t = 1\ \text{s}\ ;\ s = s_2$$

$$\therefore\ s_2 = 12 \times 1 + \frac{1}{2}\ (-2{\cdot}4) \times 1^2 = 10{\cdot}8\ \text{m}$$

$$\therefore\ S = s_1 + s_2 = 25{\cdot}2 + 10{\cdot}8 = \mathbf{36\ m}$$

68. Refer to Fig. 3(CF).31,

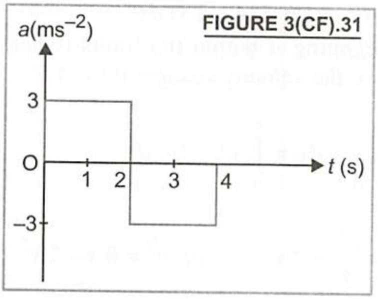


FIGURE 3(CF).31

taking the motion of particle from 0 to 2 s, we have $u = 0,\ a = 3\ \text{ms}^{-2}\ ;\ t = 2\ \text{s},\ v = ?$

$$v = u + at = 0 + 3 \times 2 = 6\ \text{ms}^{-1}$$

Taking motion of particle from 2 s to 4 s, we have

$$v = 6 + (-3) \times (2) = 0\ \text{ms}^{-1}$$

Hence, **graph (a) represents the proper choice.**

69. For the given velocity-position graph, intercept = v_0 and slope = $-v_0/x_0$

The equation of given line is, $v = -\dfrac{v_0}{x_0}x + v_0$

Differentiating it w.r.t. t, we have

$$\frac{dv}{dt} = -\frac{v_0}{x_0}\frac{dx}{dt}$$

or $\quad a = -\dfrac{v_0}{x_0}v$

$$= -\frac{v_0}{x_0}\left(-\frac{v_0}{x_0}x + v_0\right) = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}$$

It is an equation of straight line between a and x with positive slope $\left(= \dfrac{v_0^2}{x_0^2}\right)$ and negative intercept $\left(= -\dfrac{v_0^2}{x_0}\right)$.

Hence, **graph (c) is correct.**

70. Refer to Fig. 3(CF).32,

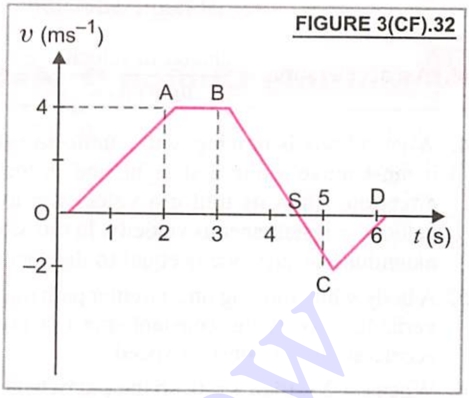


FIGURE 3(CF).32

$$4{\cdot}333\ \text{s} = 4\ \text{s} + 0{\cdot}333\ \text{s} = 4 + \frac{1}{3} = \frac{13}{3}\ \text{s}$$

Distance covered in 6 seconds

= Area of Trapezium $OABS$ + Area of ΔSCD

$$= \left[\frac{1}{2} \times 4 \times \left(\frac{13}{3} + 1\right)\right] + \left[\frac{1}{2} \times \left(6 - \frac{13}{3}\right) \times 2\right]$$

$$= 2 \times \frac{16}{3} + \frac{5}{3} = \frac{32}{3} + \frac{5}{3} = \mathbf{\frac{37}{3}}$$

71. Slope of velocity-time graph gives acceleration. For the first part, slope is negative and constant. It means, the acceleration has a steady negative value. For the second part, slope is positive and constant. It means the acceleration has a steady positive value. Thus **option (*b*) is true.**

72. The position of the particle in time $t = 5$ s is the displacement position of the particle in time 0 to 5 second.

= Area under $v - t$ graph for 5 seconds

$$= \frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = \mathbf{9\ m}$$

73. As $u = 0$, for body 1, $v_1 = at$ and for body 2, v_2 = a constant. Initially both are at $x = 0$. Relative velocity of body 1 w.r.t. body $2 = \vec{v}_1 - \vec{v}_2$. At first, the velocity v_1 is less than v_2. The distance travelled by body 1 *i.e.* $x_1 = \dfrac{1}{2}at^2$, increases with time. The distance travelled by second body $x_2 = v_2\,t$. Hence, $(x_1 - x_2) = \dfrac{1}{2}\ at^2 - v_2\,t$. This shows that the graph between $(x_1 - x_2)$ and t is a parabola before crossing x axis as well as after crossing x-axis. The **curve (*c*) satisfies this.**

HINTS/EXPLANATIONS....

For Difficult Questions

II. Multiple Choice Questions (with One or More than One Correct Answers)

74. When a body is moving with a uniform velocity, it must move along a straight line in the same direction. Then its uniform velocity = average velocity = instantaneous velocity. In this situation magnitude of distance is equal to displacement.

75. A body while moving on a circular path may have variable velocity but constant speed or non-zero acceleration but constant speed.

76. When $t = 2$ s, then $S = 0$, so the **particle is at the origin.** Compairing, the given equation with the equation of uniformly accelerated motion of a body in one dimensions, $S = ut + \frac{1}{2}At^2$, we have

Initial velocity = u, acceleration $\boldsymbol{A = 2a}$.

77. Knowledge based question.

78. Average velocity can not be compared with average speed as when body is moving with constant acceleration along a straight line, the average velocity cannot be equal to instantaneous velocity. The particle moving with uniform acceleration along straight line cannot reverse the direction of its motion.

79. When a particle while moving on a st. line path returns to its initial position after a certain time, then its displacement in that time is **zero**. Its average velocity (= displacement/time) is also **zero.**

$$\text{But its average speed} = \frac{\text{total distance travelled}}{\text{time taken}}$$

$$= \frac{20}{10} = \mathbf{2{\cdot}0\ ms^{-1}}$$

80. Let a be the retarding acceleration acting on body due to air friction. Therefore, effective value of g for upward motion, $g_1 = g + a$ and for downward motion, $g_2 = (g - a)$. We know that,

$$t = \sqrt{\frac{2h}{g_{\text{effective}}}}$$

$$\text{or} \quad t \propto \frac{1}{\sqrt{g_{\text{effective}}}}$$

$$\therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\left(\frac{g-a}{g+a}\right)} < 1 \quad \text{or} \quad t_1 < t_2$$

81. $a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt} = \dfrac{dv}{dx} \times v$

$$\therefore \quad v\frac{dv}{dx} = (4 - 2x)$$

$$\text{or} \quad v\,dv = (4 - 2x)\,dx$$

Integrating it within the limits (when $x = 0$ to $x = x$, the velocity changes 0 to v)

$$\therefore \int_0^v v\,dv = \int_0^x (4 - 2x)\,dx$$

$$\frac{v^2}{2} = 4x - x^2 \quad \text{or} \quad v^2 = 8x - 2x^2$$

$$\text{or} \quad v = (8x - 2x^2)^{1/2} \qquad \text{...(}i\text{)}$$

v is zero at $x = 0$ or $x = 4$

Acceleration of the particle is zero, at $x = 2$. Hence the particle will oscillate about $x = 2$

Velocity will be maximum, when $\dfrac{dv}{dx} = 0$

$$\text{or} \quad \frac{d}{dx}[[8x - 2x^2]^{1/2}] = 0$$

$$\text{or} \quad \frac{1}{2}(8x - 2x^2)^{-1/2} \times (8 - 4x) = 0$$

$$\text{or} \quad 8 - 4x = 0 \quad \text{or} \quad x = 2.$$

It means the velocity of particle is maximum at mean position. From (i),

$$v_{\max} = (8 \times 2 - 2 \times 2^2)^{1/2} = 2\sqrt{2}\ \text{units}$$

82. Acceleration, $= \dfrac{dv(t)}{dt} = 6{\cdot}0 - 3\,v\,(t)$ ...(i)

At $t = 0$, acceleation = $\mathbf{6{\cdot}0\ ms^{-2}}$

Thus **option (*b*) is true**

$$\text{From } (i), \quad \frac{dv}{6 - 3v} = dt$$

Integrating it within the conditions of motion, we have $\displaystyle\int_0^v \frac{dv}{6 - 3v} = \int_0^t dt$

HINTS/EXPLANATIONS

For Difficult Questions

Let $6 - 3v = y$ or $-3\,dv = dy$ or $dv = -dy/3$

$$\therefore \int_{6}^{6-3v} \frac{-dy/3}{y} = \int_{0}^{t} dt \quad \text{or} \quad \int_{6}^{6-3v} \frac{dy}{y} = -\int_{0}^{t} 3\,dt$$

$$\left(\log_e y\right)_{6}^{6-3v} = -3t$$

or $\log_e (6 - 3v) - \log_e 6 = -3t$

or $\log_e \dfrac{6-3v}{6} = -3t$

or $\dfrac{6-3v}{6} = e^{-3t}$

or $6 - 3v = 6\,e^{-3t}$

or $3v = 6 - 6\,e^{-3t}$

or $\mathbf{v = 2\,(1 - e^{-3t})}$

Thus, **option (*c*) is true**

When $\dfrac{dv(t)}{dt} = \dfrac{1}{2} \times 6 = 3{\cdot}0$, then from (*i*) we have

$$3 = 6{\cdot}0 - 3\,v\,(t)$$

or $v\,(t) = (6{\cdot}0 - 3)/3 = \mathbf{1{\cdot}0\ ms^{-1}}$

Thus, **option (*d*) is true.**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

83. $x_A = pt + qt^2\ ;\ x_B = rt^2 - st^3$

Velocity, $v_A = \dfrac{dx_A}{dt} = p + 2qt$

$v_B = \dfrac{dx_B}{dt} = 2rt - 3st^2$

Acceleration, $a_A = \dfrac{dv_A}{dt} = 2q\ ;\ a_B = 2r - 6st$

The car that initially moves ahead is the one which has larger velocity at $t = 0$. At $t = 0$, $v = p$ and $v_B = 0$.

So, **initially car *A* moves ahead.**

84. $x_A = x_B \quad \therefore \quad pt + qt^2 = rt^2 - st^3$

One solution is $t = 0$, which means both the cars start from the same point. To find the other solution, divide it by t, we have $p + qt = rt - st^2$

or $st^2 + (q - r)\,t + p = 0$

or $t = \dfrac{1}{2s}\left[-(q-r) \pm \sqrt{(q-r)^2 - 4sp}\right]$

$$= \frac{1}{2 \times 0{\cdot}20}\left[1 - (1{\cdot}20 - 2{\cdot}80) \pm \sqrt{(1{\cdot}20 - 2{\cdot}80)^2 - 4 \times 0{\cdot}20 \times 2{\cdot}60}\right]$$

$= 4{\cdot}00 \pm 1{\cdot}73 = 2{\cdot}27$ s and $5{\cdot}73$ s.

Hence, $x_A = x_B$ for $t = 0$, $t = \mathbf{2{\cdot}27}$ **s** and $t = \mathbf{5{\cdot}73}$ **s**

85. For the same acceleration ; $a_A = a_B$

$\therefore \quad 2q = 2r - 6st$

or $t = \dfrac{r - q}{3s} = \dfrac{2{\cdot}80 - 1{\cdot}20}{3 \times 0{\cdot}20} = \mathbf{2{\cdot}67\ s}$

IV. Matching Type Questions

86. Here, time of ascent + time of descent = 4 second.

Therefore, time of ascent = time of descent = 2 second.

It means the ball is at the highest position after 2 seconds.

Taking vertical upward motion of the ball upto the highest position, we have

$u = ?\ ;\ a = -9{\cdot}8\ \text{m/s}^2,\ s = ?,\ t = 2\ \text{s}\ ;\ v = 0$

As, $v = u + at$

or $0 = u - 9{\cdot}8 \times 2$

or $u = 19{\cdot}6\ \text{ms}^{-1}$

(A) $s = ut + \dfrac{1}{2}at^2 = 19{\cdot}6 \times 2 + \dfrac{1}{2}(-9{\cdot}8) \times 2^2$

$= \mathbf{19{\cdot}6\ m}$

(B) When $t = 3$ sec, then

$$s = 19{\cdot}6 \times 3 + \frac{1}{2}\,(-9{\cdot}8) \times 3^2$$

$= \mathbf{14{\cdot}7\ m}$

For speed, use $v = u + at$

(C) When $t = 3$ sec, then $v = 19{\cdot}6 + (-9{\cdot}8) \times 3$

$= \mathbf{-9{\cdot}8\ m/s}$

(D) When $t = 4$ sec, then $v = 19{\cdot}6 + (-9{\cdot}8) \times 4$

$= \mathbf{-19{\cdot}6\ m/s}$

87. Distance travelled in a given time is the total area enclosed by velocity-time graph with time axis and displacement in a given time is the area enclosed by velocity-time graph with time axis taking proper sign for area enclosed.

(A) Distance travelled in 8 seconds

= area of ΔOAB + area of ΔBCD

$$= \frac{20\times 6}{2} + \frac{20\times(8-6)}{2} = \mathbf{80\,m}$$

(B) Distance travelled in 10 seconds

$$= \frac{20\times 6}{2} + \frac{20\times(8-6)}{2} + 10\times(10-8) = \mathbf{100\ m}$$

(C) Displacement in 8 seconds

$$= \frac{20\times 6}{2} - \frac{20(8-6)}{2} = \mathbf{40\,m}$$

(D) Displacement in 10 seconds

$$= \frac{20\times 6}{2} - \frac{20\times(8-6)}{2} + 10(10-8) = \mathbf{60\,m}$$

V. Matrix-Match Type Questions

88. Motion of a particle on a straight line. A body sliding down on an inclined plane and motion of a bus on a straight path are the examples of linear motion or translatory motion.

Motion of a particular on a straight line confined in a well defined limits is the illustration of oscillatory motion.

Motion of a bullet fired directly on a target from a distance is a projectile motion.

89. The motion of a point mass body from A to B is with uniform acceleration. Using $u = 0$, $a = ?$, $s = 10$ m, $t = 2$ s, $v = ?$

As $s = ut + \frac{1}{2}at^2$

$\therefore\ 10 = 0 + \frac{1}{2} \times a \times 2^2 = 2a$ or $a = 5\ \text{m/s}^2$

Velocity at B, $v = u + at = 0 + 5 \times 2 = 10$ m/s.

For motion from B to C, 20 m is covered in 2 seconds.

$\therefore$ Velocity $= \frac{20}{2} = 10$ m/s which is the velocity at B. Thus, the motion from B to C is with a uniform velocity. The motion from A to C is with variable velocity.

For motion from C to D, there is again acceleration in the body as 50 m is covered in 2 seconds.

Using, $u = 10$ m/s, $a = ?$, $s = 80 - 30 = 50$ m ;

$t = (6 - 4) = 2$ s

As $s = ut + \frac{1}{2}at^2\quad \therefore\ 50 = 10\times 2 + \frac{1}{2}\times a\times 2^2$

On solving $a = 15\ \text{m/s}^2$

$\therefore$ For the motion from A to D, the body is moving with variable acceleration.

VI. Integer Type Questions

90. Acceleration of cyclist A, a_1 = slope of straight line $OA = \frac{MC}{4}$

Acceleration of cyclist B, a_2 = slope of straight line $DB = \frac{(MC)}{(4-3)} = \frac{MC}{1}$

$$\therefore\ \frac{a_1}{a_2} = \frac{(MC)/4}{(MC)/1} = \frac{1}{4} \qquad ...(i)$$

Let the two cyclist meet after time t, when time is noted from the start of cyclist A. At meeting point, the distance covered by both the cyclist is the same.

So, $\frac{1}{2}a_1 t^2 = \frac{1}{2}a_2 (t-3)^2$

or $\frac{a_1}{a_2} = \frac{(t-3)^2}{t^2} = \frac{1}{4}$ [from (i)]

or $\frac{t-3}{t} = \frac{1}{2}$ or $2t - 6 = t$ or $t = \mathbf{6\ min}$

91. Let the body be falling for t seconds and distance travelled be S metres before the observations are taken. Distance travelled in $(t + 2)$ seconds $= (S + 78{\cdot}4)$ m. Then

HINTS/EXPLANATIONS

For Difficult Questions

$$S = \frac{1}{2}\times 9{\cdot}8\times t^2 \qquad \text{...(i)}$$

and $$(S + 78{\cdot}4) = \frac{1}{2}\times(9{\cdot}8)(t+2)^2 \qquad \text{...(ii)}$$

Solving (*i*) and (*ii*), we get $t = 3$ **s**

92. Let t be the time interval between two successive balls while falling. When 9th ball reaches the ground, the Ist ball is just to be dropped, so the time taken by the 9th ball to reach the ground $= (9-1)\,t = 8\,t$.

$$\therefore \quad S = \frac{1}{2}\times g\times(8t)^2 = \frac{1}{2}\times 10\times 64t^2$$

$$= 320\,t^2 \qquad \text{...(i)}$$

Time taken by nth ball to fall $= (n-1)\,t$

Distance travelled $= S - 3\,S/4 = S/4$

$$\therefore \quad \frac{S}{4} = \frac{1}{2}\times 10\times[(n-1)t]^2 = 5\,(n-1)^2\,t^2$$

or $$S = 20\,(n-1)^2\,t^2 \qquad \text{...(ii)}$$

From (*i*) and (*ii*),

$$320\,t^2 = 20\,(n-1)^2\,t^2$$

On solving, $n = 5$

93. On plotting a graph between velocity (v) and time (t) we get a curve as shown in Fig. 3(CF).33. The distance covered in 4 seconds is the area under the curve ABC with time axis. So distance travelled is

S = area of ΔOAB + area of ΔBCD

$$= \frac{2\times 2}{2} + \frac{2\times 2}{2} = \mathbf{4\,m}$$

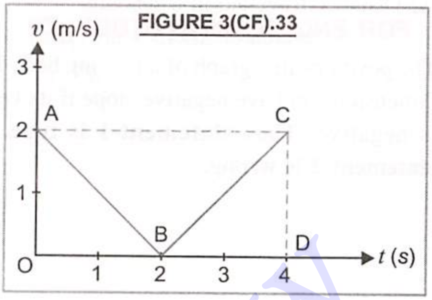


FIGURE 3(CF).33

94. Let the initial velocity be u and acceleration be a. Distance travelled in nth second by a particle moving in a straight line with a constant acceleration is

$$D_n = u = \frac{a}{2}(2n-1)$$

So, $$16 = u + \frac{a}{2}(2\times 2-1) \qquad \text{...(i)}$$

$$28 = u + \frac{a}{2}(2\times 5-1) \qquad \text{...(ii)}$$

and $$52 = u + \frac{a}{2}(2\times 11-1) \qquad \text{...(iii)}$$

On solving (*i*), (*ii*) and (*iii*), we get

$$a = 4\ \text{ms}^{-2},\ u = 10\ \text{ms}^{-1}$$

Now, $$S = ut + \frac{1}{2}at^2 = 10\times 10 + \frac{1}{2}\times 4\times 10^2$$

$$= 300\ \text{m} = 100\ a\ \text{m}$$

$$\therefore \quad a = 3$$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

95. When body while going vertically upwards, reaches at the highest point, then it is momentarily at rest and it then reverses its direction. At the highest point of motion, the velocity is zero but its acceleration is equal to acceleration due to gravity.

96. When a body is describing a uniform circular motion, its speed is constant but velocity at every point of motion represented by the tangent to the circular path, changes. Here, Assertion and Reason are true but **Reason is not correct explanation of Assertion.**

97. The speed of a body can never be negative as it is a scalar quantity. Both **Assertion and reason are false.**

98. A negative acceleration of a body can be associated with a '*speed up*', when the body is moving from extreme position to mean position in simple harmonic motion. Here **Assertion is true but Reason is false.**

HINTS/EXPLANATIONS
For Difficult Questions

FOR ENGINEERING STUDENTS

99. The position-time graph of a moving body in one dimension can have negative slope if its velocity is negative. Thus **statement-1 is true.** Here **statement-2 is wrong.**

100. A positive acceleration of a body can be associated with a '*slowing down*' of the body, if its initial velocity is negative.

Both the statements are true but statement-2 is not the correct explanation of statement-1.

101. Here **statement-1 is not correct** as the body moves in a parabolic path during angular projection of it. **Statement-2 is correct.**

4

MOTION IN A PLANE

4.1. VECTORS

In Physics, we study a large number of physical quantities, which are broadly classified into two categories : (*i*) scalars (*ii*) vectors.

Scalars are those physical quantities which have magnitude only but no direction. For example ; mass, length, time, work etc.

Vectors are those physical quantities which have both, magnitude and direction. For example ; velocity, acceleration, momentum, force etc.

Graphically, a vector can be represented by a straight line with arrow head on it, *i.e.,* an arrowed line. Here the length of line drawn on suitable scale represents the magnitude of vector and arrow head represents the direction of a vector. The starting point of arrowed line is called **tail** or **origin of vector** and the end of the arrowed line is called **tip** or **head** of the vector.

In writing, a vector can be represented by a single letter with arrow head on it. The force which is a vector quantity can be represented by $\vec{F}$.

Vectors can be divided into two types : (*a*) Polar vectors (*b*) Axial vector

(*a*) Polar vectors. ***These are those vectors which have a starting point or a point of application and act along the direction of motion of body.***

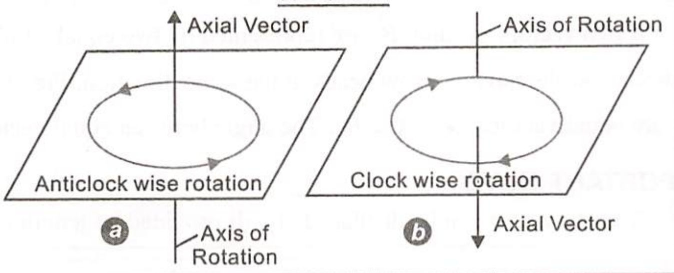
For example ; displacement, force etc. are polar vectors.

(*b*) Axial vectors. ***These are those vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule.****

For example ; angular velocity, angular acceleration torque, angular momentum etc. are axial vectors.

For a vector having anticlockwise or clockwise rotational effect, will have its direction along the axis of rotation as shown in Fig. 4.1(*a*, *b*).

FIGURE 4.1



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*For detail, refer to Art. 4.26.

4.2. IMPORTANCE OF VECTORS

Wherever possible, we express the laws of Physics in vector form, due to the following reasons.

(1) ***Many laws of Physics can be expressed in compact form by the use of vectors.***

(2) ***The derivations involving many laws of Physics can be greatly simplified by the use of vectors.***

(3) ***The laws of Physics when expressed in vector form remain invariant for translation and rotation of the coordinate system.***

4.3. A FEW DEFINITIONS IN VECTOR ALGEBRA

(i) Modulus of a vector. ***The magnitude of a vector is called modulus of that vector.***

The modulus of a vector $\vec{A}$ is represented by $|\vec{A}|$ or A.

(ii) Unit vector. ***A unit vector of the given vector is a vector of unit magnitude and has the same direction as that of the given vector.***

A unit vector of $\vec{A}$ is written as $\hat{A}$ and is read as '*A cap*' or '*A hat*'. Since, magnitude of $\vec{A}$ is A, hence,

$$\vec{A} = A\hat{A} \qquad \text{or} \qquad \hat{A} = \frac{\vec{A}}{A} = \frac{\text{vector}}{\text{modulus of the vector}}$$

Thus, ***a unit vector in a given direction is also defined as a vector in that direction divided by the magnitude of the given vector. It is unitless and dimensionless vector and represents direction only.***

In cartesian coordinates, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x-axis, y-axis and z-axis respectively.

IMPORTANT NOTES

1. Magnitude of a unit vector is unity. **2.** A unit vector has no units and no dimensions.

3. A unit vector of a given vector tells the direction of that vector.

Sample Problem **Find the unit vector of $4\hat{i} - 3\hat{j} + \hat{k}$.**

Sol. Here, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = 4\hat{i} - 3\hat{j} + \hat{k}$, therefore, $A_x = 4$, $A_y = -3$ and $A_z = 1$

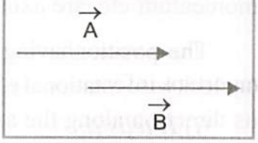
Hence, $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{4^2 + (-3)^2 + (1)^2} = \sqrt{26}$

$$\hat{A} = \frac{\vec{A}}{A} = \frac{4\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{26}}$$

(iii) Equal Vectors. ***Two vectors are said to be equal if they have equal magnitude and same direction.***

If two vectors $\vec{A}$ and $\vec{B}$ are represented by two equal parallel lines drawn with same scale, having arrow heads in the same direction, Fig. 4.2. then $\vec{A}$ and $\vec{B}$ are equal vectors, *i.e.*, $\vec{A} = \vec{B}$. The angle between equal vectors is 0°.

FIGURE 4.2



IMPORTANT NOTE

A given vector can be displaced at will provided its length and direction are not altered.

(iv) Negative Vector. ***A negative vector of a given vector is a vector of same magnitude but acting in a direction opposite to that of the given vector.***

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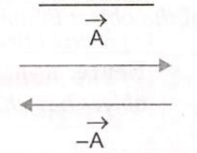
The negative vector of $\vec{A}$ is represented as $-\vec{A}$. In Fig. 4.3, $-\vec{A}$ is equal in length but opposite in direction to the vector $\vec{A}$. The angle between negative vectors is π rad or 180°.

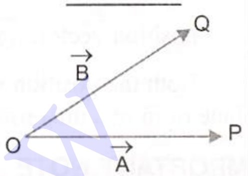
FIGURE 4.3



(v) Co-initial vectors. *The vectors are said to be co-initial, if their initial point is common.*

In Fig. 4.4, two vectors $\vec{A}$ and $\vec{B}$ have been drawn from the common initial point O. Therefore, $\vec{A}$ and $\vec{B}$ are called coinitial vectors.

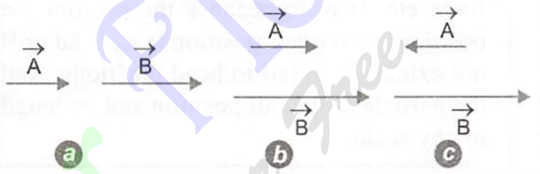
FIGURE 4.4



(vi) Collinear vectors. *These are those vectors which are having equal or unequal magnitudes and are acting along the parallel straight lines.*

If $\vec{A}$ and $\vec{B}$ are two collinear vectors, they can be represented along a line in the same direction, *Fig.* 4.5(*a*) or along the parallel lines in same direction, Fig. 4.5(*b*) or along parallel lines in opposite direction, Fig. 4.5(*c*). Angle between collinear vectors is either 0° or 180°.

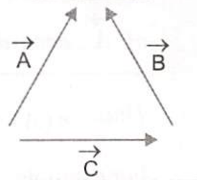
FIGURE 4.5



(vii) Coplanar Vectors. *These are those vectors which are acting in the same plane.*

In Fig. 4.6, the three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are acting in the plane of paper, hence they are coplanar vectors.

FIGURE 4.6



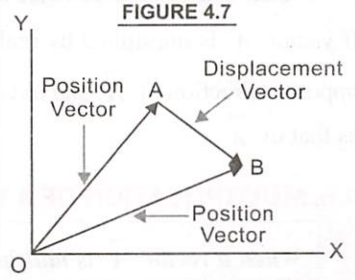
(viii) Localised Vector. *It is that vector whose initial point is fixed. It is also called* **fixed vector.**

(ix) Non–localised vector. *It is that vector whose initial point is not fixed. It is also called a* **free vector.**

4.4. POSITION VECTOR AND DISPLACEMENT VECTOR IN A PLANE

Consider the motion of an object in X-Y plane with origin at O. Let at any time t, the object be at point A. If we draw an arrow with its tail at point O and head at point A, as shown in Fig. 4.7, then $\overrightarrow{OA}$ is called **position vector of the object at point** A and is generally represented by $\vec{r}$. **Position vector of an object at an instant is a vector drawn from the origin to the position of object at that instant.**

FIGURE 4.7



The position vector $\overrightarrow{OA}$ of the object at a point provides two important informations.

(i) ***It tells the straight line distance of the object from the origin or the starting point.***

(ii) ***It tells the direction of the position of the object with respect to the origin.***

Let at time t', the object reach at point B as shown in Fig. 4.7. Then $\overrightarrow{OB}$ **is the position vector of the object at time** t'**.**

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Here, $\overrightarrow{AB}$, where tail or initial position is at A and head or tip is at B, is called the **displacement vector of** *the object in time interval* $(t' - t)$.

> **Hence, *displacement vector* is that *vector which tells how much and in which direction an object has changed its position in a given interval of time.***

If the coordinates of points A and B are (x, y) and (x', y'), then position vector, $\overrightarrow{OA} = x\,\hat{i} + y\,\hat{j}$; position vector, $\overrightarrow{OB} = x'\,\hat{i} + y'\,\hat{j}$ and displacement vector, $\overrightarrow{AB} = (x' - x)\,\hat{i} + (y' - y)\,\hat{j}$

Both the position vector and the displacement vector described above are the examples of vectors in a plane or in two dimensions.

IMPORTANT NOTE

> The position vector or displacement vector drawn is different from the vectors like velocity, force etc. It is so because the position vector is drawn from one position (*i.e.*, tail or initial position) to another position (*i.e.*, head or final position), whereas the velocity vector drawn does not extend from tail to head positions. Rather it shows the instantaneous direction of motion of the particle at the tail position and its length (representing the velocity magnitude) can be drawn to any scale.

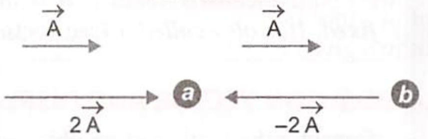
4.5. MULTIPLICATION OF A VECTOR BY A REAL NUMBER

> ***The multiplication of a vector $\vec{A}$ by a real number n becomes another vector $n\,\vec{A}$. Its magnitude becomes n times the magnitude of the given vector. Its direction is the same or opposite to that of $\vec{A}$, according as n is a positive or negative real number.***

Thus, $n(\vec{A}) = n\,\vec{A}$ and $-n(\vec{A}) = -n\,\vec{A}$.

For example, if a vector $\vec{A}$ is multiplied by a real number $n = 2$, we get $2\,\vec{A}$, which is a vector, acting in the direction of $\vec{A}$ and having magnitude twice as that of $\vec{A}$, Fig. 4.8(*a*).

FIGURE 4.8



If vector $\vec{A}$ is multiplied by real number $n = -2$, then we get $-2\,\vec{A}$, which is also a vector, acting in the opposite direction of $\vec{A}$ and having magnitude twice as that of $\vec{A}$, Fig 4.8(*b*). The unit of $n\,\vec{A}$ is the same as that of $\vec{A}$.

4.6. MULTIPLICATION OF A VECTOR BY A SCALAR

> ***When a vector $\vec{A}$ is multiplied by a scalar S, it becomes a vector $S\,\vec{A}$, whose magnitude is S times the magnitude of $\vec{A}$ and it acts along the direction of $\vec{A}$. The unit of $S\,\vec{A}$, is different from the unit of vector $\vec{A}$.***

For illustration, if $\vec{A}$ = 100 newton due west and S = 10 sec, then

$$S\,\vec{A} = 10 \text{ second} \times 100 \text{ newton due west} = 1000 \text{ newton-second due west.}$$

4.7. RESULTANT VECTOR

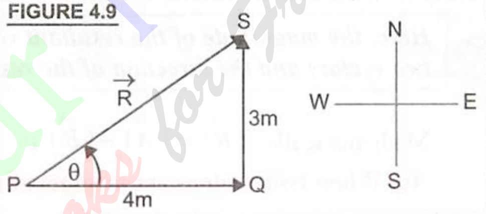
The resultant vector of two or more vectors is defined as that single vector which produces the same effect as is produced by individual vectors together.

It is to be noted that the nature of the resultant vector is the same as that of the given vectors.

4.8. VECTORS ADDITION IS NOT POSSIBLE BY SIMPLE LAWS OF ALGEBRA

Vectors can not be added by simple laws of algebra, which are applicable to scalars. To understand it, consider a particle is first displaced through 4m due east and then 3 m due north. Let these displacement be represented by $\overrightarrow{PQ}$ and $\overrightarrow{QS}$ respectively on some suitable scale in Fig. 4.9. The final displacement of the particle is found by drawing a straight line from starting point *P* to the end point *S*, *i.e.*, the displacement $\overrightarrow{PS}$. Here, one will find that the **magnitude of** $\overrightarrow{PS}$ is not equal to ; 3 + 4 = 7 m, but is equal to $= \sqrt{4^2 + 3^2} = 5\,\text{m}$ and angle θ which gives the **direction of displacement of** $\overrightarrow{PS}$ is neither towards east nor towards north but is **due north of east.** This shows that the ***vectors cannot be added by simple laws of algebra.***

FIGURE 4.9



In the above illustration, we note that the displacement 5 m along *PS* produces the same effect as the displacement 4 m along *PQ* and 3 m along *QS* produce together. It means the displacement $\overrightarrow{PS}$ is the sum (or resultant) of displacements $\overrightarrow{PQ}$ and $\overrightarrow{QS}$, *i.e.*, $\overrightarrow{PQ} + \overrightarrow{QS} = \overrightarrow{PS}$

If $\overrightarrow{PQ}, \overrightarrow{QS}$ and $\overrightarrow{PS}$ are represented by $\vec{A}, \vec{B}$ and $\vec{R}$ respectively, then $\vec{A} + \vec{B} = \vec{R}$

Here, $\vec{R}$ is called resultant of two vectors $\vec{A}$ and $\vec{B}$. It means if the tail of $\vec{B}$ coincides with the head of $\vec{A}$, then the vector drawn from the tail of the vector $\vec{A}$ to the head of vector $\vec{B}$ is called resultant vector of two given vectors.

4.9. VECTORS ADDITION BY GEOMETRICAL METHOD

General Rule for addition of vectors. ***It states that the vectors to be added are arranged in such a way that the head of first vector coincides with the tail of second vector, whose head coincides with the tail of third vector and so on, then the single vector drawn from the tail of the first vector to the head of the last vector represents their resultant vector.***

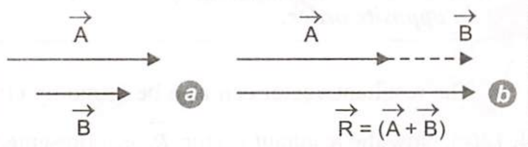
The above rule is illustrated by the following cases.

(a) When the two vectors are acting in the same direction.

Let the two vectors $\vec{A}$ and $\vec{B}$ be acting in the same direction as shown in Fig. 4. 10(*a*).

To find their resultant, coincide the tail of $\vec{B}$ on the head of $\vec{A}$ as shown in Fig. 4.10(*b*). Then the single vector $\vec{R}$ drawn from the tail of $\vec{A}$ to the head of $\vec{B}$ gives their resultant vector, *i.e.*, $\vec{R} = \vec{A} + \vec{B}$.

FIGURE 4.10



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In this case, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors and its direction is the same as that of the two given vectors. Mathematically, $|\vec{R}| = |\vec{A}| + |\vec{B}|$

IMPORTANT NOTE

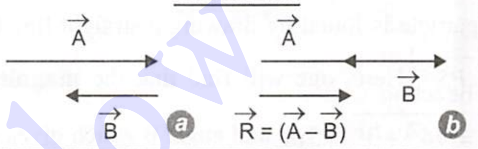
1. When a vector is displaced to any position without any change in its direction, then that vector remains the same.
2. Two vectors $\vec{A}$ and $\vec{B}$ are said to be in the same order if the tail of $\vec{B}$ coincides with the head of $\vec{A}$.

(b) When two vectors are acting in opposite directions.

Let the two vectors $\vec{A}$ and $\vec{B}$ be acting in opposite directions as shown in Fig. 4.11(*a*). To find their resultant, coincide the tail of $\vec{B}$ on the head of $\vec{A}$ as shown in Fig. 4. 11(*b*). Then the single vector $\vec{R}$ drawn from the tail of $\vec{A}$ to the head of $\vec{B}$ gives their resultant vector, *i.e.*,

$$\vec{R} = \vec{A} + (-\vec{B})$$

FIGURE 4.11



Here, the magnitude of the resultant vector is equal to the difference of the magnitudes of the two vectors and the direction of the resultant vector will be in the direction of bigger vector.

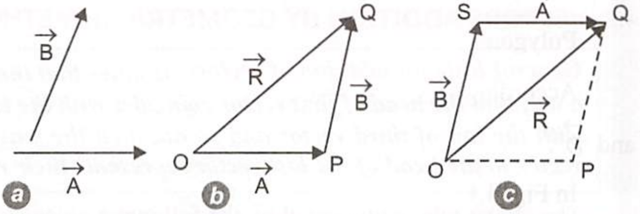
Mathematically $|\vec{R}| = |\vec{A}| - |\vec{B}|$ If $|\vec{A}| > |\vec{B}|$.

(c) When two vectors act at some angle.

Let the two vectors $\vec{A}$ and $\vec{B}$ be acting at some angle to each other as shown in Fig. 4.12 (*a*). To find their resultant $\vec{R}$, coincide the tail of $\vec{A}$ on the head of $\vec{B}$ as shown in Fig. 4. 12(*b*). Then, the single vector $\vec{R}$ drawn from the tail of $\vec{A}$ to the head of $\vec{B}$ gives the resultant vector.

As shown in Fig. 4.12(*b*), the two vectors $\vec{A}$ and $\vec{B}$ are represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ of a triangle *OPQ*, taken in the same order. Their, resultant $\vec{R}$ is represented by the third side $\overrightarrow{OQ}$ of the triangle taken in the opposite order. This method of finding the resultant of two vectors is called ***triangle law of vectors***.

FIGURE 4.12



Triangle law of vectors states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of the triangle taken in opposite order.

The resultant vector can also be found by coinciding the tail of $\vec{A}$ on the head of $\vec{B}$ as shown in Fig. 4. 12(*c*). Now the resultant vector $\vec{R}$ is represented by $\overrightarrow{OQ}$ which is the diagonal of the parallelogram *OPQS*.

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From above, it is clear that the two vectors $\vec{A}$ and $\vec{B}$ are represented by the two sides $\overrightarrow{OP}$ and $\overrightarrow{OS}$ of parallelogram *OPQS* drawn from a point *O*, then their resultant $\vec{R}$ is represented by the side $\overrightarrow{OQ}$, *i.e.*, diagonal of the parallelogram drawn from the same point. This method of finding the resultant of two vectors is called **parallelogram law of vectors.**

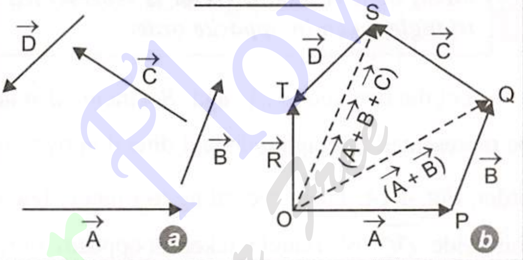
> ***Parallelogram law of vectors** states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.*

(d) When number of vectors act in different directions.

Let the number of vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ etc. be acting in different directions as shown in Fig. 4.13(*a*).

To find their resultant vector, coincide the tail of $\vec{B}$ with the head of $\vec{A}$, tail of $\vec{C}$ with the head of $\vec{B}$ and tail of $\vec{D}$ with the head of $\vec{C}$. Then, the single vector drawn from the tail of $\vec{A}$ to head of $\vec{D}$ will represent their resultant vector, Fig. 4.13(*b*).

FIGURE 4.13



Thus, it is clear that if the vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are represented in magnitude and direction by the sides $\overrightarrow{OP}$, $\overrightarrow{PQ}$, $\overrightarrow{QS}$ and $\overrightarrow{ST}$ of an open polygon taken in the same order, then their resultant vector $\vec{R}$ will be represented in magnitude and direction by the closing side $\overrightarrow{OT}$ of the polygon taken in opposite order. This method of finding the resultant is called **polygon law of vectors.**

> **Thus,** ***polygon law of vectors*** **states** ***that if any number of vectors, acting on a particle at the same time are represented in magnitude and direction by various sides of an open polygon taken in the same order, their resultant is represented in magnitude and direction by the closing side of the polygon taken in opposite order.***

Polygon law of vectors is an extension of the triangle law of vectors, which is explained below.

According to polygon law of vectors : $\vec{A}+\vec{B}+\vec{C}+\vec{D}=\vec{R}$, where $\vec{R}$ is the resultant vector of $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$.

In Fig. 4.13(*b*), $\vec{A}=(\overrightarrow{OP})$, $\vec{B}=(\overrightarrow{PQ})$, $\vec{C}=(\overrightarrow{QS})$, $\vec{D}=(\overrightarrow{ST})$ and $\vec{R}=(\overrightarrow{OT})$. Join *O* to *Q* and *O* to *S*, with straight lines.

From triangle law of vectors ; In Δ *OPQ*,

$$\vec{A}+\vec{B}=(\overrightarrow{OP})+(\overrightarrow{PQ})=(\overrightarrow{OQ})$$

In Δ *OQS*, $$(\vec{A}+\vec{B})+\vec{C}=(\overrightarrow{OQ})+(\overrightarrow{QS})=(\overrightarrow{OS})$$

In Δ *OST*, $$(\vec{A}+\vec{B}+\vec{C})+\vec{D}=(\overrightarrow{OS})+(\overrightarrow{ST})=(\overrightarrow{OT})=\vec{R}$$

Here, $\vec{R}$ is the resultant vector of $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$.

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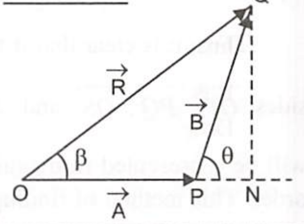
1. The two vectors can be added only if they are of the same nature. It means a displacement vector can not be added to a force vector or velocity vector but can be added to displacement vector only.
2. If a number of vectors are represented by the various sides of a closed polygon taken in one order, their resultant is zero.

4.10. ANALYTICAL METHOD OF VECTORS ADDITION

(a) Triangle law of vectors addition. ***It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of the triangle taken in opposite order.***

Let the two vector $\vec{A}$ and $\vec{B}$, inclined at an angle θ be acting on a particle at the same time. Let they be represented in magnitude and direction by two sides $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ of triangle OPQ, taken in the same order, Fig. 4.14. Then, according to triangle law of vectors addition, the resultant $\vec{R}$ is represented by the third side $\overrightarrow{OQ}$ of triangle, taken in opposite order.

FIGURE 4.14



Magnitude of $\vec{R}$.

Draw QN perpendicular to OP produced.

In Fig. 4.14 ; $OP = A$; $PQ = B$; $OQ = R$ and $\angle NPQ = \theta$.

In Δ QNP, $PN/PQ = \cos\theta$ or $PN = PQ\cos\theta = B\cos\theta$ ...(1)

and $QN/PQ = \sin\theta$ or $QN = PQ\sin\theta = B\sin\theta$ ...(2)

In right angled triangle ONQ, we have

$$OQ^2 = ON^2 + NQ^2 = (OP + PN)^2 + NQ^2$$

or $R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$ [From (1) & (2)]

or $R^2 = A^2 + 2AB\cos\theta + B^2(\cos^2\theta + \sin^2\theta)$ or $\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$ ...(3)

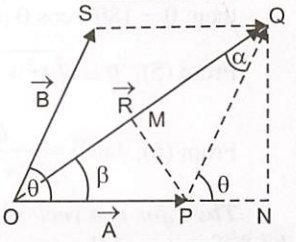
Direction of $\vec{R}$. Let the resultant $\vec{R}$ make an angle β with the direction of $\vec{A}$. Then, from right angled triangle QNO,

$$\boxed{\tan\beta = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}} \quad ...(4)$$

(b) Parallelogram law of vectors addition. ***It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.***

Let the two vectors $\vec{A}$ and $\vec{B}$, inclined at angle θ be acting on a particle at the same time. Let they be represented in magnitude and direction by two adjacent sides $\overrightarrow{OP}$ and $\overrightarrow{OS}$ of parallelogram $OPQS$, drawn from a point O. Fig. 4.15. According to parallelogram law of vectors, their resultant vector $\vec{R}$ will be represented by the diagonal $\overrightarrow{OQ}$ of the parallelogram.

FIGURE 4.15



Magnitude of $\vec{R}$.

Draw QN perpendicular to OP produced.

In Fig. 4.15, $OP = A$, $OS = PQ = B$, $OQ = R$

and $\angle SOP = \angle QPN = \theta$.

In $\Delta\, QNP$, $PN = PQ \cos\theta = B\cos\theta$

$QN = PQ \sin\theta = B \sin\theta$

In right angled triangle ONQ, we have

$$OQ^2 = ON^2 + NQ^2 = (OP + PN)^2 + (NQ)^2$$

or $$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

or $$R^2 = A^2 + 2\,AB\cos\theta + B^2(\cos^2\theta + \sin^2\theta) \quad \text{or} \quad \boxed{R = \sqrt{A^2 + B^2 + 2\,AB\cos\theta}} \quad ...(5)$$

Direction of $\vec{R}$. Let the resultant $\vec{R}$ make an angle β with the direction of $\vec{A}$. Then from right angled triangle QNO,

$$\boxed{\tan\beta = \frac{QN}{ON} = \frac{QN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}} \quad ...(6)$$

From (5), $$\cos\theta = \frac{R^2 - A^2 - B^2}{2\,AB}$$

It is known as the **law of cosines.**

Drop a perpendicular PM on OQ. Let $\angle PQO = \alpha$.

In $\Delta\, OQN$, $QN = (OQ)\sin\beta = R\sin\beta$

In $\Delta\, PQN$, $QN = (PQ)\sin\theta = B\sin\theta$

$$\therefore \quad R\sin\beta = B\sin\theta \quad \text{or} \quad \frac{R}{\sin\theta} = \frac{B}{\sin\beta} \quad ...(7)$$

Similarly, $$PM = A\sin\beta = B\sin\alpha \text{ or } \frac{A}{\sin\alpha} = \frac{B}{\sin\beta} \quad ...(8)$$

From (7) and (8), we have $$\frac{R}{\sin\theta} = \frac{A}{\sin\alpha} = \frac{B}{\sin\beta} \quad ...(9)$$

It is known as the **law of sines.**

From (9), $$\sin\beta = \frac{B\sin\theta}{R} = \frac{B\sin\theta}{(A^2 + B^2 + 2\,AB\cos\theta)^{1/2}} \quad ...(10)$$

Special cases.

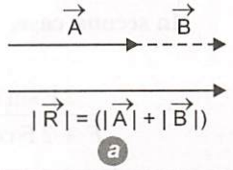
(i) When two vectors are acting in the same direction, Fig. 4.15(*a*)

then, θ = 0°, cos θ = 1 and sin θ = 0

From (5), $$R = \sqrt{A^2 + B^2 + 2\,AB(1)} = \sqrt{(A + B)^2} = (A + B)$$

From (6), $$\tan\beta = \frac{B \times 0}{A + B(1)} = 0 \quad \text{or} \quad \beta = 0^\circ.$$

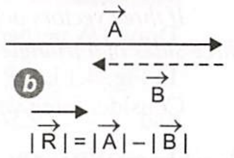
FIGURE 4.15



Thus, for two vectors acting in the same direction, the magnitude of the resultant vector is equal to the sum of the magnitudes of two vectors and acts along the direction of $\vec{A}$ and $\vec{B}$.

(ii) When two vectors are acting in opposite directions, Fig. 4.15(*b*), then, θ = 180°, cos θ = – 1 and sin θ = 0.

FIGURE 4.15



From (5), $R = \sqrt{A^2 + B^2 + 2AB(-1)} = \sqrt{(A-B)^2} = (A-B)$ or $(B-A)$

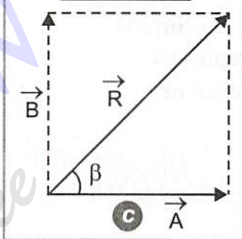
From (6), $\tan\beta = \dfrac{B \times 0}{A + B(-1)} = 0$ or $\beta = 0°$ or $180°$

Thus, for two vectors acting in opposite directions, the magnitude of the resultant vector is equal to the difference of the magnitudes of the two vectors and acts in the direction of bigger vector.

(iii) When two vectors act at right angle to each other, Fig. 4.15(*c*), then

$$\theta = 90°,\ \sin\theta = \sin 90° = 1 \text{ and } \cos\theta = \cos 90° = 0$$

FIGURE 4.15



From (5), $R = \sqrt{A^2 + B^2 + 2AB(0)} = \sqrt{A^2 + B^2}$

From (6), $\tan\beta = \dfrac{B(1)}{A + B(0)} = \dfrac{B}{A}$ or $\beta = \tan^{-1}\left(\dfrac{B}{A}\right)$

Retain in Memory

1. *The magnitude of the resultant of two vectors is* ***maximum***, *when the vectors act in the same direction, i.e., angle between them is 0° and is* ***minimum*** *when they act in opposite directions, i.e., angle between them is 180°.*
2. *While finding the resultant vector of two vectors* $\vec{A}$ *and* $\vec{B}$ *by the parallelogram law of vectors addition, both the vectors* $\vec{A}$ *and* $\vec{B}$ *should either act towards a point or away from a point.*
3. *The resultant of two equal vectors can be zero if they act in opposite directions.*
4. *The minimum number of equal vectors whose resultant can be zero is two and angle between them is 180°.*
5. *The number of three unequal vectors can be zero if they are coplanar and they can be represented by the three sides of a triangle taken in one order.*
6. *The minimum number of non-coplanar vectors whose resultant can be zero is four.*

Sample Problem **Two forces *F* and 2 F newton act on a particle. If the first force be doubled and the second force be increased by 16 newtons, the direction of the resultant remains unaltered. Find the value of *F*.**

Sol. Let the resultant make an angle β with the direction of force *F*.

In first case, $\tan\beta = \dfrac{2F\sin\theta}{F + 2F\cos\theta}$

In second case, $\tan\beta = \dfrac{(2F+16)\sin\theta}{2F + (2F+16)\cos\theta}$

$$\therefore \quad \frac{2F\sin\theta}{F + 2F\cos\theta} = \frac{(2F+16)\sin\theta}{2F+(2F+16)\cos\theta} \qquad \left[\text{since, } \frac{A}{B} = \frac{C}{D} = \frac{C-A}{D-B}\right]$$

$$\frac{2F\sin\theta}{F + 2F\cos\theta} = \frac{(2F\sin\theta + 16\sin\theta) - 2F\sin\theta}{2F + (2F+16)\cos\theta - (F + 2F\cos\theta)} = \frac{16\sin\theta}{F + 16\cos\theta}$$

From above, we have, $2F = 16$ or $F = 8$ **N**

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4.11. CONDITIONS FOR ZERO RESULTANT VECTOR

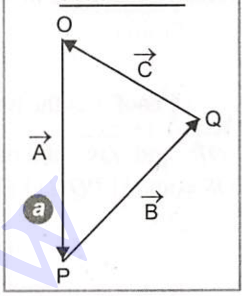
If three vectors acting on a point object at the same time are represented in magnitude and direction by the three sides of a triangle taken in the same order, their resultant is zero. The object is said to be ***in equilibrium.***

Consider three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ acting on an object at the same time. Let they be represented by $\overrightarrow{OP}, \overrightarrow{PQ}$ and $\overrightarrow{QO}$ *i.e.* the three sides of the triangle taken in one order, Fig. 4.16(*a*) ; then

$$\frac{A}{OP}=\frac{B}{PQ}=\frac{C}{QO}$$

Similarly, if any number of vectors acting on an object at the same time are represented in magnitude and direction by the various sides of a closed polygon taken in the same order, their resultant vector is zero and the object is in equilibrium.

FIGURE 4.16

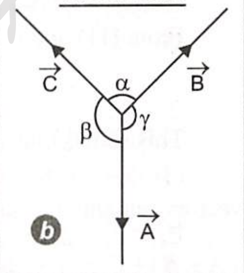


Lami's Theorem. ***It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces. i.e.,***

$$\frac{A}{\sin\alpha}=\frac{B}{\sin\beta}=\frac{C}{\sin\gamma}$$

where $\vec{A}, \vec{B}, \vec{C}$ are the three forces and α, β, γ are the angles between forces $\vec{B}$ and $\vec{C}$, $\vec{C}$ and $\vec{A}$ and $\vec{A}$ and $\vec{B}$ respectively. Fig. 4.16.(*b*).

FIGURE 4.16



Conditions for Equilibrium of an object

The object is in equilibrium if

(*i*) ***there is no linear motion of the object*** *i.e.*, the resultant force on the object is zero.

(*ii*) ***there is no rotational motion of the object*** *i.e.*, the torque* due to forces on the object is zero.

(*iii*) ***there is minimum potential energy** of the object for stable equilibrium.***

Sample Problem **A bob weighing 50 gram hangs vertically at the end of a string 50 cm long. If 20 gram force is applied horizontally, by how much distance the bob is pulled aside from its initial position when it reaches in equilibrium position ?**

Sol. Here, $m = 50$ gram ; $F = 20$ g f $= 20 \times 980$ dyne. Let the bob be in equilibrium when it is at location B, Fig. 4.17. Then

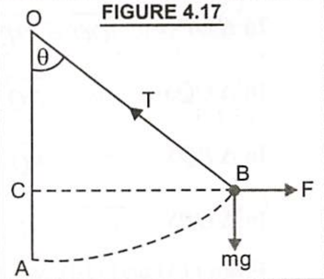
$$\frac{F}{CB}=\frac{mg}{OC}=\frac{T}{BO} \quad \text{or} \quad \frac{CB}{OC}=\frac{F}{mg}=\frac{20\times980}{50\times980}=0{\cdot}4$$

$$\tan\theta=\frac{CB}{OC}=0{\cdot}4=\tan 21°\,48' \quad \text{so} \quad \theta=21°\,48'$$

In $\Delta\, OCB$, $\sin 21°\,48' = \dfrac{CB}{OB}$

or $CB = OB \sin 21°\,48' = 50 \times 0{\cdot}3714 =$ **17·57 cm.**

FIGURE 4.17



4.12. EQUILIBRANT VECTOR

A single vector which balances two or more vectors acting on a body at the same time is called an equilibrant vector.

It is equal and opposite to the resultant of various vectors acting on the body at the same time.

---

*For torque, Refer to Art. 8.13 **For potential energy, refer to Art. 7.12.

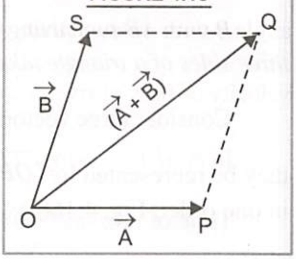
4.13. IMPORTANT FACTS OF VECTORS ADDITION

(1) *Vectors of the same nature alone can be added,* e.g., a force vector cannot be added to velocity vector but can be added to force vector only.

(2) *Vectors addition is commutative.* It states that the sum of the vectors remains the same in whatever order they may be added, *i.e.*,

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

FIGURE 4.18



Proof. Let the two vectors $\vec{A}$ and $\vec{B}$ be represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{OS}$ of a parallelogram *OPQS*, Fig. 4.18. Join *O* with *Q*. As *OS* = and | | *PQ* and *OP* = and | | *SQ*, therefore,

$$\vec{B}=\overrightarrow{OS}=\overrightarrow{PQ} \quad \text{and} \quad \vec{A}=\overrightarrow{OP}=\overrightarrow{SQ}$$

In Δ *OPQ*, $\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}$ (From Triangle law)

$$=\vec{A}+\vec{B} \quad \text{...(11)}$$

In Δ *OSQ*, $\overrightarrow{OQ}=\overrightarrow{OS}+\overrightarrow{SQ}=\vec{B}+\vec{A}$ ...(12)

From (11) and (12), we note that

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

This shows that vectors addition is commutative.

(3) *Vectors Addition is associative.* It states that the sum of the vectors remains the same in whatever, grouping they are added, *i.e.*, $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$.

Proof. Let the three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ be represented by the sides $\overrightarrow{OP}, \overrightarrow{PQ}$ and $\overrightarrow{QS}$ of a polygon *OPQS*. Fig. 4.19. Then by polygon law of vectors, $\overrightarrow{OS}$ represents the resultant $\vec{R}$ in magnitude and direction. Join *O* to *Q* and *P* to *S*.

From triangle law of vectors

In Δ *OPQ* ; $\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}=\vec{A}+\vec{B}$

In Δ *OQS* ; $\overrightarrow{OS}=\overrightarrow{OQ}+\overrightarrow{QS}=(\vec{A}+\vec{B})+\vec{C}$ ...(13)

In Δ *PQS* ; $\overrightarrow{PS}=\overrightarrow{PQ}+\overrightarrow{QS}=\vec{B}+\vec{C}$

In Δ *OPS* ; $\overrightarrow{OS}=\overrightarrow{OP}+\overrightarrow{PS}=\vec{A}+(\vec{B}+\vec{C})$ ...(14)

From (13) and (14), we note that

$$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$$

This shows that vectors addition is associative.

(4) A boat in a river always moves in the direction of resultant velocity of velocity of boat and velocity of river flow.

DO YOU KNOW ?

1. A physical quantity having magnitude and direction may or may not be a vector *e.g.*, time, pressure, surface tension, current etc. They have direction, but are not vectors.

2. A physical quantity is a vector only when it follows commutative law of addition of vectors, *i.e.*,

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

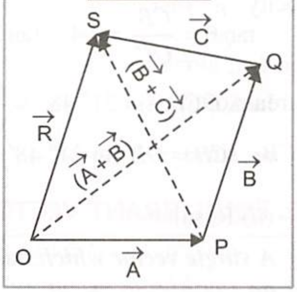
3. If the frame of reference is rotated or translated, the vector does not change but its components may change.

4. If a vector is rotated through an angle other than multiple of 2 π (or 360°), it changes.

5. A unit vector is unitless and dimensionless.

6. A vector cannot be divided by a vector, since it is not possible to divide a vector by a direction.

FIGURE 4.19



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When a boat tends to cross a river along the shortest path, it should be rowed up stream making an angle θ with AB such that $\overrightarrow{AB}$ gives the direction of resultant velocity $\vec{v}$, of velocity of boat $\vec{v}_1\ (= \overrightarrow{AD})$ and velocity of flow of river $\vec{v}_2\ (= \overrightarrow{AC} = \overrightarrow{DB})$, Fig. 4.20.

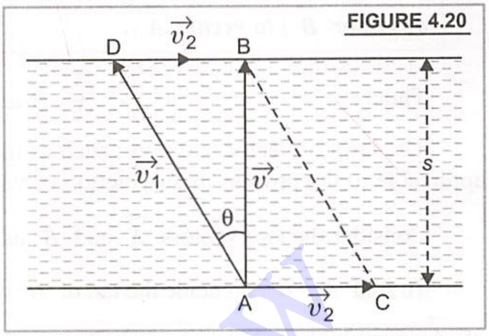


FIGURE 4.20

In Δ ABD, $\sin\theta = \dfrac{v_2}{v_1}$ and $v = \sqrt{v_1^2 - v_2^2}$

Time of crossing the river by the boat,

$$t = \frac{s}{v} = \frac{s}{\sqrt{v_1^2 - v_2^2}}$$

(5) **When a boat tends to cross a river in the shortest time,** the boat should go along AB, Fig. 4.21. Now the boat will be going along AD which is the direction of resultant velocity $\vec{v}$ of velocity of boat $\vec{v}_1\ (= \overrightarrow{AB})$ and velocity of river $\vec{v}_2\ (= \overrightarrow{AC})$.

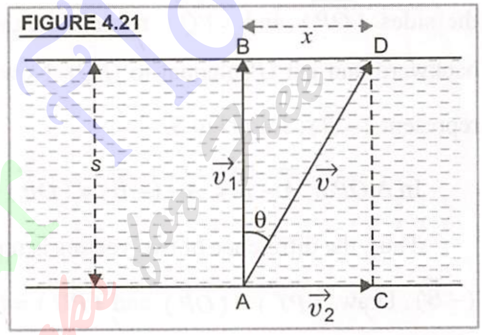


FIGURE 4.21

In Δ ABD, $\tan\theta = \dfrac{v_2}{v_1}$ and $v = \sqrt{v_2^2 + v_1^2}$

Time of crossing the river by the boat,

$$t = \frac{s}{v_1} = \frac{AD}{v} = \frac{\sqrt{s^2 + x^2}}{\sqrt{v_2^2 + v_1^2}}$$

The boat will be reaching the point D instead of point B. If $BD = x$, then $\tan\theta = \dfrac{v_2}{v_1} = \dfrac{x}{s}$ or $x = \dfrac{s\,v_2}{v_1}$.

Sample Problem **A river 500 m wide flows at a rate of 4 km h^{-1}. A swimmer who can swim at 8 km h^{-1} in still water, wishes to cross the river straight. (*i*) Along what direction must he strike ? (*ii*) What should be his resultant velocity. (*ii*) What is the time of crossing the river ?**

Sol. (*i*) Refer to Fig. 4.22,

velocity of river = 4 km h^{-1} = (OA)

Velocity of swimmer in still water

v_s = 8 km h^{-1} = (OB)

The swimmer will cross the river straight if the resultant velocity $\vec{v}$ of $\vec{v}_r$ and $\vec{v}_s$ is perpendicular to the bank of the river, *i.e.*, along $\overrightarrow{OC}$. This will be possible if the swimmer goes upstream of the river along OB, making an angle θ with OC. In right ΔOCB, $\sin\theta = \dfrac{BC}{OB} = \dfrac{v_r}{v_s} = \dfrac{4}{8} = \dfrac{1}{2} = \sin 30°$ ∴ **θ = 30°**

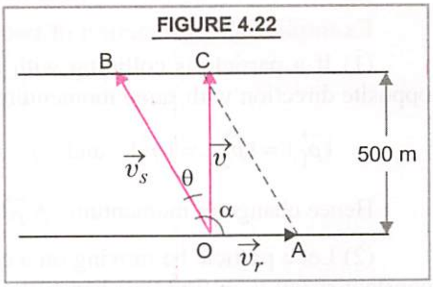


FIGURE 4.22

(*ii*) Resultant velocity of the swimmer

$$v = \sqrt{v_s^2 - v_r^2} = \sqrt{8^2 - 4^2} = \sqrt{48} = \mathbf{4\sqrt{3}\ km\ h^{-1}} = 4\sqrt{3} \times 5/18 = 1{\cdot}92\ \text{m/s}$$

(*iii*) Time taken to cross the river, $t = \dfrac{\text{width of river}}{v} = \dfrac{500\ \text{m}}{1{\cdot}92\ \text{ms}^{-1}} = \mathbf{260{\cdot}4\ s}$

4.14. SUBTRACTION OF VECTORS

Subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $(-\vec{B})$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

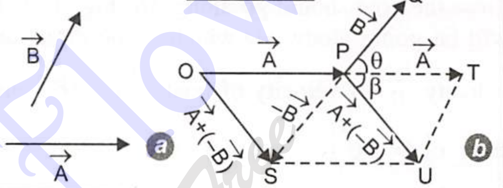
Thus, $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

The laws of vector addition ; such as triangle law of vectors, parallelogram law of vectors are equally applicable to the process of subtraction of vectors.

Consider the two vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 4.23(*a*).

To find $\vec{A}-\vec{B}$, coincide the tail of $\vec{B}$ with the head of $\vec{A}$. Fig. 4.23(*b*). Now $\vec{A}$ and $\vec{B}$ are represented by the sides $(\overrightarrow{OP})$ and $(\overrightarrow{PQ})$ respectively. Produce *QP* backward and cut (*PS*) equal to (*PQ*). Now $(\overrightarrow{PS})$ will represent $(-\vec{B})$. Join *O* to *S*.

FIGURE 4.23



In Δ *OPS*, $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})=(\overrightarrow{OP})+(\overrightarrow{PS})=(\overrightarrow{OS})=\vec{R}$ (From Δ law of vectors)

Thus, the difference of two vectors is represented by $(\overrightarrow{OS})$, which is the resultant vector $\vec{R}$ of $\vec{A}$ and $(-\vec{B})$. Draw, $(\overrightarrow{PT})=(\overrightarrow{OP})$ and $(\overrightarrow{PU})=(\overrightarrow{OS})$

Then $\vec{R}$ will be represented by $(\overrightarrow{PU})$. If θ is the angle between $\vec{A}$ and $\vec{B}$, then angle between $\vec{A}$ and $(-\vec{B})$ is (180° – θ). According to parallelogram law of vectors addition ;

$$|\vec{R}|=\sqrt{A^2+B^2+2AB\cos(180°-\theta)} \quad \text{and} \quad \tan\beta=\frac{B\sin(180°-\theta)}{A+B\cos(180°-\theta)}$$

It is **important to note** that (*i*) ***the vector subtraction does not follow commutative law, i.e.,***

$$(\vec{A}-\vec{B})\neq(\vec{B}-\vec{A})$$

(*ii*) ***the vector subtraction does not follow associative law, i.e.,*** $\vec{A}-(\vec{B}-\vec{C})\neq(\vec{A}-\vec{B})-\vec{C}$

Examples of Substraction of two vectors.

(1) If a particle is colliding with a wall in a direction normal to its surface and is reflected just in opposite direction with same momentum, then

$$|\vec{p}_1|=|\vec{p}_2|=|\vec{p}| \quad \text{and} \quad \vec{p}_2=+\vec{p};\ \vec{p}_1=-\vec{p}$$

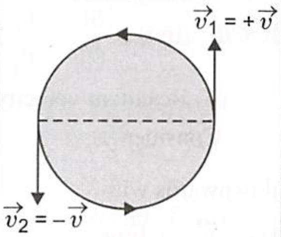
Hence change in momentum, $\Delta\vec{p}=\vec{p}_2-\vec{p}_1=\vec{p}-(-\vec{p})=2\vec{p}$

(2) Let a particle be moving on a circular path of constant radius with a constant speed v, to find the change in its velocity when it completes half of the revolution, Fig. 4.24, we note

$$\vec{v}_2=+\vec{v} \quad \text{and} \quad \vec{v}_1=-\vec{v}$$

Then, change in velocity, $\Delta\vec{v}=\vec{v}_2-\vec{v}_1=\vec{v}-(-\vec{v})=2\vec{v}$

FIGURE 4.24



(3) A particle moving with velocity v towards northward direction changes its direction and moves towards eastward with the same speed. To find the change in its velocity, Fig. 4.25.

$$\vec{v}_1 = (\overrightarrow{OA}),\ v_1 = (OA) = v\ ;\ \vec{v}_2 = (\overrightarrow{OB}),\ v_2 = (OB) = v$$

Change in velocity, $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1) = (\overrightarrow{OB}) + (\overrightarrow{OC}) = (\overrightarrow{OD})$

$$\therefore\ |\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos 90^\circ} = \sqrt{v^2 + v^2} = v\sqrt{2}$$

$$\tan\beta = \frac{|\vec{v}_1|}{|\vec{v}_2|} = \frac{v}{v} = 1 \text{ or } \beta = 45^\circ$$

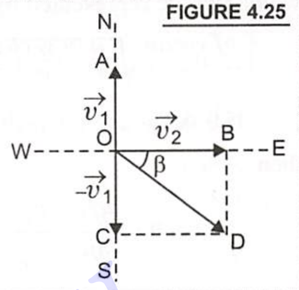


FIGURE 4.25

Thus, the direction of $\Delta\vec{v}$ will be along **south-east direction.**

Retain in Memory

Vectors addition is commutative and associative by nature. But vector subtraction is neither commutative nor associative by nature.

4.15. RELATIVE VELOCITY IN A PLANE

Refer to Article 3.20, the relative velocity of object A, moving with velocity $\vec{v}_A$ w.r.t. object B moving with velocity $\vec{v}_B$ is given by

$$\vec{v}_{AB} = \vec{v}_A + (-\vec{v}_B) = \vec{v}_A - \vec{v}_B$$

Let the two objects be moving in a plane, and θ be the angle between the directions of motion of the objects A and B, as shown in Fig. 4.26(*a*), where, $\vec{v}_A = (\overrightarrow{OQ})$ and $\vec{v}_B = (\overrightarrow{OP})$.

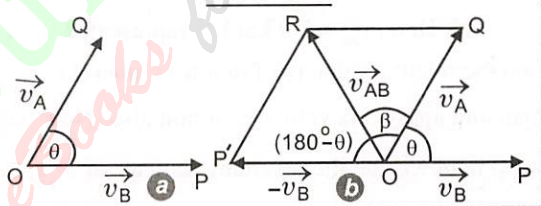


FIGURE 4.26

To find the relative velocity of object A w.r.t. B, superimpose velocity $-\vec{v}_B (= \overrightarrow{OP'})$ on both the objects A and B. The object B is brought to rest and object A possesses two velocities $\vec{v}_A$ along $\overrightarrow{OQ}$ and $-\vec{v}_B$ along OP', inclined at an angle (180° − θ). Fig. 4.26(*b*). The relative velocity of object A w.r.t. B is the resultant of velocities $\vec{v}_A$ and $-\vec{v}_B$ acting at an angle (180° − θ), which will be represented by the diagonal $\overrightarrow{OR}$ of the parallelogram $OQRP'$, according to parallelogram law of vectors addition.

In magnitude, the relative velocity of A w.r.t B is given by

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180^\circ - \theta)} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

If $\vec{v}_{AB}$ makes an angle β with the direction of $\vec{v}_A$ then

$$\tan\beta = \frac{v_B \sin(180^\circ - \theta)}{v_A + v_B \cos(180^\circ - \theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

4.16. RELATIVE VELOCITY OF RAIN W.R.T. THE MOVING MAN

Consider a man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$, Fig. 4.27. To find the relative velocity of rain w.r.t. man (*i.e.*, $\vec{v}_{rm}$), bring the man to rest by imposing a velocity $-\vec{v}_m$ on man and apply this velocity on rain too.

Now the relative velocity of rain w.r.t. man will be the resultant velocity of $\vec{v}_r(=\overrightarrow{OB})$ and $-\vec{v}_m(=\overrightarrow{OC})$, which will be represented by diagonal $\overrightarrow{OD}$ of rectangle $OBDC$.

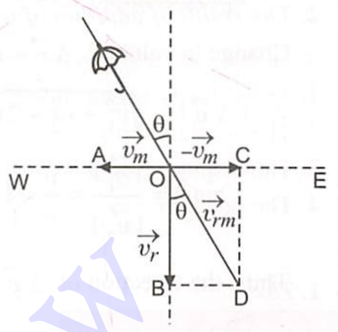
$$\therefore\ v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction, then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Here, angle θ is from vertical towards west and is written as θ, west of vertical.

FIGURE 4.27



IMPORTANT NOTE

In the above problem, if ***the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain w.r.t. man*** *i.e.* the umbrella should be held making an angle $\theta\ (= \tan^{-1} v_m/v_r)$ west of vertical.

Sample Problem **A man is walking on a level road at a speed of 3·0 km h^{-1}. Rain drops fall vertically with a speed of 4·0 km h^{-1}. Find the velocity of the raindrops with respect to the man. In which direction, the man should hold his umbrella to protect himself from rain ?**

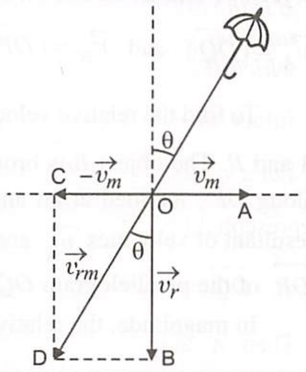
Sol. Here, $v_m = 3{\cdot}0$ km h^{-1} represented by $\overrightarrow{OA}$; $v_r = 4{\cdot}0$ km h^{-1} represented by $\overrightarrow{OB}$. Fig. 4.28. To find the relative velocity of rain w.r.t. man (*i.e.*, $\vec{v}_{rm}$) bring the man at rest by imposing a velocity $-\vec{v}_m$ on man and apply this velocity on rain also. Now the relative velocity of rain w.r.t. man will be the resultant velocity of $\vec{v}_r(=\overrightarrow{OB})$ and $-\vec{v}_m(=\overrightarrow{OC})$, which will be represented by diagonal $\overrightarrow{OD}$ of rectangle $OBDC$.

$$\therefore\quad v_{rm} = \sqrt{v_r^2 + v_m^2} = \sqrt{4^2+3^2} = \mathbf{5\ km\ h^{-1}}$$

$$\tan\theta = \frac{BD}{OB} = \frac{3}{4} = 0{\cdot}75 = \tan 36^\circ 52',$$

i.e., **θ = 36° 52′ with the vertical in forward direction.**

FIGURE 4.28



IMPORTANT NOTE

If the man wants to protect himself from the rain in the above example, he should hold his umbrella making an angle 36° 52′ with the vertical in forward direction i.e. the direction of relative velocity of rain w.r.t. man.

4.17. ZERO VECTOR OR NULL VECTOR

It is that vector which has zero magnitude and an arbitrary direction. A zero vector is represented by $\vec{0}$ (arrow over the number zero). It is also called null vector.

A zero vector being of zero magnitude must be represented by arrow of zero length. The direction of arrow head of a zero vector represented by zero length cannot be specified, hence the zero vector has an arbitrary direction.

Illustrations of zero vector

1. *When a vector is multiplied by zero, the result is a zero vector.* i.e. $0(\vec{A}) = \vec{0}$

2. *The result of addition of a vector to its own negative vector is a zero vector.* i.e. $\vec{A} + (-\vec{A}) = \vec{0}$

Examples of zero vector

1. The velocity vector of a stationary particle is a zero vector.
2. The acceleration vector of an object moving with a uniform velocity is a zero vector.
3. The displacement vector of a stationary object over any arbitrary interval of time is a zero vector.
4. The position vector of the origin of coordinate axes is a zero vector.

Properties of zero vector

1. ***The addition or subtraction of zero vector from a given vector does not alter the given vector.*** **Thus,**

$$\vec{A} + \vec{0} = \vec{A} \quad ; \quad \vec{A} - \vec{0} = \vec{A}$$

2. ***The multiplication of a non-zero real number n with a zero vector is again zero vector.*** **Thus,**

$$n\,\vec{0} = \vec{0}$$

3. ***If n_1 and n_2 are two different non zero real numbers, where $n_1 \neq n_2$, then the relation.***

$$n_1 \vec{A} = n_2 \vec{B}$$

can hold only, if both $\vec{A}$ and $\vec{B}$ are zero vectors, and are not parallel or antiparallel vectors.

A zero vector has a lot of physical significance. It is useful in describing the physical situation involving vector quantities.

4.18. RESOLUTION OF A VECTOR

> ***It is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the given single vector is splitted are called component vectors.***

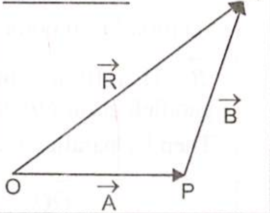
Infact, the resolution of a vector is just opposite to composition of vectors.

Let a vector $\vec{R}$ be represented by $\overrightarrow{OQ}$, Fig. 4.29. Let $\vec{A}$ and $\vec{B}$ be represented by $\overrightarrow{OP}$ and $\overrightarrow{PQ}$, so that

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} \quad \text{or} \quad \vec{R} = \vec{A} + \vec{B}$$

Then $\vec{A}$ and $\vec{B}$ are the component vectors of $\vec{R}$.

FIGURE 4.29



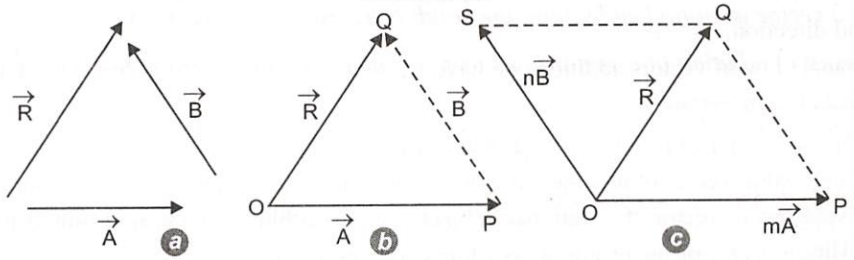
4.19. RESOLUTION OF A VECTOR INTO TWO COMPONENT VECTORS ALONG GIVEN DIRECTIONS

Suppose the given vector $\vec{R}$ has to be resolved into two component vectors so that one component vector is along the direction of $\vec{A}$ and other component vector is along the direction of $\vec{B}$, Fig. 4.30(*a*).

Draw $\overrightarrow{OQ}$ to represent $\vec{R}$ in magnitude and direction. From point *O*, draw a line parallel to the vector $\vec{A}$ and from *Q*, draw a line parallel to the vector $\vec{B}$. Let these two lines intersect at point *P*, Fig. 4.30(*b*). From triangle law of vectors addition,

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} \qquad \text{...(15)}$$

FIGURE 4.30



Therefore, $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ are the two component vectors of $\vec{R}$ in the direction of $\vec{A}$ and $\vec{B}$.

Let $\overrightarrow{OP} = m\vec{A}$ and $\overrightarrow{PQ} = n\vec{B}$, where m and n are real numbers. Therefore, the equation (15), can be written as

$$\vec{R} = m\vec{A} + n\vec{B} \qquad \text{...(16)}$$

Uniqueness of component vectors. ***It is to be noted that the resolution of vector $\vec{R}$ into two component vectors along the direction of vectors $\vec{A}$ and $\vec{B}$ is unique.*** It means, there is only one pair of real numbers m and n which satisfy the equation (16). To prove it, suppose m' and n' is another pair of real numbers such that

$$\vec{R} = m'\vec{A} + n'\vec{B} \qquad \text{...(17)}$$

From (16) and (17), we have $m\vec{A} + n\vec{B} = m'\vec{A} + n'\vec{B}$ or $(m - m')\vec{A} + (n - n')\vec{B} = \vec{0}$

Now $\vec{A} \neq \vec{0}$ and $\vec{B} \neq \vec{0}$, therefore $(m - m') = 0$ and $(n - n') = 0$. It means $m = m'$ and $n = n'$. Hence ***there is one and only one way in which a vector can be resolved into two component vectors along the directions of two given vectors.***

IMPORTANT NOTE

The resolution of vector $\vec{R}$ as given by (16) can also be obtained by following parallelogram law of vectors. From point O, draw two lines ; one parallel to the vector $\vec{A}$ and other parallel to the vector $\vec{B}$. Then from point Q, draw a line QS parallel to $\vec{A}$ and line QP parallel to vector $\vec{B}$ so that the parallelogram $OPQS$ is formed, Fig. 4.30(c).

Then by parallelogram law of vectors addition

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{OS} \quad \text{or} \quad \vec{R} = m\vec{A} + n\vec{B}$$ where m and n are real numbers.

4.20. RECTANGULAR COMPONENTS OF A VECTOR IN A PLANE

When a vector in a plane is splitted into two component vectors at right angles to each other, the component vectors are called rectangular components of that vector.

Suppose a vector $\vec{A}$ represented by $\overrightarrow{OR}$, has to be resolved into two rectangular component vectors along the directions of X-axis and Y-axis. From O, draw two axes OX and OY at rt. angles to each other. Fig. 4.31. From point R, drop RP and RQ perpendiculars on OX and OY. Then, $\overrightarrow{OP}$ $(= \vec{A}_x)$ and $\overrightarrow{OQ}$ $(= \vec{A}_y)$ are the rectangular components of $\vec{A}$. Here PR is equal and parallel to OQ, hence $\overrightarrow{PR}$ also represents $\vec{A}_y$ in magnitude and direction.

From triangle law of vectors addition, we have

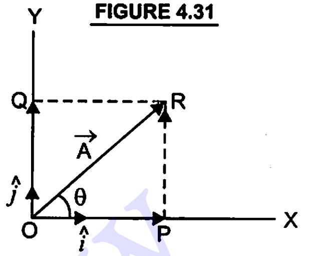


FIGURE 4.31

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} \quad \text{or} \quad \vec{A} = \vec{A}_x + \vec{A}_y \qquad \text{...(18)}$$

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along OX and OY directions respectively. Then $\vec{A}_x = A_x\,\hat{i}$ and $\vec{A}_y = A_y\,\hat{j}$.

From (18), we get $$\vec{A} = \vec{A}_x\,\hat{i} + A_y\,\hat{j} \qquad \text{...(19)}$$

Let $\angle POR = \theta$. In Fig. 4.31 ; $A_x = OP$ and $A_y = OQ = PR$

In rt. angled triangle ORP, $\dfrac{OP}{OR} = \cos\theta$

$$\therefore \quad OP = OR\cos\theta \quad \text{or} \quad A_x = A\cos\theta \qquad \text{...(20)}$$

and $\dfrac{PR}{OR} = \sin\theta$

$$\therefore \quad PR = OR\sin\theta \quad \text{or} \quad A_y = A\sin\theta \qquad \text{...(21)}$$

Also, $$OR^2 = OP^2 + PR^2 \quad \text{or} \quad A^2 = A_x^2 + A_y^2 \quad \text{or} \quad A = \sqrt{A_x^2 + A_y^2} \qquad \text{...(22)}$$

and $$\tan\theta = \frac{PR}{OP} = \frac{A_y}{A_x} \qquad \text{...(23)}$$

Sample Problem **A mass of 2 kg lies on a plane making an angle 30° to the horizontal. Resolve its weight along and perpendicular to the plane. Assume $g = 10\ \text{ms}^{-2}$.**

Sol. In Fig. 4.32, the component of weight along the plane

$= mg\sin\theta = 2 \times 10 \times \sin 30° = \mathbf{10\ N}$.

The component of weight perpendicular to plane

$= mg\cos 30° = 2 \times 10 \times \sqrt{3}/2 = \mathbf{17{\cdot}3\ N}$.

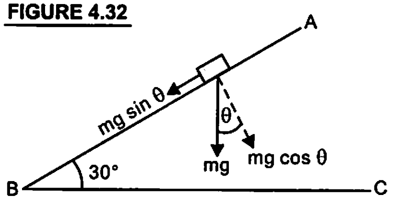


FIGURE 4.32

Curiosity Question

Q. How does the knowledge of resolution of vectors help the weight lifters in weight lifting ?

Ans. A weight lifter can lift a heavy weight easily if the vertical force he applies to lift the load is more than the weight to be lifted. It can be attained, if the lifter keeps the grips of his hands near the middle on the bar having the weights at the ends. In such a situation, the force exerted while lifting is almost perpendicular to the bar, so that the vertical component of force applied has maximum value.

If the lifter keeps the grips of his hands on the bar near the weights to be lifted, the arms of the lifter will be inclined to his body at some angle. Then for the force applied by lifter, the vertical component of the force used for lifting the weights becomes less than maximum, which makes the lifting of load difficult.

4.21. RECTANGULAR COMPONENTS OF A VECTOR IN THREE DIMENSIONS

Consider a vector $\vec{A}$ represented by $\overrightarrow{OR}$, as shown in Fig. 4.33. Taking O as origin, construct a rectangular parallelopiped with its three edges along the three rectangular axes *i.e.* X, Y and Z axes. Here, we note that $\vec{A}$ represents the diagonal of the parallelopiped whose intercepts along X, Y and Z axes are $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ respectively ; which are three rectangular components of $\vec{A}$.

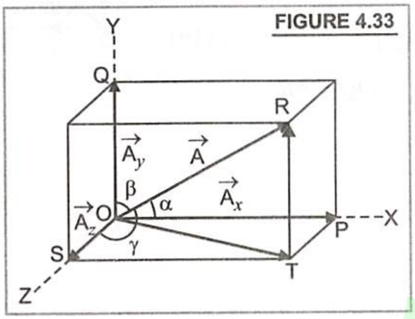


FIGURE 4.33

Using triangle law of vectors addition, $(\overrightarrow{OR}) = (\overrightarrow{OT}) + (\overrightarrow{TR})$

Using parallelogram law of vectors addition,

$$(\overrightarrow{OT}) = (\overrightarrow{OS}) + (\overrightarrow{OP})$$

$$\therefore \quad (\overrightarrow{OR}) = (\overrightarrow{OS}) + (\overrightarrow{OP}) + (\overrightarrow{TR}) = (\overrightarrow{OS}) + (\overrightarrow{OP}) + (\overrightarrow{OQ})$$

$$[\because \overrightarrow{TR} = \overrightarrow{OQ}]$$

or $$\vec{A} = \vec{A}_z + \vec{A}_x + \vec{A}_y = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

Let $\hat{i}$, $\hat{j}$, $\hat{k}$ be the unit vectors along X, Y and Z axes respectively,

then $$\vec{A}_x = A_x\,\hat{i}\,;\ \vec{A}_y = A_y\,\hat{j} \quad \text{and} \quad \vec{A}_z = A_z\,\hat{k}$$

Therefore, $$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k} \qquad \text{...(24)}$$

DO YOU KNOW ?

1. A vector can have infinite component vectors but the maximum number of rectangular component vectors are three.

2. The minimum number of vectors to give zero resultant is 2 in one plane.

3. The minimum number of vectors of unequal magnitude which can give zero resultant is three if they are coplanar.

4. The vector subtraction does not follow commutative and associative laws.

5. If the magnitude of sum of two vectors is equal to magnitude of difference of two vectors, the angle between these two vectors is 90°.

6. If $\vec{A} + \vec{B} = \vec{A} - \vec{B}$, then $\vec{B}$ is a null vector.

7. The components of a vector, perpendicular to a vector are always zero.

8. A vector cannot be zero if any of its components is not zero.

9. $(\hat{i} + \hat{j} + \hat{k})$ makes an angle 54·74° with each of X, Y and Z axes.

Magnitude of $\vec{A}$

In $\Delta\, ORT$, $(OR)^2 = (OT)^2 + (TR)^2 = (OP)^2 + (OS)^2 + (TR)^2 = (OP)^2 + (OS)^2 + (OQ)^2$

or $$A^2 = A_x^2 + A_y^2 + A_z^2 \qquad \text{...(25)}$$

or $$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad \text{...(26)}$$

Direction cosines of a vector

If α, β and γ are the angles which the $\vec{A}$ makes with X, Y and Z axes respectively, then

$$\cos\alpha = \frac{A_x}{A} \quad \text{or} \quad A_x = A\cos\alpha$$

$$\cos\beta = \frac{A_y}{A} \qquad \text{or} \qquad A_y = A\cos\beta$$

$$\cos\gamma = \frac{A_z}{A} \qquad \text{or} \qquad A_z = A\cos\gamma$$

Here, $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the **direction cosines** of the vector $\vec{A}$.

Putting the values of A_x, A_y and A_z in (25), we get

$$A^2 = A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma$$

or $$A^2 = A^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

or $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad ...(27)$$

It means ***sum of the squares of the direction cosines of a vector is always unity.***

If l, m and n are direction cosines of $\vec{A}$, then $l^2 + m^2 + n^2 = 1$

4.22. POSITION VECTOR IN SPACE

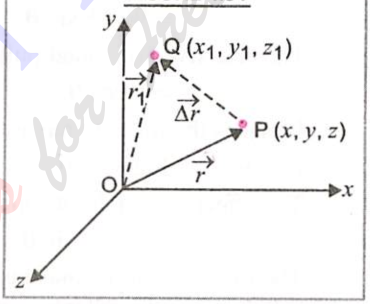
Consider a particle in a space. Let at an instant t, its position be P whose position coordinates from three mutually perpendicular axes with origin at O be (x, y, z), Fig. 4.34. The position vector of the particle at P is given by

$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

Magnitude of $\vec{r}$ is given by

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} \qquad ...(28)$$

FIGURE 4.34



Displacement vector in space. Let the particle be at $P(x, y, z)$ at time t and let it reach point $Q(x_1, y_1, z_1)$ at time t_1, Fig. 4.34. The position vector of particle at P is

$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

The position vector of particle at Q is

$$\vec{r_1} = x_1\,\hat{i} + y_1\,\hat{j} + z_1\,\hat{k}$$

The position vector of the particle in the time interval $(t_1 - t)$, (*i.e.*, displacement) is given by,

$$(\overrightarrow{PQ}) = \Delta\vec{r} = \vec{r_1} - \vec{r} = (x_1\,\hat{i} + y_1\,\hat{j} + z_1\,\hat{k}) - (x\,\hat{i} + y\,\hat{j} + z\,\hat{k})$$

$$= (x_1 - x)\,\hat{i} + (y_1 - y)\,\hat{j} + (z_1 - z)\,\hat{k}$$

Magnitude of displacement vector $\Delta\vec{r}$ is

$$|\Delta\vec{r}| = [(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2]^{1/2} \qquad ...(29)$$

Sample Problem **If $\vec{A} = 2\,\hat{i} + 4\,\hat{j} - 5\,\hat{k}$, find**

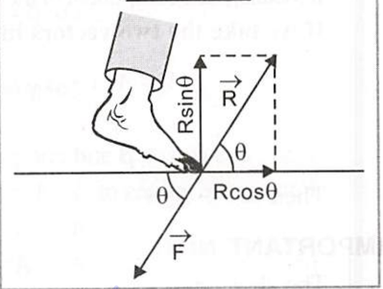
(i) $|\vec{A}|$ and (ii) the direction cosines of the vector $\vec{A}$.

Sol. (*i*) $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2)^2 + (4)^2 + (-5)^2} = \sqrt{45}$

(*ii*) $\cos\alpha = \frac{A_x}{A} = \frac{2}{\sqrt{45}}$; $\cos\beta = \frac{A_y}{A} = \frac{4}{\sqrt{45}}$ and $\cos\gamma = \frac{A_z}{A} = \frac{-5}{\sqrt{45}}$

4.23. APPLICATIONS OF RESOLUTION OF VECTORS

FIGURE 4.35



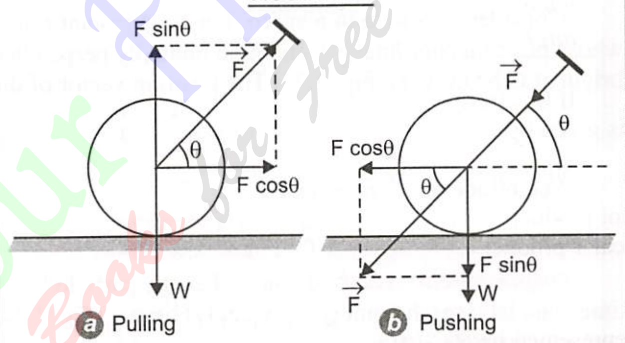
(1) Walking of a man is an example of resolution of forces. A man while walking presses the ground with his feet backward by a force F at an angle θ with ground, in action. The ground in reaction exerts an equal and opposite force R (= F) on the feet as shown in Fig. 4.35. R is resolved into two rectangular components.

(*i*) Vertical component, $R \sin\theta$ balances the weight of the man and

(*ii*) The horizontal component, $R \cos\theta$ helps the man to walk forward, if it is greater than the force of friction.

(2) It is easier to pull a lawn roller than to push it. Consider a lawn roller of weight W. Let it be pulled or pushed by a force $\vec{F}$ making an angle θ with the horizontal direction. Resolving $\vec{F}$ into two rectangular components, we get horizontal component $F \cos\theta$ and vertical component $F \sin\theta$.

FIGURE 4.36



(a) Pulling (b) Pushing

In pulling. Fig. 4.36(*a*)

The effective weight of roller

$= W - F \sin\theta$.

The effective horizontal pulling force

$= F \cos\theta$.

It makes the roller to move forward.

In pushing. Fig. 4.36(*b*).

The effective weight of roller

$= W + F \sin\theta$.

The effective horizontal pushing force $= F \cos\theta$. It makes the roller to move forward.

We know that, due to friction, the force required to move a body is directly proportional to the effective weight of the body. Since the effective weight of roller is less in pulling than in pushing, hence lesser force is required in pulling than in pushing. Therefore, it is easier to pull a lawn roller than to push it.

IMPORTANT NOTE

If $\theta = 0°$, then effective weight of roller $= mg$.
Pulling or pusing force $= F = \mu\, mg$; where μ is the coefficient of friction.
Then pulling and pushing will be no different.

4.24. VECTORS ADDITION, WHEN VECTORS ARE IN TERMS OF RECTANGULAR COMPONENTS

Consider two vectors $\vec{A}$ and $\vec{B}$ in x-y plane. Let $\vec{A} = A_x \hat{i} + A_y \hat{j}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j}$

If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, then

$$\vec{R} = \vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

If $\vec{R}_x$ and $\vec{R}_y$ are the rectangular component vectors of $\vec{R}$ in xy plane, then

$$\vec{R} = R_x \hat{i} + R_y \hat{j} \quad \therefore \quad R_x \hat{i} + R_y \hat{j} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Hence, $R_x = (A_x + B_x)$ and $R_y = (A_y + B_y)$

It means, each component of the resultant vector $\vec{R}$ is the sum of the corresponding components of $\vec{A}$ and $\vec{B}$. **If we take the two vectors in three dimensions,** then

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}\,, \quad \vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

$$\vec{R} = \vec{A} + \vec{B} \quad \text{or} \quad R_x\,\hat{i} + R_y\,\hat{j} + R_z\,\hat{k} = (A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}) + (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k})$$

Then $\quad R_x = A_x + B_x \quad ; \quad R_y = A_y + B_y \quad ; \quad R_z = A_z + B_z$

IMPORTANT NOTE

The above method can be extended to addition and subtraction of any number of vectors.

4.25. DOT PRODUCT OR SCALAR PRODUCT OF TWO VECTORS

***The dot product of two vectors* $\vec{A}$ and $\vec{B}$, represented by $\vec{A}\cdot\vec{B}$ (read as $\vec{A}$ dot $\vec{B}$) is a scalar, which is equal to the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the cosine of the smaller angle between them.**

If θ is the smaller angle between $\vec{A}$ and $\vec{B}$, then $\quad \vec{A}\cdot\vec{B} = AB\cos\theta$ **...(30)**

Since A, B and cos θ are scalars, therefore, the dot product of $\vec{A}$ and $\vec{B}$ is a scalar quantity. That is why dot product of two vectors is also called **scalar product.** Each vector, $\vec{A}$ and $\vec{B}$ has a direction, but their scalar product does not have a direction.

Geometrical interpretation of dot product of two vectors. Let $\vec{A}$ be represented by $\overrightarrow{OP}$ and $\vec{B}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$.

Draw $QR \perp OP$. In Fig. 4.37(a), $\quad \therefore\ OR = B\cos\theta$; is the projection of $\vec{B}$ onto $\vec{A}$.

Draw $PS \perp OQ$. In Fig. 4.37(b), $\quad \therefore\ OS = A\cos\theta$; is the projection of $\vec{A}$ onto $\vec{B}$.

As $\vec{A}\cdot\vec{B} = A(B\cos\theta) = B(A\cos\theta)$, therefore $\vec{A}\cdot\vec{B} = A\,(B\cos\theta)$ is the product of magnitude of $\vec{A}$ and magnitude of component of $\vec{B}$ along $\vec{A}$. Alternatively, $\vec{A}\cdot\vec{B} = B\,(A\cos\theta)$ is the product of magnitude of $\vec{B}$ and the magnitude of component of $\vec{A}$ along $\vec{B}$. Thus,

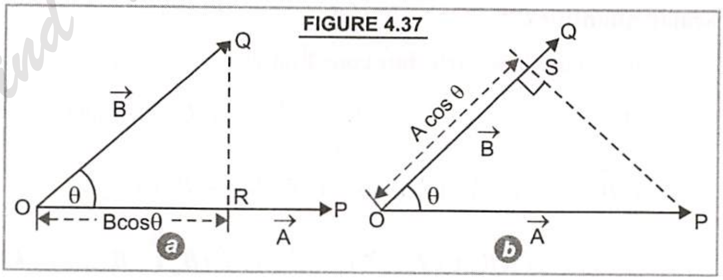


FIGURE 4.37

***Dot product of two vectors* is also defined as the product of the magnitude of one vector and the magnitude of the component of other vector in the direction of first vector.**

Some special cases :

(*i*) ***When two vectors are parallel,*** then θ = 0°, cos 0° = 1, $\vec{A}\cdot\vec{B} = AB\cos 0^\circ = AB\,(1) = AB$

For unit vectors, $\quad \hat{i}\cdot\hat{i} = 1,\ \ \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

(ii) When two vectors are mutually perpendicular, then $\theta = 90^\circ$, $\cos 90^\circ = 0$, therefore

$$\vec{A}\cdot\vec{B} = AB\cos 90^\circ = 0$$

*It means the **dot product of two perpendicular vectors is zero.***

For unit vectors, $\boxed{\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0}$

(iii) When two vectors are antiparallel, then $\theta = 180^\circ$, $\cos 180^\circ = -1$ $\therefore$ $\vec{A}\cdot\vec{B} = AB\cos 180^\circ = -AB$

Properties of Dot Product of Two Vectors

(1) Dot product or scalar product of two vectors is commutative, *i.e.,* $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

By definition, $\vec{A}\cdot\vec{B} = AB\cos\theta$ and $\vec{B}\cdot\vec{A} = BA\cos\theta = AB\cos\theta$, therefore $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$, which proves that the dot product or scalar product of two vectors is commutative.

(2) Dot product or scalar product is distributive, *i.e.,* $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

(3) Dot product of a vector with itself gives square of its magnitude, *i.e.,*

$$\vec{A}\cdot\vec{A} = A\,A\cos 0^\circ = A^2 \qquad \text{or} \qquad A = (\vec{A}\cdot\vec{A})^{1/2}$$

(4) Further, $\vec{A}\cdot(\lambda\vec{B}) = \lambda(\vec{A}\cdot\vec{B})$ where λ is a real number.

Examples of scalar product or dot product of two vectors

1. **Work done** W is defined as the dot product of force vector $\vec{F}$ and the displacement vector $\vec{S}$, *i.e.,*

$$W = \vec{F}\cdot\vec{S}$$

2. **The instantaneous power** is defined as the dot product of force vector $\vec{F}$ and the instantaneous velocity $\vec{v}$, *i.e.,* $$P = \vec{F}\cdot\vec{v}$$

3. **Magnetic flux** ϕ linked with a surface is defined as the dot product of magnetic field induction vector $\vec{B}$ and the area vector $\vec{A}$, *i.e.,* $$\phi = \vec{B}\cdot\vec{A}$$

Note that since dot product of two vectors is a scalar, hence, **work done, power** and **magnetic flux** are **scalar quantities.**

Dot product in cartesian coordinates

Let $\vec{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$= A_x\hat{i}\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})+A_y\hat{j}\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})+A_z\hat{k}\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$= A_xB_x(\hat{i}\cdot\hat{i})+A_xB_y(\hat{i}\cdot\hat{j})+A_xB_z(\hat{i}\cdot\hat{k})+A_yB_x(\hat{j}\cdot\hat{i})+A_yB_y(\hat{j}\cdot\hat{j})+A_yB_z(\hat{j}\cdot\hat{k})$$
$$+A_zB_x(\hat{k}\cdot\hat{i})+A_zB_y(\hat{k}\cdot\hat{j})+A_zB_z(\hat{k}\cdot\hat{k})$$

$$= A_xB_x(1)+A_xB_y(0)+A_xB_z(0)+A_yB_x(0)+A_yB_y(1)+A_yB_z(0)+A_zB_x(0)+A_zB_y(0)$$
$$+A_zB_z(1)$$

$$= A_xB_x+A_yB_y+A_zB_z$$

Sample Problem 1 **A constant force $(2\hat{i}+3\hat{j}+4\hat{k})$ newton produces a displacement of $(2\hat{i}+3\hat{j}+4\hat{k})$ metre. What is the work done ?**

Sol. Work done, $W = \vec{F}\cdot\vec{S} = (2\hat{i}+3\hat{j}+4\hat{k})\cdot(2\hat{i}+3\hat{j}+4\hat{k})$

$$= 2\times 2+3\times 3+4\times 4 = 4+9+16 = \mathbf{29\ J}$$

Sample Problem 2 **If the magnitudes of two vectors are 2 and 3 and the magnitude of their scalar product is $3\sqrt{2}$, then find the angle between the vectors.**

Sol. Here, $A=2\,;\ B=3,\ \vec{A}\cdot\vec{B}=3\sqrt{2}\,;$ As $\vec{A}\cdot\vec{B} = AB\cos\theta$

So $$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{3\sqrt{2}}{2\times 3} = \frac{1}{\sqrt{2}} = \cos 45^\circ\ ;\ \theta = \mathbf{45^\circ}$$

4.26. VECTOR PRODUCT OR CROSS PRODUCT OF TWO VECTORS

> ***The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$, whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them.***

It is represented as $\vec{A}\times\vec{B}$ and is read as $\vec{A}$ cross $\vec{B}$. If θ is the smaller angle between $\vec{A}$ and $\vec{B}$, then

$$\vec{A}\times\vec{B} = \vec{C} = AB\sin\theta\,\hat{C} \qquad \textbf{...(31)}$$

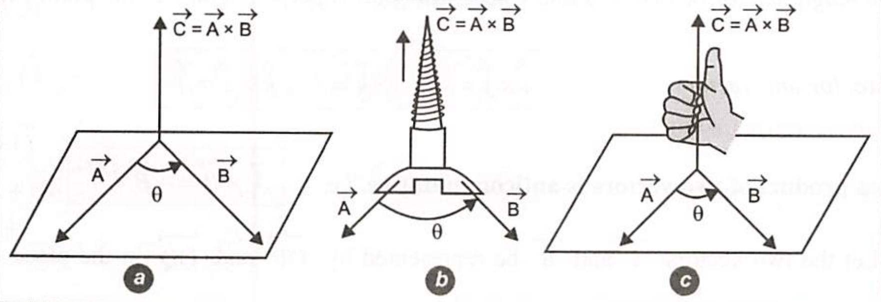
where $\hat{C}$ is a unit vector in the direction of $\vec{C}$. The direction of $\vec{C}$ or $\hat{C}$ (*i.e.* vector product of two vectors) is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and points in the direction as given by

(*i*) Right handed screw rule or (*ii*) Right hand Thumb Rule.

(i) Right handed screw rule. ***It states that if a right handed screw placed with its axis perpendicular to the plane containing the two vectors $\vec{A}$ and $\vec{B}$ is rotated from the direction of $\vec{A}$ to the direction of $\vec{B}$ through smaller angle, then the sense of the advancement of the tip of the screw gives the direction of $(\vec{A}\times\vec{B})$ or $\vec{C}$.***

If the two vectors $\vec{A}$ and $\vec{B}$ lie in the plane of paper as shown in Fig. 4.38(*a*), then the direction of cross product $\vec{A}\times\vec{B}$, according to Right handed screw rule, will be perpendicular to the plane of paper directed upwards, [Fig. 4.38(*b*)].

FIGURE 4.38



$\vec{C}=\vec{A}\times\vec{B}$
$\vec{A}$
$\vec{B}$
θ
a
$\vec{C}=\vec{A}\times\vec{B}$
$\vec{A}$
$\vec{B}$
θ
b
$\vec{C}=\vec{A}\times\vec{B}$
$\vec{A}$
$\vec{B}$
θ
c

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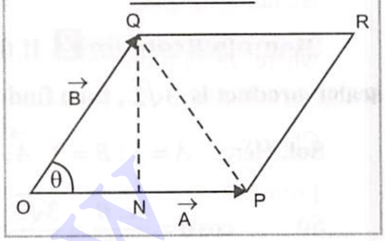
(ii) Right hand thumb rule. *It states that if we curl the fingers of right hand, keeping the thumb erect, in such a way that the fingers point in the direction of rotation from $\vec{A}$ to $\vec{B}$ through smaller angle, then the thumb points in the direction of $\vec{A}\times\vec{B}$ or $\vec{C}$*, [Fig. 4.38(c)].

Geometrical interpretation of vector product of two vectors.

FIGURE 4.39



Consider two vectors $\vec{A}$ and $\vec{B}$ represented in magnitude and direction by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ with ∠POQ = θ, Fig. 4.39. Complete the parallelogram OPRQ. Join P with Q. Drop QN ⊥ on OP. Magnitude of cross product of $\vec{A}$ and $\vec{B}$ is given by

$$|\vec{A}\times\vec{B}| = AB\sin\theta = (OP)\,(OQ\sin\theta)$$

In ΔONQ, $\sin\theta = \dfrac{NQ}{OQ}$

∴ $NQ = OQ\sin\theta$

∴ $|\vec{A}\times\vec{B}| = (OP)\,(NQ) =$ Area of parallelogram OPRQ ...(32)

Also $|\vec{A}\times\vec{B}| = 2\dfrac{[(OP)\,(NQ)]}{2} = 2$ [area of ΔOQP] ...(33)

Thus ***magnitude of vector product of two vectors***

(i) is equal to the area of the parallelogram whose two sides are represented by two vectors.

(ii) is equal to twice the area of a triangle whose two sides are represented by the two vectors.

This is the geometrical interpretation of vector product of two vectors.

Some special cases of cross product :

(i) **When vectors are parallel or antiparallel,** θ = 0° or 180° and sin θ = 0

∴ $$|\vec{A}\times\vec{B}| = AB\,(0)\,\hat{C} = \hat{0}$$

For unit vectors : $\boxed{\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \hat{0}}$

It means *cross product of two parallel or antiparallel vectors is zero.*

(ii) **When vectors are perpendicular,** θ = 90° and sin θ = sin 90° = 1

∴ $$\vec{A}\times\vec{B} = AB\,(1)\,\hat{C} = AB\,\hat{C}$$

i.e., cross product of two mutually perpendicular vectors is a vector whose magnitude is equal to product of the magnitude of two vectors and whose direction is perpendicular to the plane containing two vectors.

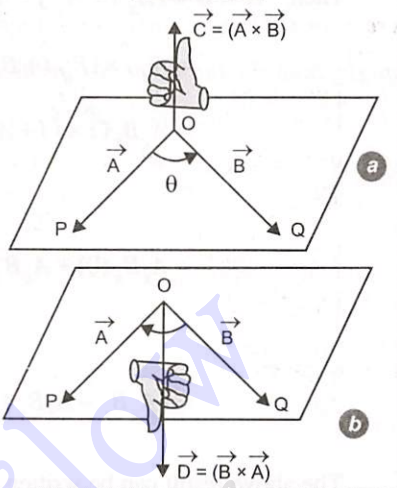
Therefore, *for unit vectors :* $\boxed{\hat{i}\times\hat{j} = \hat{k}\ ;\ \hat{j}\times\hat{k} = \hat{i}\ ;\ \hat{k}\times\hat{i} = \hat{j}}$

Properties of cross product

(1) Cross product of two vectors is anticommutative. *i.e.* $\boxed{\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}}$ ...(34)

Proof. Let the two vectors $\vec{A}$ and $\vec{B}$ be represented by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in the plane of paper and ∠POQ = θ.

FIGURE 4.40


$\vec{C} = (\vec{A} \times \vec{B})$
O
$\vec{A}$
$\vec{B}$
θ
P
Q
a
O
$\vec{A}$
$\vec{B}$
P
Q
b
$\vec{D} = (\vec{B} \times \vec{A})$

Now $\quad \vec{A} \times \vec{B} = \vec{C} = AB \sin\theta\, \hat{C}$ ...(35)

where $\hat{C}$ is a unit vector, perpendicular to the plane of paper *directed upwards,* Fig. 4.40(*a*).

Again, $\quad \vec{B} \times \vec{A} = \vec{D} = BA \sin\theta\, \hat{D}$ ...(36)

where $\hat{D}$ is a unit vector perpendicular to the plane of paper *directed downwards,* Fig. 4.40(*b*).

Clearly, $\quad \hat{C} = -\hat{D} \quad$ or $\quad \hat{D} = -\hat{C}$

From (35), $\quad \vec{A} \times \vec{B} = AB \sin\theta\, \hat{C}$

$$= AB \sin\theta\, (-\hat{D})$$

$$= -BA \sin\theta\, \hat{D} = -(\vec{B} \times \vec{A})$$

Thus, $\quad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Hence, on reversing the order of vectors, the sign of their cross product is reversed, *i.e., the cross product of two vectors is anticommutative.*

We have learnt that $\quad \hat{i} \times \hat{j} = \hat{k}\ ; \quad \hat{j} \times \hat{k} = \hat{i} \quad$ and $\quad \hat{k} \times \hat{i} = \hat{j}$

As cross product is anticommutative, therefore, $\quad \hat{j} \times \hat{i} = -\hat{k}\ ; \ \hat{k} \times \hat{j} = -\hat{i} \quad$ and $\quad \hat{i} \times \hat{k} = -\hat{j}$

(2) Cross product of two vectors is distributive, *i.e.,* $\quad \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(3) Cross product of two vectors is associative, *i.e.,*

$$(\vec{A} + \vec{B}) \times (\vec{C} + \vec{D}) = \vec{A} \times \vec{C} + \vec{A} \times \vec{D} + \vec{B} \times \vec{C} + \vec{B} \times \vec{D}$$

Examples of cross product of two vectors

1. **Torque** $\vec{\tau}$ acting on a particle is equal to the vector product or cross product of its position vector $(\vec{r})$ and force vector $(\vec{F})$, *i.e.,* $\quad \vec{\tau} = \vec{r} \times \vec{F}$

2. **Angular Momentum** $\vec{L}$ of a particle is equal to the vector product or cross product of its position vector $\vec{r}$ and its linear momentum $\vec{p}$ *i.e.* $\quad \vec{L} = \vec{r} \times \vec{p}$

3. **Linear velocity** $\vec{v}$ of a particle in rotational motion is equal to the vector product of its angular velocity $\vec{\omega}$ and its displacement vector $\vec{r}$ *i.e.* $\quad \vec{v} = \vec{\omega} \times \vec{r}$

4. **Tangential acceleration** $\vec{a}$ of a particle in rotational motion is equal to cross product of its angular acceleration vector $\vec{\alpha}$ and its position vector $\vec{r}$ *i.e.* $\quad \vec{a} = \vec{\alpha} \times \vec{r}$

5. **Centripetal acceleration** $\vec{a}_c$ of a particle is equal to the cross product of its angular velocity vector $\vec{\omega}$ and its linear velocity vector $\vec{v}$ *i.e.* $\quad \vec{a}_c = \vec{\omega} \times \vec{v}$

Cross product in Cartesian Coordinates

Let $\quad \vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k} \quad$ and $\quad \vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$

Then, $\vec{A}\times\vec{B}=(A_x\,\hat{i}+A_y\,\hat{j}+A_z\,\hat{k})\times(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k})$

$$=A_x\,\hat{i}\times(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k})+A_y\,\hat{j}\times(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k})+A_z\,\hat{k}\times(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k})$$

$$=A_xB_x(\hat{i}\times\hat{i})+A_xB_y(\hat{i}\times\hat{j})+A_xB_z(\hat{i}\times\hat{k})+A_yB_x(\hat{j}\times\hat{i})+A_yB_y(\hat{j}\times\hat{j})+A_yB_z(\hat{j}\times\hat{k})$$

$$+A_zB_x(\hat{k}\times\hat{i})+A_zB_y(\hat{k}\times\hat{j})+A_zB_z(\hat{k}\times\hat{k})$$

$$=A_xB_x(0)+A_xB_y(\hat{k})+A_xB_z(-\hat{j})+A_yB_x(-\hat{k})+A_yB_y(0)+A_yB_z(\hat{i})$$

$$+A_zB_x(\hat{j})+A_zB_y(-\hat{i})+A_zB_z(0)$$

$$=(A_yB_z-A_zB_y)\,\hat{i}+(A_zB_x-A_xB_z)\,\hat{j}+(A_xB_y-A_yB_x)\,\hat{k}$$

The above result can be written in the form of a determinant as : $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$

DO YOU KNOW ?

1. A unit vector $(\hat{n})$ perpendicular to $\vec{A}$ as well as $\vec{B}$ is $\hat{n}=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

2. The cross product of two vectors $\vec{A}$ and $\vec{B}$ does not change sign *under reflection*. This is because on reflection, $\vec{A}$ changes into $-\vec{A}$ and $\vec{B}$ changes into $-\vec{B}$. Therefore,

$$\vec{A}\times\vec{B}\xrightarrow[\text{reflection}]{\text{under}}(-\vec{A})\times(-\vec{B})=\vec{A}\times\vec{B}$$

3. Volume (*V*) of the parallelopiped whose three sides are represented by the three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as its adjacent edges is given by scalar triple product of three vectors which is a scalar, *i.e.*,

$$V=\vec{A}\times\vec{B}.\vec{C}=\vec{A}.\vec{B}\times\vec{C}=\vec{C}.\vec{A}\times\vec{B}\;=\text{volume of the parallelopiped.}$$

In terms of their rectangular components ; $\vec{A}.(\vec{B}\times\vec{C})=\begin{vmatrix}A_x & A_y & A_z\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix}$

4. If $\vec{A},\vec{B}$ and $\vec{C}$ are coplanar, then $\vec{A}.(\vec{B}\times\vec{C})=0$.

5. If any two of the given three vectors $\vec{A},\vec{B}$ and $\vec{C}$ are parallel, then

$$\vec{A}\times\vec{B}.\vec{C}=\vec{A}\times\vec{C}.\vec{B}=\vec{A}.(\vec{B}\times\vec{C})=0$$

6. Angle between $(\vec{A}+\vec{B})$ and $(\vec{A}\times\vec{B})$ is 90°.

7. Vector product of three vectors is as follows

$$\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}.\vec{C})\,\vec{B}-(\vec{A}.\vec{B})\,\vec{C}\quad\text{and}\quad(\vec{A}\times\vec{B})\times\vec{C}=(\vec{A}.\vec{C})\,\vec{B}-(\vec{B}.\vec{C})\,\vec{A}$$

Sample Problem **Calculate the area of the parallelogram whose two adjacent sides are formed by the vectors $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = -3\hat{i} + 6\hat{j}$.**

Sol. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ -3 & 6 & 0 \end{vmatrix} = \hat{i}\,(3\times0-6\times0)+\hat{j}\,[0\times(-3)-0\times4]+\hat{k}\,[4\times6-3\times(-3)] = 33\,\hat{k}$

Area of parallelogram $= |\vec{A}\times\vec{B}| = \sqrt{0^2+0^2+(33)^2}$ **= 33 sq. units**

4.27. MOTION WITH UNIFORM VELOCITY IN A PLANE

Consider a point object moving in XY plane with a uniform velocity $\vec{v}$. Suppose O is the origin for measuring time and position of the object. Let the object be at position A at time t_1 and at position B at time t_2, where $\overrightarrow{OA} = \vec{r}_1$ and $\overrightarrow{OB} = \vec{r}_2$, Fig. 4.41.

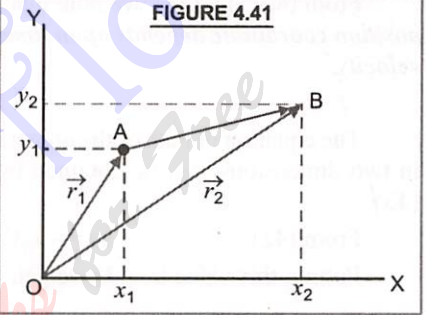


FIGURE 4.41

(a) Displacement

The displacement of the object in time interval $(t_2 - t_1)$ is $\overrightarrow{AB}$. According to triangle law of vectors :

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

or $$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1 \qquad ...(37)$$

If the position coordinates of the object at points A and B are (x_1, y_1) and (x_2, y_2), then

$$\vec{r}_1 = x_1\,\hat{i} + y_1\,\hat{j} \quad \text{and} \quad \vec{r}_2 = x_2\,\hat{i} + y_2\,\hat{j}$$

$$\therefore \quad \vec{r}_2 - \vec{r}_1 = (x_2\,\hat{i} + y_2\,\hat{j}) - (x_1\,\hat{i} + y_1\,\hat{j}) = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} \qquad ...(38)$$

$\therefore$ Displacement of object in time interval $(t_2 - t_1)$ will be

$$\overrightarrow{AB} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} \qquad ...(39)$$

Magnitude of displacement is $|\overrightarrow{AB}| = |\vec{r}_2 - \vec{r}_1| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

(b) Velocity

$$\text{Since ; velocity} = \frac{\text{displacement}}{\text{time taken}}, \quad \text{therefore} \quad \vec{v} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \qquad ...(40)$$

Since the object is moving with a uniform velocity, hence this value of velocity is the same for all values of t_1 and t_2.

(c) Equation of motion of object.

From (40), we have $$\vec{r}_2 - \vec{r}_1 = \vec{v}\,(t_2 - t_1) \quad \text{or} \quad \vec{r}_2 = \vec{r}_1 + \vec{v}\,(t_2 - t_1) \qquad ...(41)$$

Let us consider, at time $t = 0$, the object is at A where position vector with respect to O is $\vec{r}_0$ and (x_0, y_0) are the coordinates of point A. Let the object go to point B (x, y) after time t, whose position vector w.r.t. O is $\vec{r}$. Then replacing $\vec{r}_2$ by $\vec{r}$, $\vec{r}_1$ by $\vec{r}_0$, t_2 by t and t_1 by 0 in (41), we get

$$\vec{r} = \vec{r}_0 + \vec{v}\,(t - 0) = \vec{r}_0 + \vec{v}\,t \qquad ...(41)$$

Let v_x, v_y be the magnitude of the components of the velocity $\vec{v}$ of the object along x axis and y axis respectively. Then $\quad \vec{v} = v_x\,\hat{i} + v_y\,\hat{j} \quad$ where $\quad v = \sqrt{v_x^2 + v_y^2}$

Also, $\quad \vec{r}_0 = x_0\,\hat{i} + y_0\,\hat{j} \quad$ and $\quad \vec{r} = x\,\hat{i} + y\,\hat{j}$

Putting values in (41), we get

$$(x\,\hat{i} + y\,\hat{j}) = (x_0\,\hat{i} + y_0\,\hat{j}) + (v_x\,\hat{i} + v_y\,\hat{j})\,t \quad \text{or} \quad x\,\hat{i} + y\,\hat{j} = (x_0 + v_x\,t)\,\hat{i} + (y_0 + v_y t)\,\hat{j}$$

This equation is true only if the coefficients of $\hat{i}$ and $\hat{j}$ on both the sides of this equation are separately equal. Hence

$$x = x_0 + v_x\,t \qquad \text{...(42)}$$

and

$$y = y_0 + v_y\,t \qquad \text{...(43)}$$

From (42) and (43), we note that in ***two dimensional motion with uniform velocity, each rectangular position coordinate depends upon time in exactly the same way as in one dimensional motion with uniform velocity.***

(*d*) Equation of path

The equation of path of the object moving with uniform velocity in two dimensions, can be obtained by eliminating t from (42) and (43).

From (42), $\quad t = (x - x_0)/v_x$

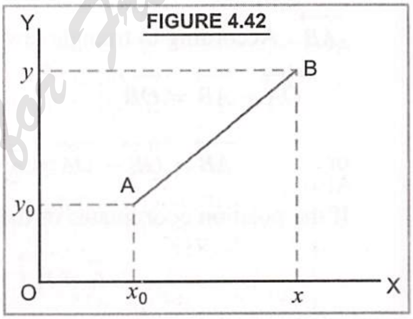
Putting this value in (43) we get,

$$y = y_0 + v_y\,(x - x_0)/v_x \qquad \text{...(44)}$$

This is the equation of a straight line, having slope v_y/v_x, which is shown by a st. line AB in Fig. 4.42.

Hence ***the path of an object moving with uniform velocity in two dimensions is a straight line.***

FIGURE 4.42



4.28. MOTION WITH UNIFORM ACCELERATION IN A PLANE

Consider a point object moving in XY plane with a uniform acceleration $\vec{a}$. Suppose O is the origin for measuring time and position of the object. Let the object be at position A at time t_1 and at B, at time t_2, where $\overrightarrow{OA} = \vec{r}_1$ and $\overrightarrow{OB} = \vec{r}_2$, Fig. 4.41.

Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the object at instants t_1 and t_2 respectively, then constant acceleration is given by

$$\vec{a} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

This value of acceleration is same for all values of t_1 and t_2. From above, we get

$$\vec{v}_2 - \vec{v}_1 = \vec{a}\,(t_2 - t_1) \quad \text{or} \quad \vec{v}_2 = \vec{v}_1 + \vec{a}\,(t_2 - t_1) \qquad \text{...(45)}$$

At time $t = 0$, let the object be at A whose position vector w.r.t. O be $\vec{r}_0$ and velocity of the object at A be $\vec{u}$. Let the object go to B after time t where its velocity becomes $\vec{v}$. Then replacing $\vec{v}_1$ by $\vec{u}$, $\vec{v}_2$ by $\vec{v}$, t_1 by 0 and t_2 by t in (45), we get,

$$\vec{v} = \vec{u} + \vec{a}\,t \qquad \text{...(46)}$$

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Equation (46) can be expressed in terms of rectangular components in *XY* plane as follows

$$\vec{u} = u_x\,\hat{i} + u_y\,\hat{j} \qquad \text{where} \quad u = \sqrt{u_x^2 + u_y^2}$$

$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} \qquad \text{where} \quad v = \sqrt{v_x^2 + v_y^2}$$

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} \qquad \text{where} \quad a = \sqrt{a_x^2 + a_y^2}$$

Putting these values in (46), we get

$$v_x\,\hat{i} + v_y\,\hat{j} = (u_x\,\hat{i} + u_y\,\hat{j}) + (a_x\,\hat{i} + a_y\,\hat{j})\,t \quad \text{or} \quad v_x\,\hat{i} + v_y\,\hat{j} = (u_x + a_x t)\,\hat{i} + (u_y + a_y t)\,\hat{j}$$

This equation is true only if the coefficients of $\hat{i}$ and $\hat{j}$ on both the sides of the equation are separately equal. Hence

$$v_x = u_x + a_x\,t \qquad \text{...(47)}$$

and

$$v_y = u_y + a_y\,t \qquad \text{...(48)}$$

These equations are similar to that of the velocity-time relation for the motion of the object with uniform acceleration in one dimension.

Expression for displacement in uniformly accelerated motion.

Average velocity during the time interval $(t_2 - t_1)$ is given by

$$\vec{v}_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \quad \text{or} \quad \vec{r}_2 = \vec{r}_1 + \vec{v}_{av}(t_2 - t_1) \qquad \text{...(49)}$$

Also, average velocity in a given interval of time is given by

$$\vec{v}_{av} = \frac{\text{initial velocity} + \text{final velocity}}{2} = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

Putting this value in (49), we get
$$\vec{r}_2 = \vec{r}_1 + \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)(t_2 - t_1)$$

Putting the value of $\vec{v}_2$ from (46) in above equation, we get

$$\vec{r}_2 = \vec{r}_1 + \frac{1}{2}\{\vec{v}_1 + \vec{v}_1 + \vec{a}\,(t_2 - t_1)\}(t_2 - t_1) \quad \text{or} \quad \vec{r}_2 = \vec{r}_1 + \vec{v}_1\,(t_2 - t_1) + \frac{1}{2}\,\vec{a}\,(t_2 - t_1)^2 \qquad \text{...(50)}$$

This is the required equation.

If $\vec{r}_0$ is the position vector of the object at time $t = 0$, where velocity of the object is $\vec{u}$, then replacing $\vec{r}_2$ by $\vec{r}$, $\vec{r}_1$ by $\vec{r}_0, \vec{v}_1$ by $\vec{u}$, t_1 by 0 and t_2 by t in (50), we get

$$\vec{r} = \vec{r}_0 + \vec{u}\,t + \frac{1}{2}\,\vec{a}\,t^2 \qquad \text{...(51)}$$

In terms of rectangular components in two dimensions, the above equation can be written as

$$x\,\hat{i} + y\,\hat{j} = (x_0\,\hat{i} + y_0\,\hat{j}) + (u_x\,\hat{i} + u_y\,\hat{j})\,t + \frac{1}{2}(a_x\,\hat{i} + a_y\,\hat{j})\,t^2$$

or

$$x\,\hat{i} + y\,\hat{j} = \left(x_0 + u_x t + \frac{1}{2}a_x t^2\right)\hat{i} + \left(y_0 + u_y t + \frac{1}{2}a_y t^2\right)\hat{j} \qquad \text{...(52)}$$

This equation is true only if the coefficients of $\hat{i}$ and $\hat{j}$ on both the sides of equation are separately equal. Hence ;

$$x = x_0 + u_x t + \frac{1}{2} a_x t^2 \qquad \text{...(53)}$$

and

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2 \qquad \text{...(54)}$$

These equations are similar to that of the position-time relation for the motion of the object with uniform acceleration in one dimension. The equation of path followed by object while moving in a plane with a uniform acceleration can be obtained by eliminating t from (53) and (54).

Sample Problem **If the position vector of a particle is given by**

$\vec{r} = (4 \cos 2t)\hat{i} + (4 \sin 2t)\hat{j} + (6t)\hat{k}$ m, calculate its acceleration at $t = \pi/4$ second.

Sol. Position, $\vec{r} = (4 \cos 2t)\,\hat{i} + (4 \sin 2t)\,\hat{j} + 6t\,\hat{k}$

Velocity, $\vec{v} = \dfrac{d\vec{r}}{dt} = [4\,(-\sin 2t)\cdot 2]\,\hat{i} + [4\,(\cos 2t)\cdot 2]\,\hat{j} + 6\,\hat{k}$

Acceleration, $\vec{a} = \dfrac{d\vec{v}}{dt} = [-8\,(\cos 2t)\cdot 2]\,\hat{i} + [8\,(-\sin 2t)\cdot 2]\,\hat{j} = (-16 \cos 2t)\,\hat{i} + (-16 \sin 2t)\,\hat{j}$

When $t = \pi/4$, $\vec{a} = (-16 \cos 2\times\pi/4)\,\hat{i} + (-16 \sin 2\pi\times\pi/4)\,\hat{j} = (-16 \cos \pi/2)\,\hat{i} + (-16 \sin \pi/2)\,\hat{j}$

$$= (-16\times 0)\,\hat{i} + (-16\times 1)\,\hat{j} = \mathbf{-16\,\hat{j}\ ms^{-2}}$$

4.29. PROJECTILE

> ***Projectile** is the name given to a body thrown with some initial velocity at some angle with the horizontal direction, and then allowed to move in two dimensions under the action of gravity alone.*

The path followed by a projectile is called its **trajectory**.

Examples of projectile are 1. A body dropped from the window of a moving train.

2. A bomb released from an aeroplane in flight.

3. A bullet fired from a rifle.

4. A piece of stone thrown in any direction.

5. A javelin or hammer thrown by an athlete.

6. A jet of water issuing from a hole near the bottom of water tank.

In the above examples, we find that a projectile moves under the combined effect of two velocities :

(i) A uniform velocity in the horizontal direction, which would not change provided there is no air resistance.

(ii) A uniformly changing velocity (i.e. increasing or decreasing) in the vertical direction due to gravity.

These two perpendicular velocities are independent of each other.

To study the motion of a projectile, we assume that

(i) there is no frictional resistance of air.

(ii) The effect due to rotation of earth and curvature of the earth is negligible.

(iii) The acceleration due to gravity is constant in magnitude and direction at all points of the motion of projectile.

4.30. HORIZONTAL PROJECTION OF A PROJECTILE

Let OX be a horizontal line parallel to ground and OY be a vertical line perpendicular to ground. Let O be the origin for X- and Y- axis and OY direction be positive for position measurement. Suppose an object is projected from the point O, above the ground with velocity u along OX such that $x_0 = 0$ and $y_0 = 0$ at $t = 0$. Soon after projection, the object moves under the combined effect of two independent perpendicular velocities.

(*i*) *Horizontal velocity* u, which remains constant if we ignore air resistance.

(*ii*) *Vertical velocity* which goes on increasing due to gravity. Initial value of this velocity at O is zero.

During the motion, the object covers horizontal distance due to horizontal velocity and vertical downward distance due to vertical velocity.

(i) Path of projectile

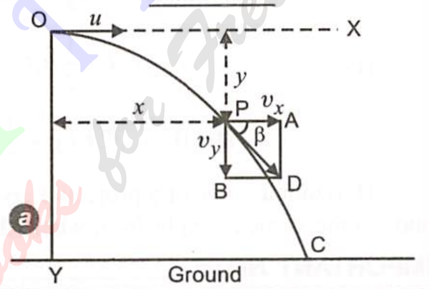
Suppose at any time t, the object reaches at $P(x, y)$. Fig. 4.43(*a*). Clearly,

x = distance travelled by object in the horizontal direction in time t.

y = distance travelled in vertical direction in time t.

Motion along horizontal direction OX of an object from O to P. Since the velocity of object in the horizontal direction is constant, so the aceleration a_x along horizontal direction is zero.

FIGURE 4.43



The position of object at any time t along the horizontal direction (*i.e.* along X-axis) is given by

$$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

Here, $x_0 = 0$, $u_x = u$ and $a_x = 0$

Putting these values in above equation, we get

$$x = 0 + ut + \frac{1}{2}(0)\,t^2 = ut$$

or $\quad t = x/u \qquad$ ...(55)

Motion along vertical direction OY of an object from O to P. Since the vertical velocity of object is increasing downwards due to gravity, hence the acceleration of the object a_y along vertical downward direction is $+ g$.

The position of object at any time t along vertical direction (*i.e.* along Y-axis) is given by

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

Here, $y_0 = 0$, $u_y = 0$ and $a_y = + g$. Putting these values in above equation, we get

$$y = 0 + (0)\,t + \frac{1}{2} g t^2 = \frac{1}{2} g t^2 \qquad \text{or} \qquad y = \frac{1}{2} g \left(\frac{x}{u}\right)^2 = \left(\frac{1}{2}\frac{g}{u^2}\right) x^2 \qquad \text{[from (55)]}$$

or $\quad \boxed{y = kx^2} \qquad$ ...(56)

(where $\frac{1}{2}\frac{g}{u^2} = k$ = a constant)

This is an equation of a parabola, which is symmetrical about Y-axis. Hence the ***path of horizontal projectile is a parabola.***

(ii) Time of flight. *It is the total time for which the projectile is in flight*

i.e., the time taken by object to go from O to C, where it strikes the ground. Let it be T. Let h be the vertical height of point of projection O from C.

Taking vertical downward motion of object from O to C (*i.e.* motion of object along OY direction), we have $\quad y_0 = 0, y = h\,; u_y = 0, a_y = g, t = T.$

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As $$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$\therefore$ $$h = 0 + 0 \times T + \frac{1}{2} g T^2 \quad \text{or} \quad \boxed{T = \sqrt{\frac{2h}{g}}}$$

IMPORTANT NOTES

1. In horizontal projection the time taken by a projectile to reach the ground is independent of the initial velocity of projection but depends upon the height of location from where the projectile is projected.

2. If two bodies of different masses are projected horizontally from the same vertical height, they reach the ground simultaneously.

(iii) Horizontal range. ***It is the horizontal distance travelled by projectile during its flight***

i.e., the horizontal distance covered by object while going from O to C. Let it be R.

Taking horizontal motion of object from O to C (*i.e.* motion of object along OX-direction), we have

$$x_0 = 0,\ x = R,\ u_x = u,\ a_x = 0,\quad t = T = \sqrt{2h/g}$$

As $$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

$\therefore$ $$R = 0 + u\sqrt{2h/g} + \frac{1}{2} \times 0 \times T^2 = u\sqrt{2h/g}$$

Horizontal range of a projectile given horizontal projection depends on (*i*) initial velocity of projection and (*ii*) the vertical height from where the projectile is projected.

IMPORTANT NOTE

Horizontal range of different projectiles given horizontal projection from the same vertical height but with different initial velocities, will be different.

(iv) Velocity of the projectile at any instant.

At the instant t, the object be at P and it possesses two perpendicular velocities.

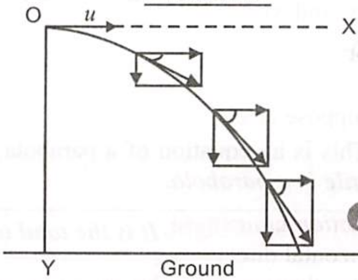
(*i*) Horizontal velocity $v_x = u$ represented by PA

(*ii*) Vertical velocity v_y, represented by PB ; which can be obtained from the relation.

$$v_y = u_y + a_y t, \text{ by putting } \quad u_y = 0,\ a_y = g\ ;$$

we have, $v_y = 0 + gt = gt$

FIGURE 4.43



The resultant velocity $\vec{v}$ is given by $\overrightarrow{PD}$ where,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2 t^2}$$

Let $\vec{v}$ make an angle β with the horizontal direction, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u}$$

or $$\beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

The velocity of the object at different locations is different and it has been shown in Fig. 4.43(*b*).

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Sample Problem **A projectile is fired horizontally with a velocity of 98 ms^{-1} from a hill 490 m high. Find (*i*) time taken to reach the ground (*ii*) the distance of the target from the hill and (*iii*) the velocity with which the body strikes the ground.**

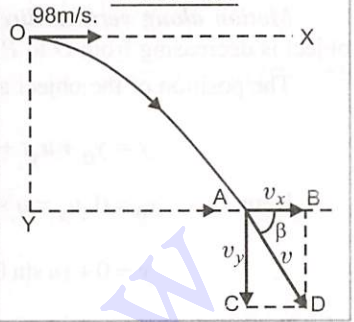


FIGURE 4.44

Sol. Here, $u = 98$ m/s, $y = 490$ m, $T = ?, x = ?, v = ?$

(*i*) Refer to Fig. 4.44, From, $y = \frac{1}{2}\,gT^2$

$$\therefore \quad 490 = \frac{1}{2} \times 9{\cdot}8 \times T^2 = 4{\cdot}9\,T^2 \quad \text{or} \quad T = \sqrt{\frac{490}{4{\cdot}9}} = \mathbf{10\,s}$$

(*ii*) From $x = uT$ $\quad\therefore\quad$ $x = 98 \times 10 = \mathbf{980\ m}$

(*iii*) Resultant velocity, $v = \sqrt{u^2 + g^2T^2} = \sqrt{98^2 + (9{\cdot}8 \times 10)^2}$

$$= \mathbf{98\sqrt{2}\ m/s}$$

If β is the angle which $\vec{v}$ makes with the horizontal direction, then

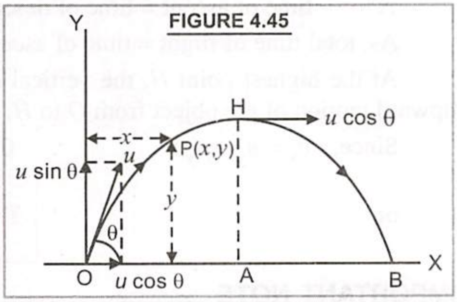
$$\tan\beta = \frac{gt}{u} = \frac{98}{98} = 1 \quad \text{or} \quad \boldsymbol{\beta = 45^\circ} \textbf{ with the horizontal.}$$

4.31. ANGULAR PROJECTION OF A PROJECTILE

Let *OX* be a horizontal line on ground and *OY* be a vertical line perpendicular to ground. *O* is the origin for *X* and *Y*-axes and *OY* direction for position measurement is taken as positive. Suppose an object is projected from the point *O* with velocity *u*, making an angle θ with the horizontal, such that $x_0 = 0$ and $y_0 = 0$ at $t = 0$.

Resolving velocity $\vec{u}$ into two rectangular components, we get (*i*) *u* cos θ, along *OX* (*ii*) *u* sin θ along *OY*. As these two component velocities act at right angles to each other, therefore they are independent of each other. The horizontal component velocity ***u* cos** θ is constant throughout the motion as there is no accelerating force in the horizontal direction and air resistance is assumed to be zero. The vertical component velocity **u sin** θ decreases continuously with height, from *O* to *H*, due to downward force of gravity and becomes zero at *H*. At point *H*, the object has only horizontal component velocity ; *u* cos θ. After this, the object moves with horizontal component velocity *u* cos θ and a continuously increasing vertical component velocity due to gravity pull and finally strikes at point *B* on *OX*. Fig. 4.45. Here *AH* is referred to as the **maximum height** attained by projectile and OB is called the **horizontal range.** The object covers horizontal distance due to its horizontal component velocity and vertical distance due to its vertical component velocity.

FIGURE 4.45



Path of projectile

Suppose at any time *t*, the object reaches at *P* (*x, y*) clearly,

x = horizontal distance travelled by object in time *t*

y = vertical distance travelled by object in time *t*.

Motion along horizontal direction OX of a projectile from O to P. Since the velocity of the object in the horizontal direction is constant, so the acceleration a_x in the horizontal direction is zero.

The position of the object at any time *t* along the horizontal direction (*i.e.* along *OX*) is given by

$$x = x_0 + u_x t + \frac{1}{2}a_x t^2$$

Here, $x_0 = 0,\ u_x = u\cos\theta,\ a_x = 0$ and $t = t$

$$x = 0 + u\cos\theta\, t + \frac{1}{2}(0)\,t^2 = (u\cos\theta\, t) \quad \text{or} \quad t = \frac{x}{v\cos\theta} \qquad ...(57)$$

Motion along vertical direction OY, of a projectile from O to P, since the vertical velocity of the object is decreasing from O to P due to gravity, hence acceleration $a_y = -g$.

The position of the object at any time t along the vertical direction is given by

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2 \qquad ...(58)$$

Here, $y_0 = 0,\ u_y = u\sin\theta,\ a_y = -g$ and $t = t$

$$\therefore \quad y = 0 + (u\sin\theta)\,t + \frac{1}{2}(-g)\,t^2 \quad \text{or} \quad y = (u\sin\theta)\,t - \frac{1}{2}gt^2 \qquad ...(59)$$

Putting the value of t from (57) in (59) we get

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2 \quad \text{or} \quad y = x\tan\theta - \left(\frac{1}{2}\frac{g}{u^2\cos^2\theta}\right)x^2 \qquad ...(60)$$

This is an equation of a parabola, hence ***the path of a projectile projected at some angle with the horizontal direction is a parabola.***

> **Time of flight.** ***It is the total time for which the object is in flight (i.e. remains in air), while going from O to B.*** It is denoted by T.

The total time of flight consists of two parts :

(*i*) Time taken by the object to go from the point O to the highest point H. It is called **time of ascent**.

(*ii*) Time taken by the object to go from the highest point H to the point B in level with O. It is called **time of descent**.

As motion of object from O to H is symmetrical to that from H to B

$\therefore$ time of ascent = time of descent = t (say)

As, total time of flight = time of ascent + time of descent $\therefore\ T = t + t = 2t$ or $t = T/2$

At the highest point H, the vertical component velocity of the object becomes zero. Taking vertical upward motion of the object from O to H, we have, $u_y = u\sin\theta,\ a_y = -g,\ t = T/2$ and $v_y = 0$.

Since, $v_y = u_y + a_y t \quad \therefore \quad 0 = u\sin\theta + (-g)\,T/2$

$$\text{or} \qquad \boxed{T = \frac{2u\sin\theta}{g}} \qquad ...(61)$$

IMPORTANT NOTE

> In angular projection of a projectile,
>
> (*i*) time of ascent = time of descent $= \dfrac{\text{time of flight}}{2}$
>
> (*ii*) time of flight depends upon the vertical component of velocity of projectile.
>
> (*iii*) time of flight depends upon, initial velocity of projection (u), angle of projection (θ).
>
> (*iv*) time of flight is independent of mass of projectile and height attained by projectile during its flight.

> **Maximum height.** ***It is the maximum vertical height attained by the object above the point of projection during its flight. It is denoted by h.***

Taking the vertical upward motion of the object from O to H, we have :

$$u_y = u \sin\theta,\ a_y = -g,\ y_0 = 0,\ y = h,\ t = T/2 = u \sin\theta/g.$$

Using the relation, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$, we have

$$h = 0 + u \sin\theta \times u \sin\theta/g + \frac{1}{2}(-g)(u \sin\theta/g)^2$$

or $$h = \frac{u^2}{g}\sin^2\theta - \frac{1}{2}\frac{u^2 \sin^2\theta}{g} = \frac{u^2 \sin^2\theta}{2g},$$

i.e., $$\boxed{h = \frac{u^2 \sin^2\theta}{2g}} \qquad \text{....(62)}$$

The maximum height attained by a projectile given angular projection depends upon (*i*) the initial velocity of projection and the angle of projection or (*ii*) the vertical component velocity of projectile.

Horizontal Range. ***It is the horizontal distance covered by the object between its point of projection and the point of hitting the ground.*** **It is denoted by *R*.**

Clearly, the horizontal range is the horizontal distance covered by the object with uniform velocity $u \cos\theta$ in the time equal to total time of flight T.

$$\therefore \qquad R = u\cos\theta \times T = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2}{g}2\sin\theta\cos\theta$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}} \qquad \text{...(63)}$$

Maximum Horizontal Range. From (63), we note that for a given speed u of the object, the value of horizontal range depends upon angle of projection θ as g is constant at a place. Therefore, horizontal range R will be maximum if

$$\sin 2\theta = \text{maximum} = 1 = \sin 90^\circ \quad \text{or} \quad 2\theta = 90^\circ \quad \text{or} \quad \theta = 45^\circ$$

$$\therefore \qquad \text{Max. horizontal range, } R_m = \frac{u^2}{g}\sin 2\times 45^\circ = \frac{u^2}{g}$$

Hence to achieve maximum horizontal range, the object should be projected at an angle of 45° with the horizontal direction. This concept has been used by athletes in long jump, javelion throw, cricket ball throw etc.

This is illustrated in Fig. 4.46(*a*).

IMPORTANT NOTES

1. Horizontal range of various projectiles of different masses is the same provided their initial velocity of projection and angle of projection with the horizontal are the same.

2. If $\theta = 45^\circ$, Max. horizontal range, $R_{\max} = \frac{u^2 \sin 2\times 45^\circ}{g} = \frac{u^2}{g}$

$$\text{Max. height, } h_{\max} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2 (1/\sqrt{2})^2}{2g} = \frac{u^2}{4g} = \frac{1}{4}R_{\max}$$

Retain in Memory

1. In projectile motion, the horizontal motion and the vertical motion are independent of each other *i.e.* neither motion affects the other.
2. A projectile moves under the combined effect of two rectangular velocities : (*i*) A uniform horizontal velocity which remains constant throughout the motion of projectile, provided we ignore air resistance (*ii*) A vertical component velocity which is uniformly changing due to gravity pull.
3. In angular projection of a projectile, at the highest point, the vertical component of its velocity is momentarily zero but the horizontal component of its velocity is not zero, but a constant.
4. Path of a projectile is determined only by magnitude of initial velocity, angle of projection, gravity and air resistance.

Sample Problem **A football player kicks a ball at an angle of 30° with the horizontal with an initial speed of 20 m/s. Assuming that the ball travels in a vertical plane, calculate (a) the time at which the ball reaches the highest point (b) the maximum height reached (c) the horizontal range of the ball (d) the time for which the ball is in air. $g = 10\ m/s^2$.**

Sol. Here, $\theta = 30°$; $u = 20$ m/s ;

(*a*) Time taken by the ball to reach the highest point,

$$t = \frac{T}{2} = \frac{u \sin\theta}{g} = \frac{20}{10} \times \sin 30° = 2 \times (1/2) = \mathbf{1\ s}$$

(*b*) The maximum height $= \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{(20)^2 \times \sin^2 30°}{2 \times 10} = \mathbf{5\,m}$

(*c*) The horizontal range $= \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(20)^2 \times \sin 2 \times 30°}{10} = \mathbf{34{\cdot}64\,m}$

(*d*) The time of flight $= \dfrac{2u \sin\theta}{g} = \dfrac{2 \times 20 \times \sin 30°}{10} = \mathbf{2\,s}$

4.32. TWO ANGLES OF PROJECTION, FOR THE SAME HORIZONTAL RANGE

When an object is projected with velocity u at an angle θ with the horizontal, then horizontal range will be,

$$R_1 = \frac{u^2 \sin 2\theta}{g} \quad \text{...(64)}$$

When another object is projected with velocity u making an angle $(90° - \theta)$ with the horizontal direction, then horizontal range will be

$$R_2 = \frac{u^2 \sin 2(90° - \theta)}{g} = \frac{u^2}{g} \sin(180° - 2\theta) = \frac{u^2}{g} \sin 2\theta\,. \quad \text{...(65)}$$

From (64) and (65), we note that $R_1 = R_2$

Thus the horizontal range is the same whether the angle of projection of an object is θ or (90° – θ) with the horizontal direction. This indicates that for a given velocity, there are two angles of projection for which the horizontal range is the same. An angle of projection (90° – θ) with the horizontal is equivalent to an angle θ with the vertical. It implies that the horizontal range is the same whether θ is the angle of projection with the horizontal or with the vertical. Fig. 4.46(*b*).

FIGURE 4.46

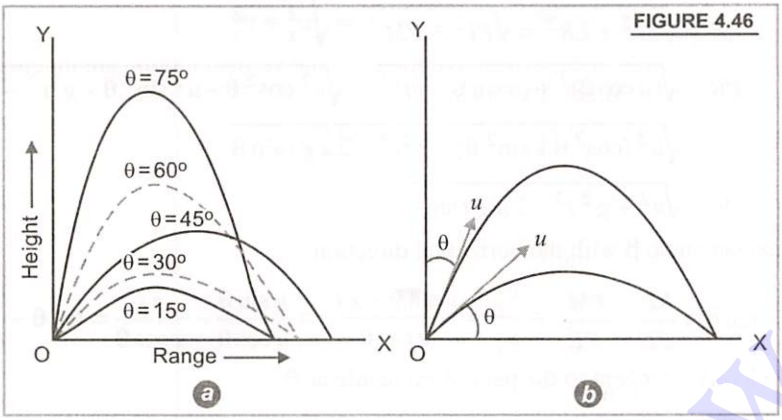


Fig. 4.46(*a*) shows that horizontal range is maximum for θ = 45°. For θ = 15° and 75°, the horizontal ranges are the same but less than the maximum horizontal range. Similarly for θ = 30° and 60°, horizontal ranges are again equal. The maximum height in every case is different.

That is why a foot ball kicked at 30° or 60° will strike the ground at the same horizontal distance although, when kicked at 60°, it will cover greater height and will remain longer in air.

IMPORTANT NOTE

From the above discussion, it follows that H.R will be same for any pair of angles whose sum is 90°. It means, H.R will be same for θ = (10° and 80°) ; (15° and 75°) ; (20° and 70°) ; (30° and 60°) ; (40° and 50°) and so on. The value of H.R would be different for each pair of angles. Further, it follows that H.R is same, when angle of projection is (*i*) greater than 45° by certain value and (*ii*) less than 45° by the same value, *i.e.*, when $\theta_1 = 45° + \theta$ and $\theta_2 = 45° - \theta$ or $\theta_1 + \theta_2 = 90°$.

4.33. VELOCITY OF PROJECTILE AT ANY TIME

Let OX be a horizontal line on ground and OY be a vertical line perpendicular to ground. The point O is taken as the origin. Let a projectile be projected from O, with a velocity *u* in XY plane at an angle θ with OX. This velocity can be resolved into two rectangular components :

(*i*) $u \cos\theta$, along OX and (*ii*) $u \sin\theta$, along OY.

If we ignore air resistance, velocity $u \cos\theta$ along OX can be taken as constant. In going from O to H, the vertical component velocity $u \sin\theta$ goes on decreasing due to opposing gravity.

At *H*, vertical component velocity becomes zero. From *H* to *B*, the vertical component velocity goes on increasing due to gravity. Fig. 4.47. At any time *t*, let the projectile be at *P*. Taking motion of the projectile along *OX* from *O* to *P*, we have $u_x = u\cos\theta$, $a_x = 0$, $t = t$ and $v_x = ?$

Using the relation, $v_x = u_x + a_x t$, we get

$$v_x = u\cos\theta + (0)\,t = u\cos\theta \qquad \text{...(66)}$$

It is represented by *PL*

Taking motion of the projectile along *OY* from *O* to *P*, we have :

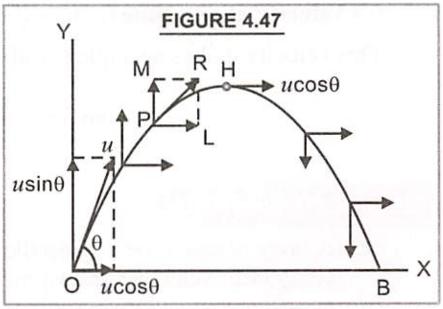
$$v_y = ?,\ u_y = u\sin\theta,\ a_y = -g,\ t = t$$

Using the relation,

$$v_y = u_y + a_y t, \text{ we get}$$

$$v_y = u\sin\theta + (-g)\,t = u\sin\theta - g\,t \qquad \text{...(67)}$$

FIGURE 4.47



It is represented by *PM*. Complete the parallelogram *PLRM* which is rectangle in this case. The resultant velocity *V* at *P* will be represented in magnitude and direction by the diagonal *PR* of the rectangle.

$$\therefore \qquad PR = \sqrt{PL^2 + LR^2} = \sqrt{PL^2 + PM^2} = \sqrt{v_x^2 + v_y^2}$$

or

$$PR = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2} = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2u\,g\,t\sin\theta}$$

$$= \sqrt{u^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2u\,g\,t\sin\theta}$$

$$\therefore \qquad V = \sqrt{u^2 + g^2t^2 - 2u\,g\,t\sin\theta}$$

Let PR make an angle β with the horizontal direction.

In $\Delta\, RPL$, $\tan\beta = \dfrac{RL}{PL} = \dfrac{PM}{PL} = \dfrac{v_y}{v_x} = \dfrac{u\sin\theta - g\,t}{u\cos\theta} = \dfrac{u\sin\theta}{u\cos\theta} - \dfrac{g\,t}{u\cos\theta} = \tan\theta - \dfrac{g\,t}{u\cos\theta}$.

$\overrightarrow{PR}$ is evidently a tangent to the path of projectile at P.

IMPORTANT NOTE

In angular projection, the velocity of projectile at the highest position of the path is not zero but is equal to horizontal component velocity = $u\cos\theta$, where u is the initial velocity of projection and θ is the angle of projection with the horizontal.

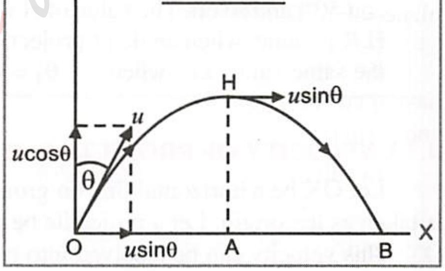
4.34. PROJECTILE FIRED AT AN ANGLE WITH THE VERTICAL

When θ is the angle of projection with the vertical Fig. 4.48, the components of initial velocity are (*i*) $u\sin\theta$ along OX. (*ii*) $u\cos\theta$ along OY.

Now proceed exactly as outlined in article 4.31.

As an angle θ with the vertical is equivalent to angle $(90° - \theta)$ with the horizontal, therefore replacing θ by $(90° - \theta)$ in the various formulae obtained in Art. 4.31, we shall obtain the corresponding expressions in this case as :

FIGURE 4.48



(*i*) **Time of flight** $= \dfrac{2u\sin(90° - \theta)}{g} = \dfrac{2u}{g}\cos\theta$

(*ii*) **Maximum height** $= \dfrac{u^2\sin^2(90° - \theta)}{2g} = \dfrac{u^2\cos^2\theta}{2g}$

(*iii*) **Horizontal range** $= \dfrac{u^2}{g}\sin 2(90° - \theta) = \dfrac{u^2}{g}\sin(180° - 2\theta) = \dfrac{u^2}{g}\sin 2\theta$

(*iv*) **Path of projectile,** $y = x\tan(90° - \theta) - \dfrac{1}{2}\dfrac{g\,x^2}{u^2\cos^2(90° - \theta)} = x\cot\theta - \dfrac{gx^2}{2u^2\sin^2\theta}$

(*v*) **Velocity at any time** t, $V = \sqrt{u^2 + g^2t^2 - 2u\,gt\sin(90° - \theta)} = \sqrt{u^2 + g^2t^2 - 2u\,gt\cos\theta}$

This velocity makes an angle β with the horizontal direction, where

$$\tan\beta = \frac{u\sin(90° - \theta) - g\,t}{u\cos(90° - \theta)} = \frac{u\cos\theta - g\,t}{u\sin\theta}$$

Retain in Memory

1. A body is said to be a projectile if it has motion in two dimensions *i.e.* a projectile should have two component velocities in two mutually perpendicular directions.
2. Horizontal range of projectile is same when angle of projection is (*i*) θ and $(90° - \theta)$ or (*ii*) $(45° + \theta)$ and $(45° - \theta)$.

3. At the highest point, the projectile possesses velocity only along horizontal direction ($u \cos \theta$).
4. At the highest point of projectile path, the velocity and acceleration are perpendicular to each other.
5. In projectile motion, the particle returns to the ground at the same angle and with the same speed with which it was projected, provided there is no loss of energy during the flight.
6. The angular momentum of the particle at highest position, given angular projection from ground is

$$= mu \cos \theta \times \text{max. height} = mu \cos\theta \times \frac{u^2 \sin^2 \theta}{2g} = \frac{mu^3 \cos\theta \sin^2 \theta}{2g}$$

7. In projectile motion, kinetic energy is maximum at the point of projection or point of reaching the ground and is minimum at the highest point.
8. There are two times for which the projectile travels the given vertical distance and the sum of these timings is equal to the total time of flight of projectile.
9. The time of flight, the horizontal range and max. height are independent of mass of projectile.
10. The maximum height attained by projectile is equal to one fourth of its maximum range.
11. The range of the long range ballistic missile Agni III of India is 3500 km.

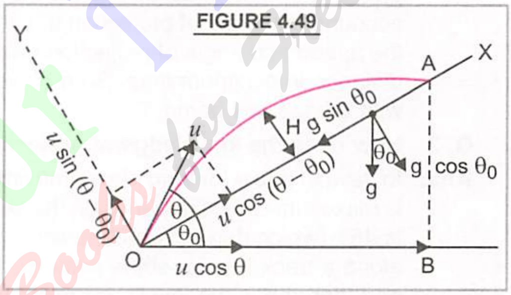
4.35. ANGULAR PROJECTION OF A PROJECTILE ON AN INCLINED PLANE

Consider an inclined plane OA of inclination θ_0 with the horizontal. Let a projectile be projected from O with velocity u making an angle θ with the horizontal direction as shown in Fig. 4.49.

Angle of projection of projectile with the inclined plane = $\theta - \theta_0$.

Resolving $\vec{u}$ into two rectangular components, we have $u \cos (\theta - \theta_0)$ acts along the inclined plane upwards and $u \sin (\theta - \theta_0)$ acts perpendicular to inclined plane upwards

FIGURE 4.49



Resolving acceleration due to gravity g into two rectangular components, we have

$g \cos \theta_0$ acts perpendicular to inclined plane downwards

and $g \sin \theta_0$ acts along the inclined plane downwards.

Time of flight. Let T be the time taken by projectile to go from O to A on the inclined plane. Displacement of projectile perpendicular to the inclined plane (*i.e.*, along Y-axis) is zero, *i.e.*, $y = 0$.

Taking motion of projectile from O to A perpendicular to plane OA, we have

$$u_y = u \sin (\theta - \theta_0), a_y = -g \cos \theta_0, t = T, y = 0$$

As $$y = u_y t + \frac{1}{2} a_y t^2$$

$\therefore$ $$0 = u \sin (\theta - \theta_0) T + \frac{1}{2} (-g \cos \theta_0) T^2 \quad \text{or} \quad \boxed{T = \frac{2u \sin (\theta - \theta_0)}{g \cos \theta_0}}$$

Maximum Height. Let H be the maximum height attained by projectile on the inclined plane. Then at the highest position, the component velocity of projectile perpendicular to plane OA becomes zero.

Taking motion of projectile perpendicular to inclined plane from O to the highest position, we have

$$u_y = u \sin (\theta - \theta_0), v_y = 0, a_y = -g \cos \theta_0, y = H$$

As $$v_y^2 = u_y^2 + 2a_y y$$

$\therefore$ $$0 = [u \sin (\theta - \theta_0)]^2 + 2(-g \cos \theta_0) H \quad \text{or} \quad \boxed{H = \frac{u^2 \sin^2 (\theta - \theta_0)}{2g \cos \theta_0}}$$

Range of projectile. The horizontal velocity of projectile $u \cos \theta$ remains constant throughout the motion of projectile from O to B. Therefore

$$\text{distance } OB = u \cos \theta \times T = u \cos \theta \times \frac{2u \sin(\theta - \theta_0)}{g \cos \theta_0} = \frac{2u^2}{g} \frac{\cos \theta \sin(\theta - \theta_0)}{\cos \theta_0}$$

Range of projectile on inclined plane

$$R = OA = \frac{OB}{\cos \theta_0} = \frac{2u^2 \cos \theta \sin(\theta - \theta_0)}{g \cos^2 \theta_0}, \quad i.e., \quad \boxed{R = \frac{2u^2 \cos \theta \sin(\theta - \theta_0)}{g \cos^2 \theta_0}}$$

Curiosity Questions

Q. 1. How does the knowledge of projectile help a player in the base ball game ?

Ans. In the base ball game, a player has to throw a ball so that it goes a certain distance in the minimum time. The time would depend on velocity of ball and angle of throw with the horizontal. For example, if a player intends to throw a ball at a distance of 20 m in one second, he has to give it a speed of 20 m/s at an angle of 15º above the horizontal. Taking air resistance into account, the angle of projection has to be increased to 18º. Thus while playing a base ball game, the speed and angle of projection have to be adjusted suitably so that the ball covers the desired distance in minimum time. So a player has to see the distance and air resistance while playing with a base ball game.

Q. 2. How does the knowledge of projectile help the atheletes in the long jump (or Javelin throw).

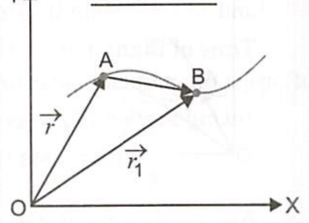
Ans. In Javelin throw (or long jump), maximum horizontal range can be covered if velocity of projection is maximum (which depends on the strength of athelete) and angle of projection with the horizontal is 45º, (which depends on practice). An athelete, taking part in javelin throw (or long jump) runs along a track fast for some distance to acquire large velocity before reaching the marking point and then throw the javelin (or takes the jump), making an angle of 45º with the horizontal, then the horizontal range of javelin throw (or long jump) is maximum.

4.36. GENERAL MOTION IN A PLANE

So far we have been studying the motion of an object in a plane with a uniform acceleration. Now we shall study the motion of the object in a plane with non-uniform acceleration.

Consider an object moving in X-Y plane with non-uniform acceleration. It follows a curved path as shown in Fig. 4.50. Let the object be at position A and B at timings t and t_1 respectively, where $\overrightarrow{OA} = \vec{r}$ and $\overrightarrow{OB} = \vec{r_1}$.

FIGURE 4.50



Displacement. The displacement of the object in time interval $(t_1 - t)$ is

$$\overrightarrow{AB} \, (= \Delta \vec{r})$$

According to triangle law of vectors

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \quad \text{or} \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r_1} - \vec{r} \quad \therefore \quad \Delta \vec{r} = \vec{r_1} - \vec{r}$$

If the position coordinates of the object at A and B are (x, y) and (x_1, y_1) then

$$\vec{r} = x\,\hat{i} + y\,\hat{j} \quad \text{and} \quad \vec{r_1} = x_1\,\hat{i} + y_1\,\hat{j}$$

Displacement, $\Delta \vec{r} = (x_1\,\hat{i} + y_1\,\hat{j}) - (x\,\hat{i} + y\,\hat{j}) = (x_1 - x)\,\hat{i} + (y_1 - y)\,\hat{j} = \Delta x\,\hat{i} + \Delta x\,\hat{j}$ ...(68)

where $(x_1 - x) = \Delta x$ and $(y_1 - y) = \Delta y$

$\therefore$ Magnitude of displacement, $\Delta r = [(x_1 - x)^2 + (y_1 - y)^2]^{1/2}$

Velocity.

The average velocity $\vec{v}_{av}$ *of an object is the ratio of the displacement* $(\Delta \vec{r})$ *and the corresponding time interval* (Δt)*, i.e.,*

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \,\hat{i} + \Delta x \,\hat{j}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = \bar{v}_x \,\hat{i} + \bar{v}_y \,\hat{j} \qquad \text{...(69)}$$

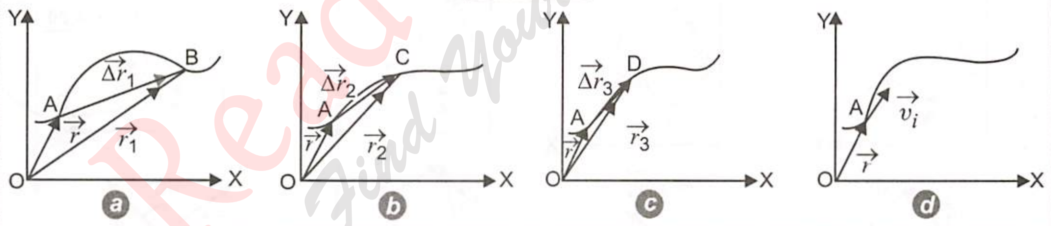
where $\bar{v}_x$ and $\bar{v}_y$ are the average component velocities of the object along X and Y directions, within time interval $(t_1 - t)$. The direction of $\vec{v}_{av}$ is the same as that of $\Delta \vec{r}$.

The instantaneous velocity $\vec{v}_i$ *of the object at an instant t is the average velocity in a very small interval of time* Δt *around time t, when* Δt *approaches to zero.*

$$\vec{v}_i = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad \text{...(70)}$$

For velocity $\vec{v}_i$, the meaning of the limiting process can be easily understood with the help of Fig. 4.51(*a*) to (*d*). In these figures, the solid curve represents the path of object. The object is at point *A* on the path at time *t*. *B*, *C* and *D* represent the positions of the object after time intervals Δt_1, Δt_2 and Δt_3, where $\Delta t_1 > \Delta t_2 > \Delta t_3$. $\Delta \vec{r}_1, \Delta \vec{r}_2$ and $\Delta \vec{r}_3$ are the displacements of the object in time intervals Δt_1, Δt_2 and Δt_3 respectively. Here, $\Delta \vec{r}_1 > \Delta \vec{r}_2 > \Delta \vec{r}_3$. It means as time interval Δt decreases, the displacement vector $\Delta \vec{r}$ decreases. When $\Delta t \to 0$, $\Delta \vec{r} \to 0$. It means, $\Delta \vec{r}$ becomes tangent to the path at *A*. Fig. 4.51(*d*). Thus, ***the direction of instantaneous velocity of object at any point on the path, is tangent to the path at that point, which is the direction of motion of the object at that instant.***

FIGURE 4.51



$\vec{v}_i$ in terms of rectangular components.

Using, $\vec{r} = x\,\hat{i} + y\,\hat{j}$ in (3), we have $\vec{v}_i = \frac{d}{dt}(x\,\hat{i} + y\,\hat{j}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\,\hat{i} + v_y\,\hat{j}$

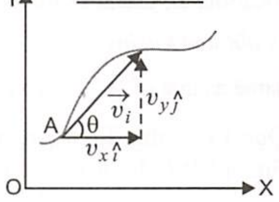
where, $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$

The magnitude of $\vec{v}_i$ is, $v_i = \sqrt{v_x^2 + v_y^2}$

If $\vec{v}_i$ makes an angle θ with the direction of *X*-axis, Fig. 4.52, then

$$\tan\theta = \frac{v_y}{v_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

FIGURE 4.52



Acceleration.

Average acceleration $\vec{a}_{av}$ *of the object for a given time interval while moving in X-Y plane is the change in velocity* $(\Delta \vec{v})$ *divided by the time interval* (Δt)*, i.e.,*

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta (v_x \hat{i} + v_y \hat{j})}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} = a_x \hat{i} + a_y \hat{j}$$

where a_x, a_y are the components of average acceleration along X and Y-directions in the interval of time Δt.

Instantaneous acceleration $(\vec{a}_i)$ *is the limiting value of the average acceleration as the time interval approaches zero, i.e.,*

$$\vec{a}_i = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

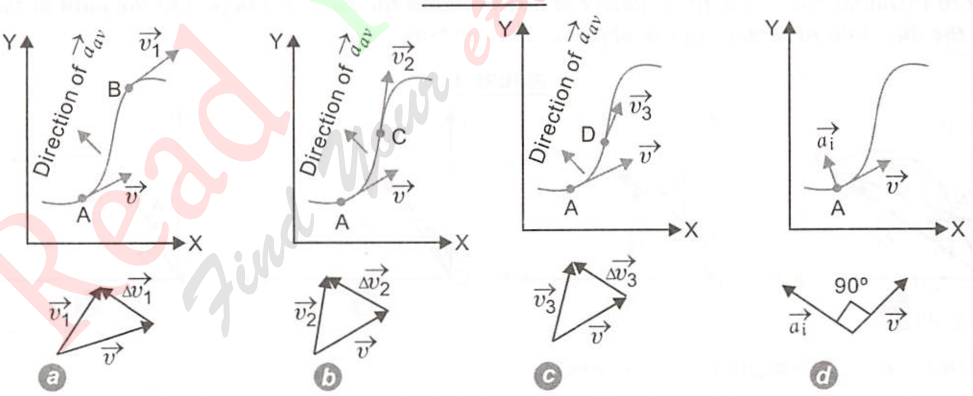
In terms of rectangular components along X and Y directions

$$a_x \hat{i} + a_y \hat{j} = \frac{d}{dt}(v_x \hat{i} + v_y \hat{j}) = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} \quad \text{or} \quad a_x = \frac{dv_x}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt}$$

Magnitude of instantaneous acceleration is, $\quad a_i = \sqrt{a_x^2 + a_y^2}$

For acceleration $\vec{a}_i$, the meaning of limiting process can be understood with the help of Fig. 4.53(a) to (d).

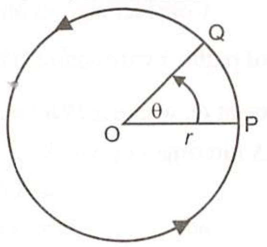
FIGURE 4.53



In the Fig. 4.53(a) to (d), the solid line curve represents the path of object. The object is at A at time t. B, C and D represent the positions of the object after time interval Δt_1, Δt_2 and Δt_3. The velocity of the object at locations B, C and D are represented by velocity vectors $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ respectively. *In each case of* Δt, $\Delta \vec{v}$ *is obtained using triangle law of vectors addition.* We know that the direction of average acceleration is the same as that of $\Delta \vec{v}$. From figures (a), (b) and (c), we note that as Δt decreases, the direction of $\Delta \vec{v}$ changes. Due to it, the direction of acceleration changes. When $\Delta t \to 0$, $\Delta \vec{v}$ becomes perpendicular to $\vec{v}$, Fig. 4.53(d). It means the ***instantaneous acceleration is perpendicular to the velocity vector at that instant.***

4.37. ANGULAR DISPLACEMENT

FIGURE 4.54



Consider an object moving along a circular path of radius r, in anticlock-wise direction in the plane of paper, with centre at O. Let the axis of circular motion pass through O in a direction perpendicular to the plane of paper. Let the position of the object change from P to Q in time t, where $\angle POQ = \theta$, Fig. 4.54. Therefore, during time interval t, the radius vector traces out an angle θ at the axis of the circular path.

Here θ is known as angular displacement of the object in time t.

> ***Thus, angular displacement in a given time of the object, moving around a circular path is defined as the angle traced out by the radius vector at the centre of the circular path in the given time.***

Since, angle = arc/radius, $\therefore$ $\theta = PQ/r$ ...(71)

In (71), the angle θ represents the magnitude of angular displacement and is expressed in radian (denoted by rad.).

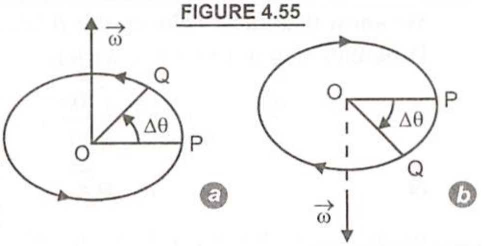
Angular displacement is a vector quantity, provided θ is small. This is because the commutative law of vectors addition for large angles is not valid, whereas for small angles, the law is valid*. The direction of angular displacement depends upon the sense of rotation of the object and is given by **Right Hand Rule.** This rule states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.

In Fig. 4.54, when particle is rotating in anticlockwise direction, angular displacement is perpendicular to the plane of the paper acting upwards.

4.38. ANGULAR VELOCITY

> ***Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.***

FIGURE 4.55



It is generally denoted by ω (omega) and is measured in radian per second (denoted by rad.s^{-1}). Its dimensional formula is $[M^\circ L^\circ T^{-1}]$.

Consider a point object moving along a circular path, with centre (*i.e.* axis of rotation) at O. Let the object move from P to Q in a small time interval Δt, where $\angle POQ = \Delta\theta$. Fig 4.55($a$).

Now angular velocity

$$\omega = \frac{\text{angle traced}}{\text{time taken}} = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular velocity is a vector quantity. Its direction is the same as that of $\vec{\Delta\theta}$.

For anticlockwise rotation of the point object on the circular path, the direction of $\vec{\omega}$, according to Right hand rule, is along the axis of circular path directed upwards. Fig. 4.55(a). ***For clockwise rotation of the point object on the circular path, the direction of $\vec{\omega}$ is along the axis of circular path directed downwards.*** Fig. 4.55(b).

It is **important to note** that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it, clockwise or anticlockwise.

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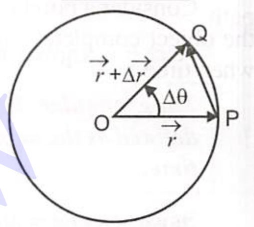
*See Conceptual Problem 42 of chapter 4.

4.39. RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY

Consider a point object moving with a uniform angular velocity ω and linear speed v, on a circular path of radius r with centre at O. At any time t let the object be at P, where $\overrightarrow{OP} = \vec{r}$. At time $(t + \Delta t)$, let the object be at Q, where $\angle POQ = \Delta\theta$, and $\overrightarrow{OQ} = \vec{r} + \Delta\vec{r}$, Fig 4.56. It means an object describes an arc PQ of length Δl in time interval Δt.

FIGURE 4.56



∴ $v = \Delta l/\Delta t$ or $\Delta l = v\,\Delta t$ ...(72)

and $\omega = \Delta\theta/\Delta t$ or $\Delta\theta = \omega\,\Delta t$ ...(73)

Since, angle = arc/radius

∴ $\Delta\theta = \Delta l/r$ or $\omega\,\Delta t = v\,\Delta t/r$ [From (72) & (73)]

or $v = \omega\, r$ ...(74)

Direction of $\vec{v}$. When $\Delta t \to 0$, then in $\Delta\, OPQ$, $\angle OPQ \to \pi/2$.

∴ $\overrightarrow{PQ}$ tends to become perpendicular to $\overrightarrow{OP}$ *i.e.* $\overrightarrow{PQ}$ is directed along the tangent to the circle at P. Hence *the velocity at P is directed along the tangent to the circle at P in the direction of motion.*

4.40. ANGULAR ACCELERATION

Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

It is generally denoted by α. Let Δω be the change in angular velocity of the object in time interval Δt from t to $(t + \Delta t)$, while moving on a circular path. Then angular acceleration of the object will be

$$\alpha = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

SI unit of angular acceleration is *rad.* s^{-2} and its dimensional formula is $[M^{\circ}\, L^{\circ}\, T^{-2}]$.

Relation between Linear acceleration and Angular acceleration

We know that linear velocity v is related with angular velocity ω by the relation, $v = \omega\, r$

Differentiating it *w.r.t.* time, we have

$$\frac{dv}{dt} = \frac{d}{dt}(\omega\, r) = \left(\frac{d\omega}{dt}\right) r \qquad (\because\ r \text{ is constant in circular motion})$$

or $$\boxed{a = \alpha\, r} \qquad ...(75)$$

where a is the linear acceleration and α is the angular acceleration.

4.41. UNIFORM CIRCULAR MOTION

When a point object is moving on a circular path with a constant speed (i.e. it covers equal distances on the circumference of the circle in equal intervals of time), then the motion of the object is said to be a uniform circular motion.

(a) Time period. ***In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.***

It is generally denoted by T and is expressed in second.

(b) Frequency. ***In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.***

It is generally denoted by v. Its unit is s^{-1} or hertz (Hz).

(c) Relation between time period and frequency

If ν is the frequency of revolution of an object in circular motion, then the object completes ν revolutions in 1 second. Therefore, the object will complete one revolution (called time period of revolution T say) in $1/\nu$ second.

$$\therefore \quad T = 1/\nu \quad \text{or} \quad \nu = 1/T \quad \text{or} \quad \boxed{\nu T = 1} \qquad \text{...(76)}$$

(d) Relation between angular velocity, frequency and time period

Consider a point object describing a uniform circular motion with frequency ν and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radian. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity

$$\boxed{\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu} \qquad (\because T = 1/\nu) \quad \text{...(77)}$$

Sample Problem **Calculate the angular speed of the flywheel making 240 revolutions per minute.**

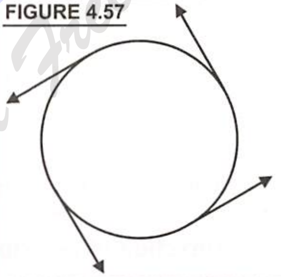
Sol. Here, $\theta = 240 \times 2\pi$ radians and $t = 60$ seconds.

$\therefore$ Angular speed, $\omega = \frac{\theta}{t} = \frac{240 \times 2\pi}{60} = \mathbf{8\pi\ rad/s}$

4.42. CENTRIPETAL ACCELERATION

When a point object is moving along a horizontal circular path with a constant speed, the direction of its velocity vector at an instant (represented by the tangent to the circular path) is changing with time, Fig 4.57. It means in a uniform circular motion, the velocity vector of the object is changing with time. This indicates that the ***uniform circular motion is an accelerated motion.***

FIGURE 4.57



> *Acceleration of an object undergoing uniform circular motion is called centripetal acceleration.*

It always acts on the object along the radius and towards the centre of the circular path.

Expression for centripetal acceleration

Consider a particle of mass m, moving with a constant speed v and uniform angular velocity ω, on a circular path of radius r with centre at O. At any time t, let the particle be at P, where $\overrightarrow{OP} = \vec{r}_1$ and at time $(t + \Delta t)$, let the particle be at Q, where $\overrightarrow{OQ} = \vec{r}_2$ and $\angle POQ = \Delta\theta$, Fig 4.58($a$). Clearly ; $|\vec{r}_1| = |\vec{r}_2| = r$

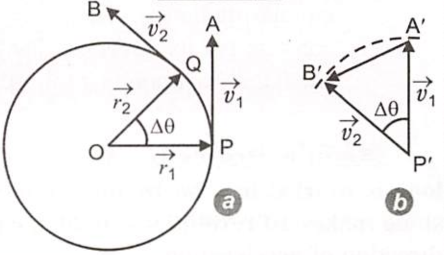
$$\text{Angular speed of the particle, } \omega = \frac{\Delta\theta}{\Delta t} \qquad \text{...(78)}$$

Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors of the particle at locations P and Q respectively.

In circular motion, the direction of velocity vector at a location is always along the tangent to the circular path at that location, hence $\vec{v}_1$ and $\vec{v}_2$ can be represented in magnitude and direction by the tangents $\overrightarrow{PA}$ and $\overrightarrow{QB}$ respectively. Since the particle is moving with a uniform speed v, the lengths of the tangents at P and Q are equal, *i.e.*,

$$|\overrightarrow{PA}| = |\overrightarrow{QB}| = |\vec{v}|.$$

FIGURE 4.58



To find the change in velocity in time interval Δt between t and $(t + \Delta t)$, take an external point P'. Draw $\overrightarrow{P'A'}$ equal and parallel to PA and $\overrightarrow{P'B'}$ equal and parallel to QB, Fig 4.58(b).

Clearly, $\angle A'P'B' = \Delta\theta$. From Δ law of vectors

$$\overrightarrow{P'A'} + \overrightarrow{A'B'} = \overrightarrow{P'B'}$$

or
$$\overrightarrow{A'B'} = \overrightarrow{P'B'} - \overrightarrow{P'A'} = \vec{v}_2 - \vec{v}_1 = \Delta\vec{v} \text{ (say)}$$

As $\Delta t \to 0$, A' lies close to B'. Then $A'B'$ can be taken as an arc $A'B'$ of circle of radius $P'A' = |\vec{v}|$

$$\therefore \quad \Delta\theta = \frac{A'B'}{P'A'} = \frac{|\Delta\vec{v}|}{|\vec{v}|} \quad \text{or} \quad \omega(\Delta t) = \frac{|\Delta\vec{v}|}{|\vec{v}|} \qquad \text{[From (78)]}$$

or
$$\frac{|\Delta\vec{v}|}{\Delta t} = |\vec{v}|\omega = (\omega r)\omega = \omega^2 r \qquad [\because v = \omega r]$$

When $\Delta t \to 0$, then $\dfrac{|\Delta\vec{v}|}{\Delta t}$ represents the magnitude of centripetal acceleration at P, which is given by

$$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \omega^2 r = \left(\frac{v}{r}\right)^2 r = \frac{v^2}{r}$$

Thus,
$$\boxed{|\vec{a}| = \omega^2 r = v^2/r} \qquad \text{...(79)}$$

Direction of centripetal acceleration. It is always the same as that of $\Delta\vec{v}$. When Δt decreases, $\Delta\theta$ also decreases. Due to it, $\Delta\vec{v}$ becomes more and more perpendicular to $\vec{v}$. When $\Delta t \to 0$, $\Delta\vec{v}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore $\Delta\vec{v}$ and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path.

Retain in Memory

1. In a uniform circular motion of an object, the direction of velocity vector of the object acting tangentially to the circular path changes continuously with time but its magnitude always remains constant ($v = r\omega$).
2. The magnitude of centripetal acceleration of a uniform circular motion is constant ($= v^2/r$) because v and r are constant but its direction changes continuously with time and it is always pointing towards the centre of circular path.

Sample Problem **A stone tied to the end of a string 2 m long is whirled in a horizontal circle with constant speed. If the stone makes 10 revolutions in 20 s, calculate the magnitude and direction of acceleration.**

DO YOU KNOW ?

1. When a body is moving with a constant angular velocity, its angular acceleration is zero.

2. A uniform circular motion is an accelerated motion.

3. The work done by centripetal force is zero.

4. The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path.

5. Without centripetal force, a body cannot move on a circular path.

6. In different types of circular motion, the centripetal force is provided by different means.

7. In projectile motion both the magnitude and direction of acceleration (g) remains constant throughout the motion, whereas in a uniform circular motion, the magnitude of acceleration remains constant but direction of acceleration changes continuously with time.

Sol. Here ; $r = 2$ m ; $v = 10/20 = 1/2$ r.p.s.

Centripetal acceleration, $a_c = r\,\omega^2 = r\,4\,\pi^2\,v^2$

$$= 2\times 4\times\left(\frac{22}{7}\right)^2\times\left(\frac{1}{2}\right)^2 = \mathbf{19{\cdot}74\ ms^{-2}}$$

Its direction is along the string directed towards the centre of rotation of stone.

4.43. TANGENTIAL ACCELERATION AND CENTRIPETAL ACCELERATION

Consider a particle describing a circular path of radius r with centre at O. Let the linear speed of the particle be changing with time. At any instant, let v be the linear speed, ω be the angular velocity and α be the angular acceleration of the particle.

The particle has two types of accelerations.

1. Centripetal acceleration (a_c) $= \dfrac{v^2}{r} = v\,\omega$

It acts along the radius and is directed towards the centre of the circular path.

2. Tangential acceleration (a_T) $= r\,\alpha$. It acts along the tangent to the circular path and is in the plane of the circular path.

$\vec{a}_c$ and $\vec{a}_T$ are perpendicular to each other. The total acceleration $\vec{a}$ of the particle is given by

$$\vec{a} = \vec{a}_c + \vec{a}_T$$

$$\therefore \qquad |\vec{a}| = \sqrt{a_c^2 + a_T^2} = \sqrt{(v\,\omega)^2 + (r\,\alpha)^2}$$

Special Cases : Case (i). If $a_T = 0$, the particle will have only centripetal acceleration. The particle will move on a circular path with constant angular velocity. The magnitude of centripetal acceleration, is

$$a_c = v^2/r.$$

Case (ii). If $a_c = 0$, the particle will have only tangential acceleration. Now the particle will accelerate along the tangent to the circular path.

Sample Problem **A motor car travelling at 30 m/s on a circular road of radius 500 m. It is increasing its speed at the rate of 2 ms^{-2}. What is its acceleration ?**

Sol. Here, $v = 30\ \text{ms}^{-1}$; $r = 500$ m ; $a_T = 2\ \text{ms}^{-2}$

Centripetal acceleration $\quad a_c = \dfrac{v^2}{r} = \dfrac{30\times 30}{500} = 1{\cdot}8\ \text{ms}^{-2}$

Here, a_T is acting tangential to the circular path and a_c is acting along the radius towards the centre of the circular path, *i.e.*, perpendicular to a_T.

The effective acceleration, $a = \sqrt{a_c^2 + a_T^2}$

$$= \sqrt{(1{\cdot}8)^2 + (2)^2} = \mathbf{2{\cdot}7\ m/s^2}$$

SOLVED EXAMPLES

TYPE I. DISPLACEMENT VECTOR, TRIANGLE LAW OF VECTORS ADDITION AND PARALLELOGRAM LAW OF VECTORS ADDITION

Formulae used.

(*i*) If $\vec{R} = \vec{A} + \vec{B}$, then

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and $$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

(*ii*) If $\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$,

then $$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

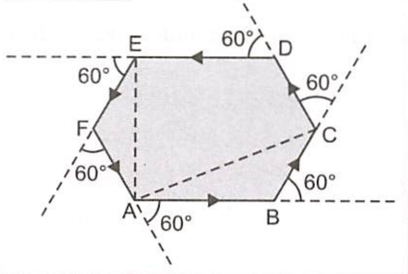
and $$\tan\beta = \frac{B\sin(180^\circ - \theta)}{A + B\cos(180^\circ - \theta)} = \frac{B\sin\theta}{A - B\cos\theta}$$

Units used. *A* and *B* are the magnitudes of vectors to be taken in the same units and θ is the angle between $\vec{A}$ and $\vec{B}$, to be taken in degrees.

Example 1 **In an open ground, a motorist follows a track that turns to his left by an angle 60° after every 200 m. Starting from a given turn, find the displacement and the total path length covered at the second, fourth and seventh turn.**

Solution. Let motorist start from *A* and follows the path *ABCDEFA* as shown in Fig. 4.59. The motorists reaches at position *C* at the end of second turn, at position *E* at the end of fourth turn and at position *B* at the end of seventh turn.

FIGURE 4.59



(*i*) **At the end of second turn.** Displacement of motorist is ;

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

Magnitude of $\overrightarrow{AC}$

$$= \sqrt{(AB)^2 + (BC)^2 + 2\,AB \times BC \times \cos 60^\circ}$$

$$= \sqrt{(200)^2 + (200)^2 + 2 \times 200 \times 200 \times 1/2}$$

$$= \mathbf{200\sqrt{3}\,m}$$

Path length at the end of second turn

$= AB + BC = 2\,AB = 2 \times 200$

$= \mathbf{400\ m}$

(*ii*) **At the end of fourth turn,** the displacement of motorist is $\overrightarrow{AE} = \overrightarrow{BD}$

Magnitude of $\overrightarrow{AE}$ = magnitude of $\overrightarrow{BD}$

$$= \sqrt{(BC)^2 + (CD)^2 + 2(BC)(CD)\cos 60^\circ}$$

$$= \sqrt{(200)^2 + (200)^2 + 2 \times 200 \times 200 \times 1/2}$$

$$= \mathbf{200\sqrt{3}\,m}$$

Path length at the end of fourth turn

$= AB + BC + CD + DE$

$= 4\,AB = 4 \times 200 = \mathbf{800\ m}$

(*iii*) **At the end of seventh turn,** the displacement of motorists is $\overrightarrow{AB}$.

Magnitude of $\overrightarrow{AB} = AB = 200$ m

Path length at the end of seventh turn

$= AB + BC + CD + DE + EF + FA + AB$

$= 7\,AB$

$= 7 \times 200 = \mathbf{1400\ m}$

Example 2 **A motor boat is racing towards North at 25 km/h and the water current in that region is 10 km/h in the direction of 60° East of South. Find the resultant velocity of the boat.** **(NCERT Solved Example)**

Solution. Refer to Fig. 4.60, here

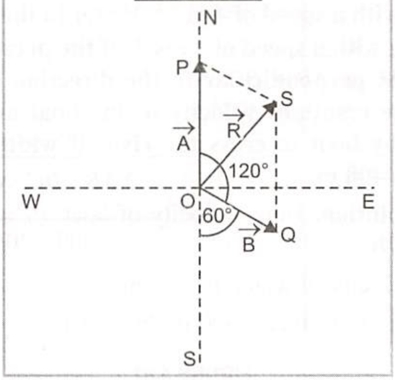
A = 25 km/h along *OP* ;

B = 10 km/h along *OQ*

Angle between $\vec{A}$ and $\vec{B}$ is

$\theta = 180^\circ - 60^\circ = 120^\circ$

FIGURE 4.60



$$R = \sqrt{A^2 + B^2 + 2\,AB\cos\theta}$$

$$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \cos 120^\circ}$$

$$= \mathbf{21{\cdot}8\ km/h}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} = \frac{10 \times \sin 120^\circ}{25 + 10\cos 120^\circ}$$

$$= 0{\cdot}433 = \tan 23{\cdot}4^\circ$$

or $\quad \beta = \mathbf{23{\cdot}4^\circ}$ **East of North**

Example 3 **Two forces F and $2\,F$ newton act on a particle. If the first force be doubled and the second force be increased by 20 N, the direction of resultant is unaltered. Find the value of bigger force.**

Solution. Let θ be the angle between two forces F and $2\,F$ and β be the angle which the resultant force makes with the force F.

In first case, $\tan\beta = \dfrac{2\,F\sin\theta}{F + 2\,F\cos\theta}$

In second case, $\tan\beta = \dfrac{(2\,F + 20)\sin\theta}{2\,F + (2\,F + 20)\cos\theta}$

Hence, $\dfrac{2\,F\sin\theta}{F + 2\,F\cos\theta} = \dfrac{(2\,F + 20)\sin\theta}{2\,F + (2\,F + 20)\cos\theta}$

$$\left[\text{As}\ ;\ \frac{A}{B} = \frac{C}{D} = \frac{C - A}{D - B}\right]$$

so $\dfrac{2F\sin\theta}{F + 2F\cos\theta}$

$$= \frac{(2F + 20)\sin\theta - 2F\sin\theta}{2F + (2F + 20)\cos\theta - (F + 2F\cos\theta)}$$

$$= \frac{20\sin\theta}{F + 20\cos\theta}$$

From above we have, $2\,F = 20$ or $F = 10$ N

Bigger force $= 2\,F = 2 \times 10 =$ **20 N**

Example 4 **Two forces whose magnitude are in the ratio 3 : 5 give a resultant of 28 N. If the angle of their inclination is 60°, find the magnitude of each force.**

Solution. Let A and B be the two forces.

Then $\quad A = 3\,x\ ;\ B = 5\,x\ ;$

$\quad R = 28$ N and $\theta = 60^\circ$

Thus, $\quad A/B = 3/5$.

Now $\quad R = \sqrt{A^2 + B^2 + 2\,AB\cos\theta}$

$$\therefore\ 28 = \sqrt{(3\,x)^2 + (5\,x)^2 + 2\,(3\,x)\,(5\,x)\cos 60^\circ}$$

or $\quad 28 = \sqrt{9\,x^2 + 25\,x^2 + 15\,x^2} = 7\,x$

or $\quad x = 28/7 = 4.$

$\therefore$ Forces are ; $A = 3 \times 4 =$ **12 N**

and $\quad B = 5 \times 4 =$ **20 N**

Example 5 **Two forces acting on a particle in opposite directions have a resultant 1 N. If they act at right angle to each other, the resultant is 5 N. Find the values of two forces.**

Solution. Let F_1 and F_2 be the two forces acting on a particle.

(*i*) When the forces act in opposite directions, then

$$F_1 - F_2 = 1 \text{ or } F_1 = 1 + F_2 \qquad ...(i)$$

(*ii*) When the forces act at right angle to each other then resultant force is

$$F = \sqrt{F_1^2 + F_2^2} = 5 \text{ or } F_1^2 + F_2^2 = 25$$

or $\quad (1 + F_2)^2 + F_2^2 = 25$

or $\quad 2\,F_2^2 + 2\,F_2 - 24 = 0$

or $\quad F_2^2 + F_2 - 12 = 0$

On solving, we get $F_2 = 3$ N and -4 N

From (*i*), $F_1 = 3 + 1 = 4$ N

or $\quad -4 + 1 = -3$ N

$\therefore$ Values of $F_1 =$ **4 N**, $F_2 =$ **3 N**

Example 6 **The greatest and the least resultant of two forces acting at a point are 25 N and 5 N respectively. If each force is increased by 5 N, find the resultant of two new forces acting at right angles to each other.**

Solution. Let A and B be the two forces. Then

greatest resultant $= A + B = 25$ ...(*i*)

Least resultant $= A - B = 5$ ...(*ii*)

On solving (*i*) and (*ii*), we get

$$A = 15\text{ N and } B = 10\text{ N}$$

When each force is increased by 5 N, then new forces are

$$A_1 = 15 + 5 = 20\text{ N}$$
$$B_1 = 10 + 5 = 15\text{ N}$$

Since the angle between new forces is 90°, so their resultant force is

$$R = \sqrt{A_1^2 + B_1^2 + 2\,A_1\,B_1\cos 90^\circ} = \sqrt{A_1^2 + B_1^2}$$
$$= \sqrt{20^2 + 15^2} = \mathbf{25\ N}$$

Let resultant $\vec{R}$ make an angle β with the direction of $\vec{A}$, then

$$\tan\beta = \frac{B_1\sin 90^\circ}{A_1 + B_1\cos 90^\circ} = \frac{B_1}{A_1} = \frac{15}{20}$$
$$= 0{\cdot}75 = \tan 36^\circ\,52'$$

or $\quad \beta = \mathbf{36^\circ\ 52'}$

Example 7 **The sum of the magnitudes of two forces acting at a point is 18 N and the magnitude of their resultant is 12 N. If the resultant makes an angle of 90° with the force of smaller magnitude, find the magnitudes of the two forces.**

Solution. Let $\vec{A}$ and $\vec{B}$ be the two forces and θ be the angle between them. Let $A < B$.

As per question, $A + B = 18$

or $\quad B = (18 - A)$ ...(*i*)

As the resultant makes an angle 90° with the force of smaller magnitude, then $\beta = 90^\circ$.

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} \quad \text{or} \quad \tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\text{or} \quad \infty = \frac{B\sin\theta}{A + B\cos\theta} \quad \text{or} \quad A + B\cos\theta = 0$$

or $\quad B\cos\theta = -A$ ...(*ii*)

Resultant, $R = \sqrt{A^2 + B^2 + 2\,AB\cos\theta}$

$(12)^2 = A^2 + (18 - A)^2 + 2\,A\,(-A)$

[From (*i*) and (*ii*)]

or $144 = A^2 + 324 - 36\,A + A^2 - 2\,A^2$

or $36\,A = 180$ or $A = \mathbf{5\ N}$; $B = 18 - 5 = \mathbf{13\ N}$

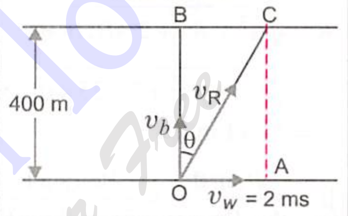
Example 8 **A person rows a boat in a water with a speed of 4 ms^{-1}. Water in the river is flowing with a speed of 2 ms^{-1}. If the person rows the boat perpendicular to the direction of flow, find the resultant velocity of the boat and time taken by boat to cross the river if width of the river is 400 m.**

Solution. Here, velocity of boat, $\upsilon_b = 4$ ms^{-1} along OB

velocity of water, $\upsilon_w = 2$ ms^{-1} along OA,

width of river = 400 m, Fig. 4.61.

FIGURE 4.61



Resultant velocity of boat υ_R is along OC. Then

$$\upsilon_R = \sqrt{\upsilon_b^2 + \upsilon_w^2} = \sqrt{4^2 + 2^2} = \mathbf{4{\cdot}472\ m/s}$$

If $\overrightarrow{OC}$ makes an angle θ with $\overrightarrow{OB}$, then

$$\sin\theta = \frac{\upsilon_w}{\upsilon_b} = \frac{2}{4} = \frac{1}{2} = \sin 30^\circ \quad \text{or} \quad \theta = \mathbf{30^\circ}$$

Time taken to cross the river,

$$t = \frac{400}{\upsilon_b} = \frac{400}{4} = \mathbf{100\ s}$$

Example 9 **A man can swim at the rate of 5 km/h in still water. A river 1 km wide flows at the rate of 3 km h^{-1}. A swimmer wishes to cross the river straight.**

(a) Along what direction must he strike ?

(b) What should be his resultant velocity ?

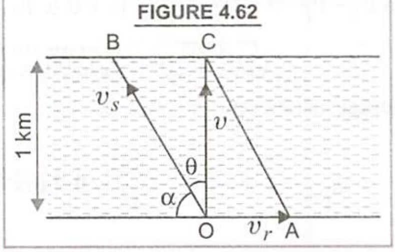
(c) How much time he would take to cross ?

Solution. Given, width of river = 1 km.

Velocity of swimmer, $\upsilon_s = 5$ km/h ; Velocity of water flowing in river, $\upsilon_r = 3$ km h^{-1} along OA.

(*a*) The swimmer will cross straight if the resultant velocity of river flow and swimmer acts perpendicular to the direction of river flow *i.e.* along OC. It will be so if swimmer moves making an angle α with upstream *i.e.* goes along OB. Refer to Fig. 4.62. Here, $\theta + \alpha = 90^\circ$ or $\theta = (90^\circ - \alpha)$

FIGURE 4.62



In Δ *OBC*, $\sin\theta = \sin(90° - \alpha)$

$$= \cos\alpha = \frac{BC}{OB} = \frac{v_r}{v_s} = \frac{3}{5} = 0{\cdot}6$$

or $\cos\alpha = \cos 53° 8'$

or **$\alpha = 53° 8'$ upstream.**

(*b*) Let v be the resultant velocity along *OC*, then

$$v = \sqrt{v_s^2 - v_r^2} = \sqrt{5^2 - 3^2} = \mathbf{4\ km/h}$$

(*c*) Time taken by swimmer to cross the river is

$$t = \frac{1\,\text{km}}{4\,\text{km/h}} = \frac{1}{4}\text{h} = \mathbf{15\ minutes}$$

Example 10 Determine the horizontal force required to displace a mass of 0·03 kg suspended by a string until the string makes an angle 30° with the vertical.

Solution. Let *OA* be the initial position of string and a mass suspended at the end *A* be displaced at an angle 30° from its vertical position by the horizontal force *F*. Fig. 4.63.

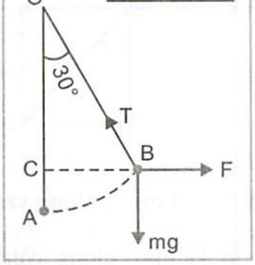
The various forces acting on the body at *B* are

(*i*) The weight = mg = 0·03 × 9·8 N acting vertically downwards,

(*ii*) The force *F* acting horizontally,

(*iii*) The tension *T* acting along the string.

FIGURE 4.63



As the body is in equilibrium, therefore, the three forces acting on the body must be represented by the three sides of the triangle *OCB* taken in one order. Hence, $\dfrac{F}{CB} = \dfrac{mg}{OC} = \dfrac{T}{BO}$

$$\therefore \quad F = mg \times \frac{CB}{OC} = (0{\cdot}03 \times 9{\cdot}8) \times \tan 30°$$

$$= 0{\cdot}03 \times 9{\cdot}8 \times 1/\sqrt{3}\ \text{N} = \mathbf{0{\cdot}1694\ N}$$

$$= 0{\cdot}03 \times 1/\sqrt{3}\ kfg = \mathbf{0{\cdot}01732\ kgf}$$

Example 11 The position of a particle is given by $\vec{r} = 3{\cdot}0t\,\hat{i} + 2{\cdot}0t^2\,\hat{j} + 5{\cdot}0\,\hat{k}$ where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres. Find the velocity and acceleration of the particle in magnitude and direction at time $t = 3{\cdot}0$ s.

(NCERT Solved Example)

Solution. Velocity; $\vec{v} = \dfrac{d(\vec{r})}{dt}$

$$= \frac{d}{dt}(3t\,\hat{i} + 2t^2\,\hat{j} + 5\,\hat{k}) = 3\,\hat{i} + 4t\,\hat{j}$$

Acceleration, $\vec{a} = \dfrac{d\vec{v}}{dt} = 0 + 4\,\hat{j} = \mathbf{4\,\hat{j}}$;

When $t = 3{\cdot}0$ s, $\vec{v} = 3\,\hat{i} + 4 \times 3\,\hat{j} = 3\,\hat{i} + 12\,\hat{j}$

$$\therefore \quad v = \sqrt{3^2 + 12^2} = \mathbf{12{\cdot}4\ ms^{-1}}$$

If θ is the angle which the direction of $\vec{v}$ makes with the *x*-axis, then

$$\tan\theta = \frac{12}{3} = 4 \approx \tan 76° \quad \text{or} \quad \theta \approx \mathbf{76°}$$

Example 12 Rain is falling vertically with a speed of 35 ms^{-1}. Winds starts blowing after sometime with a speed of 12 ms^{-1} in East to West direction. In which direction should a boy waiting at a bus stop hold his umbrella ?

(NCERT Solved Example)

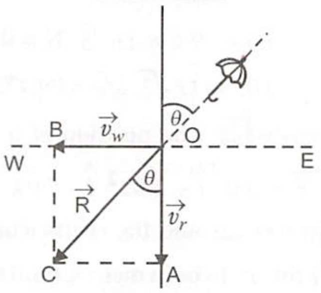
Solution. Refer Fig. 4.64,

$\vec{v}_w = (\overrightarrow{OB}) = 12$ ms^{-1}, along West

$\vec{v}_r = (\overrightarrow{OA}) = 35$ ms^{-1}, along vertical downward

The boy can protect himself from the rain if he holds his umbrella in the direction of resultant velocity $\vec{R}$ of $\vec{v}_r$ and $\vec{v}_w$, *i.e.*, along the direction OC as shown in Fig. 4.64. If θ is the angle which $\vec{R}$ makes with the vertical direction, then

FIGURE 4.64



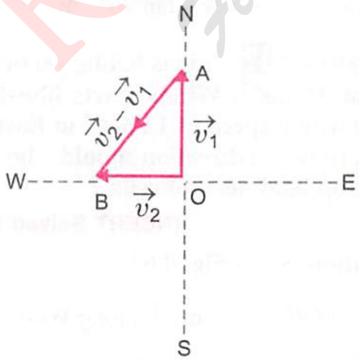
$$\tan\theta = \frac{v_w}{v_r} = \frac{12}{35} = 0{\cdot}3429 = \tan 18^\circ\ 56'$$

$\therefore$ **θ = 18° 56′ with vertical towards East.**

Example 13 **A car travelling at a speed of 30 ms^{-1} due north along the highway makes a left turn on to a side road which heads towards due west. It takes 40 s for the car to complete the turn. At the end of 40 s, the car has a speed of 20 ms^{-1} along the side road. Determine the magnitude of average acceleration over the 40 second interval.**

Solution. Refer to Fig. 4.65, initial velocity, $\vec{v}_1 = \overrightarrow{OA} = 30$ ms^{-1}, due north. Final velocity $\vec{v}_2 = \overrightarrow{OB} = 20$ ms^{-1} due west.

FIGURE 4.65



Change in velocity

$$= \vec{v}_2 - \vec{v}_1 = \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$$

$$\therefore\ |\vec{v}_2 - \vec{v}_1| = AB = \sqrt{(OA)^2 + (OB)^2}$$

$$= \sqrt{30^2 + 20^2} = \sqrt{1300} = 36\ \text{ms}^{-1}$$

Average acceleration,

$$a_{av} = \frac{|\vec{v}_2 - \vec{v}_1|}{t} = \frac{36}{40} = \mathbf{0{\cdot}9\ ms^{-2}}$$

TYPE II. RELATIVE VELOCITY IN A PLANE

Formulae used. (*i*) Relative velocity of *A* w.r.t. *B*,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B)$$

If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$v_{AB} = [v_A^2 + v_B^2 + 2v_A v_B \cos(180^\circ - \theta)]^{1/2}$$

$$= [v_A^2 + v_B^2 - 2v_A v_B \cos\theta]^{1/2}$$

If β is the angle which $\vec{v}_{AB}$ makes with the direction of $\vec{v}_A$, then,

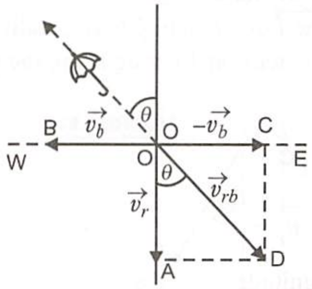
$$\tan\beta = \frac{v_B \sin(180^\circ - \theta)}{v_A + v_B \cos(180^\circ - \theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

Units used. The SI units of v_A, v_B and v_{AB} are ms^{-1} and cgs units are cms^{-1}.

Example 14 **Rain is falling vertically with a speed of 35 ms^{-1}. A woman rides a bicycle with a speed of 12 ms^{-1} in East to West direction. What is the direction in which she should hold her umbrella ?** (NCERT Solved Example)

Solution. Refer to Fig. 4.66,

FIGURE 4.66



$\vec{v}_r = (\overrightarrow{OA}) = 35$ ms^{-1}, along vertical

$\vec{v}_b = \overrightarrow{OB} = 12$ ms^{-1} along West. The woman can protect herself from the rain if she holds her umbrella in the direction of relative velocity of rain w.r.t. bicycle. To find the relative velocity of rain w.r.t. bicycle, bring the bicycle at rest by imposing a velocity $-\vec{v}_b (= \overrightarrow{OC})$ on bicycle and apply this velocity on rain also. Now the relative velocity of rain w.r.t. bicycle $(\vec{v}_{rb})$ will be the resultant of $\vec{v}_r (= \overrightarrow{OA})$ and $-\vec{v}_b (= \overrightarrow{OC})$; which will be represented by diagonal $\overrightarrow{OD}$ of rectangle *OADC*.

$$\tan\theta = \frac{AD}{OA} = \frac{v_b}{v_r} = \frac{12}{35} = 0{\cdot}3429 = \tan 18°56'$$

θ = 18° 56′ with the vertial towards the West.

Example 15 **To a person going westwards with a speed of 6 km h^{-1}, rain appears to fall vertically downwards with a speed of 8 km h^{-1}. Find the actual speed of the rain and its direction.**

Solution. Refer to 4.67, velocity of person, $\vec{v}_P = \overrightarrow{OA}$, where $v_P = 6$ km h^{-1}

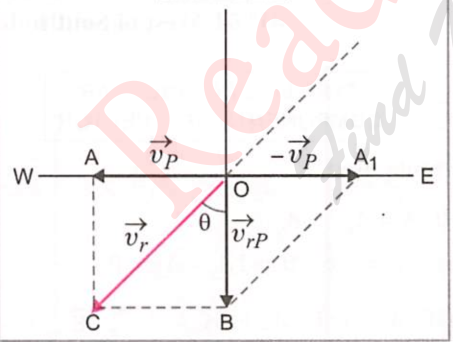
Relative velocity of rain *w.r.t.*, to person,

$$\vec{v}_{rP} = \overrightarrow{OB}, \text{ where } v_{rP} = 8 \text{ km h}^{-1}$$

Let $\vec{v}_r$ be the velocity of rain, then

$$\vec{v}_{rP} = \vec{v}_r - \vec{v}_P \quad \text{or} \quad \vec{v}_r = \vec{v}_{rP} + \vec{v}_P$$

FIGURE 4.67



or $\quad \vec{v}_r = \overrightarrow{OB} + \overrightarrow{OA} = \overrightarrow{OC}$

∴ Magnitude of $\vec{v}_r$

$$= |\overrightarrow{OC}| = \sqrt{(v_{rP})^2 + (v_P)^2}$$

$$= \sqrt{8^2 + 6^2} = \mathbf{10\ km\ h^{-1}}$$

Let θ be the angle which $\vec{v}_r$ makes with the vertical, then

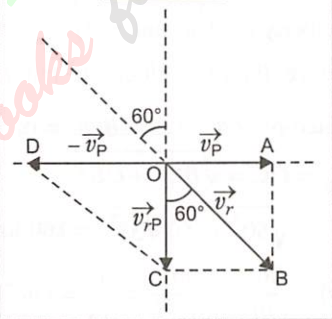
$$\tan\theta = \frac{BC}{OB} = \frac{OA}{OB} = \frac{v_P}{v_{rP}} = \frac{6}{8} = 0{\cdot}75$$

θ = 36°52′ (east of vertical)

Example 16 **A person standing on a road has to hold his umbrella at 60° with the vertical to keep the rain away. He throws the umbrella and starts running at 20 ms^{-1}. He finds that rain drops are hitting his head vertically. Find the speed of the rain drops with respect to (a) the road (b) the moving person.**

Solution. When person is at rest with respect to ground, the rain is coming to him at an angle 60° with the vertical *i.e.* along *OB*, when a person after throwing his umbrella runs on ground with velocity 20 ms^{-1}, the relative velocity of rain w.r.t. person is along *OC* as shown in Fig. 4.68.

FIGURE 4.68



Here, ∠*BOC* = 60°.

Velocity of rain w.r.t. ground, $\vec{v}_r = \overrightarrow{OB}$

Velocity of person w.r.t. ground, $\vec{v}_p = \overrightarrow{OA}$

where, $v_p = OA = 20$ ms^{-1} = *CB*

Velocity of rain w.r.t. person, $\vec{v}_{rp} = \overrightarrow{OC}$

(*a*) In Δ *OCB*, *OB* = *CB*/sin 60°

$$= 20/(\sqrt{3}/2) = 40/\sqrt{3}$$

∴ Velocity of rain w.r.t. ground,

$$v_r = 40/\sqrt{3} \text{ km/h} = \mathbf{23{\cdot}1\ km/h}$$

(*b*) In Δ *OCB*, *OC* = *CB*/tan 60°

$$= 20/\sqrt{3} = 11{\cdot}55 \text{ km/h.}$$

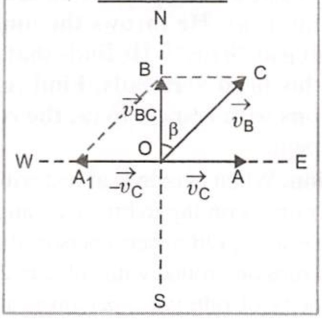
Velocity of rain w.r.t. person,

$$v_{rp} = \mathbf{11{\cdot}55\ km/h}$$

Example 17 **To a person going East in a car with a velocity of 50 km h^{-1}, a bus appears to move towards North with a velocity $50\sqrt{3}$ km h^{-1}. What is the actual velocity and direction of motion of the bus ?**

Solution. Refer to Fig. 4.69, the true velocity of car, $\vec{v}_C = \overrightarrow{OA} = 50$ km h^{-1} due East.

FIGURE 4.69



Relative velocity of bus w.r.t. car $\vec{v}_{BC} = \overrightarrow{OB} = 50\sqrt{3}$ km h^{-1}, due North. $\vec{v}_{BC}$ is the resultant velocity of $\vec{v}_B$ and $-\vec{v}_C$.

Therefore, the true velocity of bus $(\vec{v}_B)$ must be represented by $\overrightarrow{OC}$. Let $\angle BOC = \beta$. Then

$$v_B = OC = \sqrt{BC^2 + OB^2}$$

$$= \sqrt{50^2 + (50\sqrt{3})^2} = \mathbf{100\ km\ h^{-1}}$$

$$\tan\beta = \frac{BC}{OB} = \frac{50}{50\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

or **β = 30° East of North**

Example 18 **A train is moving with a velocity 72 km h^{-1} in the North-East direction. If another train is moving with a velocity 54 km h^{-1} in the North-West direction, then what is the relative velocity of the second train w.r.t. the first train ?**

Solution. Refer to Fig. 4.70,

Velocity of first train, $\vec{v}_A = \overrightarrow{OA}$

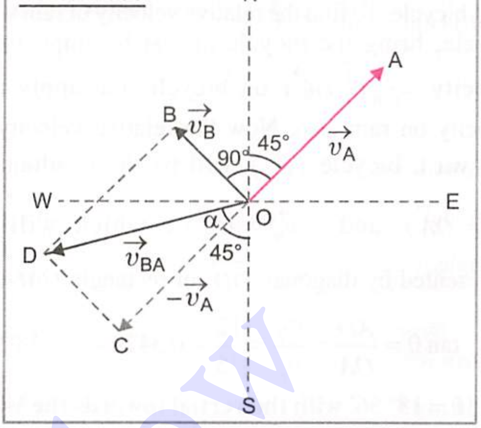
where $v_A = OA = 72$ km h^{-1}

$-\vec{v}_A = \overrightarrow{OC}$ where $(OC) = (OA) = v_A = 72$ km h^{-1}

Velocity of second train

$\vec{v}_B = \overrightarrow{OB}$, where, $v_B = 54$ km h^{-1}

FIGURE 4.70



Relative velocity of B w.r.t. A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A) = \overrightarrow{OB} + \overrightarrow{OC}$$

$$\therefore\ v_{BA} = \sqrt{(OB)^2 + (OC)^2 + 2(OB)(OC)\cos 90^\circ}$$

$$= \sqrt{v_B^2 + v_A^2 + 2 v_B v_A \times 0}$$

$$= \sqrt{54^2 + 72^2}$$

$$= \mathbf{90\ km\ h^{-1}}$$

$$\tan\alpha = \frac{CD}{OC} = \frac{OB}{OC} = \frac{v_B}{v_A} = \frac{54}{72} = \frac{3}{4} = 0{\cdot}75$$

$$= \tan 36^\circ 54'$$

$$\alpha = 36^\circ 54'$$

$$\therefore\ \angle SOD = 45^\circ + 36^\circ 54'$$

$$= \mathbf{81^\circ 54'\ West\ of\ South}$$

TYPE III. RECTANGULAR COMPONENTS OF A VECTOR

Formula used.

(i) If $\vec{A} = A_x\hat{i} + A_y\hat{j}$

then $A_x = A\cos\theta$ and $A_y = A\sin\theta$

(ii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$;

$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

(*iii*) Unit vector of $\vec{A}$ is,

$$\hat{A} = \frac{\vec{A}}{A} = \frac{A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

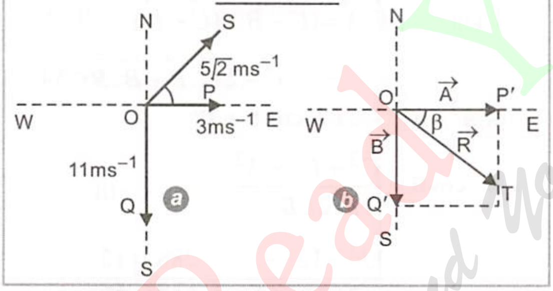
Units used. A and B are the magnitude of vectors $\vec{A}$ and $\vec{B}$, to be taken in the same units, θ is the angle which $\vec{A}$ makes with the x-axis.

Example 19 **A ball is acted upon by the following velocities. (i) 3 ms^{-1} due East (ii) 11 ms^{-1} due South and (iii) $5\sqrt{2}$ ms^{-1} due North-East. Find the magnitude and direction of resultant velocity.**

Solution. The various velocities acting on the ball are shown in Fig. 4.71(*a*). $\overrightarrow{OP}$ = 3 ms^{-1} due East, $\overrightarrow{OQ}$ = 11 ms^{-1} due South and $\overrightarrow{OS} = 5\sqrt{2}$ ms^{-1} inclined at an angle 45° with East.

Resolving $\overrightarrow{OS}$ into two rectangular components along East and North directions, we get

FIGURE 4.71



$$5\sqrt{2}\ \cos 45° = 5 \text{ ms}^{-1} \text{ along East}$$

and $5\sqrt{2}\ \sin 45° = 5$ ms^{-1} along North

∴ Effective velocity of ball along East,

$A = 3 + 5 = 8$ m/s, represented by $\overrightarrow{OP}'$

and the effective velocity of ball along South,

$B = 11 - 5 = 6$ m/s, represented by $\overrightarrow{OQ}'$.

The resultant velocity $\vec{R}$ will be represented by $\overrightarrow{OT}$, where $\angle P'OQ' = 90°$. Fig. 4.71(*b*).

Thus magnitude of $\vec{R}$ will be

$$R = \sqrt{A^2 + B^2} = \sqrt{8^2 + 6^2} = \mathbf{10\ ms^{-1}}$$

Let $\vec{R}$ make an angle β with East, then

$$\tan\beta = \frac{B}{A} = \frac{6}{8} = 0{\cdot}75 = \tan 36° \, 52'$$

or **β = 36° 52′ South of East**

Example 20 **What is the distance travelled by a point during the time τ, if it moves in X–Y plane, according to the relation**

$x = a \sin \omega t$ and $y = a\,(1 - \cos \omega t)$?

Solution. Velocity of the point along x-axis,

$$v_x = \frac{dx}{dt} = a\omega \cos \omega t$$

Velocity of the point along y-axis,

$$v_y = \frac{dy}{dt} = a\omega \cos \omega t$$

As, $\vec{v} = v_x\,\hat{i} + v_y\,\hat{j}$ and $v = \sqrt{v_x^2 + v_y^2}$

so, $v = \sqrt{a^2\,\omega^2 \cos^2 \omega t + a^2\,\omega^2 \sin^2 \omega t}$

$= a\,\omega$

∴ Distance travelled in time $\tau = v\tau = \boldsymbol{a\,\omega\,\tau}$

Example 21 **A vector $\vec{C}$, when added to the resultant of the vectors $\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ gives a unit vector along y-axis. Find the magnitude of $\vec{C}$.**

Solution. Resultant of $\vec{A}$ and $\vec{B}$ is

$$\vec{R} = \vec{A} + \vec{B} = (3\hat{i} - 4\hat{j} + 5\hat{k}) + (2\hat{i} + 3\hat{j} - 4\hat{k})$$
$$= 5\hat{i} - \hat{j} + \hat{k}$$

Unit vector along y-axis $= \hat{j}$

As per question

$$\vec{C} + \vec{R} = \hat{j} \text{ or } \vec{C} = \hat{j} - \vec{R} = \hat{j} - (5\hat{i} - \hat{j} + \hat{k})$$
$$= -5\hat{i} + 2\hat{j} - \hat{k}$$

$$|\vec{C}| = \sqrt{(-5)^2 + (2)^2 + (-1)^2} = \sqrt{30}$$

Example 22 **If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as $\vec{A}$ and parallel to $\vec{B}$.**

Solution. Here, $|\vec{A}| = \sqrt{3^2 + 4^2} = 5$;

$$|\vec{B}| = \sqrt{7^2 + 24^2} = 25$$

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Unit vector in the direction of $\vec{B}$ is

$$\hat{B} = \frac{\vec{B}}{B} = \frac{7\hat{i}+24\hat{j}}{25} = \frac{1}{25}(7\hat{i}+24\hat{j})$$

The vector having the same magnitude as $\vec{A}$ and parallel to $\vec{B}$ is

$$= |\vec{A}|\,\hat{B} = 5\times\frac{1}{25}(7\hat{i}+24\hat{j})$$

$$= \mathbf{\frac{1}{5}(7\hat{i}+24\hat{j})}$$

TYPE IV. DOT PRODUCT OR SCALAR PRODUCT OF TWO VECTORS

Formulae used. (*i*) $\vec{A}.\vec{B} = AB\cos\theta$ where θ is the smaller angle between $\vec{A}$ and $\vec{B}$.

(*ii*) If θ is smaller angle between $\vec{A}$ and $\vec{B}$,

then $\cos\theta = \dfrac{\vec{A}.\vec{B}}{AB}$

(*iii*) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Example 23 **For what value of m, the vector $\vec{A} = 2\hat{i}+3\hat{j}-6\hat{k}$ is perpendicular to $\vec{B} = 3\hat{i}-m\hat{j}+6\hat{k}$?**

Solution. Here, $\vec{A} = 2\hat{i}+3\hat{j}-6\hat{k}$;

$$\vec{B} = 3\hat{i}-m\hat{j}+6\hat{k}\ ;\ \theta = 90^\circ$$

$\therefore\quad \vec{A}.\vec{B} = AB\cos 90^\circ = 0$

Hence, $(2\hat{i}+3\hat{j}-6\hat{k}).(3\hat{i}-m\hat{j}+6\hat{k}) = 0$

or $\quad 2\times3 + 3\times(-m) + (-6)\times6 = 0$

or $\quad 6 - 3m - 36 = 0$ or $\mathbf{m = -10}$

Example 24 **Find the component of a vector $\vec{A} = 3\hat{i}+4\hat{j}$ along the direction of $2\hat{i}-3\hat{j}$.**

Solution. Here, $\vec{A} = (3\hat{i}+4\hat{j})$; $\vec{B} = (2\hat{i}-3\hat{j})$

Unit vector of $\vec{B}$,

$$\hat{B} = \frac{\vec{B}}{B} = \frac{2\hat{i}-3\hat{j}}{\sqrt{(2)^2+(-3)^2}} = \frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$$

Let θ be the angle between $\vec{A}$ and $\vec{B}$. The component of $\vec{A}$ along the direction of $\vec{B}$ is

$$= (A\cos\theta)\,\hat{B} = (\vec{A}\cdot\hat{B})\,\hat{B}$$

$$= \left[(3\hat{i}+4\hat{j})\cdot\left(\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}\right)\right]\left(\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}\right)$$

$$= \frac{(6-12)}{13}(2\hat{i}-3\hat{j}) = \mathbf{-\frac{6}{13}(2\hat{i}-3\hat{j})}$$

Example 25 **If vectors $\vec{A}, \vec{B}$ and $\vec{C}$ have magnitudes 5, 12 and 13 units and $\vec{A}+\vec{B}=\vec{C}$, find the angle between $\vec{B}$ and $\vec{C}$.**

Solution. As $\vec{A}+\vec{B}=\vec{C}$

$\therefore\quad \vec{A} = \vec{C} - \vec{B}$

Then $\quad \vec{A}.\vec{A} = (\vec{C}-\vec{B}).(\vec{C}-\vec{B})$

$$= \vec{C}.\vec{C} - 2\vec{C}.\vec{B} + \vec{B}.\vec{B}$$

or $A^2 = C^2 - 2\,CB\cos\theta + B^2$

or $\quad \cos\theta = \dfrac{C^2+B^2-A^2}{2CB}$

$$= \frac{13^2+12^2-5^2}{2\times13\times12} = \frac{288}{312} = \frac{12}{13}$$

$$\mathbf{\theta = \cos^{-1}(12/13)}$$

Example 26 **If $\vec{A}+\vec{B} = 3\hat{i}+6\hat{j}+2\hat{k}$ and $\vec{A}-\vec{B} = 6\hat{i}+3\hat{j}-\hat{k}$. Find the magnitude of $\vec{A}$ and $\vec{B}$ and their scalar product $\vec{A}.\vec{B}$.**

Solution. Here, $\vec{A}+\vec{B} = 3\hat{i}+6\hat{j}+2\hat{k}$

and $\quad \vec{A}-\vec{B} = 6\hat{i}+3\hat{j}-\hat{k}$

On solving, we get $\vec{A} = \dfrac{1}{2}[9\hat{i}+9\hat{j}+\hat{k}]$

and $\quad \vec{B} = \dfrac{1}{2}[-3\hat{i}+3\hat{j}+3\hat{k}]$

$$\therefore \quad |\vec{A}| = \sqrt{\frac{81}{4}+\frac{81}{4}+\frac{1}{4}} = \sqrt{\frac{163}{4}}$$

$$= \frac{1}{2}\sqrt{163} = \frac{12{\cdot}77}{2} = \mathbf{6{\cdot}39}$$

$$|\vec{B}| = \sqrt{\frac{9}{4}+\frac{9}{4}+\frac{9}{4}} = \frac{3}{2}\sqrt{3} \approx \mathbf{2{\cdot}6}$$

$$\vec{A}\,.\,\vec{B} = \frac{1}{2}[9\,\hat{i}+9\,\hat{j}+\hat{k}]\,.\,\frac{1}{2}[-3\,\hat{i}+3\,\hat{j}+3\,\hat{k}]$$

$$= \frac{1}{4}[9\times(-3)+9\times3+1\times3] = \frac{3}{4} = \mathbf{0{\cdot}75}$$

Example 27 **If unit vectors $\hat{A}$ and $\hat{B}$ are inclined at an angle θ, then prove that**

$$|\hat{A}-\hat{B}| = \mathbf{2\sin\theta/2}$$

Solution. We know that

$$|\hat{A}-\hat{B}|^2 = (\hat{A}-\hat{B}).(\hat{A}-\hat{B})$$

$$\text{or}\quad |\hat{A}-\hat{B}|^2 = \hat{A}.\hat{A}+\hat{B}.\hat{B}-2\,\hat{A}.\hat{B}$$

$$= 1 + 1 - 2\,(1)\,(1)\cos\theta = 2\,(1-\cos\theta)$$

$$= 2\,[1-(1-2\sin^2\theta/2)] = 4\sin^2\theta/2$$

$$(\because \cos 2\theta = 1 - 2\sin^2\theta)$$

$$\therefore \quad |\hat{A}-\hat{B}| = \mathbf{2\sin\theta/2}$$

Example 28 **Find the angle between force $\vec{F} = (3\,\hat{i}+4\,\hat{j}-5\,\hat{k})$ and displacement $\vec{d} = (5\,\hat{i}+4\,\hat{j}+3\,\hat{k})$ unit. Also, find the projection of $\vec{F}$ on $\vec{d}$.** **(NCERT Solved Example)**

Solution. Here, $\vec{F} = (3\,\hat{i}+4\,\hat{j}-5\,\hat{k})$;

$$\vec{d} = (5\,\hat{i}+4\,\hat{j}+3\,\hat{k})$$

$$\vec{F}.\vec{d} = (3\,\hat{i}+4\,\hat{j}-5\,\hat{k})\cdot(5\,\hat{i}+4\,\hat{j}+3\,\hat{k})$$

$$= 3\,(5) + 4\,(4) - 5\,(3)$$

$$= 16 \text{ units}$$

$$F = \sqrt{F_x^2+F_y^2+F_z^2}$$

$$= \sqrt{3^2+4^2+(-5)^2} = \sqrt{50}$$

$$d = \sqrt{d_x^2+d_y^2+d_z^2} = \sqrt{5^2+4^2+3^2} = \sqrt{50}$$

$$\text{Now,}\ \cos\theta = \frac{\vec{F}\,.\,\vec{d}}{F\,d} = \frac{16}{\sqrt{50}\,\sqrt{50}} = \frac{16}{50} = 0{\cdot}32$$

$$\theta = \cos^{-1}(0{\cdot}32) = \mathbf{71{\cdot}3^\circ}$$

Unit vector along $\vec{d}$ is

$$\hat{d} = \frac{5\,\hat{i}+4\,\hat{j}+3\,\hat{k}}{\sqrt{5^2+4^2+3^2}} = \frac{5\,\hat{i}+4\,\hat{j}+3\,\hat{k}}{\sqrt{50}}$$

$$\vec{F}.\hat{d} = (3\,\hat{i}+4\,\hat{j}-5\,\hat{k}).\,\frac{(5\,\hat{i}+4\,\hat{j}+3\,\hat{k})}{\sqrt{50}}$$

$$= \frac{3\,(5)+4\,(4)-5\,(3)}{\sqrt{50}} = \frac{16}{\sqrt{50}}$$

∴ Projection of $\vec{F}$ on $\vec{d}$ = component vector of $\vec{F}$ along $\vec{d}$. It is

$$= (\vec{F}.\hat{d})\,\hat{d} = \frac{16}{\sqrt{50}}\,\frac{(5\,\hat{i}+4\,\hat{j}+3\,\hat{k})}{\sqrt{50}}$$

$$= \mathbf{0{\cdot}32\,(5\,\hat{i}+4\,\hat{j}+3\,\hat{k})}$$

TYPE V. CROSS PRODUCT OR VECTOR PRODUCT OF TWO VECTORS

Formulae used

(*i*) $\vec{A}\times\vec{B} = \vec{C} = AB\sin\theta\ \hat{C}$; where $\hat{C}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, θ is the smaller angle between $\vec{A}$ and $\vec{B}$.

(*ii*) If two vectors are parallel or antiparallel then

$$\vec{A}\times\vec{B} = 0$$

(*iii*) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$;

$$\hat{i}\times\hat{j} = \hat{k}\ ;\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$$

(*iv*) If $\vec{A} = A_x\,\hat{i}+A_y\,\hat{j}+A_z\,\hat{k}$,

and $\vec{B} = B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k}$, then

$$\vec{A}\times\vec{B} = \hat{i}\,(A_yB_z - A_zB_y) + \hat{j}\,(A_zB_x - A_xB_z) + \hat{k}\,(A_xB_y - A_yB_x)$$

(*v*) Unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$ is $\hat{C}=\dfrac{\vec{A}\times\vec{B}}{AB\sin\theta}=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

(*vi*) $|\vec{A}\times\vec{B}|$ = area of parallelogram whose two sides are represented by two vectors $\vec{A}$ and $\vec{B}$
= twice the area of a triangle whose two sides are represented by two vectors $\vec{A}$ and $\vec{B}$.

Example 29 **Find the scalar and vector products of two vectors**

$$\vec{a}=(3\hat{i}-4\hat{j}+5\hat{k})\text{ and }\vec{b}=(-2\hat{i}+\hat{j}-3\hat{k}).$$

(NCERT Solved Example)

Solution. Here,

$$\vec{a}=3\hat{i}-4\hat{j}+5\hat{k}\ ,\ \vec{b}=-2\hat{i}+\hat{j}-3\hat{k}$$

$$\vec{a}\cdot\vec{b}=(3\hat{i}-4\hat{j}+5\hat{k})\cdot(-2\hat{i}+\hat{j}-3\hat{k})$$
$$=3(-2)-4(1)+5(-3)$$
$$=-6-4-15=\mathbf{-25}$$

$$\vec{a}\times\vec{b}=(3\hat{i}-4\hat{j}+5\hat{k})\times(-2\hat{i}+\hat{j}-3\hat{k})$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -4 & 5\\ -2 & 1 & -3\end{vmatrix}$$

$$=\hat{i}\,(12-5)+\hat{j}\,(-10+9)+\hat{k}\,(3-8)$$

$$=\mathbf{7\hat{i}-\hat{j}-5\hat{k}}$$

Example 30 **Calculate the area of a parallelogram whose two adjacent sides are formed by the vectors**

$$\vec{A}=3\hat{i}+5\hat{j}\ \text{ and }\ \vec{B}=-3\hat{i}+7\hat{j}.$$

Solution. $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 5 & 0\\ -3 & +7 & 0\end{vmatrix}$

$$=\hat{i}\,[0-0]+\hat{j}\,[0-0]+\hat{k}\,[21+15]$$
$$=+36\hat{k}$$

Area of parallelogram $=|\vec{A}\times\vec{B}|$

$$=\sqrt{0^2+0^2+(+36)^2}$$
$$=\mathbf{36\ sq.\ unit}$$

Example 31 **The diagonals of a parallelogram are represented by $\vec{d}_1=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{d}_2=6\hat{i}+5\hat{j}-3\hat{k}$. Find the area of the parallelogram.**

Solution. Here, $\vec{d}_1=2\hat{i}+3\hat{j}-5\hat{k}$;

$$\vec{d}_2=6\hat{i}+5\hat{j}-3\hat{k}$$

$$\vec{d}_1\times\vec{d}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -5\\ 6 & 5 & -3\end{vmatrix}$$

$$=\hat{i}\,(-9+25)+\hat{j}\,(-30+6)+\hat{k}\,(10-18)$$

$$=16\hat{i}-24\hat{j}-8\hat{k}$$

$$|\vec{d}_1\times\vec{d}_2|=\sqrt{16^2+(-24)^2+(-8)^2}$$
$$=\sqrt{256+576+64}=\sqrt{896}$$
$$=29\cdot93\text{ sq. unit}$$

Area of parallelogram $=\dfrac{1}{2}|\vec{d}_1\times\vec{d}_2|$

$$=\frac{1}{2}\times29\cdot93\ =\ \mathbf{14\cdot97\ sq.\ unit}$$

Example 32 **Determine the sine of the angle between the vectors**

$$\mathbf{2\hat{i}+3\hat{j}-4\hat{k}\ \text{and}\ (\hat{i}-2\hat{j}+2\hat{k}).}$$

Solution. Here $\vec{A}=(2\hat{i}+3\hat{j}-4\hat{k})$

and $\vec{B}=(\hat{i}-2\hat{j}+2\hat{k})$

$$(\vec{A}\times\vec{B})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -4\\ 1 & -2 & 2\end{vmatrix}$$

$$=\hat{i}\,(6-8)+\hat{j}\,(-4-4)+\hat{k}\,(-4-3)$$

$$=-2\hat{i}-8\hat{j}-7\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(-2)^2+(-8)^2+(-7)^2} = \sqrt{117}$$

$$|\vec{A}| = \sqrt{2^2+3^2+(-4)^2} = \sqrt{29} \;;$$

$$|\vec{B}| = \sqrt{1^2+(-2)^2+2^2} = \sqrt{9}$$

$$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}|\,|\vec{B}|} = \frac{\sqrt{117}}{\sqrt{29}\sqrt{9}} = \sqrt{\frac{13}{29}}$$

or $$\theta = \sin^{-1}\sqrt{\frac{13}{29}}$$

Example 33 **Determine a unit vector which is perpendicular to both**

$$\vec{A} = 2\hat{i}+\hat{j}+\hat{k}$$

and $$\vec{B} = \hat{i}-\hat{j}+2\hat{k}.$$

Solution. Let $\vec{C}$ be perpendicular to $\vec{A}$ and $\vec{B}$, then $\vec{C}$ can be taken as the cross product of $\vec{A}$ and $\vec{B}$

$$\therefore\quad \vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}\,\{1\times 2 - 1\times(-1)\} + \hat{j}\,\{1\times 1 - 2\times 2\}$$
$$+\hat{k}\,\{2\times(-1) - 1\times 1\}$$

$$= \hat{i}\,(3) + \hat{j}\,(-3) + \hat{k}\,(-3) = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\therefore\quad |\vec{C}| = \sqrt{3^2+(-3)^2+(-3)^2}$$

$$= \sqrt{27} = 3\sqrt{3}$$

$\therefore$ unit vector along $\vec{C}$ is, $\hat{C} = \dfrac{\vec{C}}{|\vec{C}|}$

$$= \frac{3\hat{i}-3\hat{j}-3\hat{k}}{3\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}\,(\hat{i}-\hat{j}-\hat{k})$$

TYPE VI. HORIZONTAL PROJECTILE

Formulae used. (*i*) For motion along *x*-axis.

$$v_x = u_x + a_x t \quad\text{and}\quad x = x_0 + u_x t + \frac{1}{2}a_x t^2$$

(*ii*) For motion along *y*-axis $\quad v_y = u_y + a_y t$

and $$y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

(*iii*) Time of flight, $T = \sqrt{2h/g}$

(*iv*) Horizontal range, $R = u_x T = u_x\sqrt{2h/g}$

(*v*) Velocity of projectile at an instant of its flight is

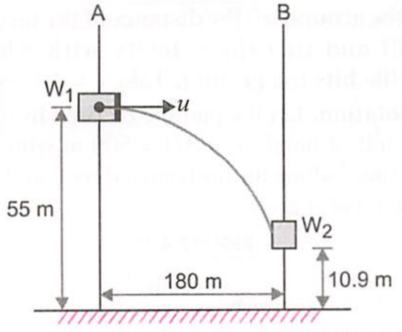
$$v = \sqrt{v_x^2 + v_y^2} \quad\text{and}\quad \tan\beta = v_y/v_x$$

Units used. Here, u_x, u_y, v_x, v_y and v are in ms^{-1} ; a_x, a_y in ms^{-2}, t in second and R in metre.

Example 34 **Two tall buildings face each other and are at a distance of 180 m from each other. With what velocity must a ball be thrown horizontally from a window 55 m above the ground in one building, so that it enters a window 10·9 m above the ground in second window. g = 9·8 ms^{-2}**

Solution. Refer to Fig. 4.72, *A* and *B* are two tall buildings which are 180 m apart. W_1 and W_2 are the two windows in buildings *A* and *B* respectively.

FIGURE 4.72



Vertical distance between two windows

$$= 55 - 10{\cdot}9$$
$$= 44{\cdot}1 \text{ m}$$

Taking vertical downward motion of ball from W_1 to W_2,

we have, $u_y = 0$, $a_y = g = 9{\cdot}8$ ms^{-2}, $t = ?$,
$y = 44{\cdot}1$ m

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As $y = u_y\,t + \frac{1}{2}\,a_y\,t^2$

$$\therefore\ 44{\cdot}1 = 0 + \frac{1}{2} \times 9{\cdot}8 \times t^2 \text{ or } t^2 = \frac{44{\cdot}1\times 2}{9{\cdot}8} = 9$$

or $t = 3$ s

Required horizontal velocity of ball

$$= \frac{\text{horizontal distance}}{\text{time taken}} = \frac{180}{3} = \mathbf{60\ ms^{-1}}$$

Example 35 **A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. Take $g = 9{\cdot}8$ m/s^2.**

(NCERT Solved Example)

Solution. Here, $h = 490$ m ; $u_x = 15$ ms^{-1}, $a_y = 9{\cdot}8$ ms^{-1} ; $a_x = 0$

$$t = \sqrt{\frac{2\,h}{g}} = \sqrt{\frac{2\times 490}{9{\cdot}8}} = \mathbf{10\,s}$$

$$v_x = u_x + a_x\,t = 15 + 0 \times 10 = 15 \text{ m/s}$$

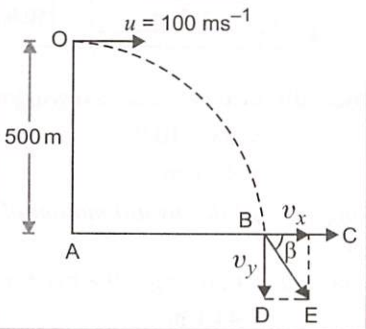
$$v_y = u_y + a_y\,t = 0 + 9{\cdot}8 \times 10 = 98 \text{ m/s}$$

$$\therefore\quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = \mathbf{99{\cdot}1\,m/s}$$

Example 36 **A particle is fired horizontally with a velocity of 100 ms^{-1} from the top of a hill 500 m high. Find (*i*) the time taken to reach the ground (*ii*) the distance of the target from the hill and (*iii*) the velocity with which the projectile hits the ground. Take $g = 10$ ms^{-2}.**

Solution. Let the particle be fired from the top O of a hill of height $h = AO = 500$ m with velocity $u = 100$ ms^{-1} along the horizontal direction. It reaches the ground at B after time t.

FIGURE 4.73



(*i*) **Taking vertical downward motion of the particle from O to B,** we have,

$$y_0 = 0,\ y = 500 \text{ m} ;\ u_y = 0,\ a_y = 10 \text{ ms}^{-2},\ t = ?$$

As $y = y_0 + u_y\,t + \frac{1}{2}\,a_y\,t^2$

$$\therefore\quad 500 = 0 + 0 \times t + \frac{1}{2} \times 10 \times t^2$$

or $t^2 = 100$ or $t = \mathbf{10\ s}$

(*ii*) **Distance of the target from the hill = $AB = x$**

Taking horizontal motion of particle from O to B, we have

$$x_0 = 0,\ x = ?,\ u_x = 100 \text{ ms}^{-1},\ a_x = 0,\ t = 10 \text{ s}$$

As, $x = x_0 + u_x\,t + \frac{1}{2}\,a_x\,t^2$

$$\therefore\ x = 0 + 100 \times 10 + \frac{1}{2} \times 0 \times 10^2 = \mathbf{1000\ m}$$

(*iii*) Let v_x and v_y be the horizontal and vertical components of velocity v of the particle at B.

Then $v_x = u_x + a_x\,t = u_x = 100$ ms^{-1}

$$v_y = u_y + a_y\,t = 0 + 10 \times 10 = 100 \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{100^2 + 100^2} = \mathbf{100\sqrt{2}\ ms^{-1}}$$

Let the resultant velocity at B make an angle β with the horizontal direction, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{100}{100} = 1 = \tan 45^\circ \text{ or } \beta = \mathbf{45^\circ}$$

Example 37 **A body is thrown horizontally from the top of a tower and strikes the ground after two seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. $g = 9{\cdot}8$ ms^{-2}.**

Solution. Here, $u = ?$, $t = 2$ sec., $\beta = 45^\circ$, $y = ?$

$$\tan\beta = \frac{v_y}{v_x} = \frac{v_y}{u} = \tan 45^\circ = 1$$

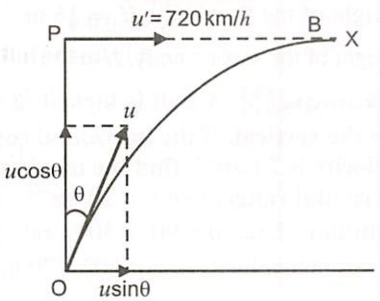
$$\therefore\quad u = v_y = gt = 9{\cdot}8 \times 2 = \mathbf{19{\cdot}6\ m/s}$$

$$y = \frac{1}{2}\,gt^2 = \frac{1}{2} \times 9{\cdot}8\,(2)^2 = \mathbf{19{\cdot}6\,m}$$

Example 38 **A fighter plane flying horizontally at an altitude of 1·5 km with speed of 720 km h^{-1} passes directly overhead an anticraft gun. At what angle from the vertical should the shell be fired from the gun with muzzle speed 400 ms^{-1} to hit the plane in shortest time ?**

Solution. Let the figher plane be flying horizontally with a speed u' at height $OP = 1{\cdot}5$ km. Let P be directly overhead an anticraft gun at O. Let the shell be fired with velocity u making an angle θ with the vertical direction so that it hits the fighter plane at B in shortest time, Fig. 4.74.

FIGURE 4.74



Resolving u into two rectangular components, we have :

$u \cos \theta$, vertically upwards and $u \sin \theta$, horizontally. If t is the time taken by the shell to hit the fighter plane, then horizontal distance travelled by the fighter plane in time t with velocity u' is equal to the horizontal distance travelled by the shell in time t with velocity $u \sin \theta$ *i.e.*,

$$u' t = u \sin \theta \, t$$

or $\quad \sin \theta = u'/u$

Here, $\quad u' = 720 \text{ km } h^{-1}$

$$= 720 \times (1000 \text{ m}) \times (60 \times 60 \text{ s})^{-1}$$

$$= 200 \text{ ms}^{-1}, \text{ and } u = 400 \text{ ms}^{-1}$$

$\therefore \quad \sin \theta = 200/400 = 1/2$

or $\quad$ **$\theta = 30°$ with vertical**

TYPE VII. ANGULAR PROJECTILE

Formulae used. (*i*) Time of flight, $T = \dfrac{2 u \sin \theta}{g}$

(*ii*) Maximum height, $h = \dfrac{u^2 \sin^2 \theta}{2 g}$

(*iii*) Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

where θ is the angle of projection of the projectile with the horizontal

(*iv*) Maximum horizontal range, $R_{\max} = \dfrac{u^2}{g}$

(*v*) Path of projectile, $y = x \tan \theta - \dfrac{g x^2}{(2 u^2 \cos^2 \theta)}$

Units used. u is in m/s ; g in m/s^2, θ in degrees ; h and R in metres.

Example 39 **A bullet fired at an angle of 30° with the horizontal hits the ground $3\sqrt{3}$ km away. Can we hit a target at a distance of $6\sqrt{2}$ km by adjusting its angle of projection ?**

Solution. Here, $\theta = 30°$,

$$R = 3\sqrt{3} \text{ km} = 3000\sqrt{3} \text{ m}$$

As, $\quad R = \dfrac{u^2 \sin 2\theta}{g}$

$\therefore \quad 3000\sqrt{3} = \dfrac{u^2 \sin 2 \times 30°}{g}$

or $\quad \dfrac{u^2}{g} = \dfrac{3000\sqrt{3}}{\sin 60°} = \dfrac{3000\sqrt{3}}{\sqrt{3}/2}$

$$= 6000 \text{ m} = 6 \text{ km}$$

The maximum horizontal range $= \dfrac{u^2}{g} = 6$ km, which is less than $6\sqrt{2}$ km. Thus the bullet **will never hit the target at a** distance $6\sqrt{2}$ km by adjusting angle of projection.

Example 40 **How much high above the ground a man can throw a ball if he is able to throw the same ball upto maximum distance of 60 m ?**

Solution. Here, $R_{\max} = 60$ m.

As, $\quad R_{\max} = \dfrac{u^2}{g}$; so $\quad 60 = \dfrac{u^2}{g}$ $\quad$ ...(*i*)

Let h be the maximum height attained by the ball.

Taking vertical upward motion of ball upto maximum height h, we have

$$u_y = u, \; a_y = -g, \; v_y = 0, \; y = h$$

As $\quad v_y^2 = u_y^2 + 2 a_y \, y$

$\therefore \quad 0 = u^2 + 2(-g) h$

or $\quad h = \dfrac{u^2}{2g} = \dfrac{1}{2} \times 60$ **= 30 m**

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Example 41 **Calculate the horizontal distance travelled by a ball thrown with a velocity $20\sqrt{2}$ ms^{-1} without hitting the ceiling of an auditorium of height 20 m. Use $g = 10$ ms^{-2}.**

Solution. Here, $u = 20\sqrt{2}\ \text{ms}^{-1}$, $H = 20$ m ;

$$g = 10\ \text{ms}^{-2}$$

$$\text{As, } H = \frac{u^2 \sin^2\theta}{2g} \quad \therefore \quad 20 = \frac{(20\sqrt{2})^2 \sin^2\theta}{2\times 10}$$

$$\text{or } 400 = 800 \sin^2\theta \text{ or } \sin^2\theta = \frac{400}{800} = \frac{1}{2}$$

$$\text{or } \sin\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

$$\text{or } \theta = 45^\circ$$

$$\text{Horizontal range } = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(20\sqrt{2})^2 \sin 2\times 45^\circ}{10} = \frac{800\times 1}{10} = \mathbf{80\ m}$$

Example 42 **One body is thrown at an angle θ with the horizontal and another similar body is thrown at an angle θ with the vertical direction from the same point with some velocity 40 ms^{-1}. The second body reaches 50 metres higher than the first body. Determine their individual heights. Take $g = 10$ ms^{-2}.**

Solution. For the first body, angle of projection is θ with the horzontal direction. Velocity of projection $u = 40$ m/s.

Maximum height,

$$H_1 = \frac{u^2 \sin^2\theta}{2g} = \frac{(40)^2 \sin^2\theta}{2\times 10} \qquad ...(i)$$

For the second body, angle of projection = (90° – θ) with the horizontal direction, velocity of projection, $u = 40$ ms^{-1}.

Maximum height reached, $H_2 = H_1 + 50$ ...(*ii*)

Where

$$H_2 = \frac{(40)^2 \sin^2(90^\circ - \theta)}{2\times 10} = \frac{(40)^2 \cos^2\theta}{2\times 10}$$

From (*ii*), we have

$$\therefore H_1 + 50 = \frac{(40)^2 \cos^2\theta}{2\times 10} \qquad ...(iii)$$

Adding (*i*) and (*iii*), we have

$$2H_1 + 50 = \frac{(40)^2}{2\times 10}[\sin^2\theta + \cos^2\theta]$$

$$= \frac{(40)^2}{20} = 80 \text{ or } 2H_1 = 80 - 50 = 30$$

or $H_1 = 15$ m

Height of the first body, $H_1 = \mathbf{15\ m}$

Height of the second body, $H_2 = 15 + 50 = \mathbf{65\ m}$

Example 43 **A ball is kicked at an angle 30° with the vertical. If the horizontal component of its velocity is 20 ms^{-1}, find the maximum hight and horizontal range. Use $g = 10$ ms^{-2}.**

Solution. Here, θ = 90° – 30° = 60°

Horizontal velocity, $u \cos 60^\circ = 20$ ms^{-1}

$$\text{or } u = \frac{20}{\cos 60^\circ} = \frac{20}{1/2} = 40\ \text{ms}^{-1}$$

Maximum height,

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{40^2 \sin^2 60^\circ}{2\times 10}$$

$$= \frac{1600}{20}\times\left[\frac{\sqrt{3}}{2}\right]^2 = \mathbf{60\ m}$$

Horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{40^2 \sin 2\times 60^\circ}{10}$$

$$= \frac{1600}{10}(\sqrt{3}/2) = \mathbf{138{\cdot}6\ m}$$

Example 44 **A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate (*a*) the maximum height (*b*) the time taken by the ball to return to the same level, and (*c*) the distance from the thrower to the point where the ball returns to the same level.**

(NCERT Solved Example)

Solution. Here, $u = 28$ ms^{-1} ; θ = 30°

(*a*) Maximum height,

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{(28)^2(\sin 30^\circ)^2}{2\times 9{\cdot}8} = \mathbf{10{\cdot}0\ m}$$

(*b*) Time of flight,

$$T = \frac{2u\sin\theta}{g} = \frac{2\times 28\times \sin 30^\circ}{9{\cdot}8} = \mathbf{2{\cdot}86\ s}$$

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(*c*) Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

$$= \frac{(28)^2 \times \sin(2\times 30°)}{9{\cdot}8} = \mathbf{69{\cdot}3\ m}$$

Example 45 A boy stands at 78·4 m from a building and throws a ball which just enters a window 39·2 m above the ground. Calculate the velocity of projection of the ball. Fig. 4.75.

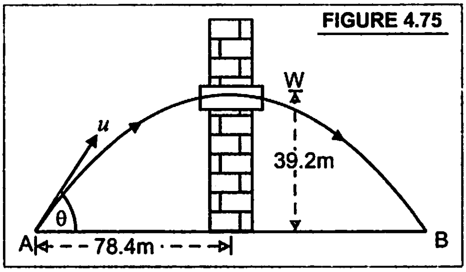


FIGURE 4.75

Solution. Let a boy standing at *A* throw a ball with a velocity *u* at an angle θ with the horizontal, which just enters window *W*.

As the boy is at 78·4 m from the building and the ball just enters the window 39·2 m above the ground, therefore

Max. height, $\dfrac{u^2 \sin^2\theta}{2g} = 39{\cdot}2$ m ...(*i*)

and horizontal range,

$$\frac{u^2 \sin 2\theta}{g} = 2 \times 78{\cdot}4 \text{ m} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{u^2 \sin^2\theta}{2g} \times \frac{g}{u^2\, 2\sin\theta\cos\theta} = \frac{39{\cdot}2}{2\times 78{\cdot}4}$$

or $\dfrac{\sin^2\theta}{2\times 2\sin\theta\cos\theta} = \dfrac{1}{4}$

or $\tan\theta = 1$ or $\theta = 45°$

Substituting in (*ii*), we get

$$\frac{u^2 \sin 90°}{9{\cdot}8} = 2 \times 78{\cdot}4$$

or $u = \sqrt{2\times 78{\cdot}4\times 9{\cdot}8} = \mathbf{39{\cdot}2\ ms^{-1}}$

Example 46 From a point on the ground at a distance 15 m from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance 5 m on the other side. Find the height of the wall.

Solution. Let *u* be the velocity of projection from *O* at an angle 45°, which just clears the top *A* of wall of height *h*. Fig. 4.76.

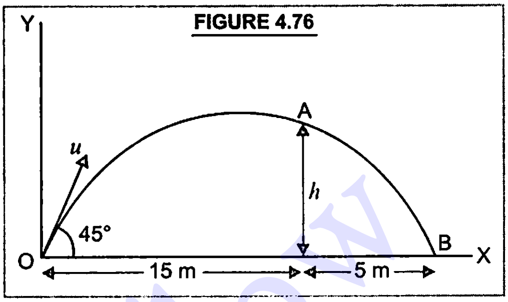


FIGURE 4.76

Given, horizontal range = 15 + 5 = 20 m

So $\dfrac{u^2 \sin 2\theta}{g} = 20$

or $\dfrac{u^2 \sin 2\times 45°}{g} = 20$

or $\dfrac{u^2}{g} = 20$ ...(i)

Let (*x*, *y*) be the coordinates of *A*, then

x = 15 m and *y* = *h*.

Using the equation of trajectory, we have

$$y = x\tan\theta - \frac{1}{2}\frac{g\,x^2}{u^2\cos^2\theta}$$

$$h = 15\tan 45° - \frac{1}{2}\times\frac{15^2}{20\times\frac{1}{2}} = \mathbf{3{\cdot}75\ m}$$

Example 47 At what angle should a body be projected with a velocity 20 ms^{-1} just to pass over the obstacle 12 m high at a horizontal distance of 24 m ? Take $g = 10$ ms^{-2}.

Solution. Refer to Fig. 4.77,

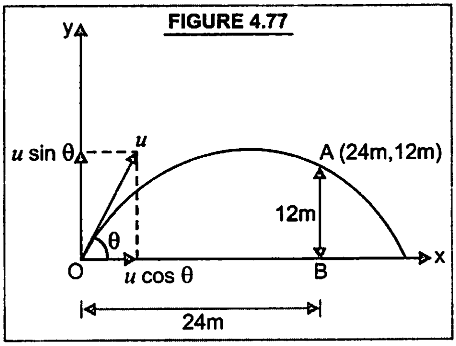


FIGURE 4.77

Taking horizontal motion of body projected from O, from O to A

we have, $u_x = u\cos\theta = 20\cos\theta\ \text{ms}^{-1}$,

$x_0 = 0,\ x = 24\ \text{m},\ a_x = 0,\ t = t$ (say)

As, $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

$\therefore\quad 24 = 0 + (u\cos\theta) \times t = (20\cos\theta) \times t$

or $\quad t = \dfrac{24}{20\cos\theta} = \dfrac{6}{5\cos\theta}$

Taking vertical upward motion of body from O to A,

we have : $u_y = u\sin\theta = 20\sin\theta,\ y_0 = 0,$

$y = 12\ \text{m},\ a_y = -10\ \text{ms}^{-2},\ t = t$

As, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$\therefore\quad 12 = 0 + (20\sin\theta)\, t + \frac{1}{2} \times (-10)\, t^2$

or $\quad 12 = (20\sin\theta) \times \dfrac{6}{5\cos\theta} - 5 \times \left(\dfrac{6}{5\cos\theta}\right)^2$

or $\quad 12 = 24\tan\theta - \dfrac{5 \times 36}{25\cos^2\theta}$

$= 24\tan\theta - \dfrac{36}{5}\sec^2\theta$

or $\quad 12 = 24\tan\theta - \dfrac{36}{5}(1 + \tan^2\theta)$

or $\quad 60 = 120\tan\theta - 36 - 36\tan^2\theta$

or $\quad 36\tan^2\theta - 120\tan\theta + 96 = 0$

or $\quad 3\tan^2\theta - 10\tan\theta + 8 = 0$

or $\quad (\tan\theta - 2)(3\tan\theta - 4) = 0$

or $\quad \tan\theta = 2$ or $4/3$ $\quad$ or $\quad$ $\boldsymbol{\theta = 63° 26'}$ **or** $\boldsymbol{53° 4'}$

Example 48 **A body is projected with a velocity of 40 ms^{-1}. After 2 s it crosses a vertical pole of height 20·4 m. Find the angle of projection and horizontal range of projectile. ($g = 9{\cdot}8$ ms^{-2}).**

Solution. Here, $u = 40\ \text{ms}^{-1}$;

$t = 2\ \text{s}\ ;\ y = 20{\cdot}4\ \text{m}$

Taking vertical upward motion of the projectile from point of projection up to the top of vertical pole we have

$u_y = 40\sin\theta\ ;\ a_y = -9{\cdot}8\ \text{m/s}^2,$

$t = 2\ \text{s}\ ;\ y = 20{\cdot}4\ \text{m}.$

As, $\quad y = u_y\, t + \frac{1}{2} a_y\, t^2$

$\therefore\quad 20{\cdot}4 = 40\sin\theta \times 2 + \frac{1}{2}(-9{\cdot}8) \times 2^2$

$= 80\sin\theta - 19{\cdot}6$

or $\quad \sin\theta = (20{\cdot}4 + 19{\cdot}6)/80 = 1/2$

or $\quad \theta = 30°$

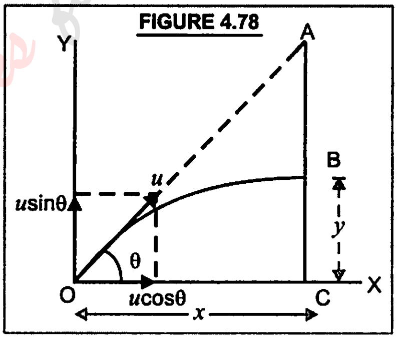
Horizontal range $= \dfrac{u^2}{g}\sin 2\theta$

$= \dfrac{40^2}{9{\cdot}8}\sin 2 \times 30° = \mathbf{141{\cdot}4\ m}$

Example 49 **A hunter aims his gun and fires a bullet directly at a monkey in a tree. At the instant the bullet leaves the barrel of the gun, the monkey drops. Will the bullet hit the monkey. Substantiate your answer with proper reasoning.**

Solution. Let the monkey stationed at *A*, be fired with a gun from *O* with a velocity *u* at an angle θ with the horizontal direction *OX*. Draw *AC*, perpendicular to *OX*. Let the bullet cross the vertical line *AC* at *B* after time *t* and coordinates of *B* be (*x*, *y*) *w.r.t.* origin *O*. Fig. 4.78.

FIGURE 4.78



$\therefore\quad t = \dfrac{OC}{u\cos\theta} = \dfrac{x}{u\cos\theta}$ $\quad$ ...(*i*)

(where $OC = x$)

In Δ *OAC*, $\quad AC = OC\tan\theta = x\tan\theta$ $\quad$ ...(*ii*)

Clearly $\quad CB = y$ = the vertical distance travelled by the bullet in time *t*.

Taking motion of the bullet from O to B along Y-axis we have

$y_0 = 0,\ y = y,\ u_y = u\sin\theta,\ a_y = -g,\ t = t$

As, $\quad y = y_0 + u_y\, t + \frac{1}{2} a_y\, t^2$

$\therefore\quad y = 0 + u\sin\theta\, t + \frac{1}{2}(-g)\, t^2$

$= u\sin\theta\, t - \frac{1}{2} g t^2$ $\quad$ ...(*iii*)

$\therefore AB = AC - BC = x \tan\theta - y$

$$= x\tan\theta - \left(u\sin\theta\, t - \frac{1}{2}gt^2\right)$$

$$= x\tan\theta - \left(u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2}gt^2\right)$$

[from (*i*)]

$$= x\tan\theta - x\tan\theta + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

It means the bullet will pass through the point B on vertical line AC at a vertical distance $\frac{1}{2}gt^2$ below point A.

The distance through which the monkey falls vertically in time $t = \frac{1}{2}gt^2 = AB$.

It means the bullet and monkey will pass through the point B simultaneously. Therefore, **the bullet will hit the monkey.**

Example 50 **The maximum height attained by a projectile is increased by 10% by increasing its speed of projection, without changing the angle of projection. What will the percentage increase in the horizontal range.**

Solution. As max. height,

$$H = \frac{u^2}{2g}\sin^2\theta \qquad \ldots(i)$$

Let ΔH be the increase in H when u changes by Δu. Differentiating (*i*), we get

$$\Delta H = \frac{2u\,\Delta u \sin^2\theta}{2g}$$

$$\therefore \qquad \frac{\Delta H}{H} = \frac{2\Delta u}{u}$$

Given, % increase in H is 10%,

so $\frac{\Delta H}{H} = \frac{10}{100} = 0{\cdot}1$. Therefore, $\frac{2\Delta u}{u} = 0{\cdot}1$

As $\qquad R = \frac{u^2 \sin 2\theta}{g}$

Therefore, $\quad \Delta R = \frac{2u\,\Delta u}{g} \times \sin 2\theta$

and $\qquad \frac{\Delta R}{R} = \frac{2\Delta u}{u} = 0{\cdot}1$

$\therefore$ % increase in horizontal range

$$= \frac{\Delta R}{R} \times 100 = 0{\cdot}1 \times 100 = \mathbf{10\%}$$

Example 51 **From the top of a tower 156·8 m high, a projectile is thrown up with a velocity of 39·2 ms^{-1}, making an angle 30° with horizontal direction. Find the distance from the foot of tower where it strikes the ground and the time taken by it to do so.**

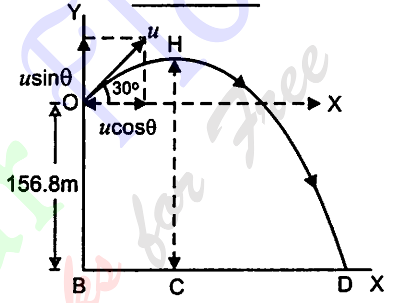
Solution. In Fig. 4.79, height of tower

$OB = 156{\cdot}8$ m ; $u = 39{\cdot}2$ m/s ; $\theta = 30°$

Component of velocity along OX

$= u\cos\theta = 39{\cdot}2 \cos 30° = 33{\cdot}947$ ms^{-1}

FIGURE 4.79



Component of velocity along OY

$= u\sin\theta = 39{\cdot}2 \sin 30° = 19{\cdot}6$ ms^{-1}

Let t be the total time of flight (*i.e.*, time in going from O to D). Consider the vertical downward direction OB as the positive direction of Y-axis. Taking motion of a projectile from O to D along Y-axis, we have $\quad y_0 = 0,\ y = 156{\cdot}8$ m,

$u_y = -u\sin 30° = -19{\cdot}6$ m/s,

$a_y = 9{\cdot}8$ m/s^2, $t = t$

As, $\qquad y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$\therefore \quad 156{\cdot}8 = 0 + (-19{\cdot}6)\,t + \frac{1}{2} \times 9{\cdot}8 \times t^2$

or $\quad 156{\cdot}8 = -19{\cdot}6\,t + 4{\cdot}9\,t^2$

or $\quad 4{\cdot}9\,t^2 - 19{\cdot}6\,t - 156{\cdot}8 = 0$

or $\quad t^2 - 4t - 32 = 0$

or $\quad t^2 - 8t + 4t - 32 = 0$

or $\quad t(t-8) + 4(t-8) = 0$

or $\quad (t+4)(t-8) = 0 \quad$ or $\quad t = -4$ or 8.

As $t = -4$ s is not possible, therefore $t = \mathbf{8}$ **s**

Distance from the foot of tower where it strikes the ground is

$BD = u\cos 30° \times t = 33{\cdot}947 \times 8 = \mathbf{271{\cdot}57\ m}$

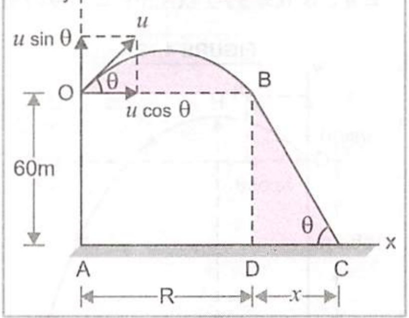
Example 52 **A machine gun is mounted on the top of a tower 60 m high. At what angle should the gun be inclined to cover a maximum range of firing on the ground below ? The muzzle speed of the bullet is 90 ms^{-1}, take g = 10 ms^{-2}.**

Solution. Let the machine gun be mounted at O and fired with a speed of 90 ms^{-1} at an angle θ with the horizontal. Fig. 4.80.

Total range of firing on ground

$$R' = R + x = \frac{u^2 \sin 2\theta}{g} + 60 \cot\theta$$

FIGURE 4.80



$$\frac{dR'}{d\theta} = \frac{u^2\, 2\cos 2\theta}{g} + 60 \times (-\operatorname{cosec}^2\theta)$$

$$= \frac{2u^2}{g}(1 - 2\sin^2\theta) - \frac{60}{\sin^2\theta}$$

$$= \frac{2 \times 90^2}{10}(1 - 2\sin^2\theta) - \frac{60}{\sin^2\theta}$$

$$= 1620 - 3240 \sin^2\theta - \frac{60}{\sin^2\theta}$$

For R' to be maximum, $\frac{dR'}{d\theta} = 0$

$$\therefore \quad 1620 - 3240\sin^2\theta - \frac{60}{\sin^2\theta} = 0$$

or $54 \sin^4\theta - 27 \sin^2\theta + 1 = 0$

or $\sin^2\theta = \frac{27 \pm \sqrt{(-27)^2 - 4 \times 54 \times 1}}{2 \times 54}$

$$= \frac{27 \pm (22{\cdot}65)}{108}$$

Taking only positive sign,

$$\sin^2\theta = \frac{49{\cdot}65}{108} = 0{\cdot}4597$$

$\sin\theta = 0{\cdot}6780$ or $\theta = \mathbf{42^\circ\, 41'}$

TYPE VIII. CIRCULAR MOTION

Formulae used. (*i*) $\omega = \theta/t$ (*ii*) $\nu = 1/T$
(*iii*) $\omega = 2\pi\nu = 2\pi/T$ (*iv*) $a = \omega^2 r = \omega v = v^2/r$.
Units used. The centripetal acceleration, a in ms^{-2} and θ in radian ; ν in Hz.

Example 53 **An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (*a*) What is the angular speed, and the linear speed of the motion ? (*b*) Is the acceleration vector a constant vector ? What is its magnitude ?**

(NCERT Solved Example)

Solution. Here, T = 100/7 s ; r = 12 cm

(*a*) Angular speed, $\omega = \frac{2\pi}{T} = \frac{2 \times (22/7)}{(100/7)}$

$$= \mathbf{0{\cdot}44\ rad/s}$$

Linear speed, $v = r\,\omega = 12 \times 0{\cdot}44$

$$= \mathbf{5{\cdot}3\ cm\ s^{-1}}$$

(*b*) The given motion is a uniform circular motion. Here the direction of velocity is along the tangent to the circle at every point. The acceleration in this motion is centripetal acceleration. It is always directed towards the centre of the circle. Since this direction changes continuously at different points on circular motion, therefore, the acceleration here is **not a constant vector.** The magnitude of acceleration is

$$a_c = \omega^2 r = (0{\cdot}44)^2 \times 12 = \mathbf{2{\cdot}3\ cm\ s^{-2}}$$

Example 54 **The radius of the earth's orbit around the sun 1·5 × 10^{11} m. Calculate the angular velocity and linear velocity of the earth. Through how much angle does the earth revolve in 2 days ?**

Solution. Here, $r = 1{\cdot}5 \times 10^{11}$ m ; Time period of revolution of earth around the sun is 1 year *i.e.*

$$T = 1 \text{ year} = 365 \times 24 \times 60 \times 60 \text{ s}.$$

$\therefore$ Angular velocity, $\omega = \frac{2\pi}{T}$

$$= \frac{2 \times (22/7)}{365 \times 24 \times 60 \times 60}$$

$$= \mathbf{1{\cdot}99 \times 10^{-7}\ rad\ s^{-1}}$$

Linear velocity, $v = \omega\, r$

$$= 1{\cdot}99 \times 10^{-7} \times 1{\cdot}5 \times 10^{11}$$

$$= \mathbf{2{\cdot}99 \times 10^4\ ms^{-1}}$$

In 365 days, the earth completes one revolution. Therefore in 365 days the earth revolves through an angle = 2 π radian. In 2 days, the earth revolves through an angle

$$= \frac{2\pi}{365} \times 2 = \frac{2 \times 22 \times 2}{7 \times 365} = \mathbf{0{\cdot}034\ rad}$$

Example 55 **A body of mass 2 kg revolves in a circle of diameter 40 cm, making 120 revolutions per minute. Calculate its linear velocity and centripetal acceleration.**

Solution. Here, $2\,r = 40$ cm

or $r = 20 \text{ cm} = 0{\cdot}20$ m ;

Frequency $\nu = \frac{120}{60} = 2$ rps

Angular velocity, $\omega = 2\,\pi\nu = 2\,\pi \times 2 = 4\,\pi$ rad/s

Linear velocity, $v = r\omega = 0{\cdot}2 \times 4\,\pi = \mathbf{0{\cdot}8\,\pi\ ms^{-1}}$

Centripetal acceleration $= r\omega^2 = 0{\cdot}2 \times (4\,\pi)^2$

$$= \mathbf{3{\cdot}2\,\pi^2\ ms^{-2}}$$

Example 56 **Calculate the magnitude of linear acceleration of a particle moving in a circle of radius 0·5 m at the instant when its angular velocity is 2·5 rad s^{-1} and its angular acceleration is 6 rad s^{-2}.**

Solution. Here, $r = 0{\cdot}5$ m ; $\omega = 2{\cdot}5$ rad s^{-1}, $\alpha = 6$ rad s^{-2}.

Tangential acceleration,

$$a_T = r\alpha = 0{\cdot}5 \times 6 = 3{\cdot}0\ \text{ms}^{-2}$$

Centripetal acceleration,

$$a_C = r\omega^2 = 0{\cdot}5 \times (2{\cdot}5)^2 = 3{\cdot}125\ \text{ms}^{-2}$$

Magnitude of total linear acceleration,

$$a = \sqrt{a_T^2 + a_C^2} = \sqrt{(3{\cdot}0)^2 + (3{\cdot}125)^2}$$

$$= \mathbf{4{\cdot}33\ ms^{-2}}$$

TYPE IX. TYPICAL EXAMPLES

Example 57 **The edges of a parallelopiped are given by the vectors $(2\,\hat{i} + 3\,\hat{j} + 4\,\hat{k})$, $4\,\hat{j}$ and $(5\,\hat{j} + m\,\hat{k})$. What should be the value of m in order that the volume of the parallelopiped be 24 ?**

Solution. We know that the volume (*V*) of the parallelopiped is the scalar triple product of three given vectors,

$$\vec{A} = (2\,\hat{i} + 3\,\hat{j} + 4\,\hat{k})\ ;\ \vec{B} = 4\,\hat{j}$$

and $\vec{C} = (5\,\hat{j} + m\,\hat{k})$.

Here, $\vec{B} = 0\,\hat{i} + 4\,\hat{j} + 0\,\hat{k}\ ;\ \vec{C} = 0\,\hat{i} + 5\,\hat{j} + m\,\hat{k}$;

Volume $V = 24$.

Clearly volume of parallelopiped is given by

$$V = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} 2 & 3 & 4 \\ 0 & 4 & 0 \\ 0 & 5 & m \end{vmatrix}$$

$$= 2\,[4\,m - 0] + 3\,[0 - 0] + 4\,[0 - 0]$$

$$= 8\,m$$

$\therefore\ 24 = 8$ m or $m = 3$

Example 58 **A particle starts from origin at $t = 0$ with a velocity $5{\cdot}0\,\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3{\cdot}0\,\hat{i} + 2{\cdot}0\,\hat{j})\,\text{m/s}^2$. (*a*) What is the y-coordinate of the particle at the instant its x-coordinate is 84 m ? (*b*) What is the speed of the particle at this time ?**

(NCERT Solved Example)

Solution. *Taking motion along x-direction*, we have $u_x = 5{\cdot}0$ m/s ; $a_x = 3{\cdot}0$ m/s^2,

$x = 84$ m, $t = ?$; $v_x = ?$

As, $x = u_x t + \frac{1}{2} a_x t^2$

$$\therefore\ 84 = 5{\cdot}0\,t + \frac{1}{2} \times 3 \times t^2$$

or $168 = 10\,t + 3\,t^2$ or $3\,t^2 + 10\,t - 168 = 0$

On solving, $t = 6$ **s**

$$v_x = u_x + a_x t = 5{\cdot}0 + 3 \times 6 = 23\ \text{m/s}$$

Taking motion along y-direction, we have

$u_y = 0$, $a_y = 2{\cdot}0$ m/s^2, $y = ?$, $t = 6$ s ; $v_y = ?$

As, $y = \frac{1}{2} a_y t^2 = \frac{1}{2} \times 2 \times 6^2 = \mathbf{36\,m}$

$$v_y = u_y + a_y t = 0 + 2 \times 6 = 12\ \text{ms}^{-1}$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{23^2 + 12^2}$$

$$= \mathbf{25{\cdot}9\ ms^{-1}}$$

Example 59 **If $\vec{A}=\hat{i}+2\hat{j}-3\hat{k}$; $\vec{B}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{C}=\hat{i}-3\hat{j}+2\hat{k}$, then find $\vec{A}\times(\vec{B}\times\vec{C})$.**

Solution. $\vec{B}\times\vec{C}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & -3 & 2\end{vmatrix}$

$$=\hat{i}(-2+3)+\hat{j}(1-4)+\hat{k}(-6+1)$$

$$=\hat{i}-3\hat{j}-5\hat{k}$$

$$\vec{A}\times(\vec{B}\times\vec{C})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 1 & -3 & -5\end{vmatrix}$$

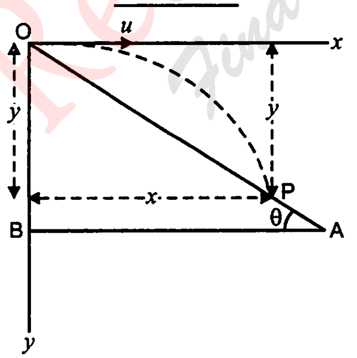
$$=\hat{i}(-10-9)+\hat{j}(-3+5)+\hat{k}(-3-2)$$

$$=\mathbf{-19\hat{i}+2\hat{j}-5\hat{k}}$$

Example 60 **A particle is projected horizontally with a speed *u* from top of a plane inclined at an angle θ with the horizontal direction. How far from the point of projection will the particle strike the plane ?**

Solution. Let the particle projected from *O* strike the inclined plane *OA* at *P* after time *t* and coordinates of *P* be (*x*, *y*), Fig. 4.81.

FIGURE 4.81



Taking motion of projectile from *O* to *P* along *x*-axis we have $x_0 = 0, x = x, u_x = u, a_x = 0, t = t$.

Using the relation, $x = x_0 + u_x t + \frac{1}{2} a_x t^2$,

we get $x = ut$ or $t = x/u$ ...(*i*)

Taking motion of projectile along Y-axis, we have

$$y_0 = 0,\ y = y,\ u_y = 0,\ a_y = g,\ t = t.$$

Using the relation, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$,

we have $y = 0 + 0 + \frac{1}{2} g t^2$

$$= \frac{1}{2} g t^2 = \frac{1}{2} g \frac{x^2}{u^2} \qquad \text{[From } (i)]$$

Here, $y = x \tan\theta$, so $\frac{g x^2}{2u^2} = x \tan\theta$

or $x = \frac{2u^2 \tan\theta}{g}$

and $y = x \tan\theta = \frac{2u^2 \tan^2\theta}{g}$

∴ Distance $OP = \sqrt{x^2 + y^2}$

$$= \frac{2u^2 \tan\theta}{g}\sqrt{1+\tan^2\theta}$$

$$= \mathbf{\frac{2u^2 \tan\theta \sec\theta}{g}}$$

Example 61 **A football is kicked at a speed of 20 m/s at a projection angle of 45°. A receiver on the goal line 25 metres away in the direction of the kick runs the same instant to meet the ball. What must be his speed, if he is to catch the ball before it hits the ground ?**

Solution. Here ; $u = 20$ m/s ;

$\theta = 45°$, $d = 25$ m.

Horizontal range ;

$$R = \frac{u^2}{g}\sin 2\theta = \frac{20^2}{9{\cdot}8}\sin 2\times 45° = 40{\cdot}82 \text{ m}$$

$$\text{Time of flight, } T = \frac{2u\sin\theta}{g} = \frac{20^2}{9{\cdot}8}\times \sin 45°$$

$$= 2{\cdot}886 \text{ s}$$

Since the goal man is already 25 m away in the direction of the ball, so to catch the ball, he is to cover a distance = 40·82 – 25 = 15·82 m, in time 2·886 s. Therefore, the velocity of goal man to catch the ball is $v = \frac{15{\cdot}82}{2{\cdot}886}$ = **5·481 ms^{-1}**

Example 62 **A stone is thrown by a student from the bottom of a hill with a velocity 30 ms^{-1} making an angle of 60° with the horizontal. If the slope of the hill is 30° with the horizontal, find the distance from the student to a point at which the stone falls on hill, use g = 10 ms^{-2}.**

Solution. Refer to Art. 4.9, the distance covered on slope is $R = \dfrac{2u^2 \cos\theta \sin(\theta - \theta_0)}{g \cos^2 \theta_0}$

Here, $u = 30 \text{ ms}^{-1}$,

$$\theta = 60°, \theta_0 = 30°, g = 10 \text{ ms}^{-2}$$

$$\therefore \quad R = \frac{2 \times 30^2 \cos 60° \sin(60° - 30°)}{10 \times \cos^2 30°}$$

$$= \frac{2 \times 900 \times (1/2) \times (1/2)}{10 \times (\sqrt{3}/2)^2}$$

$$= \mathbf{60 \text{ m}}$$

REVISION AT A GLANCE

1. **Scalars.** These are those quantities which have only magnitudes but no direction. For example, mass, length, time, speed, work, temperature etc.
2. **Vectors.** These are those quantities which have magnitude as well as direction. For example , displacement, velocity, acceleration, force, momentum etc.
3. **Unit vector.** A unit vector of a given vector $\vec{A}$ is a vector of unit magnitude and has the same direction as that of the given vector. A unit vector of $\vec{A}$ is written as $\hat{A}$, where $\hat{A} = \vec{A}/A$. A unit vector is unitless and dimensionless vector and represents direction only.
4. **Equal vectors.** The two vectors are said to be equal if they have equal magnitude and same direction.
5. **Negative vectors.** A negative vector of a given vector is a vector of same magnitude but acting in a direction opposite to that of the given vector. The negative vector of $\vec{A}$ is represented by $-\vec{A}$.
6. **Collinear vectors.** These are those vectors which are acting along parallel straight lines.
7. **Coplanar vectors.** These are those vectors which are acting in the same plane.
8. **Localised and Non-localised vectors.** A vector whose initial point is fixed is called a localised vector and whose initial point is not fixed is called non-localised vector.
9. **Multiplication of a vector by a real number.** When a vector $\vec{A}$ is multiplied by a real number n, it becomes another vector $n\vec{A}$. Its magnitude becomes n times the magnitude of $\vec{A}$. Its direction is same or opposite as that of $\vec{A}$, according as n is positive or negative real number. The unit of $n\vec{A}$ is the same as that of $\vec{A}$.
10. **Multiplication of a vector by a scalar.** When a vector $\vec{A}$ is multiplied by a scalar S, it becomes a vector $S\vec{A}$, whose magnitude is S times the magnitude of $\vec{A}$ and it acts along the direction of $\vec{A}$. The unit of $S\vec{A}$ is different from the unit of vector $\vec{A}$.
11. **Resultant vector.** The resultant vector of two or more vectors is defined as that single vector which produces the same effect as is produced by individual vectors together. The resultant vector can be obtined from the rules for **geometric addition of vectors.**

    (i) For the addition of two vectors, represent these two vectors by arrowed lines using the same suitable scale. Displace the second vector such that its tail coincides with the head of first vector. Then the single vector, drawn from the tail of the first vector to the head of the second vector represents the resultant vector.

    (ii) For the addition of three or more vectors, represent these vectors by arrowed lines using the same suitable scale. Displace these vectors such that the head of the first vector coincides with the tail of second vector and whose head coincides with the tail of third vector and so on, then the single vector drawn from the tail of the first vector to head of last vector represents resultant vector.

12. Analytical method of vectors addition.

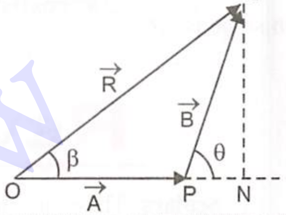
(i) Triangle law of vectors. It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of the triangle taken in the opposite order :

If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, Fig.4.82, then

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and $$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

FIGURE 4.82



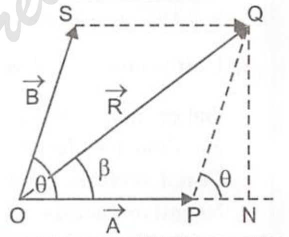
(ii) Parallelogram law of vectors. It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.

If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, Fig. 4.83, then

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and $$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

FIGURE 4.83



Note. The magnitude of the resultant vector is maximum if the two vectors are acting in the same direction and is minimum if the two vectors are acting in the opposite direction.

13. Polygon law of vectors. It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector is represented in magnitude and direction by the closing side of the polygon taken in opposite order.

In fact polygon law of vectors is the outcome of triangle law of vectors.

14. Important points of vector addition.

(*i*) *Vectors of same nature alone can be added i.e.* a force vector can not be added to velocity vector but can be added to force vector only.

(*ii*) *Vector addition is commutative i.e.* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(*iii*) *Vector addition is associative i.e.* $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

15. Substraction of vectors. Subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$. Thus $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

If θ is the angle between $\vec{A}$ and $\vec{B}$, and $\vec{R} = (\vec{A} - \vec{B})$, then

$$R = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$$

and $$\tan\beta = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)}$$

The vector subtraction does not obey, commutative law and associative law.

16. Zero vector. It is that vector which has zero magnitude and an arbitrary direction. A zero vector is represented by $\vec{0}$. It is also called a **null vector.**

17. **Rectangular components of a vector in a plane.** When a vector is splitted into two component vectors at right angles to each other, the component vectors are called rectangular components of a vector. If $\vec{A}$ makes an angle θ with x-axis and $\vec{A_x}$ and $\vec{A_y}$ be the rectangular components of $\vec{A}$ along X-axis and Y-axis respectively, then $\vec{A} = \vec{A_x} + \vec{A_y} = A_x \hat{i} + A_y \hat{j}$

Here, $A_x = A\cos\theta$

and $A_y = A\sin\theta.$

$\therefore \quad A^2(\cos^2\theta + \sin^2\theta) = A_x^2 + A_y^2$

or $A = (A_x^2 + A_y^2)^{1/2}$

and $\tan\theta = A_y / A_x$

18. **Addition of vectors in rectangular coordinates.**

If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

Then, $\vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) + (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}\,.$$

19. **The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar,** which is equal to the product of the magnitude of two vectors and cosine of the smaller angle θ between them, *i.e.,*

$$\vec{A}.\vec{B} = AB\cos\theta$$

20. **The vector or cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector,** whose magnitude is equal to the product of the magnitude of two vectors and sine of the smaller angle θ between them and its direction is given by a unit vector $\hat{c}$, *i.e.,*

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{c}$$

The direction of $\hat{c}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$. It is directed as given by Right hand rule. Thus

$$|\vec{A} \times \vec{B}| = AB\sin\theta$$

21. **Scalar product of three vectors**

$\vec{A}.(\vec{B} \times \vec{C}) = \vec{B}.(\vec{C} \times \vec{A}) = \vec{C}.(\vec{A} \times \vec{B})$

= volume of parallelopiped whose three sides are represented by the three vectors

22. **Vector product of three vectors**

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

and $$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A}.\vec{C})\vec{B} - (\vec{B}.\vec{C})\vec{A}$$

23. **Projectile.** Projectile is the name given to a body which is thrown with some initial velocity with the horizontal direction and then it is allowed to move under the effect of gravity alone, without being propelled by any engine or fuel. A projectile during its flight must posses two component velocities (*i*) in the horizontal direction and (*ii*) in the vertical direction.

If a projectile is given horizontal projection with velocity u from a height h, then its

time of flight, $$T = \sqrt{2h/g}$$

Horizontal range, $$R = uT = u\sqrt{2h/g}$$

Velocity on reaching the ground, $$v = \sqrt{u^2 + g^2T^2}$$

If a projectile is given angular projection of angle θ from the horizontal ground with velocity u, then its

time of flight, $$T = \frac{2u\sin\theta}{g}$$

Maximum height, $$H = \frac{u^2\sin^2\theta}{2g}$$

Horizontal range, $$R = \frac{u^2\sin 2\theta}{g}$$

There are two angles of projections (α and β) for which the horizontal range will be the same provided $\alpha + \beta = 90°$.

24. **Angular displacement (θ).** The angular displacement of an object moving around a circular path is defined as the angle traced out by the radius vector at the axis of the circular path in the given time.

Angular displacement is a vector quantity provided angle θ is small. It is expressed in radian.

25. **Angular velocity (ω).** It is defined as the time rate of change of angular displacement of the object, *i.e.*, $\omega = d\theta/dt$. Its SI unit is rad/*s*.

26. **Uniform circular motion.** When a point object is moving on a circular path with a constant speed, then the motion of the object is said to be a uniform circular motion.

27. **Centripetal acceleration.** It is defined as the acceleration acting on the object undergoing uniform circular motion. It always acts on the object along the radius towards the centre of the circular path. The magnitude of centripetal acceleration is,

$$a = \frac{v^2}{r} = r\omega^2 = r(2\pi\nu)^2 = 4\pi^2 r\nu^2 = \frac{4\pi^2 r}{T^2}$$

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OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. The relation between linear velocity and angular velocity is

(*a*) $v = \omega r$ (*b*) $\omega = v r$

(*c*) $v = \omega / r$ (*d*) $\omega = v^2 / r$

2. When a particle moves in circular path, then the direction of centripetal acceleration is

(*a*) Towards the centre

(*b*) perpendicular to velocity

(*c*) away from the centre

(*d*) parallel to velocity

3. Which force is responsible for the circular path of an object

(*a*) Friction force (*b*) mechanical force

(*c*) centripetal force (*d*) gravitational force

4. When a particle is accelerated with a linear acceleration of 5 m/s^2 and moves in a circular path of radius 2 m. Then value of angular acceleration is

(*a*) 0 (*b*) 2·5 rad/s^2

(*c*) – 2·5 rad/s^2 (*d*) none of these

5. What is the value of linear velocity, if $\vec{\omega} = (2\hat{i} - 4\hat{j} + \hat{k})$ rad/*s* and $\vec{r} = (\hat{i} + \hat{j} + \hat{k})$ m ?

(*a*) $(\hat{i} + 3\hat{j} + 6\hat{k})$ m/s (*b*) $(-5\hat{i} - \hat{j} + 6\hat{k})$ m/s

(*c*) $(3\hat{i} - 3\hat{j} + 6\hat{k})$ m/s (*d*) $(\hat{i} + \hat{j} + 6\hat{k})$ m/s

6. The angular displacement of a particle following circular path is given by $\theta(t) = 25t^3 + 3t + 7$, then angular velocity will be at $t = 2$ sec is

(*a*) 303 rad/sec (*b*) 400 rad/sec

(*c*) 100 rad/sec (*d*) zero

7. What is the relation between angular acceleration and linear acceleration

(*a*) $\vec{a}_T = \vec{\alpha} \cdot \vec{r}$ (*b*) $\vec{\alpha} = \vec{a}_T \cdot \vec{r}$

(*c*) $\vec{\alpha} = \vec{a}_T \times \vec{r}$ (*d*) $\vec{a}_T = \vec{\alpha} \times \vec{r}$

8. If a particle of mass 5 g moves with linear velocity 5 m/sec in a circular path of radius is 10 cm, then centripetal force will be

(*a*) 1·25 N (*b*) 12·5 N

(*c*) 125 N (*d*) none of these

9. The component of a vector r along x axis will have maximum value if

(*a*) $\vec{r}$ is a long + y axis

(*b*) $\vec{r}$ is along + x axis

(*c*) $\vec{r}$ makes an angle of 45 degree with the x-axis

(*d*) $\vec{r}$ is along negative y axis

10. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, what is the angle between $\vec{a}$ and $\vec{b}$?

(*a*) 0° (*b*) 180°

(*c*) 90° (*d*) 45°

11. Cross product of two vectors $\vec{a}$ and $\vec{b}$ is maximum in magnitude when angle between them is

(*a*) 0° (*b*) 180°

(*c*) 90° (*d*) 45°

12. For two vectors $\vec{a}$ and $\vec{b}$, choose the correct option

(*a*) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (*b*) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(*c*) $\vec{a} \perp \vec{a} \times \vec{b}$ (*d*) All of the above

13. If $\vec{a}_c$ and $\vec{v}$ be centripetal acceleration and tangential velocity respectively in uniform circular motion then the angle between $\vec{a}_c$ and $\vec{v}$ will be

(*a*) 0° (*b*) 180°

(*c*) 90° (*d*) 270°

14. In what direction should a boy at rest hold his umbrella if rain falls vertically with a speed of 30 m/s and wind blows towards west with a speed of 10 m/s.

(*a*) $\tan^{-1} 3$ towards west from vertical

(*b*) $\tan^{-1} 3$ towards east from vertical

(*c*) $\tan^{-1}\left(\frac{1}{3}\right)$ towards west from vertical

(*d*) $\tan^{-1}\left(\frac{1}{3}\right)$ towards east from vertical

15. At some point of time, when a motor car is travelling at 30 m/s on a circular road of radius

500 m. If it is increasing speed at the rate of 2 m/s. The acceleration is

(*a*) 2·7 m/s (*b*) 2·5 m/s

(*c*) 2·9 m/s (*d*) 2·3 m/s

16. In uniform circular motion

(*a*) Both velocity and acceleration are constant

(*b*) Velocity changes and acceleration is constant

(*c*) Velocity is constant and acceleration changes

(*d*) Both velocity and acceleration change

17. What is the angular velocity of the Earth

(*a*) 2 π/86400 rad/s (*b*) 2 π/3600 rad/s

(*c*) 2π/24 rad/s (*d*) 2 π/6400 rad/s

18. The ratio of angular speed of second hand to that to the minute hand of a clock is

(*a*) 60 : 1 (*b*) 1 : 60

(*c*) 1 : 1 (*d*) 1 : 6

19. A car sometimes overturns while taking a sharp turn when it overturns

(*a*) The inner wheel which leaves the ground first

(*b*) The outer wheel which leaves the ground first

(*c*) Both the wheels leave the ground simultaneously

(*d*) Either wheel leaves the ground first

20. Two quantities *A* and *B* have different dimensions. Which of the following operation is correct ?

(*a*) A/B (*b*) $A\times B$

(*c*) $A+B$ (*d*) both '1' and '2'

21. If $\vec{A}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{B}=2\hat{i}+3\hat{k}$ and

$\vec{C}=-2\hat{i}+4\hat{k}$,

then magnitude of vector $\vec{A}+\vec{B}-\vec{C}$ is

(*a*) $\sqrt{20}$ (*b*) $\sqrt{29}$

(*c*) $\sqrt{65}$ (*d*) $\sqrt{69}$

22. The length of the sum of the vectors $\vec{A}=3\hat{i}$ and $\vec{B}=4\hat{j}$ is

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

23. Let $\vec{A}=\frac{1}{\sqrt{2}}\cos\theta\,\hat{i}+\frac{1}{\sqrt{2}}\sin\theta\,\hat{j}$ be any vector.

What will be the unit vector $\vec{n}$ in the direction of $\vec{A}$?

(*a*) $\cos\theta\,\hat{i}+\sin\theta\,\hat{j}$

(*b*) $-\cos\theta\,\hat{i}+\sin\theta\,\hat{j}$

(*c*) $\frac{1}{\sqrt{2}}(\cos\theta\,\hat{i}+\sin\theta\,\hat{j})$

(*d*) $\frac{1}{\sqrt{2}}(\cos\theta\,\hat{i}-\sin\theta\,\hat{j})$

24. If a unit vector is represented by $0{\cdot}5\,\hat{i}+0{\cdot}8\,\hat{j}+c\hat{k}$, then the value of '*c*' is

(*a*) 1 (*b*) $\sqrt{0{\cdot}11}$

(*c*) $\sqrt{0{\cdot}1}$ (*d*) $\sqrt{0{\cdot}01}$

25. When a unit vector $\vec{n}=a\hat{i}+b\hat{j}$ is perpendicular to $(\hat{i}+\hat{j})$, then the values of *a* and *b* are

(*a*) 1 and 0 (*b*) – 2 and 0

(*c*) 3 and 0 (*d*) $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$

26. A boat is sent across a river with a velocity of 8 kmh^{-1}. If the resultant velocity of boat is 10 kmh^{-1}, then velocity of the river is

(*a*) 4 kmh^{-1} (*b*) 6 kmh^{-1}

(*c*) 8 kmh^{-1} (*d*) 10 kmh^{-1}

27. The position vector of a particle is

$$\vec{r}=(a\cos\omega t)\,\hat{i}+(a\sin\omega t)\,\hat{j}.$$

The velocity of particle is

(*a*) directed towards the origin

(*b*) directed away from the origin

(*c*) parallel to the position vector

(*d*) perpendicular to the position vector

28. If $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then

(*a*) $\vec{A}\cdot\vec{B}=0$ (*b*) $\vec{A}\times\vec{B}=0$

(*c*) $\vec{A}\cdot\vec{B}=\vec{A}\times\vec{B}$ (*d*) None of these

29. If $|\vec{A}\times\vec{B}|=\sqrt{3}\,\vec{A}\cdot\vec{B}$,

then the value of $|\vec{A}+\vec{B}|$ is

(*a*) $A+B$ (*b*) $(A^2+B^2+AB)^{1/2}$

(*c*) $(A^2+B^2+\sqrt{3}\,AB)^{1/2}$

(*d*) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$

30. Three vectors satisfy the relations $\vec{A}\cdot\vec{B}=0$ and $\vec{A}\cdot\vec{C}=0$. The vector $\vec{A}$ is parallel to

(*a*) $\vec{B}$ (*b*) $\vec{C}$

(*c*) $\vec{B}\cdot\vec{C}=0$ (*d*) $\vec{B}\times\vec{C}$

31. The maximum range of projectile fired with some initial velocity is found to be 1000 m, in the absence of wind and air resistance. The maximum height reached by this projectile is

(*a*) 250 m (*b*) 300 m

(*c*) 400 m (*d*) 500 m

32. Four balls *A*, *B*, *C* and *D* are projected with the same velocity *u* making angles of 30°, 45°, 60° and 75° respectively with the horizontal. Which ball will strike the ground at the farthest point ?

(*a*) *A* (*b*) *B*

(*c*) *C* (*d*) *D*

33. A projectile can have the same range '*R*' for two angles of projection. If 't_1' and 't_2' be the times of flights in the two cases, then the product of the two time of flights is proportional to

(*a*) *R* (*b*) R^2

(*c*) $\frac{1}{R}$ (*d*) $\frac{1}{R^2}$

34. If 3 ms^{-1}, 4 ms^{-1} and 12 ms^{-1} are respectively *x*, *y* and *z*-components of the velocity of an object at any time '*t*', in the space, then resultant velocity of the object at that time is

(*a*) 9 ms^{-1} (*b*) 13 ms^{-1}

(*c*) 20 ms^{-1} (*d*) 25 ms^{-1}

35. The parametric equations of a curve traced by a projectile on a certain planet with no surrounding are given by : $y = (8\,t - 5\,t^2)$ metre and $x = 6\,t$ meter, where *t* is in seconds. The velocity with which the projectile is projected, is

(*a*) 6 ms^{-1} (*b*) 8 ms^{-1}

(*c*) 10 ms^{-1} (*d*) 16 ms^{-1}

36. Given, $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$, what is the component of $\vec{A}$ along vector $\vec{B}$ is

(*a*) $\frac{(\hat{i}+\hat{j})}{2}$ (*b*) $\frac{3}{2}(\hat{i}+\hat{j})$

(*c*) $\frac{5}{2}(\hat{i}+\hat{j})$ (*d*) $\frac{7}{2}(\hat{i}+\hat{j})$

37. If $\vec{A}=\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$, then the vector parallel to $\vec{A}$ and having the magnitude $\vec{B}$ is

(*a*) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (*b*) $\sqrt{2}(\hat{i}+\hat{j})$

(*c*) $2(\hat{i}+\hat{j})$ (*d*) $(\hat{i}+\hat{j})$

38. A particle moves along a circle with constant speed 2 ms^{-1}. If it completes one-fourth of rotation, change in its velocity is

(*a*) $\sqrt{2}$ ms^{-1} (*b*) 2 ms^{-1}

(*c*) $2\sqrt{2}$ ms^{-1} (*d*) 4 ms^{-1}

39. A 200 m train moving at 11 ms^{-1} crosses a man running on the platform along the same direction in 20 s. Then the speed of the man is

(*a*) 8 ms^{-1} (*b*) 6 ms^{-1}

(*c*) 4 ms^{-1} (*d*) 1 ms^{-1}

40. The speed of projectile at its maximum is 6 ms^{-1}. If it stays in the air for a total time of 10 seconds, its horizontal range is

(*a*) 20 m (*b*) 40 m

(*c*) 60 m (*d*) 80 m

41. A projectile is fired from a gun with a muzzle velocity of 2000 ms^{-1} at an elevation of 30°. The maximum height reached would be the same as that by an identical particle fired vertically with a muzzle velocity of

(*a*) 500 ms^{-1} (*b*) 1000 ms^{-1}

(*c*) 1500 ms^{-1} (*d*) 1414 ms^{-1}

42. The equation of trajectory of a projectile thrown from a point on the ground is : $y = (x - x^2/40)$ m. If g = 10 ms^{-2}, the maximum height reached is

(*a*) 5 m (*b*) 10 m

(*c*) 20 m (*d*) 15 m

43. A ball thrown by a girl is caught 2 seconds later by another at some distance away on the same level. If the angle of projection is 30°, the velocity of projection is : (use g = 10 ms^{-2})

(*a*) 10 ms^{-1} (*b*) 5 ms^{-1}

(*c*) 20 ms^{-1} (*d*) 12·5 ms^{-1}

44. Calculate the magnitude of linear acceleration of a particle moving in a circle of radius 0·5 m at the instant when its angular velocity is 2·5 rad s^{-1} and its angular acceleration is 6 rad s^{-2}.

(*a*) 3·0 ms^{-2} (*b*) 3·125 ms^{-2}

(*c*) 6·125 ms^{-2} (*d*) 4·33 ms^{-2}

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true, but R is a false.

(*d*) A is false, and R is also false.

45. Assertion. The component of a vector can be equal to the magnitude of the given vector.

Reason. Vector component of $\vec{A}$ in the direction making an angle θ with the given vector will be $A\cos\theta$.

46. Assertion. If $(\vec{A}\times\vec{B})=(\vec{B}\times\vec{A})$, then angle between $\vec{A}$ and $\vec{B}$ is π rad.

Reason. $\vec{P}\times\vec{Q}=PQ\sin\theta\,\hat{n}$.

47. Assertion. Two vectors have magnitudes 4 and 5. If the angle between them is 60°, then the dot product of two vectors will be 10.

Reason. $\vec{A}\cdot\vec{B}=AB\cos\theta$.

48. Assertion. The minimum number of vectors of unequal magnitude required to produce a zero resultant is four.

Reason. The vectors must not be coplanar.

49. Assertion. If a unit vector represented by $0{\cdot}5\,\hat{i}+0{\cdot}8\,\hat{j}+c\,\hat{k}$, then the value of c is $\sqrt{0{\cdot}11}$.

Reason. Unit vector of $\vec{A}$ has magnitude one.

50. Assertion. The angle between $(\vec{A}+\vec{B})$ and $(\vec{A}\times\vec{B})$ is 180°.

Reason. $(\vec{A}+\vec{B})$ and $(\vec{A}\times\vec{B})$ are acting in opposite directions.

51. Assertion. If $\vec{C}=(\vec{A}+\vec{B})$, then the necessarily valid condition is $A = B$.

Reason. $\vec{A}$ and $\vec{B}$ should be perpendicular to each other.

52. Assertion. The angle between $\vec{A}$ and the resultant of $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ is zero.

Reason. Resultant of $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ acts along the direction of $\vec{A}$.

53. Assertion. A room has a dimensions $3\text{ m}\times 4\text{ m}\times 5\text{ m}$. A fly walks starting at one corner and ends at the diametrically opposite corner. The length of the shortest path is nearly 7 m.

Reason. Length of the shortest path will be $=\sqrt{x^2+y^2+z^2}$.

54. Assertion. An aeroplane takes off at an angle of 60° to the horizontal. If the component of its velocity along the horizontal is 240 km h^{-1}, then its vertical component velocity is $240\sqrt{3}\text{ km h}^{-1}$.

Reason. Horizontal component velocity of $\vec{A}$ is $A\cos\theta$ and vertical component velocity is $A\sin\theta$.

55. Assertion. A projectile has a constant initial velocity and angle of projection. The changes in the horizontal range due to change in acceleration due to gravity is $=\frac{-dg}{g}R$.

Reason. Horizontal range, $R=\frac{u^2\sin 2\theta}{g}$.

56. Assertion. A particle is projected with velocity u so that its horizontal range is thrice the greatest height attained. The horizontal range of the particle is $\frac{24}{25}\frac{u^2}{g}$.

Reason. Horizontal range, $R=\frac{u^2\sin 2\theta}{g}$

and max. height, $H=\frac{u^2\sin^2\theta}{2g}$.

57. Assertion. A uniform circular motion is an accelerated motion.

Reason. The speed of the body in a uniform circular motion is constant but the direction of motion of the body changes at every instant of time.

58. Assertion. $(\vec{A}-\vec{B})-\vec{C}=\vec{A}-(\vec{B}-\vec{C})$.

Reason. The associative law is applicable to vectors subtration.

59. Assertion. Minimum number of unequal forces whose sum is zero are three.

Reason. The forces represented by three sides of a triangle taken in one order have zero resultant.

ANSWERS

I. Multiple Choice Questions

1. (*a*) **2.** (*a*) **3.** (*c*) **4.** (*b*) **5.** (*b*) **6.** (*a*) **7.** (*d*) **8.** (*a*) **9.** (*b*) **10.** (*c*)
11. (*c*) **12.** (*d*) **13.** (*c*) **14.** (*d*) **15.** (*a*) **16.** (*d*) **17.** (*a*) **18.** (*a*) **19.** (*a*) **20.** (*d*)
21. (*b*) **22.** (*c*) **23.** (*a*) **24.** (*b*) **25.** (*d*) **26.** (*b*) **27.** (*d*) **28.** (*a*) **29.** (*b*) **30.** (*d*)
31. (*a*) **32.** (*b*) **33.** (*a*) **34.** (*b*) **35.** (*c*) **36.** (*c*) **37.** (*d*) **38.** (*c*) **39.** (*d*) **40.** (*c*)
41. (*b*) **42.** (*b*) **43.** (*c*) **44.** (*d*)

II. Assertion-Reason Type Questions

45. (*d*) **46.** (*a*) **47.** (*a*) **48.** (*d*) **49.** (*a*) **50.** (*d*) **51.** (*d*) **52.** (*a*) **53.** (*d*) **54.** (*a*)
55. (*b*) **56.** (*a*) **57.** (*a*) **58.** (*d*) **59.** (*a*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

5. $\vec{v} = \vec{\omega} \times \vec{r} = (2\hat{i} - 4\hat{j} + \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\,(-4-1) + \hat{j}\,(1-2) + \hat{k}\,(2+4)$$

$$= \mathbf{(-5\hat{i} - \hat{j} + 6\hat{k})\ m/s}$$

10. $[\vec{a} + \vec{b}]^2 = |\vec{a} - \vec{b}|$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

or $\vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$

$$= \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$$

or $4\vec{a} \cdot \vec{b} = 0$ or $\vec{a} \cdot \vec{b} = 0$

or $ab \cos\theta = 0$ or $\cos\theta = 0$ or $\theta = \mathbf{90^\circ}$

15. $a_c = \dfrac{v^2}{r} = \dfrac{(30)^2}{500} = 1{\cdot}8\ \text{ms}^{-2}$; $a_T = 2$ m/s

$\therefore\quad a = \sqrt{a_c^2 + a_T^2} = \sqrt{(1{\cdot}8)^2 + (2)^2} = \mathbf{2{\cdot}7\ m/s^2}$

18. second hand of a clock completes one rotation in 60 seconds and minute hand completes one rotation in 60 minutes

23. $$\hat{A} = \frac{\vec{A}}{A} = \frac{\left(\dfrac{1}{\sqrt{2}}\cos\theta\,\hat{i} + \dfrac{1}{\sqrt{2}}\sin\theta\,\hat{j}\right)}{\sqrt{\left(\dfrac{1}{\sqrt{2}}\cos\theta\right)^2 + \left(\dfrac{1}{\sqrt{2}}\sin\theta\right)^2}}$$

$$= \mathbf{\cos\theta\,\hat{i} + \sin\theta\,\hat{j}}$$

24. $\vec{A} = 0{\cdot}5\,\hat{i} + 0{\cdot}8\,\hat{j} + c\hat{k} = \hat{A}$ (Given)

$\therefore\quad \sqrt{(0{\cdot}5)^2 + (0{\cdot}8)^2 + c^2} = 1$

or $0{\cdot}25 + 0{\cdot}64 + c^2 = 1$ or $c = \mathbf{\sqrt{0{\cdot}11}}$

25. $\vec{n} = (a\,\hat{i} + b\hat{j})$; $\vec{A} = (\hat{i} + \hat{j})$

As $\vec{n}$ is perpendicular to $\vec{A}$,

so $\vec{n} \cdot \vec{A} = 0$ or $(a\,\hat{i} + b\hat{j}) \cdot (\hat{i} + \hat{j}) = 0$

or $a + b = 0$ or $a = -b$.

Thus **option (*d*) is true.**

26. Here, $v_b = 8\ \text{km h}^{-1}$; $v_{br} = 10\ \text{km h}^{-1}$; $v_r = ?$

$$v_{br}^2 = v_b^2 + v_r^2$$

or $v_r = \sqrt{v_{br}^2 - v_b^2} = \sqrt{10^2 - 8^2} = \mathbf{6\ km\ h^{-1}}$

27. The given particle will describe a circular path. The velocity of this particle at an instant is represented by the tangent to the circular path at that instant, which is perpendicular to the position vector.

29. Given, $|\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A} \cdot \vec{B}$

or $AB\sin\theta = \sqrt{3}\ AB\cos\theta$

or $\tan\theta = \sqrt{3}$ or $\theta = 60^\circ$

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 60^\circ}$$

$$= \mathbf{\sqrt{A^2 + B^2 + AB}}$$

30. As $\vec{A} \cdot \vec{B} = 0$; so $\vec{A}$ is perpendicular to $\vec{B}$ and $\vec{A} \cdot \vec{C} = 0$, so $\vec{A}$ is perpendicular to $\vec{C}$.

Now $(\vec{B}\times\vec{C})$ will be perpendicular to the plane containing $\vec{B}$ and $\vec{C}$ *i.e.*, parallel to $\vec{A}$.

31. $\frac{u^2\sin 2\times 45°}{g} = 1000$ or $\frac{u^2}{g} = 1000$

Max. height, $H = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{g}\left[\frac{\sin^2 45°}{2}\right]$

$$= 1000\times\frac{1/2}{2} = \mathbf{250\ m}$$

33. $t_1 = \frac{2u\sin\theta}{g}$ and $t_2 = \frac{2u\cos\theta}{g}$

$$\therefore\ t_1 t_2 = \frac{2u\sin\theta}{g}\times\frac{2u\cos\theta}{g} = 2\frac{u^2\sin 2\theta}{g}$$

$$= 2R \quad \text{or} \quad t_1 t_2 \propto R$$

35. $v_y = \frac{dy}{dt} = 8-10t \quad v_x = \frac{dx}{dt} = 6$

When $t = 0$, then $v_y = 8$; $v_x = 6$

$$\therefore\ v_0 = \sqrt{v_y^2+v_x^2} = \sqrt{8^2+6^2} = \mathbf{10\ ms^{-1}}$$

36. Given, $\vec{A} = 2\hat{i}+3\hat{j}$,

then $A = \sqrt{2^2+3^2} = 5$

$$\vec{B} = \hat{i}+\hat{j}\ ;\ B = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\hat{B} = \frac{\vec{B}}{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(2\hat{i}+3\hat{j})\cdot(\hat{i}+\hat{j})}{5\times\sqrt{2}}$$

$$= \frac{2+3}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45° \quad \text{or} \quad \theta = 45°$$

Component of $\vec{A}$ along $\vec{B}$ is

$$= A\cos\theta\,\hat{B} = 5\cos 45°\times\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

$$= 5\times\frac{1}{\sqrt{2}}\times\frac{(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{5}{2}(\hat{i}+\hat{j})$$

37. Unit vector of $\vec{A}$,

$$\hat{A} = \frac{\vec{A}}{A} = \frac{\sqrt{2}\,\hat{i}+\sqrt{2}\,\hat{j}}{\sqrt{(\sqrt{2})^2+(\sqrt{2})^2}} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

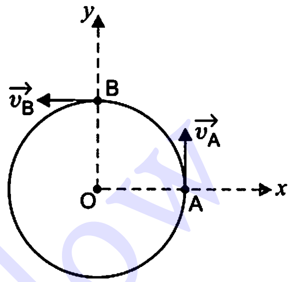
Magnitude of $\vec{B}$, $B = \sqrt{1^2+1^2} = \sqrt{2}$

$\therefore$ Vector parallel to $\vec{A}$ and having the magnitude $\vec{B}$

$$= B\hat{A} = \sqrt{2}\times\frac{(\hat{i}+\hat{j})}{\sqrt{2}} = \hat{i}+\hat{j}$$

38. Refer to Fig. 4(Q).1,

FIGURE 4(Q).1



$$\vec{v}_A = v\hat{j} = 2\hat{j}\ ;\ \vec{v}_B = -v\hat{i} = -2\hat{i}$$

Change in velocity $= |\vec{v}_B - \vec{v}_A| = |-2\hat{i}-2\hat{j}|$

$$= \sqrt{(-2)^2+(-2)^2}$$

$$= \mathbf{2\sqrt{2}\ ms^{-1}}$$

39. Length of train, l = 200 m ; velocity of train, v_t = 11 ms^{-1}, time of crossing the man, t = 20 s. Let v_m be the speed of the man in the direction of moving train. Then

$$t = \frac{l}{v_t - v_m} \quad \text{or} \quad 20 = \frac{200}{11-v_m}$$

or $10 = 11 - v_m$ or $v_m = 11 - 10 = \mathbf{1\ ms^{-1}}$

40. Let θ be the angle of projection of projectile at the origin and u be its initial velocity of projection. Then, $u\cos\theta = 6$ and time of flight,

$$T = \frac{2u\sin\theta}{g} \quad \text{or} \quad \frac{u\sin\theta}{g} = \frac{T}{2} = \frac{10}{2} = 5$$

Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$

$$= 2\frac{(u\sin\theta)}{g}u\cos\theta$$

$$= 2\times 5\times 6 = \mathbf{60\ m}$$

41. Here, $u = 2000$ ms^{-1} ; $\theta = 30°$

Max. height reached, $H = \frac{u^2\sin^2\theta}{2g}$ ...(*i*)

When the particle is fired vertically with muzzle velocity v, then maximum height reached is

$$h = \frac{v^2}{2g} \quad \text{...(ii)}$$

As per question, $H = h$

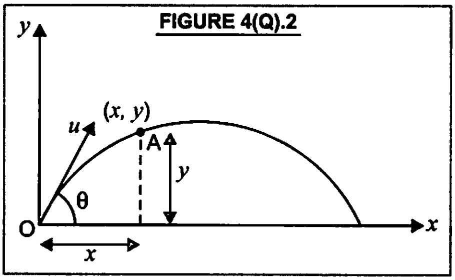
$$\therefore \quad \frac{u^2 \sin^2\theta}{2g} = \frac{v^2}{2g} \quad \text{or} \quad v^2 = u^2 \sin^2\theta$$

or $v = u \sin\theta = 2000 \times \sin 30° = 2000 \times 1/2$

$$= \mathbf{1000\ ms^{-1}}$$

42. Let a projectile projected from O reaches at A after time t and coordinates of point A be (x, y).

FIGURE 4(Q).2



Then, $x = u \cos\theta\, t$ or $t = \dfrac{x}{u\cos\theta}$

$$y = u\sin\theta \times t - \frac{1}{2}gt^2$$

$$= u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \qquad ...(i)$$

Comparing (i) with the given equation of trajectory, *i.e.*, $y = x - \dfrac{x^2}{40}$, we have

$$\tan\theta = 1 \text{ or } \theta = 45° \text{ and } \frac{g}{2u^2\cos^2\theta} = \frac{1}{40}$$

$$\text{or} \quad \frac{u^2\cos^2\theta}{g} = \frac{40}{2} = 20 \quad \text{or} \quad \frac{u^2}{g}\cos^2 45° = 20$$

$$\text{or} \quad \frac{u^2}{g} = \frac{20}{(1/\sqrt{2})^2} = 40$$

Max. height reached,

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{1}{2}\left(\frac{u^2}{g}\right)\sin^2 45°$$

$$= \frac{1}{2} \times 40 \times \left(\frac{1}{\sqrt{2}}\right)^2 = \mathbf{10\,m}$$

43. Here, time of flight, $T = 2$ s ; $\theta = 30°$, $u = ?$

$$\text{As,} \quad T = \frac{2u\sin\theta}{g} \quad \therefore \quad 2 = \frac{2u\sin 30°}{10}$$

$$\text{or} \quad u = \frac{2 \times 10}{2\sin 30°} = \frac{20}{2 \times 1/2} = \mathbf{20\ ms^{-1}}$$

44. Here, $r = 0{\cdot}5$ m ; $\omega = 2{\cdot}5$ rad s^{-1}, $\alpha = 6$ rad s^{-2}

Tangential acceleration,

$$a_T = r\,\alpha = 0{\cdot}5 \times 6 = 3{\cdot}0\ \text{ms}^{-2}$$

Centripetal acceleration,

$$a_c = r\,\omega^2 = 0{\cdot}5 \times (2{\cdot}5)^2 = 3{\cdot}125\ \text{ms}^{-2}$$

Magnitude of total linear acceleration

$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{(3{\cdot}0)^2 + (3{\cdot}125)^2} = \mathbf{4{\cdot}33\ ms^{-2}}$$

II. Assertion-Reason Type Questions

45. Vector component of $\vec{A}$ is also a vector. Hence **Reason is wrong.** The component of a vector is always less than the magnitude of the given vector. Hence **Assertion is also wrong.**

46. If $\theta = \pi$ rad, then $\vec{A} \times \vec{B} = AB \sin\pi\,\hat{n} = \vec{0}$

and $\vec{B} \times \vec{A} = BA\sin\pi(-\hat{n}) = \vec{0}$

Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

47. $\vec{A} \cdot \vec{B} = AB\cos\theta = 4 \times 5 \times \cos 60°$

$$= 4 \times 5 \times 1/2 = 10$$

Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

48. The minimum number of vectors of unequal magnitude required to produce a zero resultant is three and they must be represented by the three sides of a triangle taken in same order and they should be in the same plane. **Both Assertion and Reason are false.**

49. Unit vector of the given vector $\vec{A}$ is

$$\sqrt{(0{\cdot}5)^2 + (0{\cdot}8)^2 + c^2} = 1$$

or $\quad 0{\cdot}25 + 0{\cdot}64 + c^2 = 1$

or $\quad c^2 = 1 - 0{\cdot}89 = 0{\cdot}11 \quad$ or $\quad C = \sqrt{0{\cdot}11}$

Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

50. $(\vec{A} + \vec{B})$ is acting in the plane of $\vec{A}$ and $\vec{B}$.

$(\vec{A} \times \vec{B})$ is acting ⊥r to the plane of $\vec{A}$ and $\vec{B}$.

Thus angle between $(\vec{A} + \vec{B})$ and $(\vec{A} \times \vec{B})$ is 90°. Hence **both Assertion and Reason are false.**

51. If $\vec{C} = \vec{A} + \vec{B}$, then the relation is valid whether $A = B$ or $A > B$ or $B < A$. **Thus both the Assertion and Reason are wrong.**

52. Resultant, $\vec{R} = (\vec{A}+\vec{B}) + (\vec{A}-\vec{B}) = 2\vec{A}$

$\therefore$ Angle between $\vec{A}$ and $\vec{R}$ is zero. **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

53. When the fly walks then shortest distance

travelled $= x + \sqrt{y^2+z^2} = 3 + \sqrt{4^2+5^2}$

$= 3 + \sqrt{41} = 9{\cdot}4$ m.

Thus, **both Assertion and Reason are false.**

54. $240 = A \cos 60°$

or $A = \dfrac{240}{\cos 60°} = \dfrac{240}{\frac{1}{2}} = 480 \text{ km h}^{-1}$

Vertical component of $\vec{A} = A \sin 60°$

$= 480 \times \sqrt{3}/2$

$= 240\sqrt{3} \text{ km h}^{-1}$

Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

55. Horizontal rang $R = \dfrac{u^2 \sin 2\theta}{g}$

Differentiating it w.r.t. g we have

$$\frac{dR}{dg} = -\frac{u^2}{g^2}\sin 2\theta \quad [\because u \text{ and } \theta \text{ are constant}]$$

$$\therefore \; dR = -\frac{u^2}{g^2}\sin 2\theta \, dg$$

$$= -\left(\frac{u^2 \sin 2\theta}{g}\right)\frac{dg}{g} = -R\frac{dg}{g}$$

Thus both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

56. Given, $R = 3$ H

$$\therefore \; \frac{2u^2 \sin\theta\cos\theta}{g} = 3 \times \frac{u^2 \sin^2\theta}{2g}$$

or $2 \sin\theta \cos\theta = \dfrac{3}{2}\sin^2\theta$ or $\tan\theta = \dfrac{4}{3}$

So, $\sin\theta = \dfrac{4}{5}$ and $\cos\theta = \dfrac{3}{5}$

Horizontal range, $R = \dfrac{2u^2}{g}\sin\theta\cos\theta$

$$= 2 \times \frac{u^2}{g} \times \frac{4}{5} \times \frac{3}{5} = \mathbf{\frac{24}{25}\frac{u^2}{g}}$$

Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

57. In a uniform circular motion, the direction of motion changes with time. Hence the velocity of body changes with time. As the rate of change of velocity is acceleration, so uniform circular motion is an accelerated motion.

58. $(\vec{A}-\vec{B})-\vec{C} \neq \vec{A}-(\vec{B}-\vec{C})$. Hence associative law is not applicable to vectors subtraction. Thus **both Assertion and Reason are false.**

59. Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

CONCEPTUAL PROBLEMS

I. Vectors and Vectors addition

1. Can three vectors not in one plane give a zero resultant ? Can four vectors do ?

Sol. Three vectors which are not in one plane can not give zero resultant. This is because resultant of two vectors (in a plane) lies in their plane. It can not balance the third vector which is in a different plane.

The resultant of four coplanar vectors can be zero ; if they are represented in magnitude and direction by four sides of a polygon taken in the same order. The resultant of four non-coplanar vectors may be zero.

2. We can order events in time and there is a sense of time, distinguishing past, present and future. Is therefore, time a vector ?

Sol. Time always flows on and on *i.e.* from past to present and then to future. Therefore, a direction can be assigned to time. Since, the direction of time is unique, it is not to be specified or stated. It is due to this reason that time can not be a vector though it has a direction.

3. Is $|\vec{A}+\vec{B}|$ greater than or less than $|\vec{A}|+|\vec{B}|$? Explain.

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Sol. $|\vec{A}+\vec{B}|^2-(|\vec{A}|+|\vec{B}|)^2$

$$=|\vec{A}|^2+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos\theta$$

$$-|\vec{A}|^2-|\vec{B}|^2-2|\vec{A}||\vec{B}|$$

$$=-2|\vec{A}||\vec{B}|(1-\cos\theta)$$

$$=-2|\vec{A}||\vec{B}|(2\sin^2\theta/2)$$

$$=-4|\vec{A}||\vec{B}|\sin^2\theta/2$$

It is a negative quantity for all values of θ and is zero if $\theta = 0°$. Hence, $|\vec{A}+\vec{B}|\le|\vec{A}|+|\vec{B}|$

4. Is $|\vec{A}-\vec{B}|$ greater than or less than $|\vec{A}|+|\vec{B}|$? Explain.

Sol. $|\vec{A}-\vec{B}|^2-(|\vec{A}|+|\vec{B}|^2)^2$

$$=|\vec{A}|+|\vec{B}|^2-2|\vec{A}||\vec{B}|\cos\theta$$

$$-|\vec{A}|^2-|\vec{B}|^2-2|\vec{A}||\vec{B}|$$

$$=-2|\vec{A}||\vec{B}|(1+\cos\theta)$$

$$=-2|\vec{A}||\vec{B}|(2\cos^2\theta/2)$$

$$=-4|\vec{A}||\vec{B}|\cos^2\theta/2$$

It is a negative quantity for all values of θ and is zero value for $\theta = 180°$. Hence,

$$|\vec{A}-\vec{B}|\le|\vec{A}|+|\vec{B}|$$

5. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and its magnitude is half that of $\vec{B}$. What is the angle between $\vec{A}$ and $\vec{B}$?

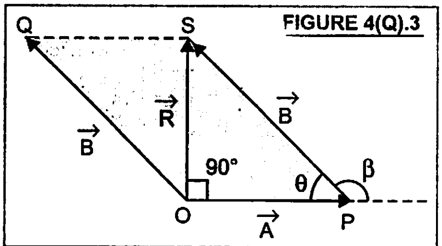
Sol. Refer to Fig. 4(Q).3,

Here, $|\vec{A}| = OP$, $|\vec{B}| = OQ = PS$,

$$|\vec{R}| = OS = B/2$$

In ΔOPS, $(OP)^2+(OS)^2=(PS)^2$

FIGURE 4(Q).3



$$A^2+(B/2)^2=B^2 \text{ or } A^2=\frac{3}{4}B^2 \text{ or } A=\frac{\sqrt{3}}{2}B$$

Let $\angle OPS = \theta$, then $\tan\theta = \dfrac{OS}{OP} = \dfrac{B/2}{A}$

or $\tan\theta = \dfrac{B}{2A} = \dfrac{B}{2\times(\sqrt{3}/2)B} = \dfrac{1}{\sqrt{3}} = \tan 30°$

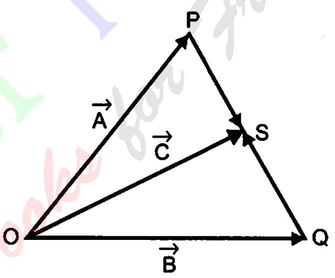
or $\theta = 30°$

Therefore angle between $\vec{A}$ and $\vec{B}$,

$$\beta = (180° - \theta) = (180° - 30°) = 150°$$

6. The three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are represented in magnitude and direction by $\overrightarrow{OP}, \overrightarrow{OQ}$ and $\overrightarrow{OS}$ Fig. 4(Q).4. If $\vec{A}+\vec{B}=2\vec{C}$, show that *S* is the mid point of *PQ*.

FIGURE 4(Q).4



Sol. In Δ *OPS*, by vectors addition,

$$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS} \quad \text{...(i)}$$

Similarly, in Δ *OQS*, we have

$$\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{QS} \quad \text{...(ii)}$$

Adding (*i*) and (*ii*), we have

$$2\,\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{OQ} + \overrightarrow{PS} + \overrightarrow{QS}$$

or $2\vec{C} = \vec{A} + \vec{B} + \overrightarrow{PS} + \overrightarrow{QS}$

or $(2\vec{C} - \vec{A} - \vec{B}) = \overrightarrow{PS} + \overrightarrow{QS}$

or $0 = \overrightarrow{PS} + \overrightarrow{QS}$ $(\because 2\vec{C} = \vec{A} + \vec{B})$

or $\overrightarrow{PS} = -\overrightarrow{QS}$

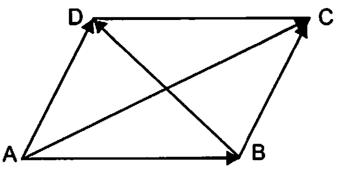
So, *S* is the mid point of *PQ*

7. *ABCD* is a parallelogram Fig. 4(Q).5. *AC* and *BD* are its diagonals. Show that

(*a*) $\overrightarrow{AC} + \overrightarrow{BD} = 2\,\overrightarrow{BC}$

(*b*) $\overrightarrow{AC} - \overrightarrow{BD} = 2\,\overrightarrow{AB}$

FIGURE 4(Q).5



Sol. (*a*) Refer to Fig. 4(Q).5, using triangle law of vectors, we have,

$$\vec{AC} + \vec{BD} = (\vec{AB} + \vec{BC}) + (\vec{BC} + \vec{CD})$$

$$= \vec{AB} + 2\,\vec{BC} + \vec{CD}$$

$$= \vec{AB} + 2\,\vec{BC} - \vec{AB}$$

$$(\because \vec{CD} = -\vec{AB})$$

or $$\vec{AC} + \vec{BD} = 2\,\vec{BC}$$

(*b*) $$\vec{AC} - \vec{BD} = (\vec{AB} + \vec{BC}) - (\vec{BC} + \vec{CD})$$

$$= \vec{AB} - \vec{CD}$$

$$\vec{AC} - \vec{BD} = \vec{AB} - (-\vec{AB}) = 2\,\vec{AB}$$

8. The greatest resultant of two vectors $\vec{P}$ and $\vec{Q}$ is n times their least resultant. Given $|\vec{P}| > |\vec{Q}|$. When θ is the angle between the two vectors, their resultant is half the sum of the two vectors. Show that,

$$\cos\theta = -(n^2+2)/(n^2-1)$$

Sol. The greatest resultant of given two vectors $= (P+Q)$

and the least resultant of given two vectors $= (P-Q)$

According to questions ;

$$(P+Q) = n\,(P-Q) \quad \text{or} \quad Q = \frac{(n-1)}{(n+1)}P$$

If angle between the vectors is θ, then the resultant is given by

$$R^2 = P^2 + Q^2 + 2\,PQ\cos\theta \qquad ...(i)$$

Given, $$R = \frac{P+Q}{2} = \frac{1}{2}\left[P + \left(\frac{n-1}{n+1}\right)P\right]$$

$$= \frac{nP}{(n+1)}$$

Putting the values in (*i*), we get

$$\frac{n^2P^2}{(n+1)^2} = P^2 + \frac{(n-1)^2}{(n+1)^2}P^2 + 2P\frac{(n-1)}{(n+1)}P\cos\theta$$

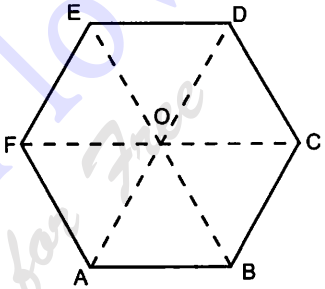
On solving we get,

$$\cos\theta = -\frac{(n^2+2)}{(n^2-1)}$$

9. *ABCDEF* is a regular hexagon, Fig. 4(Q).6. What is the value of

$$(\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF})\ ?$$

FIGURE 4(Q).6



Sol. $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$

$$= \vec{AB} + (\vec{AD} + \vec{DC}) + \vec{AD} + (\vec{AD} + \vec{DE}) + \vec{AF}$$

$$= 3\,\vec{AD} + (\vec{AB} + \vec{DE}) + (\vec{DC} + \vec{AF}) = 3\,\vec{AD}$$

$$= 3 \times (2\,\vec{AO}) = 6\,\vec{AO}$$

$$[\because \vec{AB} = -\vec{DE}, \text{ and } \vec{DC} = -\vec{AF}\,]$$

10. A vector has magnitude and direction (i) Does it have a location in the space ? (ii) Can it vary with time ? (iii) Will two equal vectors $\vec{a}$ and $\vec{b}$ at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.

Sol. (*i*) **A vector in general has no definite location in space** because a vector remains unaffected whenever it is displaced anywhere in space provided its magnitude and direction do not change. However a **position vector has a definite location in space.**

(*ii*) **A vector can vary with time** *e.g.* the velocity vector of an accelerated particle varies with time.

(*iii*) Two equal vectors at different locations in space do not necessarily have same physical effects. For example, two equal forces acting at

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two different points on a body which can cause the rotation of a body about an axis will not produce equal turning effect.

11. A vector has both magnitude and direction. Does that mean anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation and the angle of rotation about the axis. Does that make any rotation a vector ?

Sol. **No.** There are certain physical quantities which have both magnitude and direction, but they are not vectors as they do not follow the laws of vectors addition, which is essential for vectors. The finite rotation of a body about an axis is not a vector because the finite rotations do not obey the laws of vectors addition. However, the small rotation of a body (*i.e.* small angle of rotation) is a vector quantity as it obeys the law of vectors addition.

12. Can you associate vectors with (a) the length of a wire bent into a loop (b) a plane area (c) a sphere ? Explain.

Sol. (*a*) We cannot associate a vector with the length of a wire bent into a loop.

(*b*) We can associate a vector with a plane area. Such a vector is called **area vector** and its direction is represented by outward drawn normal to the area.

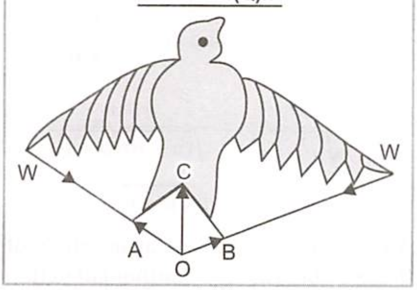
(*c*) We can not associate a vector with volume of sphere however a vector can be associated with the area of sphere.

II. Components of Vectors and Relative Velocity

13. Can a flight of a bird be an example of composition of vectors ?

Sol. **Yes,** the flight of a bird is an example of composition of vectors as is clear from the Fig. 4(Q).7. As the bird flies, it strikes the air downwards with forces *W*, *W* with its wings, along *WO*. According to Newton's third Law of motion, air strikes the wings in opposite directions with the same force in reaction. These reactions are represented in Fig. 4(Q).7 along $\overrightarrow{OA}$ and $\overrightarrow{OB}$. According to parallelogram law of vectors, the resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\overrightarrow{OC}$. It is this resultant upward force $\overrightarrow{OC}$ which is responsible for the flight of the bird.

FIGURE 4(Q).7



14. A room has dimensions 3 m × 4 m × 5 m. A fly starting at one corner ends up at the diametrically opposite corner. (a) What is the magnitude of its displacement ? (b) If the fly were to walk, what is the length of the shortest path it can take ?

Sol. (*a*) If the starting point of fly which is one corner of room is taken as origin of coordinates, then the coordinates of diametrically opposite corner of room are (3, 4, 5). So displacement is

$$\vec{r} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

and $$r = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} \approx \mathbf{7\ m}$$

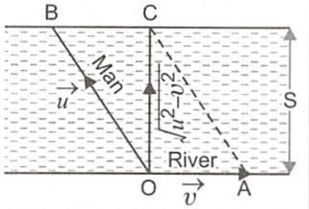
(*b*) When the fly were to walk, then shortest distance travelled is

$$= 3 + \sqrt{4^2 + 5^2} = 3 + \sqrt{41} = \mathbf{9{\cdot}4\ m}$$

15. A man rows directly across a flowing river in time t_1 and rows an equal distance down the stream in time t_2. If u is the speed of the man in still water and v that of stream, then find the ratio of t_1 and t_2 in terms of u and v.

Sol. A man will row directly across a flowing river if his resultant velocity of river flow and the man is along *OC*, which is perpendicular to the river velocity *v*. Fig. 4(Q).8.

FIGURE 4(Q).8



Resultant velocity of man across the river

$$= \sqrt{u^2 - v^2}$$

Resultant velocity of man down the stream

$$= u + v$$

If S is the distance covered in each case, then

$$t_1 = \frac{S}{\sqrt{u^2 - v^2}} \quad \text{and} \quad t_2 = \frac{S}{u+v}$$

$$\frac{t_1}{t_2} = \frac{u+v}{\sqrt{u^2-v^2}} = \frac{u+v}{\sqrt{(u-v)(u+v)}} = \frac{\sqrt{u+v}}{\sqrt{u-v}}$$

$$\therefore\ t_1 : t_2 = \sqrt{u+v} : \sqrt{u-v}$$

16. An aeroplane takes off at an angle of 30° to the horizontal. If the component of its velocity along the horizontal is 240 km h^{-1}, what is the actual velocity ? Also find the vertical component of its velocity ?

Sol. Let u be the actual velocity of aeroplane while taking off. As per question

$$u \cos 30° = 240$$

$$\text{or} \quad u = \frac{240}{\cos 30°} = \frac{240}{\sqrt{3}/2} = \frac{480}{\sqrt{3}} = \frac{480\sqrt{3}}{3}$$

$$= \mathbf{160\sqrt{3}\,km\,h^{-1}}$$

Vertical component velocity of aeroplane

$$= u \sin 30° = 160\sqrt{3} \times 1/2$$

$$= \mathbf{80\sqrt{3}\,km\,h^{-1}}$$

III. Scalar product and vector product of vectors

17. If $\vec{A}.\vec{B} = \vec{A}.\vec{C}$, is it correct to conclude that $\vec{B} = \vec{C}$?

Sol. Given $\quad \vec{A}.\vec{B} = \vec{A}.\vec{C}$,

i.e., $AB \cos \theta_1 = AC \cos \theta_2$ ...(*i*)

where θ_1 is smaller angle between $\vec{A}$ and $\vec{B}$; and θ_2 is the smaller angle between $\vec{A}$ and $\vec{C}$.

From (*i*), $\quad B \cos \theta_1 = C \cos \theta_2$ ...(*ii*)

If $\theta_1 = \theta_2$, then from (*ii*), $B = C$ or $\vec{B} = \vec{C}$

If $\theta_1 \neq \theta_2$, then from (*ii*), $B \neq C$ or $\vec{B} \neq \vec{C}$

18. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$. To which vector, the vector $\vec{A}$ is parallel ?

Sol. As $\vec{A}.\vec{B} = 0$; so $\vec{A}$ is perpendicular to $\vec{B}$.

As $\vec{A}.\vec{C} = 0$; so $\vec{A}$ is perpendicular to $\vec{C}$

As $\vec{B} \times \vec{C}$ is perpendicular to both $\vec{B}$ and $\vec{C}$, so $\vec{B} \times \vec{C}$ is parallel to $\vec{A}$

19. If $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$, is it correct to conclude that $\vec{B} = \vec{C}$?

Sol. Let θ_1 be the smaller angle between $\vec{A}$ and $\vec{B}$; and θ_2 be the smaller angle between $\vec{A}$ and $\vec{C}$

Given, $\quad \vec{A} \times \vec{B} = \vec{A} \times \vec{C}$

$\therefore \quad AB \sin \theta_1\, \hat{n}_1 = AC \sin \theta_2\, \hat{n}_2$ ...(*i*)

where $\hat{n}_1$ and $\hat{n}_2$ are unit vectors in the direction of vectors $(\vec{A} \times \vec{B})$ and $(\vec{A} \times \vec{C})$ respectively.

From (*i*), $\quad B \sin \theta_1\, \hat{n}_1 = C \sin \theta_2\, \hat{n}_2$ ...(*ii*)

If $\theta_1 = \theta_2$ and $\hat{n}_1 = \hat{n}_2$,

then $\quad B = C$ and $\vec{B} = \vec{C}$

If $\theta_1 \neq \theta_2$ and $\hat{n}_1 \neq \hat{n}_2$,

then $\quad B \neq C$ and $\vec{B} \neq \vec{C}$

If $\theta_1 \neq \theta_2$ and $\hat{n}_1 = \hat{n}_2$,

then $\quad B \neq C$ and $\vec{B} \neq \vec{C}$

If $\theta_1 = \theta_2$ and $\hat{n}_1 \neq \hat{n}_2$,

then $\quad B = C$ and $\vec{B} \neq \vec{C}$

20. If $\vec{A} \times \vec{B} = \vec{C} \times \vec{B}$, show that $\vec{C}$ need not be equal to $\vec{A}$.

Sol. $\quad \vec{A} \times \vec{B} = \vec{C} \times \vec{B}$ or $\vec{A} \times \vec{B} - \vec{C} \times \vec{B} = 0$

or $\quad (\vec{A} - \vec{C}) \times \vec{B} = 0.$ ...(*i*)

To satisfy (*i*), the three possibilities can be there

(*i*) $\vec{A} - \vec{C} = 0$ or $\vec{A} = \vec{C}$

(*ii*) $\vec{B} = \vec{0}$

(*iii*) $\vec{A} - \vec{C}$ and $\vec{B}$ are parallel to each other *i.e.* $\vec{A} - \vec{C} = n\,\vec{B}$, where n is a non zero real number.

or $\vec{A} = \vec{C} + n\,\vec{B}$

Thus, if $\vec{A}\times\vec{B} = \vec{C}\times\vec{B}$, $\vec{C}$ need not be equal to $\vec{A}$. The given statement is true if $\vec{B}$ is a zero vector or $\vec{A}$ is equal to $\vec{C}+n\,\vec{B}$.

21. If three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that

$$\vec{A}.\vec{B} = \vec{A}.\vec{C}, \quad \vec{A}\times\vec{B} = \vec{A}\times\vec{C}, \quad \vec{A}\neq\vec{0}$$

then prove that $\vec{B} = \vec{C}$.

Sol. Given, $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ or $\vec{A}.\vec{B} - \vec{A}.\vec{C} = 0$

or $\quad \vec{A}.(\vec{B}-\vec{C}) = 0 \quad$ ...(*i*)

But $\vec{A}\neq\vec{0}$, so **either** $\vec{B}-\vec{C}=\vec{0}$ or $\vec{B}=\vec{C}$

or $\vec{A}$ is perpendicular to $(\vec{B}-\vec{C})$

Also $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ or $\vec{A}\times\vec{B} - \vec{A}\times\vec{C} = \vec{0}$

or $\quad \vec{A}\times(\vec{B}-\vec{C}) = \vec{0} \quad$ ...(*ii*)

But $\vec{A}\neq\vec{0}$ therefore **either** $\vec{B}-\vec{C}=\vec{0}$ or $\vec{B}=\vec{C}$ or $\vec{A}$ is parallel to $(\vec{B}-\vec{C})$

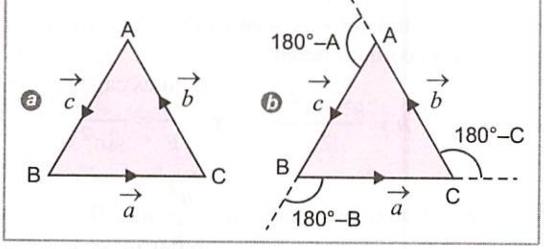
But at a time $\vec{A}$ cannot be perpendicular to $(\vec{B}-\vec{C})$ and parallel to $(\vec{B}-\vec{C})$. So equations (*i*) and (*ii*) will be true at a time if $\vec{B}=\vec{C}$

22. In any ΔABC as shown in Fig. 4(Q).9(*a*) prove that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

Sol. Refer to Fig. 4(Q).9(*b*),

vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by the three sides of a triangle taken in one order. Their resultant is zero. So

FIGURE 4(Q).9



$$\vec{a}+\vec{b}+\vec{c} = 0$$

or $\quad \vec{a}+\vec{b} = -\vec{c}$

or $\quad (\vec{a}+\vec{b})\times\vec{c} = -\vec{c}\times\vec{c} = 0$

or $\quad \vec{a}\times\vec{c}+\vec{b}\times\vec{c} = 0$ or $-\vec{c}\times\vec{a}+\vec{b}\times\vec{c} = 0$

or $\quad \vec{b}\times\vec{c} = \vec{c}\times\vec{a} \quad$ ...(*i*)

Similarly we can get

$$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} \quad ...(ii)$$

From (*i*) and (*ii*),

$$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$$

or $\quad |\vec{a}\times\vec{b}| = |\vec{b}\times\vec{c}| = |\vec{c}\times\vec{a}|$

or $ab\sin(180^\circ - C) = bc\sin(180^\circ - A)$

$= ca\sin(180^\circ - B)$

or $ab\sin C = bc\sin A = ca\sin B$

Dividing it by abc, we get

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

or $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \quad$ (Proved)

IV. Horizontal projectile

23. A body slides down a smooth inclined plane when released from the top, while another body falls freely from the same point. Which one will strike the ground earlier ?

Sol. A body falling freely will reach the ground earlier because its acceleration is g (*i.e.* acceleration due to gravity) which is greater than the acceleration of other body = $g\sin\theta$; where θ is the inclination of the plane with the horizontal.

24. A stone droped from the window of a stationary bus takes 4 seconds to reach the ground. In what time the stone will reach the ground when the bus is moving with (*a*) constant velocity of 108 km h^{-1} (*b*) constant acceleration of 2 km h^{-2}.

Sol. (*a*) **4 seconds** (*b*) **4 seconds.** In both the cases the initial vertical downward velocity is zero and the vertical downward acceleration is acceleration due to gravity g and the stone is to cover the same vertical height in each case.

25. When a rifle is fired at a distant target, the barrel is not lined up exactly on the target. Why ?

Sol. As soon as a bullet is fired from a gun whose barrel is lined up exactly on the target, it starts falling downwards on account of acceleration due to gravity. Due to it, the bullet will hit below the target. Just to avoid it, the barrel of the gun is lined up little above the target, so that the bullet, after travelling in parabolic path hits the distant target.

26. A railway carriage moves over a straight track with acceleration a. Passenger in the carriage drops a stone. What is the acceleration of the stone w.r.t. the carriage and the earth ?

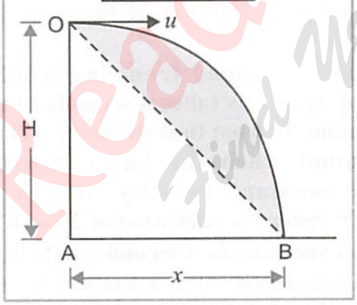
Sol. When a stone is dropped from a railway carriage, it will fall vertically downwards with acceleration due to gravity g. Therefore, **with respect to earth,** the acceleration of the stone will be **g only.**

Inside the carriage the stone has two accelerations : (*i*) horizontal acceleration a, due to the motion of the carriage and (*ii*) vertical acc. due to gravity g. Thus the acceleration of the stone **with respect to carriage is** $\sqrt{a^2 + g^2}$

27. A helicopter on a flood relief mission flying horizontally with a speed u at an altitude H, has to drop a food packet for a victim standing on the ground. At what distance from the victim should the food packet be dropped ?

Sol. Refer to Fig. 4(Q).10, O is the position of helicopter from where a food packet is dropped. B be the position of victim. The food packet follows a parabolic path while going from O to B.

FIGURE 4(Q).10



Taking vertical downward motion of food packet from O to B, we have

$$u_y = 0,\ y_0 = 0,\ y = H,\ a_y = g,\ t = ?$$

As $\quad y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$$\therefore \quad H = 0 + 0 + \frac{1}{2} g t^2 \quad \text{or} \quad t = \sqrt{\frac{2H}{g}}$$

Taking horizontal motion of food packet from O to B, we have

$$x_0 = 0,\ x = x,\ u_x = u,\ a_x = 0,\ t = \sqrt{\frac{2H}{g}}$$

As $\quad x = x_0 + u_x t + \frac{1}{2} a_x t^2$

$$\therefore \quad x = 0 + u\sqrt{\frac{2H}{g}} + 0 = \sqrt{\frac{2u^2 H}{g}}$$

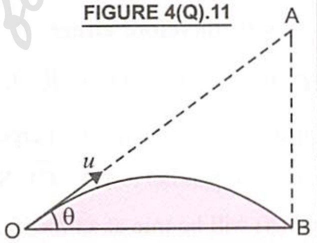
$$\therefore \quad AB = x = \sqrt{\frac{2u^2 H}{g}}$$

Distance $\quad OB = \sqrt{OA^2 + AB^2} = \sqrt{H^2 + \frac{2u^2 H}{g}}$

V. Angular projectile

28. A projectile is fired at an angle θ with the horizontal direction from O, Fig. 4(Q).11. Neglecting the air friction, it hits the ground at B after 3 seconds. What is the height of point A from ground ?
[Use $g = 10\ \text{m/s}^2$]

FIGURE 4(Q).11



Sol. Height of point A from B is the vertical distance a body will cover in three seconds. Thus,

$$AB = S = \frac{1}{2} g t^2 \simeq \frac{1}{2} \times 10 \times 3^2 = \mathbf{45\ m}$$

29. In long jump, does it matter how high you jump ? What factors determine the span of the jump ?

Sol. **Yes,** in long jump, it matters how high one jumps. It is explained below.

For initial velocity u and angle of projection θ, the maximum height,

$$h = \frac{u^2 \sin^2\theta}{2g} \quad \text{or} \quad \frac{u^2}{g} = \frac{2\text{h}}{\sin^2\theta}$$

and, horizontal range, $R = \frac{u^2}{g} \sin 2\theta$

$$= \frac{2\text{h}}{\sin^2\theta}\ 2\sin\theta\cos\theta = 4\text{h}\cot\theta$$

Thus the span of jump depends upon (*i*) height h attained or initial velocity of projection (*ii*) angle of projection, θ.

30. What are the two angles of projection of a projectile projected with velocity of 30 m/s, so that the horizontal range is 45 m. Take, $g = 10\ m/s^2$.

Sol. Horizontal range $= \frac{u^2 \sin 2\theta}{g}$

$$45 = \frac{30^2 \sin 2\theta}{10}$$

$$\sin 2\theta = \frac{45\times10}{30\times30} = \frac{1}{2} = \sin 30° \text{ or } \sin 150°$$

$$2\theta = 30° \quad \text{or} \quad 150° \quad \text{or} \quad \boldsymbol{\theta = 15° \text{ or } 75°}$$

31. If there are two values of time for which a projectile given angular projection is at the same height, then show that the sum of these two times is equal to the time of flight.

Sol. Refer to Fig. 4(Q).12, let the projectile projected from O reach the locations A and B after times t_1 and t_2 respectively, where $AA_1 = BB_1 = h$.

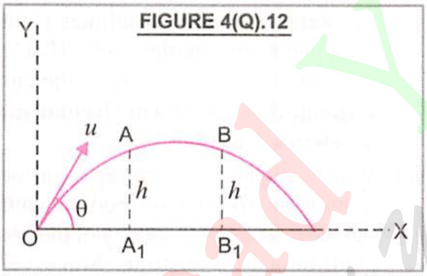


FIGURE 4(Q).12

If the projectile reaches at a height h after time t, then $u = u \sin\theta$, $a = -g$, $S = h$, $t = t$

$$S = ut + \frac{1}{2}at^2$$

$$\therefore \quad h = u \sin\theta\, t + \frac{1}{2}(-g)\, t^2$$

$$gt^2 - 2u \sin\theta\, t + 2h = 0$$

$$\text{or} \quad t = \frac{2u\sin\theta \pm \sqrt{4u^2\sin^2\theta - 4g\times 2h}}{2g}$$

$$\therefore \quad t_1 = \frac{u\sin\theta - \sqrt{u^2\sin^2\theta - 2gh}}{g}$$

$$\text{and} \quad t_2 = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta - 2gh}}{g}$$

$$\therefore \quad t_1 + t_2 = \frac{u\sin\theta - \sqrt{u^2\sin^2\theta - 2gh}}{g} + \frac{u\sin\theta + \sqrt{u^2\sin^2\theta - 2gh}}{g}$$

$$= \frac{2u\sin\theta}{g} = T = \textbf{time of flight}$$

32. If a projectile has a constant initial speed and angle of projection, find the relation between the changes in the horizontal range due to change in acceleration due to gravity.

Sol. Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

Differentiating it *w.r.t.* g, we have

$$\frac{dR}{dg} = -\frac{u^2}{g^2}\sin 2\theta \quad [\because\ u \text{ and } \theta \text{ are constant}]$$

$$\text{or} \quad dR = -\frac{u^2\sin 2\theta}{g}\frac{dg}{g} = -R\frac{dg}{g}$$

$$\text{or} \quad \boldsymbol{\frac{dR}{R} = -\frac{dg}{g}}$$

33. There are two angles of projection for which the horizontal range is the same. Prove that the sum of the maximum heights for these two angles does not depend upon the angle of projection.

Sol. If a projectile is projected with velocity u, making an angle θ with the horizontal direction,

then Horizontal range, $R = \frac{u^2}{R}\sin 2\theta$

and Max. height, $H = \frac{u^2\sin^2\theta}{2g}$

Case (i) If $\theta = \alpha$, let $R = R_1$ and $H = H_1$

$$\text{then} \quad R_1 = \frac{u^2\sin 2\alpha}{g} \qquad \textbf{...(i)}$$

$$\text{and} \quad H_1 = \frac{u^2}{2g}\sin^2\alpha \qquad \textbf{...(ii)}$$

Case (ii) If $\theta = (90° - \alpha)$,
let $R = R_2$ and $H = H_2$, then

$$R_2 = \frac{u^2\sin 2(90° - \alpha)}{g} = \frac{u^2}{g}\sin(180° - 2\alpha)$$

$$= \frac{u^2}{g}\sin 2\alpha \qquad \textbf{...(iii)}$$

$$H_2 = \frac{u^2}{2g}\sin^2(90° - \alpha) = \frac{u^2}{2g}\cos^2\alpha \qquad ...(iv)$$

From (*i*) and (*iii*) ; $R_1 = R_2$

From (*ii*) and (*iv*) ;

$$H_1 + H_2 = \frac{u^2}{2g}(\sin^2\theta + \cos^2\theta) = \frac{u^2}{2g}$$

which is independent of angle of projection θ. Thus **proved.**

34. A particle is projected with a velocity *u* so that its horizontal range is thrice the greatest height attained. What is its horizontal range ?

Sol. Given, $\dfrac{u^2\sin 2\theta}{g} = 3 \times \dfrac{u^2\sin^2\theta}{2g}$

or $2\sin\theta\cos\theta = \dfrac{3}{2}\sin^2\theta$; or $\tan\theta = 4/3$;

so $\sin\theta = 4/5$ and $\cos\theta = 3/5$

$\therefore$ Horizontal range $= \dfrac{u^2}{g}\sin 2\theta$

$$= \frac{2u^2}{g}\sin\theta\cos\theta = \frac{2u^2}{g}\times\frac{4}{5}\times\frac{3}{5} = \frac{\mathbf{24}\,u^2}{\mathbf{25}\,g}$$

35. A bob hung from the ceiling of a room by a string is performing simple harmonic oscillations. What will be the trajectory of the bob, if the string is cut. When bob is (*i*) at one of its extreme positions (*ii*) at its mean position ?

Sol. (*i*) When a bob hung from a ceiling, while performing simple harmonic motion reaches at one of its extreme position, the bob is momentarily at rest. If the string is cut there, the bob will fall freely vertically downwards under the effect of gravity.

(*ii*) When a bob hung from a ceiling while performing a simple harmonic motion reaches at mean position, it has maximum horizontal velocity. If the string is cut there, the bob will follow a parabolic path as it is similar to the case of projectile given a horizontal projection.

36. A projectile can have the same range *R* for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then find the relation between t_1, t_2 and *R*.

Sol. A projectile will have same horizontal range for two angles of projection, θ and (90° – θ).

Then horizontal range, $R = \dfrac{u^2\sin 2\theta}{g}$

$$t_1 = \frac{2u\sin\theta}{g} \text{ and } t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore\ t_1 t_2 = \frac{2u\sin\theta}{g}\times\frac{2u\cos\theta}{g} = \frac{2}{g}\frac{u^2\sin 2\theta}{g} = \frac{2R}{g}$$

Thus $\boldsymbol{t_1 t_2 = 2R/g}$

VI. Circular motion

37. Can a particle accelerate if its speed is constant ? Can it accelerate if its velocity is constant ? Explain.

Sol. When the particle describes a uniform circular motion, its speed is constant but it has centripetal acceleration acting along the radius directed towards the centre of the circular path. When the particle is moving with a constant velocity, there is no change in velocity with time and hence its acceleration is zero.

38. A stone tied at the end of string is whirled in a circle. If the string breaks, the stone flies away tangentially. Why ?

Sol. When a stone is going around a circular path, the instantaneous velocity of stone is acting along tangent to the circle. When the string breaks, the centripetal force stops acting. Due to inertia, the stone continues to move along the tangent to circular path. That is why, the stone flies off tangentially to the circular path.

39. Explain that a uniform circular motion is an accelerated motion.

Sol. When a body is describing a uniform circular motion, the speed of the body remains the same but the direction of velocity of the body changes continuously with time. Since velocity is a vector quantity, the continuous change in the direction of velocity of the body means the continuous change in velocity of the body with time. As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.

40. When a body is in a uniform circular motion, what will be its direction of linear velocity, angular velocity and acceleration at any instant.

Sol. When a body is in a uniform circular motion, at any instant, its direction of linear velocity is tangent to the circular path at that instant. The direction of angular velocity is perpendicular to the plane of motion of body passing through its centre directed upwards for anticlockwise rotation of body and directed downwards for clockwise rotation of body.

The direction of acceleration in a uniform circular motion is along the radius directed towards the centre.

41. In a non-uniform circular motion, what is the direction of acceleration ? Discuss.

Sol. In a non-uniform circular motion of an object, the object has two types of acceleration :

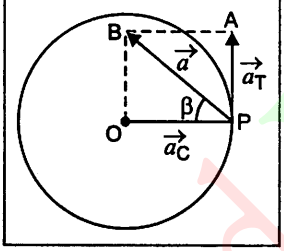
(*i*) Tangential acceleration $(\vec{a}_T)$. It acts tangentially to the circular path at a given instant of the object.

(*ii*) Centripetal acceleration $(\vec{a}_C)$. It acts along the radius directed towards the centre of the circular path. Here $\vec{a}_T$ and $\vec{a}_C$ are perpendicular to each other at every location on circular path. Fig. 4(Q).13. The resultant acceleration of object is

$$\vec{a} = \vec{a}_T + \vec{a}_C$$

where $|\vec{a}| = \sqrt{a_T^2 + a_C^2}$

FIGURE 4(Q).13



If $\vec{a}$ make an angle β with $\vec{a}_C$ then

$$\tan\beta = \frac{a_T}{a_C} \quad \text{or} \quad \beta = \tan^{-1}\left(\frac{a_T}{a_C}\right)$$

42. Discuss whether or not ; angular displacement is a vector quantity ?

Sol. Angular displacement is a vector quantity provided angle θ is small because the commutative law of vector addition for large angle is not valid, where as for small angles, the law is valid *i.e.* $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$, but $\Delta\theta_1 + \Delta\theta_2 = \Delta\theta_2 + \Delta\theta_1$. The same is explained below.

Consider a book with front page *F* lying in the plane of paper. It can be rotated about two mutually perpendicular axes 1 and 2 as shown in Fig. 4(Q).14(*a*).

(*i*) Let the book be rotated through an angle θ_1 (= 90°) in the clockwise direction about axis 1. It takes the position as shown in Fig. 4(Q).14(*b*). On further rotating the book through an angle θ_2 (= 90°) in anticlockwise direction about axis 2, the book occupies the position as shown in Fig. 4(Q).14(*c*).

(*ii*) From the initial position of book, [Fig 4(Q).14(*a*)], if the book is rotated through θ_1 (= 90°) in the anticlockwise direction first about the axis 2, it occupies the position as shown in Fig. 4(Q).14(*d*). On further rotating the book through θ_2 (= 90°) in clockwise direction about axis 1, the book occupies the position shown in Fig. 4(Q).14(*e*).

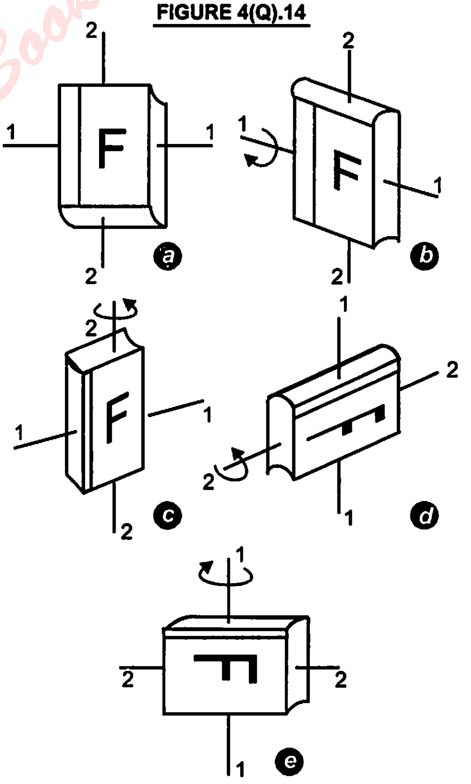
From above we note that the final positions of the book shown in Fig. 4.33(c) and Fig. 4(Q).14(*e*) are not the same. Hence $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$.

This shows that θ_1 and θ_2 are not vectors as they do not obey the commutative law of vector addition.

If the book is rotated through a smaller angle $\Delta\theta_1$ and $\Delta\theta_2$ (say 2° or 3°), the final positions of the book in the two cases discussed above would almost be the same. As $\Delta\theta \to 0$, the final positions of the book will become indistinguishable, hence

$$\Delta\theta_1 + \Delta\theta_2 = \Delta\theta_2 + \Delta\theta_1.$$

FIGURE 4(Q).14



QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Vectors and Vectors addition

1. Give an example of a physical quantity (*i*) which has neither unit nor direction (*ii*) has a direction but not a vector (*iii*) can be either a vector or a scalar.

Ans. (*i*) Specific gravity (*ii*) pressure or current (*iii*) angular displacement.

2. Is finite rotation a vector ?

Ans. The finite rotation about an axis is not a vector because its addition to another finite rotation about a different axis does not obey commutative law of addition.

3. Consider a vector $\vec{F} = 4\hat{i} - 3\hat{j}$. What is a unit vector perpendicular to $\vec{F}$?

Ans. $\hat{k}$; It will be a unit vector along Z-axis as $\vec{F}$ lies in *X*-*Y* plane.

4. State the most basic condition for the addition of vectors.

Ans. The most basic condition for the addition of vectors is that they must represent the physical quantities of the same nature.

5. Vectors can not be added algebraically. Why ?

Ans. It is so because, the vectors possess direction as well as magnitude.

6. Does it make a sense to call a physical quantity a vector, when its magnitude is zero ?

Ans. Yes, null vector has a definite physical significance.

7. Are the magnitude and direction of $(\vec{A} - \vec{B})$ same as that of $(\vec{B} - \vec{A})$? (CBSE Delhi 2010)

Ans. The vectors $(\vec{A} - \vec{B})$ and $(\vec{B} - \vec{A})$ have the same magnitude but opposite directions.

8. Is the magnitude of $(\vec{A} - \vec{B})$ is the same as that of $(\vec{A} + \vec{B})$?

Ans. Magnitude is the same when θ between $\vec{A}$ and $\vec{B}$ is 90° and is different if θ is other than 90°.

9. Give two necessary conditions for a given quantity to be a vector.

Ans. The quantity can be taken as a vector if it obeys the following conditions :

(*i*) The quantity has both magnitude and direction.

(*ii*) The quantity obeys the laws of vectors addition.

10. What is the property of two vectors if,

$$\vec{A} + \vec{B} = \vec{A} - \vec{B} \text{ ?}$$

Ans. Given, $\vec{A} + \vec{B} = \vec{A} - \vec{B}$ or $2\vec{B} = 0$ or $B = 0$

i.e. $\vec{B}$ is a null vector.

11. Are the commutative law and associative law applicable to vectors subtraction.

Ans. Commutative law is not applicable to vectors subtraction. It is so because

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

Associative law is also not applicable to vectors subtraction. It is so because

$$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$$

12. If the magnitude of vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively, and if $\vec{A} + \vec{B} = \vec{C}$, find the angle between $\vec{A}$ and $\vec{C}$.

Ans. Since $A^2 + B^2 = C^2$; therefore angle between $\vec{A}$ and $\vec{B}$ is 90°. Let θ be the angle between $\vec{A}$ and $\vec{C}$, then, $\cos\theta = 3/5$ or $\theta = \cos^{-1}$ **(3/5)**

13. Find the vector sum of *n* coplanar forces, each of magnitude *F*, when each force is making an angle of 2 π/*n* with the proceeding one.

Ans. Zero. Since the total angle between *n* coplanar forces = (2 π/*n*) × *n* = 2 π. It means the forces can be represented by the various sides of a closed polygon taken in one order, hence their resultant force is **zero.**

14. Is $\hat{i} - \hat{j}$ a unit vector ? Explain.

Ans. $(\hat{i} - \hat{j})$ **is not a unit vector.** If $\vec{R} = \hat{i} - \hat{j}$, then $R = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$ and angle β which $\vec{R}$ makes with *X*-axis, is given by,

$$\cos\beta = 1/\sqrt{2} = \cos 45^\circ$$

or **β = 45° below the *x*-axis**

15. Under what condition, the three vectors (*i*) cannot give zero resultant (*ii*) can give zero resultant ?

Ans. (*i*) When three vectors are not lying in one plane, they **can not** produce zero resultant.

(*ii*) When three vectors are lying in a plane and are represented in magnitude and direction by the three sides of a triangle taken in the same order, they **can produce** zero resultant.

16. Can the magnitude of the resultant vector of two given vectors be less than the magnitude of any of the given vector ?

Ans. Yes, if the angle between the two vectors is more than 90° but less than 270°, because in that case cos θ is negative.

17. Under what conditions the directions of sum and difference of two vectors will be the same.

Ans. The direction of sum and difference of two vectors will be the same, when the two vectors are unequal in magnitude and acting in the same direction.

18. What are minimum number of unequal forces whose vector sum is zero ?

Ans. Three forces, provided they can be represented by the three sides of a triangle taken in the same order.

19. What are the minimum number of forces (all numerically equal) whose vector sum can be zero ?

Ans. Two only, provided they are acting in opposite directions.

II. Components of Vectors and Relative Velocity

20. A unit vector is represented by $a\hat{i} + b\hat{j} + c\hat{k}$. If the values of a and b are 0·6 and 0·8 respectively, find the value of c. (CBSE Delhi 2008)

Ans. Given, $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k} = 0{\cdot}6\,\hat{i} + 0{\cdot}8\,\hat{j} + c\,\hat{k}$.

Here $|\vec{A}| = 1 = \sqrt{(0{\cdot}6)^2 + (0{\cdot}8)^2 + c^2}$

or $1 = 0{\cdot}36 + 0{\cdot}64 + c^2$ or $c^2 = 0$ or $c = 0$.

21. A vector $\vec{A}$ is expressed as $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$ where A_x and A_y are its components along x-axis and y-axis respectively. If vector $\vec{A}$ makes an angle θ with x-axis, then θ is given by which expression ? (CBSE Delhi 2008)

Ans. Here $A_x = A\cos\theta$ and $A_y = A\sin\theta$

$$\therefore\ \frac{A_y}{A_x} = \frac{A\sin\theta}{A\cos\theta} = \tan\theta \text{ or } \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

22. What are the maximum number of (i) rectangular component vectors (ii) component vectors, into which a vector can be resolved in a plane ?

Ans. (*i*) Two only in a plane (*ii*) Any number of component vectors in a plane.

23. Can a vector be multiplied with both dimensional and non-dimensional scalar ?

Ans. Yes, when a vector is multiplied with a dimensional scalar, the resultant vector will have different dimensions. *For example,* if acceleration vector is multiplied with mass (a dimensional scalar), the resultant vector has the dimensions of force.

When a vector is multiplied with non-dimensional scalar, it will be a vector having dimensions as that of the given vector.

24. If a vector is added or subtracted from a vector, the resultant is a vector. Is this also true in case of multiplication of two vectors ?

Ans. May or may not be true. If the multiplication of two vectors is a scalar product or dot product, then the new physical quantity is called scalar. If the multiplication of two vectors is a vector product or cross product, then the new physical quantity is called vector.

25. A boat is moving with a velocity $(3\hat{i} + 4\hat{j})$ with respect to ground. The water in the river is moving with a velocity $-3\hat{i} - 4\hat{j}$ with respect to ground. What is the relative velocity of boat with respect to water ?

Ans. Velocity of boat *w.r.t.* ground, $\vec{v}_B = 3\hat{i} + 4\hat{j}$.

Velocity of water *w.r.t.* ground, $\vec{v}_w = -3\hat{i} - 4\hat{j}$.

Relative velocity of boat *w.r.t.* water is,

$$\vec{v}_{BW} = \vec{v}_B - \vec{v}_W = (3\hat{i} + 4\hat{j}) - (-3\hat{i} - 4\hat{j})$$
$$= 6\hat{i} + 8\hat{j}$$

III. Scalar product and vector product of vectors

26. If $\vec{A}, \vec{B}$ and $\vec{C}$ are non-zero vectors and $\vec{A}.\vec{B} = 0$ and $\vec{B}.\vec{C} = 0$, then find out the value of $\vec{A}.\vec{C}$.

Ans. As $\vec{A}.\vec{B} = 0$;

$\therefore\ AB\cos\theta = 0$ or $\cos\theta = 0$ or $\theta = 90°$

Similarly, $\vec{B}.\vec{C} = 0$;

$\therefore$ $BC \cos \theta_1 = 0$ or $\cos \theta_1 = 0$ or $\theta_1 = 90°$,

i.e., $\vec{A}$ and $\vec{C}$ are parallel to each other.

$\therefore$ $\vec{A}.\vec{C} = AC \cos 0° = AC$

27. Can the scalar product of two vectors be negative ?

Ans. **Yes,** it will be negative if the angle between the two vectors is between 90° to 270°.

28. If $\vec{A}, \vec{B}$ and $\vec{C}$ are mutually perpendicular vectors, then find the value of $\vec{A}.(\vec{B}+\vec{C})$.

Ans. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B}+\vec{A}.\vec{C}$

$$= AB \cos 90° + AC \cos 90° = \mathbf{0}$$

29. Find the value of $\hat{i}.(\hat{j}\times\hat{k})$.

Ans. $\hat{i}.(\hat{j}\times\hat{k}) = \hat{i}.(\hat{i}) = (1)\,(1) \cos 0° = \mathbf{1}$

30. Show that $A = (\vec{A}.\vec{A})^{1/2}$

Ans. As $\vec{A}.\vec{A} = A\,A \cos 0° = A^2$ or $A = (\vec{A}.\vec{A})^{1/2}$

31. If $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$, find the value of angle between $\vec{A}$ and $\vec{B}$.

Ans. As $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$, therefore,

$AB \cos \theta = AB \sin \theta$ or $\tan \theta = 1$ or $\theta = \pi/4$

32. A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards east. What is the direction of $\vec{A}\times\vec{B}$?

Ans. The direction of $\vec{A}\times\vec{B}$ is **along north** according to Right handed screw rule.

33. What is the angle between $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ denote the adjacent sides of a parallelogram drawn from a point and the area of the parallelogram is $\frac{1}{2}AB$.

Ans. Area of parallelogram $= |\vec{A}\times\vec{B}|$

$$= AB \sin \theta = \frac{1}{2}AB \quad \text{(Given)}$$

$$\therefore \quad \sin \theta = \frac{1}{2} = \sin 30° \quad \text{or} \quad \theta = \mathbf{30°}$$

IV. Horizontal projectile

34. Is the rocket in flight is an illustration of projectile ?

Ans. **No,** because it is propelled by combustion of fuel and does not move under the effect of gravity alone.

35. Can there be a motion in two dimensions with an acceleration only in one direction ?

Ans. **Yes,** it is so in case of a projectile motion, where the acceleration acts vertically downwards while the projectile follows a parabolic path.

36. A body projected horizontally moves with the same horizontal velocity throughout the motion although it is under the effect of force of gravity. Why ?

Ans. In horizontal projection of a projectile, the force of gravity acts in a vertically downward direction which cannot affect the horizontal component velocity of the body. Due to it, the body moves with a constant horizontal velocity in horizontally projected body.

V. Angular projectile

37. A projectile is shot in air with velocity v at an angle θ with the horizontal. Neglecting the air resistance, what are the vertical acceleration and horizontal acceleration at the highest point of its flight ?

Ans. At the highest point, the vertical downward acceleration of a projectile is g ; due to gravity pull. The horizontal acceleration is zero because no horizontal force acts on it.

38. Is the maximum height attained by projectile is largest when its horizontal range is maximum ?

Ans. No ; horizontal range is maximum when $\theta = 45°$ and maximum height attained by projectile is largest when $\theta = 90°$.

39. A ball is thrown upwards and returns to the ground describing parabolic path. Which of the quantities remain constant throughout the motion.

Ans. Horizontal component velocity and total mechanical energy of projectile remain constant throughout the motion.

40. Two bodies are projected at angle θ and $(90° - \theta)$ to the horizontal with the same speed. Find the ratio of their time of flight.

Ans. $T_1 = 2\,u \sin \theta/g$

and $T_2 = 2\,u \sin (90° - \theta)$

so $T_1/T_2 = \sin \theta/\cos \theta = \tan \theta$.

41. A body is projected with a speed u at an angle to the horizontal to have maximum range. What is its velocity at the highest point ?

Ans. For range to be maximum $\theta = 45°$. At the highest point, the vertical component velocity is zero but the horizontal component velocity $= u \cos \theta = u \cos 45° = u/\sqrt{2}$

42. The direction of the oblique projectile becomes horizontal at the maximum height. What is the cause of it ?

Ans. At the maximum height of projectile, the vertical component velocity becomes zero and only horizontal component velocity of projectile is there.

43. A bomb thrown as projectile with angular projection explodes in mid-air. What is the path traced by the centre of mass of the fragments assuming the friction to be negligible ?

Ans. The path traced by the centre of mass of the fragments is a parabolic path, because the centre of mass of the fragments of the bomb will follow the path which the bomb would have followed if remains unexploded.

44. What will be the effect on maximum height of a projectile when its angle of projection is changed from 30° to 60°, without changing its initial velocity of projection ?

Ans. Maximum height, $H = \dfrac{u^2 \sin^2 \theta}{2g}$

i.e., $H \propto \sin^2 \theta$

$$\therefore \quad \frac{H_2}{H_1} = \frac{\sin^2 60°}{\sin^2 30°} = \frac{(\sqrt{3}/2)^2}{(1/2)^2} = 3$$

or $\boldsymbol{H_2 = 3\, H_1}$

Thus the maximum height reached becomes three times the original height of projectile.

45. Two bombs of 2 kg and 4 kg are thrown from a canon with the same velocity in the same direction. Which bomb will reach the ground first ?

Ans. Both the bombs will reach the ground simultaneously, because the time of flight is independent of mass of bombs but depends upon the initial velocity of projection and angle of projection, which is same for both the bombs.

46. A projectile is fired with kinetic energy 1 kJ. If the range is maximum, what is its K.E. at the highest point ?

Ans. Here, $\dfrac{1}{2} mv^2 = 1\,\text{kJ} = 1000\,\text{J}$.

For maximum range, $\theta = 45°$ and velocity of projectile at the highest point $= v \cos 45°$.

At the highest point, $KE = \dfrac{1}{2} m (v \cos 45°)^2$

$$= \frac{1}{2}\, m\, \frac{v^2}{2} = \frac{1000}{2} = \mathbf{500\,J}$$

47. A body of mass m is projected with a speed v making an angle θ with the vertical. What is the change in momentum of the body along (*i*) the X-axis (*ii*) the Y-axis ; between the starting point and the highest point of its path.

Ans. (*i*) Zero.

(*ii*) $-\,m\, v \cos \theta$.

48. A projectile of mass m is thrown with velocity u from the ground at an angle of 60° with the horizontal. What is the magnitude of change in momentum between leaving and arriving back at the ground ?

Ans. In angular projection of a projectile, the horizontal component velocity $u \cos \theta$ remains constant at all instants of its motion whereas vertical component velocity at the point of projection is $v \sin \theta$ and at the point of return is $-v \sin \theta$. Therefore, magnitude of change in momentum

$$= mv \sin \theta + mv \sin \theta = 2\, mv \sin \theta$$

$$= 2\, mv \sin 60° = 2\, mv \times \sqrt{3}/2 = \boldsymbol{\sqrt{3}\, mv}$$

acting in vertical direction.

VI. Circular motion

49. Can a body move on a curved path without having acceleration ?

Ans. No. It is so because while moving on a curved path, the velocity of the body (which is represented by tangent to the curved path at a point) changes with time and the body is in the accelerated state.

50. The velocity of a particle is constant in magnitude but not in a direction. What is the nature of trajectory followed by the motion of particle ?

Ans. In this case, the particle is moving on a circular path with uniform speed. Here the direction of velocity at an instant is tangent to the circular path.

51. What will the effect on the centripetal acceleration, if both the speed and the radius of the circular path of a body are doubled ?

Ans. Original centripetal acceleration,

$$a_C = \frac{v^2}{r}$$

New centripetal acceleration,

$$a'_C = \frac{(2v)^2}{2r} = \frac{2v^2}{r} = 2\,a_C$$

52. A body in a uniform horizontal circular motion possesses a variable velocity. Does it mean that the K.E. of the body is also variable ?

Ans. No ; K.E. of body $= \frac{1}{2} m v^2 = \frac{1}{2} m\,(\vec{v}\,.\,\vec{v})$, which is scalar. In uniform circular motion, speed of body v remains constant, hence K.E. of body remains constant in uniform circular motion.

53. When a knife is sharpened with the help of a rotating grinding stone, the spark always travel tangentially to it. Why ?

Ans. The sparks are infact red hot particles of sand which get separated from grinding stone, when knife is sharpened. Since particles on the rotating grinding stone describe circular motions and their velocities are directed along the tangent to the circular path, hence they move along the tangent to the grinding stone.

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Vectors and Vectors addition

1. Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the equation $\vec{A} + \vec{B} = \vec{C}$, and their magnitudes are related by the equation $A + B = C$. How is the vector $\vec{A}$ oriented with respect to vector $\vec{B}$? Explain your reasoning.

Ans. If θ is the angle between $\vec{A}$ and $\vec{B}$, then $C = (A^2 + B^2 + 2\,AB\cos\theta)^{1/2} = (A + B)$ [Given]

$$\therefore \quad A^2 + B^2 + 2\,AB\cos\theta = (A + B)^2 = A^2 + 2\,AB + B^2$$

or $\cos\theta = 1$ or $\theta = 0°$

2. Do $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ lie in the same plane. Explain.

Ans. Yes, $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ lie in the same plane because $(\vec{A} + \vec{B})$ is represented by the diagonal of a parallelogram where two adjacent sides are represented by vectors $\vec{A}$ and $\vec{B}$. The diagonal passes through the common tail of $\vec{A}$ and $\vec{B}$.

$(\vec{A} - \vec{B})$ is represented by the other diagonal of the same parallelogram. It is not passing through the common tail of $\vec{A}$ and $\vec{B}$.

From above we note that both $(\vec{A} + \vec{B}), (\vec{A} - \vec{B})$ lie in the plane of same parallelogram.

3. Can we add a vector representing a force of 10 N to a vector of force 200 dyne.

Ans. Force is a vector quantity. The two forces can be added by laws of vectors; parallelogram law of vectors or triangle law of vectors which is possible if we know the angle between the two forces. As the angle between the two given forces is not known, hence, they can not be added.

4. Given $\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{0}$, can the magnitude of $\vec{A} + \vec{B} + \vec{C}$ be equal to the magnitude of $\vec{D}$. Explain.

Ans. Yes. Given $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$

or $\vec{A} + \vec{B} + \vec{C} = \vec{0} - \vec{D} = -\vec{D}$

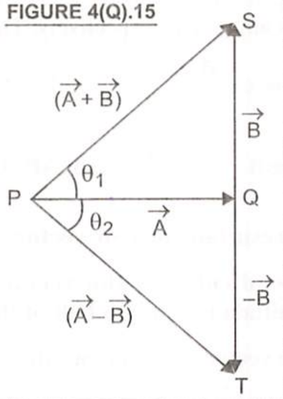
Hence magnitude of $(\vec{A} + \vec{B} + \vec{C})$ must be equal to the magnitude of $\vec{D}$

5. Two vectors $\vec{A}$ and $\vec{B}$ are of equal lengths $(A = B)$ and mutually perpendicular. Show by vector diagram that their vector sum $(\vec{A} + \vec{B})$ and vector difference $(\vec{A} - \vec{B})$ will be of the same length and mutually perpendicular.

Ans. Draw $(\overrightarrow{PQ}) = \vec{A}$ and from the arrow head of $\vec{A}$, draw $(\overrightarrow{QS}) = \vec{B}$, of the same length (*i.e.*, $QS = PQ$) and perpendicular to $\vec{A}$. Fig. 4(Q).15. Now $(\overrightarrow{PS})$ will represent $(\vec{A} + \vec{B})$. Here,

$$\tan\theta_1 = \frac{QS}{PQ} = 1 \quad \text{or} \quad \theta_1 = 45^\circ$$

FIGURE 4(Q).15



Now draw $(\overrightarrow{QT}) = -\vec{B}$,

where $QT = QS$. Now $(\overrightarrow{PT})$ will represent $(\vec{A}-\vec{B})$. Here, $\tan\theta_2 = \frac{QT}{PQ} = 1$ or $\theta_2 = 45^\circ$

On measuring, the lengths of $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ come out to be the same and angle between them, $(\theta_1 + \theta_2) = 45^\circ + 45^\circ = \mathbf{90^\circ}$

6. Two vectors $\vec{A}$ and $\vec{B}$ are added. Prove that the magnitude of the resultant vector can not be greater than $(A + B)$ and smaller than $(A - B)$ or $(B - A)$.

Ans. The magnitude of resultant vector $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Case (i). R will be maximum if $\cos\theta = 1$ or $\theta = 0^\circ$.

Then $R = \sqrt{A^2 + B^2 + 2AB(1)} = (A + B)$

Case (ii). R will be minimum if $\cos\theta = -1$ or $\theta = 180^\circ$

$$\text{Then, } R = \sqrt{A^2 + B^2 + 2AB(-1)} = (A - B) \text{ or } (B - A)$$

7. Suppose you have two forces $\vec{F}$ and $\vec{F}$. How would you combine them in order to have resultant force of magnitudes (*a*) zero (*b*) $2\vec{F}$ and (*c*) $\vec{F}$. **(CBSE Delhi 2003)**

Ans. The magnitude of resultant $\vec{R}$ of the addition of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

where θ is the angle between $\vec{A}$ and $\vec{B}$.

(*a*) If $\vec{A} = \vec{F}$, $\vec{B} = \vec{F}$ and $R = 0$, then

$$0 = \sqrt{F^2 + F^2 + 2FF\cos\theta}$$

or $2F^2 + 2F^2\cos\theta = 0$

or $\cos\theta = -1 = \cos 180^\circ$ or $\theta = \mathbf{180^\circ}$

i.e., **two vectors are acting in opposite directions.**

(*b*) If $\vec{A} = \vec{F}$, $\vec{B} = \vec{F}$ and $R = 2F$, then

$(2F)^2 = F^2 + F^2 + 2FF\cos\theta$

or $\cos\theta = 1$ or $\theta = \mathbf{0^\circ}$

i.e., **two vectors are acting in the same direction.**

(*c*) $\vec{A} = \vec{F}$, $\vec{B} = \vec{F}$

and $R = F$, then

$F^2 = F^2 + F^2 + 2FF\cos\theta = 2F^2(1 - \cos\theta)$

or $1 - \cos\theta = \frac{1}{2}$ or $\cos\theta = 1 - \frac{1}{2} = \frac{1}{2} = \cos 60^\circ$

or $\theta = \mathbf{60^\circ}$

i.e., **two vectors are inclined at an angle of 60°.**

8. What is the difference between the following data ? (*i*) 3 (5 km h^{-1}, west)
(*ii*) 3 hour (5 km h^{-1}, west).

Ans. (*i*) It is the product of a pure number and a velocity vector, hence the unit of product is same as that of velocity vector *i.e.* the product is a velocity of magnitude 15 km h^{-1} towards west.

(*ii*) It is the product of a scalar (time) and velocity vector. The unit of this product will be hour × km h^{-1} = km. Thus the product is a displacement of magnitude **15 km towards west.**

9. What is the property of two vectors $\vec{A}$ and $\vec{B}$, if $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$.

Ans. We know that

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\text{and} \quad |\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

As per questions,

$$\sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Squaring both the sides, we get

$4AB\cos\theta = 0$ or $\cos\theta = 0$ or $\theta = 90°$

It means, the vectors $\vec{A}$ and $\vec{B}$ are **perpendicular to each other.**

10. Given that $\vec{A}+\vec{B}=\vec{R}$ and $A^2+B^2=R^2$, find the angle between $\vec{A}$ and $\vec{B}$.

Ans. $\cos\theta = \frac{R^2-A^2-B^2}{2AB} = \frac{R^2-R^2}{2AB} = 0$

or $\theta = \mathbf{90°}$

II. Components of Vectors and Relative Velocity

11. Find a vector $\vec{A}$ and its magnitude as well as direction with x-axis having initial point P (1, 2, – 1) and terminal point Q (3, 2, 2).

Ans. $\vec{A} = (3-1)\hat{i} + (2-2)\hat{j} + [2-(-1)]\hat{k}$

$= 2\hat{i} + 0\hat{j} + 3\hat{k}$

$= 2\hat{i} + 3\hat{k}$ It means $\vec{A}$ **lies in x-z plane.**

$|\vec{A}| = \sqrt{2^2+3^2} = \mathbf{\sqrt{13}}$

If θ is the angle which $\vec{A}$ makes with x-axis, then $\tan\theta = \frac{3}{2}$ or $\theta = \tan^{-1}\left(\frac{3}{2}\right)$

12. The velocity of a body is 100 kmh^{-1}, 30° west of south. Find the north and east components of the vector.

Ans. The angle which the given velocity makes with the north direction is 180° – 30° = 150°. The component velocity along north = $v\cos 150°$

$= 100\times(-\sqrt{3}/2)$

$= \mathbf{-86{\cdot}6\ km\ h^{-1}}$

The angle which the given velocity makes with the east direction is 90° + 30° = 120°. The component velocity along east = $v\cos 120°$

$= 100 \times (-1/2)$

$= \mathbf{-50\ km\ h^{-1}}$

13. What are the angles made by vector $\vec{A} = \hat{i} + \sqrt{3}\,\hat{j}$ with x-axis and y-axis ?

Ans. Comparing the given vector $\vec{A} = \hat{i} + \sqrt{3}\,\hat{j}$ with vector, $\vec{A} = A_x\hat{i} + A_y\hat{j}$,

we have $A_x = 1$ and $A_y = \sqrt{3}$

$\therefore\quad A = \sqrt{A_x^2 + A_y^2} = \sqrt{1^2 + (\sqrt{3})^2} = 2$

Let α, β are the angles which $\vec{A}$ make with x-axis and y-axis respectively. Then

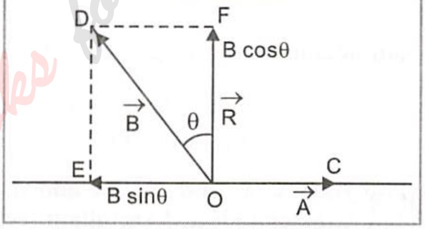
$\cos\alpha = \frac{A_x}{A} = \frac{1}{2} = \cos 60°$ or $\alpha = \mathbf{60°}$

$\cos\beta = \frac{A_y}{A} = \frac{\sqrt{3}}{2} = \cos 30°$ or $\beta = \mathbf{30°}$

14. The resultant of two vectors $\vec{A}$ and $\vec{B}$ perpendicular to the vector $\vec{A}$ and its magnitude is equal to half of the magnitude of the vector $\vec{B}$. Find out the angle between $\vec{A}$ and $\vec{B}$.

Ans. Refer to Fig. 4(Q).16, let $\vec{A} = (\overrightarrow{OC})$; $\vec{B} = (\overrightarrow{OD})$ and $\vec{R} = (\overrightarrow{OF})$; $\angle DOF = \theta$.

FIGURE 4(Q).16



Resolving $\vec{B}$ into two rectangular components, we have : $B\cos\theta$ along OF and $B\sin\theta$ along OE. Here resultant vector is along OF.

$\therefore\quad R = B\cos\theta$. As per question,

$R = B/2 = B\cos\theta$

or $\cos\theta = 1/2$ or $\theta = 60°$.

Hence, angle between $\vec{A}$ and $\vec{B}$

$= \angle COD = 90° + 60° = \mathbf{150°}$

15. A man moving in rain holds his umbrella inclined to the vertical even though the rain drops are falling vertically downwards. Why ?

Ans. A man walking in a rain can protect himself if he holds his umbrella in the direction of relative velocity of rain w.r.t. himself.

Consider a man is moving due east with velocity $\vec{v}_m$. Suppose the rain is falling vertically downwards with velocity $\vec{v}_r$. See Fig. 4(Q).17. The relative velocity of rain w.r.t. man is

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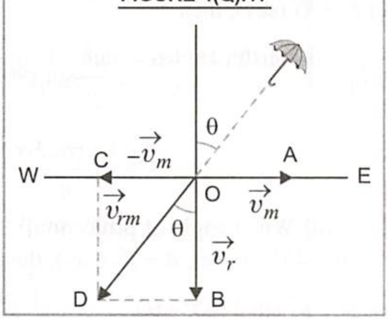
FIGURE 4(Q).17



$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m = \vec{v}_r + (-\vec{v}_m)$$
$$= \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OD}$$

Let $\overrightarrow{OD}$ make an angle θ with the vertical, then

$$\tan\theta = \frac{DB}{OB} = \frac{v_m}{v_r} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Thus a man can protect himself if he holds his umbrella at an angle $\theta = \left[\tan^{-1}\left(\frac{v_m}{v_r}\right)\right]$ with the vertical in the direction of motion.

III. Scalar product and vector product of vectors

16. If $\vec{R} = (\vec{A} + \vec{B})$, show that

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

where, θ is the smaller angle between $\vec{A}$ and $\vec{B}$.

Ans. Given $\vec{R} = (\vec{A} + \vec{B})$; Taking dot product of $\vec{R}$ with itself, we have,

$$\vec{R}.\vec{R} = (\vec{A} + \vec{B}).(\vec{A} + \vec{B})$$

or $$R^2 = \vec{A}.\vec{A} + 2\,\vec{A}.\vec{B} + \vec{B}.\vec{B}$$

or $$R^2 = A^2 + 2AB\cos\theta + B^2$$

17. If $\vec{A} = \vec{B} - \vec{C}$, then determine the angle between $\vec{A}$ and $\vec{B}$.

Ans. Given, $\vec{A} = \vec{B} - \vec{C}$ or $\vec{C} = \vec{B} - \vec{A}$

$$\vec{C}.\vec{C} = (\vec{B} - \vec{A}).(\vec{B} - \vec{A})$$
$$= \vec{B}.\vec{B} - \vec{A}.\vec{B} - \vec{A}.\vec{B} + \vec{A}.\vec{A}$$

or $$C^2 = B^2 - 2\,\vec{A}.\vec{B} + A^2$$
$$= A^2 + B^2 - 2AB\cos\theta$$

where θ is the angle between $\vec{A}$ and $\vec{B}$

or $$2AB\cos\theta = A^2 + B^2 - C^2$$

or $$\cos\theta = \frac{A^2 + B^2 - C^2}{2AB}$$

or $$\theta = \cos^{-1}\left(\frac{A^2 + B^2 - C^2}{2AB}\right)$$

18. The sum and difference of two vectors are perpendicular to each other. Prove that the vectors are equal in magnitude.

Ans. Let the two vectors be $\vec{A}$ and $\vec{B}$. As the vectors $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are perpendicular to each other so their dot product is zero, *i.e.*,

$$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$$

or $$\vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$$
$$(\because \vec{A}.\vec{B} = \vec{B}.\vec{A})$$

or $A^2 - B^2 = 0$ or $A^2 = B^2$ or $A = B$

IV. Horizontal projectile

19. What are the assumptions made in the study of a projectile motion ?

Ans. To study the motion of a projectile, we assume that (*i*) there is no frictional resistance of air. (*ii*) The effect due to rotation of earth and curvature of the earth is negligible. (*iii*) The acceleration due to gravity is constant in magnitude and direction at all points of motion of projectile.

20. A stone is thrown vertically upwards and then it returns to the thrower. Is it a projectile ? Explain.

Ans. It is not a projectile, because a projectile should have two component velocities in two mutually perpendicular directions, but in this case the stone has velocity only in one direction while going up or coming downwards.

21. A stone is thrown horizontally with a velocity $2\sqrt{gh}$ from the top of a tower of height h. It strikes the ground level through the foot of tower at a distance x from it. What is the value of x ?

Ans. Taking vertical downward motion of stone from starting point to striking point on ground level, we have

$$y_0 = 0,\ y = h,\ u_y = 0,\ a_y = g,\ t = t \text{ (say)}$$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

$$\therefore\ h = 0 + 0 + \frac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}$$

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Taking horizontal motion of stone from starting point to striking point on ground level, we have,

$$x_0 = 0,\ x = x,\ u_x = 2\sqrt{gh},\ a_x = 0,\ t = \sqrt{2h/g}$$

As $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

$$\therefore\ x = 0 + 2\sqrt{gh} \times \sqrt{\frac{2h}{g}} + \frac{1}{2} \times 0 \times t^2 = 2\sqrt{2}\,h$$

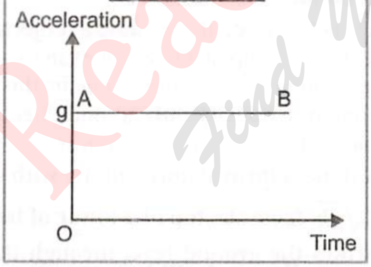
22. A projectile is given a horizontal projection from a height *h* with velocity *u*. What is the nature of trajectory of the projectile ? Draw the acceleration-time graph of this trajectory. What does its slope indicate ?

Ans. When a projectile is given a horizontal projection from a height, soon after the projection it will be having two mutually perpendicular velocities (*i*) Horizontal velocity *u* which remains constant throughout the motion and (*ii*) vertical downward velocity which increases with time, due to gravity pull.

Under the effect of these velocities, the projectile will describe a parabolic path.

The acceleration of projectile is constant with time acting vertically downwards (= *g*). Therefore, the graph between acceleration (*a*) – time (*t*) of this projectile motion is a straight line *AB* parallel to time axis. Fig. 4(Q).18. The slope of this straight line is zero with time axis. It shows that there is no change in acceleration of projectile with time

FIGURE 4(Q).18



V. Angular projectile

23. Prove that the horizontal range is same when angle of projection is (*i*) greater than 45° by certain value and (*ii*) less than 45° by the same value. (NCERT Solved Example)

Ans. Horizontal range for angular projection of a projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Case (i) When angle of projection, $\theta = 45° + \alpha$, let $R = R_1$ (say), then

$$R_1 = \frac{u^2 \sin 2\,(45° + \alpha)}{g} = \frac{u^2}{g} \sin\,(90° + 2\alpha)$$

$$= \frac{u^2 \cos 2\alpha}{g} \qquad ...(i)$$

Case (ii) When angle of projection, $\theta = 45° - \alpha$, let, $R = R_2$ (say), then

$$R_2 = \frac{u^2 \sin 2\,(45° - \alpha)}{g}$$

$$= \frac{u^2}{g} \sin\,(90° - 2\alpha) = \frac{u^2}{g} \cos 2\alpha \qquad ...(ii)$$

From equation (*i*) and (*ii*),

$$R_1 = R_2$$

24. Two bodies are thrown with the same initial velocity at angles α and (90° – α) with the horizontal. What will be the ratio of (*i*) maximum heights attained by them and (*ii*) of horizontal ranges ?

Ans. Horizontal range, $R = \frac{u^2}{g} \sin 2\theta$

and Max. height, $H = \frac{u^2 \sin^2 \theta}{2g}$

Case (i) when $\theta = \alpha$;

$$R_1 = \frac{u^2}{g} \sin 2\alpha \quad \text{and} \quad H_1 = \frac{u^2 \sin^2 \alpha}{2g}$$

Case (ii) When $\theta = (90° - \alpha)$;

$$R_2 = \frac{u^2 \sin 2\,(90° - \alpha)}{g} = \frac{u^2 \sin 2\alpha}{g}$$

and $$H_2 = \frac{u^2 \sin^2 (90° - \alpha)}{g} = \frac{u^2 \cos^2 \alpha}{g}$$

$$\therefore\ \frac{H_1}{H_2} = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \tan^2 \alpha \quad \text{and} \quad \frac{R_1}{R_2} = 1$$

25. Prove that there are two times for which the projectile travels the same vertical distance. Also prove that the sum of the two times is equal to the time of flight.

Ans. In angular projection of a projectile, let *h* be the height attained by projectile in timings t_1 and t_2 respetively. Then,

$$h = u \sin\theta\, t_1 - \frac{1}{2} g t_1^2 = u \sin\theta\, t_2 - \frac{1}{2} g t_2^2$$

$$\text{or } u\sin\theta\,(t_2-t_1)=\frac{1}{2}g\,(t_2^2-t_1^2)$$

$$=\frac{1}{2}g\,(t_2-t_1)\,(t_2+t_1)$$

$$\text{or }\ (t_2+t_1)=\frac{2u\sin\theta}{g}=\text{total time of flight.}$$

26. Show that a gun will shoot three times as high when elevated at an angle of 60° as when fired at an angle 30° but will have the same horizontal range.

Ans. Let u be the velocity of shot fired from a gun at an angle θ with the horizontal. Then

$$\text{Max. height, } H=\frac{u^2\sin^2\theta}{2g}\ ;$$

$$\text{Horizontal range, } R=\frac{u^2\sin 2\theta}{g}$$

When $\theta = 30°$,

$$H_1=\frac{u^2\sin^2 30°}{2g}=\frac{u^2\times 1/4}{2g}=\frac{u^2}{8g}$$

$$R_1=\frac{u^2\sin 2\times 30°}{g}=\frac{u^2\sin 60°}{g}=\frac{u^2\sqrt{3}}{g\,2}$$

When $\theta = 60°$,

$$H_2=\frac{u^2\sin^2 60°}{2g}=\frac{u^2(\sqrt{3}/2)^2}{2g}=\frac{3u^2}{8g}$$

$$R_2=\frac{u^2\sin 2\times 60°}{g}=\frac{u^2\sin 120°}{g}=\frac{u^2\sqrt{3}}{g\,2}$$

From above, we note that $R_1 = R_2$ and

$$\frac{H_2}{H_1}=\frac{3u^2/8g}{u^2/8g}=3 \text{ or } H_2=3H_1 \text{ (Proved)}$$

27. The greatest height to which a boy can throw a stone is *h*. What will be the greatest distance on horizontal surface upto which the boy can throw the stone with the same speed ? Neglect the air friction.

Ans. Maximum height attained, $H=\dfrac{u^2\sin^2\theta}{2g}$

H will be maximum, when $\sin\theta = 1$ or $\theta = 90°$

$$\therefore\ \ H_{max}=\frac{u^2}{2g}=h \text{ (given)}$$

Horizontal range will be maximum if angle of projection is 45° with the horizontal.

Max. Horizontal range,

$$R_{max}=\frac{u^2\sin 2\times 45°}{g}=\frac{u^2}{g}=2\left(\frac{u^2}{2g}\right)=2h$$

28. A projectile is projected from horizontal with velocity *u* making an angle 45° with the horizontal direction. Find the distance of the highest point of the projectile from its starting point.

Ans. Maximum height,

$$H=\frac{u^2\sin^2 45°}{2g}=\frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2=\frac{u^2}{4g}$$

$$\text{Horizontal range, } R=\frac{u^2\sin 2\times 45°}{g}=\frac{u^2}{g}$$

In Fig. 4(Q).19, $AC = H = \dfrac{u^2}{4g}$

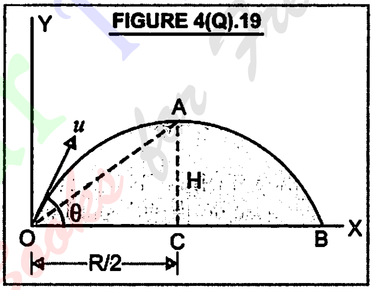


FIGURE 4(Q).19

$$OC=\frac{R}{2}=\frac{u^2}{2g}$$

$$OA=\sqrt{OC^2+AC^2}=\sqrt{\left(\frac{u^2}{2g}\right)^2+\left(\frac{u^2}{4g}\right)^2}$$

$$=\frac{u^2}{2g}\sqrt{1+\frac{1}{4}}=\frac{\sqrt{5}}{4}\frac{u^2}{g}$$

29. At which point of projectile motion (*i*) potential energy is maximum (*ii*) kinetic energy is maximum (*iii*) total mechanical energy is maximum.

Ans. In case of a projectile, the potential energy is maximum at the highest point, given by

$$\therefore\ \ (\text{P.E.})_H=mg\,H=mg\frac{u^2\sin^2\theta}{2g}$$

$$=\frac{1}{2}m\,u^2\sin^2\theta$$

Kinetic energy of projectile will be minimum (but not zero) at the highest point, because only vertical component velocity is zero there. Therefore

$$(\text{K.E.})_H = \frac{1}{2} m u_H^2 = \frac{1}{2} m u^2 \cos^2\theta$$

$$\therefore \quad (\text{P.E.})_H + (\text{K.E.})_H = \frac{1}{2} m u^2 \left(\sin^2\theta + \cos^2\theta\right) = \frac{1}{2} m u^2$$

which is the total mechanical energy of projectile at the point of projection. So in projectile motion, **total mechanical energy is conserved,** *i.e.*, same at all locations of its motion. **K.E. of projectile is maximum at place of projection.**

30. Find the angle of projection for a projectile motion whose range R is n times the maximum height H.

Ans. Given ; $R = n\,H$; so, $\dfrac{u^2 \sin 2\theta}{g} = n \dfrac{u^2 \sin^2\theta}{2g}$

or $\tan\theta = \dfrac{4}{n}$ or $\boldsymbol{\theta = \tan^{-1}(4/n)}$

31. A ball is projected with velocity u at an angle α with horizontal plane. What is its speed when it makes an angle β with the horizontal plane ?

Ans. Let v be the velocity of ball at an instant, when it makes an angle β with the horizontal. The horizontal component velocity of the ball $= v\cos\beta$.

Initial horizontal component velocity of ball $= u\cos\alpha$

In angular projection of a projectile, the horizontal component velocity remains unchanged, hence $v\cos\beta = u\cos\alpha$

or $\boldsymbol{v = u\cos\alpha/\cos\beta}$

32. A paricle is projected at an angle θ from the horizontal with kinetic energy T. What is the kinetic energy of the particle at the highest point ?

Ans. At highest point, velocity of projectile $= u\cos\theta$.

K.E. of projectile at highest point

$$= \frac{1}{2} \text{m} \,(u\cos\theta)^2 = \left(\frac{1}{2} m u^2\right) \cos^2\theta$$

$$= \boldsymbol{T\cos^2\theta}$$

33. A ball of mass m is thrown vertically up. Another ball of mass 2 m is thrown at an angle θ with the vertical. Both of them stay in air for the same period of time. What is the ratio of the height attained by the two balls.

Ans. For the ball thrown vertically upwards, the time taken by the ball to come back is,

$$T_1 = 2\,u_1/g.$$

For the ball projected at an angle θ with their vertical, the time of flight is,

$$T_2 = 2\,u_2\cos\theta/g$$

Since time for both the balls is same, so

$$\frac{2u_1}{g} = \frac{2u_2\cos\theta}{g} \quad \text{or} \quad u_1 = u_2\cos\theta.$$

Now, $h_1 = \dfrac{u_1^2}{2g}$ and $h_2 = \dfrac{u_2^2}{2g}\cos^2\theta$;

Hence, $\dfrac{h_1}{h_2} = \dfrac{u_1^2}{u_2^2\cos^2\theta} = \dfrac{u_2^2\cos^2\theta}{u_2^2\cos^2\theta} = \mathbf{1}$

VI. Circular motion

34. Which is greater ; the angular velocity of the hour hand of a watch or angular velocity of earth around its own axis ?

Ans. For hour hand of watch,
time period, $T_h = 12$ h ;
For earth, $T_e = 24$ h.

Angular velocity, $\omega = \dfrac{2\pi}{T}$

$$\therefore \quad \frac{\omega_h}{\omega_e} = \frac{T_e}{T_h} = \frac{24}{12} = 2 \quad \text{or} \quad \boldsymbol{\omega_h = 2\,\omega_e}$$

Thus **angular velocity of hour hand of a watch is greater than the angular velocity of earth around its own axis.**

35. What are the angular velocities of a second hand, minute hand and hour hand of a clock ?

Ans. *Second hand of a clock* completes one rotation in 60 seconds. So $T = 60$ s, $\theta = 2\,\pi$ rad.

$$\omega = \frac{\theta}{T} = \frac{2\pi}{60} = \boldsymbol{\frac{\pi}{30}\,\text{rad/s}}$$

For Minute hand of a clock,

$$T = 60 \text{ min} = 60 \times 60 \text{ s} = 3600 \text{ s}$$

$$\omega = \frac{2\pi}{3600} = \boldsymbol{\frac{\pi}{1800}\,\text{rad/s}}$$

For hour hand of a clock,

$$T = 12\,h = 12 \times 60 \times 60 \text{ s}$$

$$\omega = \frac{2\pi}{12\times 60\times 60} = \boldsymbol{\frac{\pi}{21600}\,\text{rad/s}}$$

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SHORT ANSWER QUESTIONS

Carrying 3 marks

1. What do you understand by (*i*) position vector and (*ii*) displacement vector. Distinguish them with illustration. **[Art. 4.3]**

2. Explain multiplication of a vector by (*i*) a real number (*ii*) by a scalar. **[Arts. 4.5 & 4.6]**

3. What do you understand by resultant vector ? Show that vector addition of two vectors is different from scalar addition of two scalars. **[Arts. 4.7 & 4.8]**

4. Explain the rules for addition of vectors geometrically with illustrations. **[Art. 4.9]**

5. State polygon law of vectors and show that it can be deduced from triangle law of vectors. **[Art. 4.9]**

6. Explain subtraction of two vectors with illustration. **[Art. 4.14]**

7. What is relative velocity ? Explain how a man can hold his umbrella while walking on ground in a rain. **[Arts. 4.15 and 16]**

8. What is a zero vector ? How can you obtain zero vectors. Give examples and properties of zero vectors. **[Art. 4.17]**

9. What do you understand by resolution of a vector ? Show that there is only one way in which a vector can be resolved into two component vectors along the directions of two given vectors. **[Arts. 4.18 & 19]**

10. Explain rectangular components of a vector with illustration. Show that walking of a man is accounted by resolution of vectors. **[Arts. 4.20 & 23]**

11. It is easier to pull than to push a lawn roller. **[Art. 4.23]**

12. Explain vectors addition when vectors are in terms of rectangular component vectors. **[Art. 4.24]**

13. What do you understand by motion in two dimensions ? When an object is moving with uniform velocity in two dimensions, explain displacement, velocity and find the equations of motion of the object. **[Art. 4.27]**

14. Find the relation for

(*i*) velocity and time

(*ii*) displacement and time,

when an object is moving with uniform acceleration in two dimensions. **[Art. 4.28]**

15. What is a projectile ? Give its examples. Show that the path of projectile is a parabolic path when projected horizontally from a certain height. **[Art. 4.29 & 4.30]**

16. Define trajectory of a projectile and hence derive equation of motion of the projectile when projected at an angle θ with horizontal direction. (J & K Board 2011) **[Art. 4.31]**

17. Show that there are two angles of projection for which the horizontal range is the same. **[Art. 4.32]**

18. Discuss the general relations for velocity and acceleration for motion of an object in a plane. **[Art. 4.36]**

19. What do you understand by angular displacement and angular velocity in a circular motion. **[Arts. 4.37 & 4.38]**

20. What is a uniform circular motion ? Explain the terms ; time period, frequency and angular velocity. Establish relation between them. **[Art. 4.41]**

21. Define angular velocity and derive a relation which connects it with linear velocity. Give units of angular velocity. (J & K Board 2011) **[Arts. 4.38 & 39]**

22. Establish a relation between linear velocity and angular velocity in a uniform circular motion and explain the direction of linear velocity. **[Art. 4.40]**

23. Explain angular acceleration. Establish its relation with linear acceleration. **[Art. 4.41]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Explain (*i*) Unit vector (*ii*) Equal vectors (*iii*) Negative vectors, (*iv*) Coinitial vectors (*v*) Collinear vectors and (*vi*) Coplanar vectors. **[Art. 4.3]**

2. State triangle law of vectors addition. Find analytically the magnitude and direction of resultant vector. **[Art. 4.10]**

3. State parallelogram law of vectors addition. Find analytically the magnitude and direction of resultant vector, when (*i*) two vectors are parallel to each other (*ii*) two vectors are perpendicular to each other. **[Art. 4.10]**

4. State and explain the various facts of vectors addition. **[Art. 4.13]**

5. Explain dot product of two vectors. Give examples and properties. **[Refer to Ans. 4.25]**
6. Explain cross product of two vectors. Give its examples and properties. **(J & K Board 2011)** **[Refer to Art. 4.26]**
7. Find (*a*) time of flight, (*b*) Max. height (*c*) Horizontal range of projectile projected with speed v making an angle θ with the horizontal direction from ground. **[Art. 4.31]**
8. Find the magnitude and direction of the velocity of an object at any instant during the oblique projection of projectile. **[Art. 4.33]**
9. Find (*i*) the path of projectile (*ii*) time of flight (*iii*) horizontal range (*iv*) maximum height, when projectile is projected with velocity v making an angle θ with the vertical direction. **[Art. 4.34]**
10. Find (*i*) the time of flight (*ii*) maximum height and (*iii*) horizontal range of a projectile given angular projection on an inclined plane where angle of inclination with horizontal is θ_0. **[Art. 4.35]**
11. What is centripetal acceleration ? Find its magnitude and direction in case of a uniform circular motion of an object. **[Art. 4.42]**

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. A vector is a physical quantity which has both a magnitude as well as direction. A vector can be represented by a letter with an arrow head on it. For example, a velocity of a body which is a vector quantity can be represented by $\vec{v}$. Its magnitude is denoted by v or $|\vec{v}|$, called modulus of velocity vector which is a scalar. The direction of $\vec{v}$ is denoted by $\hat{v}$, which is a unit vector of $\vec{v}$. Thus $\vec{v} = v\hat{v}$ or $\hat{v} = \vec{v}/v$.

Vectors can be added or subtracted by the laws of vectors. If two vectors $\vec{A}$ and $\vec{B}$ are inclined at an angle θ, then the magnitude of resultant vector $\vec{R}$ is

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Zero vector is a vector of zero magnitude and has an arbitrary direction. A zero vector is denoted by $\vec{0}$. When two equal and opposite vectors are added, the resultant is a zero or null vector. When three vectors lying in a plane are represented in magnitude and direction by the three sides of a triangle taken in one order, their resultant is zero vector or null vector.

Read the above paragraph carefully and answer the following very short and short answer questions :

1. Does it make a sense to call a physical quantity a vector, when its magnitude is zero ?
2. Under what conditions the direction of sum and difference of two vectors will be the same ?
3. We can order events in time and there is a sense of time, distinguishing past, present and future. Is therefore, time a vector ?
4. Vectors $\vec{A}, \vec{B}$ and $\vec{C}$ satisfy the equation, $\vec{A} + \vec{B} = \vec{C}$, and their magnitudes are related by the equation, $A + B = C$. How is the vector $\vec{A}$ oriented with respect to $\vec{B}$? Explain your reasoning.

CASE 2. In a uniform circular motion, a point object is moving on a circular path with a constant speed. The angular velocity of the object is constant and its angular acceleration is zero. Without centripetal force an object cannot move on a circular path. The centripetal force in different circular motions is provided by different means. The centripetal force does not increase the kinetic energy and angular momentum of the object in a circular motion.

A uniform circular motion is an accelerated motion. The acceleration of a particle in a uniform circular motion is called centripetal acceleration. If a particle is moving with a constant speed v, on a circle of radius r with angular velocity ω, then the centripetal acceleration of particle is $a_c = v\omega = r\,\omega^2 = v^2/r$.

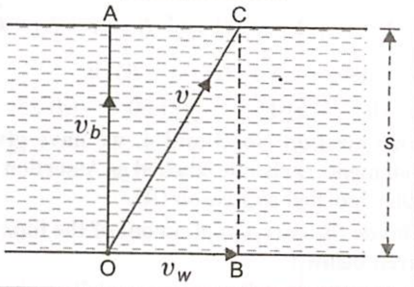
The direction of this centripetal acceleration is along the radius and acting towards the centre of the circular path.

Read the above paragraph carefully and answer the following very short and short answer questions :

5. Can a body move on a circular path without having acceleration ? Comment.
6. A body in a uniform circular motion possesses a variable velocity. Does it mean that the kinetic energy of the body is also variable ?
7. Can a particle accelerate if its speed is constant ? Can it accelerate if its velocity is constant ? Discuss.
8. A body of mass 1 kg revolves in a circle of diameter 40 cm, making 120 revolutions per minute. What is its frequency and centripetal acceleration ?

CASE 3. A boat in a river always moves in the direction of resultant velocity of boat and velocity of river flow. There is a river of width s and water is flowing in river with velocity v_w. A boatman can row a boat in still water with velocity v_b. If a boatman tends to cross the river in the shortest time, he should go perpendicular to the direction of river flow, Fig. 4(Q).20. In this case the resultant velocity of boat and river flow is $v = \sqrt{v_b^2 + v_w^2}$. Time of crossing the river, $t = \dfrac{s}{v_b}$.

FIGURE 4(Q).20



In this case, the boat will be reaching the opposite bank of river at point C,

where $AC = v_w t$

and $OC = vt$.

A river 1 km wide flows at the rate of 3 km h^{-1} from west to east. A boatman on the north bank capable of rowing his boat at the rate of 4 km h^{-1} intends to cross the river in the shortest time.

Read the above paragraph carefully and answer the following very short and short answer questions :

9. What is the resultant velocity of the boat ?

10. What is the time of crossing the river ?

11. What is the distance covered by the boatman from starting point to the reaching point on the opposite bank of river ?

12. A boat travels a distance in 4 hrs upstream and the same distance downstream in 2 hrs. Find the ratio of velocity of boat to that of water flow.

ANSWERS

1. Yes, zero or null vector has a definite physical significance.

2. The directions of sum and difference of two vectors will be the same, when two vectors are unequal in magnitude and acting in the same direction.

3. Time always flows on and on *i.e.*, from past to present and then to future. Therefore a direction can be assigned to time. Since, the direction of time is unique, it is not to be specified or stated. It is due to this reason that time cannot be a vector though it has a direction.

4. If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$C = (A^2 + B^2 + 2\,AB\cos\theta)^{1/2} = (A + B) \quad \text{[Given]}$$

$$\therefore \quad A^2 + B^2 + 2\,AB\cos\theta = (A+B)^2 = A^2 + 2\,AB + B^2$$

or $2\,AB\cos\theta = 2\,AB$ or $\cos\theta = 1$ or $\theta = 0°$

5. **No.** It is because while moving on a circular path, the velocity of the body (which is represented by tangent to the circular path at a point) changes with time. Hence the body is in accelerated state.

6. **No.** KE of body $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m(\vec{v}\cdot\vec{v})$, which is a scalar. In uniform circular motion, speed of body v, remains constant, hence KE of body remains constant in a uniform circular motion.

7. When a particle describes a uniform circular motion, its speed is constant but it has centripetal acceleration acting along the radius directed towards the centre of the circular path.

When the particle is moving with a constant velocity, there is no change in velocity with time. Hence its acceleration is zero.

8. Here, $2\,r = 40$ cm or $r = 20$ cm $= 0{\cdot}20$ m.

Frequency, $\nu = \dfrac{120}{60} = 2\,\text{rps}$;

$$\omega = 2\,\pi\nu = 2\,\pi \times 2 = \mathbf{4\,\pi\ rad/s}$$

Centripetal acceleration,

$$a_c = r\,\omega^2 = 0{\cdot}20 \times (4\,\pi)^2 = \mathbf{3{\cdot}2\,\pi^2\ ms^{-2}}$$

9. Here, $v_b = 4$ km h^{-1}, $v_w = 3$ km h^{-1},

width of river, $s = 1$ km

Resultant velocity of boat along OC,

$$v = \sqrt{v_b^2 + v_w^2} = \sqrt{4^2 + 3^2} = \mathbf{5\ km\ h^{-1}}$$

10. Time of crossing the river,

$$t = \frac{s}{v_b} = \frac{1\text{ km}}{4\text{ km h}^{-1}} = 0{\cdot}25\ h$$

$$= 0{\cdot}25 \times 60\text{ min} = \mathbf{15\ min}$$

11. Here actual distance covered by the boatman $= OC = v \times t = 5 \times 0{\cdot}25 =$ **1·25 km**

12. **Case (*i*),**

$$t_1 = \frac{s}{v_b - v_w} = 4 \quad \text{or} \quad s = 4\,(v_b - v_w) \qquad ...(i)$$

Case (*ii*),

$$t_2 = \frac{s}{v_b + v_w} = 2 \quad \text{or} \quad s = 2\,(v_b + v_w) \qquad ...(ii)$$

From (*i*) and (*ii*), we have

$$4\,(v_b - v_w) = 2\,(v_b + v_w)$$

or $$4\,v_b - 4\,v_w = 2\,v_b + 2\,v_w$$

or $$2\,v_b = 6\,v_w$$

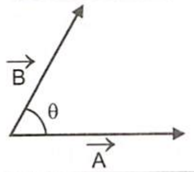
or $$v_b/v_w = 6/2 = 3/1$$

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. A dot product of two vectors $\vec{A}$ and $\vec{B}$ is equal to the product of the magnitudes of the two vectors and cosine of the smaller angle between them,

i.e., $$\vec{A}\cdot\vec{B} = AB\cos\theta$$

FIGURE 4(Q).21



Dot product of two vectors is a scalar.

Dot product of two vectors may be positive, zero or negative. Dot product of two equal vectors is equal to the square of the magnitude of either vector, *i.e.*,

$$\vec{A}\cdot\vec{A} = A^2$$

Dot product of two perpendicular vectors is zero,

i.e., $$\vec{A}\cdot\vec{B} = AB\cos 90^\circ = AB \times 0 = 0.$$

Vector component of $\vec{A}$ along $\vec{B}$ is

$$= (A\cos\theta)\,\hat{B} = \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B}$$

Vector component of $\vec{B}$ along $\vec{A}$ is

$$= (B\cos\theta)\,\hat{A} = \left(\frac{\vec{A}\cdot\vec{B}}{A}\right)\hat{A}.$$

Based on the above paragraph, answer questions no. 1 to 4 :

1. $\vec{A} = (\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{B} = (-\hat{i} + \hat{j} - 2\hat{k})$, then the angle (in rad) between $\vec{A}$ and $\vec{B}$ is

(*a*) π/2 (*b*) π/3

(*c*) π/4 (*d*) π/6

2. If $\vec{A} = (\hat{i} + 2\hat{j} - 4\hat{k})$ and $\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$, then the value of $(\vec{A} + \vec{B})\cdot(\vec{A} - \vec{B})$ is

(*a*) 10 (*b*) 15 (*c*) 20 (*d*) 25

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

3. **Assertion.** Dot product of two vectors is minimum.

Reason. For the angle θ between two vectors, the value of cos θ is less than the value of sin θ.

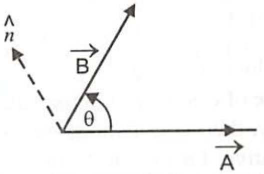
4. **Assertion.** Dot product of two vectors is commutative.

Reason. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

CASE 2. Cross product of two vectors is a vector whose magnitude is equal to the product of magnitudes of two vectors and sine of the smaller angle between them. Direction of this vector is perpendicular to the plane containing two vectors and is directed as given by Right Hand Screw rule, *i.e.*, $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$. It is directed upwards the plane as per Right hand screw rule.

FIGURE 4(Q).22



Cross product of two vectors is anticommutative, *i.e.*, $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

Magnitude of the vector product of two vectors is equal to (*i*) the area of a parallelogram whose two sides are represented by two vectors (*ii*) twice the area of a triangle whose two sides are represented by two vectors.

Volume V of the parallelopiped whose three sides are represented by the theree vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as its adjacent edges is given by scalar triple product of three vectors, *i.e.*,

$$V=\vec{A}\times\vec{B}\cdot\vec{C}=\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{C}\cdot(\vec{A}\times\vec{B})$$

Based on the above paragraph, answer questions no. 5 to 8 :

5. Two adjacement sides of a parallelogram are found by the vectors,

$$\vec{A}=(4\hat{i}+3\hat{j})\text{ and }\vec{B}=(-3\hat{i}+6\hat{j}),$$

then the area of parallelogram is :

(*a*) 6 units (*b*) 12·5 units

(*c*) 18·5 units (*d*) 33·0 units

6. Determine a unit vector which is perpendicular to both, $\vec{A}=(2\hat{i}+\hat{j}+\hat{k})$ and $\vec{B}=(\hat{i}-\hat{j}+2\hat{k})$.

(*a*) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}+\hat{k})$ (*b*) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

(*c*) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}-\hat{k})$ (*d*) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true, but R is not the true explanation of R.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

7. Assertion. The cross product of two vectors $\vec{A}$ and $\vec{B}$ does not change under reflection.

Reason. On reflection, $\vec{A}$ changes into $-\vec{A}$ and $\vec{B}$ changes into $-\vec{B}$.

8. Assertion. If $\vec{A},\vec{B}$ and $\vec{C}$ are coplanar, then $\vec{A}\cdot(\vec{B}\times\vec{C})=0$.

Reason $(\vec{B}\times\vec{C})$ is parallel to $\vec{A}$.

ANSWERS

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*a*) **5.** (*d*) **6.** (*d*) **7.** (*a*) **8.** (*c*)

HINTS/EXPLANATIONS For Difficult Questions

1. $$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}$$

$$=\frac{(\hat{i}+2\hat{j}-\hat{k})\cdot(-\hat{i}+\hat{j}-2\hat{k})}{\sqrt{1^2+2^2+(-1)^2}\sqrt{(-1)^2+(1)^2+(-2)^2}}$$

$$=\frac{-1+2+2}{6}=\frac{3}{6}=\frac{1}{2}=\cos\frac{\pi}{3}\text{ or }\theta=\frac{\pi}{3}\textbf{ rad.}$$

2. $$(\vec{A}+\vec{B})=(\hat{i}+2\hat{j}-4\hat{k})+(\hat{i}+2\hat{j}-\hat{k})$$
$$=2\hat{i}+4\hat{j}-5\hat{k}$$

$$(\vec{A}-\vec{B})=(\hat{i}+\hat{2}-4\hat{k})-(\hat{i}+2\hat{j}-\hat{k})=-3\hat{k}$$

$$\therefore\ (\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=(2\hat{i}+4\hat{j}-5\hat{k})\cdot(-3\hat{k})=\mathbf{15}$$

3. Dot product of two vectors $\vec{A}$ and $\vec{B}$ is $AB\cos\theta$ and value of $\cos\theta$ can be positive, zero or negative Similarly, the value of $\sin\theta$ can be zero, positive and negative. Therefore, dot product of two vectors is not minimum in all situation. Thus, **both Assertion and Reason are false.**

4. Both Assertion and Reason are true and Reason is the correct explanation of Assertion as

$$\vec{A}\cdot\vec{B}=AB\cos\theta\text{ and }\vec{B}\cdot\vec{A}=BA\cos\theta=AB\cos\theta$$

5. Area of parallelogram

$$|\vec{A}\times\vec{B}|=|(4\hat{i}+3\hat{j})\times(-3\hat{i}+6\hat{j})|$$

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\4&3&0\\-3&6&0\end{vmatrix}$$

$$=|\hat{i}(0-0)+\hat{j}(0-0)+\hat{k}(24+9)|$$

$$=\mathbf{33\ units}$$

6. Let $\vec{C}$ be perpendicular to both $\vec{A}$ and $\vec{B}$, then

$$\vec{C}=(\vec{A}\times\vec{B})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&1&1\\1&-1&2\end{vmatrix}$$

$$= \hat{i}\,[1\times2-1\times(-1)]+\hat{j}\,[1\times1-2\times2] + \hat{k}\,[2\times(-1)-1\times1]$$

$$= \hat{i}\,(3)+\hat{j}\,(-3)+\hat{k}\,(-3)$$

$$= 3\hat{i}-3\hat{j}-3\hat{k}$$

$$|\vec{C}| = \sqrt{(3)^2+(-3)^2+(-3)^2} = \sqrt{27} = 3\sqrt{3}$$

Unit vector along $\vec{C}$ is

$$\hat{C} = \frac{\vec{C}}{|\vec{C}|} = \frac{3\hat{i}-3\hat{j}-3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$$

7. $\vec{A}\times\vec{B}$ under reflection $= (-\vec{A})\times(-\vec{B}) = \vec{A}\times\vec{B}$

Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

8. $(\vec{B}\times\vec{C})$ is acting perpendicular to the plane containing $\vec{B}$ and $\vec{C}$ and is directed as given by Right Hand Rule. As, $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar, so $\vec{A}$ is perpendicular to $(\vec{B}\times\vec{C})$. Hence

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = |\vec{A}|\,|\vec{B}\times\vec{C}|\,\cos 90° = 0$$

Thus **Assertion is true but Reason is false.**

PROBLEMS FOR PRACTICE

TYPE I. DISPLACEMENT VECTOR, TRIANGLE LAW OF VECTORS ADDITION AND PARALLELOGRAM LAW OF VECTORS ADDITION

1. Two vectors, both equal in magnitude, have their resultant equal in magnitude of the either. Find the angle between the two vectors.
(CBSE Delhi 2002, 2014) **[Ans. 120°]**

2. A man walks 20 m towards east and then turns at an angle of 60° to the north of east and walks 20 m. Calculate the net displacement of the man. Also find the direction of net displacement.
[Ans. $20\sqrt{3}$ m; 30°]

3. Two persons are raising a load pulling at an angle of each other. If they exert forces of 30 N and 60 N respectively and their effective pull is at right angles to the direction of the pull of the first person, what is the angle between their pulling forces ? What is the effective pull ?
[Ans. 120°, $30\sqrt{3}$ N]

4. Two forces $2P$ and $4P$ newton act on a particle. If the first force is doubled and the second force is increased by 30 N, the direction of resultant is unaltered. Find the value of smaller force.
[Ans. 15 N]

5. Two equal forces act at a point. The square of their resultant is 3 times their product. Find the angle between them. **[Ans. 60°]**

6. At what angle two forces $(P+Q)$ and $(P-Q)$ act so that resultant is (*i*) $\sqrt{3P^2+Q^2}$ (*ii*) $\sqrt{2(P^2+Q^2)}$ (*iii*) $\sqrt{(P^2+Q^2)}$
[Ans. (*i*) 60° (*ii*) 90° (*iii*) $\cos^{-1}\left[\dfrac{P^2+Q^2}{2(Q^2-P^2)}\right]$]

7. A particle has the following displacements in succession (*i*) 12 m towards East (*ii*) 5 m towards North and (*iii*) 6 m vertically. Find the magnitude of the resultant displacement.
[Ans. 14·32 m]

8. Two forces acting on a particle in opposite directions, have the resultant of 10 N. If they act at right angles to each other, the resultant is 50 N. Find the two forces. **[Ans. 30 N, 40 N]**

9. Two forces equal to F and 3 F newton act on a particle. If the first force be tripled and the second force be increased by 30 newton, the direction of the resultant is unaltered. Find the value of F. **[Ans. 5 N]**

10. A motor cyclist is riding North in still air at 20 km h^{-1}. If the wind starts blowing westward with a velocity of 45·8 km h^{-1}, find the apparent velocity with which the motor cyclist moves, time it takes to cover 100 km and its direction of motion. **[Ans. 49·97 km h^{-1}, 2 hour, 66° 25′ West of North]**

11. Given that $\vec{A}+\vec{B}+\vec{C}=0$. Two out of the three vectors are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of the either out of the other two. Find the angle between these vectors. **[Ans. 90°, 135°, 135°]**

12. Two buses start from a bus stand with velocities 10 km h^{-1} and 30 km h^{-1} along two tracks inclined at an angle 60°. Find the distance between them after 12 minutes. **[Ans. $2\sqrt{7}$ km]**

13. A bob weighing 0·06 kg hangs vertically at the end of a string 0·30 m long. Find the force which when applied horizontally, will pull the bob 0·10 m aside from its initial position.
[Ans. 0·021 kgf]

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14. A ship is streaming due West at 12 ms^{-1}. A boy runs across the deck at 5 ms^{-1} in a direction at right angles to the direction of motion of the ship towards South. Calculate the velocity of the boy relative to sea.

[Ans. 13 ms^{-1} ; 22° 37′ South of West]

15. A river 800 m wide flows at the rate of 6 km h^{-1}. A swimmer who can swim at 10 km h^{-1} in still water, wishes to cross the river straight.

(*a*) Along what direction must he strike ?

(*b*) What should be his resultant velocity ?

(*c*) How much time he would take ?

[Ans. (*a*) 53° 8′ up stream (*b*) 8 km h^{-1} (*c*) 6 minutes]

16. A river is flowing steadily with a speed 5 km h^{-1}. A boat man can row with a speed of 10 km h^{-1} in still water. He rows his boat in river at right angles to the bank of river. If width of river is 800 m, (*i*) how much time the boat man will take to cross the river ? (*ii*) How far away from a point just opposite to the bank of river he will be reaching there ? (*iii*) What will be his effective speed ?

[Ans. (*i*) 4·8 min (*ii*) 400 m (*iii*) $5\sqrt{5}\,\text{km h}^{-1}$]

17. A swimmer croses a flowing river of width d to and fro in time t_1. The time taken to cover the same distance up and down the stream is t_2. If t_3 is the time the swimmer would take to swim a distance 2 d in still water, then prove that

$$t_1^2 = t_2\, t_3 .$$

18. A car travelling at 36 km h^{-1} due North turns West in 5 seconds and maintains the same speed. What is the acceleration of the car ?

[Ans. $2\sqrt{2}\,\text{ms}^{-2}$ South West]

TYPE II. RELATIVE VELOCITY IN A PLANE

19. A boat is moving with a velocity $(3\hat{i}+4\hat{j})\,\text{ms}^{-1}$ with respect to ground. The water in the river is moving with a velocity $(-3\hat{i}-4\hat{j})\,\text{ms}^{-1}$ with respect to ground. What is the magnitude of relative velocity of boat with respect to river ? **[Ans. 10 ms^{-1}]**

20. A train is moving with a velocity 40 km h^{-1} due East and a car is moving with a velocity 60 km h^{-1} due North. What is the velocity of train as appears to a passenger in the car ?

[Ans. 72·11 km h^{-1}, 56° 19′ South of East]

21. A man is walking due East at a rate of 5 km h^{-1} and the rain appears to be falling vertically with a speed of 12 km h^{-1}. Find the actual speed and direction of rain with vertical.

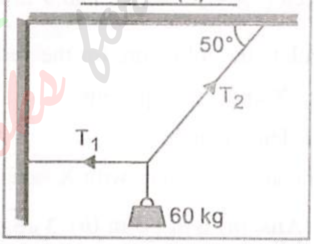
$$\left[\textbf{Ans. } 13\,\text{km h}^{-1}\,;\ \tan^{-1}\left(\frac{5}{12}\right)\right]$$

22. A person standing on a road has to hold his umbrella at 45° with the vertical to keep the rain away. He throws the umbrella and starts running at 30 ms^{-1}. He find that the rain drops are hitting his head vertically. Find the speed of the rain drops with respect to (*a*) the road (*b*) the moving person. **[Ans. (*a*) $30\sqrt{2}$ ms^{-1} (*b*) 30 ms^{-1}]**

TYPE III. RECTANGULAR COMPONENTS OF A VECTOR

23. Find the values of T_1 and T_2 for the system shown in Fig. 4(Q).23 ; $g = 10\ \text{m/s}^2$.

FIGURE 4(Q).23



[Ans. 503 N ; 783 N]

24. A plane is inclined at an angle of 30° with horizontal. Find the component of a force $\vec{F} = -10\hat{k}$ N perpendicular to the plane. Given that z-direction is vertically upwards.

[Ans. $5\sqrt{3}$ N]

25. Find the resultant force of the following forces which act upon a particle.

(*a*) 30 N due East (*b*) 20 N due North

(*c*) 50 N due West (*d*) 40 N due South.

[Ans. $20\sqrt{2}$ N, 45° South of West]

26. A point moves in x–y plane according to the law $x = 4 \sin 6t$ and $y = 4(1 - \cos 6t)$. Find distance traversed by the particle in 5 seconds, when x and y are in metres. **[Ans. 120 m]**

27. Given three coplanar vectors

$$\vec{A} = 2\hat{i} - \hat{j};\quad \vec{B} = -3\hat{i} + 2\hat{j} \quad\text{and}\quad \vec{C} = 3\hat{j}.$$

Find the magnitude of the sum of three vectors.

[Ans. $\sqrt{17}$]

28. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$. Find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$. **[Ans. $15\hat{i} + 20\hat{j}$]**

29. A bird moves with velocity 10 ms^{-1} in a direction making an angle of 60° with the eastern direction and 45° with vertical upward direction. Represent the velocity vector of bird in rectangular form. **[Ans. $5\hat{i} + 5\hat{j} + 5\sqrt{2}\hat{k}$]**

30. Find a unit vector parallel to the resultant of vectors $\vec{A} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{B} = \hat{i} - 5\hat{j} + \hat{k}$.

$$\left[\textbf{Ans. } \frac{4\hat{i} - 2\hat{j} - \hat{k}}{\sqrt{21}}\right]$$

31. The X and Y components of vector $\vec{A}$ are 4 and 6 m respectively. The X and Y components of vector $\vec{A} + \vec{B}$ are 10 and 9 m respectively. Calculate the following for the vector $\vec{B}$,

(*a*) its X and Y components

(*b*) its length, and

(*c*) the angle it makes with X -axis.

[Ans. (*a*) 6 m ; 3 m (*b*) $3\sqrt{5}$ m (*c*) 26·6°]

32. If $\vec{R} = \vec{A} - \vec{B}$, and θ is the smaller angle between $\vec{A}$ and $\vec{B}$, show that

$$R^2 = A^2 + B^2 - 2\,AB\cos\theta.$$

TYPE IV. DOT PRODUCT OR SCALAR PRODUCT OF TWO VECTORS

33. Find the angle between the vectors $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} - 2\hat{k}$.

[Ans. 60°]

34. For what value of *a* are the vectors $\vec{A} = 2a\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = a\hat{i} + a\hat{j} - 4\hat{k}$ perpendicular to each other ?

[Ans. 2, – 1]

35. Show that $|\vec{A} + \vec{B}|^2 - |\vec{A} - \vec{B}|^2 = 4\,\vec{A}.\vec{B}$

36. Show that $\vec{A} = \hat{i} - 5\hat{j}$ and $\vec{B} = 2\hat{i} - 10\hat{j}$ are parallel to each other.

37. Find the component of a vector $\vec{A} = 3\hat{i} + 2\hat{j}$ along the direction of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$.

$$\left[\textbf{Ans. } \frac{5}{2}(\hat{i} + \hat{j})\,;\, \frac{1}{2}(\hat{i} - \hat{j})\right]$$

38. If $\vec{a}$ and $\vec{b}$ are two non collinear unit vectors and if $|\vec{a} + \vec{b}| = \sqrt{3}$, then find the value of $(\vec{a} - \vec{b})\cdot(2\vec{a} + \vec{b})$. **[Ans. 1/2]**

39. A particle moves from position vector $\vec{r_1} = (3\hat{i} + 2\hat{j} - 6\hat{k})$ to position vector, $\vec{r_2} = (14\hat{i} + 13\hat{j} + 9\hat{k})$ in metre under the action of a constant force of $\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})$ N. Calculate work done by the force. **[Ans. 100 J]**

TYPE V. CROSS PRODUCT OR VECTOR PRODUCT OF TWO VECTORS

40. Determine the sine of the angle between the vectors $(3\hat{i} + 2\hat{j} + 4\hat{k})$ and $(2\hat{i} - 2\hat{j} - 4\hat{k})$.

[Ans. 0·8476]

41. Calculate the area of a parallelogram whose adjacent sides are given by the vectors :

$$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}\,;\ \text{and}\ \vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}.$$

[Ans. $7\sqrt{3}$ units of area]

42. Find the area of the triangle formed by the tips of the vectors,

$\vec{a} = (2\hat{i} - \hat{j} + 3\hat{k})$,

$\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$,

$\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$. **[Ans. 3 sq. units]**

43. A vector of magnitude 100 N is inclined at an angle of 30° to another vector of magnitude 50 N. Calculate the magnitude of dot product and cross product of two vectors.

[Ans. 2500 $\sqrt{3}$ N^2, 2500 N^2]

44. Find the unit vector perpendicular to each of the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$.

$$\left[\textbf{Ans. } \frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})\right]$$

45. Given that $\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{0}$. If $\vec{A}, \vec{B}$ and $\vec{C}$ are not null vectors, find the value of $\vec{C} \times \vec{A}$. **[Ans. $\vec{0}$]**

46. For any two vectors $\vec{A}$ and $\vec{B}$, prove that

$$|\vec{A} \times \vec{B}|^2 = A^2B^2 - (\vec{A}.\vec{B})^2$$

47. Find the area of a triangle formed by the tips of the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$; $\vec{b} = (\hat{i} - \hat{j} - 3\hat{k})$ and $\vec{c} = (4\hat{i} - 3\hat{j} + \hat{k})$. **[Ans. 10·87 sq. units]**

TYPE VI. HORIZONTAL PROJECTILE

48. A bomb is flying horizontally at a height of 2000 m with a speed of 200 ms^{-1}. When a bomb is released from it. Find (*i*) the time taken by bomb to reach the ground (*ii*) the velocity with which the bomb hits the target and the distance of the target from where the bomb is dropped. Take $g = 10$ m/s^2.

[Ans. (*i*) 20 s, (*ii*) $200\sqrt{2}$ ms^{-1}, 45° (*iii*) $2000\sqrt{5}$ m]

49. Two tall buildings are situated 200 m apart. With what speed must a ball be thrown horizontally from the window 550 m above the ground in one building so that it will enter a window 50 m above the ground in the other building ? Take $g = 10$ ms^{-2}. **[Ans. 20 ms^{-1}]**

50. A ball is thrown horizontally from the top of a tower with a speed of 50 ms^{-1}. Find the velocity and position at the end of 3 second. $g = 9{\cdot}8$ ms^{-2}.

[Ans. 58 ms^{-1} ; 30° 27′ with horizontal ; 44·1 m below and 150 m horizontally away from the starting point]

51. A body is thrown horizontally from the top of a tower and strikes the ground in 3 seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which it was projected. **[Ans. 44·1 m ; 29·4 ms^{-1}]**

52. An air craft is flying at a height of 3500 m above the ground. If the angle subtended at a ground observation point by the air craft positions 10s apart is 30° while passing over his head, what is the speed of the air craft ? **[Ans. 187·5 ms^{-1}]**

53. A body is projected downwards at an angle of 30° to the horizontal with a velocity of 9·8 m/s from the top of a tower 29·4 m high. How long will it take before striking the ground ? **[Ans. 2·0 s]**

TYPE VII. ANGULAR PROJECTILE

54. A person can throw a ball to a maximum horizontal distance of 90 m. Calculate the maximum vertical height to which he can through the ball. Given $g = 10$ ms^{-2} **[Ans. 45 m]**

55. A ball is kicked at an angle of 30° with the horizontal. If the horizontal component of its velocity is 20 ms^{-1}, find the maximum height and horizontal range. Take $g = 10$ m/s^2. **[Ans. 6·67 m ; 46·19 m]**

56. A particle is projected from the ground with an initial velocity of 20 m/s at an angle of 30° with the horizontal. What is the magnitude of change in velocity in 0·5 second ? ($g = 10$ m/s^2) **[Ans. 5 m/s]**

57. A bullet fired at an angle of 60° with the vertical hits the ground at a distance of 2·5 km., Calculate the distance at which the bullet will hit the ground when fired at an angle of 45° with vertical, assuming the speed to be the same. **[Ans. 2·89 km]**

58. A projectile has the same range R when the maximum height attained by it is either H_1 or H_2. Find the relation between R, H_1 and H_2. **[Ans. $R = 4\sqrt{H_1 H_2}$]**

59. A ball is thrown at an angle θ and another ball is thrown at an angle (90° – θ) with the horizontal direction from the same point, each with velocity 40 ms^{-1}. The second ball reaches 40 m higher than the first ball. Find their individual heights. Take $g = 10$ m/s^2. **[Ans. 20 m ; 60 m]**

60. A person sees an object on a tree at a height of 40 m and at a distance of 60 m. With what velocity he should throw an arrow at an angle of 45° so that it may hit the object ? Take $g = 10$ ms^{-2}. **[Ans. $30\sqrt{2}$ ms^{-1}]**

61. A football is kicked with speed 20 ms^{-1} at a projection angle of 45° from the ground. A receiver on the goal line 20 m away in the direction of the kick runs the same instant to meet the ball. What must be his speed, if he is to catch the ball before it hits the ground ? Take $g = 10\ \text{ms}^{-2}$. **[Ans. 7·07 ms^{-1}]**

62. For the angular projection θ, the velocity of projectile is *u*. Let *H* be the maximum height reached by the projectile and *R* be its horizontal range ; show that $\frac{R^2}{8H} + 2H$ is equal to its maximum range.

63. A projectile is projected in the upward direction making an angle of 60° with the horizontal direction with a velocity of 150 ms^{-1}. After what time will its inclination with the horizontal be 45° ? Take $g = 10\ \text{ms}^{-2}$. **[Ans. 5·49 s]**

64. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$, where $\hat{i}$ is a unit vector along horizontal and $\hat{j}$ is unit vector vertically upwards. Find the cartesian equation of its path. ($g = 10\ \text{m/s}^2$) **[Ans. $y = 2x - 5x^2$]**

65. A ball is thrown upwards with a velocity of 80 m/s at an angle of 30° to the horizontal. Find its velocity after one second.
[Ans. 75·57 m/s, 23°33′ with horizontal]

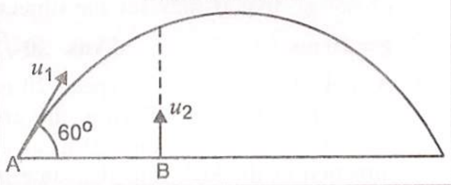
66. Show that the maximum range of a projectile in any direction is described in the same time in which it would fall freely under gravity through this distance starting from rest.

67. A shot is fired at an angle of 30° with horizontal from the top of a tower 182·88 metres high. The velocity of projection is 60·96 m/s. Find where from the foot of tower it strikes the ground.
[Ans. 526·3 m]

68. A particle is projected with a velocity *u* so that its horizontal range is twice the greatest height attained. Find the horizontal range of it.
[Ans. 4 u^2/5 g]

69. A body is projected with the velocity u_1 from the point *A* as shown in Fig. 4(Q).24. At the same time another body is projected vertically upwards with the velocity u_2 from the point *B*. What should be the value of u_1/u_2 for both the bodies to collide ?

FIGURE 4(Q).24



[Ans. $2/\sqrt{3}$]

TYPE VIII. CIRCULAR MOTION

70. Calculate the angular velocity of the minute's hand of a clock. **[Ans. π/1800 rad. s^{-1}]**

71. The radius of the earth's orbit around the sun is 1 A.U. Calculate the angular velocity and linear velocity of the earth. Also calculate the angle through which the earth revolve around the sun in 4 days. (Given 1 AU = $1·5 \times 10^{11}$ m)
[Ans. $1·99 \times 10^{-7}$ rad/s; $2·99 \times 10^4\ \text{ms}^{-1}$, 0·0688 rad]

72. A body of mass 5 kg is revolving in a circle of diameter 0·30 m making 2000 revolutions in 2 minutes. Calculate the linear velocity and centripetal acceleration.
[Ans. 15·71 ms^{-1}, 1646·3 ms^{-2}]

73. The angular velocity of a particle moving in a circle of radius 50 cm is increased in 5 minutes from 100 revolutions per minute to 400 revolutions per minute. Find (*a*) angular acceleration (*b*) linear acceleration.
[Ans. π/30 rad. s^{-2}, 5 π/3 cm s^{-2}]

74. Calculate the linear acceleration of a particle moving in a circle of radius 0·5 m at the instant when its angular velocity is 2 rad s^{-1} and its angular acceleration is 6 rad s^{-2}.
[Ans. 3·6 ms^{-2} ; 33° 42′ with tangential acceleration]

TYPE IX. TYPICAL PROBLEMS

75. Find the sum of N-coplanar vectors of equal lengths, each making angle 2 π/N with that of preceding. **[Ans. zero]**

76. A vector $\vec{A}$ when added to the vector $\vec{B} = 3·0\,\hat{i} + 4\,\hat{j}$ yields a resultant vector that is in the positive *y*-direction and has a magnitude equal to that of $\vec{B}$. Find the magnitude of $\vec{A}$.
[Ans. $\sqrt{10}$]

77. A vector $\vec{A}$ of magnitude *A* is turned through an angle θ. Calculate the change in the magnitude of vector. **[Ans. 2 *A* sin θ/2]**

78. Two vectors $\vec{P}$ and $\vec{Q}$ act at a point and have a resultant $\vec{R_1}$. If $\vec{Q}$ is replaced by the vector $\overrightarrow{\frac{(R_1^2 - P^2)}{Q}}$, acting in the opposite direction to that of $\vec{Q}$, show that the resultant is still of magnitude $\vec{R_1}$.

79. Two cars starts together from the same point and move along two straight lines inclined at an angle θ, one moving with velocity u and the other from rest with a uniform acceleration a. Show that the least relative velocity between them is $u \sin \theta$ and it occurs after a time $t = (u \cos \theta)/a$.

80. A grass hopper can jump a maximum horizontal distance of 20·4 cm. If it spends negligible time on the ground, what is its speed of travel along the road. $g = 10 \text{ m/s}^2$. **[Ans. 1 ms^{-1}]**

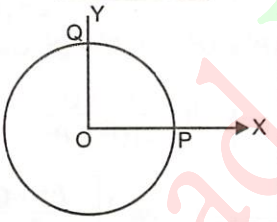
81. A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80·0 m, he applies brakes and reduces his speed at a constant rate of 0·5 ms^{-1} per second. Find the magnitude of the net acceleration of the cyclist. **[Ans. 0·86 ms^{-2}]**

82. A target is fixed on the top of a pole 13 m high. A person standing at a distance 50 m from the pole is capable of projecting a stone with a velocity $10\sqrt{g}$ m/s. If he wants to strike the target in shortest possible time, at what angle should he project the stone. **[Ans. 30°58′]**

83. A particle moves in a circle of radius 4·0 cm clockwise at constant speed of 2 cm s^{-1}. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X-axis and Y-axis respectively, find the acceleration of the particle at the instant half way between PQ. Fig. 4(Q).25.

[Ans. $-(\hat{x}+\hat{y})/\sqrt{2}$ cm/s^2]

FIGURE 4(Q).25



84. Two bodies were thrown simultaneously from the same point ; one straight up and the other, at an angle $\theta = 60°$ to the horizontal. The initial velocity of each body is equal to $u = 30$ m/s. Neglecting the air resistance, find the distance between the bodies after 2 seconds.

[Ans. 31 m]

85. The height y and the distance x along the horizontal plane of projectile on a certain planet (with no surrounding atmosphere) are given by ; $y = (8\,t - 5\,t^2)$ metre and $x = 6\,t$ metre, where t is in seconds. Find the initial velocity with which the projectile is projected.

[Ans. 10 ms^{-1}, 53°8′]

86. A ball is thrown from the ground into air. At a height of 9·0 m, the velocity is observed to be $\vec{v} = 7{\cdot}0\,\hat{i} + 6{\cdot}0\,\hat{j}$. Find the maximum height to which the ball will rise. $g = 10 \text{ ms}^{-2}$.

[Ans. 10·8 m]

87. The speed of a projectile when it is at its greatest height is $\sqrt{2/5}$ times its speed at half the maximum height. What is its angle of projection ? **[Ans. 60°]**

88. A body of mass m is projected with a velocity v at an angle of 45° with the horizontal. Find the change in its momentum at the end of its flight.

[Ans. $\sqrt{2}\,mv$]

89. A projectile is fired at an angle of 45° with the horizontal. Find the elevation angle of the projectile at its highest point as seen from the point of projection. **[Ans. tan^{-1} (1/2)]**

HINTS

For Difficult Problems

1. Here, $A = B = R = x$ (say)

As, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\therefore\quad x^2 = x^2 + x^2 + 2\,xx \cos\theta$

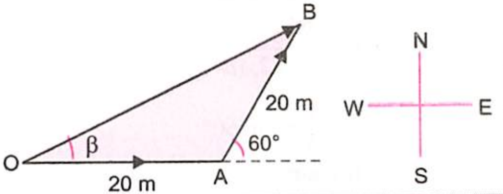
or $\quad 1 = 1 + 1 + 2 \cos\theta$

or $\quad \cos\theta = -\dfrac{1}{2} = \cos 120°$

or $\quad \theta = 120°$s

2. Refer to Fig. 4(Q).26, the net displacement of the man is

FIGURE 4(Q).26



$$= \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

where $OA = 20\text{ m} = AB$ and angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is 60°.

Magnitude of $\overrightarrow{OB}$ is

$$|\overrightarrow{OB}| = \sqrt{(OA)^2 + (AB)^2 + 2(OA)(AB)\cos 60°}$$

$$= \sqrt{20^2 + 20^2 + 2\times 20\times 20\times(1/2)}$$

$$= \mathbf{20\sqrt{3}\ m}$$

Direction of $\overrightarrow{OB}$

$$\tan\beta = \frac{20\sin 60°}{20 + 20\cos 60°}$$

$$= \frac{20\times\sqrt{3}/2}{20 + 20\times 1/2} = \frac{1}{\sqrt{3}} = \tan 30°.$$

$$\beta = \mathbf{30°}$$

3. Here, $A = 30$ N, $B = 60$ N, $\beta = 90°$

Let θ be the angle between two forces. Then

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

or $$\tan 90° = \frac{60\sin\theta}{30 + 60\cos\theta}$$

or $$\infty = \frac{60\sin\theta}{30 + 60\cos\theta}$$

or $30 + 60\cos\theta = 0$

or $\cos\theta = -30/60 = -1/2 = \cos 120°$

or $\theta = \mathbf{120°}$

Effective pulling force is

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{30^2 + 60^2 + 2\times 30\times 60\times\cos 120°}$$

$$= \sqrt{900 + 3600 + 2\times 30\times 60\times(-1/2)}$$

$$= \mathbf{30\sqrt{3}\ N}$$

4. Refer to Example 3.

5. Here, $A = B = x$ and $R^2 = 3AB = 3x\ x = 3x^2$

$$\cos\theta = \frac{R^2 - A^2 - B^2}{2AB} = \frac{3x^2 - x^2 - x^2}{2\times x\times x}$$

$$= \frac{1}{2} = \cos 60°$$

$$\theta = \mathbf{60°}$$

6. (*i*) $A = (P + Q)$; $B = (P - Q)$

and $R = \sqrt{3P^2 + Q^2}$

Using ; $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

we have, $\sqrt{3P^2 + Q^2}$

$$= \sqrt{(P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta}$$

or $3P^2 + Q^2 = P^2 + 2PQ + Q^2 + P^2 - 2PQ + Q^2 + 2(P^2 - Q^2)\cos\theta.$

or $P^2 - Q^2 = 2(P^2 - Q^2)\cos\theta$

or $\cos\theta = 1/2 = \cos 60°$ or $\theta = \mathbf{60°}$

(*ii*) $\sqrt{2(P^2 + Q^2)}$

$$= \sqrt{(P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta}$$

or $2P^2 + 2Q^2$

$= P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ + 2(P^2 - Q^2)\cos\theta.$

or $0 = 2(P^2 - Q^2)\cos\theta$ or $\cos\theta = 0$ or $\theta = \mathbf{90°}$

(*iii*) $\sqrt{P^2 + Q^2}$

$$= \sqrt{(P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta}$$

or $P^2 + Q^2 = P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ + 2(P^2 - Q^2)\cos\theta$

or $-(P^2 + Q^2) = 2(P^2 - Q^2)\cos\theta$

or $$\cos\theta = \frac{P^2 + Q^2}{2(Q^2 - P^2)}$$

or $$\theta = \cos^{-1}\left[\frac{P^2 + Q^2}{2(Q^2 - P^2)}\right]$$

7. $x = 12$ m ; $y = 5$ m ; $z = 6$ m.

Then, $r = (x^2 + y^2 + z^2)^{1/2}$

$= \mathbf{(12^2 + 5^2 + 6^2)^{1/2} = 14{\cdot}32\ m}$

8. As per question ; $A - B = 10$ ...(*i*)

and $\sqrt{A^2 + B^2} = 50$ ...(*ii*)

From (*i*), $A = B + 10$. Putting this value in (*ii*) ; $(B + 10)^2 + B^2 = 50^2$. On solving we get, $B = 30$ N or 40 N. Then $A = \mathbf{40\ N}$ or **30 N**

9. Let β be the angle which the resultant force make with the force F and θ be the angle between two forces.

In first case : $$\tan\beta = \frac{3F\sin\theta}{F + 3F\cos\theta}$$

In second case : $$\tan\beta = \frac{(3F + 30)\sin\theta}{3F + (3F + 30)\cos\theta}$$

Hence $\frac{3F\sin\theta}{F+3F\cos\theta}=\frac{(3F+30)\sin\theta}{3F+(3F+30)\cos\theta}$

or $\frac{3}{1+3\cos\theta}=\frac{3F+30}{3F+3F\cos\theta+30\cos\theta}$

or $9F+9F\cos\theta+90\cos\theta$
$=3F+30+9F\cos\theta+90\cos\theta$

or $9F-3F=30$ or $F=5$ N

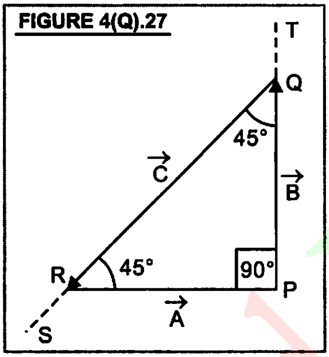
10. Here, $A = 20$ km h^{-1} ; $B = 45{\cdot}8$ km h^{-1} ;
$\theta = 90°$; $R = ?$; $\beta = ?$

Time, t = distance/resultant velocity.

11. Let $A = B$ and $C = \sqrt{2}\,A$

Since $A^2 + B^2 = C^2$, therefore angle between $\vec{A}$ and $\vec{B}$ is **90°**. Since $A = B$, so
$\angle PQR = \angle QRP = 45°$.

FIGURE 4(Q).27



Therefore, angle between $\vec{B}$ and $\vec{C}$
$= 180° - 45° = $ **135°**

Angle between $\vec{C}$ and $\vec{A}$ $= 180° - 45° = $ **135°**

12. Displacement of 1st bus in time 12 minute,

$$\overrightarrow{OP} = 10\times\frac{12}{60} = 2\,\text{km}$$

Displacement of 2nd bus in time 12 minute,

$$\overrightarrow{OQ} = 30\times\frac{12}{60} = 6\,\text{km}.$$

Refer to Fig. 4(Q).28, Separation between the two buses after 12 minutes = $\overrightarrow{PQ}$; where

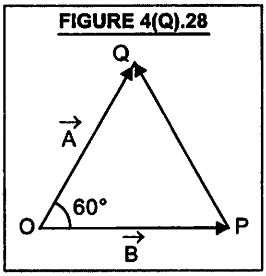
$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

or $PQ = \sqrt{A^2 + B^2 - 2AB\cos 60°}$

$= \sqrt{6^2 + 2^2 - 2\times 6\times 2\times\cos 60°}$

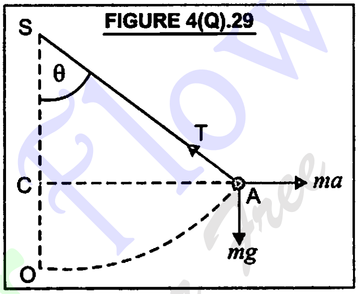
$= \mathbf{2\sqrt{7}\,km\,h^{-1}}$

FIGURE 4(Q).28



13. In the Fig 4(Q).29 ; $F = ?$; $CA = 0{\cdot}10$ m ;
$SO = SA = 0{\cdot}30$ m.

FIGURE 4(Q).29



$\therefore\ SC = (SA^2 - CA^2)^{1/2} = [(0{\cdot}3)^2 - (0{\cdot}1)^2]^{1/2}$

$= 2\sqrt{2}/10$ m

As, $\frac{mg}{SC} = \frac{F}{CA} = \frac{T}{AS}$

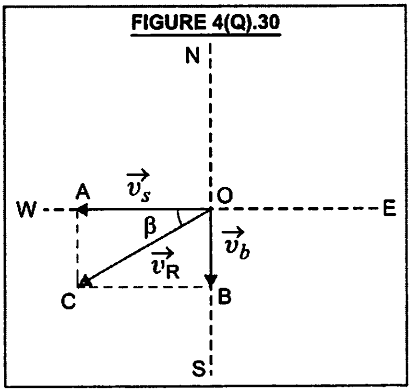
$$F = mg\frac{CA}{SC} = 0{\cdot}06\times 9{\cdot}8\times\frac{0{\cdot}1}{2\sqrt{2}}\times 10\,\text{N}$$

$= 0{\cdot}06\times 1/2\sqrt{2}$ kgf $=$ **0·021 kg f**

14. Boy has two velocities ; velocity of ship and his own velocity acting at right angles to each other.

Refer to Fig. 4(Q).30,

FIGURE 4(Q).30



$\vec{v}_s = \overrightarrow{OA}$, where $v_s = 12$ ms^{-1}

$\vec{v}_b = \overrightarrow{OB}$, when $v_b = 5$ ms^{-1}

$\angle AOB = 90°$

Resultant velocity,

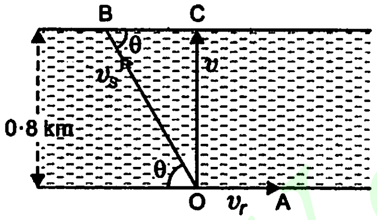
$$|\vec{v}_R| = \sqrt{v_s^2+v_b^2} = \sqrt{12^2+5^2} = 13\ \text{ms}^{-1}$$

$$\tan\beta = \frac{v_b}{v_s} = \frac{5}{12} = 0{\cdot}4167 = \tan 22° 37'$$

$$\beta = 22° 37' \text{ south of west}$$

15. Width of river = 800 m = 0·8 km ; velocity of water flowing in river, $v_r = 6\ \text{km h}^{-1}$, velocity of swimmer, $v_s = 10$ km/h. The swimmer will cross the river straight if the resultant velocity of river flow and swimmer acts perpendicular to the direction of river flow. It will be so if swimmer moves making an angle θ with upstream of river flow. Fig. 4(Q).31(*a*).

FIGURE 4(Q).31



Then, $\cos\theta = 6/10 = 0{\cdot}6$ or $\theta = 53° 8'$

Resultant velocity of swimmer,

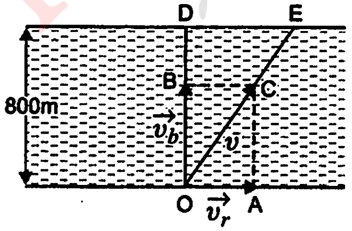
$$v = \sqrt{v_s^2 - v_r^2} = (10^2 - 6^2)^{1/2} = 8\ \text{km h}^{-1}$$

Time taken to cross the river,

$$t = \frac{S}{v} = \frac{0{\cdot}8}{8} = 0{\cdot}1\ \text{h} = 0{\cdot}1 \times 60\ \text{min} = 6\ \text{min}$$

16. Refer to Fig. 4(Q).32,

FIGURE 4(Q).32



Velocity of river flow, $\vec{v}_r = \overrightarrow{OA}$,

where $v_r = 5\ \text{km h}^{-1}$

Velocity of boat man, $\vec{v}_b = \overrightarrow{OB}$,

where $v_b = 10\ \text{km h}^{-1}$

(*i*) Time taken by boat man to cross the river

$$t = \frac{(OD)}{v_b} = \frac{0{\cdot}8\,\text{km}}{10\,\text{km}\,\text{h}^{-1}} = 0{\cdot}08\ h$$

$$= 0{\cdot}08 \times 60\ \text{min} = 4{\cdot}8\ \text{min}$$

(*ii*) Distance $DE = v_r \times t = 5\ (\text{km h}^{-1}) \times 0{\cdot}08\ h$
$= 0{\cdot}40\ \text{km} = 400\ \text{m}$

(*iii*) Resultant velocity of boat man

$$v = \sqrt{v_r^2 + v_b^2} = \sqrt{5^2+10^2} = 5\sqrt{5}\,\text{km}\,\text{h}^{-1}$$

The boat will row along *OC* and reaches the opposite bank at point *E*.

17. Let *u* and *v* be the velocity of a swimmer in still water and river water respectively. Then

$$t_1 = 2\left(\frac{d}{\sqrt{u^2 - v^2}}\right)$$

$$t_2 = \frac{d}{u+v} + \frac{d}{u-v} = \frac{2ud}{u^2-v^2}$$

and $t_3 = \dfrac{2d}{u}$

$$\therefore\ t_2 t_3 = \frac{2ud}{u^2-v^2} \times \frac{2d}{u} = \frac{4d^2}{u^2-v^2} = t_1^2$$

18. Refer Fig. 4(Q).33. Initial velocity, $\vec{v}_A = 36$ km h^{-1} due North = 10 ms^{-1} due North. Final velocity, $\vec{v}_B = 36\ \text{km h}^{-1} = 10\ \text{ms}^{-1}$ due West. Therefore,

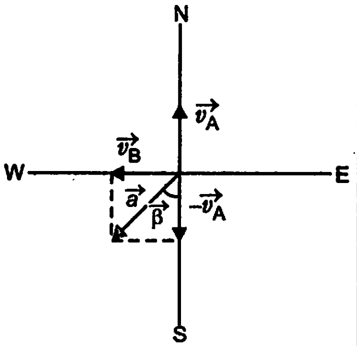
$-\vec{v}_A = -10\ \text{ms}^{-1}$ due North = 10 ms^{-1} due South.

$$\therefore\ \text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\vec{a} = \frac{\vec{v}_B - \vec{v}_A}{t} = \frac{\vec{v}_B + (-\vec{v}_A)}{t}$$

$$a = \frac{\sqrt{10^2+10^2}}{5} = 2\sqrt{2}\,\text{ms}^{-2}$$

FIGURE 4(Q).33



$$\tan\beta = \frac{v_B}{v_A} = \frac{10}{10} = 1 = \tan 45^\circ$$

$\beta = 45^\circ$ *i.e.* **South-West direction**

19. Velocity of boat, $\vec{v}_B = 3\hat{i} + 4\hat{j}$

velocity of water, $\vec{v}_W = -3\hat{i} - 4\hat{j}$

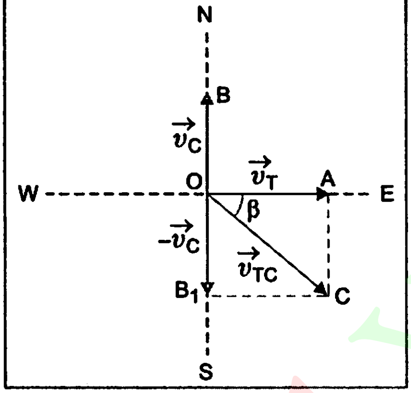
Relative velocity of boat w.r.t. water

$$\vec{v}_{BW} = \vec{v}_B - \vec{v}_W = (3\hat{i}+4\hat{j}) - (-3\hat{i}-4\hat{j})$$
$$= 6\hat{i} + 8\hat{j}$$

Magnitude of $\vec{v}_{BW} = \sqrt{6^2+8^2} = \mathbf{10\ ms^{-1}}$

20. Refer to Fig. 4(Q).34 ;

FIGURE 4(Q).34



Velocity of train, $\vec{v}_T = \overrightarrow{OA}$;

where $v_T = 40$ km h^{-1}

Velocity of car, $\vec{v}_C = \overrightarrow{OB}$;

where $v_C = 60$ km h^{-1}

Velocity of train as appears to a passenger in the car is the relative velocity of train w.r.t. car is, $\vec{v}_{TC} = \vec{v}_T - \vec{v}_C = \vec{v}_T + (-\vec{v}_C)$

$$= (\overrightarrow{OA}) + (\overrightarrow{OB_1}) = \overrightarrow{OC}$$

where $|\vec{v}_{TC}| = \sqrt{v_T^2 + v_C^2} = \sqrt{40^2+60^2}$

$= \mathbf{72{\cdot}11\ km\ h^{-1}}$

If $\vec{v}_{TC}$ make an angle β with East direction,

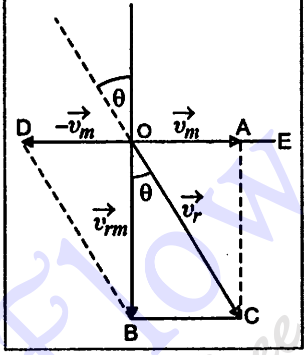
then $\tan\beta = \frac{v_C}{v_T} = \frac{60}{40} = \frac{3}{2} = 1{\cdot}5 = \tan 56^\circ\,19'$

$\beta = $ **56° 19′ South of East**

21. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along East and vertically downward directions respectively. In Fig. 4(Q).35, velocity of man,

$\vec{v}_m = 5\hat{i}$ (represented by $\overrightarrow{OA}$)

FIGURE 4(Q).35



Velocity of rain w.r.t. man,

$\vec{v}_{rm} = 12\hat{j}$ (represented by $\overrightarrow{OB}$)

If $\vec{v}_r$ is the actual velocity of rain (represented by $\overrightarrow{OC}$) then, $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

or $\vec{v}_r = \vec{v}_{rm} + \vec{v}_m = 12\hat{j} + 5\hat{i}$

or $v_r = \sqrt{12^2+5^2} = \mathbf{13\ km\ h^{-1}}$

If θ is the angle which the rain is making with the vertical then

$$\tan\theta = \frac{v_m}{v_{rm}} = \frac{5}{12} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{5}{12}\right)$$

22. When person is at rest with respect to ground, the rain is coming to him at an angle 45° with the vertical, *i.e.*, along *OB*. When a person after throwing his umbrella runs on ground at 30 ms^{-1}, the relative velocity of rain w.r.t. person, v_{rp} is along *OC* as shown in Fig. 4(Q).36. Here $\angle BOC = 45^\circ$

velocity of rain w.r.t. ground, $\vec{v}_r = \overrightarrow{OB}$

velocity of person w.r.t. ground, $\vec{v}_p = \overrightarrow{OA}$

where $v_p = (OA) = 30$ ms^{-1} $= CB$

velocity of rain w.r.t. person, $\vec{v}_{rp} = \overrightarrow{OC}$

In ΔOCB, $\angle COB = 45^\circ = \angle OBC$;

so $OC = CB$.

$\therefore\ v_{rp} = OC = \mathbf{30\ ms^{-1}}$

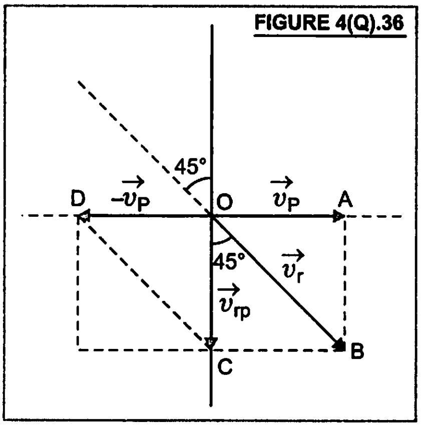


FIGURE 4(Q).36

In ΔOCB,

$$OB = \sqrt{OC^2 + CB^2} = \sqrt{OC^2 + OC^2}$$

$$= OC\sqrt{2} = 30\sqrt{2}\,\text{ms}^{-1}$$

$$\therefore \qquad v_r = \mathbf{30\sqrt{2}\,ms^{-1}}$$

23. $T_2 \sin 50° = mg = 60 \times 10 = 600$ N

or $T_2 = 600/\sin 50° = 600/0{\cdot}766 = \mathbf{783\ N}$

$T_1 = T_2 \cos 50° = 783 \times 0{\cdot}643 = \mathbf{503\ N}$

24. Refer to Fig. 4(Q).37, it is given that the Z-direction is vertically upwards. Therefore, the force $\vec{F} = -10\hat{k}$ N must be acting vertically downwards, *i.e.*, along AB. The component of $\vec{F}$ perpendicular to the plane, *i.e.*, along AC, is

$$= 10\cos 30° = 10 \times \sqrt{3}/2 = \mathbf{5\sqrt{3}\ N}$$

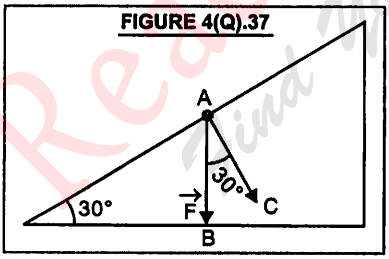


FIGURE 4(Q).37

25. Effective force along West, $A = 50 - 30 = 20$ N

Effective force along South, $B = 40 - 20 = 20$ N

$$\therefore\ R = \sqrt{A^2 + B^2} = \sqrt{20^2 + 20^2} = \mathbf{20\sqrt{2}\ N}\ ;$$

$$\tan\beta = \frac{B}{A} = \frac{20}{20} = 1 \quad \text{or} \quad \beta = \mathbf{45°}$$

26. $v_x = \dfrac{dx}{dt} = 4 \times 6 \cos 6t = 24 \cos 6t$

$v_y = \dfrac{dy}{dt} = 4 \times 6 \sin 6t = 24 \sin 6t$

$$\therefore\ \text{Velocity, } v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(24\cos 6t)^2 + (24\sin 6t)^2} = 24\,\text{ms}^{-1}.$$

It means, velocity of point is constant. The distance travelled by particle in 5 sec

$= 24 \times 5 = \mathbf{120\ m}$

27. Sum of vectors, $\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$$= (2\hat{i} - \hat{j}) + (-3\hat{i} + 2\hat{j}) + 3\hat{j} = -\hat{i} + 4\hat{j}$$

$$|\vec{R}| = \sqrt{(-1)^2 + (4)^2} = \mathbf{\sqrt{17}}$$

28. The magnitude of $\vec{A}$ is,

$$A = \sqrt{3^2 + 4^2} = 5\ ;$$

A unit vector parallel to $\vec{A}$ will be

$$= \hat{A} = \vec{A}/A = (3\hat{i} + 4\hat{j})/5$$

Now magnitude of $\vec{B}$ is,

$$B = \sqrt{7^2 + (24)^2} = 25\,.$$

Therefore a vector parallel to $\vec{A}$ and having magnitude of $\vec{B}$ will be

$$= 25\hat{A} = 25(3\hat{i} + 4\hat{j})/5$$

$$= \mathbf{(15\hat{i} + 20\hat{j})}$$

29. Let the eastern line be taken as X-axis, the northern line be taken as Y-axis and vertical upward direction as Z-axis. Let the velocity vector $\vec{v}$ of the moving bird makes angles α, β and γ with X-, Y- and Z-axes respectively. Then

$$\alpha = 60°,\ \gamma = 45°,$$

As $\qquad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \quad \cos^2 60° + \cos^2\beta + \cos^2 45° = 1$

$$\text{or} \quad \left(\frac{1}{2}\right)^2 + \cos^2\beta + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\text{or} \quad \cos^2\beta = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4} \quad \text{or} \quad \cos\beta = \frac{1}{2}$$

$$\therefore \quad \vec{v} = v\cos\alpha\,\hat{i} + v\cos\beta\,\hat{j} + v\cos\gamma\,\hat{k}$$

$$= 10 \times \frac{1}{2}\hat{i} + 10 \times \frac{1}{2}\hat{j} + 10 \times \frac{1}{\sqrt{2}}\hat{k}$$

$$= \mathbf{5\hat{i} + 5\hat{j} + 5\sqrt{2}\hat{k}}$$

30. The resultant of $\vec{A}$ and $\vec{B}$ is

$$\vec{R} = \vec{A}+\vec{B} = (3\hat{i}+3\hat{j}-2\hat{k})+(\hat{i}-5\hat{j}+\hat{k})$$
$$=(4\hat{i}-2\hat{j}-\hat{k})$$
$$R=\sqrt{4^2+(-2)^2+(-1)^2}=\sqrt{21}$$
$$\hat{R}=\frac{\vec{R}}{R}=\frac{4\hat{i}-2\hat{j}-\hat{k}}{\sqrt{21}}$$

31. (*a*) Let $\vec{A}=A_x\,\hat{i}+A_y\,\hat{j}=4\hat{i}+6\hat{j}$

and $\vec{B}=B_x\,\hat{i}+B_y\,\hat{j}$

$$\therefore\ \vec{A}+\vec{B}=(4\hat{i}+6\hat{j})+(B_x\,\hat{i}+B_y\,\hat{j})$$
$$=(4+B_x)\,\hat{i}+(6+B_y)\,\hat{j}$$

According to given problem

$4+B_x=10$ or $B_x=10-4=\mathbf{6\ m}$

and $6+B_y=9$ or $B_y=9-6=\mathbf{3\ m}$

(*b*) $B=\sqrt{B_x^2+B_y^2}=\sqrt{6^2+3^2}=\mathbf{3\sqrt{5}\ m}$

(*c*) $\tan\theta=\dfrac{B_y}{B_x}=\dfrac{3}{6}=\dfrac{1}{2}=0{\cdot}5=\tan 26{\cdot}6°$

or $\theta=\mathbf{26{\cdot}6°}$

32. Given, $\vec{R}=\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$;

As θ is the angle between $\vec{A}$ and $\vec{B}$, therefore, angle between $\vec{A}$ and $-\vec{B}$ is $(180°-\theta)$. Using parallelogram law of vectors addition

or $R^2=A^2+B^2+2\,AB\cos(180°-\theta)$

$=\mathbf{A^2+B^2-2\,AB\cos\theta}$

33. $\vec{A}=\hat{i}+2\hat{j}-\hat{k}$

$$\therefore\ |\vec{A}|=\sqrt{(1)^2+(2)^2+(-1)^2}=\sqrt{6}$$
$$\vec{B}=-\hat{i}+\hat{j}-2\hat{k}$$
$$\therefore\ |\vec{B}|=\sqrt{(-1)^2+(1)^2+(-2)^2}=\sqrt{6}$$
$$\vec{A}.\vec{B}=(\hat{i}+2\hat{j}-\hat{k}).(-\hat{i}+\hat{j}-2\hat{k})$$
$$=-1+2+2=3$$
$$\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}|\,|\vec{B}|}=\frac{3}{\sqrt{6}\sqrt{6}}=\frac{1}{2}=\cos 60°$$

or $\theta=\mathbf{60°}$

34. If $\vec{A}$ is perpendicular to $\vec{B}$, then $\vec{A}.\vec{B}=0$

$$(2a\,\hat{i}-2\,\hat{j}+\hat{k}).(a\,\hat{i}+a\,\hat{j}-4\,\hat{k})=0$$

or $2\,a^2-2\,a-4=0$ or $2\,a^2-4\,a+2\,a-4=0$

or $2\,a\,(a-2)+2\,(a-2)=0$

or $(2\,a+2)\,(a-2)=0$

If $a-2=0$ then $a=\mathbf{2}$.

If $2\,a+2=0$ then $a=\mathbf{-1}$

35. $|\vec{A}+\vec{B}|^2=(\vec{A}+\vec{B}).(\vec{A}+\vec{B})$

and $|\vec{A}-\vec{B}|^2=(\vec{A}-\vec{B}).(\vec{A}-\vec{B})$

36. $\cos\theta=\dfrac{\vec{A}.\vec{B}}{AB}$

$$=\frac{(\hat{i}-5\hat{j}).(2\hat{i}-10\hat{j})}{(\sqrt{1^2+(-5)^2})\,(\sqrt{2^2+(-10)^2})}$$
$$=\frac{2+50}{\sqrt{26}\sqrt{104}}=\frac{52}{52}=1 \text{ or } \theta=\mathbf{0°}$$

37. Unit vector along $(\hat{i}+\hat{j})$ is

$$\hat{a}=\frac{\hat{i}+\hat{j}}{|\hat{i}+\hat{j}|}=\frac{\hat{i}+\hat{j}}{\sqrt{1^2+1^2}}=\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

Magnitude of component vector $\vec{A}$ along $(\hat{i}+\hat{j})$ is

$$\vec{A}.\hat{a}=(3\hat{i}+2\hat{j}).\frac{(\hat{i}+\hat{j})}{\sqrt{2}}=\frac{1}{\sqrt{2}}(3+2)=\frac{5}{\sqrt{2}}$$

Therefore, component vector of $\vec{A}$ along $(\hat{i}+\hat{j})$ is $(\vec{A}.\hat{a})\,\hat{a}=\dfrac{5}{\sqrt{2}}\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)=\mathbf{\dfrac{5}{2}(\hat{i}+\hat{j})}$

Unit vector along $(\hat{i}-\hat{j})$ is ;

$$\hat{b}=\frac{\hat{i}-\hat{j}}{|\hat{i}-\hat{j}|}=\frac{\hat{i}-\hat{j}}{\sqrt{2}}$$

Component of $\vec{A}$ along $(\hat{i}-\hat{j})$ is

$$(\vec{A}.\hat{b})\,\hat{b}=\left\{(3\hat{i}+2\hat{j}).\frac{(\hat{i}-\hat{j})}{\sqrt{2}}\right\}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$
$$=\frac{1}{\sqrt{2}}(3-2)\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)=\mathbf{\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})}$$

38. Given $a = b = 1$ and $|\vec{a} + \vec{b}| = \sqrt{3}$

$$\therefore \; a^2 + b^2 + 2ab\cos\theta = (\sqrt{3})^2 = 3$$

or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = 1/2$

Now $(\vec{a} - \vec{b})\cdot(2\vec{a} + \vec{b}) = 2a^2 - b^2 - ab\cos\theta$

$$= 2 - 1 - \frac{1}{2} = \mathbf{\frac{1}{2}}$$

39. Here, $\vec{r_1} = (3\hat{i} + 2\hat{j} - 6\hat{k})$

$$\vec{r_2} = (14\hat{i} + 13\hat{j} + 9\hat{k})$$

$$\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})\,\text{N} \;;\; W = ?$$

Displacement $\vec{r}_{12} = \vec{r_2} - \vec{r_1}$

$$= (14\hat{i} + 13\hat{j} + 9\hat{k}) - (3\hat{i} + 2\hat{j} - 6\hat{k})$$

$$= 11\hat{i} + 11\hat{j} + 15\hat{k}$$

$$W = \vec{F}\cdot\vec{r}_{12} = (4\hat{i} + \hat{j} + 3\hat{k})\cdot(11\hat{i} + 11\hat{j} + 15\hat{k})$$

$$= 44\,\hat{i}\cdot\hat{i} + 11\,\hat{j}\cdot\hat{j} + 45\,\hat{k}\cdot\hat{k}$$

$$= \mathbf{100\ J}$$

40. Here, $\vec{A} = (3\hat{i} + 2\hat{j} + 4\hat{k})$;

$$\vec{B} = (2\hat{i} - 2\hat{j} - 4\hat{k})$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 4 \\ 2 & -2 & -4 \end{vmatrix}$$

$$= \hat{i}\,(-8 + 8) + \hat{j}\,(8 + 12) + \hat{k}\,(-6 - 4)$$

$$= \hat{i}\,(0) + 20\,\hat{j} - 10\,\hat{k}$$

$$= 20\,\hat{j} - 10\,\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{20^2 + (-10)^2} = \sqrt{500} = 10\sqrt{5}$$

$$|\vec{A}| = \sqrt{3^2 + 2^2 + 4^2} = \sqrt{29}$$

$$|\vec{B}| = \sqrt{2^2 + (-2)^2 + (-4)^2} = \sqrt{24}$$

$$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}|\,|\vec{B}|} = \frac{10\sqrt{5}}{\sqrt{29}\,\sqrt{24}}$$

$$= \frac{10\times 2{\cdot}236}{5{\cdot}385\times 4{\cdot}899} = \mathbf{0{\cdot}8476}$$

41. Area of the parallelogram $= |\vec{A}\times\vec{B}|$

Here, $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix}$

$$= \hat{i}\,(2 - 9) + \hat{j}\,(6 + 1) + \hat{k}\,(3 + 4)$$

$$= -7\,\hat{i} + 7\,\hat{j} + 7\,\hat{k}$$

$\therefore$ Area of parallelogram

$$|\vec{A}\times\vec{B}| = \sqrt{(-7)^2 + (7)^2 + (7)^2}$$

$$= \mathbf{7\sqrt{3}\ \text{units of area}}$$

42. Let ABC be the triangle formed by the tips of the given vectors. Then

$$\overrightarrow{BA} = \vec{a} - \vec{b} = (2\hat{i} - \hat{j} + 3\hat{k}) - (4\hat{i} + 3\hat{j} - \hat{k})$$

$$= -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = (3\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} + 3\hat{j} - \hat{k})$$

$$= -\hat{i} - 4\hat{j} + 3\hat{k}$$

$$\overrightarrow{BA}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -4 & 4 \\ -1 & -4 & 3 \end{vmatrix}$$

$$= \hat{i}(-12 + 16) + \hat{j}(-4 + 6) + \hat{k}\,(8 - 4)$$

$$= 4\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|\overrightarrow{BA}\times\overrightarrow{BC}| = \sqrt{(4)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{16 + 4 + 16} = 6$$

Area of triangle ABC $= \dfrac{1}{2}|\overrightarrow{BA}\times\overrightarrow{BC}| = \dfrac{6}{2}$

$$= \mathbf{3\ \text{sq. units}}$$

43. Here, $A = 100$ N, $B = 50$ N, $\theta = 30°$

$$\vec{A}\cdot\vec{B} = AB\cos\theta = 100\times 50\times\cos 30°$$

$$= 100\times 50\times\sqrt{3}/2 = \mathbf{2500\ \sqrt{3}\ N^2}$$

$$|\vec{A}\times\vec{B}| = AB\sin\theta = 100\times 50\sin 30°$$

$$= 100\times 50\times 1/2 = \mathbf{2500\ N^2}$$

44. Here, $\vec{A}\times\vec{B} = (3\hat{i} + \hat{j} + 2\hat{k})\times(2\hat{i} - 2\hat{j} + 4\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$$= (4 + 4)\,\hat{i} + (4 - 12)\,\hat{j} + (-6 - 2)\,\hat{k}$$

$$= 8\hat{i} - 8\hat{j} - 8\hat{k}.$$

$$\therefore \quad |\vec{A}\times\vec{B}| = \sqrt{8^2+(-8)^2+(-8)^2}$$

$$= 8\sqrt{3}$$

Since a vector perpendicular to each of the vectors $\vec{A}$ and $\vec{B}$ will be given by cross product of these two vectors, therefore the given unit vector,

$$\hat{n} = \frac{(\vec{A}\times\vec{B})}{|\vec{A}\times\vec{B}|} = \frac{8\hat{i}-8\hat{j}-8\hat{k}}{8\sqrt{3}} = \frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$$

45. Given, $\vec{A}\times\vec{B}=\vec{0}$, it means $\vec{A}$ is parallel or antiparallel to $\vec{B}$. As, $\vec{B}\times\vec{C}=\vec{0}$, it means $\vec{B}$ is parallel or antiparallel to $\vec{C}$. Hence, $\vec{A}, \vec{B}$ and $\vec{C}$ are all parallel or antiparallel to each other. Therefore, $\vec{A}$ and $\vec{C}$ are parallel or antiparallel. Hence,

$$\vec{C}\times\vec{A} = CA\sin 0^\circ\,\hat{n} = \vec{\mathbf{0}}$$

46. Taking L.H.S. $= |\vec{A}\times\vec{B}|^2 = (AB\sin\theta)^2$

$= A^2B^2\sin^2\theta = A^2B^2(1-\cos^2\theta)$

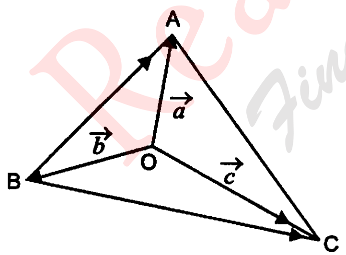
$= A^2B^2 - A^2B^2\cos^2\theta = A^2B^2-(AB\cos\theta)^2$

$= \mathbf{A^2B^2-(\vec{A}.\vec{B})^2}$ **proved.**

47. Let ABC be the triangle formed by the tips of the given vectors. Then

Refer to Fig. 4(Q).38.

FIGURE 4(Q).38



$$\overrightarrow{BA}+\vec{b}=\vec{a},$$

or $\overrightarrow{BA} = \vec{a}-\vec{b} = (3\hat{i}+\hat{j}+2\hat{k})-(\hat{i}-\hat{j}-3\hat{k})$

$$= 2\hat{i}+2\hat{j}+5\hat{k}$$

$$\overrightarrow{BC}+\vec{b}=\vec{c}$$

or $\qquad \overrightarrow{BC}=\vec{c}-\vec{b}$

or $\qquad \overrightarrow{BC} = (4\hat{i}-3\hat{j}+\hat{k})-(\hat{i}-\hat{j}-3\hat{k})$

$$= 3\hat{i}-2\hat{j}+4\hat{k}$$

Now, $\overrightarrow{BA}\times\overrightarrow{BC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 5\\ 3 & -2 & 4\end{vmatrix}$

$= \hat{i}(8+10)+\hat{j}(15-8)+\hat{k}(-4-6)$

$= 18\hat{i}+7\hat{j}-10\hat{k}$

$|\overrightarrow{BA}\times\overrightarrow{BC}| = \sqrt{18^2+7^2+(-10)^2}$

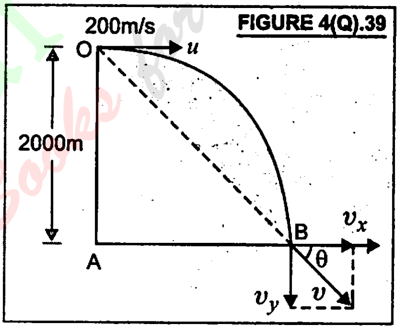
$= \sqrt{324+49+100} = 21{\cdot}75$

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{BA}\times\overrightarrow{BC}| = \frac{1}{2}\times 21{\cdot}75$

$= \mathbf{10{\cdot}87}$ **sq. units**

48. Let bomb be released from O and target be at B and bomb hits the target at B after time t. Fig. 4(Q).39.

FIGURE 4(Q).39



(*i*) *Taking vertical downward motion of bomb from O to B, we have*

$u_y = 0, y_0 = 0, y = 2000\text{ m}; a_y = 10\text{ ms}^{-2}, t = ?,$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

$$\therefore \quad 2000 = 0+0+\frac{1}{2}\times 10t^2$$

or $\quad t^2 = \frac{4000}{10} = 400$

or $\quad t = \mathbf{20\ s}$

(*ii*) Horizontal velocity at B,

$v_x = u_x = 200\text{ ms}^{-1}$;

Vertical velocity at B,

$v_y = a_y t = 10\times 20 = 200\text{ ms}^{-1}$

Velocity with which the bomb hits the target is

$$v = \sqrt{v_x^2+v_y^2} = \sqrt{(200)^2+(200)^2}$$

$$= \mathbf{200\sqrt{2}\ ms^{-1}}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{200}{200} = 1 \quad \text{or} \quad \theta = \mathbf{45°}$$

(*iii*) Horizontal distance $AB = u_x\, t$

$$= 200 \times 20 = 4000 \text{ m}$$

Distance of target from $O = OB$

$$= \sqrt{2000^2 + 4000^2} = \mathbf{2000\sqrt{5}\, m}$$

49. Vertical distance to be travelled by ball,

$$y = 550 - 50 = 500 \text{ m}$$

Horizontal distance to be travelled by ball,

$$x = 200 \text{ m}$$

Let u be the horizontal velocity of projection of ball and it reaches in time t in the other building.

Then, $y = \frac{1}{2} g t^2$

or $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10 \text{ s}$

Now $x = ut$ or $200 = u \times 10$ or $u = \mathbf{20\ ms^{-1}}$

50. Here, $u = 50 \text{ ms}^{-1}$; $t = 3$ s, $x = ?$; $y = ?$; $v = ?$

Now, $x = ut = 50 \times 3 = \mathbf{150\ m}$

$$y = \frac{1}{2} g t^2 = \frac{1}{2} \times 9{\cdot}8 \times 3^2 = \mathbf{44{\cdot}1\, m}$$

$v_x = u_x + a_x t = 50 + 0 \times 3 = 50$ m/s ; acting horizontally

$v_y = u_y + a_y t = 0 + 9{\cdot}8 \times 3 = 29{\cdot}4$ m/s

acting vertically

$\therefore$ Magnitude of velocity after 3 sec,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{50^2 + 29{\cdot}4^2} = \mathbf{58\ m/s}$$

If θ is the angle of $\vec{v}$ with horizontal, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{29{\cdot}4}{50} = 0{\cdot}588 = \mathbf{\tan 30°27'}$$

51. Let u be the horizontal velocity of projected body from the top of a tower. The initial vertical velocity of the body is zero *i.e.* $u_y = 0$. Let the body reach the ground after 3 seconds with horizontal component velocity v_x $(= u)$ and vertical component velocity v_y.

Taking vertical downward motion of the body from top of tower to ground, we have,

$u_y = 0$, $y = ?$, $y_0 = 0$, $a_y = 9{\cdot}8 \text{ ms}^{-2}$, $t = 3$ s , $v_y = ?$

As, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$$= 0 + 0 \times 3 + \frac{1}{2} \times 9{\cdot}8 \times 3^2 = \mathbf{44{\cdot}1\ m}$$

$$v_y = u_y + a_y t = 0 + 9{\cdot}8 \times 3 = 29{\cdot}4 \text{ ms}^{-1}$$

Let θ be the angle which the resultant velocity of body while striking on ground makes with X-axis, then $v_y / v_x = \tan\theta = \tan 45° = 1$

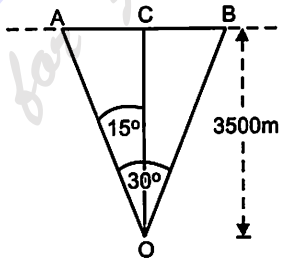
or $v_y = v_x = 29{\cdot}4 \text{ ms}^{-1}$

Hence initial horizontal velocity of projection,

$$u = v_x = \mathbf{29{\cdot}4\ ms^{-1}}$$

52. In Fig. 4(Q).40, O is the observation point at the ground. A and B are the positions of the air craft for which $\angle AOB = 30°$. Draw a perpendicular OC on AB. Here, $OC = 3500$ m and $\angle AOC = \angle COB = 15°$. Time taken by air craft from A to B is 10 sec.

FIGURE 4(Q).40



In $\Delta\, OAC$, $AC = OC \tan 15°$

$$= 3500 \times 0{\cdot}26795 = 937{\cdot}65 \text{ m}$$

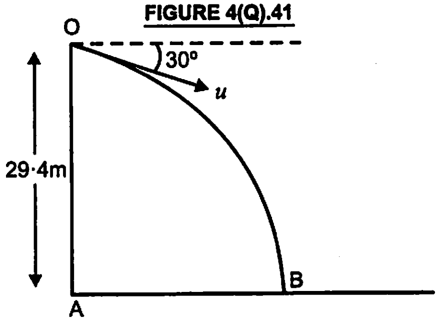
So, $AB = 2 \times AC = 2 \times 937{\cdot}65 = 1875{\cdot}3$.

$$\therefore \text{ Speed of air craft, } v = \frac{1875{\cdot}3}{10} = \mathbf{187{\cdot}5\ ms^{-1}}$$

53. The situation is shown in Fig. 4(Q).41. Resolving $\vec{u}$ into two rectangular components we have the component velocity along vertically downwards,

$$u_y = u \sin 30° = 9{\cdot}8 \times 1/2 = 4{\cdot}9 \text{ m/s}$$

FIGURE 4(Q).41



Taking vertical downward motion of the body from O to B, we have

$y_0 = 0$; $y = 29.4$ m ; $u_y = 4.9$ m/s ;
$a_y = 9.8$ m/s^2, $t = ?$

As, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

or $29.4 = 0 + 4.9 \times t + \frac{1}{2} \times 9.8 \times t^2$

or $4.9\,t^2 + 4.9\,t - 29.4 = 0$

or $t^2 + t - 6 = 0$

On solving we get,

$t = 2$ s or -3 s

∴ Time taken to reach the ground = **2 s**

54. Max. Horizontal distance, $R_{max} = \frac{u^2}{g} = 90$

or $u^2 = 90\,g = 90 \times 10 = 900$ so $u = 30$ m/s

Max. vertical height, $H = \frac{u^2}{2g} = \frac{900}{2 \times 10} = \mathbf{45\ m}$

55. Here, $\theta = 30°$, $u \cos\theta = 20$ ms^{-1}

or $u \cos 30° = 20$

or $u = \frac{20}{\cos 30°} = \frac{20}{\sqrt{3}/2} = \frac{40}{\sqrt{3}}$ ms^{-1}

Maximum height

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(40/\sqrt{3})^2 \sin^2 30°}{2 \times 10} = \mathbf{6.67\ m}$$

Horizontal range

$$= \frac{u^2 \sin 2\theta}{g} = \frac{(40/\sqrt{3})^2 \sin 2 \times 30°}{10} = \mathbf{46.19\ m}$$

56. The change in velocity is due to vertical motion of projectile which will be given by

$|\Delta \vec{v}| = g\,\Delta t = 10 \times 0.5 = \mathbf{5\ m/s}$

57. Angle of projection with horizontal,

$\theta_1 = 90° - 60° = 30°$.

Horizontal range, $R_1 = 2.5$ km $= 2500$ m

As $R_1 = \frac{u^2 \sin 2\theta}{g}$; so, $2500 = \frac{u^2 \sin 2 \times 30°}{g}$

or $\frac{u^2}{g} = \frac{2500}{\sin 60°} = \frac{2500}{\sqrt{3}/2} = \frac{2500 \times 2}{\sqrt{3}}$

When angle of projection is 45° with vertical, the angle of projection with horizontal,

$\theta_2 = 90° - 45 = 45°$.

Horizontal range,

$$R_2 = \frac{u^2 \sin 2 \times 45°}{g} = \frac{u^2}{g} = \frac{2500 \times 2}{\sqrt{3}}$$

$= 2886.8$ m $=$ **2.89 km**

58. The horizontal range is same for two angles of projection θ and $(90° - \theta)$. For these angles of projection, the maximum heights are

$$H_1 = \frac{u^2 \sin^2\theta}{2g}$$

and $H_2 = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$

$$\therefore\ H_1 H_2 = \frac{u^4}{4g^2} \sin^2\theta \cos^2\theta$$

$$= \frac{1}{16}\left[\frac{2u^2}{g} \sin\theta \cos\theta\right]^2 = \frac{1}{16} R^2$$

or $\mathbf{R = 4\sqrt{H_1 H_2}}$

59. *For the first ball*, angle of projection $= \theta$,
$u = 40$ ms^{-1}

Maximum height,

$$H_1 = \frac{u^2 \sin^2\theta}{2g} = \frac{(40)^2 \sin^2\theta}{2 \times 10} \qquad ...(i)$$

For second ball, angle of projection $= (90° - \theta)$, $u = 40$ ms^{-1}

Maximum height,

$$H_2 = H_1 + 40 = \frac{u^2 \sin^2(90° - \theta)}{2g}$$

$$= \frac{(40)^2 \cos^2\theta}{2 \times 10} \qquad ...(ii)$$

Adding (*i*) and (*ii*)

$$2H_1 + 40 = \frac{(40)^2}{20}[\sin^2\theta + \cos^2\theta] = 80$$

or $2H_1 = 80 - 40 = 40$ or $\mathbf{H_1 = 20\ m}$

$H_2 = H_1 + 40 = 20 + 40 = \mathbf{60\ m}$

60. Here, $y = 40$ m ; $x = 60$ m ; $u = ?$, $\theta = 45°$

Let the arrow hit the object on tree in time t.

Then $x = u_x t + \frac{1}{2} a_x t^2$

$\therefore\ 60 = u \cos 45°\, t + \frac{1}{2} \times 0 \times t^2 = u \times \frac{1}{\sqrt{2}} t$

or $u = \frac{60\sqrt{2}}{t}$ ...(*i*)

$$y = u_y\, t + \frac{1}{2}\, a_y\, t^2$$

$$40 = (u \sin 45^\circ)\, t - \frac{1}{2}\, g t^2$$

$$= u \times \frac{1}{\sqrt{2}} t - \frac{1}{2} g t^2$$

or $$40 = \frac{60\sqrt{2}}{t} \times \frac{1}{\sqrt{2}} t - \frac{1}{2} \times 10 t^2 \quad \text{[From } (i)\text{]}$$

or $5\, t^2 = 60 - 40 = 20$ or $t^2 = 4$ or $t = 2$ s

From (*i*), $u = \dfrac{60\sqrt{2}}{2} = \mathbf{30\sqrt{2}\ ms^{-1}}$

61. Here, $u = 20\ \text{ms}^{-1}$, $\theta = 45^\circ$,

Horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \sin 2 \times 45^\circ}{10} = 40\ \text{m}$$

Time of flight, $$T = \frac{2 u \sin\theta}{g} = \frac{2 \times 20 \sin 45^\circ}{10}$$

$$= 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}\ \text{s}$$

Distance required to be covered by goal man to catch the ball = 40 – 20 = 20 m.

∴ Velocity of goal man to catch the ball is

$$= \frac{20}{2\sqrt{2}} = 5\sqrt{2}\ \text{ms}^{-1}$$

$$= 5 \times 1{\cdot}414 = \mathbf{7{\cdot}070\ ms^{-1}}$$

62. Maximum height, $$H = \frac{u^2 \sin^2\theta}{2g}$$

Horizontal range, $$R = \frac{u^2 \sin 2\theta}{g}$$

$$\therefore \frac{R^2}{8H} + 2H = \frac{[u^2 \sin 2\theta / g]^2}{8(u^2 \sin^2\theta / 2g)} + 2\left(\frac{u^2 \sin^2\theta}{2g}\right)$$

$$= \frac{u^2 (2 \sin\theta \cos\theta)^2}{4 g \sin^2\theta} + \frac{u^2}{g} \sin^2\theta$$

$$= \frac{u^2}{g} \cos^2\theta + \frac{u^2}{g} \sin^2\theta$$

$$= \frac{u^2}{g} (\cos^2\theta + \sin^2\theta) = \frac{u^2}{g}$$

Max. Horizontal range, $$R_{\max} = \frac{u^2}{g}$$

$$\therefore \frac{R^2}{8H} + 2H = R_{\max} \quad \textbf{(Proved)}$$

63. Here, $\theta = 60^\circ$, $u = 150\ \text{ms}^{-1}$. Let after time *t*, its velocity become v making an angle 45° with the horizontal. Since the horizontal component velocity of projectile remains unchanged during angular projection, hence

$$u \cos 60^\circ = v \cos 45^\circ \quad \text{or} \quad v = \frac{u \cos 60^\circ}{\cos 45^\circ}$$

Taking vertical upward motion of projectile for time *t*, we have, $u = u \sin 60^\circ$, $a = -g$, $t = t$ and $v = v \sin 45^\circ$.

As, $v = u + at$

so $v \sin 45^\circ = u \sin 60^\circ - gt$

or $$\frac{u \cos 60^\circ}{\cos 45^\circ} \times \sin 45^\circ = u \sin 60^\circ - gt$$

or $$t = \frac{u}{g}[\sin 60^\circ - \cos 60^\circ] = \frac{150}{10}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

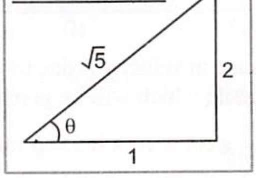
$$= \mathbf{5{\cdot}49\ s}$$

64. Here, $u \cos\theta = 1$ and $u \sin\theta = 2$

So, $$\tan\theta = \frac{u \sin\theta}{u \cos\theta} = \frac{2}{1}$$

and $$\cos\theta = \frac{1}{\sqrt{5}}\ ; \quad u = \sqrt{1^2 + 2^2} = \sqrt{5}$$

FIGURE 4(Q).42



The equation of path of projectile is

$$y = x \tan\theta - \frac{g x^2}{2 u^2 \cos^2\theta}$$

$$= x\,(2) - \frac{10 x^2}{2 \times (\sqrt{5})^2 \times (1/\sqrt{5})^2} = \mathbf{2x - 5x^2}$$

65. Let v_x, v_y be the rectangular component velocities of ball at a point after 1 second of projection, where $v_x = u \cos 30^\circ$

$$= 80 \times \sqrt{3}/2 = 40\sqrt{3}\ \text{ms}^{-1}$$

Taking vertical upward motion of ball for 1 sec, we have

$v_y = ?,\ u_y = u \sin 30° = 80 \times 1/2 = 40$ m/s ;

$a_y = -9\cdot8$ ms^{-2} ; $t = 1$ s

As, $v_y = u_y + a_y t = 40 + (-9\cdot8)\ 1$

$= 40 - 9\cdot8 = 30\cdot2$ ms^{-1}

Resultant velocity, $v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(40\sqrt{3})^2 + (30\cdot2)^2} = \mathbf{75\cdot57\ ms^{-1}}$

If θ is the angle which the resultant velocity makes with the x-axis then

$$\tan\theta = \frac{v_y}{v_x} = \frac{30\cdot2}{40\sqrt{3}} = 0\cdot4359$$

$$= \tan 23°\,33' \quad \text{or} \quad \theta = \mathbf{23°\,33'}$$

66. Max. Horizontal range $= \dfrac{u^2}{g}\sin 2\times45° = \dfrac{u^2}{g}$

Time of flight $= \dfrac{2u}{g}\sin 45° = \dfrac{2u}{g\sqrt{2}} = \dfrac{\sqrt{2}\,u}{g}$

Taking vertical downward motion of body for time equal to time of flight, we have :

$$u = 0\ ;\ a = g\ ;\ S = ?\ ;\ t = \sqrt{2}\,u/g$$

$$\text{As, } S = ut + \frac{1}{2}at^2 = \frac{0\times\sqrt{2}\,u}{g} + \frac{1}{2}g\times\left(\frac{\sqrt{2}\,u}{g}\right)^2$$

$$= \frac{u^2}{g} = \text{Max. H.R.}$$

67. Follow example 51.

68. Max. height, $H = \dfrac{u^2\sin^2\theta}{2g}$

Horizontal range,

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

As per question, $R = 2H$

$$\frac{2u^2\sin\theta\cos\theta}{g} = \frac{2u^2\sin^2\theta}{2g}$$

Solving we get, $\tan\theta = 2$; therefore

$$\sin\theta = (2/\sqrt{5})$$

and $\quad \cos\theta = (1/\sqrt{5})$

∴ Horizontal range,

$$R = \frac{2u^2}{g}\times\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}} = \frac{\mathbf{4u^2}}{\mathbf{5g}}$$

69. The two bodies will collide, if they reach at a point covering the same vertical distance in the same time. Therefore

$$y = u_1\sin 60°\times t - \frac{1}{2}g\,t^2 = u_2\,t - \frac{1}{2}g\,t^2$$

or $\quad u_1\sin 60°\times t = u_2\,t$

or $\quad \dfrac{u_1}{u_2} = \dfrac{1}{\sin 60°} = \dfrac{2}{\sqrt{3}}$

70. The minute's hand of a clock completes one rotation in 60 minutes. *i.e.*

$T = 60$ min $= 60 \times 60$ s

and $\quad \theta = 2\pi$ rad.

∴ $\omega = \theta/T = 2\pi/(60\times60)$ rad. s^{-1}

71. Here, Radius of earth, $R = 1$ AU $= 1\cdot5\times10^{11}$ m.

Period of revolution, of earth around to sun

$T = 365$ days $= 365\times24\times60\times60\ s$

Angular velocity, $\omega = \dfrac{2\pi}{T} = \dfrac{2\times3\cdot14}{365\times24\times60\times60}$

$= \mathbf{1\cdot99\times10^{-7}\ rad/s}$

Linear velocity,

$v = R\omega = (1\cdot5\times10^{11})\times(1\cdot99\times10^{-7})$

$= \mathbf{2\cdot99\times10^4\ ms^{-1}}$

In 365 days, the earth revolves through an angle of 2π radians.

Angle through which the earth revolves in

4 days $= \dfrac{2\pi}{365}\times4 = \dfrac{2\times3\cdot14\times4}{365} = \mathbf{0\cdot0688\ rad}$

72. Here, $m = 5$ kg, $r = 0 = 0\cdot15$ m ;

$$v = \frac{2000}{2\times60} = \frac{50}{3}\,\text{rps}$$

Linear velocity, $v = r\omega = r\times2\pi v$

$$= 0\cdot15\times2\times\frac{22}{7}\times\frac{50}{3} = \mathbf{15\cdot71\ m/s}$$

Centripetal acceleration, $a_c = r\omega^2 = r\,4\,\pi^2\,v^2$

$$= (0\cdot15)\times4\times\left(\frac{22}{7}\right)^2\times\left(\frac{50}{3}\right)^2 = \mathbf{1646\cdot3\ ms^{-2}}$$

73. $v_1 = 100/60$ rps, $v_2 = 400/60$ rps

Angular acceleration,

$$\alpha = \frac{\omega_2-\omega_1}{t} = \frac{2\pi v_2 - 2\pi v_1}{t}$$

Linear acceleration, $a = r\,\alpha$

74. Here, $r = 0{\cdot}5$ m ; $\omega = 2$ rad/s ; $\alpha = 6$ rad s^{-2}

Tangential acceleration, $a_T = r\alpha = 0{\cdot}5 \times 6$
$= 3{\cdot}0 \text{ ms}^{-2}$

Centripetal acceleration, $a_C = r\omega^2 = 0{\cdot}5 \times 2^2$
$= 2{\cdot}0 \text{ ms}^{-2}$

Total linear acceleration, $a = \sqrt{a_T^2 + a_C^2}$

$$\therefore \quad a = \sqrt{(3{\cdot}0)^2 + (2{\cdot}0)^2} = \mathbf{3{\cdot}6 \text{ ms}^{-2}}$$

If $\vec{a}$ makes an angle θ with $\vec{a}_T$, then

$$\tan\theta = \frac{a_C}{a_T} = \frac{2}{3} = 0{\cdot}6667 = \tan 33°\,42'$$

$$\boldsymbol{\theta = 33°\,42' \text{ with } a_T}$$

75. Let A be the magnitude of each vector and the first vector is along x-axis. Then the various vectors will be making angles with the x-axis as $0, \frac{2\pi}{N}, \frac{4\pi}{N}, \frac{6\pi}{N}, \ldots\ldots, \frac{(N-1)}{N}2\pi.$

Let R_x and R_y be the X and Y-components of resultant vector $\vec{R}$. Then

$$R_x = A\cos 0° + A\cos\frac{2\pi}{N} + A\cos\frac{4\pi}{N} + \ldots\ldots$$

$$+ A\cos(N-1)\frac{2\pi}{N} = A\sum_{i=0}^{(N-1)} \cos\frac{2\pi i}{N}$$

$$\text{and } R_y = A\sum_{i=0}^{N-1} \sin\frac{2\pi i}{N}$$

$$\text{But } \sum_{i=0}^{(N-1)} \cos\frac{2\pi i}{N} = \sum_{i=0}^{(N-1)} \sin\frac{2\pi i}{N} = 0$$

$$\therefore \quad R_x = R_y = 0.$$

Hence, $R = \sqrt{R_x^2 + R_y^2} = \mathbf{0}$

Note. The total angle subtended by N vectors with the initial vector $= (2\pi/N) \times N = 2\pi$. It means these N vectors can be represented by the N sides of a closed polygon taken in one order. Hence their vector sum is **zero**.

76. As, $\vec{B} = 3\hat{i} + 4\hat{j}$; so $B = \sqrt{3^2 + 4^2} = 5$

Let $\vec{A} = x\hat{i} + y\hat{j}$;

$$\therefore \quad \vec{A} + \vec{B} = (3+x)\hat{i} + (4+y)\hat{j}$$

But $\vec{A} + \vec{B} = 5\hat{j}$

$$\therefore \quad (3+x)\hat{i} + (4+y)\hat{j} = 5\hat{j}$$

Hence $3 + x + 0$ or $x = -3$

and $4 + y = 5$ or $y = 1$

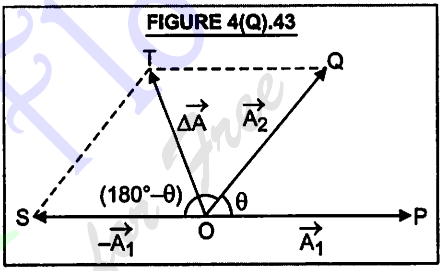
$$\therefore \quad \vec{A} = -3\hat{i} + \hat{j}$$

so $|\vec{A}| = \sqrt{(-3)^2 + (1)^2} = \mathbf{\sqrt{10}}$

77. $\vec{A}_1$ and $\vec{A}_2$ be the initial and final vector, where $A_1 = A = A_2$. Fig 4(Q).43, then change in vector

$$|\Delta\vec{A}| = |\vec{A}_2 - \vec{A}_1| = |\vec{A}_2 + (-\vec{A}_1)|$$

FIGURE 4(Q).43



Since the angle between $\vec{A}_1$ and $\vec{A}_2$ is θ, therefore angle between $\vec{A}_2$ and $(-\vec{A}_1)$ is $(180° - \theta)$.

$\therefore$ Change in magnitude $= \Delta A$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(180° - \theta)}$$

$$= \sqrt{A^2 + A^2 - 2AA\cos\theta}$$

$$= \sqrt{2A^2 - 2A^2\cos\theta} = \sqrt{2A^2(1 - \cos\theta)}$$

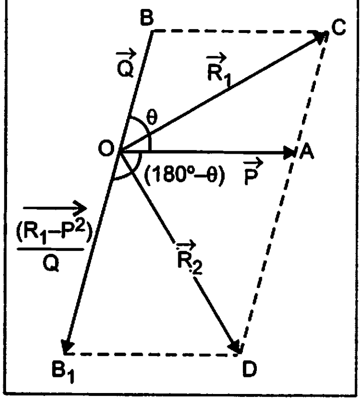
$$= \sqrt{2A^2 \times 2\sin^2\theta/2} = \mathbf{2A\sin\theta/2}$$

78. Refer Fig. 4(Q).44, let θ be the angle between $\vec{P}$ and $\vec{Q}$. Then $R_1^2 = P^2 + Q^2 + 2PQ\cos\theta$

or $\quad 2PQ\cos\theta = R_1^2 - P^2 - Q^2 \quad \ldots(i)$

When the vector $\vec{Q}$ is replaced by the vector $\left(\overrightarrow{\frac{R_1^2 - P^2}{Q}}\right)$ in a direction opposite to that of $\vec{Q}$, then the angle between the vectors is $(180° - \theta)$. If $\vec{R}_2$ is the magnitude of the new resultant, then

FIGURE 4(Q).44



$$R_2^2 = P^2 + \left(\frac{R_1^2 - P^2}{Q}\right)^2 + 2P\left(\frac{R_1^2 - P^2}{Q}\right)\cos(180° - \theta)$$

$$= \frac{1}{Q^2}[(P^2Q^2 + R_1^4 - 2R_1^2P^2 + P^4 - 2PQ(R_1^2 - P^2)\cos\theta]$$

$$= \frac{1}{Q^2}[P^2Q^2 + R_1^4 - 2R_1^2P^2 + P^4 - (R_1^2 - P^2)(R_1^2 - P^2 - Q^2)] \text{ [From (i)]}$$

$$= \frac{1}{Q^2}[P^2Q^2 + R_1^4 - 2R_1^2P^2 + P^4 - R_1^4 + 2P^2R_1^2 - P^4 - P^2Q^2 + R_1^2Q^2]$$

$$= \frac{R_1^2Q^2}{Q^2} = R_1^2$$

∴ $R_2 = R_1$. Thus the magnitude of resultant in the second case is also R_1.

79. Let the first car be moving with uniform velocity u along OA and the second car be moving with uniform acceleration a along OB, Fig. 4(Q).45. After t seconds the velocity of second car,

$$v = at$$

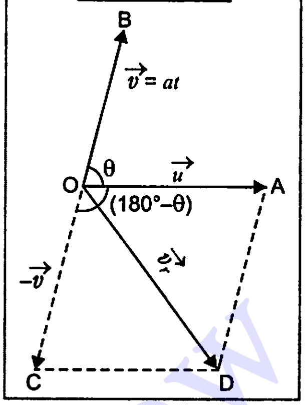
Relative velocity of first car w.r.t. the second car is $\vec{v}_r = \vec{u} - \vec{v} = \vec{u} + (-\vec{v})$

$$= (\overrightarrow{OA}) + (\overrightarrow{OC}) = (\overrightarrow{OD})$$

Magnitude of $\vec{v}_r$ is given by

$$v_r^2 = u^2 + v^2 + 2\,uv\cos(180° - \theta)$$
$$v_r = [u^2 + a^2t^2 - 2\,u\,at\cos\theta]^{1/2}$$

FIGURE 4(Q).45



v_r is minimum if $\frac{dv_r}{dt} = 0$

$$\frac{dv_r}{dt} = \frac{1}{2}\,(u^2 + a^2t^2 - 2\,u\,at\cos\theta)^{-1/2} \times (2\,a^2t - 2\,u\,a\cos\theta) = 0.$$

Hence, $2\,a^2t - 2\,u\,a\cos\theta = 0$

or $t = \frac{u\cos\theta}{a}$

$$\therefore\ v_r = \left[u^2 + a^2 \times \frac{u^2\cos^2\theta}{a^2} - 2ua\frac{u\cos\theta}{a} \times \cos\theta\right]^{1/2}$$

$$= [u^2 - u^2\cos^2\theta]^{1/2}$$

$$= u\sin\theta \text{ (Proved)}$$

80. For maximum horizontal range, $\theta = 45°$;

$R = 0{\cdot}204$ m.

So, $0{\cdot}204 = u^2\sin(2 \times 45°)/g$

or $u^2 = g \times 0{\cdot}204 = 10 \times 0{\cdot}204 = 2{\cdot}04\ \text{m}^2/\text{s}^2$

or $u = \sqrt{2{\cdot}04} = 1{\cdot}414$ m/s

Velocity along the road

$$= u\cos 45° = 1{\cdot}414 \times (1/\sqrt{2}) = \mathbf{1\ ms^{-1}}$$

81. Here, $v = 27\ \text{km h}^{-1} = 27 \times 1000/(60 \times 60)$

$= 15/2\ \text{ms}^{-1}$;

$r = 80$ m

Centipetal acceleration,

$$a_c = \frac{v^2}{r} = \left(\frac{15}{2}\right)^2 \times \frac{1}{80} = 0{\cdot}7\ \text{ms}^{-2}$$

Tangential acceleration, $a_T = 0{\cdot}5\ \text{ms}^{-2}$ (given)

∴ Effective accelertion,

$$a = \sqrt{a_c^2 + a_T^2} = \sqrt{(0{\cdot}7)^2 + (0{\cdot}5)^2}$$
$$= \mathbf{0{\cdot}86\ m/s^2}$$

82. From the equation of path of projectile,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}, \text{ we have}$$

$$13 = 50\tan\theta - \frac{g\,(50)^2}{2\times(10\sqrt{g})^2\cos^2\theta}$$

$$= 50\tan\theta - \frac{25}{2}\sec^2\theta$$

or $13 = 50\tan\theta - \frac{25}{2}(1+\tan^2\theta)$

or $25\tan^2\theta - 100\tan\theta + 51 = 0$

$$\tan\theta = \frac{100 \pm \sqrt{(100)^2 - 4\times25\times51}}{50}$$

$$= \frac{17}{5} \text{ or } \frac{3}{5}$$

The horizontal distance covered in time t is given by $x = u\cos\theta\cdot t$

or $50 = (10\sqrt{g})\cos\theta\, t$

or $t = \dfrac{50}{10\sqrt{g}\cos\theta}$

Hence, t will be minimum if the value of $\cos\theta$ is maximum. It will be so if the value of $\tan\theta$ is least

i.e. $\tan\theta = \frac{3}{5} = 0{\cdot}6$ or $\theta = \mathbf{30° 58'}$

83. Let the particle be at R, where $\angle XOR = 45°$. Magnitude of acceleration at R

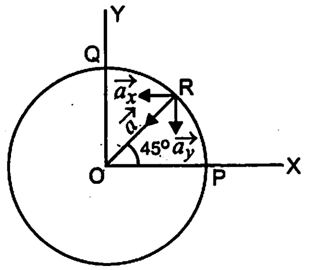
$$= \frac{v^2}{r} = \frac{2\times2}{4} = 1\text{ cm } s^{-2}$$

It is acting along RO. Fig. 4(Q).46.

$\therefore$ Component of acceleration along X-axis,

$$a_x = 1\cos45° = 1/\sqrt{2}$$

FIGURE 4(Q).46



So, $\vec{a}_x = -\frac{1}{\sqrt{2}}\hat{x}$

Component of acceleration along Y axis,

$$a_y = 1\sin45° = 1/\sqrt{2}$$

so, $\vec{a}_y = -\frac{1}{\sqrt{2}}\hat{y}$

$$\therefore \quad \vec{a} = \vec{a}_x + \vec{a}_y = -\frac{(\hat{x}+\hat{y})}{\sqrt{2}}\ \textbf{cm s}^{-2}$$

84. *For first body*, the distance moved in vertical upward direction in time t is $y = ut - \frac{1}{2}gt^2$

For second body, the horizontal dsitance,

$$x' = u\cos\theta\times t = 30\times\frac{1}{2}\times2 = 30\text{ m}$$

and vertical distance, $y' = u\sin\theta\, t - \frac{1}{2}gt^2$

Vertical distance between two bodies after time t

$$= y - y' = \left(ut - \frac{1}{2}gt^2\right) - \left(u\sin\theta t - \frac{1}{2}gt^2\right)$$

$= ut - u\sin\theta\, t$

$= 30\times2 - 30\times(\sqrt{3}/2)\times2 \approx 8\text{ m}$

The distance between two bodies after time t is

$$= \sqrt{(y-y')^2 + x'^2} = \sqrt{8^2+30^2} \approx \mathbf{31m}$$

85. $u_{x_0} = \left(\frac{dx}{dt}\right)_{t=0} = \frac{d}{dt}(6t) = 6$

$$u_y = \left(\frac{dy}{dt}\right) = \frac{d}{dt}(8t - 5t^2) = 8 - 10t$$

$$u_{y_0} = \left(\frac{dy}{dt}\right)_{t=0} = 8 - 10\times0 = 8\text{ ms}^{-1}$$

$$\therefore \quad u = \sqrt{u_{x_0}^2 + u_{y_0}^2} = \sqrt{6^2+8^2} = \mathbf{10\ ms^{-1}}$$

If θ is the angle of projection of projectile, then

$$\tan\theta = \frac{u_{y_0}}{u_{x_0}} = \frac{8}{6} = \frac{4}{3} = 1{\cdot}3333 = \tan 53° 8'$$

or $\theta = \mathbf{53° 8'}$

86. Here, $v_x = 7\text{ ms}^{-1}$; $v_y = 6{\cdot}0\text{ ms}^{-1}$; $S = 9{\cdot}0$ m ; $u_y = ?$

Taking vertical upward motion of the ball up to given height, we have

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$$v_y^2 = u_y^2 + 2aS$$

$$6^2 = u_y^2 + 2(-10)\times 9$$

or $$u_y^2 = 36 + 180 = 216$$

Maximum height attained by the ball,

$$h_{\max.} = \frac{u_y^2}{2g} = \frac{216}{2\times 10} = \mathbf{10{\cdot}8\ m}$$

87. Max. height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

or $$gH = \frac{u^2\sin^2\theta}{2}$$

Velocity at highest point, $v_H = u\cos\theta$

Let v_x, v_y be the horizontal and vertical velocity of projectile at height $H/2$. Then

$$v_x = u\cos\theta \text{ and } v_y^2 = u^2\sin^2\theta - 2g\times H/2$$

$$= u^2\sin^2\theta - gH$$

$$= u^2\sin^2\theta - \frac{u^2\sin^2\theta}{2} = \frac{u^2\sin^2\theta}{2}$$

$\therefore$ Effective velocity at height $H/2$

$$= (v_x^2 + v_y^2)^{1/2}$$

As per question ;

$$\sqrt{\frac{2}{5}}\,(v_x^2+v_y^2)^{1/2} = v_H \text{ or } \frac{2}{5}(v_x^2+v_y^2) = v_H^2$$

or $$\frac{2}{5}\left[u^2\cos^2\theta + \frac{u^2}{2}\sin^2\theta\right] = u^2\cos^2\theta$$

or $\sin^2\theta = 3\cos^2\theta$ or $\sin\theta = \sqrt{3}\cos\theta$

or $\tan\theta = \sqrt{3} = \tan 60°$ or $\boldsymbol{\theta = 60°}$

88. Initial momentum of body along horizontal direction, $p_x = mv\cos 45° = \dfrac{mv}{\sqrt{2}}$,

Initial momentum along vertical upward direction, $p_y = mv\sin 45° = \dfrac{mv}{\sqrt{2}}$

$\therefore$ Total momentum,

$$\vec{p} = p_x\hat{i} + p_y\hat{j} = \frac{mv}{\sqrt{2}}\hat{i} + \frac{mv}{\sqrt{2}}\hat{j}$$

The body will strike the horizontal surface with velocity v, hence its momentum along vertical downward direction,

$$p_y' = mv\sin 45° = \frac{mv}{\sqrt{2}}$$

momentum along horizontal direction,

$$p_x' = mv\cos 45° = \frac{mv}{\sqrt{2}}$$

$\therefore$ Total momentum,

$$\vec{p}\,' = p_x'\hat{i} + p_y'(-\hat{j}) = \frac{mv}{\sqrt{2}}\hat{i} - \frac{mv}{\sqrt{2}}\hat{j}$$

$\therefore$ Change in momentum, $\Delta\vec{p} = \vec{p} - \vec{p}\,'$

$$= \left(\frac{mv}{\sqrt{2}}\hat{i} + \frac{mv}{\sqrt{2}}\hat{j}\right) - \left(\frac{mv}{\sqrt{2}}\hat{i} - \frac{mv}{\sqrt{2}}\hat{j}\right)$$

$$= \frac{2mv}{\sqrt{2}}\hat{j}$$

$$\therefore\ |\Delta\vec{p}| = \frac{2mv}{\sqrt{2}} = \boldsymbol{\sqrt{2}\,mv}$$

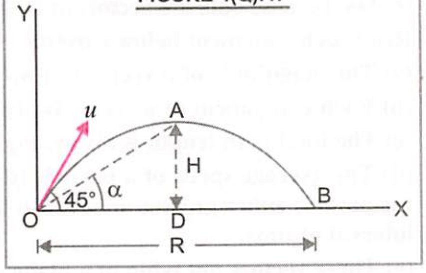
89. Refer to Fig. 4(Q).47, let u be the initial velocity of projectile at O. It then follows a path OAB, where $AD = H$ = Maximum height.

Now, Max. height, $H = \dfrac{u^2\sin^2 45°}{2g} = \dfrac{u^2}{4g}$

Horizontal range, $R = \dfrac{u^2\sin 2\times 45°}{g} = \dfrac{u^2}{g}$

If α is the angle of elevation of the projectile at the highest point from the point of projection, then $\angle AOD = \alpha$.

FIGURE 4(Q).47



From ΔAOD,

$$\tan\alpha = \frac{H}{R/2} = \frac{2H}{R} = \frac{2(u^2/4g)}{u^2/g} = \frac{1}{2}$$

$$\therefore\quad \boldsymbol{\alpha = \tan^{-1}\left(\frac{1}{2}\right)}$$

NCERT EXERCISES

WITH SOLUTIONS

Q. 1. State, for each of the following physical quantities, if it is a scalar or a vector. Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Sol. Scalars. Volume, mass, speed, density, number of moles, angular frequency.

Vectors. Acceleration, velocity, displacement, angular velocity.

Q. 2. Pick out the two scalar quantities in the following lists : force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Sol. Work and current are the scalar quantities in the given list.

Q. 3. Pick out the only vector quantity in the following list : temperature, pressure, impluse, time, power, total path-length, energy, gravitational potential, coefficient of friction, charge.

Sol. Since, Impulse = change in momentum = force × time. As momentum and force are vector quantities, hence impulse is a vector quantity.

Q. 4. State with reasons, whether the following algebraic operations with scalars and vectors are meaningful.

(a) Adding any two scalars.

(b) Adding a scalar to a vector of the same dimension

(c) Multiplying any vector by any scalar.

(d) Multiplying any two scalars.

(e) Adding any two vectors.

(f) Adding a component of a vector to the same vector.

Sol. (*a*) **No,** because only the scalars of same dimensions can be added.

(*b*) **No,** because a scalar cannot be added to a vector.

(*c*) **Yes,** when acceleration $\vec{A}$ is multiplied by mass m, we get a force $\vec{F} = m\vec{A}$, which is a meaningful operation.

(*d*) **Yes,** when power P is multiplied by time t, we get work done = Pt, which is a useful operation.

(*e*) **No,** because the two vectors of same dimensions can be added.

(*f*) **Yes,** because both are vectors of the same dimensions.

Q. 5. Read each statement below carefully and state with reasons, if it is true or false :

(a) The magnitude of a vector is always a scalar.

(b) Each component of a vector is always a scalar.

(c) The total path length is always equal to the magnitude of the displacement vector of a particle.

(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.

(e) Three vectors not lying in a plane can never add up to give a null vector.

Sol. (*a*) **True** ; because magnitude is a pure number.

(*b*) **False** ; each component of a vector is also a vector.

(*c*) **True only** if the particle moves along a straight line in the same direction, **otherwise false.**

(*d*) **True** ; because the total path length is either greater than or equal to the magnitude of the displacement vector.

(*e*) **True** ; as they can not be represented by the three sides of a triangle taken in the same order.

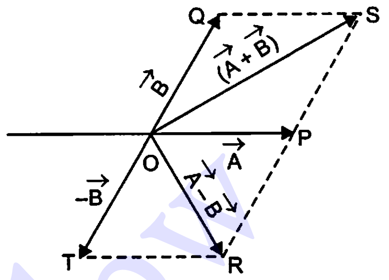
Q. 6. Establish the following inequalities geometrically or otherwise :

(a) $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$ **(b)** $|\vec{A}+\vec{B}| \ge \big||\vec{A}|-|\vec{B}|\big|$

(c) $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$ **(d)** $|\vec{A}-\vec{B}| \ge \big||\vec{A}|-|\vec{B}|\big|$

When does the equality sign above apply ?

FIGURE 4(N).1



Sol. Consider two vectors $\vec{A}$ and $\vec{B}$ be represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of a parallelogram $OPSQ$. According to parallelogram law of vector addition ; $(\vec{A}+\vec{B})$ will be represented by $\overrightarrow{OS}$ as shown in Fig. 4(N).1. Thus $OP = |\vec{A}|$, $OQ = PS = |\vec{B}|$ and $OS = |\vec{A}+\vec{B}|$

(a) To prove $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$

We know that the length of one side of a triangle is always less than the sum of the lengths of the other two sides. Hence from Δ OPS, we have

$$OS < OP + PS \quad \text{or} \quad OS < OP + OQ$$

or $$|\vec{A}+\vec{B}| < |\vec{A}|+|\vec{B}| \qquad ...(i)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line and in the same direction

then $$|\vec{A}+\vec{B}| = |\vec{A}|+|\vec{B}| \qquad ...(ii)$$

Combining the conditions mentioned in (*i*) and (*ii*) we have $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$

(b) To prove $|\vec{A}+\vec{B}| \ge \big||\vec{A}|+|\vec{B}|\big|$

From Δ OPS, we have $OS + PS > OP$ or $OS > |OP - PS|$ or $OS > |OP - OQ|$ ...(*iii*)

$(\because PS = OQ)$

The modulus of $(OP - PS)$ has been taken because the L.H.S. is always positive but the R.H.S. may be negative if $OP < PS$. Thus from (*iii*) we have.

$$|\vec{A}+\vec{B}| > \big||\vec{A}|-|\vec{B}|\big| \qquad ...(iv)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along a straight line in opposite directions, then

$$|\vec{A}+\vec{B}| = \big||\vec{A}|-|\vec{B}|\big| \qquad ...(v)$$

Combining the conditions mentioned in (*iv*) and (*v*) we get

$$|\vec{A}+\vec{B}| \ge \big||\vec{A}|-|\vec{B}|\big|$$

(c) To Prove $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$

In Fig. 4(N).1 $\vec{A} = (\overrightarrow{OP})$ and $-\vec{B} = \overrightarrow{OT} = \overrightarrow{PR}$ and $(\vec{A}-\vec{B}) = \overrightarrow{OR}$

From Δ ORP we note that $OR < OP + PR$.

or $|\vec{A}-\vec{B}| < |\vec{A}|+|-\vec{B}|$ or $|\vec{A}-\vec{B}| < |\vec{A}|+|\vec{B}|$ ...(*vi*)

If the two vectors are acting along a straight line

but in the opposite direction, then $|\vec{A}-\vec{B}| = |\vec{A}|+|\vec{B}|$ ...(*vii*)

Combining the conditions mentioned in (*vi*) and (*vii*) we get $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$

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(d) To prove $|\vec{A}-\vec{B}| \geq \left| |\vec{A}| - |\vec{B}| \right|$

In Fig. 4(N).1, from Δ *OPR*, we note that

$$OR + PR > OP \quad \text{or} \quad OR > |OP - PR| \quad \text{or} \quad OR > |OP - OT| \qquad ...(viii)$$

$$(\because OT = PR)$$

The modulus of ($OP - OT$) has been taken because L.H.S. is positive and R.H.S. may be negative if $OP < OT$.

From (*viii*), $$|\vec{A}-\vec{B}| > \left| |\vec{A}| - |\vec{B}| \right| \qquad ...(ix)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line in the same direction, then

$$|\vec{A}-\vec{B}| = \left| |\vec{A}| - |\vec{B}| \right| \qquad ...(x)$$

Combining the conditions mentioned in (*ix*) and (*x*) we get $|\vec{A}-\vec{B}| \geq \left| |\vec{A}| - |\vec{B}| \right|$

Q. 7. Given $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, which of the following statements are correct ?

(a) $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ must each be a null vector.

(b) The magnitude of $(\vec{A}+\vec{C})$ equals the magnitude of $(\vec{B}+\vec{D})$.

(c) The magnitude of $\vec{A}$ can never be greater than the sum of the magnitude of $\vec{B}, \vec{C}$ and $\vec{D}$.

(d) $\vec{B}+\vec{C}$ must lie in the plane of $\vec{A}+\vec{D}$, if $\vec{A}$ and $\vec{D}$ are not collinear and in the line of $\vec{A}$ and $\vec{D}$, if they are collinear.

Sol. (*a*) **Not correct,** because $\vec{A}+\vec{B}+\vec{C}+\vec{D}$ can be zero in many ways other than $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ must each be a null vector.

(*b*) **Correct.** Since, $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$; $\therefore \vec{A}+\vec{C}=-(\vec{B}+\vec{D})$ or $|\vec{A}+\vec{C}|=(\vec{B}+\vec{D})$.

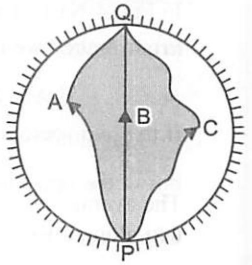
(*c*) **Correct.** Since $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$ or $\vec{A}=-(\vec{B}+\vec{C}+\vec{D})$

It means the magnitude of $\vec{A}$ is equal to the magnitude of vector $(\vec{B}+\vec{C}+\vec{D})$. Since the sum of the magnitudes of $\vec{B}, \vec{C}$ and $\vec{D}$ may be equal or greater than the magnitude of $\vec{A}$, hence the magnitude of $\vec{A}$ can never be greater than the sum of the manitude of $\vec{B}, \vec{C}$ and $\vec{D}$.

(*d*) **Correct.** Since $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$; or $\vec{A}+(\vec{B}+\vec{C})+\vec{D}=0$. The resultant sum of three vectors $\vec{A}, (\vec{B}+\vec{C})$ and $\vec{D}$ can be zero only if $(\vec{B}+\vec{C})$ lies in the plane of $\vec{A}$ and $\vec{D}$ and these three vectors are represented by the three sides of a triangle taken in one order. If $\vec{A}$ and $\vec{D}$ are collinear, then $(\vec{B}+\vec{C})$ must be in line of $\vec{A}$ and $\vec{D}$, only then the vector sum of all the vectors will be zero.

Q. 8. Three girls skating on a circular ice ground of radius 200 m start from a point *P* on the edge of the ground and reach a point *Q* diametrically opposite to *P* following different paths as shown in Fig. 4(N).2. What is the magnitude of the displacement vector for each ? For which girl is this equal to the actual length of path skated ?

FIGURE 4(N).2



Sol. Displacement for each girl = $\overrightarrow{PQ}$.

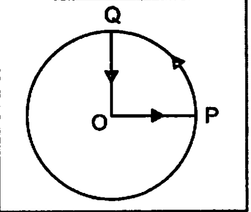
∴ Magnitude of the displacement for each girl

$$= PQ = \text{diameter of circular ice ground} = 2 \times 200 = 400 \text{ m.}$$

For girl *B*, the magnitude of displacement is equal to the actual length of path skated.

Q. 9. A cyclist starts from the centre *O* of a circular park of radius 1 km reaches the edge *P* of the park, then cycles along the circumference, and returns to the centre along *QO* as shown in Fig. 4(N).3. If the round trip takes 10 minutes, what is the (*a*) net displacement (*b*) average velocity and (*c*) average speed of the cyclist.

FIGURE 4(N).3



Sol. (*a*) Here, net displacement = zero.

(*b*) Average velocity $= \dfrac{\text{net displacement}}{\text{time taken}} = \dfrac{0}{10/60\,\text{h}} = 0$

(*c*) Average speed $= \dfrac{\text{total path length}}{\text{time taken}} = \dfrac{1\,\text{km} + \dfrac{2\pi r}{4} + 1\,\text{km}}{(10/60)\ \text{h}} = \dfrac{1 + 2\times\dfrac{22}{7}\times\dfrac{1}{4} + 1}{1/6} = \mathbf{21{\cdot}4\ km\ h^{-1}}$

Q. 10. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Sol. In this problem, the path is a regular hexagon *ABCDEF* of side length 500 m. Fig. 4(N).4. Let the motorist start from *A*.

Third turn. The motor cyclist will take the third turn at *D*.

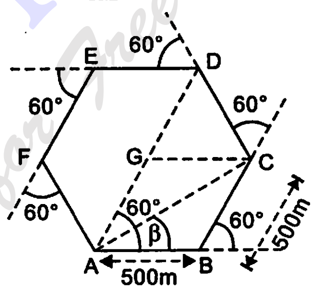
Displacement vector at $D = \overrightarrow{AD}$

Magnitude of this displacement = 500 + 500 = **1000 m**

Total path length from *A* to *D* = *AB* + *BC* + *CD* = 500 + 500 + 500 = **1500 m**

Sixth turn. The motor cyclist takes the sixth turn at *A*. So displacement vector is **null vector.** The total path length = *AB* + *BC* + *CD* + *DE* + *EF* + *FA* = 6 × 500 = **3000 m**

FIGURE 4(N).4



Eighth turn. The motor cyclist takes the eighth turn at *C*. The displacement vector $= \overrightarrow{AC}$, which is represented by the diagonal of the parallelogram *ABCG*.

So, $|\overrightarrow{AC}| = \sqrt{(500)^2 + (500)^2 + 2\times(500)\times(500)\times\cos 60^\circ} = \sqrt{(500)^2 + (500)^2 + 250000} = \mathbf{866{\cdot}03\ m}$

$$\tan\beta = \frac{500\sin 60^\circ}{500 + 500\cos 60^\circ} = \frac{500\times\sqrt{3}/2}{500\,(1+1/2)} = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad \text{or} \quad \beta = 30^\circ$$

It means $\overrightarrow{AC}$ makes an angle **30° with the initial direction.** Total path length = 8 × 500 = **4000 m**

Q. 11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes. What is (a) the average speed of the taxi, (b) the magnitude of average velocity ? Are the two equal ?

Sol. Here, actual path length travelled, *S* = 23 km ; Displacement = 10 km ; Time taken, *t* = 28 min = 28/60 h

(*a*) Average speed of the taxi $= \dfrac{\text{actual path length}}{\text{time taken}} = \dfrac{23}{28/60} = \mathbf{49{\cdot}3\ km/h}$

(*b*) Magnitude of average velocity $= \dfrac{\text{displacement}}{\text{time taken}} = \dfrac{10}{(28/60)} = \mathbf{21{\cdot}4\ km/h}$

The average speed is not equal to the magnitude of average velocity. The two are equal for the motion of taxi along a straight path in one direction.

Q. 12. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ?

Sol. Here, $u = 40$ ms^{-1} ; $H = 25$ m ; $R = ?$

Let θ be the angle of projection with the horizontal direction to have the maximum range, with maximum height = 25 m.

$$\text{Maximum height, } H = 25 = \frac{u^2 \sin^2 \theta}{2g} = \frac{(40)^2 \sin^2 \theta}{2 \times 9{\cdot}8}$$

or $$\sin \theta = \left(\frac{25 \times 2 \times 9{\cdot}8}{40^2}\right)^{\frac{1}{2}} = 0{\cdot}5534 = \sin 33{\cdot}6^\circ \quad \text{or} \quad \theta = 33{\cdot}6^\circ$$

$$\text{Horizontal range, } R = \frac{u^2 \sin 2\theta}{g} = \frac{(40)^2 \times \sin 2 \times 33{\cdot}6^\circ}{9{\cdot}8} = \frac{1600 \times 0{\cdot}9219}{9{\cdot}8} = \mathbf{150{\cdot}5\ m}$$

Q. 13. A cricketer can throw a ball to a maximum horizontal distance of 100 m. With the same speed how high above the ground can the cricketer throw the same ball ?

Sol. Let u be the velocity of projection of the ball. The ball will cover maximum horizontal distance when angle of projection with horizontal, θ = 45°. Then, $R_{max} = u^2/g$

Here, $u^2/g = 100$ m. ...(*i*)

In order to study the motion of the ball along vertical direction, consider a point on the surface of earth as the origin and vertical upward direction as the positive direction of Y axis. Taking motion of the ball along vertical upward direction we have $u_y = u,\ a_y = -g,\ v_y = 0,\ t = ?,\ y_0 = 0,\ y = ?$

As, $v_y = u_y + a_y t \quad \therefore \quad 0 = u + (-g)\,t \quad \text{or} \quad t = u/g$

Also, $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$$\therefore \quad y = 0 + u\,(u/g) + \frac{1}{2}(-g)\,u^2/g^2 = \frac{u^2}{g} - \frac{1}{2}\frac{u^2}{g} = \frac{1}{2}\frac{u^2}{g} = \frac{100}{2} = \mathbf{50\ m} \qquad \text{[from } (i)]$$

Q. 14. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone ?

Sol. Here, $r = 80$ cm $= 0{\cdot}8$ m ; $\nu = 14/25$ s^{-1}. $\therefore \quad \omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25}$ rad. s^{-1}.

$$\text{The centripetal acceleration, } a = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 0{\cdot}80 = \mathbf{9{\cdot}90\ ms^{-2}}$$

The direction of centripetal acceleration is along the string directed towards the centre of circular path.

Q. 15. An air craft executes a horizontal loop of radius 1 km with a steady speed of 900 km h^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.

Sol. Here, $r = 1$ km $= 1000$ m ; $v = 900$ km h^{-1} $= 900 \times (1000\text{ m}) \times (60 \times 60\text{ s})^{-1} = 250$ ms^{-1}

$$\text{Centripetal acceleration, } a = \frac{v^2}{r} = \frac{(250)^2}{1000}$$

$$\text{Now, } \frac{a}{g} = \frac{(250)^2}{1000} \times \frac{1}{9{\cdot}8} = \mathbf{6{\cdot}38}$$

Q. 16. Read each statement below carefully and state, with reasons, if it is true or false :

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

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Sol. (*a*) **False.** The net acceleration of a particle is towards the centre only in case of a uniform circular motion.

(*b*) **True**, because while leaving the circular path, the particle moves tangentially to the circular path.

(*c*) **True.** The direction of acceleration vector in a uniform circular motion is directed towards the centre of circular path. It is constantly changing with time. The resultant of all these vectors will be a zero vector.

Q. 17. The position of a particle is given by $\vec r = 3{\cdot}0\,t\,\hat i - 2{\cdot}0\,t^2\,\hat j + 4{\cdot}0\,\hat k$ m, where t is in seconds and the coefficients have the proper units for $\vec r$ to be in metres. (*a*) Find the $\vec v$ and $\vec a$ of the particle ? (*b*) What is the magnitude and direction of velocity of the particle at $t = 2$ s ?

Sol. (*a*) Velocity, $\vec v = \frac{d\vec r}{dt} = \frac{d}{dt}(3{\cdot}0\,t\,\hat i - 2{\cdot}0\,t^2\,\hat j + 4{\cdot}0\,\hat k) = \mathbf{[3{\cdot}0\,\hat i - 4{\cdot}0\,t\,\hat j]\ ms^{-1}}$

Acceleration, $\vec a = \frac{d\vec v}{dt} = \frac{d}{dt}(3{\cdot}0\,\hat i - 4{\cdot}0\,t\,\hat j) = 0 - 4{\cdot}0\,\hat j = \mathbf{-4{\cdot}0\,\hat j\ ms^{-2}}$

(*b*) At time $t = 2$ s, $\vec v = 3{\cdot}0\,\hat i - 4{\cdot}0 \times 2\,\hat j = 3{\cdot}0\,\hat i - 8{\cdot}0\,\hat j$ $\therefore$ $v = \sqrt{(3{\cdot}0)^2 + (-8)^2} = \sqrt{73} = 8{\cdot}54\ \text{ms}^{-1}$

If θ is the angle which $\vec v$ makes with x-axis, then $\tan\theta = \frac{v_y}{v_x} = \frac{-8}{3} = -2{\cdot}667 = -\tan 69{\cdot}5°$

$\therefore$ $\qquad$ **θ = 69·5° below the x-axis**

Q. 18. A particle starts from the origin at $t = 0$ with a velocity of $10{\cdot}0\,\hat j$ m/s and moves in the *X-Y* plane with a constant acceleration of $(8{\cdot}0\,\hat i + 2{\cdot}0\,\hat j)$ ms^{-2}. (*a*) At what time is the x-coordinate of the particle 16 m ? What is the y-coordinate of the particle at that time ? (*b*) What is the speed of the particle at that time ?

Sol. Here, $\vec u = 10{\cdot}0\,\hat j$ ms^{-1} at $t = 0$.

$$\vec a = \frac{d\vec v}{dt} = (8{\cdot}0\,\hat i + 2{\cdot}0\,\hat j)\ \text{ms}^{-2}$$

So $\qquad d\vec v = (8{\cdot}0\,\hat i + 2{\cdot}0\,\hat j)\,dt$

Integrating it within the limits of motion *i.e.* as time changes from 0 to t, velocity changes from u to v, we have $\qquad \vec v - \vec u = (8{\cdot}0\,\hat i + 2{\cdot}0\,\hat j)\,t$ $\quad$ or $\quad$ $\vec v = \vec u + 8{\cdot}0\,t\,\hat i + 2{\cdot}0\,t\,\hat j$

As $\qquad \vec v = \frac{d\vec r}{dt}$ $\quad$ or $\quad$ $d\vec r = \vec v\,dt$

So, $\qquad d\vec r = (\vec u + 8{\cdot}0\,t\,\hat i + 2{\cdot}0\,t\,\hat j)\,dt$

Integrating it within the conditions of motion *i.e.* as time changes from 0 to t, displacement changes from 0 to r, we have :

$$\vec r = \vec u\,t + \frac{1}{2} \times 8{\cdot}0\,t^2\,\hat i + \frac{1}{2} \times 2{\cdot}0\,t^2\,\hat j \quad \text{or} \quad x\,\hat i + y\,\hat j = 10\,\hat j\,t + 4{\cdot}0\,t^2\,\hat i + t^2\,\hat j = 4{\cdot}0 t^2\,\hat i + (10\,t + t^2)\,\hat j$$

Here, we have, $x = 4{\cdot}0\,t^2$ and $y = 10\,t + t^2$ $\quad\therefore\quad$ $t = (x/4)^{1/2}$

(*a*) At $x = 16$ m ; $t = (16/4)^{1/2} = \mathbf{2\ s}$

$$y = 10 \times 2 + 2^2 = \mathbf{24\ m}$$

(*b*) Velocity of the particle at time t is $\quad \vec v = 10\,\hat j + 8{\cdot}0\,t\,\hat i + 2{\cdot}0\,t\,\hat j$

When $t = 2$ s, then, $\quad \vec v = 10\,\hat j + 8{\cdot}0 \times 2\,\hat i + 2{\cdot}0 \times 2\,\hat j = 16\,\hat i + 14\,\hat j$

Speed $= |\vec v| = \sqrt{16^2 + 14^2} = \mathbf{21{\cdot}26\ ms^{-1}}$

Q. 19. $\hat{i}$ **and** $\hat{j}$ **are unit vectors along** x**- and** y**-axis respectively.**

(a) What is the magnitude and direction of the vectors $(\hat{i}+\hat{j})$ **and** $(\hat{i}-\hat{j})$ **?**

(b) What are the components of a vector $\vec{B}=2\hat{i}+3\hat{j}$ **along the directions of** $(\hat{i}+\hat{j})$ **and** $(\hat{i}-\hat{j})$ **?**

Sol. First method : (*a*) Magnitude of $(\hat{i}+\hat{j})=|\hat{i}+\hat{j}|=\sqrt{(1)^2+(1)^2}=\sqrt{2}$

Let the vector $(\hat{i}+\hat{j})$ make an angle θ with the direction of $\hat{i}$, then

$$\cos\theta=\frac{|\hat{i}+\hat{j}|.\hat{i}}{|\hat{i}+\hat{j}||\hat{i}|}=\frac{1}{(\sqrt{2})(1)}=\frac{1}{\sqrt{2}}=\cos 45^\circ \quad \text{or} \quad \theta=45^\circ \qquad \left[\because \cos\theta=\frac{\vec{A}.\vec{B}}{AB}\right]$$

Magnitude of $(\hat{i}-\hat{j})=|\hat{i}-\hat{j}|=\sqrt{(1)^2+(-1)^2}=\sqrt{2}$

Similarly, if θ is the angle which the vector $(\hat{i}-\hat{j})$ makes with the direction of $\hat{i}$, then

$$\cos\theta=\frac{(\hat{i}-\hat{j}).\hat{i}}{|\hat{i}-\hat{j}||\hat{i}|}=\frac{1}{\sqrt{2}}=\cos 45^\circ \quad \text{or} \quad \theta=45^\circ$$

Here, $\theta=-45^\circ$ **with** $\hat{i}$

(*b*) Here, $\vec{B}=2\hat{i}+3\hat{j}$

To find the component vectors of $\vec{B}$ along the vectors $(\hat{i}+\hat{j})$ we first find the unit vector along the direction of vector $(\hat{i}+\hat{j})$. Let $\hat{a}$ be the unit vector along the direction of vector $(\hat{i}+\hat{j})$.

Then, $$\hat{a}=\frac{\hat{i}+\hat{j}}{|\hat{i}+\hat{j}|}=\frac{\hat{i}+\hat{j}}{\sqrt{1^2+1^2}}=\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

Magnitude of the component vector of $\vec{B}$ along $(\hat{i}+\hat{j})=\vec{B}.\hat{a}=(2\hat{i}+3\hat{j}).\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

$$=\frac{1}{\sqrt{2}}(2+3)=\frac{5}{\sqrt{2}}$$

$\therefore$ Component of $\vec{B}$ along $(\hat{i}+\hat{j})=(\vec{B}.\hat{a})\,\hat{a}=\frac{5}{\sqrt{2}}\left(\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\right)=\frac{5}{2}(\hat{i}+\hat{j})$

Let $\hat{b}$ be the unit vector along the direction of $(\hat{i}-\hat{j})$.

Then, $$\hat{b}=\frac{(\hat{i}-\hat{j})}{|\hat{i}-\hat{j}|}=\frac{\hat{i}-\hat{j}}{\sqrt{(1)^2+(-1)^2}}=\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$$

Similarly component of $\vec{B}$ along $(\hat{i}-\hat{j})$ will be $=(\vec{B}.\hat{b})\,\hat{b}$

$$=[(2\hat{i}+3\hat{j}).\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})]\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})=\frac{1}{\sqrt{2}}(2-3)\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})=-\frac{1}{2}(\hat{i}-\hat{j})$$

Second method :

(*a*) Refer to Fig. 4(N).5(a) ;

Given, $\vec{A} = (\hat{i} + \hat{j}) = (A_x \hat{i} + A_y \hat{j})$

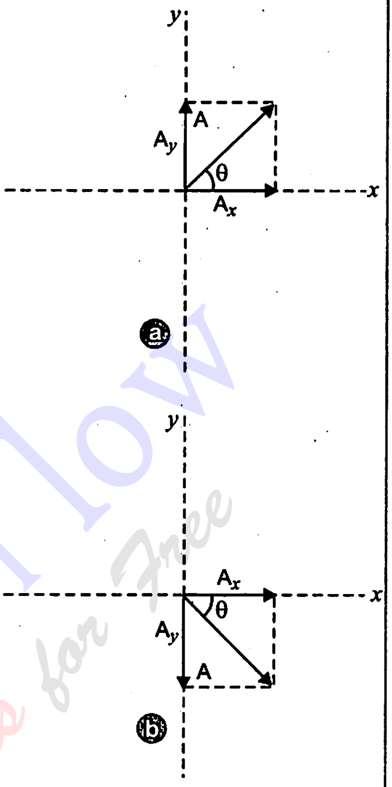
Then, $A_x = 1$ and $A_y = 1$

$\therefore \quad A = \sqrt{A_x^2 + A_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

If θ is the angle which $\vec{A}$ makes with the *x*-axis, then

$$\tan\theta = \frac{A_y}{A_x} = \frac{1}{1} = 1 = \tan 45° \text{ or } \theta = \mathbf{45°}$$

FIGURE 4(N).5



Refer to Fig. 4(N).5(b) ;

If, $\vec{A} = \hat{i} - \hat{j} = A_x \hat{i} + A_y \hat{j}$, then, $A_x = 1$ and $A_y = -1$

Here, $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

If θ is the angle which $\vec{A}$ makes with the *x*-axis.

Then $\tan\theta = \frac{A_y}{A_x} = \frac{-1}{1} = -1 = -\tan\ 45° = \tan(-45°)$

or $\qquad \theta = \mathbf{-45°}$

(*b*) Component of $\vec{B}$ along the direction of $\vec{A}(= \hat{i} + \hat{j})$ is

$$= (\vec{B}.\hat{A})\hat{A} = \left[(2\hat{i} + 3\hat{j}).\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\right]\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

$$= \frac{5}{2}(\hat{i} + \hat{j}) \quad = \mathbf{2{\cdot}5(\hat{i} + \hat{j})}$$

Component of $\vec{B}$ along the direction of $\vec{A}(= \hat{i} - \hat{j})$ is

$$= (\vec{B}.\hat{A})\hat{A} = \left[(2\hat{i} + 3\hat{j}).\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\right]\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right) = \frac{2-3}{\sqrt{2}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right) = \mathbf{-0{\cdot}5(\hat{i} - \hat{j})}$$

Q. 20. For an arbitrary motion in space, which of the following relations are true :

(a) $\vec{v}_{\text{average}} = \frac{1}{2}[\vec{v}(t_1) + \vec{v}(t_2)]$ **(b)** $\vec{v}_{\text{average}} = [\vec{r}(t_2) - \vec{r}(t_1)]/(t_2 - t_1)$

(c) $\vec{v}(t) = \vec{v}(0) + \vec{a}\,t$ **(d)** $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + (1/2)\vec{a}\,t^2$

(e) $\vec{a}_{\text{average}} = [\vec{v}(t_2) - \vec{v}(t_1)]/(t_2 - t_1)$

[The average stands for the average of the quantity over the time interval t_1 and t_2]

Sol. The relations (*b*) and (*e*) are true ; others are false because relations (*a*), (*c*) and (*d*) hold only for uniformly accelerated motion.

Q. 21. Read each statement below carefully and state with reason and examples, if it is true or false. A scalar quantity is one that (a) is conserved in a process (b) can never take negative values (c) must be dimensionless (d) does not vary from one point to another in space (e) has the same value for observers with different orientations of axes.

Sol. (*a*) **False**, because energy (being a scalar quantity) is not conserved during inelastic collisions.

(*b*) **False**, because the temperature (being a scalar quantity) can be negative.

(*c*) **False**, because the density (being a scalar quantity) has dimensions.

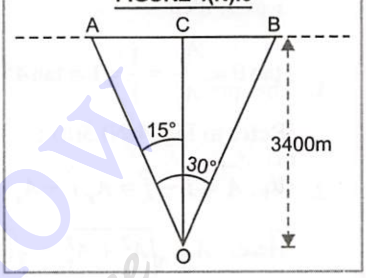
(*d*) **False**, because gravitational potential (being a scalar quantity) vary from point to point in space.

(*e*) **True**, because the value of scalar does not change with orientation of axes.

Q. 22. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft ?

Sol. In Fig 4(N).6, O is the observation point at the ground. A and B are the positions of air craft for which $\angle AOB = 30°$. Draw a perpendicular OC on AB. Here, $OC = 3400$ m and $\angle AOC = \angle COB = 15°$. Time taken by aircraft from A to B is 10 s.

FIGURE 4(N).6



In ΔAOC, $AC = OC \tan 15° = 3400 \times 0{\cdot}2679 = 910{\cdot}86$ m.

$$AB = AC + CB = AC + AC = 2\ AC = 2 \times 910{\cdot}86 \text{ m}$$

$$\text{Speed of the aircraft, } v = \frac{\text{distance } AB}{\text{time}} = \frac{2 \times 910{\cdot}86}{10}$$

$$= 182{\cdot}17 \text{ ms}^{-1} = \mathbf{182{\cdot}2\ ms^{-1}}$$

NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

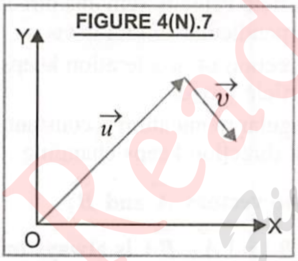
MULTIPLE CHOICE QUESTIONS-I

1. **The angle between $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$ is**
   (*a*) 45° (*b*) 90° (*c*) – 45° (*d*) 180°

2. **Which one of the following statements is true ?**
   (*a*) A scalar quantity is the one that is conserved in a process
   (*b*) A scalar quantity is the one that can never take negative values
   (*c*) A scalar quantity is the one that does not vary from one point to another in space
   (*d*) A scalar quantity has the same value for observers with different orientations of the axes

3. **Figure 4(N).7 shows the orientations of two vectors $\vec{u}$ and $\vec{v}$ in the *XY* plane.**
   If $\vec{u} = a\hat{i} + b\hat{j}$ and $\vec{v} = p\hat{i} + q\hat{j}$ which of the following is correct ?

FIGURE 4(N).7



   (*a*) *a* and *p* are positive while *b* and *q* are negative
   (*b*) *a*, *p* and *b* are positive while *q* is negative
   (*c*) *a*, *q* and *b* are positive while *p* is negative
   (*d*) *a*, *b*, *p* and *q* are all positive

4. **The component of a vector $\vec{r}$ along *X*-axis will have maximum value if**
   (*a*) $\vec{r}$ is along positive *Y*-axis
   (*b*) $\vec{r}$ is along positive *X*-axis
   (*c*) $\vec{r}$ makes an angle of 45° with the *X*-axis
   (*d*) $\vec{r}$ is along negative *Y*-axis

5. **The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be**
   (*a*) 60 m (*b*) 71 m
   (*c*) 100 m (*d*) 141 m

6. **Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are**
   (*a*) Impulse, pressure and area
   (*b*) Impulse and area
   (*c*) Area and gravitational potential
   (*d*) Impulse and pressure

7. **In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true ?**
   (*a*) The average velocity is not zero at any time
   (*b*) Average acceleration must always vanish
   (*c*) Displacements in equal time intervals are equal
   (*d*) Equal path lengths are traversed in equal intervals

8. **In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true ?**
   (*a*) The acceleration of the particle is zero
   (*b*) The acceleration of the particle is bounded
   (*c*) The acceleration of the particle is necessarily in the plane of motion
   (*d*) The particle must be undergoing a uniform circular motion

9. **Three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ add up to zero. Find which is false.**
   (*a*) $(\vec{A} \times \vec{B}) \times \vec{C}$ is not zero unless $\vec{B}, \vec{C}$ are parallel
   (*b*) $(\vec{A} \times \vec{B}) \cdot \vec{C}$ is not zero unless $\vec{B}, \vec{C}$ are parallel

(*c*) If $\vec{A}, \vec{B}, \vec{C}$ define a plane, $(\vec{A}\times\vec{B}\times\vec{C})$ is in that plane

(*d*) $(\vec{A}\times\vec{B})\cdot\vec{C} = |\vec{A}||\vec{B}||\vec{C}| \rightarrow C^2 = A^2 + B^2$

10. It is found that $|\vec{A}+\vec{B}| = |\vec{A}|$. This necessarily implies

(*a*) $\vec{B} = 0$

(*b*) $\vec{A}, \vec{B}$ are antiparallel

(*c*) $\vec{A}, \vec{B}$ are perpendicular

(*d*) $\vec{A}\cdot\vec{B} \le 0$

MULTIPLE CHOICE QUESTIONS-II

11. Two particles are projected in air with speed v_0 at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices

(*a*) angle of projection : $\theta_1 > \theta_2$

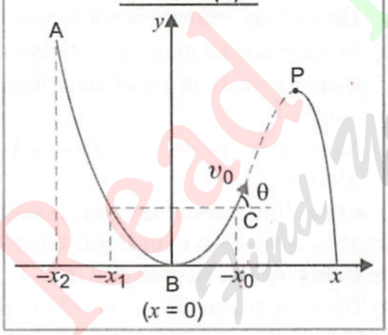
(*b*) time of flight : $T_1 > T_2$

(*c*) horizontal range : $R_1 > R_2$

(*d*) total energy : $U_1 > U_2$.

12. A particle slides down a frictionless parabolic ($y = x^2$) track ($A - B - C$) starting from rest at point A (Fig. 4(N).8). Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then

FIGURE 4(N).8



(*a*) KE at P = KE at B

(*b*) height at P = height at A

(*c*) total energy at P = total energy at A

(*d*) time of travel from A to B = time of travel from B to P.

13. Following are four different relations about displacement, velocity and acceleration for the motion of a particle in general. Choose the incorrect one (s) :

(*a*) $\vec{v}_{av} = \frac{1}{2}[\vec{v}(t_1) + \vec{v}(t_2)]$

(*b*) $\vec{v}_{av} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$

(*c*) $\vec{r} = \frac{1}{2}(\vec{v}(t_2) - \vec{v}(t_1))(t_2 - t_1)$

(*d*) $\vec{a}_{av} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$

14. For a paricle performing uniform circular motion, choose the correct statement(s) from the following :

(*a*) Magnitude of particle velocity (speed) remains constant

(*b*) Particle velocity remains directed perpendicular to radius vector

(*c*) Direction of acceleration keeps changing as particle moves

(*d*) Angular momentum is constant in magnitude but direction keeps changing.

15. For two vectors $\vec{A}$ and $\vec{B}$, $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$ is always true when

(*a*) $|\vec{A}| = |\vec{B}| \neq 0$ (*b*) $\vec{A} \perp \vec{B}$

(*c*) $|\vec{A}| = |\vec{B}| \neq 0$ and $\vec{A}$ and $\vec{B}$ are parallel or antiparallel

(*d*) When either $|\vec{A}|$ or $|\vec{B}|$ is zero.

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*b*)	**7.** (*d*)	**8.** (*c*)
9. (*b, d*)	**10.** (*a, b*)	**11.** (*a, b*)	**12.** (*c*)	**13.** (*a,c*)	**14.** (*a, b, c*)	**15.** (*b, d*)	

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{(\hat{i}+\hat{j})\cdot(\hat{i}-\hat{j})}{\sqrt{1^2+1^2}\times\sqrt{1^2+(-1)^2}} = \dfrac{1-1}{2} = 0 = \cos 90^\circ \quad \therefore \quad \boldsymbol{\theta = 90^\circ}$

2. A scalar quantity has the same value for observers with different orientations of the axes.

3. As per fig, in $\vec{u} = a\,\hat{i} + b\,\hat{j}$, both a and b are positive. In $\vec{v} = p\,\hat{i} + q\,\hat{j}$, p is positive and q is negative. Thus a, b and p are positive and q is negative.

4. If $\vec{r}$ makes an angle θ with x-axis, then component of $\vec{r}$ along x-axis $= r\cos\theta$.

It will be maximum if $\cos\theta = \max = 1$ or $\theta = 0^\circ$ *i.e.*, $\vec{r}$ is along positive x-axis.

5. $50 = \dfrac{u^2 \sin 2\times 15^\circ}{g}$ or $\dfrac{u^2}{g} = \dfrac{50}{\sin 30^\circ} = \dfrac{50}{1/2} = 100$

$$R = \frac{u^2 \sin 2\times 45^\circ}{g} = \frac{u^2}{g} = \mathbf{100\,m}$$

6. Out of the given quantities impulse and area are vector quantities others are scalar quantities.

7. In two dimensional motion, if instantaneous speed is a positive constant then equal path lengths are traversed in equal intervals of time.

8. In two dimensional motion, if instantaneous speed is a positive constant, then the acceleration of the particle is necessarily in the plane of motion.

9. Given, $\vec{A}+\vec{B}+\vec{C}=0$, then $\vec{A}$, $\vec{B}$ and $\vec{C}$ are in one plane and are represented by the three sides of a triangle taken in one order.

(i) $\therefore \quad \vec{B}\times(\vec{A}+\vec{B}+\vec{C}) = \vec{B}\times 0 = 0 \quad$ or $\quad \vec{B}\times\vec{A}+\vec{B}\times\vec{B}+\vec{B}\times\vec{C}=0$

or $\quad \vec{B}\times\vec{A}+0+\vec{B}\times\vec{C}=0 \quad$ or $\quad \vec{A}\times\vec{B}=\vec{B}\times\vec{C}$ ...(*i*)

$\therefore \quad (\vec{A}\times\vec{B})\times\vec{C} = (\vec{B}\times\vec{C})\times\vec{C}$; It cannot be zero.

If $\vec{B}\parallel\vec{C}$, then $\vec{B}\times\vec{C}=0$, then $(\vec{B}\times\vec{C})\times\vec{C}=0$

Thus, option (*a*) is correct.

(ii) $(\vec{A}\times\vec{B})\cdot\vec{C} = (\vec{B}\times\vec{C})\cdot\vec{C} = 0$ whatever be the positions of $\vec{A}$, $\vec{B}$ and $\vec{C}$

If $\vec{B}\parallel\vec{C}$, then $\vec{B}\times\vec{C}=0$, then $(\vec{B}\times\vec{C})\times\vec{C}=0$

Thus, **option (*b*) is false**

(iii) $(\vec{A}\times\vec{B})=\vec{D}=AB\sin\theta\,\hat{D}$ .The direction of $\hat{D}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

$(\vec{A}\times\vec{B})\times\vec{C}=\vec{D}\times\vec{C}$. Its direction is in the plane of $\vec{A}$, $\vec{B}$ and $\vec{C}$. Thus, option (*c*) is correct.

(iv) If $C^2 = A^2 + B^2$, then angle between $\vec{A}$ and $\vec{B}$ is 90°.

$\therefore \; (\vec{A}\times\vec{B})\cdot\vec{C} = (AB\sin 90^\circ\,\hat{D})\cdot\vec{C} = AB(\hat{D}\cdot\vec{C}) = ABC\cos 90^\circ = 0$. Thus, **option (*d*) is false**

10. If $|\vec{A}+\vec{B}| = |\vec{A}|$, then either $|\vec{B}|=0$ or $\vec{A}$ and $\vec{B}$ will be antiparallel, where $|\vec{B}| = 2|\vec{A}|$.

Multiple Choice Questions – II

11. Height, $h = \dfrac{v_0^2 \sin^2\theta}{2g}$, *i.e.*, $h \propto \sin^2\theta$

$$\therefore \qquad \frac{h_1}{h_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} > 1 \qquad \text{so} \qquad \sin^2\theta_1 > \sin^2\theta_2 \quad \text{or} \quad \boldsymbol{\theta_1 > \theta_2}$$

Time of flight, $T = \dfrac{2 v_0 \sin\theta}{g}$ or $T \propto \sin\theta$ $\quad\therefore\quad \dfrac{T_1}{T_2} = \dfrac{\sin\theta_1}{\sin\theta_2} > 1$ or $\boldsymbol{T_1 > T_2}$

Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$ or $R \propto \sin 2\theta$ $\quad\therefore\quad \dfrac{R_1}{R_2} = \dfrac{\sin 2\theta_1}{\sin 2\theta_2} \le 1$ or $R_1 \le R_2$

Total energy of each particle will be equal to KE of each particle at the time of its projection.

12. Since $y = x^2$, the motion is in two dimensions as shown in figure. Velocity at B is greater than at P. In the given motion of a particle, the law of conservation of energy is obeyed. Therefore, **total energy at P = total energy at A.** As vertical distance $AB > BP$, time of travel from A to B is greater than that from B to P.

13. For the motion of a particle if acceleration is not uniform then the relations

$$\vec{v}_{av} = \frac{1}{2}[\vec{v}(t_1) + \vec{v}(t_2)] \text{ is incorrect.}$$

$$\vec{v}_{av} = \frac{\vec{r}(t_2) - \vec{r}_1(t_1)}{(t_2 - t_1)} \text{ is correct.}$$

$$\vec{r} = \frac{1}{2}[\vec{v}(t_2) - \vec{v}(t_1)]\,(t_2 - t_1) \text{ is incorrect.} \qquad \text{and} \qquad \vec{a}_{av} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1} \text{ is correct.}$$

14. Knowledge based question.

15. When $\vec{A}$ is perpendicular to $\vec{B}$, then $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ} = \sqrt{A^2 + B^2}$

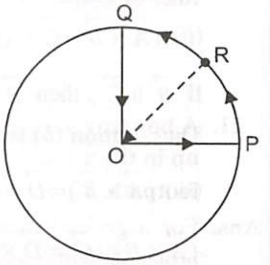
$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 90^\circ} = \sqrt{A^2 + B^2}$$

When $|\vec{A}| = 0$ or $|\vec{B}| = 0$ then $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

VERY SHORT ANSWER QUESTIONS

16. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path *OPRQO* as shown Fig. 4(N).9. If he maintains constant speed of 10 ms^{-1}, what is his acceleration at point *R* in magnitude and direction ?

FIGURE 4(N).9



Ans. Centripetal acceleration, $a_c = \dfrac{v^2}{r} = \dfrac{10^2}{1000} = \mathbf{0{\cdot}1\ m/s^2}$ **along RO**

17. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig. 4(N).10, where *X* and *Y* indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points *A*, *B* and *C*.

Ans. Let u be the velocity of projection at O and θ be the angle of projection. Let the particle reach at A after time t_1, at B after time t_2 and at C after time t_3.

At A, $\quad v_{xA} = u\cos\theta,\ v_{yA} = u\sin\theta - g t_1$

$$v_A = \sqrt{v_{xA}^2 + v_{yA}^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - g t_1)^2}$$

If $\vec{v}_A$ makes an angle β_1 with the horizontal direction (*i.e.*, *x*-axis) then

$$\tan\beta_1 = \frac{v_{yA}}{v_{xA}} = \frac{u\sin\theta - gt_1}{u\cos\theta}$$

Acceleration at A, $a_A = -g$

B is the highest point of trajectory of projectile. Then

At B, $v_{xB} = u\cos\theta,\ v_{yB} = 0$

$$\therefore \quad v_B = \sqrt{v_{xB}^2 + v_{yB}^2} = \sqrt{(u\cos\theta)^2 + 0^2} = u\cos\theta$$

$$\tan\beta_2 = \frac{v_{yB}}{v_{xB}} = \frac{0}{u\cos\theta} = 0\ ;\ \text{so } \beta_2 = 0$$

Acceleration $a_B = -g$.

At C, $v_{xC} = u\cos\theta,\ v_{yC} = u\sin\theta - gt_3$

$$\therefore \quad v_C = \sqrt{v_{xC}^2 + v_{yC}^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt_3)^2}$$

$$\tan\beta_3 = \frac{v_{yC}}{v_{xC}} = \frac{u\sin\theta - gt_3}{u\cos\theta}$$

Acceleration, $a_c = -g$.

FIGURE 4(N).10

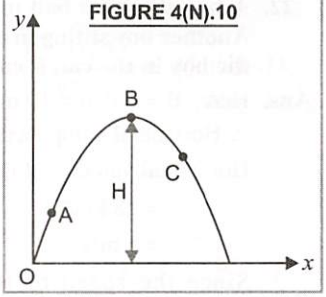
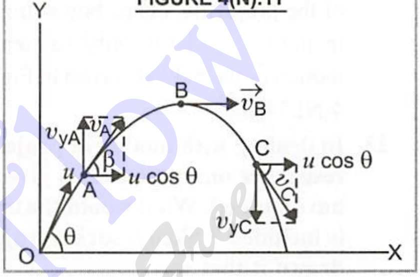


FIGURE 4(N).11



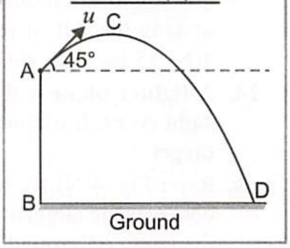
18. A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have

(*a*) greatest speed (*b*) smallest speed (*c*) greatest acceleration ? Explain

Ans. Refer Fig. 2(N).12, (*a*) at *D*, just before it hits the ground

(*b*) at *C*, the highest point reached (*c*) $a = g = a$ constant.

FIGURE 4(N).12



19. A football is kicked into the air vertically upwards. What is its (*a*) acceleration, and (*b*) velocity at the highest point ?

Ans. At highest point, acceleration = $-g$ and velocity = zero.

20. $\vec{A}$, $\vec{B}$ **and** $\vec{C}$ **are three non-collinear, non co-planar vectors. What can you say about direction of** $\vec{A}\times(\vec{B}\times\vec{C})$ **?**

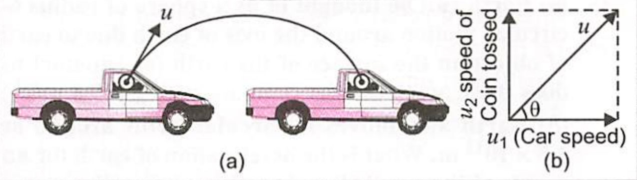
Ans. $(\vec{B}\times\vec{C})$ will act perpendicular to the plane containing $\vec{B}$ and $\vec{C}$ and is directed as given by right hand rule. $\vec{A}\times(\vec{B}\times\vec{C})$ will lie in the plane of $\vec{B}$ and $\vec{C}$ and is perpendicular to vector $\vec{A}$.

SHORT ANSWER QUESTIONS

21. A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.

Ans. For a ground observer, the ball is a projectile with speed *u* making an angle of projection θ with the horizontal direction as shown in Fig. 4(N).13(*b*). The ball will follow a parabolic path. Fig. 4(N).13(*a*)

FIGURE 4(N).13



(a) (b)

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22. A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10 m/s (36 km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18 km/h). Give explanation to support your diagram.

Ans. Here, $\theta = 60°$, $u = 10$ m/s

$\therefore$ Horizontal component velocity of ball $= u \cos\theta = 10 \cos 60° = 10 \times ½ = 5$ m/s

Horizontal velocity of car

$= 18$ km/h

$= 5$ m/s

Since the speed of the car is matching with the horizontal speed of the projectile, hence boy sitting in the car will see only vertical motion of the ball, as shown in Fig. 4(N).14(*b*).

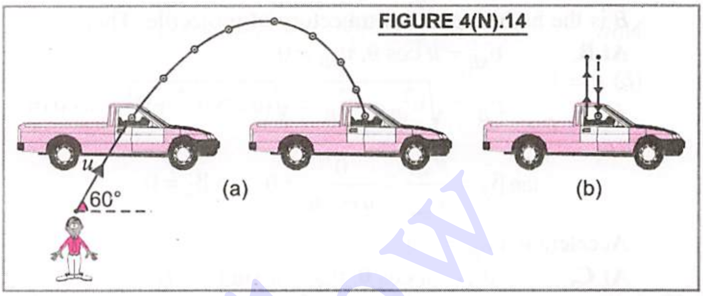


FIGURE 4(N).14

23. In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included ? Sketch such a trajectory and explain why you have drawn it that way.

Ans. Due to air resistance, the total energy of particle as well as its horizontal component velocity will go on decreasing with time, making the fall of projectile steeper than its rise, as shown in Fig. 4(N).15 by thick solid curve.

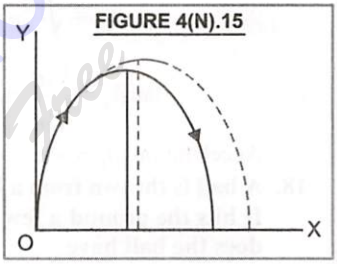


FIGURE 4(N).15

24. A fighter plane is flying horizontally at an altitude of 1·5 km with speed 720 km/h. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target ?

Ans. Refer Fig. 4(N).16, when the fighter plane is at *A* moving with horizontal speed *u* (=720 km/h = 200 m/s) observes the target at *C*. Let $\angle CAB = \theta$. He releases a bomb at *A* with horizontal velocity 200 m/s and it hits the target on ground after time *t*.

Taking horizontal motion of fighter plane from *A* to *B*, where *B* is vertically above the target, we have

$$AB = ut = 200 \times t \qquad \ldots(i)$$

Taking vertical downward motion of bomb from *A* to *C*, we have

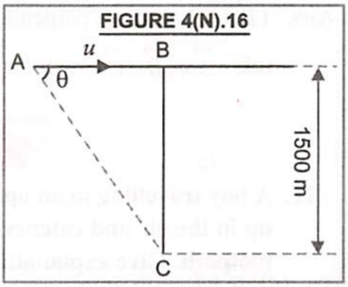


FIGURE 4(N).16

$$BC = \frac{1}{2} \times 9{\cdot}8 \times t^2 = 4{\cdot}9\,t^2$$

or $\quad 1500 = 4{\cdot}9\,t^2 \quad$ or $\quad t = \sqrt{\frac{1500}{4{\cdot}9}} = 17{\cdot}49\,\text{s}$

$\therefore \quad AB = 200 \times 17{\cdot}49$ m

$$\tan\theta = \frac{BC}{AB} = \frac{1500}{200 \times 17{\cdot}49} = 0{\cdot}4287 = \tan 23°\,12'$$

$\therefore \quad \theta = 23°\,12'$

25. (*a*) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of earth due to earth's rotation (period 1 day). What is acceleration of object on the surface of the earth (at equator) towards its centre ? What is its latitude θ ? How does these accelerations compare with $g = 9{\cdot}8$ m/s^2 ?

(*b*) Earth also moves in circular orbit around sun once every year with on orbital radius of $1{\cdot}5 \times 10^{11}$ m. What is the acceleration of earth (or any object on the surface of the earth) towards the centre of the sun ? How does this acceleration compare with $g = 9{\cdot}8$ m/s^2 ?

Ans. (*a*) Here, $R = 6400 \text{ km} = 6{\cdot}4 \times 10^6 \text{ m}, T = 24 \times 60 \times 60 \text{ s}$

$$a_c = R\omega^2 = \frac{R4\pi^2}{T^2} = \frac{(6{\cdot}4\times10^6)\times4\times(22/7)^2}{(24\times60\times60)^2} = 0{\cdot}034 \text{ m/s}^2$$

At latitude θ (=0°), the value of R remains the same, hence no change in the value of a_c (= 0·034 m/s^2)

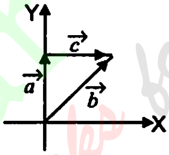
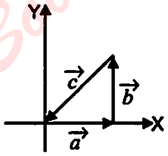
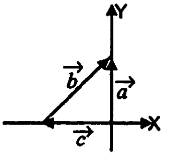
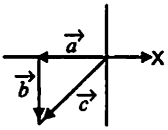
Now, $\frac{a_c}{g} = \frac{0{\cdot}034}{9{\cdot}8} = \frac{1}{288}$ which is much smaller than 1

(*b*) $R = 1{\cdot}5 \times 10^{11}$ m, $T = 365 \times 24 \times 60 \times 60 \text{ s} = 3{\cdot}15 \times 10^7$ s

$$a_c = \frac{R4\pi^2}{T^2} = \frac{(1{\cdot}5\times10^{11})\times4\times(3{\cdot}142)^2}{(3{\cdot}15\times10^7)^2} = \mathbf{5{\cdot}97 \times 10^{-3} \text{ m/s}^2}$$

$$\frac{a_c}{g} = \frac{5{\cdot}97\times10^{-3}}{9{\cdot}8} = \frac{1}{1642}$$

26. Given below in column I are the relations between vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and in column II are the orientations of $\vec{a}, \vec{b}$ and $\vec{c}$ in the *XY* plane. Match the relation in column I to correct orientations in column II.

	Column I		**Column II**
(*a*)	$\vec{a} + \vec{b} = \vec{c}$	(*i*)	
(*b*)	$\vec{a} - \vec{c} = \vec{b}$	(*ii*)	
(*c*)	$\vec{b} - \vec{a} = \vec{c}$	(*iii*)	
(*d*)	$\vec{a} + \vec{b} + \vec{c} = 0$	(*iv*)	

Ans. (*a*) $\vec{a} + \vec{b} = \vec{c}$, it matches with option (*iv*).

(*b*) $\vec{a} - \vec{c} = \vec{b}$, it matches with option (*iii*).

(*c*) $\vec{b} - \vec{a} = \vec{c}$, it matches with option (*i*).

(*d*) $\vec{A} + \vec{B} + \vec{C} = 0$, it matches with option (*ii*).

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27. If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, then match the relations in column I with the angle θ between $\vec{A}$ and $\vec{B}$ in column II.

	Column I		Column II
(a)	$\vec{A}.\vec{B} = 0$	(i)	$\theta = 0$
(b)	$\vec{A}.\vec{B} = +8$	(ii)	$\theta = 90°$
(c)	$\vec{A}.\vec{B} = 4$	(iii)	$\theta = 180°$
(d)	$\vec{A}.\vec{B} = -8$	(iv)	$\theta = 60°$

Ans. Given $|\vec{A}| = 2, |\vec{B}| = 4$

(a) $\vec{A}\cdot\vec{B} = 0 = AB\cos\theta = 2\times 4\cos\theta$

or $0 = 8\cos\theta$ or $\cos\theta = 0$ or $\theta = 90°$. It matches with option (ii).

(b) $\vec{A}\cdot\vec{B} = 8 = AB\cos\theta = 2\times 4\times\cos\theta$ or $\cos\theta = 1, \theta = 0°$. It matches with option (i).

(c) $\vec{A}\cdot\vec{B} = 4 = AB\cos\theta = 2\times 4\times\cos\theta$ or $\cos\theta = \frac{1}{2}$ or $\theta = 60°$. It matches with option (iv).

(d) $\vec{A}\cdot\vec{B} = -8 = 2\times 4\cos\theta$ or $\cos\theta = -1$ or $\theta = 180°$. It matches with option (iii).

28. If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, then match the relations in column I with the angle θ between $\vec{A}$ and $\vec{B}$ in column II.

	Column I		Column II
(a)	$|\vec{A}\times\vec{B}| = 0$	(i)	$\theta = 30°$
(b)	$|\vec{A}\times\vec{B}| = 8$	(ii)	$\theta = 45°$
(c)	$|\vec{A}\times\vec{B}| = 4$	(iii)	$\theta = 90°$
(d)	$|\vec{A}\times\vec{B}| = 4\sqrt{2}$	(iv)	$\theta = 0°$

Ans. Here $A = 2$ and $B = 4$.

(a) $|\vec{A}\times\vec{B}| = 0 = AB\sin\theta = 2\times 4\sin\theta$ or $\sin\theta = 0$ and $\theta = 0°$. It matches with option (iv).

(b) $|\vec{A}\times\vec{B}| = 8 = AB\sin\theta = 2\times 4\sin\theta$ or $\sin\theta = 1$ and $\theta = 90°$. It matches with option (iii).

(c) $|\vec{A}\times\vec{B}| = 4 = 2\times 4\sin\theta$ or $\sin\theta = \frac{1}{2}$ or $\theta = 30°$. It matches with option (i).

(d) $|\vec{A}\times\vec{B}| = 4\sqrt{2} = 2\times 4\sin\theta$ or $\sin\theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$. It matches with option (ii).

LONG ANSWER QUESTIONS

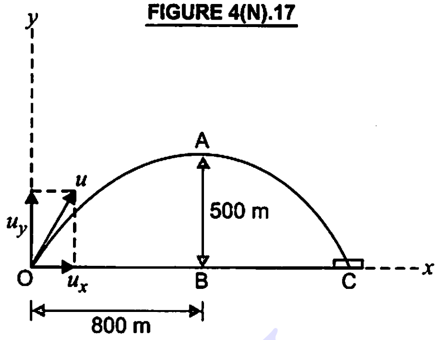
29. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 m/s ; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill ? Take $g = 10\ m/s^2$.

Ans. The initial minimum vertical velocity (u_y) required to just cross the hill is

$$u_y \geq \sqrt{2gh} \geq \sqrt{2\times 10\times 500} \geq 100\,\text{m/s}.$$

The initial horizontal velocity, $u_x = \sqrt{u^2 - u_y^2} = \sqrt{(125)^2 - (100)^2} = 75$ m/s

FIGURE 4(N).17



Time taken to reach the top of the hill,

$$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times500}{10}}=10\text{ s}$$

Horizontal distance covered in 10 s $= u_x \times t$

$$= 75 \times 10 = 750 \text{ m}$$

So, canon has to be moved through a distance

$$= 800 - 750 = 50 \text{ m}$$

Time taken to move the canon through 50 m on ground

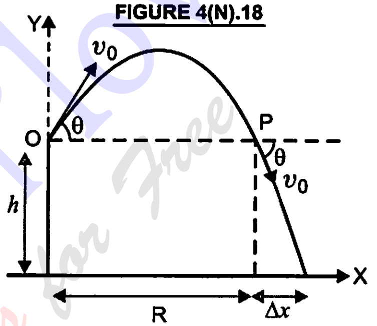
$$=\frac{50}{2}=25\text{s}$$

Total time taken by packet to reach ground the hill = 25 + 10 + 10 = 45 s

30. A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = v_0^2/g$. If a target farther away by distance Δx (beyond R) has to be hit with the same gun (Fig. 4(N).18), show that it could be achieved by raising the gun to a height at least

$$h=\Delta x\left[1+\frac{\Delta x}{R}\right].$$

FIGURE 4(N).18



Ans. As per question, the maximum horizontal range on ground is

$$R_{\max}=\frac{v_0^2}{g}$$

It is so if θ = 45°. The shell fired at O from height h can hit the same target at B. Fig. 4(N).19

Taking vertical downward motion from O to B, we have

$u_y = -v_0 \sin\theta,\ a_y = g,\ y_0 = 0,\ y = h,\ t = ?$

As, $\quad y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$$\therefore \quad h = 0 + (-v_0 \sin\theta)t + \frac{1}{2}gt^2 \qquad \text{...}(i)$$

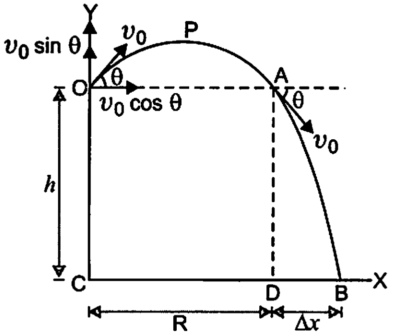
Taking horizontal motion from O to B, we have

$u_x = v_0 \cos\theta,\ a_x = 0,\ x_0 = 0,\ x = (R + \Delta x),\ t = t$

As, $\quad x = x_0 + u_x t + \frac{1}{2} a_x t^2$

$$\therefore \quad (R + \Delta x) = 0 + v_0 \cos\theta\, t \quad \text{or} \quad t=\frac{(R+\Delta x)}{v_0\cos\theta}$$

FIGURE 4(N).19



Putting value of (t) in (i), we have

$$h=-v_0\sin\theta\times\frac{R+\Delta x}{v_0\cos\theta}+\frac{1}{2}g\frac{(R+\Delta x)^2}{v_0^2\cos^2\theta}=-(R+\Delta x)\tan\theta+\frac{1}{2}g\frac{(R+\Delta x)^2}{v_0^2\cos^2\theta}$$

As $\quad \theta = 45°$, so $\quad h=-(R+\Delta x)\tan 45°+\frac{1}{2}\frac{g(R+\Delta x)^2}{v_0^2\cos^2 45°}$

or $\quad h=-(R+\Delta x)\times 1+\frac{1}{2}\frac{g(R+\Delta x)^2}{v_0^2}\times 2=-(R+\Delta x)+\frac{1}{R}\times(R+\Delta x)^2 \qquad \left[\because v_0^2/g=R\right]$

$$=-(R+\Delta x)+R+2\Delta x+\Delta x^2/R=\Delta x+\Delta x^2/R=\Delta x\,(1+\Delta x/R)$$

31. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal (Fig. 4(N).20).

(*a*) Find an expression for range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).

(*b*) Time of flight.

(*c*) β at which range will be maximum.

FIGURE 4(N).20

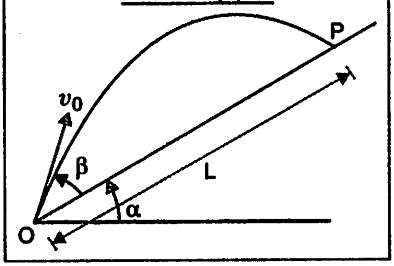
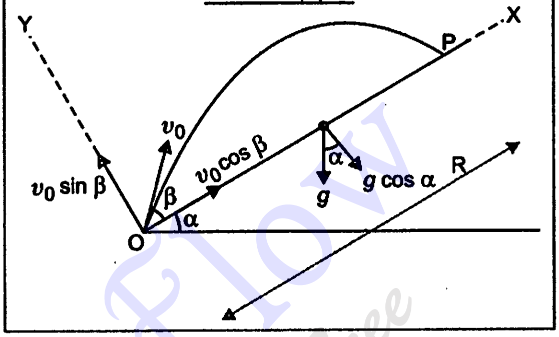


FIGURE 4(N).21



Ans. Let the particle go from *O* to *P* in time *T*. Fig. 4(N).21.

(*b*) Taking vertical upward motion of particle perpendicular to plane *OX*, from *O* to *P*, we have

$$y_0 = 0,\ y = 0,\ u_y = v_0 \sin\beta,\ a_y = (-g\cos\alpha),\ t = T$$

As $$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$$\therefore \quad 0 = 0 + (v_0 \sin\beta)\, T + \frac{1}{2}(-g\cos\alpha)\, T^2 \quad \text{or} \quad T = \frac{2v_0 \sin\beta}{g\cos\alpha}$$

(*a*) For horizontal range *OP* (= *R*).

Taking motion of particle from *O* to *P* along *OX* direction.

Here, $x_0 = 0,\ x = R,\ u_x = v_0\cos\beta,\ a_x = -g\sin\alpha,\ t = T = \dfrac{2v_0\sin\beta}{g\cos\alpha}$

As, $$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

$$R = 0 + v_0\cos\beta \times \frac{2v_0\sin\beta}{g\cos\alpha} + \frac{1}{2}(-g\sin\alpha)\left(\frac{2v_0\sin\beta}{g\cos\alpha}\right)^2$$

$$= \frac{2v_0^2}{g\cos\alpha}\sin\beta\cos\beta - \frac{g\sin\alpha}{2} \times \frac{4v_0^2\sin^2\beta}{g^2\cos^2\alpha} = \frac{2v_0^2}{g\cos^2\alpha}[\sin\beta\cos\beta\cos\alpha - \sin\alpha\sin^2\beta]$$

$$= \frac{2v_0^2\sin\beta}{g\cos^2\alpha}[\cos\beta\cos\alpha - \sin\alpha\sin\beta] = \frac{2v_0^2\sin\beta}{g\cos^2\alpha}\cos[\alpha+\beta]$$

(*c*) For maximum horizontal range

$$R = \frac{v_0^2}{g\cos^2\alpha}[\sin 2\beta\cos\alpha - 2\sin\alpha\sin^2\beta] = \frac{v_0^2}{g\cos^2\alpha}[\sin 2\beta\cos\alpha - \sin\alpha(1-\cos 2\beta)]$$

$$= \frac{v_0^2}{g\cos^2\alpha}[\sin 2\beta\cos\alpha - \sin\alpha + \sin\alpha\cos 2\beta] = \frac{v_0^2}{g\cos^2\alpha}[\sin(2\beta+\alpha) - \sin\alpha]$$

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R is maximum if $\sin(2\beta+\alpha)=1$ or $2\beta+\alpha=\pi/2$ or $\beta=\dfrac{(\pi/2)-\alpha}{2}=\dfrac{\pi}{4}-\dfrac{\alpha}{2}$

32. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_0 and rebounds elastically (Fig. 4(N).22). Find the distance along the plane where it will hit second time.

FIGURE 4(N).22

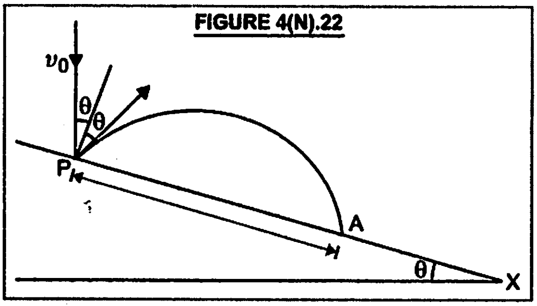
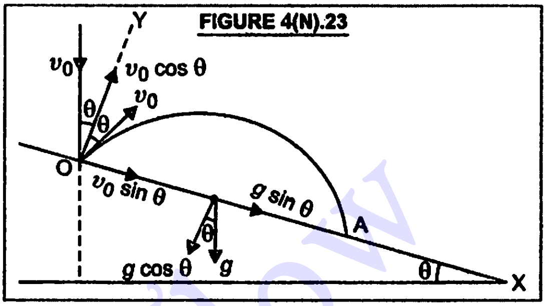


FIGURE 4(N).23



Ans. Refer Fig. 4(N).23. Taking motion of particle from O to A along Y-axis (*i.e.*, ⊥r to plane OA upwards), we have $y=0,\ y_0=0,\ u_y=v_0\cos\theta,\ a_y=-g\cos\theta,\ t=T$

As, $y=y_0+u_y t+\frac{1}{2}a_y t^2$, we have $0=0+v_0\cos\theta T+\frac{1}{2}(-g\cos\theta)T^2$

On solving, $T=\dfrac{2v_0\cos\theta}{g\cos\theta}=\dfrac{2v_0}{g}$

Taking motion of particle along X-axis (*i.e.*, along the plane from O to A), we have

$$x_0=0,\quad x=R,\quad u_x=v_0\sin\theta,\quad a_x=g\sin\theta,\quad t=T=\frac{2v_0}{g}$$

As, $$x=x_0+u_x t+\frac{1}{2}a_x t^2$$

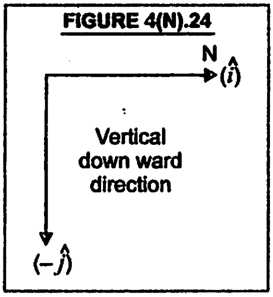
$\therefore$ $$R=0+v_0\sin\theta\times\frac{2v_0}{g}+\frac{1}{2}(g\sin\theta)\times\left(\frac{2v_0}{g}\right)^2=\frac{2v_0^2}{g}\sin\theta+\frac{1}{2}g\sin\theta\times\frac{4v_0^2}{g^2}$$

$$=\frac{2v_0^2}{g}[\sin\theta+\sin\theta]=\frac{4v_0^2}{g}\sin\theta$$

33. A girl riding a bicycle with a speed of 5 m/s towards North direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain ? In what direction does rain fall as observed by a ground based observer ?

Ans. Refer Fig. 4(N).24,

FIGURE 4(N).24



Let velocity of rain be, $\vec{v}_r=a\,\hat{i}+b\,\hat{j}$ ...(*i*)

Ist case : Velocity of girl, $v_g=(5\,\text{ms}^{-1})\,\hat{i}$

Velocity of rain w.r.t. girl, $\vec{v}_{rg}=\vec{v}_r-\vec{v}_g=(a\,\hat{i}+b\,\hat{j})-5\,\hat{i}=(a-5)\,\hat{i}+b\,\hat{j}$

Since, rain appears to fall vertically downwards, so $(a-5)=0$ or $a=5$.

2nd case : $\vec{v}_g=(10\,\text{ms}^{-1})\,\hat{i}$

$\therefore$ $$\vec{v}_{rg}=\vec{v}_r-10\,\hat{i}=(a\,\hat{i}+b\,\hat{j})-10\,\hat{i}=(a-10)\,\hat{i}+b\,\hat{j}$$

Since, rain appears to fall at 45° to the vertical, therefore, $\tan 45°=\dfrac{b}{a-10}=\dfrac{b}{5-10}=\dfrac{b}{-5}$

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or $$b = -5 \tan 45° = -5 \times 1 = -5$$

Hence, $\vec{v}_r = 5\hat{i} - 5\hat{j}$ and $|\vec{v}_r| = \sqrt{(5)^2 + (-5)^2} = 5\sqrt{2}\,\text{ms}^{-1}$

34. A river is flowing due east with a speed 3 m/s. A swimmer can swim in still water at a speed of 4 m/s (Fig. 4(N).25).

(*a*) If swimmer starts swimming due North, what will be his resultant velocity (magnitude and direction) ?

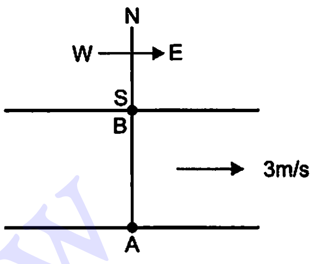
(*b*) If he wants to start from point *A* on South bank and reach opposite point *B* on North bank,

(*i*) Which direction should he swim ?

(*ii*) What will be his resultant speed ?

(*c*) From two different cases as mentioned in (*a*) and (*b*) above, in which case will he reach opposite bank in shorter time ?

FIGURE 4(N).25



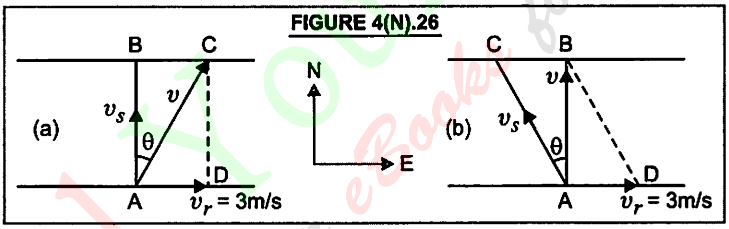
Ans. (*a*) Refer Fig. 4(N).26(a), $v_r = 3$ m/s along *AD*
$v_s = 4$ m/s along *AB*.

Resultant velocity $v = \sqrt{v_r^2 + v_s^2} = \sqrt{3^2 + 4^2} = 5\,\text{m/s}$

$$\tan\theta = \frac{v_r}{v_s} = \frac{3}{4} = 0{\cdot}75 = \tan 36° 54' \approx \tan 37°$$

$\theta = 37°$ with north towards the direction of river flow.

FIGURE 4(N).26



(*b*) Refer Fig. 4(N).26(b), the swimmer will go along *AB* if he goes along *AC*, so that resultant of river velocity and swimmer velocity may act along *AB*.

Velocity along *AB*, $v = \sqrt{v_s^2 - v_r^2} = \sqrt{4^3 - 3^2} = \sqrt{7}$ m/s

Let $\angle BAC = \theta$, then $\tan\theta = \frac{v_r}{v} = \frac{3}{\sqrt{7}}$ or $\theta = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$ **West of North**

(*c*) In case (*a*), time taken to cross the river, $t_1 = \frac{d}{v_s} = \frac{d}{4}$

In case (*b*), time taken to cross the river, $t_2 = \frac{d}{v} = \frac{d}{\sqrt{7}}$

As $\frac{d}{4} < \frac{d}{\sqrt{7}}$, so $t_1 < t_2$, therefore the swimmer will cross the river in shorter time in case (*a*).

35. A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed *u* at angle θ to the horizontal, find

(*a*) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.

(*b*) what will be time of flight ?

(*c*) what is the distance (horizontal range) from the point of projection at which the ball will land ?

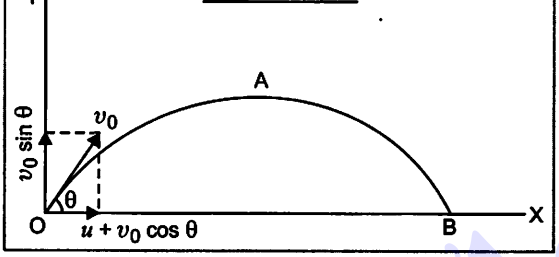
(*d*) find θ at which he should throw the ball that would maximise the horizontal range as found in (*c*).

(*e*) how does θ for maximum range change if $u > v_0$, $u = v_0$, $u < v_0$?

(*f*) how does θ in (*e*) compare with that for $u = 0$ (*i.e.*, 45°) ?

Ans. Refer Fig. 4(N).27,

FIGURE 4(N).27



(*a*) Here, at *O*, $u_x = u + v_0 \cos\theta$

$$u_y = v_0 \sin\theta$$

$$\tan\theta = \frac{u_y}{u_x} = \frac{v_0 \sin\theta}{u + v_0 \cos\theta}$$

or
$$\theta = \tan^{-1}\left(\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right)$$

(*b*) Let *T* be the time of flight of cricket ball from *O* to *B*, then taking vertical motion of ball from *O* to *B*, we have $y_0 = 0$, $y = 0$, $u_y = v_0 \sin\theta$, $a_y = -g$, $t = T$

As,
$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

∴
$$0 = 0 + v_0 \sin\theta T + \frac{1}{2}(-g)T^2 \quad \text{or} \quad T = \frac{2v_0 \sin\theta}{g}$$

(*c*) Horizontal range, $R = (u + v_0 \cos\theta)\, T = (u + v_0 \cos\theta)\dfrac{2v_0 \sin\theta}{g} = \dfrac{v_0}{g}[2u\sin\theta + v_0 \sin 2\theta]$

(*d*) Horizontal range will be maximum if $\dfrac{dR}{d\theta} = 0$

∴
$$\frac{dR}{d\theta} = \frac{v_0}{g}[2u\cos\theta + v_0 \cos 2\theta \times 2] = 0$$

or
$$2u\cos\theta + 2v_0[2\cos^2\theta - 1] = 0$$

or
$$4v_0\cos^2\theta + 2u\cos\theta - 2v_0 = 0$$

or
$$\cos\theta = \frac{-2u \pm \sqrt{4u^2 + 32v_0^2}}{8v_0} = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$$

or
$$\theta_{\max} = \cos^{-1}\left[\frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}\right]$$

(*e*) (*i*) If $u = v_0$, then $\cos\theta = \dfrac{-v_0 \pm \sqrt{v_0^2 + 8v_0^2}}{4v_0} = \dfrac{-1+3}{4} = \dfrac{1}{2} = \cos 60°$ or $\theta = 60°$

(*ii*) If $u < v_0$, then $8v_0^2 + u^2 \approx 8v_0^2$

$$\theta_{\max} = \cos^{-1}\left[\frac{-u \pm 2\sqrt{2}\,v_0}{4v_0}\right] = \cos^{-1}\left[\frac{1}{\sqrt{2}} - \frac{u}{4v_0}\right]$$

If $u << v_0$ then $\theta_{\max} = \cos^{-1}\left[\dfrac{1}{\sqrt{2}}\right] = \dfrac{\pi}{4}$

(*iii*) If $u > v$ and $u >> v_0$

$$\theta_{\max} = \cos^{-1}\left[\frac{-u \pm u}{4v_0}\right] = \cos^{-1}(0) = \frac{\pi}{2}$$

(*f*) If $u = 0$, $\theta_{\max} = \cos^{-1}\left[\dfrac{0 \pm \sqrt{8v_0^2}}{4v_0}\right] = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

36. Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in Cartesian co-ordinates $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$ where $\hat{i}$ and $\hat{j}$ are unit vector along x and y directions, respectively and A_x and A_y are corresponding components of $\vec{A}$ (Fig. 4(N).28). Motion can also be studied by expressing vectors in circular polar co-ordinates as $\vec{A} = A_r\,\hat{r} + A_\theta\,\hat{\theta}$ where $\hat{r} = \dfrac{\vec{r}}{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ and $\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ are unit vectors along direction in which 'r' and 'θ' are increasing.

(*a*) Express $\hat{i}$ and $\hat{j}$ in terms of $\hat{r}$ and $\hat{\theta}$

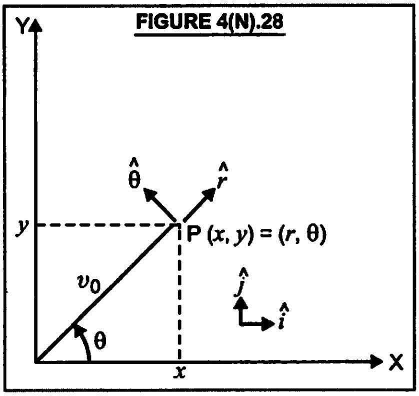
(*b*) Show that both $\hat{r}$ and $\hat{\theta}$ are unit vectors and are perpendicular to each other.

(*c*) Show that $\dfrac{d}{dt}(\hat{r}) = \omega\hat{\theta}$ where $\omega = \dfrac{d\theta}{dt}$ and $\dfrac{d}{dt}(\hat{\theta}) = -\omega\hat{r}$

(*d*) For a particle moving along a spiral given by $\vec{r} = \alpha\theta\,\hat{r}$, where $\alpha = 1$ (unit), find dimensions of 'α'.

(*e*) Find velocity and acceleration in polar vector representation for particle moving along spiral described in (*d*) above.

FIGURE 4(N).28



Ans. (*a*) $\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ ...(*i*)

$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ ...(*ii*)

Multiplying (*i*) by sin θ and (*ii*) with cos θ and adding, we get

$$\hat{r}\sin\theta + \hat{\theta}\cos\theta = (\sin^2\theta + \cos^2\theta)\,\hat{j} = \hat{j}$$

or $\hat{r}\,\sin\theta + \hat{\theta}\,\cos\theta = \hat{j}$ ...(*iii*)

Multiplying (*i*) by cos θ and (*ii*) by sin θ and subtracting, we get

$$(\hat{r}\cos\theta - \hat{\theta}\sin\theta) = \hat{i} \qquad ...(iv)$$

(*b*) $\hat{r}.\hat{\theta} = (\cos\theta\,\hat{i} + \sin\theta\,\hat{j}).(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

(1) (1) $\cos\theta = -\cos\theta\sin\theta + \sin\theta\cos\theta = 0 = \cos 90^\circ$ or $\theta = 90^\circ$

(*c*) $\frac{d}{dr}(\hat{r}) = \frac{d}{dt}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) = -\sin\theta\frac{d\theta}{dt}\hat{i} + \cos\theta\frac{d\theta}{dt}\hat{j} = \omega[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}] = \omega\hat{\theta}$

(*d*) $\vec{r} = \alpha\theta\hat{r}$ or $|\vec{r}| = \alpha\theta$ or $\alpha = \frac{|\vec{r}|}{\theta}$

Dimensions of $\alpha = \frac{L}{1} = L = [M^0 L^1 T^0]$

(*e*) Given, α = 1 unit, from above, we have $\vec{r} = \theta\hat{r} = \theta[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\theta}{dt}\hat{r} + \theta[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}]\frac{d\theta}{dt} = \frac{d\theta}{dt}\hat{r} + \theta\hat{\theta}\omega$ $\left[\because \frac{d\theta}{dt} = \omega\right]$

$$= \omega\hat{r} + \omega\theta\hat{\theta}$$

Acceleration, $\vec{a} = \frac{d}{dt}[\omega\hat{r} + \omega\theta\hat{\theta}] = \frac{d}{dt}\left[\frac{d\theta}{dt}\hat{r} + \frac{d\theta}{dt}\times(\theta\hat{\theta})\right] = \frac{d^2\theta}{dt^2}\hat{r} + \frac{d\theta}{dt}\frac{d\hat{r}}{dt} + \frac{d^2\theta}{dt^2}\times\theta\hat{\theta} + \frac{d\theta}{dt}\frac{d}{dt}(\theta\hat{\theta})$

$$= \frac{d^2\theta}{dt^2}\hat{r} + \omega\frac{d}{dt}[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}] + \frac{d^2\theta}{dt^2}\theta\hat{\theta} + \omega\frac{d}{dt}(\theta\hat{\theta})$$

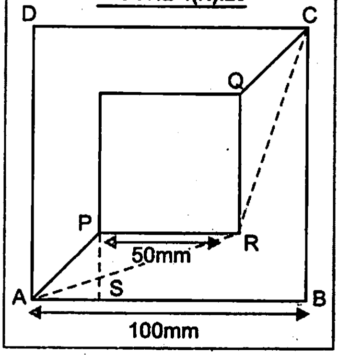
$$= \frac{d^2\theta}{dt^2}\hat{r} + \omega[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}]\frac{d\theta}{dt} + \frac{d^2\theta}{dt^2}\theta\hat{\theta} + \omega\left[\frac{d\theta}{dt}\hat{\theta} + \theta\frac{d\hat{\theta}}{dt}\right]$$

$$= \frac{d^2\theta}{dt^2}\hat{r} + \omega^2\hat{\theta} + \frac{d^2\theta}{dt^2}\times\theta\hat{\theta} + \omega^2\hat{\theta} + \omega^2\theta(-\hat{r}) \qquad \left[\because \frac{d\hat{\theta}}{dt} = -\omega\hat{r}\right]$$

$$= \left(\frac{d^2\theta}{dt^2} - \omega^2\theta\right)\hat{r} + \left(2\omega^2 + \frac{d^2\theta}{dt^2}\theta\right)\hat{\theta}$$

37. A man wants to reach from *A* to the opposite corner of the square *C* (Fig. 4(N).29). The sides of the square are 100 m. A central square of 50 m × 50 m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he can walk only at a speed of *v* m/s (*v* < 1). What is smallest value of *v* for which he can reach faster via a straight path through the sand than any path in the square outside the sand ?

FIGURE 4(N).29



Ans. In Fig. 4(N). 29, $AR = \sqrt{AT^2 + TR^2} = \sqrt{75^2 + 25^2} = 25\sqrt{10} = RC$

Total path $= AR + RC = 2\,AR = 50\sqrt{10}$ m

Time outside sand, $T_{out} = \frac{50\sqrt{10}\,\text{m}}{1\,\text{m/s}} = 50\sqrt{10}$ s

$$AP = QC = \sqrt{AS^2 + SP^2} = \sqrt{25^2 + 25^2} = 25\sqrt{2}\text{ m}$$

$$PQ = \sqrt{PR^2 + RQ^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}\text{ m}$$

In sand, $$T_{\text{sand}} = \frac{AP + QC}{1} + \frac{PQ}{v}$$

$$= \frac{25\sqrt{2} + 25\sqrt{2}}{1} + \frac{50\sqrt{2}}{v} = 50\sqrt{2}\left[1 + \frac{1}{v}\right]$$

Since $T_{\text{sand}} < T_{\text{out}}$

$$\therefore\ 50\sqrt{2}\left(1 + \frac{1}{v}\right) < 50\sqrt{10}$$

or $\sqrt{2}\left(1 + \frac{1}{v}\right) < \sqrt{10}$ or $\left(1 + \frac{1}{v}\right) < \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$ or $\frac{1}{v} < \sqrt{5} - 1$ or $v < \frac{1}{\sqrt{5} - 1}$ $\approx$ **0·81 m/s**

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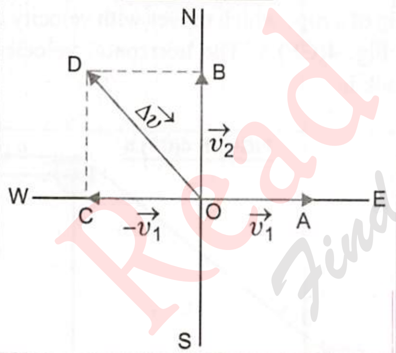
I. Multiple Choice Questions (with One Correct Answer)

I. Vectors and their additions and products

1. A particle is moving eastward with a velocity of 5 m/s in 10 s the velocity changes to 5 m/s northward. The average acceleration in this time is

FIGURE 4(CF).1



(*a*) $\frac{1}{\sqrt{2}}$ ms^{-2}, 45° West of North

(*b*) $\frac{1}{2}$ ms^{-2}, 60° West of North

(*c*) 2 ms^{-2}, 60° East of South

(*d*) $\frac{1}{\sqrt{2}}$ ms^{-2}, 30° West of South

2. A person moves 30 m North, then 20 m East, then $30\sqrt{2}$ m South-West. His displacement from the original position is

(*a*) 14 m South-West (*b*) 28 m South

(*c*) 10 m West (*d*) 15 m East

3. A vector of length *a* is turned through an angle (α) about its tail. The change in the position vector of its head is

(*a*) $2a \sin(\alpha/2)$ (*b*) $2a \cos(\alpha/2)$

(*c*) $2a \tan(\alpha/2)$ (*d*) $2a \cot(\alpha/2)$

4. An aeroplane flies a straight path from town *A* to town *B*, 500 km away. Town *B* is due East of town *A* and a strong wind blows from South to North at 300 km/h. If the plane's air speed is 900 km/h. Which of the following statements is true ?

(*a*) Trip time is $\frac{5}{6\sqrt{2}}$ hour

(*b*) Plane's ground speed is 600 km/h

(*c*) Plane's heading is 30° South of East

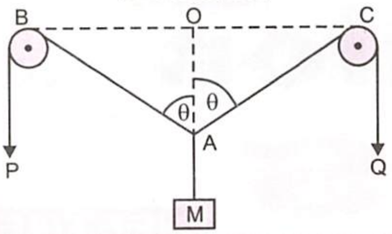
(*d*) Plane's heading is 60° South of East.

5. In the arrangement shown in Fig. 4(CF).2 the ends *P* and *Q* of an unstretchable string move downwards with speed *u*. Pulleys are fixed. Mass *M* moves upwards with speed of

ANSWERS

1. (*a*) **2.** (*c*) **3.** (*a*) **4.** (*a*)

FIGURE 4(CF).2



(*a*) $2u\cos\theta$ (*b*) $u/\cos\theta$

(*c*) $2u/\cos\theta$ (*d*) $u\cos\theta$.

6. A body is moving with velocity 30 m/s towards East. After 10 seconds its velocity becomes 40 m/s towards North. The average acceleration of the body is

(*a*) 1 m/s^2 (*b*) 7 m/s^2

(*c*) $\sqrt{7}\text{ m/s}^2$ (*d*) 5 m/s^2

(AIPMT 2011)

7. The simple sum of two forces acting at a point is 16 newton and their vector sum is 8 newton and its direction is perpendicular to the smaller force, then the forces are

(*a*) 6 newton and 10 newton

(*b*) 8 newton and 8 newton

(*c*) 4 newton and 12 newton

(*d*) 2 newton and 14 newton

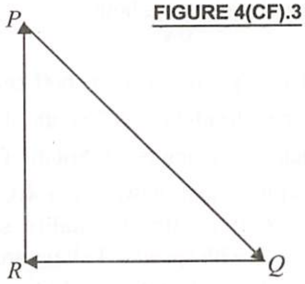
8. An object of mass 3 kg is at rest. Now a force of $\vec{F} = 6t^2\,\hat{i} + 4t\,\hat{j}$ is applied on the object, then velocity of object at $t = 3$ s is

(*a*) $18\,\hat{i} + 3\,\hat{j}$ (*b*) $18\,\hat{i} + 6\,\hat{j}$

(*c*) $3\,\hat{i} + 18\,\hat{j}$ (*d*) $18\,\hat{i} + 4\,\hat{j}$

9. A particle moving with velocity $\vec{v}$ is acted by three forces shown by the vector triangle *PQR*. The velocity of the particle will

FIGURE 4(CF).3



(*a*) increase

(*b*) decrease

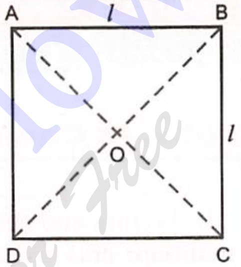
(*c*) remain constant

(*d*) change according to the smallest force $\overrightarrow{QR}$

(NEET 2019)

10. Four athletes are standing at the corners *A*, *B*, *C* and *D* of a square of side *l*. They simultaneously start running, such that *A* runs towards *B*, *B* runs towards *C*, *C* runs towards *D* and *D* runs towards *A*, each with velocity *v*. They will meet at *O*, after a time

FIGURE 4(CF).4

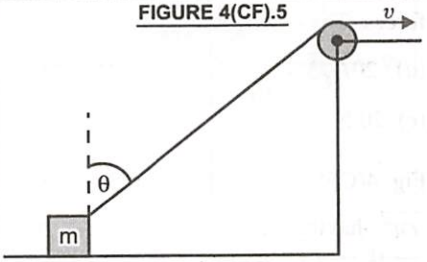


(*a*) $l/2v$ (*b*) $\sqrt{2}\,l/v$

(*c*) $l/\sqrt{2}\,v$ (*d*) l/v.

11. A block is dragged on a smooth plane with the help of a rope which moves with velocity *v* shown in Fig. 4(CF).5. The horizontal velocity of the block is

FIGURE 4(CF).5



(*a*) v (*b*) $v\sin\theta$

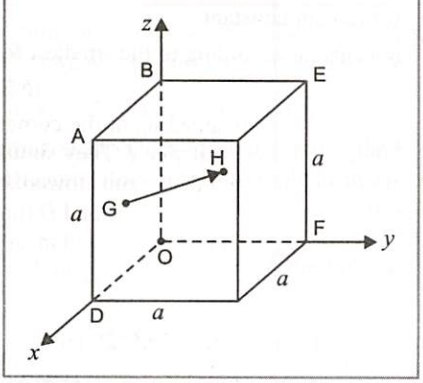
(c) $v/\sin\theta$ (*d*) $v/\cos\theta$.

12. In the cube of side *a* shown in Fig. 4(CF).6, the vector from the central point of the face *ABOD* to the central point of the face *BEFO* will be :

ANSWERS

5. (*b*) **6.** (*d*) **7.** (*a*) **8.** (*b*) **9.** (*c*) **10.** (*d*) **11.** (*c*)

FIGURE 4(CF).6



(*a*) $\frac{1}{2}a(\hat{k}-\hat{i})$ (*b*) $\frac{1}{2}a(\hat{i}-\hat{k})$

(*c*) $\frac{1}{2}a(\hat{j}-\hat{i})$ (*d*) $\frac{1}{2}a(\hat{j}-\hat{k})$

(JEE Main 2019)

13. A mass of 10 kg suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviates at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is : ($g = 10$ m/s^2)

(*a*) 100 N (*b*) 200 N

(*c*) 70 N (*d*) 140 N

(JEE Main 2019)

14. The resultant of two forces acting at an angle of 150° is 10 N and is perpendicular to one of the forces. The other force is

(*a*) $20/\sqrt{3}$ N (*b*) $10/\sqrt{3}$ N

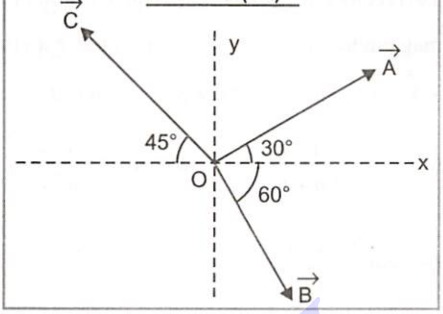
(*c*) 20 N (*d*) $20/\sqrt{3}$ N

15. Fig. 4(CF).7 shows three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ having same magnitude. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$, with *X*-axis is

(*a*) $\tan^{-1}\left(\frac{2-\sqrt{3}-1}{2+\sqrt{3}+1}\right)$

(*b*) $\tan^{-1}\left(\frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}\right)$

FIGURE 4(CF).7



(*c*) $\tan^{-1}\left(\frac{1-\sqrt{3}+\sqrt{2}}{1-\sqrt{3}-\sqrt{2}}\right)$

(*d*) $\tan^{-1}\left(\frac{1+\sqrt{3}+\sqrt{2}}{1-\sqrt{3}+\sqrt{2}}\right)$

16. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$. Then the magnitude of $\vec{R}$ is

(*a*) $P+Q$ (*b*) Q

(*c*) P (*d*) $\frac{P+Q}{2}$

(*e*) $P-Q$ **(Kerala PET 2009)**

17. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A}+\vec{B}$ is equal to 4 times the magnitude of $\vec{A}-\vec{B}$, then angle between $\vec{A}$ and $\vec{B}$ is

(*a*) $\cos^{-1}\left(\frac{3}{4}\right)$ (*b*) $\cos^{-1}\left(\frac{15}{17}\right)$

(*c*) $\sin^{-1}\left(\frac{3}{4}\right)$ (*d*) $\sin^{-1}\left(\frac{15}{17}\right)$

(Odisha JEE 2010)

18. The magnitude of the component of the vector $(2\hat{i}+3\hat{j}+\hat{k})$ along $(3\hat{i}+4\hat{k})$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{14}{5}$

(*c*) 3 (*d*) $\frac{6}{5}$

(*e*) 2 **(Kerala PMT 2010)**

ANSWERS

12. (*c*) 13. (*a*) 14. (*c*) 15. (*b*) 16. (*b*) 17. (*b*) 18. (*e*)

19. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitudes of $(\vec{A}+\vec{B})$ is n times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is

(*a*) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ (*b*) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(*c*) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (*d*) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(JEE Main 2019)

20. Let $|\vec{A}_1| = 3$, $|\vec{A}_2| = 5$ and $|\vec{A}_1+\vec{A}_2| = 5$, the value of $(2\vec{A}_1+3\vec{A}_2)\cdot(3\vec{A}_1-2\vec{A}_2)$ is :

(*a*) – 106·5 (*b*) – 118·5

(*c*) – 99·5 (*d*) – 112·5

(JEE Main 2019)

21. Two forces *P* and *Q*, of magnitude 2 *F* and 3 *F*, respectively are at an angle θ with each other. If the force *Q* is doubled then their resultant is also gets doubled. Then the angle θ is

(*a*) 30° (*b*) 90°

(*c*) 60° (*d*) 120°

(JEE Main 2019)

22. Two vectors $\vec{A}$ and $\vec{B}$ having same magnitude are at an angle 120° with each other. Find the angle between $\vec{A}$ and $(\vec{A}-\vec{B})$.

(*a*) 45° (*b*) 60°

(*c*) 30° (*d*) 120°

(JEE Main 2021)

23. Which of the following relations is true for two unit vectors $\hat{A}$ and $\hat{B}$ making an angle θ to each other ?

(*a*) $|\hat{A}+\hat{B}| = |\hat{A}-\hat{B}| \tan\theta/2$

(*b*) $|\hat{A}-\hat{B}| = |\hat{A}+\hat{B}| \tan\theta/2$

(*c*) $|\hat{A}+\hat{B}| = |\hat{A}-\hat{B}| \cos\theta/2$

(*d*) $|\hat{A}-\hat{B}| = |\hat{A}+\hat{B}| \cos\theta/2$

(JEE Main 2022)

II. Motion of Boat and Relative Motion

24. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}$ kmh^{-1}, the rain drops hit her head vertically. The speed of the rain drops with respect to moving girl is :

(*a*) 30 kmh^{-1} (*b*) $\frac{25}{\sqrt{2}}$ kmh^{-1}

(*c*) $\frac{30}{\sqrt{2}}$ kmh^{-1} (*d*) 25 kmh^{-1}

(JEE Main 2022)

25. The speed of a swimmer in still water is 20 ms^{-1}. The speed of river water is 10 ms^{-1} and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is given by :

(*a*) 30° west (*b*) 0°

(*c*) 60° west (*d*) 45° west

(NEET 2019)

26. A boat which has a speed of 5 km h^{-1} in still water crosses a river of width 1 km along the shortest path in 15 minutes. The velocity of the river water in km/h is

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) $\sqrt{41}$. **(IIT)**

27. Wind is blowing West to East along two parallel tracks. Two trains moving with same speed in opposite directions have the steam track of one double than other. The speed of each train is

(*a*) equal to that of wind

(*b*) double that of wind

(*c*) three times that of wind

(*d*) half that of wind.

28. A ship travels downstream from point *A* to point *B* in two hours and upstream in three hours. Then the time taken by log of wood to cover the distance from *A* to *B* is

(*a*) 5 h (*b*) 9 h

(*c*) 12 h (*d*) 1 h

29. A ship *A* is moving Westwards with a speed of 10 km h^{-1} and a ship *B*, 100 km South of *A* is

ANSWERS

19. (*b*) **20.** (*b*) **21.** (*d*) **22.** (*c*) **23.** (*b*) **24.** (*c*) **25.** (*a*) **26.** (*b*) **27.** (*c*) **28.** (*c*) moving Northwards with a speed of 10 kmh^{-1}. Find the time after which the distance between them is shortest and the shortest distance between them are

(*a*) 0 h, 100 km (*b*) $5\,\text{h}, 50\sqrt{2}\,\text{km}$

(*c*) $5\sqrt{2}\,\text{h}, 50\,\text{km}$ (*d*) $10\sqrt{2}\,\text{h}, 50\sqrt{2}\,\text{km}$

(AIPMT 2015)

30. Ship *A* is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/h, where $\hat{i}$ points east and $\hat{j}$, north. Ship *B* is at a distance of 80 km east and 150 km north of ship *A* and is sailing towards west at 10 kmh^{-1}. A will be at minimum distance from *B* in :

(*a*) 2·2 *h* (*b*) 4·2 *h*

(*c*) 2·6 *h* (*d*) 3·2 *h*

(JEE Main 2019)

31. Two cars are moving in the same direction with the same speed of 30 km/h. They are separated by a distance of 5 km. What is the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 minutes ?

(*a*) 40 km h^{-1} (*b*) 45 km h^{-1}

(*c*) 50 km h^{-1} (*d*) 35 km h^{-1}

32. A man running on the horizontal road at 8 km h^{-1} finds the rain appears to be falling vertically. He increases his speed to 12 km h^{-1} and finds that the drops make an angle 30° with the vertical. The speed of the rain with respect to road is

(*a*) 10 km/h (*b*) 9·5 km/h

(*c*) 10·58 km/h (*d*) 6 km/h

33. A train is moving at a constant speed v. Its driver sights another train infront of him on the same track and moving in the same direction with constant speed v_1. If the distance between the trains be d, then what should be the minimum retardation of the train so as to avoid collision ?

(*a*) $(v + v_1)^2\, d$ (*b*) $(v - v_1)^2\, d$

(*c*) $(v + v_1)^2/2\, d$ (*d*) $(v - v_1)^2/2\, d$

34. A boat can go across a lake and return in time T_0 at a speed v. On a rough day there is a uniform current at speed v_1 to help the onward journey and impede the return journey. If the time taken to go across and return on the same day be T, then T/T_0 is

(*a*) $(1 - v_1^2/v^2)$ (*b*) $\dfrac{1}{(1 - v_1^2/v^2)}$

(*c*) $\left(1 + \dfrac{v_1^2}{v^2}\right)$ (*d*) $\dfrac{1}{(1 + v_1^2/v^2)}$

35. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car ?

(*a*) 105 ms^{-1} (*b*) 210 ms^{-1}

(*c*) 315 ms^{-1} (*d*) 205 ms^{-1}

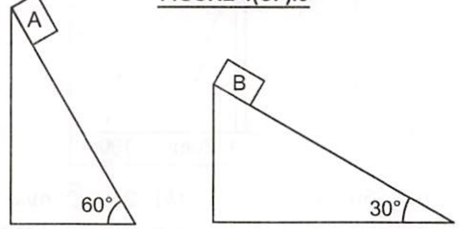
36. Two particles having position vectors $\vec{r}_1 = (3\hat{i} + 5\hat{j})$ metres and $\vec{r}_2 = (-5\hat{i} - 3\hat{j})$ metres are moving with velocites $\vec{v}_1 = (4\hat{i} + 3\hat{j})$ ms^{-1} and $\vec{v}_2 = (a\hat{i} + 7\hat{j})$ m/s. If they collide after 2 seconds, the value of a is

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8

37. Two frictionless inclined planes making an angle 30° and 60° with the horizontal are shown in the Fig. 4(CF).8. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?

FIGURE 4(CF).8



(*a*) 9·8 ms^{-2} in vertical direction

(*b*) Zero

(*c*) 4·9 ms^{-2} in vertical direction

(*d*) 4·9 ms^{-2} in horizontal direction

(AIEEE 2010)

38. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time

ANSWERS

29. (*b*) **30.** (*c*) **31.** (*b*) **32.** (*c*) **33.** (*d*) **34.** (*b*) **35.** (*a*) **36.** (*d*) **37.** (*c*) taken by her to walk up on the moving escalator will be

(*a*) $\frac{t_1 + t_2}{2}$ (*b*) $\frac{t_1 t_2}{t_2 - t_1}$

(*c*) $\frac{t_1 t_2}{t_2 + t_1}$ (*d*) $t_1 - t_2$ **(NEET 2017)**

III. Horizontal projectile

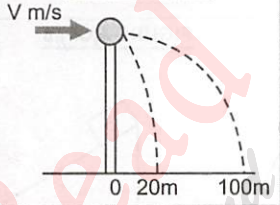
39. A stone is dropped into a well in which the level of water is at a distance h below the top of well. If v is the velocity of sound, the time T after which the splash is heard is given by

(*a*) $T = 2\,h/v$ (*b*) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

(*c*) $T = \sqrt{\frac{2h}{v}} + \frac{h}{g}$ (*d*) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

40. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is

FIGURE 4(CF).9



(*a*) 250 m/s (*b*) $250\sqrt{2}$ m/s

(*c*) 400 m/s (*d*) 500 m/s **(IIT 2011)**

41. A car starts from rest and accelerates at 5 ms^{-2}. At $t = 4$ s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at $t = 6$ s ?

(*a*) $20\sqrt{2}$ ms^{-1}, 10 ms^{-2}

(*b*) 20 ms^{-1}, 5 ms^{-2}

(*c*) 20 ms^{-1}, 0

(*d*) $20\sqrt{2}$ ms^{-1}, 0 **(NEET 2021)**

42. A particle moves from the point $(2{\cdot}0\,\hat{i} + 4{\cdot}0\,\hat{j})$ m, at $t = 0$, with an initial velocity $(5{\cdot}0\,\hat{i} + 4{\cdot}0\,\hat{j})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4{\cdot}0\,\hat{i} + 4{\cdot}0\,\hat{j})$ ms^{-2}. What is the distance of the particle from the origin at time 2 s ?

(*a*) $20\sqrt{2}$ m (*b*) 15 m

(*c*) $10\sqrt{2}$ m (*d*) 5 m

(JEE Main 2019)

43. A helicopter flying with horizontal velocity u at a height h drops a particle to a man on ground. How far the helicopter should be from the man so that particle reaches the man.

(*a*) $h\sqrt{1 + \frac{u^2}{gh}}$ (*b*) $h\sqrt{1 + \frac{u^2}{2\,gh}}$

(*c*) $h\sqrt{1 + \frac{2u^2}{gh}}$ (*d*) $h\sqrt{1 + \frac{u^2}{4\,gh}}$

(JEE Main 2021)

IV. Angular projectile

44. A body of mass m is projected with a velocity v at an angle of 45° with horizontal. The change in momentum at the end of this flight is

(*a*) *mv* (*b*) 2 *mv*

(*c*) $\sqrt{2}\,mv$ (*d*) zero

45. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be :

(*a*) $1 : \sqrt{2}$ (*b*) $\sqrt{2} : 1$

(*c*) $1 : \sqrt{3}$ (*d*) $1 : 2\sqrt{3}$

(NEET 2019)

46. A football is kicked 20 m/s at a projection angle of 45°. A receiver on the goal line 25 metres away in the direction of kick runs the same instant to meet the ball. If he is to catch the ball before it hits the ground, his speed must be ?

ANSWERS

38. (*c*) 39. (*b*) 40. (*d*) 41. (*a*) 42. (*a*) 43. (*c*) 44. (*c*) 45. (*c*)

(*a*) 5·483 m/s (*b*) 6·283 m/s
(*c*) 5·112 m/s (*d*) 6·112 m/s

47. A projectile is projected in the upward direction making an angle of 60° with horizontal direction with a velocity of 147 ms^{-1}. Then, the time after which its inclination with the horizontal is 45° is
(*a*) 2·74 s (*b*) 5·49 s
(*c*) 10·98 s (*d*) 15 s.

48. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in the two cases then the initial velocity of projectile is
(*a*) $\frac{1}{4} g t_1 t_2$ (*b*) $\frac{1}{2} g t_1 t_2$
(*c*) $\frac{1}{2} g (t_1 + t_2)^2$ (*d*) $\frac{1}{2} g (t_1^2 + t_2^2)^{1/2}$.

49. A gun mounted on the top of a moving truck is aimed in the backward direction at an angle of 30° to the vertical. If the muzzle velocity of the bullet fired from the gun is 4 ms^{-1}, then the speed of the truck that will make the bullet come out vertically is
(*a*) 0·5 ms^{-1} (*b*) 1·0 ms^{-1}
(*c*) 1·5 ms^{-1} (*d*) 2·0 ms^{-1}

50. Two guns A and B can fire bullets at speed 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by two guns on the ground is :
(*a*) 1 : 16 (*b*) 1 : 2
(*c*) 1 : 4 (*d*) 1 : 8

(JEE Main 2019)

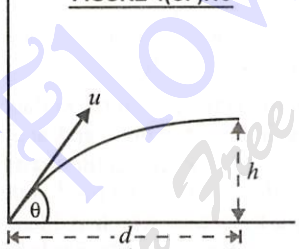
51. A body is projected with a velocity of 40 ms^{-1}. After 2 s it crosses a vertical pole of height 20·4 m. The angle of projection be ($g = 9{\cdot}8 \text{ m/s}^2$)
(*a*) 15° (*b*) 30°
(*c*) 45° (*d*) 60°.

52. A projectile of mass 100 g is fired with a velocity of 20 ms^{-1} making an angle of 30° with the horizontal. As it rises to the highest point of its path, its momentum changes by
(*a*) 1/2 kg ms^{-1} (*b*) 1 kg ms^{-1}
(*c*) 2 kg ms^{-1} (*d*) none of these.

53. If at a height of 40 m, the direction of motion of a projectile makes an angle $\pi/4$ with the horizontal, then its initial velocity and angle of projection are, respectively
(*a*) $30, \frac{1}{2}\cos^{-1}\left(-\frac{4}{5}\right)$ (*b*) $30, \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right)$
(*c*) $50, \frac{1}{2}\cos^{-1}\left(-\frac{8}{25}\right)$ (*d*) $60, \frac{1}{2}\cos^{-1}\left(-\frac{1}{4}\right)$

54. If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u if stone is launched at an angle θ ?

FIGURE 4(CF).10



(*a*) $\frac{g}{\cos\theta}\sqrt{\frac{d}{2(d\tan\theta - h)}}$

(*b*) $\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

(*c*) $\sqrt{\frac{gd^2}{h\cos^2\theta}}$

(*d*) $\sqrt{\frac{gd^2}{(d-h)}}$

55. A projectile can have the same range R for two angle of projections. If t_1 and t_2 be the times of flight in the two cases, then the product of the two times of flight is directly proportional to
(*a*) R (*b*) $1/R$
(*c*) R^2 (*d*) $1/R^2$

56. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the Fig. 4(CF).11. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is

ANSWERS

46. (*a*) 47. (*b*) 48. (*d*) 49. (*d*) 50. (*a*) 51. (*b*) 52. (*b*) 53. (*c*) 54. (*b*) 55. (*a*)

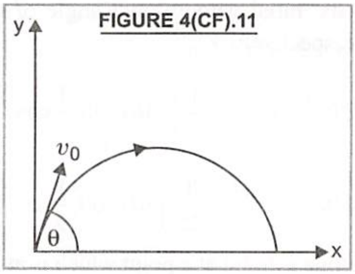


FIGURE 4(CF).11

(*a*) $mg\, v_0\, t \cos\theta\ \hat{k}$ (*b*) $-\frac{1}{2} mg\, v_0\, t^2 \cos\theta\, \hat{k}$

(*c*) $\frac{1}{2} mg\, v_0\, t^2 \cos\theta\, \hat{i}$ (*d*) $-\, mg\, v_0\, t^2 \cos\theta\ \hat{j}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along *x*, *y* and *z*-axis respectively. **(AIEEE 2010)**

57. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is (given $g = 9{\cdot}8\ ms^{-2}$)

(*a*) 3·5 (*b*) 5·9
(*c*) 16·3 (*d*) 110·8

(AIPMT 2014)

58. A particle is projected with an angle of projection θ to the horizontal line passing through the points (*P*, *Q*) and (*Q*, *P*) referred to horizontal and vertical axes (can be treated as *x*-axis and *y*-axis respectively). The angle of projection can be given by :

(*a*) $\tan^{-1}\left[\frac{P^2 + PQ + Q^2}{PQ}\right]$

(*b*) $\tan^{-1}\left[\frac{P^2 + Q^2 - PQ}{PQ}\right]$

(*c*) $\tan^{-1}\left[\frac{P^2 + Q^2}{2PQ}\right]$

(*d*) $\sin^{-1}\left[\frac{P^2 + Q^2 + PQ}{2PQ}\right]$ **(AIIMS 2015)**

59. A particle of mass *m* is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial velocity u_0. The angle that the composite system makes with the horizontal immediately after the collision is

(*a*) π/4 (*b*) (π/4) + α
(*c*) (π/2) – α (*d*) (π/2)

(JEE Advanced 2013)

60. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ ms^{-2}$, the equation of its trajectory is

(*a*) $y = x - 5x^2$ (*b*) $y = 2x - 5x^2$
(*c*) $4y = 2x - 5x^2$ (*d*) $4y = 2x - 25x^2$

(JEE Main 2013)

61. A small block is connected to one end of a massless spring of un-stretched length 4·9 m. The other end of the spring (Fig. 4(CF).12) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0·2 m and released from rest at *t* = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously, at *t* = 0, a small pebble is projected with speed *v* from point *P* at an angle of 45° as shown in the figure. Point *P* is at a horizontal distance of 10 m from *O*. If the pebble hits the block at *t* = 1 s, the value of *v* is (take $g = 10\ m/s^2$)

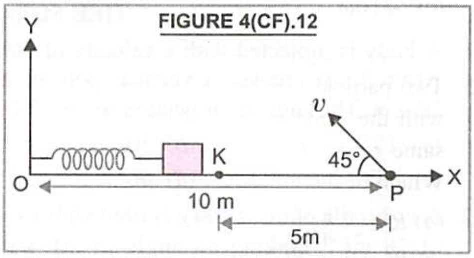


FIGURE 4(CF).12

(*a*) $\sqrt{50}$ m/s (*b*) $\sqrt{51}$ m/s
(*c*) $\sqrt{52}$ m/s (*d*) $\sqrt{53}$ m/s

(IIT 2012)

62. A projectile is projected in the upward direction making an angle of 45° with horizontal direction

ANSWERS

56. (*b*) **57.** (*a*) **58.** (*a*) **59.** (*a*) **60.** (*b*) **61.** (*a*)

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with a velocity of 150 ms^{-1}. Then the time after which its inclination with the horizontal is 30° is ($g = 10$ ms^{-2})

(*a*) 2·24 s (*b*) 4·48 s

(*c*) 9·98 s (*d*) 12·23 s

63. A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution. If this particle were projected with the same speed at an angle 'θ' to the horizontal, the maximum height attained by it equals 4 R. The angle of projection θ is then given by

(*a*) $\theta = \sin^{-1}\left[\dfrac{2gT^2}{\pi^2 R}\right]^{1/2}$

(*b*) $\theta = \cos^{-1}\left[\dfrac{gT^2}{\pi^2 R}\right]^{1/2}$

(*c*) $\theta = \cos^{-1}\left[\dfrac{\pi^2 R}{gT^2}\right]^{1/2}$

(*d*) $\theta = \sin^{-1}\left[\dfrac{\pi^2 R}{gT^2}\right]^{1/2}$ **(NEET 2021)**

64. The position co-ordinates of a particle moving in a 3-D co-ordinate system is given by

$x = a \cos \omega t$; $y = a \sin \omega t$ and $z = a \omega t$

The speed of the particle is

(*a*) $2\, a\omega$ (*b*) $\sqrt{2}\, a\omega$

(*c*) $\sqrt{3}\, a\omega$ (*d*) $a\omega$

(JEE Main 2019)

65. Two particles are projected from the same point with the same speed u such that they have the same range R, but different heights h_1 and h_2. Which of the following is correct ?

(*a*) $R^2 = 4\, h_1 h_2$ (*b*) $R^2 = 16\, h_1 h_2$

(*c*) $R^2 = 2\, h_1 h_2$ (*d*) $R^2 = h_1 h_2$

(JEE Main 2019)

66. A particle at origin (0, 0) moving with initial velocity $u = 5$ ms^{-1} $\hat{j}$ and acceleration $(10\,\hat{i} + 4\,\hat{j})$ ms^{-2}. After time t seconds, it reaches at position (20, y) m. Find t and y.

(*a*) $t = 2$ s ; $y = 18$ m (*b*) $t = 4$ s ; $y = 16$ m

(*c*) $t = 6$ s ; $y = 12$ m (*d*) $t = 8$ s ; $y = 10$ m

(JEE Main 2020)

V. Circular motion

67. A particle moves so that its position vector is given by $\vec{r} = \cos \omega t\, \hat{x} + \sin \omega t\, \hat{y}$, where ω is a constant. Which of the following is true ?

(*a*) velocity and acceleration both are perpendicular to $\vec{r}$

(*b*) velocity and acceleration both are parallel to $\vec{r}$

(*c*) velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin

(*d*) velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

(AIPMT 2016)

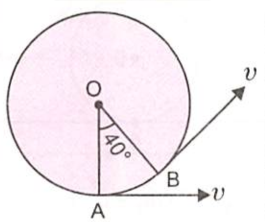
68. The length of seconds hand of a watch is 1 cm. The change in velocity of its tip in 15 seconds in cm/s is

(*a*) zero (*b*) $\pi/(30\sqrt{2})$

(*c*) $\pi/30$ (*d*) $2\pi/(30\sqrt{2})$

69. A particle is moving in a circle of radius r centred at O with constant speed v. The change in velocity in moving from A to B is [Given $\angle AOB = 40°$]

FIGURE 4(CF).13



(*a*) $2\, v \cos 40°$ (*b*) $2\, v \sin 20°$

(*c*) $2\, v$ cosec $40°$ (*d*) $2\, v \sec 20°$

70. A particle is moving in a circular path with a constant speed v. If θ is the angular displacement, then starting from $\theta = 0$, the maximum and minimum changes in the momentum will occur, when value of θ is respectively

(*a*) 45° and 90° (*b*) 90° and 180°

(*c*) 180° and 360° (*d*) 90° and 270°

ANSWERS

62. (*b*) **63.** (*a*) **64.** (*b*) **65.** (*b*) **66.** (*a*) **67.** (*c*) **68.** (*d*) **69.** (*b*) **70.** (*c*)

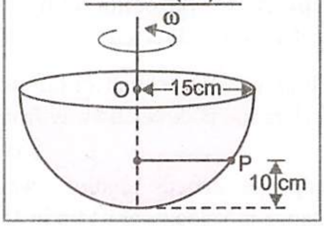
71. The angular speed of a fly wheel moving with uniform angular acceleration changes from 1200 rpm to 3120 rpm in 16 seconds. The angular acceleration in rad/s^2 is

(*a*) 2 π (*b*) 4 π

(*c*) 12 π (*d*) 10 4 π **(NEET 2022)**

72. A smooth hemispherical bowl 30 cm diameter rotates with a constant angular velocity ω, about its vertical axis of symmetry. A particle at *P* of weighing 5 kg is observed to remain at rest relative to the bowl at a height 10 cm above the base, Fig. 4(CF).14. The magnitude of speed of rotation of the bowl is

FIGURE 4(CF).14

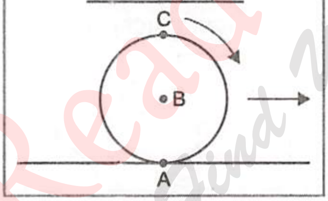


(*a*) 15 rad/s (*b*) 13 rad/s

(*c*) 14 rad/s (*d*) 16 rad/s

73. A sphere is rolling without slipping on a fixed horizontal plane surface. In the Fig. 4(CF).15, A is a point of contact, *B* is the centre of the sphere and *C* is its topmost point. Then

FIGURE 4(CF).15



(*a*) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$

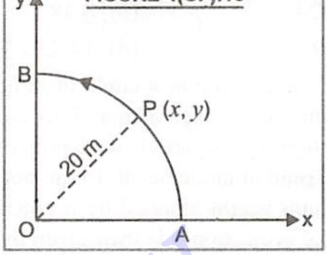
(*b*) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

(*c*) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$

(*d*) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$ **(IIT 2009)**

74. A point P moves in counter-clockwise direction on a circular path as shown in the Fig. 4(CF).16. The movement of '*P*' is such that it sweeps out a length $s = t^3 + 5$; where *s* is in metres and *t* is in seconds. The radius of the path is 20 m. The acceleration of '*P*' when $t = 2$ s is nearly

FIGURE 4(CF).16



(*a*) 12 m/s^2 (*b*) 7·2 m/s^2

(*c*) 14 m/s^2 (*d*) 13 m/s^2

(AIEEE 2010)

75. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point *P* (*R*, θ) on the circle of radius *R* is (Here θ is measured from the *x*-axis)

(*a*) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(*b*) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

(*c*) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(*d*) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ **(AIEEE 2010)**

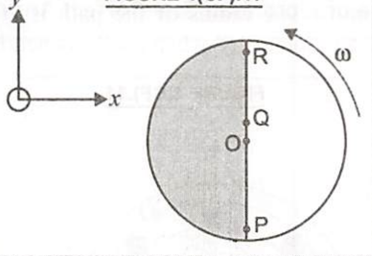
76. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre *O*. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the Fig. 4(CF).17. When the disc is in the orientation as shown, two pebbles *P* and *Q* are simultaneously projected at an angle towards *R*. The velocity of projection is in the *y*-*z* plane and is same for both pebbles with respect to the disc. Assume that (*i*) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (*ii*) their range is less than half the disc radius, and (*iii*) ω remains constant throughout. Then

(*a*) *P* lands in the shaded region and *Q* in the unshaded region

ANSWERS

71. (*b*) 72. (*c*) 73. (*c*) 74. (*c*) 75. (*b*)

FIGURE 4(CF).17



(*b*) P lands in the unshaded region and Q in the shaded region

(*c*) Both P and Q land in the unshaded region

(*d*) Both P and Q land in the shaded region

(IIT JEE 2012)

77. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the Fig. 4(CF).18. The relative speed between the two points P and Q is v_r. As a function of time, it is best represented by

FIGURE 4(CF).18

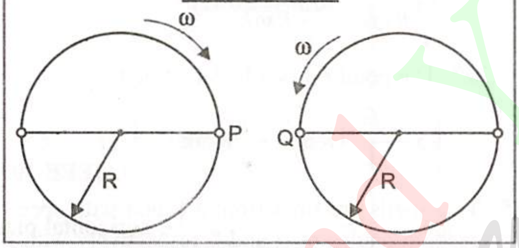
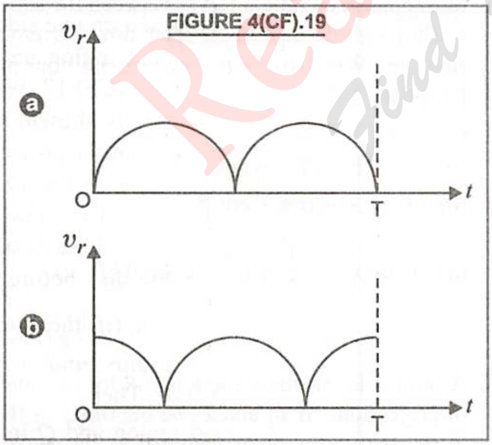
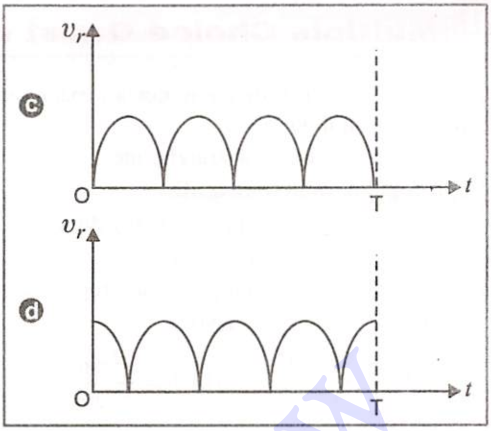


FIGURE 4(CF).19

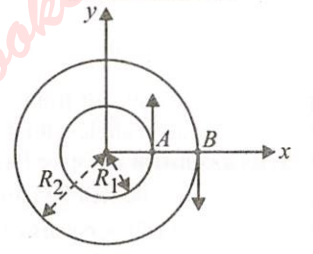




(IIT JEE 2012)

78. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and directions of motion are shown in the Fig. 4(CF).20. The relative velocity $(\vec{v}_A - \vec{v}_B)$ at $t = \frac{\pi}{2\omega}$ is given by

FIGURE 4(CF).20



(*a*) $\omega(R_2 - R_1)\hat{i}$
(*b*) $\omega(R_1 - R_2)\hat{i}$
(*c*) $-\omega(R_1 + R_2)\hat{i}$
(*d*) $\omega(R_1 + R_2)\hat{i}$

(JEE Main 2019)

79. A clock has a continuously moving second's hand of 0·1 m length. The instantaneous accelerating of the tip of the hand (in units of ms^{-2}) is of the order of

(*a*) 10^{-3}
(*b*) 10^{-1}
(*c*) 10^{-4}
(*d*) 10^{-2}

(JEE Main 2020)

ANSWERS

76. (*b*) **77.** (*a*) **78.** (*a*) **79.** (*a*)

II. Multiple Choice Questions (with One or More than One Correct Answers)

80. Which of the following is not an example of projectile motion

(*a*) A car moving in a straight line

(*b*) A bullet fired from a rifle

(*c*) A piece of stone thrown in any direction

(*d*) Second's hand of a clock

81. Which of the following doesn't represent the relation of angular projection

(*a*) $R = \dfrac{u^2 \sin^2 \theta}{g}$ (*b*) $h = \dfrac{u^2 \sin 2\theta}{2g}$

(*c*) $T = \dfrac{2u \sin \theta}{g}$ (*d*) $v = \sqrt{v_x^2 + v_y^2}$

82. A particle leaves the origin with an initial velocity $\vec{u} = (3\hat{i})\,\text{ms}^{-1}$ and a constant acceleration $\vec{a} = (-1{\cdot}0\,\hat{i} - 0{\cdot}5\,\hat{j})\,\text{ms}^{-1}$. Its velocity $\vec{v}$ and position vector $\vec{r}$ when it reaches its maximum *x*-coordinate are

(*a*) $\vec{v} = (-3\hat{j})\,\text{ms}^{-1}$ (*b*) $\vec{v} = \left(-\dfrac{3}{2}\hat{j}\right)\text{ms}^{-1}$

(*c*) $\vec{r} = (3\hat{i} - 2\hat{j})$ m (*d*) $\vec{r} = \left(\dfrac{9}{2}\hat{i} - \dfrac{9}{4}\hat{j}\right)$ m

83. A projectile is hurled into air from a point on the horizontal ground at an angle with the vertical. If the air exerts a constant resistive force,

(*a*) the path of projectile will be parabolic path

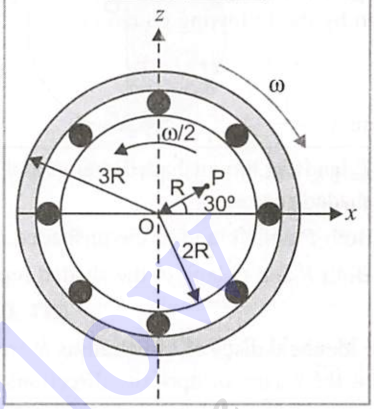
(*b*) the time of ascent will be equal to time of decent

(*c*) the total energy of the projectile is not conserved

(*d*) at the highest point, the velocity of projectile is horizontal.

84. The Fig. 4(CF).21. shows a system consisting of (*i*) a ring of outer radius 3 *R* rolling clockwise without slipping on a horizontal surface with angular speed ω and (*ii*) an inner disc of radius 2 *R* rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearing. The system is in the *x-z* plane. The point *P* on the inner disc is at a distance *R* from the origin, where *OP* makes an angle 30° with the horizontal. Then with respect to the horizontal surface

FIGURE 4(CF).21



(*a*) The point *O* has a linear velocity $3R\omega\hat{i}$

(*b*) The point *P* has a linear velocity $\dfrac{11}{4}R\omega\hat{i} + \dfrac{\sqrt{3}}{4}R\omega\hat{k}$

(*c*) The point *P* has a linear velocity $\dfrac{13}{4}R\omega\hat{i} - \dfrac{\sqrt{3}}{4}R\omega\hat{k}$

(*d*) The point *P* has a linear velocity $\left(3 - \dfrac{\sqrt{3}}{4}\right)R\omega\hat{i} + \dfrac{1}{4}R\omega\hat{k}$

85. Two shells are fired from a canon with speed u each, at angles of α and β respectively with the horizontal. The time interval between the shots is *t*. They collide in mid air after time *T* from the first shot. Which of the following conditions must be satisfied ?

(*a*) $\alpha > \beta$

(*b*) $T \cos \alpha = (T - t) \cos \beta$

(*c*) $(T - t) \cos \alpha = T \cos \beta$

(*d*) $u \sin \alpha\, T - \dfrac{1}{2} g T^2 = u \sin \beta\, (T - t) - \dfrac{1}{2} g (T - t)^2$

86. A projectile has the same range *R* for two angles of projections. If T_1 and T_2 be the times of flight in the two cases, then (using θ as the angle of

ANSWERS

80. (*a,d*) **81.** (*a,b*) **82.** (*b,d*) **83.** (*a,c,d*) **84.** (*a,b*) **85.** (*a,b,d*) projection corresponding to T_1)

(*a*) $T_1 T_2 \propto R$ (*b*) $T_1 T_2 \propto R^2$

(*c*) $T_1/T_2 = \tan\theta$ (*d*) $T_1/T_2 = 1$

87. The position vector $\vec{r}$ of a particle of mass *m* is given by the following equation

$$\vec{r}(t) = \alpha t^3\,\hat{i} + \beta t^2\,\hat{j}$$

where $\alpha = \frac{10}{3}\,\text{ms}^{-3}$, $\beta = 5\ \text{ms}^{-2}$ and $m = 0{\cdot}1$ kg.

At $t = 1$ s, which of the following statement(s) is (are) true about the particle ?

(*a*) The velocity $\vec{v}$ is given by

$$\vec{v} = (10\,\hat{i} + 10\,\hat{j})\,\text{ms}^{-1}$$

(*b*) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\,\hat{k}$ N ms

(*c*) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\,\hat{j})$ N

(*d*) The torque with respect to the origin is given by $\vec{\tau} = -\frac{20}{3}\hat{k}$ Nm **(JEE Advanced 2016)**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1 **An aeroplane is flying with velocity $\vec{v}_p$ (= 100 ms^{-1} towards East) with respect to ground through motionless air and $\vec{v}_w$ is the wind velocity with respect to ground. The total velocity of aeroplane is $\vec{v} = \vec{v}_p + \vec{v}_w$**

The magnitude of the velocity is often called speed. The heading of the plane is the direction in which the nose of the plane points. In fact, it is the direction in which the engine propels the plane. Answer the following questions :

88. If the wind blows with velocity 25 ms^{-1} Northwards, the plane velocity is deflected from East by an angle

(*a*) $\sin^{-1}\frac{25}{100}$ (*b*) $\cos^{-1}\frac{25}{100}$

(*c*) $\tan^{-1}\frac{25}{100}$ (*d*) $\cot^{-1}\frac{25}{100}$

89. If θ is the answer to question 77, the total speed of the plane in ms^{-1} is

(*a*) 100 sin θ (*b*) 100 cos θ

(*c*) 100 cosec θ (*d*) 100 sec θ

90. If the wind blows with velocity 25 ms^{-1} Northwards and the pilot adjusts the heading of the plane so that the total velocity of plane is Eastwards. By what angle does the new heading differ from due East ?

(*a*) $\sin^{-1}$ (25/100) (*b*) $\cos^{-1}$ (25/100)

(*c*) $\tan^{-1}$ (25/100) (*d*) $\cot^{-1}$ (25/100)

Comprehension 2 **A person is interested to find experimentally a relation between range of the ball and velocity of projection of ball. The range is the horizontal distance upto which the ball goes on ground after projection with some velocity on ground. Let the ground be plane where the person is performing the experiment. In each projection, the person uses the same ball and projects it at the same angle. The table below shows the experi- mental detail.**

Projection	Projection speed (m/s)	Range (m)
1	5	4
2	10	16
3	15	36
4	20	64

From above observations, the person concludes that the range *R* depends on the initial speed *v* according to the equation ; $R = C\,v^{n-1}$ where, *C* is a constant and *n* is another constant. Answer the followings :

ANSWERS

86. (*a,c*) 87. (*a,b,d*) 88. (*c*) 89. (*d*) 90. (*a*)

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91. On the basis of above observations, what is the value of n ?

(*a*) 2·5 (*b*) 3·0

(*c*) 3·5 (*d*) 4·0

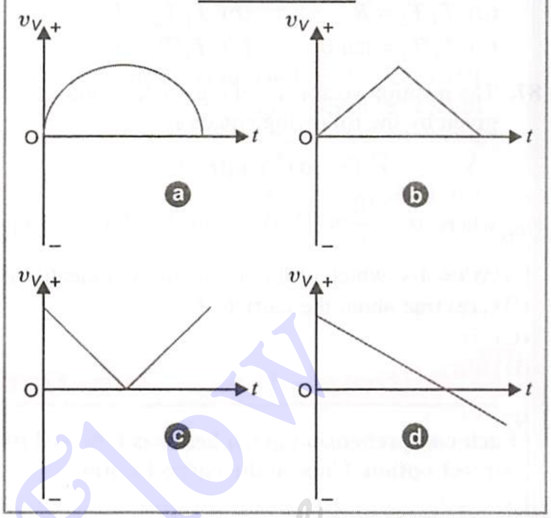
92. The person performs another experiment in which the ball is projected at a speed of 4 ms^{-1}, its range is approximately

(*a*) 2·12 m (*b*) 3·45 m

(*c*) 2·56 m (*d*) 4·02 m

93. For projection 3 (as given in observation table) which of the following graphs best represents the vertical component of the ball's velocity (v_V) as a function of time (t), assuming upward direction is positive.

FIGURE 4(CF).22



IV Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices.

94. Two vectors are given as, $\vec{A} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} - 2\hat{k}$. Then

Column I	Column II
(A) $\lvert \vec{A} + \vec{B} \rvert$	(*p*) $\sqrt{14}$
(B) $\lvert \vec{A} - \vec{B} \rvert$	(*q*) $\sqrt{10}$
(C) $(\vec{A} \cdot \vec{B})$	(*r*) 1
(D) $\lvert \vec{A} \times \vec{B} \rvert$	(*s*) $\sqrt{26}$

(*a*) (A)-(*p*) ; (B)-(*q*) ; (C)-(*r*) ; (D)-(*s*) (*b*) (A)-(*q*) ; (B)-(*r*) ; (C)-(*s*) ; (D)-(*p*)

(*c*) (A)-(*r*) ; (B)-(*s*) ; (C)-(*p*) ; (D)-(*q*) (*d*) (A)-(*s*) ; (B)-(*p*) ; (C)-(*q*) ; (D)-(*r*)

95. Two particles of equal mass are projected in air with speed u at angles $\theta_1 = 30°$ and $\theta_2 = 60°$ (both acute) to the horizontal, respectively. Then ratio of

Column I	Column II
(A) Maximum height	(*p*) 1 : 3
(B) Total Time of flight	(*q*) $1 : \sqrt{3}$
(C) Horizontal range	(*r*) $\sqrt{3} : 1$
(D) Total energy	(*s*) 1 : 1

(*a*) (A)-(*p*) ; (B)-(*q*) ; (C)-(*r*) ; (D)-(*s*) (*b*) (A)-(*q*) ; (B)-(*r*) ; (C)-(*s*) ; (D)-(*p*)

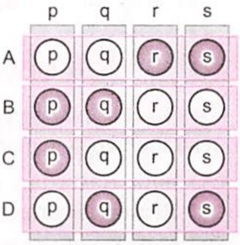
(*c*) (A)-(*r*) ; (B)-(*s*) ; (C)-(*p*) ; (D)-(*q*) (*d*) (A)-(*s*) ; (B)-(*p*) ; (C)-(*q*) ; (D)-(*r*)

ANSWERS

91. (*b*) **92.** (*c*) **93.** (*d*) **94.** (*a*) **95.** (*a*)

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answer to these questions have to be appropriately bubbled. If the correct matches are A-*r* ; A-*s* ; B-*p* ; B-*q* ; C-*p*, and D-*q* ; D-*s*, then the correctly bubbled matrix will look like the one shown here.



	p	q	r	s
A	○	○	●	●
B	●	●	○	○
C	●	○	○	○
D	○	●	○	●

96.

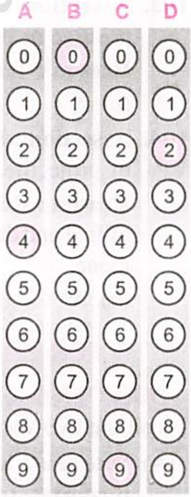
Column I		Column II
(A) One dimensional motion	(*p*)	The motion of earth around the sun in an orbit.
(B) Two dimensional motion	(*q*)	Motion of the gas molecules in the given region.
(C) Three dimensional motion	(*r*)	Sommer sault by a swimmer from a height before entering the water.
(D) Rotational motion	(*s*)	Oscillations of a mass suspended from a vertical spring.

97. Match the column I with column II, for magnitude component of a vector $\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$,

Column I		Column II
(A) along *y*-axis	(*p*)	> 1 less than 5
(B) along *z*-axis	(*q*)	< 1
(C) along $(\hat{i}+\hat{j})$	(*r*)	> 3 less than 5
(D) along *x*-axis	(*s*)	> 2 less than 5

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :



A	B	C	D
○ 0	● 0	○ 0	○ 0
○ 1	○ 1	○ 1	○ 1
○ 2	○ 2	○ 2	● 2
○ 3	○ 3	○ 3	○ 3
● 4	○ 4	○ 4	○ 4
○ 5	○ 5	○ 5	○ 5
○ 6	○ 6	○ 6	○ 6
○ 7	○ 7	○ 7	○ 7
○ 8	○ 8	○ 8	○ 8
○ 9	○ 9	● 9	○ 9

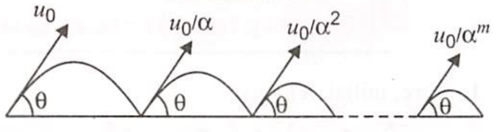
98. The sum of the magnitude of two forces acting at a point is 18 N and magnitude of their resultant is 12 N. If the resultant is at 90° with the force of smaller magnitude, what is the magnitude of smaller force in newton ?

99. A ball rolls of the top of a stair way with horizontal velocity of magnitude 1·8 ms^{-1}. The steps are 0·20 m high and 0·20 m wide. Which step will the ball hit first ? ($g = 10$ m/s^2)

100. A train is moving along a straight line with a constant acceleration *a*. A boy standing in the train throws a ball forward with speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is : **(IIT JEE 2011)**

101. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion the magnitude of the average velocity of the ball up to the point when it hits the ground for the first time is v_1. After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in Fig. 4(CF).23. If the magnitude of average velocity of the ball for the entire duration of motion is $0{\cdot}8\ v_1$, the value of α is **(JEE Advanced 2019)**

FIGURE 4(CF).23



ANSWERS

96. A-*s*, B-*p*, *r*, C-*q*, D-*r** **97.** A-*p*, *r*, *s*, B-*q*, C-*p*, *r*, *s*, D-*p*, *s* **98.** (5) **99.** (4)
100. (5) **101.** (4)

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

(A) both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) both Assertion and Reason are true but the Reason is not a correct explanation of the Assertion.

(C) Assertion is true but the Reason is false.

(D) both Assertion and Reason are false.

102. Assertion : When a body is projected at an angle 45°, its range is maximum.

Reason : For maximum range, the value of sin 2 θ should be equal to one.

(*a*) A (*b*) B (*c*) C (*d*) D

103. Assertion : The value of temperature can be positive or negative.

Reason : Temperature is a vector quantity

(*a*) A (*b*) B (*c*) C (*d*) D

104. Assertion : Rocket in flight is not an illustration of projectile.

Reason : Rocket takes flight due to combustion of fuel and does not move under the gravity effect alone.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

(A) Statement-1 is true ; Statement-2 is true ; Statement-2 is correct explanation of Statement-1.

(B) Statement-1 is true ; Statement-2 is true ; Statement-2 is not correct explanation of Statement-1.

(C) Statement-1 is true ; Statement-2 is false.

(D) Statement-1 is false ; Statement-2 is true.

105. Statement-1 : Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.

Statement-2 : The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.

(*a*) A (*b*) B (*c*) C (*d*) D

106. Statement-1 : The outside horses on a merry-go-round get more acceleration than inside ones.

Statement-2 : This is because of linear acceleration is directly proportional to the distance.

(*a*) A (*b*) B (*c*) C (*d*) D

107. Statement-1 : The three vectors not lying in a plane can never add up to give a null vector.

Statement-2 : The three vectors not lying in a plane can not be represented by the three sides of a triangle taken in the same order.

(*a*) A (*b*) B (*c*) C (*d*) D

108. Statement-1 : The magnitude of the resultant vector of two given vectors can never be less than the magnitude of any of the given vector.

Statement-2 : The resultant vector is the vector sum of two vectors.

(*a*) A (*b*) B (*c*) C (*d*) D

HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Here, initial velocity

$$\vec{v}_1 = 5\,\text{ms}^{-1} \text{ along East } = 5\,\hat{i}$$

Final velocity $\vec{v}_2 = 5\,\text{ms}^{-1}$ along North $= 5\,\hat{j}$

Change in velocity,

$$\Delta v = \vec{v}_2 - \vec{v}_1 = 5\,\hat{j} - 5\,\hat{i} = -5\,\hat{i} + 5\,\hat{j}$$

Magnitude of $\Delta v = \sqrt{(-5)^2 + 5^2} = 5\sqrt{2}$ m/s

ANSWERS

102. (*a*) **103.** (*c*) **104.** (*a*) **105.** (*a*) **106.** (*a*) **107.** (*a*) **108.** (*d*)

HINTS/EXPLANATIONS....

For Difficult Questions

$$\text{Acceleration, } |\vec{a}| = \frac{\Delta v}{t} = \frac{5\sqrt{2}}{10}$$

$$= \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\,\mathbf{ms^{-2}}$$

$$\tan\theta = \frac{BD}{OB} = \frac{5}{5} = 1 = \tan 45^\circ$$

θ = 45° West of North

2. Resolving displacement $30\sqrt{2}$ m South West in two rectangular components. We have

$$30\sqrt{2}\cos 45^\circ = 30\sqrt{2}\times\frac{1}{\sqrt{2}} = 30\,\text{m}$$

towards South

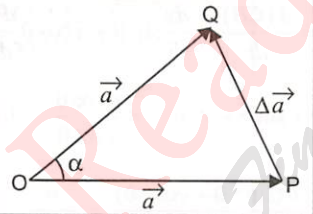
$$\text{and}\quad 30\sqrt{2}\sin 45^\circ = 30\sqrt{2}\times\frac{1}{\sqrt{2}} = 30\,\text{m}$$

towards West.

The resultant of 30 m North will neutrilise the displacement of 30 m South. Hence, the effective displacement is the resultant of 30 m West and 20 m East = **10 m West.**

3. Refer to Fig. 4(CF).24, the change in position vector $\Delta\vec{a}$ is represented by $\overrightarrow{PQ}$.

FIGURE 4(CF).24



From Δ law of vectors,

$$(\overrightarrow{PQ}) = (\overrightarrow{OQ}) - (\overrightarrow{OP}) = (\overrightarrow{OQ}) + (-\overrightarrow{OP})$$

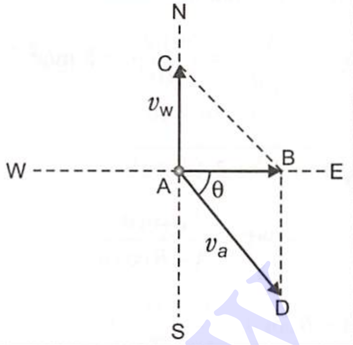
Angle between $(\overrightarrow{OQ})$ and $(-\overrightarrow{OP})$ is $(180^\circ - \alpha)$

$$\therefore (\Delta a)^2 = a^2 + a^2 + 2a^2\cos(180^\circ - \alpha)$$
$$= 2a^2 - 2a^2\cos\alpha$$
$$= 2a^2(1-\cos\alpha)$$
$$= 2a^2[1 - 1 + 2\sin^2\alpha/2]$$
$$= 4a^2\sin^2\alpha/2$$

$$\therefore\quad \Delta a = \mathbf{2\,a\sin\alpha/2}$$

4. Refer to Fig. 4(CF).25, AB = 500 km.

FIGURE 4(CF).25



Velocity of wind, v_w = 300 km/h along AC. The air plane can fly a st. path AB if the resultant velocity of aeroplane and wind acts along AB. It will be so if the velocity of aeroplane is represented by AD.

$$\text{Velocity along } AB,\ v = \sqrt{v_a^2 - v_w^2}$$

$$= \sqrt{900^2 - 300^2} = \sqrt{72000} = 600\sqrt{2}\ \text{km/h}$$

$$\text{Time taken to reach } B,\ t = \frac{S}{v} = \frac{500\,\text{km}}{600\sqrt{2}\,\text{km/h}}$$

$$= \frac{\mathbf{5}}{\mathbf{6\sqrt{2}}}\,\mathbf{h}$$

$$\sin\theta = \frac{v_w}{v_a} = \frac{300}{900} = \frac{1}{3} = 0{\cdot}3333 = \sin 19^\circ 28'$$

θ = 19° 28′ South of East

5. Let $AB = r$, $OB = x$ and $AO = y$.

In Δ ABO ; $r^2 = x^2 + y^2$

Differentiating it w.r.t. time we get ;

$$2r\frac{dr}{dt} = 0 + 2y\frac{dy}{dt}$$

$$\text{or}\quad \frac{dy}{dt} = \frac{r}{y}\frac{dr}{dt} = \frac{dr/dt}{y/r} = \frac{\mathbf{u}}{\mathbf{\cos\theta}}$$

6. Velocity towards East, $\vec{v}_1 = 30\,\hat{i}$ m/s

Velocity towards North, $\vec{v}_2 = 40\,\hat{j}$ m/s

Change in velocity,

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = (40\,\hat{j} - 30\,\hat{i})$$

$$|\Delta\vec{v}| = |40\,\hat{j} - 30\,\hat{i}| = \sqrt{40^2 + (-30)^2} = 50\ \text{m/s}$$

HINTS/EXPLANATIONS

For Difficult Questions

Average acceleration,

$$a_{av} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{50\,\text{m/s}}{10\,\text{s}} = \mathbf{5\ m/s^2}$$

7. Here, $A + B = 16$ ...(*i*)

$$\sqrt{A^2 + B^2 + 2AB\cos\theta} = 8 \quad \text{...(ii)}$$

and $\tan 90^\circ = \dfrac{B\sin\theta}{A + B\cos\theta}$

or $A + B\cos\theta = \dfrac{B\sin\theta}{\tan 90^\circ} = 0$

or $B\cos\theta = -A$ or $\cos\theta = -B/A$

From (*ii*), $A^2 + B^2 + 2AB(-B/A) = 64$

or $A^2 - B^2 = 64$ ...(*iii*)

Solving (*i*) and (*iii*), we get

$A = \mathbf{10\ N}$ and $B = \mathbf{6\ N}$

8. Acceleration, $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{\vec{F}}{m}$

$$= \frac{6t^2\,\hat{i} + 4t\,\hat{j}}{3} = 2t^2\,\hat{i} + \frac{4}{3}t\,\hat{j}$$

or $d\vec{v} = \left(2t^2\,\hat{i} + \dfrac{4}{3}t\,\hat{j}\right)dt$

Integrating it within the given limits

$$\vec{v} = \int_0^3 \left(2t^2\,\hat{i} + \frac{4}{3}t\,\hat{j}\right)dt$$

$$= \left[\frac{2}{3}t^3\,\hat{i} + \frac{4}{3}\frac{t^2}{2}\,\hat{j}\right]_0^3 = \mathbf{18\,\hat{i} + 6\,\hat{j}}$$

9. As the three forces acting on a particle are represented by the three sides of a triangle, taken in the same order, their resultant is zero, *i.e.*,

$$\vec{F}_{net} = 0 \text{ or } m\frac{d\vec{v}}{dt} = 0 \text{ or } \vec{v} = \textbf{a constant}$$

10. At the start, the velocity of athlete at A is v along AB. The component of this velocity along

$$AO = v\cos 45^\circ = v/\sqrt{2}.$$

Distance $AO = \dfrac{1}{2}AC = \dfrac{1}{2}(\sqrt{l^2 + l^2}) = \dfrac{l}{\sqrt{2}}$

$\therefore$ Time taken by athletes to meet at O is

$$t = \frac{\text{distance } AO}{\text{speed along } AO} = \frac{l/\sqrt{2}}{v/\sqrt{2}} = \frac{l}{v}$$

11. Let $AB = x$, then $AC = x\cos\theta$

$$\therefore \frac{d(AC)}{dt} = \frac{d}{dt}(x\cos\theta) = \frac{dx}{dt}\cos\theta + x(-\sin\theta)\frac{d\theta}{dt}$$

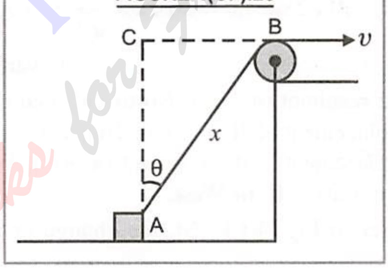
But $\dfrac{d(AC)}{dt} = 0$,

so $0 = \dfrac{dx}{dt}\cos\theta - x\sin\theta\dfrac{d\theta}{dt}$

or $v\cos\theta = x\sin\theta\dfrac{d\theta}{dt}$

or $\dfrac{d\theta}{dt} = \dfrac{v\cos\theta}{x\sin\theta}$.

FIGURE 4(CF).26



Also, $CB = x\sin\theta$.

Velocity of block will be

$$u = \frac{d(CB)}{dt} = \frac{dx}{dt}\sin\theta + x\cos\theta\frac{d\theta}{dt}$$

$$= v\sin\theta + x\cos\theta \times \frac{v\cos\theta}{x\sin\theta}$$

$$= \frac{v(\sin^2\theta + \cos^2\theta)}{\sin\theta} = \mathbf{\frac{v}{\sin\theta}}$$

12. The central point G of force $ABOD$ has coordinates $\left(\dfrac{a}{2}, 0, \dfrac{a}{2}\right)$. The central point H of face $BEFO$ has coordinates $\left(0, \dfrac{a}{2}, \dfrac{a}{2}\right)$.

$$\therefore \overrightarrow{GH} = \left(0 - \frac{a}{2}\right)\hat{i} + \left(\frac{a}{2} - 0\right)\hat{j} + \left(\frac{a}{2} - \frac{a}{2}\right)\hat{k}$$

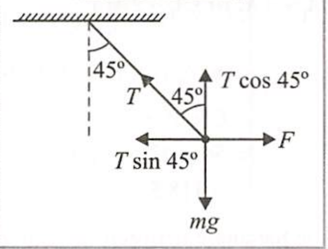
$$= \mathbf{\frac{1}{2}a(\hat{j} - \hat{i})}$$

HINTS/EXPLANATIONS

For Difficult Questions

13. Refer to Fig. 4(CF).27, at equilibrium

FIGURE 4(CF).27



$$T\cos 45^\circ = mg \qquad ...(i)$$

and $$T\sin 45^\circ = F \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{\sin 45^\circ}{\cos 45^\circ} = \frac{F}{mg} \quad \text{or} \quad \tan 45^\circ = \frac{F}{mg}$$

or $F = mg\tan 45^\circ = mg = 10 \times 10 = \mathbf{100\ N}$

14. $10^2 = A^2 + B^2 + 2\,AB\cos 150^\circ$

$$= A^2 + B^2 - 2\,AB\,(\sqrt{3}/2) \qquad \textbf{...(i)}$$

$$\tan 90^\circ = \frac{B\sin 150^\circ}{A + B\cos 150^\circ}$$

or $$A + B\cos 150^\circ = 0$$

or $$A = B \times \sqrt{3}/2 \qquad \textbf{...(ii)}$$

From (*i*), $100 = \frac{3}{4}B^2 + B^2 - \frac{3\,B^2}{2} = \frac{B^2}{4}$

or $\mathbf{B = 20\ N}$

15. Let $OA = OB = OC = P$

Component of $\overrightarrow{OA}$ along *x*-axis

$$= P\cos 30^\circ = \frac{P\sqrt{3}}{2}$$

Component of $\overrightarrow{OA}$ along *y*-axis $= P\sin 30^\circ = \frac{P}{2}$

Component of $\overrightarrow{OB}$ along *x*-axis $= P\cos 60^\circ = \frac{P}{2}$

Component of *OB* along *y*-axis

$$= -P\sin 60^\circ = \frac{-P\sqrt{3}}{2}$$

Component of $\overrightarrow{OC}$ along *x*-axis

$$= -P\sin 45^\circ = \frac{-P}{\sqrt{2}}$$

Component of $\overrightarrow{OC}$ along *y*-axis

$$= P\cos 45^\circ = \frac{P}{\sqrt{2}}$$

$\therefore$ *x*-component of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$,

$$|\vec{P}_x| = \frac{P\sqrt{3}}{2} + \frac{P}{2} - \left(\frac{-P}{\sqrt{2}}\right) = \frac{P}{2}(\sqrt{3} + 1 + \sqrt{2})$$

y-component of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$,

$$|\vec{P}_y| = \frac{P}{2} + \left(\frac{-P\sqrt{3}}{2}\right) - \frac{P}{\sqrt{2}} = \frac{P}{2}(1 - \sqrt{3} - \sqrt{2})$$

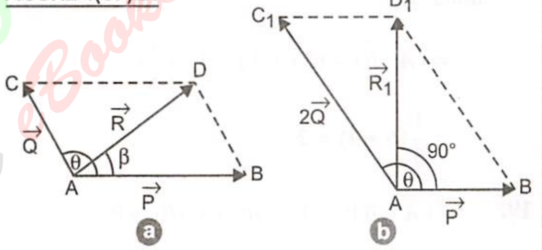
$\therefore$ Angle of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with *x*-axis is

$$\theta = \tan^{-1}(P_y/P_x) = \tan^{-1}\left[\frac{\frac{P}{2}(1 - \sqrt{3} - \sqrt{2})}{\frac{P}{2}(1 + \sqrt{3} + \sqrt{2})}\right]$$

$$= \mathbf{\tan^{-1}\left(\frac{1 - \sqrt{3} - \sqrt{2}}{1 + \sqrt{3} + \sqrt{2}}\right)}$$

16. Refer to Fig. 4(CF).28 (*a*)

FIGURE 4(CF).28



$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \text{and}$$

$$R^2 = P^2 + Q^2 + 2\,PQ\cos\theta \qquad \textbf{...(i)}$$

When Q is doubled, resultant $\vec{R_1}$ is perpendicular to $\vec{P}$. Fig. 4(CF).28 (*b*). Then

$$R_1^2 = P^2 + 4\,Q^2 + 4\,PQ\cos\theta \qquad \textbf{...(ii)}$$

From right angled triangle BAD_1, we have

$$R_1^2 = (2\,Q)^2 - P^2 = 4\,Q^2 - P^2$$

Putting this value in (*ii*), we get

$$4\,Q^2 - P^2 = P^2 + 4\,Q^2 + 4\,PQ\cos\theta$$

or $P^2 + 2\,PQ\cos\theta = 0$

Putting this value in (*i*), we get

$$R^2 = (P^2 + 2\,PQ\cos\theta) + Q^2 = 0 + Q^2 = Q^2$$

or $R = Q$

HINTS/EXPLANATIONS

For Difficult Questions

17. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Given,

$$|\vec{A}+\vec{B}| = 4\,|\vec{A}-\vec{B}|$$

$$\therefore \quad |\vec{A}+\vec{B}|^2 = 16\,|\vec{A}-\vec{B}|^2$$

or $A^2+B^2+2AB\cos\theta$

$$= 16\,[A^2+B^2-2AB\cos\theta]$$

or $A^2+A^2+2A^2\cos\theta = 16\,[A^2+A^2-2A^2\cos\theta]$

(∵ $A = B$)

or $2+2\cos\theta = 16\,(2-2\cos\theta)$

or $(16+1)\cos\theta = (16-1)$

or $$\cos\theta = \left(\frac{\mathbf{16-1}}{\mathbf{16+1}}\right) = \frac{\mathbf{15}}{\mathbf{17}}$$

18. Here, $\vec{A} = 2\hat{i}+3\hat{j}+\hat{k}$; $\vec{B} = 3\hat{i}+4\hat{k}$

$$\therefore \quad \hat{B} = \frac{\vec{B}}{B} = \frac{3\hat{i}+4\hat{k}}{\sqrt{3^2+4^2}} = \frac{1}{5}[3\hat{i}+4\hat{k}]$$

Magnitude of the component of the vector $\vec{A}$ along $\vec{B}$

$$= (\vec{A}\cdot\hat{B}) = (2\hat{i}+3\hat{j}+\hat{k})\cdot\left[\frac{1}{5}(3\hat{i}+4\hat{k})\right]$$

$$= \frac{1}{5}(6+4) = \mathbf{2}$$

19. $$|\vec{A}+\vec{B}|^2 = A^2+B^2+2AB\cos\theta$$

and $|\vec{A}-\vec{B}|^2 = A^2+B^2-2AB\cos\theta$

As $|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$

so $|\vec{A}+\vec{B}|^2 = n^2|\vec{A}-\vec{B}|^2$

∴ $A^2+B^2+2AB\cos\theta = n^2[A^2+B^2-2AB\cos\theta)$

$$= n^2A^2+n^2B^2-2n^2AB\cos\theta$$

or $\cos\theta\times 2AB(1+n^2) = A^2(n^2-1)+B^2(n^2-1)$

or $$\cos\theta = \frac{(n^2-1)(A^2+B^2)}{2AB(1+n^2)} = \frac{(n^2-1)(A^2+A^2)}{2A\times A(1+n^2)}$$

[∵ $A = B$]

$$= \frac{n^2-1}{1+n^2} \quad \therefore \quad \theta = \cos^{-1}\left(\frac{\mathbf{n^2-1}}{\mathbf{n^2+1}}\right)$$

20. Given, $A_1 = 3$, $A_2 = 5$, $|\vec{A}_1+\vec{A}_2| = 5$

As, $|\vec{A}_1+\vec{A}_2|^2 = |\vec{A}_1|^2+|\vec{A}_2|^2+2\vec{A}_1\cdot\vec{A}_2$

$(5)^2 = 3^2+5^2+2(\vec{A}_1\cdot\vec{A}_2)$ or $\vec{A}_1\cdot\vec{A}_2 = -9/2$

∴ $(2\vec{A}_1+3\vec{A}_2)\cdot(3\vec{A}_1-2\vec{A}_2)$

$$= 6|\vec{A}_1|^2+5(\vec{A}_1\cdot\vec{A}_2)-6|\vec{A}_2|^2$$

$$= 6\times 3^2+5\times(-9/2)-6\times 5^2$$

$$= 54-22{\cdot}5-150$$

$$= \mathbf{-118{\cdot}5}$$

21. Let θ be the angle between $\vec{P}$ and $\vec{Q}$, then

$$|\vec{P}+\vec{Q}|^2 = P^2+Q^2+2PQ\cos\theta$$

$$F_R^2 = 4F^2+9F^2+2\times 2F\times 3F\times\cos\theta$$

$$= 13F^2+12F^2\cos\theta \quad \text{...}(i)$$

Also, $|\vec{P}+2\vec{Q}|^2$

$$= 4F^2+36F^2+2\times 2F\times 6F\cos\theta$$

$$= 40F^2+24F^2\cos\theta$$

or $4F_R^2 = 40F^2+24F^2\cos\theta$

or $F_R^2 = 10F^2+6F^2\cos\theta \quad \text{...}(ii)$

From (*i*) and (*ii*), we get

$$13F^2+12F^2\cos\theta = 10F^2+6F^2\cos\theta$$

or $13+12\cos\theta = 10+6\cos\theta$

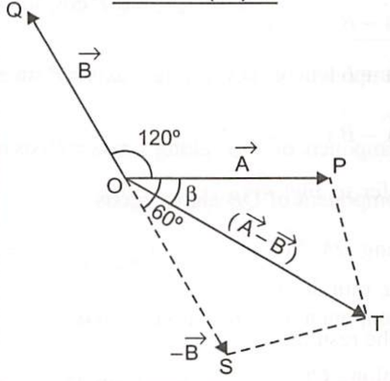
or $6\cos\theta = -3$

or $\cos\theta = -\frac{3}{6} = -\frac{1}{2} = \cos 120^\circ$

or $\mathbf{\theta = 120^\circ}$

22. Refer to Fig. 4(CF).29

FIGURE 4(CF).29



HINTS/EXPLANATIONS For Difficult Questions

$$\vec{A} = (\overrightarrow{OP}),\ \vec{B} = (\overrightarrow{OQ}),\ \angle POQ = 120^\circ$$

Now $-\vec{B} = \overrightarrow{OS}$, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$$= (\overrightarrow{OP}) + (\overrightarrow{OS}) = (\overrightarrow{OT})$$

$\angle POS$, $\theta = 180^\circ - 120^\circ = 60^\circ$,

i.e., angle between $\vec{A}$ and $(-\vec{B})$ is 60°.

Let β be the angle between $\vec{A}(= \overrightarrow{OP})$ and $\vec{A} - \vec{B}(= \overrightarrow{OT})$. Then

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} = \frac{A\sin 60^\circ}{A + A\cos 60^\circ}$$

$$= \frac{\sqrt{3}/2}{1 + 1/2} = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad \text{or} \quad \beta = \mathbf{30^\circ}$$

23. $|\hat{A} + \hat{B}| = \sqrt{|\hat{A}|^2 + |\hat{B}|^2 + 2|\hat{A}||\hat{B}|\cos\theta}$

$$= \sqrt{1 + 1 + 2 \times 1 \times 1 \times \cos\theta}$$

$$= \sqrt{2(1 + \cos\theta)}$$

$$= \sqrt{2 \times 2\cos^2\theta/2} = 2\cos\theta/2 \qquad \text{...(1)}$$

$$|\hat{A} - \hat{B}| = \sqrt{|\hat{A}|^2 + |\hat{B}|^2 - 2|\hat{A}||\hat{B}|\cos\theta}$$

$$= \sqrt{1 + 1 - 2\cos\theta} = \sqrt{2(1 - \cos\theta)}$$

$$= \sqrt{2 \times 2\sin^2\theta/2} = 2\sin\theta/2 \qquad \text{...(2)}$$

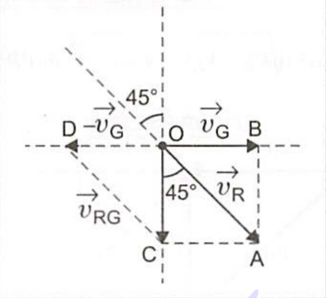
Dividing (2) by (1), we get

$$\frac{|\hat{A} - \hat{B}|}{|\hat{A} + \hat{B}|} = \tan\theta/2 \qquad \text{or}$$

$$\mathbf{|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}|\tan\theta/2}$$

24. Refer to Fig. 4(CF).30, velocity of rain, $\vec{v}_R$ is along *OA*. The velocity of girl $\vec{v}_G$ is along *OB*. The rain drops will hit the head of girl vertically if the resultant velocity $\vec{v}_{RG}$ of $\vec{v}_R$ and $(-\vec{v}_G)$ is along *OC*.

FIGURE 4(CF).30



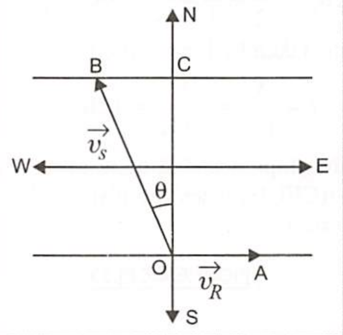
In ΔOAC, $\quad \tan 45^\circ = \dfrac{v_G}{v_{RG}}$

or $\quad v_{RG} = \dfrac{v_G}{\tan 45^\circ} = \dfrac{15\sqrt{2}}{1}$

$$= 15\sqrt{2} = \frac{\mathbf{30}}{\mathbf{\sqrt{2}}}\ \mathbf{kmh^{-1}}$$

25. Here, $\vec{v}_R = 10\ \text{ms}^{-1}$ due east $= \overrightarrow{OA}$. The swimmer will cross the river through shortest path, if the resultant of $\vec{v}_R$ and $\vec{v}_S$ is along $(\overrightarrow{OC})$. It will be so if the swimmer goes along *OB*. In Fig. 4(CF).31, $|\vec{v}_s| = OB = 20\ \text{ms}^{-1}$. Let $\angle COB = 30^\circ$.

FIGURE 4(CF).31



In ΔOBC,

$$\sin\theta = \frac{(CB)}{(OB)} = \frac{v_R}{v_s} = \frac{10}{20} = \frac{1}{2} = \sin 30^\circ$$

$\therefore \qquad \mathbf{\theta = 30^\circ\ west}$

26. If v_R is the resultant velocity of river water (v_r) and of boat (v_b), then

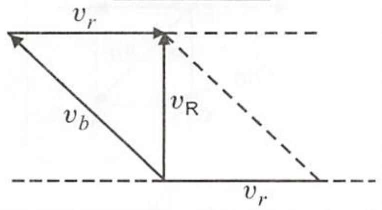
$$v_R = \frac{1\,\text{km}}{15\,\text{min.}} = \frac{1\,\text{km}}{(1/4)\,\text{h}} = 4\,\text{km}\,\text{h}^{-1}$$

HINTS/EXPLANATIONS

For Difficult Questions

$$v_r = \sqrt{v_b^2 - v_R^2} = \sqrt{5^2 - 4^2} = \mathbf{3\,km/h}$$

FIGURE 4(CF).32



27. Let u and v be the speed of train and wind respectively. The speed of steam track of train moving in the direction of wind = $u - v$

The speed of steam track of train moving in the opposite direction of wind = $u + v$.

As per question, $(u + v) = 2\,(u - v)$ or $\mathbf{u = 3\,v}$.

28. Let v_s, v_w = velocity of ship and river water w.r.t. ground,

S = distance between two points A and B

Log of wood will move with the velocity of river water.

As per question, $v_s + v_w = S/2$ ...(*i*)

$v_s - v_w = S/3$ ...(*ii*)

Subtracting (*ii*) from (*i*), we have

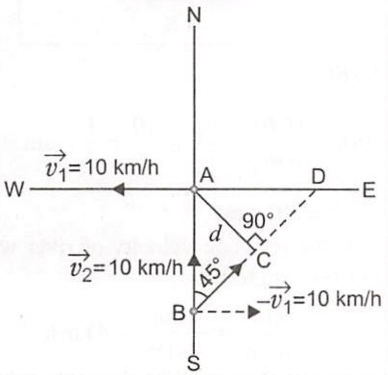
$$2v_w = \frac{S}{2} - \frac{S}{3} = \frac{S}{6} \quad \text{or} \quad v_w = \frac{S}{12}$$

∴ time taken by log of wood,

$$t = \frac{S}{v_w} = \frac{S}{S/12} = \mathbf{12\,h}$$

29. Let the ships A and B be at positions shown in Fig. 4(CF).33, when the distance between them is shortest.

FIGURE 4(CF).33



Relative velocity of B w.r.t. A is

$$v_r = \sqrt{v_1^2 + v_2^2} = \sqrt{10^2 + 10^2}$$

$$= 10\sqrt{2}\text{ km/h along } BC.$$

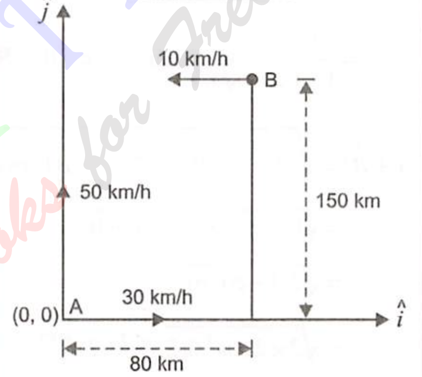
The shortest distance between A and C is d given by, $d = AC = AB \sin 45°$

$$= 100 \times \frac{1}{\sqrt{2}} = \mathbf{50\sqrt{2}\,km}$$

$$\text{Shortest time } t = \frac{d}{v_r} = \frac{50\sqrt{2}}{10\sqrt{2}} = \mathbf{5\,h}$$

30. Refer to Fig. 4(CF).34, let position of ship A be origin, then positions and velocities of both ships can be given as

FIGURE 4(CF).34



$$\vec{v}_A = (30\hat{i} + 50\hat{j})\text{ km/h},$$

$$\vec{v}_B = -10\hat{i}\text{ km/h};$$

$$\vec{r}_A = 0\hat{i} + 0\hat{j}$$

$$\vec{r}_B = (80\hat{i} + 150\hat{j})\text{ km}$$

$$\therefore \quad \vec{r}_{BA} = (80 - 0)\,\hat{i} + (150 - 0)\,\hat{j}$$

$$= (80\,\hat{i} + 150\hat{j})\text{ km}$$

$$\vec{v}_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j})$$

$$= (-40\hat{i} - 50\hat{j})\text{ km/h}$$

Time after which distance between them will be minimum [In this activation $\vec{r}_{BA}$ will be with a negative sign]

HINTS/EXPLANATIONS

For Difficult Questions

$$t = -\frac{(\vec{r}_{BA})}{(\vec{v}_{BA})} = -\frac{(\vec{r}_{BA})\cdot(\vec{v}_{BA})}{|\vec{v}_{BA}|^2}$$

$$= -\frac{(80\hat{i}+150\hat{j})\cdot(-40\hat{i}-50\hat{j})}{|-40\hat{i}-50\hat{j}|^2}$$

$$= \frac{3200+7500}{4100}\,h = \frac{10700}{4100} = \mathbf{2\cdot 6\ h}$$

31. Let car C be moving in opposite direction to A and B with velocity $\vec{v}_C$ relative to ground. Then relative velocity of C relative to A and B will be

$\vec{v}_{\text{rel}} = \vec{v}_C - \vec{v}$, As $\vec{v}$ is opposite to $\vec{v}_C$

So $v_{\text{rel}} = v_C - (-30) = (v_C + 30)$ km h^{-1}

Hence, time taken by car C to cross the cars A and B will be $t = \frac{S}{v_{\text{rel}}}$ or $\frac{4}{60} = \frac{5}{v_C+30}$

or $\quad v_C = \mathbf{45\ km\ h^{-1}}$

32. Let $\vec{u}$ be the velocity of man and $\vec{v}$ be the velocity of rain. If $\hat{i}$ and $\hat{j}$ are unit vectors along horizontal road and vertical direction respectively. According to question

$$\vec{u} = 8\hat{i} \quad \text{and} \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$$

If $\vec{v}_{rm}$ is the velocity of rain relative to man, then

$$\vec{v}_{rm} = \vec{v} - \vec{u} = (v_x\hat{i} + v_y\hat{j}) - 8\hat{j}$$

$$= (v_x - 8)\hat{i} + v_y\hat{j}$$

As rain appears to be falling vertically downwards.

$$\vec{v}_{rm} = -v_r\hat{j} \quad \therefore \quad -\vec{v}_r\hat{j} = (v_x-8)\hat{i} + v_y\hat{j}$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$ on both the sides, we get

$$v_x - 8 = 0 \quad \text{or} \quad v_x = 8 \quad \text{and} \quad -v_r = v_y$$

When man increases his speed, then $\vec{u} = 12\hat{i}$.

Let the relative velocity of rain w.r.t. man be $\vec{v}_{r'm}$,

then $\vec{v}_{r'm} = \vec{v} - \vec{u} = (v_x\hat{i} + v_y\hat{j}) - 12\hat{i}$

$$= (8\hat{i} + v_y\hat{j}) - 12\hat{i} = -4\hat{i} + v_y\hat{j}$$

As rain appears to be falling at an angle 30° with the vertical, therefore angle with the road *i.e.* horizontal direction $\theta = (90^\circ - 30^\circ) = 60^\circ$. Now

$$\tan 60^\circ = \frac{v_y}{-4} \quad \text{or} \quad v_y = -4\tan 60^\circ = -4\sqrt{3}\ \text{m/s}$$

$$\therefore \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = 8\hat{i} - 4\sqrt{3}\,\hat{j}$$

$$|\vec{v}| = \sqrt{(8)^2 + (-4\sqrt{3})^2} = \sqrt{64+48}$$

$$= \mathbf{10{\cdot}58\ km\ h^{-1}}$$

33. Relative velocity of a train with respect to another train, $\quad u = (v - v_1)$

using, $\quad v^2 - u^2 = 2\,aS$, we have

$$0 - (v - v_1)^2 = 2\,a \times d \quad \text{or} \quad a = -\frac{(v-v_1)^2}{2d}$$

34. Let d be the width of lake to be crossed. Then

$$T_0 = \frac{2d}{v}$$

and $\quad T = \frac{d}{v+v_1} + \frac{d}{v-v_1}$

$$= \frac{2dv}{v^2 - v_1^2} = \frac{2d}{v(1 - v_1^2/v^2)}$$

$$\therefore \quad \frac{T}{T_0} = \frac{1}{(1 - v_1^2/v^2)}$$

35. Speed of thief's car w.r.t. ground,

$$v_{TG} = 192\ \text{km h}^{-1}$$

Speed of police van w.r.t. ground,

$$v_{PG} = 30\ \text{km h}^{-1}$$

Speed of bullet w.r.t. police van,

$$v_{BP} = 150\ \text{ms}^{-1} = 540\ \text{km h}^{-1}$$

Speed with which the bullet will hit the thief's car will be

$$v_{BT} = v_{BG} + v_{GT} = v_{BP} + v_{PG} + v_{GT}$$

$$= 540 + 30 - 192 \qquad [\because\ v_{GT} = -v_{TG}]$$

$$= 378\ \text{km h}^{-1} = 378 \times \frac{1000}{3600}\ \text{ms}^{-1} = \mathbf{105\ ms^{-1}}$$

36. $\vec{r}_1 = (3\hat{i} + 5\hat{j})\,m$; $\vec{r}_2 = (-5\hat{i} - 3\hat{j})\,m$

$$\therefore\ \Delta\vec{r} = \vec{r}_1 - \vec{r}_2$$

$$= (3\hat{i} + 5\hat{j}) - (-5\hat{i} - 3\hat{j}) = (8\hat{i} + 8\hat{j})\,\text{m}$$

$$\vec{v}_1 = (4\hat{i} + 3\hat{j})\,\text{m/s} \ ; \ \vec{v}_2 = (a\hat{i} + 7\hat{j})\,\text{m/s}$$

$$\therefore \quad \Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = (a\hat{i} + 7\hat{j}) - (4\hat{i} + 3\hat{j})$$

$$= (a-4)\hat{i} + 4\hat{j}$$

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As, $\Delta \vec{v} = \frac{\Delta \vec{r}}{\Delta t}$;

so $(a-4)\hat{i} + 4\hat{j} = (8\hat{i} + 8\hat{j})/2$

On solving, we get, $a = \mathbf{8}$

37. Acceleration of the body down the inclined plane of inclination θ with horizontal direction is,

$$a = g \sin\theta$$

It is along the inclined plane.

The vertical component of acceleration a is

$$a_{(\text{along vertical})} = a\cos(90^\circ - \theta) = a \sin\theta$$
$$= (g\sin\theta)\sin\theta = g\sin^2\theta$$

For block A ; $a_{A\,(\text{along vertical})} = g\sin^2 60^\circ$

For block B ; $a_{B\,(\text{along vertical})} = g\sin^2 30^\circ$

The relative vertical acceleration of A w.r.t. B is

$$a_{AB\,(\text{along vertical})} = a_{A\,(\text{along vertical})} - a_{B\,(\text{along vertical})}$$
$$= g\sin^2 60^\circ - g\sin^2 30^\circ$$
$$= g\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right] = \frac{g}{2}$$
$$= \mathbf{4{\cdot}9\ ms^{-2}}\ \textbf{in vertical direction}$$

38. Velocity of girl w.r.t. escalator, $v_{ge} = \frac{d}{t_1}$

velocity of escalator w.r.t. ground, $v_{eG} = \frac{d}{t_2}$

velocity of girl on moving escalator w.r.t. ground

$$\vec{v}_{gG} = \vec{v}_{ge} + \vec{v}_{eG}$$

i.e., $v_{gG} = v_{ge} + v_{eG}$ or $\frac{d}{t} = \frac{d}{t_1} + \frac{d}{t_2}$

or $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ or $t = \frac{t_1 t_2}{t_1 + t_2}$

39. Let t_1 be the time taken by the stone to fall through a vertical distance h and t_2 be the time taken by the sound to travel distance h.

Since sound moves with a constant velocity,

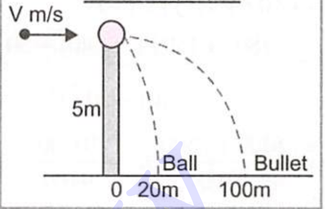
∴ $t_2 = \frac{h}{v}$

Taking downward motion, $t_1 = \sqrt{\frac{2h}{g}}$

total time $= t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

40. Refer to Fig. 4(CF).35, taking vertical downward motion of ball or bullet, we have

FIGURE 4(CF).35



time of flight, $T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5\,m}{10\,m/s^2}} = 1\,s$

After collision, the velocity of ball,

$$V_{ball} = 20/1 = 20\ \text{m/s}.$$

the velocity of bullet, $V_{bullet} = 100/1 = 100$ m/s.

According to law of conservation of linear momentum, we get

$$m_{\text{ball}} \times 0 + m_{\text{bullet}} \times V = m_{\text{ball}} \times V_{\text{ball}} + m_{\text{bullet}} \times V_{\text{bullet}}$$
$$0 + 0{\cdot}01 \times V = 0{\cdot}2 \times 20 + 0{\cdot}01 \times 100$$

or $\mathbf{V = 500\ m/s}$

41. Here, $u_x = 0$, $a_x = 5\ \text{ms}^{-2}$, $t = 4$ s, $v_x = ?$

As, $v_x = u_x + a_x t = 0 + 5 \times 4 = 20\ \text{ms}^{-1}$

At $t = 4$ s, a ball is dropped out of a window of car. Now the ball has two dimensional motion. A horizontal motion with a constant velocity 20 ms^{-1} and zero acceleration and vertical downward motion with zero initial velocity and acceleration, $g = 10\ \text{ms}^{-2}$.

Taking motion of the ball for 2 seconds in horizontal direction, we have,

$$u_x = 20\ \text{ms}^{-1},\ a_x = 0,\ t = 2\ \text{s}$$
$$v_x = u_x + a_x t = 20 + 0 \times 2 = 20\ \text{ms}^{-1}$$

Taking motion of the ball for 2 seconds in vertical downward direction, we have,

$$v_x = 0,\ a_y = g = 10\ \text{ms}^{-2},\ t = 2\text{s}$$
$$v_y = u_y + a_y t = 0 + 10 \times 2 = 20\ \text{ms}^{-1}$$

So magnitude of velocity of the ball is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (20)^2}$$
$$= \mathbf{20\sqrt{2}\ ms^{-2}}$$

Since the ball is under the free fall, so the acceleration of the ball at $t = 6$ s is,

$$a_y = \mathbf{10\ ms^{-2}}$$

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42. As $\vec{r} = \vec{r}_0 + \vec{u}_0 t + \frac{1}{2}\vec{a}t^2$

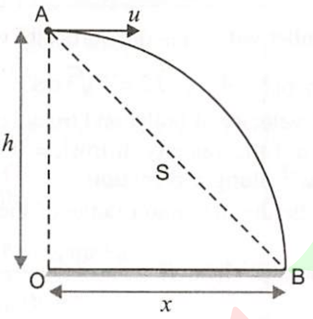
$$= (2{\cdot}0\,\hat{i} + 4{\cdot}0\,\hat{j}) + (5{\cdot}0\,\hat{i} + 4{\cdot}0\,\hat{j})2 + \frac{1}{2}(4{\cdot}0\,\hat{i} + 4{\cdot}0\,\hat{j})4$$

$$= (20\,\hat{i} + 20\,\hat{j})\,\text{m}$$

$$\therefore \; |\vec{r}| = \sqrt{20^2 + 20^2} = \mathbf{20\sqrt{2}\,m}$$

43. Refer to Fig. 4(CF).36, Let T be the time taken by particle to go from A to B.

FIGURE 4(CF).36



Taking vertical downward motion of particle from A to B, we have

$u_y = 0,\ a_y = g,\ y = h,\ y_0 = 0,\ t = T$

As, $\quad y = y_0 + u_y t + \frac{1}{2}a_y t^2$

$$\therefore \quad h = 0 + 0 + \frac{1}{2}gT^2 \text{ or } T = \sqrt{\frac{2h}{g}}$$

$$\text{Distance, } OB = x = uT = u\sqrt{\frac{2h}{g}}$$

$$\therefore \text{ Distance, } AB = S = \sqrt{h^2 + x^2}$$

$$= \sqrt{h^2 + u^2\left(\frac{2h}{g}\right)} = h\sqrt{\mathbf{1} + \frac{\mathbf{2u^2}}{\mathbf{gh}}}$$

44. Initial momentum of body along horizontal direction, $p_x = mv \cos 45° = \frac{mv}{\sqrt{2}}$

Initial momentum along vertical upward direction

$$p_y = mv \sin 45° = \frac{mv}{\sqrt{2}}$$

$$\therefore \text{ Total momentum } = \vec{p}_x\,\hat{i} + \vec{p}_y\,\hat{j}$$

$$= \frac{mv}{\sqrt{2}}\hat{i} + \frac{mv}{\sqrt{2}}\hat{j}$$

The body will strike the horizontal surface with velocity v, hence its momentum along vertical downward direction $P_y' = mv \sin 45° = \frac{mv}{\sqrt{2}}$

momentum along horizontal direction

$$= p_x' = mv \cos 45° = \frac{mv}{\sqrt{2}}$$

Total momentum

$$\vec{p}\,' = \vec{p}_x\,'\,\hat{i} + \vec{p}_y\,'\,\hat{j} = \frac{mv}{\sqrt{2}}\hat{i} - \frac{mv}{\sqrt{2}}\hat{j}$$

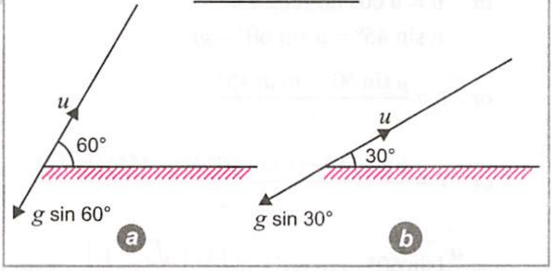
$$\text{Change in momentum } = \Delta\vec{p} = \vec{p} - \vec{p}\,'$$

$$= \left(\frac{mv}{\sqrt{2}}\hat{i} + \frac{mv}{\sqrt{2}}\hat{j}\right) - \left(\frac{mv}{\sqrt{2}}\hat{i} - \frac{mv}{\sqrt{2}}\hat{j}\right) = \frac{2mv}{\sqrt{2}}\hat{j}$$

$$= \mathbf{\sqrt{2}\,mv}$$

45. Refer to Fig. 4(CF).37 (a), stopping distance,

FIGURE 4(CF).37



$$x_1 = \frac{u^2}{2g \sin 60°}$$

Refer to Fig. 4(CF).37 (b), stopping distance,

$$x_2 = \frac{u^2}{2g \sin 30°}$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{\sin 30°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = \frac{\mathbf{1}}{\mathbf{\sqrt{3}}}$$

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46. Here $u = 20$ m/s, $\theta = 45°$, $d = 25$ m

Horizontal range ;

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \sin 2\times 45°}{9{\cdot}8}$$

$$= \frac{400}{9{\cdot}8}\times \sin 90° = \frac{400}{9{\cdot}8} = 40{\cdot}82 \text{ m}$$

Time of flight,

$$T = \frac{20\sin\theta}{g} = \frac{2\times 20 \sin 45}{9{\cdot}8} = \frac{2\times 20\times 1}{9{\cdot}8\times\sqrt{2}}$$

or $T = 2{\cdot}886$ s

Since the goal man is already 25 m away in the direction of the ball. So to catch the ball, he is to cover a distance = 40·82 – 25 = 15·82 m, in time 2·886 sec.

∴ the velocity of goal man to catch the ball is

$$v = \frac{15{\cdot}82}{2{\cdot}886} = \mathbf{5{\cdot}483 \text{ m/s}}$$

47. Let the body be projected from O with velocity u making an angle 60° with the horizontal and it reaches at A after time t with velocity v making an angle 45° with the horizontal direction. Resolving u and v into horizontal and vertical components we have

$$u \cos 60° = v \cos 45°$$

or $v = u \cos 60°/\cos 45°$

$$v \sin 45° = u \sin 60° - gt$$

or $t = \dfrac{u \sin 60° - v \sin 45°}{g}$

or $t = \dfrac{u \sin 60° - (u \cos 60°/\cos 45°) \sin 45°}{g}$

$$= \frac{u}{g}[\sin 60° - \cos 60°] = \frac{147}{9{\cdot}8}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right] = \mathbf{5{\cdot}49 \text{ s}}$$

48. Horizontal range is same when angle of projection is α or (90° – α).

$$\therefore \quad R = \frac{u^2 \sin 2\alpha}{g} \; ; \; t_1 = \frac{2u \sin\alpha}{g} \qquad ...(i)$$

and $$t_2 = \frac{2u \sin(90° - \alpha)}{g} = \frac{2u \cos\alpha}{g} \qquad ...(ii)$$

From (*i*), $\sin\alpha = gt_1/2u$;

From (*ii*), $\cos\alpha = gt_2/2u$

$$\therefore \quad \sin^2\alpha + \cos^2\alpha = \left(\frac{g}{2u}\right)^2 (t_1^2 + t_2^2)$$

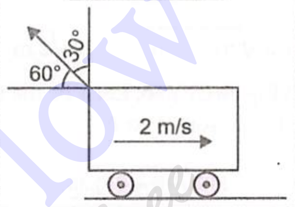
or $u = \dfrac{g}{2}(t_1^2 + t_2^2)^{1/2}$

49. Refer to Fig. 4(CF).38, the horizontal component velocity of bullet

= 4 cos 60° with – x direction

$= 4 \times 1/2 = -2 \text{ ms}^{-1}$ along x-direction

FIGURE 4(CF).38



The bullet will come out vertically with velocity $= 4 \sin 60° = 4\times\sqrt{3}/2 = 2\sqrt{3} \text{ ms}^{-1}$ if the horizontal velocity of bullet and truck is zero. It will be zero if the velocity of truck $= 0 - (-2 \text{ ms}^{-1})$ **$= 2 \text{ ms}^{-1}$ along x-direction.**

50. Let R be the maximum range of the bullet fired from a gun. Then, $R = \dfrac{u^2 \sin 2\times 45°}{g} = \dfrac{u^2}{g}$

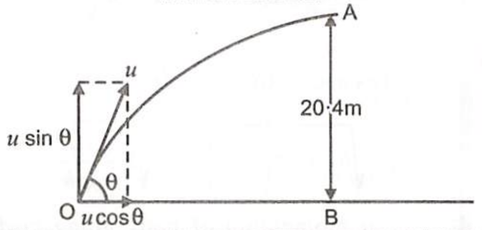
Area covered, $A = \pi R^2 = \pi\left(\dfrac{u^2}{g}\right)^2 = \dfrac{\pi u^4}{g^2}$

i.e., $A \propto u^4$

$$\therefore \quad \frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left(\frac{1}{2}\right)^4 = \mathbf{\frac{1}{16}}$$

51. Here, $u = 40$ m/s ; $BA = 20{\cdot}4$ m. Let θ be the angle of projection at O. Taking vertical motion of projectile from O to A we have

FIGURE 4(CF).39



$u = u \sin\theta = 40 \sin\theta$; $a = -9{\cdot}8 \text{ m/s}^2$,

$S = 20{\cdot}4$ m, $t = 2$ s. As $S = ut + \dfrac{1}{2}at^2$

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$$\therefore\ 20{\cdot}4 = (40\sin\theta)\times 2 + \frac{1}{2}(-9{\cdot}8)\times 2^2$$

or $20{\cdot}4 = 80\sin\theta - 19{\cdot}6$

or $80\sin\theta = 20{\cdot}4 + 19{\cdot}6 = 40$

or $\sin\theta = \dfrac{40}{80} = \dfrac{1}{2} = \sin 30°$ or $\theta = 30°$

52. **Ist method.** Initial momentum of the body,

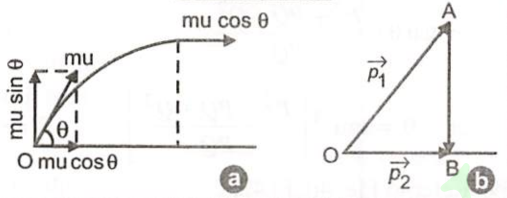
$$\vec{p}_1 = m\vec{u} = (0{\cdot}1\times 20) \text{ along } OA$$

Final momentum, $\vec{p}_2 = mu\cos\theta$ along OB

$$= 0{\cdot}1\times 20\times\cos 30°$$

$$= 0{\cdot}1\times 20\times\frac{\sqrt{3}}{2} = \sqrt{3}\ \text{kg ms}^{-1} \text{ along } OB.$$

FIGURE 4(CF).40



Change in momentum

$$= \vec{p}_2 - \vec{p}_1 = \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$$

Now $|\overrightarrow{AB}| = \sqrt{(OA)^2 - (OB)^2}$

$$= \sqrt{2^2 - (\sqrt{3})^2} = \mathbf{1\ kg\ ms^{-1}}$$

2nd method. Horizontal momentum of the body at O and at highest point is same. The vertical momentum at O = $mu\sin\theta$ and at highest point is zero. Therefore, charge in momentum

$$= mu\sin\theta = 0{\cdot}1\times 20\times\sin 30° = \mathbf{1\ kg\ ms^{-1}}$$

53. Let u be the initial velocity of projectile and θ be the angle of projection. Taking vertical upward motion for height h, we have

$u = u\sin\theta,\ a = -g,\ S = h,\ v = v_y$, As

$$v^2 = u^2 + 2aS\ ;$$

so $$v_y^2 = u^2\sin^2\theta - 2gh \qquad ...(i)$$

At height h, $\tan\dfrac{\pi}{4} = \dfrac{v_y}{v_x}$ or $v_y = v_x = u\cos\theta$

$\therefore\ u^2\cos^2\theta = u^2\sin^2\theta - 2gh$

or $u^2(\cos^2\theta - \sin^2\theta) = -2gh$

or $u^2\cos 2\theta = -2gh \qquad ...(ii)$

The equation (*ii*) is valid only if u = **50 m/s** and $\cos 2\theta = -8/25$

or $$\theta = \frac{1}{2}\cos^{-1}(-8/25)$$

54. $$h = (u\sin\theta)t - \frac{1}{2}gt^2 \qquad ...(i)$$

$d = u\cos\theta\times t$ or $t = d/(u\cos\theta)$

From (*i*), $$h = u\sin\theta\times\frac{d}{u\cos\theta} - \frac{1}{2}g\frac{d^2}{u^2\cos^2\theta}$$

or $$u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$$

55. Horizontal range is same when angle of projection is θ or $(90° - \theta)$,

$$R = \frac{u^2\sin 2\theta}{g},\quad t_1 = \frac{2u\sin\theta}{g}$$

and $$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore\ t_1 t_2 = \frac{2u\sin\theta}{g}\times\frac{2u\cos\theta}{g} = \frac{2u^2(2\sin\theta\cos\theta)}{g^2}$$

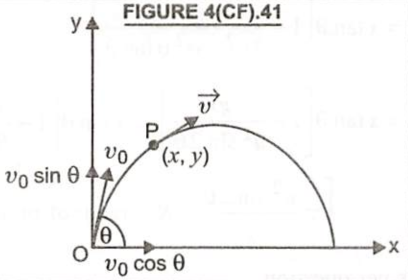
$$= \frac{2u^2\sin 2\theta}{g} = \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right)$$

or $$t_1 t_2 = \frac{2}{g}R$$

$$\therefore\ t_1 t_2 \propto R$$

56. Let the particle projected from O, reach P in time t with velocity $\vec{v} = (v_x\hat{i} + v_y\hat{j})$ and coordinates of P be (x, y).

FIGURE 4(CF).41



Then $v_x = v_0\cos\theta$, $v_y = v_0\sin\theta - gt$

$$x = v_0\cos\theta t,\quad y = v_0\sin\theta t - \frac{1}{2}gt^2$$

$$\vec{r} = (x\hat{i} + y\hat{j})$$

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Momentum of particle at P

$$\vec{p} = m\vec{v} = m(v_x\hat{i} + v_y\hat{j})$$

Angular momentum of particle at P about origin O is

$$\vec{L} = \vec{r}\times\vec{p} = (x\hat{i} + y\hat{j})\times m(v_x\hat{i} + v_y\hat{j})$$

$$= m y v_x(-\hat{k}) + m x v_y(\hat{k})$$

$$= m\left(v_0 \sin\theta t - \frac{1}{2}gt^2\right)v_0\cos\theta(-\hat{k})$$

$$+ m(v_0\cos\theta\, t)(v_0\sin\theta - gt)\ (\hat{k})$$

$$= -mv_0^2\cos\theta\sin\theta t\,\hat{k} + mv_0\cos\theta\times\frac{1}{2}gt^2\,\hat{k}$$

$$+ mv_0^2\sin\theta\cos\theta t\,\hat{k} - mv_0\cos\theta\, gt^2\,\hat{k}$$

$$= -\frac{1}{2}mv_0\,gt^2\cos\theta\,\hat{k}$$

57. $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

For equal trajectories for same angle of projection

$$\frac{g}{u^2} = \text{constant} \Rightarrow \frac{9{\cdot}8}{5^2} = \frac{g'}{3^2}$$

$$g' = \frac{9{\cdot}8\times 9}{25} = 3{\cdot}528\ \text{m/s}^2 = \mathbf{3{\cdot}5\ m/s^2}$$

58. For angular projection of a projectile, equation of trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta\tan\theta}\right]$$

$$= x\tan\theta\left[1 - \frac{gx}{u^2\sin 2\theta}\right] = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$\left[\because \frac{u^2\sin 2\theta}{g} = R = \text{range of projectile}\right]$$

As per question

$$Q = P\tan\theta\left[1 - \frac{P}{R}\right] \qquad \text{...(i)}$$

and

$$P = Q\tan\theta\left[1 - \frac{Q}{R}\right] \qquad \text{...(ii)}$$

Dividing (*i*) by (*ii*), we get

$$\frac{Q}{P} = \frac{P(R-P)}{Q(R-Q)} \quad\text{or}\quad Q^2R - Q^3 = P^2R - P^3$$

or $R(P^2 - Q^2) = P^3 - Q^3$

or $R = \dfrac{P^3 - Q^3}{P^2 - Q^2} = \dfrac{P^2 + PQ + Q^2}{(P+Q)}$

Putting this value in (*i*), we get

$$Q = P\tan\theta\left[1 - \frac{P(P+Q)}{P^2 + PQ + Q^2}\right]$$

$$= P\tan\theta\left[\frac{P^2 + PQ + Q^2 - P^2 - PQ}{P^2 + PQ + Q^2}\right]$$

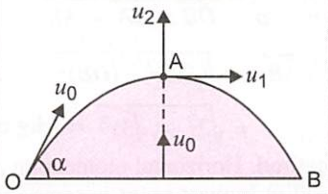
$$= P\tan\theta\left[\frac{Q^2}{P^2 + PQ + Q^2}\right]$$

or $\tan\theta = \dfrac{P^2 + PQ + Q^2}{PQ}$

or $\boldsymbol{\theta = \tan^{-1}\left[\dfrac{P^2 + PQ + Q^2}{PQ}\right]}$

59. Refer to Fig. 4(CF).42,

FIGURE 4(CF).42



speed of the first particle at highest point A, $u_1 = u_0\cos\alpha$

Speed of the second particle at the highest point A is $\quad u_2 = \sqrt{u_0^2 - 2gH}$

where $\quad H = \dfrac{u_0^2\sin^2\alpha}{2g}$

$$\therefore \quad u_2 = \sqrt{u_0^2 - 2gu_0^2\sin^2\alpha/2g} = u_0\cos\alpha.$$

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along horizontal and vertical directions respectively. Then

$$\vec{u_1} = u_0\cos\alpha\,\hat{i} \quad\text{and}\quad \vec{u_2} = u_0\cos\alpha\,\hat{j}$$

Let $\vec{v}$ be the velocity of composite system after inelastic collision and θ is the angle which it

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For Difficult Questions

makes with the horizontal direction. According to law of conservation of linear momentum

$$2m\,(v_x\,\hat{i}+v_y\,\hat{j}) = mu_0\cos\alpha\,\hat{i}+mu_0\cos\alpha\,\hat{j}$$

We have, $v_x = \dfrac{u_0\cos\alpha}{2}$ and $v_y = \dfrac{u_0\cos\alpha}{2}$

Then $\tan\theta = \dfrac{v_y}{v_x} = \dfrac{(u_0\cos\alpha)/2}{(u_0\cos\alpha)/2} = 1$ or $\theta = \dfrac{\pi}{4}$

60. Here, initial velocity, $\vec{u} = \hat{i}+2\,\hat{j}$. Comparing it, with $\vec{u} = u_x\,\hat{i}+u_y\,\hat{j}$, we have

$u_x = 1$ m/s and $u_y = 2$ m/s

Let the projectile at time t be at location whose position coordinates from origin are (x, y). Then

$$x = u_x\,t = 1\,.\,t = t$$

and $\quad y = u_y t - \dfrac{1}{2}\,gt^2 = 2\,t - \dfrac{1}{2}\times 10\times t^2$

$$= 2\,x - 5\,x^2 \qquad (\because t = x)$$

Thus $\boldsymbol{y = 2\,x - 5\,x^2}$**, i.e., option (b) is true**

61. For the pebble, $u = v$, $\theta = 45°$, $t = 1$ s

Time of flight, when the pebble will be at the horizontal level OX, where the block is hit

$$T = \frac{2\,u\sin\theta}{g} = \frac{2\,v\sin 45°}{10} = \frac{v}{5\sqrt{2}} = 1\,\text{sec}$$

$\therefore \quad v = 5\sqrt{2} = \boldsymbol{\sqrt{50}}$ **m/s**

If the block is hit at any point K on OX, then PK = horizontal range

$$= \frac{u^2\sin 2\theta}{g} = \frac{v^2\sin 90°}{10} = \frac{(\sqrt{50})^2\times 1}{10} = 5\text{ m}$$

This is likely, as unstretched length of spring is 4·9 m and it is stretched by 0·2 m.

62. Here, $\quad u = 150\text{ ms}^{-1}, \quad g = 10\text{ m/s}^2$

Let the body be projected from O with velocity u making an angle 45° with the horizontal and it reaches at A after time t with velocity v making an angle 30° with horizontal direction. Resolving u and v into horizontal and vertical components, we have $u\cos 45° = v\cos 30°$

[As in horizontal direction acceleration is zero]

or $\quad v = \dfrac{u\cos 45°}{\cos 30°}$

In vertical direction ; $v\sin 30° = u\sin 45° - gt$

$$t = \frac{1}{g}\,[u\cos 45° - v\sin 30°]$$

$$= \frac{1}{g}\left[u\cos 45° - \frac{u\cos 45°}{\cos 30°}\times\sin 30°\right]$$

$$= \frac{u}{g}\,[\cos 45° - \cos 45°\tan 30°]$$

$$= \frac{150}{10}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{3}}\right]$$

$$= \frac{15}{\sqrt{2}}\times\frac{\sqrt{3}-1}{\sqrt{3}} = \frac{15}{1{\cdot}414}\times\frac{(1{\cdot}732-1)}{1{\cdot}732} = \mathbf{4{\cdot}48\ s}$$

63. Here, radius of circular path = R,

Time period of revolution of particle = T

So, the speed of the particle, $u = \dfrac{2\pi R}{T}$

When the particle is projected with the same speed u at an angle θ to the horizontal, then

Maximum height, $H = \dfrac{u^2\sin^2\theta}{2\,g}$

Given, $\quad H = 4\,R,$

so $\quad \dfrac{u^2\sin^2\theta}{2\,g} = 4\,R \quad$ or $\quad \sin^2\theta = \dfrac{8\,g\,R}{u^2}$

or $\quad \sin^2\theta = \dfrac{8\,g\,R}{(2\,\pi\,R/T)^2} = \dfrac{2\,g\,T^2}{\pi^2\,R}$

or $\sin\theta = \left[\dfrac{2\,g\,T^2}{\pi^2\,R}\right]^{1/2}$ or $\boldsymbol{\theta = \sin^{-1}\left[\dfrac{2\,g\,T^2}{\pi^2\,R}\right]^{1/2}}$

64. $\quad v_x = \dfrac{dx}{dt} = -a\omega\sin\omega t\ ;$

$$v_y = \frac{dy}{dt} = a\omega\cos\omega t\ ;$$

$$v_z = \frac{dz}{dt} = a\omega$$

Speed of the particle,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$= \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + a^2\,\omega^2}$$

$$= \sqrt{2}\,a\omega$$

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65. Horizontal range will be same when angle of projection is θ and (90° – θ). Then

Horizontal range, $R = \dfrac{2u^2 \sin\theta\cos\theta}{g}$

$$h_1 = \frac{u^2 \sin^2\theta}{2g} \; ;$$

$$h_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$h_1 \times h_2 = \frac{u^2}{2g}\sin^2\theta \times \frac{u^2}{2g}\cos^2\theta$$

$$= \frac{1}{16}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]^2 = \frac{1}{16}R^2$$

$\therefore$ $\boldsymbol{R^2 = 16\, h_1\, h_2}$

66. Taking motion of the particle along the *x*-axis, we have :

$$x = 20\text{ m} \; ; x_0 = 0, u_x = 0, a_x = 10\text{ ms}^{-2}, t = t$$

As, $x = x_0 + u_x t + \dfrac{1}{2}a_x t^2$

$\therefore$ $20 = 0 + 0 + \dfrac{1}{2}\times 10 \times t^2 = 5\,t^2$

or $t^2 = 4$ or $\boldsymbol{t = 2\text{ s}}$

Taking motion of the particle along the *y*-axis, we have :

$$y = y \; ; y_0 = 0 \; ; u_y = 5\text{ ms}^{-1}, a_y = 4\text{ ms}^{-2}, t = 2\text{ s}$$

As, $y = y_0 + u_y t + \dfrac{1}{2}a_y t^2$

$\therefore$ $y = 0 + 5\times 2 + \dfrac{1}{2}\times 4\times 2^2 = \mathbf{18\ m}$

67. Here, $\vec r = \cos\omega t\,\hat x + \sin\omega t\,\hat y$ ...*(i)*

velocity, $\vec v = \dfrac{d\vec r}{dt} = -\omega\sin\omega t\,\hat x + \omega\cos\omega t\,\hat y$ ...*(ii)*

Acceleration, $\vec a = -\omega^2\cos\omega t\,\hat x - \omega^2\sin\omega t\,\hat y$

$= -\omega^2\,\vec r$ ...*(iii)*

From *(iii)*, we note that **accelerated is directed towards the origin** because $\vec a$ is in opposite direction to $\vec r$.

Here, $\vec v \cdot \vec r = (-\omega\sin\omega t\,\hat x + \omega\cos\omega t\,\hat y) . (\cos\omega t\,\hat x + \sin\omega t\,\hat y)$

$= -\,\omega\sin\omega t\cos\omega t + \omega\sin\omega t\cos\omega t = 0$

or $v\,r\cos\theta = 0$ or $\cos\theta = 0$ or $\theta = 90^\circ$

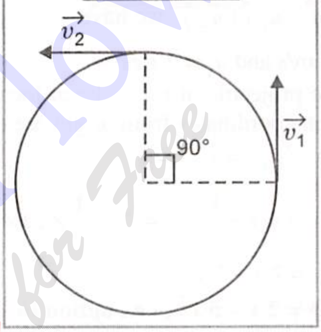
i.e., $\vec v$ **and** $\vec r$ **are perpendicular to each other.**

68. The angle described at the centre by the length of seconds hand of a watch in 15 seconds = 90° = π/2 radians Fig. 4(CF).43.

Linear speed, $v = r\,\omega = r\,\theta/t$

$= 1\times(\pi/2)/15 = \pi/30\text{ cm s}^{-1}$

FIGURE 4(CF).43



Magnitude of change in velocity in 15 sec;

$$|\Delta\vec v| = |\vec v_2 - \vec v_1| = \sqrt{v_2^2 + v_1^2}$$

$$= \sqrt{v^2 + v^2} = \sqrt{2}\,v$$

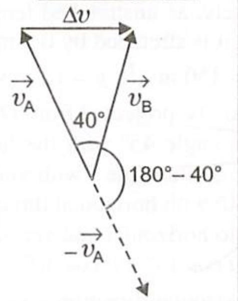
$$= \sqrt{2}\,\frac{\pi}{30} = \frac{\mathbf{2\pi}}{\mathbf{30\sqrt{2}}}\ \textbf{cm/s}$$

69. Here, $|\vec v_A| = |\vec v_B| = v$

Change in velocity

$$\Delta\vec v = \vec v_B - \vec v_A = \vec v_B + (-\vec v_A)$$

FIGURE 4(CF).44



Angle between $\vec v_B$ and $(-\vec v_A)$

$= 180^\circ - 40^\circ = 140^\circ$

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$$\Delta v = \sqrt{v^2 + v^2 + 2v.v\cos 140^\circ}$$

$$= \sqrt{v^2 + v^2 + 2v^2\cos(180^\circ - 40^\circ)}$$

$$= \sqrt{v^2 + v^2 - 2v^2\cos 40^\circ}$$

$$= \sqrt{2v^2(1-\cos 40^\circ)}$$

$$= \sqrt{2v^2 \times 2\sin^2 20^\circ} = \mathbf{2\,v\,\sin 20^\circ}$$

70. When $\theta = 180^\circ$, the particle will be at diametrically opposite point, where its velocity is opposite to the initial direction of motion. The change in momentum $= mv - (-mv) = \mathbf{2\,mv}$ **; (max).** When $\theta = 360^\circ$, the particle is at initial position with momentum mv. Change in momentum $= mv - mv = \mathbf{0}$ **(minimum).**

71. Here, $n_1 = \frac{1200}{60}$ rps ; $n_2 = \frac{3120}{60}$ rps,

$t = 16$ s, $\alpha = ?$

$$\omega_1 = 2\pi\, n_1 = 2\pi \times \frac{1200}{60} \text{ rad/s}$$

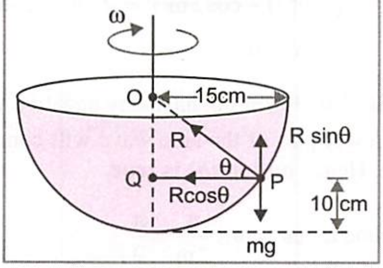
$$\omega_2 = 2\pi\, n_2 = 2\pi \times \frac{3120}{60} \text{ rad/s}$$

Angular acceleration, $\alpha = \frac{\omega_2 - \omega_1}{t}$

$$= \frac{2\pi}{60}\frac{(3120-1200)}{16} = \frac{2\pi}{60} \times \frac{1920}{16} = \mathbf{4\,\pi\ rad/s^2}$$

72. Let R be the force exerted by the bowl on the particle. It will be as a reactional force R. Various forces acting on particle at P will be (*i*) weight Mg (downwards) (*ii*) R (Normal Reaction) along PO.

FIGURE 4(CF).45



Resolving R into its two rectangular components we have $R \sin\theta$ acts vertically upwards and $R\cos\theta$ acts horizontally along PQ.

As horizontal component $R\cos\theta$ will provide required centripetal force.

$\therefore\ R\cos\theta = ma_c$

[$a_c \rightarrow$ centripetal acceleration]

$$R\cos\theta = mr\omega^2 \quad \text{or} \quad \omega = \sqrt{\frac{R\cos\theta}{mr}}$$

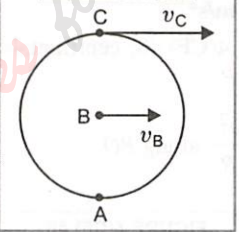
$$\cos\theta = \frac{r}{OP} = \frac{r}{0{\cdot}15}$$

$$\left[\begin{array}{l} \because R\sin\theta = mg \\ R = \dfrac{mg}{\sin\theta} = \dfrac{mg}{OQ/OP} = \dfrac{5\times 9{\cdot}8}{0{\cdot}05/0{\cdot}15} \\ R = 15 \times 9{\cdot}8\ \text{N} \end{array}\right]$$

$$\therefore\ \omega = \sqrt{\frac{15\times 9{\cdot}8 \times r/0{\cdot}15}{5\times r}} = \mathbf{14\ rad/s}$$

73. Let ω be the angular velocity of rotation of sphere. $\vec{v}_A, \vec{v}_B, \vec{v}_C$ be the velocities at A, B and C respectively. Then A is at rest, *i.e.* $|\vec{v}_A| = 0$.

FIGURE 4(CF).46



$|\vec{v}_C| = 2\omega R$ about A, $|\vec{v}_B| = \omega R$ about A

$\therefore\ |\vec{v}_C - \vec{v}_B| = 2\omega R - \omega R = \omega R$

$|\vec{v}_B - \vec{v}_A| = \omega R - 0 = \omega R$

$\therefore\ |\vec{v}_C - \vec{v}_B| = |\vec{v}_B - \vec{v}_A|$

Also, $|\vec{v}_C - \vec{v}_A| = (2\omega R - 0) = 2\omega R$

$|\vec{v}_B - \vec{v}_C| = |\omega R - 2\omega R| = |-\omega R|$

$\therefore\ |\vec{v}_C - \vec{v}_A| = \mathbf{2}|\vec{v}_B - \vec{v}_C|$

74. $s = t^3 + 5$; $r = 20$ m

velocity, $v = \frac{ds}{dt} = 3t^2$

When, $t = 2$ s, $v = 3 \times 2^2 = 12\ \text{ms}^{-1}$

Tangential acceleration, $a_T = \frac{dv}{dt} = 6t$

When, $t = 2$ s, $a_T = 6 \times 2 = 12\ \text{m/s}^2$

HINTS/EXPLANATIONS

For Difficult Questions

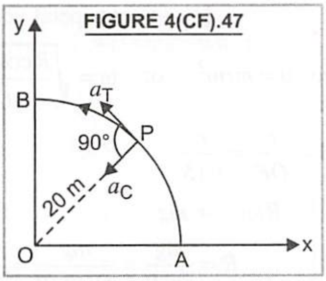


FIGURE 4(CF).47

Centripetal acceleration,

$$a_c = \frac{v^2}{r} = \frac{12^2}{20} = 7{\cdot}2\,\text{m/s}^2$$

Effective acceleration,

$$a = \sqrt{a_T^2 + a_c^2}$$

$$= \sqrt{12^2 + (7{\cdot}2)^2}$$

$$= \mathbf{14\ m/s^2}$$

75. Refer Fig. 4(CF).48, centripetal acceleration of particle at P

$$a_c = \frac{v^2}{R} \quad \text{along } PO$$

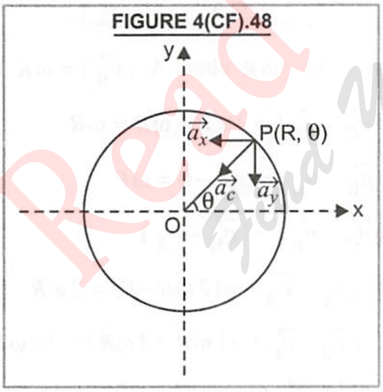


FIGURE 4(CF).48

As $\quad \vec{a}_c = \vec{a}_x + \vec{a}_y$

where, $a_x = a_c \cos\theta = \dfrac{v^2}{R}\cos\theta$

and $\quad a_y = a_c \sin\theta = \dfrac{v^2}{R}\sin\theta$

$$\therefore\ \vec{a}_c = a_x(-\hat{i}) + a_y(-\hat{j})$$

$$= -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$

76. Time of flight of each pebble,

$$T = \text{time for } \frac{1}{8}\text{th rotation of disc}$$

$$= \frac{2\pi}{\omega} \times \frac{1}{8} = \frac{\pi}{4\omega}$$

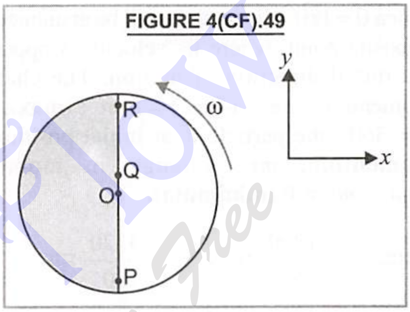


FIGURE 4(CF).49

Given range of each pebble < $r/2$.

Therefore, **pebble P lands in unshaded region and pebble Q in the shaded region. Thus option (*b*) is true.**

77. **At an instant,** speed of $P = v$, going in clockwise direction

Speed of $Q = v$, going in anticlockwise direction

Relative angular velocity of P w.r.t. $Q = \omega - (-\omega)$
$= 2\omega$

Relative angular separation of P and Q in time t
$\theta = 2\omega t$.

Relative speed between the points P and Q at time t

$$|\vec{v}_r| = \sqrt{v^2 + v^2 - 2vv\cos(2\omega t)}$$

$$= \sqrt{2v^2(1-\cos 2\omega t)} = \sqrt{2v^2 \times 2\sin^2 \omega t}$$

$$= 2v\sin\omega t$$

Since, $|\vec{v}_r|$ will not have any negative value so the lower part of the sine wave will come upper side. Hence **option (*a*) is true.**

78. At time t, $\quad \omega t = \omega \times \dfrac{\pi}{2\omega} = \dfrac{\pi}{2}$

The position of two particles will be as shown in Fig. 4(CF).50.

HINTS/EXPLANATIONS

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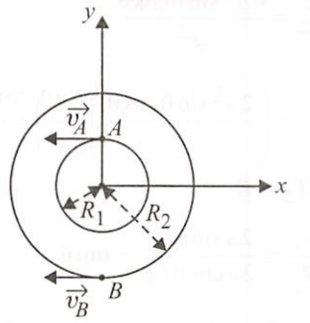
FIGURE 4(CF).50



Then, $\vec{v}_A = \omega R_1 (-\hat{i})$

and $\vec{v}_B = \omega R_2 (-\hat{i})$

$$\therefore\ \vec{v}_A - \vec{v}_B = \omega R_1 (-\hat{i}) - \omega R_2 (-\hat{i})$$

$$= \omega (R_2 - R_1)\hat{i}$$

79. Instantaneous acceleration of the tip of the hand is

$$a_i = \omega^2 r = \left(\frac{2\pi}{60}\right)^2 \times 0{\cdot}1 = \frac{4 \times 10 \times 0{\cdot}1}{3600}$$

$$= \frac{1}{900} \approx \mathbf{10^{-3}\ ms^{-2}}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

80. A bullet fired from gun and a piece of stone thrown in any direction are the example of projectile motion because their motion is under the action of gravity alone as well as motion is in one plane and not in a straight line.

81. $R = \dfrac{u^2 \sin^2\theta}{g}$

and $h = \dfrac{u^2 \sin 2\theta}{2g}$.

These two are incorrect relations while relations given in option (*c*) and (*d*) are correct.

82. Here, $u_x = 3\ \text{ms}^{-1}$; $a_x = -1{\cdot}0\ \text{ms}^{-2}$;

$u_y = 0$; $a_y = -0{\cdot}5\ \text{ms}^{-2}$

For maximum *x*-coordinates, the final horizontal velocity becomes zero, *i.e.* $v_x = 0$

As $v_x^2 = u_x^2 + 2a_x x$ so $0 = 3^2 + 2(-1)x$

or $x = 4{\cdot}5$ m

Also, $v_x = u_x + a_x t$ or $0 = 3 + (-1)t$

or $t = 3$ s

Taking motion along vertical direction, we have

$$y = u_y t + \frac{1}{2} a_y t^2 = 0 \times 3 + \frac{1}{2}(-0{\cdot}5) \times 3^2$$

$$= -2{\cdot}25\ \text{m}$$

and $v_y = u_y + a_y t = 0 + (-0{\cdot}5)\,3 = -1{\cdot}5\ \text{ms}^{-1}$

$$\therefore\ \vec{v} = v_x \hat{i} + v_y \hat{j} = 0 - 1{\cdot}5\,\hat{j}$$

$$= \mathbf{(-1{\cdot}5\,\hat{j})\,ms^{-1}}$$

and $\vec{r} = x\hat{i} + y\hat{j} = \mathbf{(4{\cdot}5\,\hat{i} - 2{\cdot}25\,\hat{j})\,m}$

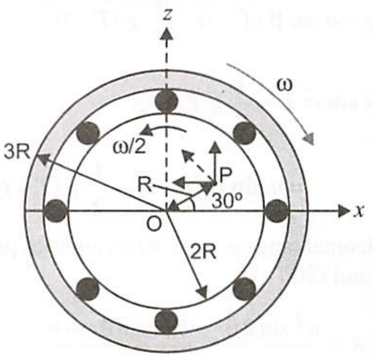
83. Since, the projectile has two component velocities *i.e.*, a horizontal velocity which is constant without air friction and a vertical velocity which changes with air friction and gravity pull, hence the path of projectile is a parabolic path.

Due to air friction, the energy spent by projectile against air friction is not conserved, so the total energy of projectile is not conserved.

At the highest point, the projectile has horizontal component velocity only.

84. As is clear from Fig. 4(CF).51 ;

FIGURE 4(CF).51



linear velocity of point $O = r\,\omega$

$$\vec{v}_0 = (3R)\,\omega\,\hat{i},$$

along positive direction of *x*-axis.

Choice (*a*) is correct.

Linear velocity of point P w.r.t. point O is

$$\vec{v}_{P_0} = \vec{v}_P - \vec{v}_0$$

or $$\vec{v}_P = \vec{v}_0 + \vec{v}_{P_0}$$

$$= (3R\omega)\,\hat{i} + \frac{\omega}{2}R\sin 30^\circ(-\hat{i}) + \frac{\omega}{2}R\cos 30^\circ\,\hat{k}$$

$$= (3R\omega)\,\hat{i} + \frac{\omega R}{4}(-\hat{i}) + \frac{\omega R}{2}\frac{\sqrt{3}}{2}\hat{k}$$

$$\vec{v}_P = \frac{11}{4}R\omega\,\hat{i} + \frac{\sqrt{3}}{4}R\omega\,\hat{k}$$

Choices (*a*) and (*b*) are correct.

85. Since the first shot must have travelled for more time than the second shot before colliding in mid air. It will be so if the angle of projection of first short α is greater than the angle of projection of second shot β *i.e.* $\boldsymbol{\alpha > \beta}$.

At the colliding position of two shots, the horizontal distance (x) covered by each shot is same so $x = u\cos\alpha\, T = u\cos\beta\,(T - t)$

or $\boldsymbol{T\cos\alpha = (T - t)\cos\beta}$

At the colliding position of two shots, the vertical distance (y) covered by first shot is

$$y = u\sin\alpha T - \frac{1}{2}gT^2$$

The vertical distance y covered by second shot is

$$y = u\sin\beta\,(T - t) - \frac{1}{2}g\,(T - t)^2$$

$$\therefore\ \boldsymbol{u\sin\alpha\, T - \frac{1}{2}gT^2}$$

$$\boldsymbol{= u\sin\beta\,(T - t) - \frac{1}{2}g\,(T - t)^2}$$

86. Horizontal range is same when angle of projection is θ and $(90^\circ - \theta)$

$$\therefore\ R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} \qquad ...(1)$$

When angle of projection is θ, then

$$T_1 = \frac{2u\sin\theta}{g}$$

When angle of projection is $(90^\circ - \theta)$, then

$$T_2 = \frac{2u\sin(90^\circ - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore\ T_1T_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$$

$$= \left(\frac{2u^2\sin\theta\cos\theta}{g}\right)\left(\frac{2}{g}\right) = \frac{2R}{g}$$

$$\therefore\ \boldsymbol{T_1T_2 \propto R}$$

And $$\frac{T_1}{T_2} = \frac{2u\sin\theta/g}{2u\cos\theta/g} = \boldsymbol{\tan\theta}$$

87. $\vec{r} = \alpha t^3\,\hat{i} + \beta t^2\,\hat{j}$

velocity, $$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2\,\hat{i} + 2\beta t\,\hat{j}$$

Acceleration, $$\vec{a} = \frac{d\vec{v}}{dt} = 6\alpha t\,\hat{i} + 2\beta\,\hat{j}$$

At $t = 1$ s,

(*a*) $$\vec{v} = 3\times\frac{10}{3}\times(1)^2\,\hat{i} + 2\times 5\times 1\times\hat{j}$$

$$\boldsymbol{= (10\,\hat{i} + 10\,\hat{j})\,\mathrm{ms^{-1}}}$$

(*b*) Angular momentum $\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times m\vec{v}$

$$\therefore\ \vec{L} = \left[\frac{10}{3}(1)^3\,\hat{i} + 5(1)^2\,\hat{j}\right]\times 0{\cdot}1\,(10\,\hat{i} + 10\,\hat{j})$$

$$= \left(-\frac{5}{3}\hat{k}\right)\mathbf{N\,ms}$$

(*c*) Force $$\vec{F} = m\vec{a} = 0{\cdot}1\left(6\times\frac{10}{3}\times\hat{i} + 2\times 5\,\hat{j}\right)$$

$$= (2\,\hat{i} + \hat{j})\,\mathrm{N}$$

(*d*) Torque $\vec{\tau} = \vec{r}\times\vec{F}$

$$= \left[\frac{10}{3}(1)^3\,\hat{i} + 5(1)^2\,\hat{j}\right]\times(2\,\hat{i} + \hat{j})$$

$$= \left(\frac{10}{3}\hat{i} + 5\,\hat{j}\right)\times(2\,\hat{i} + \hat{j})$$

$$= \frac{10}{3}\hat{k} + 10(-\hat{k}) = \left(-\frac{20}{3}\hat{k}\right)\mathbf{N\text{-}m}$$

HINTS/EXPLANATIONS....

For Difficult Questions

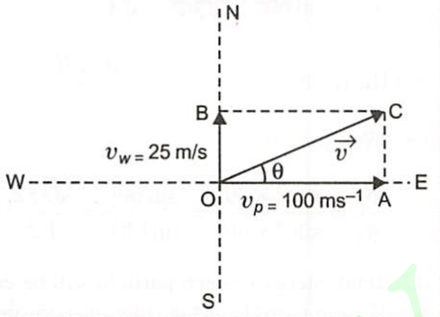
III. Multiple Choice Questions (Based on the given Passage/Comprehension)

88. Refer to Fig. 4(CF).52, the resultant velocity of plane will be along OC. In $\Delta\ OCA$

$$\tan\theta = \frac{AC}{OA} = \frac{v_w}{v_p} = \frac{25}{100}$$

or $$\boldsymbol{\theta = \tan^{-1}\frac{25}{100}}$$

FIGURE 4(CF).52



89. Given, $\tan\theta = \frac{25}{100} = \frac{1}{4}$;

$$\cos\theta = \frac{4}{\sqrt{4^2+1^2}} = \frac{4}{\sqrt{17}}$$

Resultant speed of the plane,

$$v = \sqrt{v_p^2 + v_w^2} = \sqrt{100^2 + 25^2}$$

$$= \sqrt{25^2 \times 4^2 + 25^2} = 25\sqrt{17}\ \text{ms}^{-1}$$

$$\therefore \quad v = \frac{25\sqrt{17}}{\cos\theta} \times \cos\theta$$

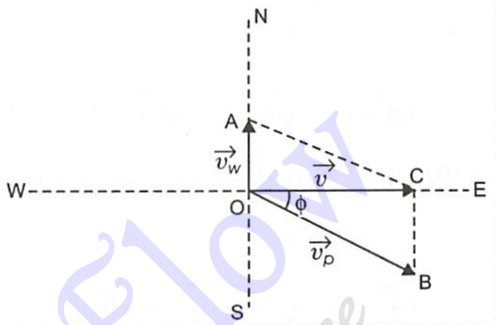
$$= \frac{25\sqrt{17}}{\cos\theta} \times \frac{4}{\sqrt{17}} = \frac{100}{\cos\theta}$$

$$\boldsymbol{= 100 \sec\theta}$$

90. Refer to Fig. 4(CF).53, as the resultant velocity is along OC, so the plane velocity w.r.t. ground must be along OB. Here,

$$v_p = 100\ \text{ms}^{-1} \text{ and } v_w = 25\ \text{ms}^{-1}.$$

FIGURE 4(CF).53



In $\Delta\ OCB$,

$$\sin\phi = \frac{CB}{OB} = \frac{v_w}{v_p} = \frac{25}{100}$$

or $$\boldsymbol{\phi = \sin^{-1}\left(\frac{25}{100}\right)}$$

91. From first two observations, using $R = C\,v^{n-1}$, we have $4 = C\,5^{n-1}$ and $16 = C\,10^{n-1}$

$$\therefore \quad \frac{16}{4} = \left(\frac{10}{5}\right)^{n-1} = 2^{n-1}$$

or $2^2 = 2^{n-1}$ or $2 = n - 1$ or $\boldsymbol{n = 3}$

92. When speed is 4 ms^{-1}, then range

$$R = C\,4^{3-1} = C \times 4^2$$

and $$4 = C\,5^{3-1} = C \times 5^2$$

$$\therefore \quad \frac{R}{4} = \left(\frac{4}{5}\right)^2 = 0{\cdot}64$$

or $$R = 4 \times 0{\cdot}64 = \boldsymbol{2{\cdot}56\ \text{m}}$$

93. During projectile motion as the ball goes upwards its vertical velocity decreases upwards, upto the highest point as it is retarded by acceleration due to gravity and when ball falls downwards, its velocity increases downwards due to gravity pull. Hence, the **graph (*d*) is correct.**

HINTS/EXPLANATIONS

For Difficult Questions

IV. Matching Type Questions

94. (A) $(\vec{A}+\vec{B}) = (\hat{i}+\hat{j}-\hat{k})+(\hat{i}-2\hat{j}-2\hat{k})$

$$= 2\hat{i}-\hat{j}-3\hat{k}$$

$$\therefore \quad |\vec{A}+\vec{B}| = \sqrt{(2)^2+(-1)^2+(-3)^2} = \sqrt{\mathbf{14}}$$

(B) $(\vec{A}-\vec{B}) = (\hat{i}+\hat{j}-\hat{k})-(\hat{i}-2\hat{j}-2\hat{k})$

$$= 3\hat{j}+\hat{k}$$

$$|\vec{A}-\vec{B}| = \sqrt{(3)^2+(1)^2} = \sqrt{\mathbf{10}}$$

(C) $(\vec{A}\cdot\vec{B}) = (\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}-2\hat{j}-2\hat{k})$

$$= 1-2+2 = \mathbf{1}$$

(D) $(\vec{A}\times\vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & -2 \end{vmatrix}$

$$= \hat{i}\,(-2-2)+\hat{j}\,(-1+2)+\hat{k}\,(-2-1)$$

$$= -4\hat{i}+\hat{j}-3\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{(-4)^2+(1)^2+(-3)^2} = \sqrt{\mathbf{26}}$$

95. (A) Max height, $h = \dfrac{u^2\sin^2\theta}{2g}$ *i.e.,* $h \propto \sin^2\theta$

$$\therefore \quad \frac{h_1}{h_2} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{\mathbf{1}}{\mathbf{3}}$$

(B) Time of flight, $T = \dfrac{2u\sin\theta}{g}$ *i.e.,* $T \propto \sin\theta$

$$\therefore \quad \frac{T_1}{T_2} = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$$

(C) Horizontal range, $R = \dfrac{u^2\sin 2\theta}{g}$

i.e., $R \propto \sin 2\theta$

$$\therefore \quad \frac{R_1}{R_2} = \frac{\sin 2\times 30^\circ}{\sin 2\times 60^\circ} = \frac{\sin 60^\circ}{\sin 120^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{\mathbf{3}}$$

(D) Total energy of each particle will be equal to *KE* of each particle at the time of its projection.

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2}mu^2}{\frac{1}{2}mu^2} = \mathbf{1}$$

V. Matrix-Match Type Questions

96. Oscillations of a mass suspended from a vertical spring is one dimensional motion as it is confined along a st. line path.

The motion of earth around the sun in an orbit has orbital motion and spinning motion. It is two dimensional motion being in a definite orbit along with a rotational motion.

The motion of the gas molecules in the given region is a three dimensional motion.

Sommer-sault by a swimmer from a height before entering the water is two dimensional motion along with a rotational motion.

97. (A) Given $\vec{A} = 3\hat{i}+4\hat{j}-5\hat{k}$

$$= A_x\,\hat{i}+A_y\,\hat{j}+A_z\,\hat{k} \quad \therefore \; A_y = 5 > 1$$

(B) $A_z = -5 < 1$

(C) $A = \sqrt{3^2+4^2+(-5)^2} = 5\sqrt{2}$; $\vec{B} = (\hat{i}+\hat{j})$;

$$B = \sqrt{1^2+1^2} = \sqrt{2} \;;\quad \hat{B} = \frac{\vec{B}}{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

$\therefore$ Component vector of $\vec{A}$ along $\vec{B}$ is

$$= (\vec{A}\cdot\hat{B})\,\hat{B} = \left[(3\hat{i}+4\hat{j}-5\hat{k})\cdot\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

$$= \left(\frac{3+4}{\sqrt{2}}\right)\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{7}{2}\,(\hat{i}+\hat{j})$$

Magnitude of component vector $= \dfrac{7}{2}\sqrt{1^2+1^2} = \dfrac{7}{\sqrt{2}}$

$$= 4{\cdot}95$$

(D) $A_x = 3$, *i.e.,* $>$ **2** but less than 5

HINTS/EXPLANATIONS

For Difficult Questions

VI. Integer Type Questions

98. Let A and B be two forces acting at a point and θ be the angle between them. Then

$$A + B = 18 \qquad ...(i)$$

and $A^2 + B^2 + 2\,AB\cos\theta = 12^2 = 144 \qquad ...(ii)$

If A is smaller force, then as per question

$$\tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta} \quad \text{or} \quad \infty = \frac{B\sin\theta}{A + B\cos\theta}$$

or $A + B\cos\theta = 0$ or $\cos\theta = -A/B$

From (ii), $A^2 + B^2 + 2\,AB\,(-A/B) = 144$

or $B^2 - A^2 = 144$

or $(B - A)(B + A) = 144$

or $$B - A = \frac{144}{B + A} = \frac{144}{18} = 8 \qquad ...(iii)$$

Solving (i) and (iii), we shall get, $A = \mathbf{5\ N}$

99. Let the ball strike the nth step in time t, where vertical height of each step is h and horizontal length of each step is x. So ;

$$nh = \frac{1}{2}gt^2$$

or $$t = \sqrt{\frac{2nh}{g}}$$

Now $ut = nx$

or $$n = \frac{ut}{x} = \frac{u}{x}\sqrt{\frac{2nh}{g}}$$

Squaring both the sides, we have

$$n = \frac{2u^2h}{x^2g} = \frac{2\times(1{\cdot}8)^2\times 0{\cdot}2}{(0{\cdot}2)^2\times 10} = 3{\cdot}24$$

$$\approx \mathbf{4\ (fourth\ step)}$$

100. Here, $u = 10$ m/s, $\theta = 60^\circ$

Time of flight, $$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times 10\times \sin 60^\circ}{10} = \sqrt{3}\ s$$

Taking horizontal motion of ball in train, we have

$$u_x = u\cos 60^\circ = 10\times 1/2 = 5\text{ m/s},$$

$$a_x = -a,\ s = 1{\cdot}15\text{ m},\quad t = T = \sqrt{3}\ s.$$

Using, $$s = u_x t + \frac{1}{2}a_x t^2$$

we have, $$1{\cdot}15 = 5\times\sqrt{3} + \frac{1}{2}(-a)\times(\sqrt{3})^2$$

On solving, $a = \mathbf{5\ m/s^2}$

101. Let t_1, t_2, t_3 be the times of flight in successive parabolic paths and v_1, v_2, v_3 be the average velocities during these paths. Then

Total time taken, $T = t_1 + t_2 + t_3 + ...$

$$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + ...$$

$$= \frac{t_1}{1 - \dfrac{1}{\alpha}}$$

Total displacement,

$$S = v_1 t_1 + v_2 t_2 + v_3 t_3 + ...$$

$$= v_1 t_1 + \frac{v_1}{\alpha}\frac{t_1}{\alpha} + \frac{v_1}{\alpha^2}\times\frac{t_1}{\alpha^2} + ...$$

$$= \frac{v_1 t_1}{1 - 1/\alpha^2}$$

$$\therefore \text{Average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

$$= \frac{v_1 t_1/(1 - 1/\alpha^2)}{t_1/(1 - 1/\alpha)}$$

$$= \frac{v_1\alpha}{\alpha + 1}$$

As per question, $$0{\cdot}8\,v_1 = \frac{v_1\alpha}{1 + \alpha}$$

or $$0{\cdot}8 = \frac{\alpha}{1 + \alpha} \quad \text{or} \quad \alpha = \mathbf{4{\cdot}00}$$

HINTS/EXPLANATIONS
For Difficult Questions

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

102. Horizontal range, $R = \frac{u^2}{g} \sin 2\theta$. It is maximum, when $\sin 2\theta = 1 = \sin 90°$ or $\theta = 45°$.

103. Here, **the Assertion is true** but temperature is a scalar quantity and not a vector, hence **Reason is wrong.**

104. A projectile is the name given to a body which is once projected can move under gravity in two dimensions without consuming any fuel or propelled by any engine. Hence **both Assertion and Reason are correct.**

FOR ENGINEERING STUDENTS

105. Both the statements are true and **statement-2 is the correct explanation of statement-1.**

106. In merry-go-round, linear acceleration of a horse is $a = \alpha r$, where α is the angular acceleration and r is the distance of horse from the axis of rotation. So $a \propto r$.

107. Here, both statements are true and **statement-2 is the correct explanation of statement-1.**

108. The magnitude of the resultant vector of two given vectors can be less than the magnitude of individual vectors if the angle between two vectors is in between 90° to 270°. Thus **statement-1 is false and statement-2 is true.**

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UNIT-3

LAWS OF MOTION

CHAPTER 5.

LAWS OF MOTION

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5

LAWS OF MOTION

5.1. INTRODUCTION

In the previous unit, we have learnt the concept of velocity to describe uniform motion of a body. An additional concept of acceleration was introduced to describe non-uniform motion. In the present chapter, we shall investigate as to what causes and governs the motion of bodies.

5.2. FORCE

Our common experience tells us that to move a football at rest, someone must kick it. Similarly, to throw a stone upwards, we have to give an upward push. To stop a ball rolling down an inclined plane, we have to apply some force against its direction of motion. Similarly, when a ball is hit by a bat, its direction of motion changes. We may, therefore, define

> ***Force** as an external effort in the form of push or pull, which (i) produces or tries to produce motion in a body at rest, or (ii) stops or tries to stop a moving body, or (iii) changes or tries to change the direction of motion of the body.*

The following examples illustrate the above definition :

(*i*) When we push a ball lying on the ground, it starts rolling. The force exerted has thus produced motion in the ball. However, when we push a heavy stone, it does not move. The effort made in this case has only tried to produce motion, but has not succeeded.

(*ii*) A ball falling downwards can be caught easily by our hands. The motion of the ball has thus been destroyed. However, a big piece of rock rolling down a hill cannot be stopped even when we try our best to stop it.

(*iii*) When a piece of stone tied to one end of a string is whirled in a circle, a constant force has to be exerted by the hand along the string. This is because the natural tendency of a body is to move along a straight line. Force is spent in changing the direction of motion of the body from straight line path to the circular path.

It should be clearly understood that some external agency is always needed to provide the force. In the examples given above, force is applied by our hands. Often, the external agency is in contact with the object on which force is applied. But it is not always necessary. For example, a ball released from some height accelerates downward due to the gravitational pull of earth, which is not in contact with the ball. A magnet attracts an iron nail from a distance.

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This shows that external agencies of force (e.g. gravitational and magnetic) can exert force on a body even from a distance.

They need not necessarily be in contact with the object.

5.3. ARISTOTLE'S FALLACY

The Greek thinker Aristotle (384 B.C. – 322 B.C.), held the view that *if a body is moving , some external force is required to keep it moving.* Thus, the Aristotelian law of motion is : ***An external force is required to keep a body in uniform motion.*** It is a natural view as external forces seem to be needed to keep bodies in motion. Left to themselves, all bodies eventually come to rest.

But there is **flaw in Aristotle's argument.** A ball rolling on the floor stops because an external force of friction between the ball and the floor and also resistance of air oppose the motion of the ball. If there were no opposing forces, the rolling ball would never stop.

Infact, the opposing forces such as friction (in case of solids) and viscous forces (in case of fluids) are always present in the natural world. That is why forces by external agencies are necessary to overcome the opposing forces to keep bodies in uniform motion. ***If we were to imagine a world without any frictional forces, no force would be required to keep a body moving uniformly along a straight line.*** This is the true law of nature for forces and motion.

5.4. GALILEO'S LAW OF INERTIA

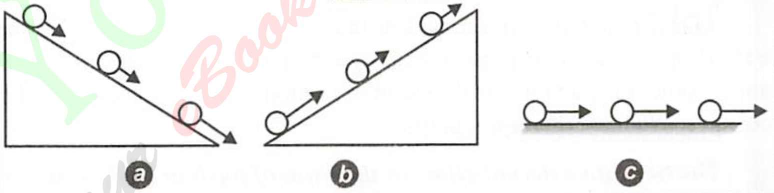
Galileo studied motion of objects on an inclined plane. He found that

(*i*) objects moving down a smooth inclined plane accelerate, Fig. 5.1(*a*).

(*ii*) objects moving up a smooth inclined plane retard, Fig. 5.1(*b*).

(*iii*) objects moving on a frictionless horizontal plane move with a constant velocity, having neither acceleration nor retardation, Fig. 5.1(*c*).

FIGURE 5.1



In another experiment using a double inclined plane, Galileo observed that

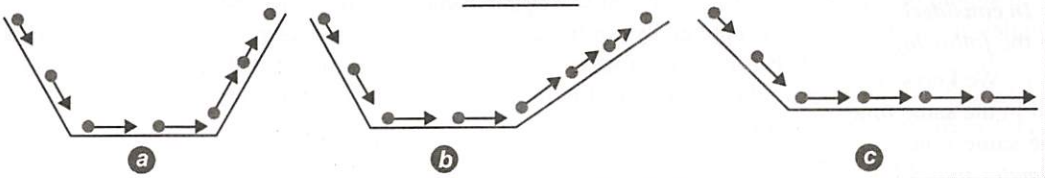
A ball released from rest on one smooth inclined plane rolls down and climbs up the other smooth inclined plane. He found that

(*i*) in ideal situation, when there is no friction, the final height of the ball is the same as its initial height. In actual practice, when some friction is there, final height is somewhat less than the initial height. In no case, final height is ever greater than the initial height. When the slopes of the two planes are same, distance covered in rolling down one incline is the same as the distance covered in climbing up the other incline. This is shown in Fig. 5.2(*a*).

(*ii*) When the slope of second smooth inclined plane is decreased, and the experiment is repeated, the ball still reaches the same final height. But in doing so, it travels a larger distance as shown in Fig. 5.2(*b*).

(*iii*) When the slope of second smooth inclined plane is made zero (*i.e.*, the second plane is made horizontal), the ball travels an infinite distance in the ideal situation (when there is no friction) *i.e.*, motion of the ball never ceases, Fig. 5.2(*c*).

FIGURE 5.2



In actual practice, the ball comes to rest after moving some finite distance on the horizontal plane. This is because the opposing force of friction can never be totally eliminated.

From his experiments, Galileo concluded that ***the state of rest and the state of motion with constant velocity are equivalent. In both cases, no net force is acting on the body.***

Galileo emphasized that *it is incorrect to assume that a net force is needed to keep a body in uniform motion along a straight line.*

To maintain a body in uniform motion along a straight line, we need to apply an external force to counter the frictional force. Infact, *the external force applied and frictional force sum up to zero net external force, and the body moves uniformly along a straight line.*

Thus, Galileo concluded that

If net external force is zero, a body at rest continues to be at rest, and a body in uniform motion continues to move uniformly along the same straight line. This is Galileo's law of inertia.

This, inherent property of all bodies, by virtue of which they cannot change by themselves their state of rest or state of uniform motion along a straight line is called Inertia.

5.5. LINEAR MOMENTUM

The linear momentum of a body is the quantity of motion possessed by the body. Quantitatively,

the linear momentum of a body is defined as the product of the mass of the body and its velocity i.e.

$$\text{Linear momentum} = \text{mass} \times \text{velocity}$$

If a body of mass m is moving with a velocity $\vec{v}$, its linear momentum $\vec{p}$ is given by

$$\boxed{\vec{p} = m\vec{v}}$$

Linear momentum is a **vector quantity.** Its direction is the same as the direction of velocity of the body. **The SI unit** of linear momentum is **kg ms^{-1}** and the cgs unit of linear momentum is **g cm s^{-1}.**

The dimensional formula of linear momentum is $[M^1 L^1 T^{-1}]$. From $\vec{p} = m\vec{v}$, we find that

(*i*) when m is constant, $p \propto v$. This is shown in Fig. 5.3(*a*).

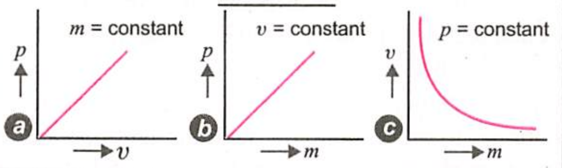
(*ii*) when v is constant, $p \propto m$. This is shown in Fig. 5.3(*b*).

(*iii*) Further, when two bodies of unequal masses m_1, m_2 have the same linear momentum *i.e.*,

$$p_1 = p_2 \quad \text{or} \quad m_1 v_1 = m_2 v_2$$

$$\therefore \frac{v_2}{v_1} = \frac{m_1}{m_2}$$

FIGURE 5.3



i.e. velocities of the bodies having equal linear momenta vary inversely as their masses *i.e.* the heavier body has smaller velocity and the lighter body has greater velocity. This is shown in Fig. 5.3(*c*).

In considering the effect of force on motion, linear momentum is very important. This can be understood from the *following examples :*

(*i*) We know that a much greater force is required to push a truck than a car to bring them to the same speed in the same time. Similarly, a greater opposing force is needed to stop a heavy body than a light body in the same time, when they are moving with the same speed. *Thus, the mass of a body is an important parameter that determines the effect of force on its motion.*

(*ii*) Speed is another important parameter to consider. A bullet fired from a gun can easily pierce through a target like human tissue, before it stops, resulting in grave injury. The same bullet, thrown with a moderate speed can be easily stopped. Thus for a given mass, greater is the speed, greater is the opposing force needed to stop the body in a certain time.

Taken together, the product of mass and velocity, *i.e.*, linear momentum is an important quantity, when we consider effect of force on its motion.

5.6. NEWTON'S LAWS OF MOTION

Newton gave three laws governing the motion of bodies. We proceed with the statement of these three laws:

First Law. According to this law, *a body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some net unbalanced external force to change the state.*

Second Law. According to this law, the *rate of change of linear momentum of a body is directly proportional to the external force applied on the body, and this change takes place always in the direction of the force applied.*

Third Law. According to this law, *to every action, there is always an equal and opposite reaction i.e. the forces of action and reaction are always equal and opposite.*

These laws can also be stated as under :

1st Law. *If no net force acts on a body, then the velocity of the body cannot change, i.e., the body cannot accelerate.*

2nd Law. *The net force on a body is equal to the product of mass of the body and acceleration of the body.*

3rd Law. *When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.*

5.7. EXPLANATION OF NEWTON'S FIRST LAW

As stated in Art. 5.6, Newton's first law consists of three parts :

(*i*) Without the application of an external force, a body at rest continues to be at rest. This part is self evident. For example, we find in day to day life that every object continues to lie where it is, unless we move it.

(*ii*) Without the application of an external force, a body in uniform motion continues to move uniformly. This part is slightly difficult to realise, because in every day life, we find that a ball rolling on the ground does stop after some time. Similarly, when the engine of a moving car is switched off, it stops after travelling some distance. Infact, motion of bodies is being opposed by the invisible forces like air resistance and force of friction between the body and the ground. If these forces were removed, a body moving uniformly shall never stop on its own.

(*iii*) Without the application of an external force, a body cannot change its direction of motion, *i.e.*, it continues to move along the same straight line. For example, to turn a car moving over a straight road, we have to apply the force on the steering wheel of the car. Similarly, a bike cannot change its straight line path on its own. We have to turn its handle.

It should be clearly understood that the state of rest or uniform linear motion, both imply zero acceleration. Therefore, first law of motion can be expressed as :

If the net external force on a body is zero, its acceleration is zero. Acceleration can be non-zero only if there is a net external force on the body.

If an object on earth is at rest or in uniform linear motion, it does not mean that no forces are acting on it. Rather, it means that various external forces acting on the body cancel out, *i.e.*, *these forces add up to zero net external force.*

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IMPORTANT NOTES

1. *Note that when several forces act simultaneously on a body, the net force is the vector sum of all of them. Individual forces matter only to the extent that they contribute to the total.*

2. Remember that state of rest or state of uniform motion of a body along a straight line cannot be changed, even when any number of forces are acting on the body, but the resultant of these forces is zero. For change of state of a body, some net external force is a must.

3. Newton's first law of motion is valid only in an inertial frame of reference. This law is not valid in a non-inertial frame of reference.

5.8. NEWTON'S FIRST LAW DEFINES FORCE

According to Newton's first law of motion, a body continues to be in a state of rest or of uniform motion along a straight line, unless it is acted upon by an external force to change the state. This means force applied on a body alone, can change its state of rest or state of uniform motion along a straight line. Hence, we define

Force as an external effort in the form of a push or pull which moves or tries to move a body at rest ; stops or tries to stop a body in motion ; changes or tries to change the direction of motion of a body.

This is how Newton's first law of motion defines force.

5.9. NEWTON'S FIRST LAW DEFINES INERTIA

According to Newton's first law of motion, a body continues to be in a state of rest or of uniform motion along a straight line, unless it is acted upon by an external force to change its state. This means a body, on its own, cannot change its state of rest or state of uniform motion along a straight line.

This inability of a body to change by itself its state of rest or state of uniform motion along a straight line is called inertia of the body.

Infact, a body opposes any external force that tries to change its state of rest or state of uniform motion along a straight line.

Hence Newton's first law defines inertia and is rightly called the **law of inertia.**

Quantitatively, inertia of a body in linear motion is measured by the mass of the body. Heavier the body, greater is the force required to change its state and hence greater is its inertia. *The reverse is also true.*

The SI unit of inertia of linear motion is same as the SI unit of mass (*i.e.*, kg).

5.10. THREE TYPES OF INERTIA

Inertia of a body is of three types : **1.** Inertia of rest, **2.** Inertia of motion, **3.** Inertia of direction.

(*a*) Inertia of rest

It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving on its own. Rather, a body at rest opposes the force that tries to move it.

For example :

1. Suppose we are standing in a stationary bus and the driver starts the bus suddenly. We get thrown backwards with a jerk. Let us understand why ?

Our feet are in touch with the floor of the bus. There is some friction between the feet and the floor. If start of the bus is not that sudden, the frictional force would be enough to accelerate our feet alongwith the bus. But our body is not strictly a rigid body. It is **deformable,** allowing some relative displacement between different parts. Therefore, while our feet go with the bus, the rest of the body remains where it is, due to inertia. Therefore, relative to the bus, we are thrown backwards. At this instant, the muscular forces on the rest of the body by our feet come into play and move the body alongwith the bus.

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2. When a horse starts suddenly, the rider tends to fall backwards on account of inertia of rest of upper part of the body as explained above.

3. The dust particles in a durree fall off when it is beaten with a stick. This is because the beating sets the durree in motion whereas the dust particles tend to remain at rest and hence separate.

4. When we place a coin on smooth piece of cardboard covering a glass and strike the cardboard piece suddenly with a finger, the cardboard slips away and the coin falls into the glass. This happens on *account of inertia of rest of the coin.*

5. When we shake a branch of a mango tree, the mangoes fall down. This is because the branch comes in motion and the mangoes tend to remain at rest. Hence they get detached.

(*b*) Inertia of motion

> ***It is the inability of a body to change by itself, its state of uniform motion i.e. a body in uniform motion can neither accelerate nor retard on its own and come to rest. Rather, a body in uniform motion opposes the force that tries to stop it.***

For example :

1. Suppose we are standing in a moving bus, and the driver stops the bus suddenly. We are thrown forward with a jerk. Let us understand why.

As the bus is suddenly stopped, our feet stop due to friction which does not allow relative motion between the feet and the floor of the bus. But the rest of our body continues to move forward due to inertia. That is why we are thrown forward. The restoring muscular forces exerted on our body by our feet come into play and bring the body to rest.

2. When a horse at full gallop stops suddenly, the rider falls forward on account of inertia of motion as explained above.

3. A person jumping out of a speeding train may fall forward. This is because his feet come to rest on touching the ground and the remaining body continues to move due to inertia of motion.

4. An athelete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to the velocity of the athelete at the time of jump. Hence he can jump over a longer distance.

(*c*) Inertia of direction

> ***It is the inability of a body to change by itself, its direction of motion i.e. a body continues to move along the same straight line unless compelled by some external force to change it. Rather, a body opposes the force that tries to change its direction of motion.***

For example :

1. An umbrella protects us from rain. It is based on the property of inertia of direction. The rain drops falling vertically downwards cannot change their direction of motion and wet us, with the umbrella on.

2. When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off *along the tangent to the circle.* This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

3. When a car rounds a curve suddenly,the person *sitting inside is thrown outwards.* This is because the person tries to maintain his direction of motion due to directional inertia while the car turns.

4. The rotating wheels of any vehicle throw out mud, if any, tangentially, due to directional inertia.

The mud guards over the wheels stop this mud, protecting the clothes etc. of the driver of the bike or motor bike.

5. When a knife is sharpened by pressing it against a grinding stone, the sparks fly off along the tangent to the grinding stone, on account of directional inertia.

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5.11. EXPLANATION OF NEWTON'S SECOND LAW

According to Newton's second law of motion, ***the rate of change of linear momentum of a body is directly proportional to the external force applied on the body, and this change takes place always in the direction of the applied force.***

Suppose two bodies of different masses are initially at rest, and a fixed force is applied on them for a certain interval of time. To start with, the lighter body picks up a greater speed than the heavier body. However, at the end of the time interval, observations show that each body acquires the same linear momentum. It means that *the same force applied for the same time causes the same change in linear momentum in bodies of different masses.*

The law implies that when a bigger force is applied on a body of given mass, its linear momentum changes faster and vice-versa. The momentum will change in the direction of the applied force. To understand it further, suppose

m = mass of a body, $\vec{v}$ = velocity of the body

$\therefore$ The linear momentum of the body $\vec{p} = m\vec{v}$ ...(1)

Let $\vec{F}$ = external force applied on the body in the direction of motion of the body.

$\Delta\vec{p}$ = a small change in linear momentum of the body in a small time Δt.

Rate of change of linear momentum of the body $= \dfrac{\Delta\vec{p}}{\Delta t}$

According to Newton's second law,

$$\frac{\Delta\vec{p}}{\Delta t} \propto \vec{F} \quad \text{or} \quad \vec{F} \propto \frac{\Delta\vec{p}}{\Delta t} \quad \text{or} \quad \vec{F} = k\frac{\Delta\vec{p}}{\Delta t} \qquad \text{...(2)}$$

where k is a constant of proportionality.

Taking the limit $\Delta t \to 0$, the term $\dfrac{\Delta\vec{p}}{\Delta t}$ becomes the derivative or differential coefficient of $\vec{p}$ w.r.t. time t. It is denoted by $\dfrac{d\vec{p}}{dt}$. $\quad \therefore \quad \vec{F} = k\dfrac{d\vec{p}}{dt}$

Using (1), $\vec{F} = k\dfrac{d}{dt}(m\vec{v}) = k\,m\dfrac{d\vec{v}}{dt}$ As $\dfrac{d\vec{v}}{dt} = \vec{a} =$ acceleration of the body,

$$\therefore \quad \vec{F} = k\,m\,\vec{a} \qquad \text{...(3)}$$

The value of constant of proportionality k depends on the units adopted for measuring the force. Both in SI and cgs systems, the unit of force is selected in such a manner that $k = 1$.

Putting this value of k in (3), we get

$$\boxed{\vec{F} = m\,\vec{a}} \qquad \text{...(4)}$$

As acceleration is a vector quantity and mass is scalar, therefore force $\vec{F}$ being the product of m and $\vec{a}$ is a **vector**.

The direction of $\vec{F}$ is the same as the direction of $\vec{a}$.

Eqn. (4) represents of the *equation of motion of the body.*

We can rewrite eqn. (4) in scalar form as $F = ma$ ...(5)

Thus magnitude of force can be calculated by multiplying mass of the body and the acceleration produced in it. Hence second law of motion gives us a measure of force.

IMPORTANT NOTE

From eqn. (5), $F = ma = \frac{dp}{dt} = \frac{d}{dt}(mv)$ or $F = m\frac{dv}{dt} + v\frac{dm}{dt}$

If m is constant, $\frac{dm}{dt} = 0$ $\therefore$ $\boxed{F = m\frac{dv}{dt}}$; If v is constant, $\frac{dv}{dt} = 0$ $\therefore$ $\boxed{F = v\frac{dm}{dt}}$

5.12. DIMENSIONS AND UNITS OF FORCE

As $F = ma$ $\therefore$ $F = (M)(LT^{-2}) = [M^1 L^1 T^{-2}]$

This is the dimensional formula of force.

The units of force are of two types : Absolute units and Gravitational units.

(*a*) **Absolute units**

(*i*) The absolute unit of force on SI is newton (represented by *N*).

***One newton* force is that much force which produces an acceleration of 1 ms^{-2} in a body of mass 1 kg.**

As $F = ma$ $\therefore$ $\boxed{1\text{ N} = 1\text{ kg} \times 1\text{ ms}^{-2} = 1\text{ kg ms}^{-2}}$

(*ii*) The absolute unit of force on cgs system is **dyne.**

***One dyne* force is that much force which produces an acceleration of 1 cm s^{-2} in a body of mass one gram.**

As $F = ma$ $\therefore$ $\boxed{1\text{ dyne} = 1\text{ g} \times 1\text{ cm s}^{-2} = 1\text{ g cm s}^{-2}}$

Relation between newton and dyne

As $1\text{ N} = 1\text{ kg} \times 1\text{ ms}^{-2} = 10^3\text{ g} \times 10^2\text{ cm s}^{-2}$ $\therefore$ $\boxed{1\text{ N} = 10^5\text{ dyne}}$

(*b*) **Gravitational units**

(*i*) The gravitational unit of force on SI is **kilogram weight** or **kilogram force.**

***1 kilogram weight* (kg wt.) or 1 kilogram force (kg f) is that much force which produces an acceleration of 9·8 ms^{-2} in a body of mass 1 kg.**

Thus, $\boxed{1\text{ kg wt. or } 1\text{ kg f} = 1\text{ kg} \times 9{\cdot}8\text{ ms}^{-2} = 9{\cdot}8\text{ N}}$

(*ii*) The gravitational unit of force on cgs system is **gram weight** or **gram force.**

***1 gram weight* (g wt.) or 1 gram force (1 g f) is that much force which produces an acceleration of 980 cm s^{-2} in a body of mass 1 gram.**

Thus, $\boxed{1\text{ g wt.} = 1\text{ g f} = 1\text{ g} \times 980\text{ cm s}^{-2} = 980\text{ dyne}}$

Note that the gravitational units of force are used to express weight of a body. For example, weight of a body of mass 5 kg is 5 kg f or 5 kg wt. These units are, therefore, called the *practical units.*

Retain in Memory

Remember that in all numerical problems, we have to use only the absolute units of force. Gravitational units are the practical units. They have to be converted into absolute units for use in any problem.

Sample Problem 1 **A constant force acting on a body of mass 3 kg changes its speed from 2 ms^{-1} to 3·5 ms^{-1} in 25 s. The direction of motion of the body remains unchanged. Calculate magnitude and direction of the force.**

Sol. Here $m = 3$ kg, $u = 2$ m/s, $v = 3{\cdot}5$ m/s, $t = 25$ s, $F = ?$

As $$F = m\,a = m\frac{(v-u)}{t} \quad \therefore \quad F = \frac{3(3{\cdot}5-2)}{25} = \mathbf{0{\cdot}18\,N}$$

As direction of motion does not change and magnitude of velocity increases, therefore, force must be along the direction of motion.

Sample Problem 2 **A Car of mass 1000 kg is moving with a velocity of 10 m/s under the action of a forward force of 1000 N and retarding force of 500 N due to friction. What will be its velocity after 5 seconds ?**

Sol. Here, $m = 1000$ kg, $u = 10$ m/s

Net forward force, $F = 1000\text{ N} - 500\text{ N} = 500$ N

$$v = ?\ t = 5\text{ s} \qquad a = \frac{F}{m} = \frac{500}{1000} = 0{\cdot}5\text{ m/s}^2$$

From $v = u + \text{at} = 10 + 0{\cdot}5 \times 5 = \mathbf{12{\cdot}5\ m/s}$

5.13. CONSEQUENCES OF NEWTON'S SECOND LAW OF MOTION

Following are some of the important consequences of Newton's second law of motion.

1. No force is required to move a body uniformly along a straight line

According to Newton's second law of motion $F = ma$

When body is moving *uniformly* along a straight line, and there is no force of friction, acceleration/retardation of the body, $a = 0$

$\therefore$ $F = ma = 0$, *i.e.,* no external force is required.

The second law is obviously consistent with the first law.

2. Accelerated motion is always due to an external force

The motion of a body may be accelerated under the following three conditions :

(*i*) *Speed of a body may change.* For this, force has to be applied in the direction of motion or in a direction opposite to that of motion.

(*ii*) *Direction of motion of a body may change.* For this, force has to be applied in a direction perpendicular to the direction of motion of the body.

(*iii*) *Velocity of the body may change.* For this, force has to be applied in an inclined direction. The tangential component of force changes the speed and the normal component of force changes the direction of motion of the body.

3. Measurement of force

From Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

By knowing the mass (m) of the body and measuring change in its velocity (dv) in a particular time (dt), force F applied on the body can be calculated.

DO YOU KNOW ?

The forces acting on a car being driven along a straight horizontal road are :

(*i*) *Weight* of car acts through its centre of mass, which is normally closer to front of car.

(*ii*) *Normal Reaction force* pushes up on all four wheels of the car.

(*iii*) *The driving force* is provided by the road and acts on the driving wheels (front/rear) when motor of car turns the driving wheels. As the tyres push back on the road, the road pushes forward on the tyres.

(*iv*) *Road Friction.* Rolling friction acts on the non-driving wheels in a direction opposite to the direction of car movement. However, when the driving wheels are not being turned by the motor, rolling friction opposes the forward movement of all the four wheels.

(*v*) *Air Resistance* is the opposition posed by the air as the car moves forward. It is more when the car moves faster and vice-versa.

When the car moves along the road with a constant speed, net force acting on the car is zero.

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4. The second law of motion is a vector law. It is equivalent to three equations, one for each component of the vector :

$$F_x = \frac{dp_x}{dt} = ma_x\,,\quad F_y = \frac{dp_y}{dt} = ma_y\,,\quad F_z = \frac{dp_z}{dt} = ma_z$$

It means that if a force is not parallel to the velocity of the body, but makes some angle with it, then the ***force changes only the component of velocity along the direction of force.*** The component of velocity normal to the direction of force would remain unchanged.

5. The second law of motion is applicable to *a single point particle*, *i.e.*, $\vec{F}$ in the law is the net external force on the particle and $\vec{a}$ is acceleration of the particle.

However, the law in the same form can be applied to a rigid body or more generally, it can be applied to a system of particles. In that case, $\vec{F}$ refers to the total external force acting on the system and $\vec{a}$ would be acceleration of *centre of mass* of the system. Note that* any internal forces in the system are not to be included in $\vec{F}$.

6. The second law of motion, $\vec{F} = m\vec{a}$ is a *local relation*, *i.e.*, $\vec{F}$ is force acting on a particle at a point in space, at a certain instant of time and $\vec{a}$ is the acceleration of the particle at the same point and at the same instant of time.

It means acceleration here and now is determined by the force here and now—and not by any history of motion of the particle.

For example, when a piece of stone is dropped out of an accelerated train, initial velocity of stone along the horizontal is equal to velocity of train. But no horizontal force or horizontal acceleration is acting on the stone, *i.e.*, the *stone carries no memory of its acceleration with the train, a moment ago.*

7. Concept of inertial mass

From Newton's second law of motion,

$$F = ma \quad \text{or} \quad a = F/m$$

Larger the mass of a body, smaller is the acceleration produced in it by a given force and hence greater is its inertia. Thus mass of a body is a measure of inertia of the body in linear motion.

Such a mass which accounts for linear inertia of the body is called* inertial mass *of the body.

If $a = 1$, $F = m \times 1 = m = m_i$, the inertial mass. We may define

inertial mass of a body as the force required to produce unit acceleration in the body.

Retain in Memory

1. Force is a vector quantity. It is defined completely when its magnitude, direction and point of application are given.
2. The straight line along which a force is directed is called **line of action** of force.
3. Forces acting together on a body at the same point are called **concurrent forces.**
4. Forces acting along parallel lines are called **collinear forces.**
5. Forces acting in the same plane are called **coplanar forces.**
6. Gravitational forces between two bodies are **position dependent forces.**
7. Viscous force ($F = 6\pi\eta r v$) and force on a charged particle in a magnetic field ($F = B q v \sin\theta$) are **velocity dependent forces.**

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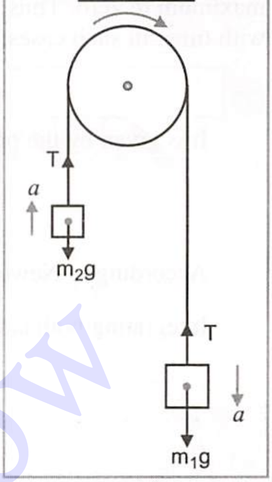
*For details, refer to chapter 7 of this book.

5.14. CONNECTED MOTION

FIGURE 5.4



Consider two bodies of masses m_1 and m_2 tied at the ends of an inextensible string, which passes over a light and frictionless pulley. Let $m_1 > m_2$. The heavier body will move downwards and the lighter body will move upwards. Let a be the common acceleration of the system of two bodies, Fig. 5.4.

Since the pulley is light and frictionless, therefore, the tension in the string shall be the same on both the sides of the pulley, Fig. 5.4.

On the heavier body, the various forces are :

(*i*) Its weight $m_1 g$ acting downwards,

(*ii*) The tension T in the string acting upwards.

As this body moves downwards with acceleration a, the net downward force on it is $m_1 a$.

Clearly,
$$m_1 g - T = m_1 a \qquad \text{...(6)}$$

On the lighter body, the various forces are :

(*i*) Its weight $m_2 g$ acting downwards,

(*ii*) The tension T in the string acting upwards.

As this body moves upwards with acceleration a, the net upward force on it is $m_2 a$.

Clearly,
$$T - m_2 g = m_2 a \qquad \text{...(7)}$$

Adding (6) and (7), we get
$$m_1 g - m_2 g = (m_1 + m_2)\, a$$
$$(m_1 - m_2)\, g = (m_1 + m_2)\, a$$

or
$$\boxed{a = \frac{(m_1 - m_2)\, g}{m_1 + m_2}} \qquad \text{...(8)}$$

Clearly $a < g$ *i.e.* acceleration a of the system of two connected bodies is always less than acceleration due to gravity g.

Dividing (6) by (7), we get
$$\frac{m_1 g - T}{T - m_2 g} = \frac{m_1 a}{m_2 a} = \frac{m_1}{m_2}$$
$$m_1 m_2 g - T m_2 = m_1 T - m_1 m_2 g$$
$$2 m_1 m_2 g = T (m_1 + m_2) \quad \text{or} \quad \boxed{T = \frac{2 m_1 m_2 \times g}{m_1 + m_2}} \qquad \text{...(9)}$$

Hence, tension T in the string can be calculated.

Sample Problem **Two masses of 10 kg and 6 kg connected at the two ends of an inextensible string pass over a smooth frictionless pulley. Calculate acceleration of the system and tension in the string.**

Sol. Here, $m_1 = 10$ kg, $m_2 = 6$ kg, $a = ?$ $T = ?$

$$a = \frac{(m_1 - m_2)\, g}{m_1 + m_2} = \left(\frac{10 - 6}{10 + 6}\right) \times 9{\cdot}8 \text{ m/s}^2 = 2{\cdot}45 \text{ m/s}^2$$

$$T = \frac{2 m_1 m_2\, g}{m_1 + m_2} = \frac{2 \times 10 \times 6 \times 9{\cdot}8}{(10 + 6)} = \mathbf{73{\cdot}5\ N}$$

5.15. IMPULSE

The forces which act on bodies for short time are called impulsive forces. For example : Hitting, jumping, diving, catching, kicking, etc. are all examples of impulsive forces.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. Thus it is not possible to measure easily the value of impulsive force because it changes with time. In such cases, we measure the total effect of the force, called impulse. Hence

impulse of a force is a measure of total effect of the force.

It is given by the product of average force and the time for which the force acts on the body ; *i.e.*,

$$\boxed{\text{Impulse} = \text{average force} \times \text{time}} \qquad \text{...(10)}$$

According to Newton's 2nd law of motion, $\vec{F} = \dfrac{d\vec{p}}{dt}$ $\quad \therefore \quad \vec{F}\,dt = d\vec{p}$

Integrating both sides within the limits indicated, we get

$$\int_0^t \vec{F}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} \qquad \text{...(11)}$$

where $\vec{p}_1$ is initial linear momentum at $t = 0$ and $\vec{p}_2$ is final linear momentum at time t.

If $\vec{F}_{av}$ is the (constant) force during this time, we may rewrite (11) as

$$\vec{F}_{av}\,[t]_0^t = [\vec{p}\,]_{\vec{p}_1}^{\vec{p}_2}$$

or

$$\vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1 \qquad \text{...(12)}$$

i.e., Impulse,

$$\boxed{\vec{I} = \vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1} \qquad \text{...(13)}$$

Hence, impulse of a force, which is the product of average force during impact and the time for which the impact lasts, is measured by the total change in linear momentum produced during the impact.

Impulse of a force is positive, negative or zero according as the momentum of the body increases, decreases or remains unchanged under the effect of force.

Impulse is a ***vector quantity***. Its direction is the same as that of the force.

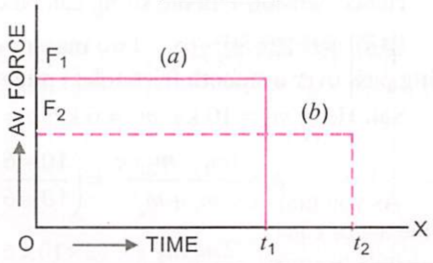
The **dimensional formula** of impulse is the same as that of linear momentum *i.e.*, $[M^1 L^1 T^{-1}]$.

The SI units of impulse are $(N\text{–}s)$ and kg ms^{-1} and the c.g.s. units of impulse are dyne-sec and g cm s^{-1}.

If we plot a graph between average force and time, the area under the curve and time axis gives the value of impulse. In Fig. 5.5, if the areas under the curves (*a*) and (*b*) are equal, they represent equal impulse.

From graph, we note that the force (F_1) for curve (*a*) is greater than the force (F_2) for curve (*b*). The time t_2 for which the force (F_2) acts is greater in case of curve (*b*) than the time t_1 in case of curve (*a*).

FIGURE 5.5



i.e.,

$$\boxed{F_1 \times t_1 = F_2 \times t_2} \qquad \text{...(14)}$$

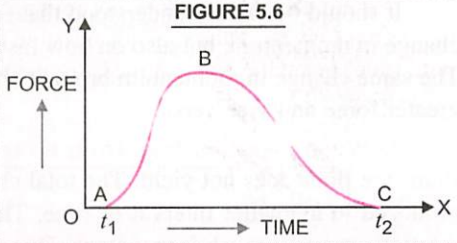
Hence, a given change in linear momentum can be produced by applying a larger force for a smaller time or by applying a smaller force for a larger time. This is sometimes called impulse-momentum theorem.

According to impulse-momentum theorem, the linear momentum can be produced by applying a smaller force for a longer time or by applying a larger force for a shorter time.

In Fig. 5.6, we have shown a **variable force,** *i.e.*, a force that changes with time. The impulse of variable force

$$\boxed{\text{Impulse} = \int_{t_1}^{t_2} \vec{F}\, dt = \text{area ABCA}}$$

FIGURE 5.6



i.e., impulse of a variable force is equal to area enclosed by the force curve and the time axis.

Retain in Memory

Examples of impulsive forces are in hitting, kicking, jumping, diving, catching, and colliding etc.

From (13), $\vec{F}_{av} = \dfrac{\vec{p}_2 - \vec{p}_1}{t}$ ...(15)

Therefore, for a given change in momentum $(\vec{p}_2 - \vec{p}_1)$, if time of contact (t) is increased, average force decreases and vice-versa. For example, in hitting or kicking a ball, time of contact is decreased so that a large force acts on the ball producing greater acceleration.

Sample Problem 1 **A hammer of mass 1 kg moving with a speed of 6 ms^{-1} strikes a wall and comes to rest in 0·1 s. Calculate (i) the impulse of force (ii) the retardation of the hammer, and (iii) the retarding force that stops the hammer.**

Sol. Here, $m = 1$ kg, $u = 6$ m/s, $v = 0$, $t = 0{\cdot}1$ s

Impulse $= F \times t = m\,(v - u) = 1\,(0 - 6) = \mathbf{-6\ Ns}$

Retarding force that stops the hammer,

$$F = \frac{F \times t}{t} = \frac{6}{0{\cdot}1} = \mathbf{60\ N}$$

Retardation of hammer, $a = \dfrac{F}{m} = \dfrac{60}{1} = \mathbf{60\ m/s^2}$

DO YOU KNOW ?

Impulsive force is like any other force — except that it is large and acts for a short time.

Sample Problem 2 **While launching a rocket of mass 2 × 10^4 kg, a force of 5 × 10^5 N is applied for 10 seconds. What is the velocity attained by the rocket at the end of 10 s ?**

Sol. Here, $m = 2 \times 10^4$ kg ; $F = 5 \times 10^5$ N, $t = 10$ s, $u = 0$, $v = ?$

Impulse of force $= F \times t = m\,(v - u)$

$$5 \times 10^5 \times 10 = 2 \times 10^4\,(v - 0)$$

$$v = \frac{5 \times 10^6}{2 \times 10^4} = \mathbf{250\ m/s}$$

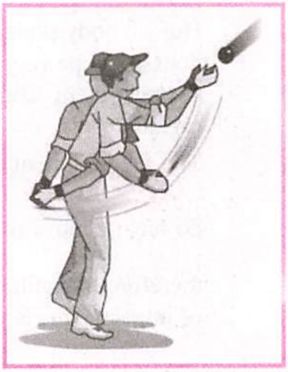
5.16. APPLICATIONS OF THE CONCEPT OF IMPULSE

1. A cricket player lowers his hands while catching a cricket ball. As you may have noticed, the player draws in his hands backwards in the act of catching the ball, Fig. 5.7. Thus he allows a longer time for his hands to stop the ball.

As Impulse = force × time = change in linear momentum.

or force = change in linear momentum/time, therefore for a given change in linear momentum, by increasing the time of a catch, the player has to apply a smaller force against the ball in order to stop it. The ball in turn, exerts a smaller force on his hands and his hands are not injured.

FIGURE 5.7



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It should be clearly understood that force depends not only on the change in momentum, but also on how fast the change is brought about. The same change in momentum brought about in a shorter time needs a greater force and vice-versa.

2. When a person falls from a certain height on a cemented floor, the floor does not yield. The total change in linear momentum is produced in a smaller interval of time. Therefore, as explained above, the floor exerts a much larger force. Due to it, a person receives more injury.

On the other hand, when a person falls on a heap of sand, the sand yields. The same change in linear momentum is produced in much longer time. The average force exerted on the person by the heap of sand is, therefore, much smaller and hence the person is not hurt.

3. China wares and glasswares are wrapped in paper or straw pieces while packing. In the event of fall, impact will take a longer time to reach the glass/chinawares through paper/straw. As a result, the average force exerted on the china or glasswares is small and chances of their breaking reduce.

DO YOU KNOW ?

Seat belts in cars are rightly called safety belts.

In a high speed head on car collision, each car comes to a stop rapidly. An unrestrained occupant of car moves at high speed and crashes through wind screen into the object collided with or into the road or into another occupant close by. An occupant restrained properly with a seat belt stops with the car. Further, properly fitted seat belt (*i*) spreads the force over a larger area of the body, (*ii*) reduces the likelihood of a collision between the body and interior of the vehicle.

4. The vehicles like scooter, car, bus, truck etc. are provided with shockers. When they move over an uneven road, impulsive forces are exerted by the road. The function of shockers is to increase the time of impact. This would reduce the force/jerk experienced by the rider of the vehicle.

5. Bogies of a train are provided with the buffers. They avoid severe jerks during shunting of the train.

Due to presence of buffers, time of impact increases. Therefore, force during jerks decreases. Hence the chances of damage decrease.

6. It is difficult to catch a cricket ball than to catch a tennis ball. The cricket ball being heavier has much larger momentum and therefore, exerts a much larger force on the hands during catch, in comparison to the force exerted by tennis ball.

7. In a head on collision between two vehicles, change in linear momentum is equal to sum of the linear momenta of the two vehicles. As time of impact is small, an extremely large force developes which causes damage to the vehicles. The damage to the lighter vehicle will obviously be more than the damage to the heavier vehicle.

8. An athlete is advised to come to stop slowly, after finishing a fast race, so that time of stop increases and hence force experienced by him decreases.

Retain in Memory

Human body skeleton is a fairly rigid frame that protects the vital organs inside. But this skeleton does not cope very well with sudden blows. These blows may push or pull the bones hard enough to break them. They may tear or strain the ligaments/muscles and even push the bones into vital organs.

The magnitude of change in our body's momentum is usually beyond our control. However, we do have control over the time interval during which the momentum changes. As $\vec{F}_{\text{net}} = \dfrac{\Delta \vec{p}}{\Delta t}$, therefore, magnitude of net force developed on our body can decrease by increasing Δt, the time of impact. This is what is being done in all the applications of the concept of impulse.

Curiosity Questions

Q. 1. How does a bike helmet protect our head ?

Ans. A bike helmet protects our head using two simple principles of Physics :

(*i*) The foam lining in the interior of the helmet increases the time over which the impulse acts, reducing the force thereby (Impulse = force × time = constant).

(*ii*) The rigid helmet spreads this force, applying it over a wider area. Therefore, pressure exerted (= force/area) is reduced. As is known, it is the applied pressure that breaks the bones and the skin, rather than the applied force.

Q. 2. What part does Physics play in the design of running shoes ?

Ans. Each person's foot is different and each person runs in a different way. Each time an average runner's foot strikes the ground, the vertical component of its momentum reduces to zero, requiring a large impulse. To reduce the stress of this load on the leg and foot of the runner, the time of impact has to be increased.

Athletic shoes have four parts : *the upper section, the insole, the midsole* and *the out sole.* The uppers are usually made of light weight materials that breathe.

The midsole, which is sandwiched between the insole and outsole is the most important part of the shoe. It cushions the foot increasing the time of impact and absorbs part of the force on the runner's body. A recent design provides two sets of air cylinders placed beneath the insole, over the midsole.

5.17. EXPLANATION OF NEWTON'S THIRD LAW OF MOTION

We have seen above that Newton's first law of motion defines force. The second law establishes a relation between the external force on a body and its acceleration. ***The third law tells us what is the origin of external force on the body.*** The law implies that the external force on a body arises always due to some other body.

For example, when we press a coiled spring, the spring is compressed by the force of our hand. In turn, the compressed spring exerts a force on our hand and we can feel it. Again, the earth pulls a stone downwards due to gravity. According to Newton, the stone exerts an equal and opposite force on the earth. We do not notice this force as the mass of earth is too large and the effect of the force exerted by stone, on motion of earth is negligible.

Thus, according to Newton, ***force never occurs singly in nature. Forces always occur in pairs as a result of mutual interaction between two bodies.***

According to Newton's third law, to every action, there is always an equal and opposite reaction.

Here, the term *action* means the force exerted by one body on the other body, while the term *reaction* means the force exerted by the second body on the first. If $\vec{F}_{AB}$ is the force exerted on body *A* by body *B* (*i.e.* action) and $\vec{F}_{BA}$ is the force exerted on body *B* by body *A* (*i.e.* reaction), then according to Newton's third law of motion,

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}} \qquad ...(16)$$

To prove this law, consider an isolated system consisting of two bodies *A* and *B* moving along the same straight line.

Let them collide so that they mutually act and react upon each other. Due to this, their velocities will change and hence their linear momenta will also change.

Let $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$ be the change in linear momenta of bodies *A* and *B* respectively. According to law of conservation of linear momentum, *the net change in linear momentum of the system is zero. i.e.,*

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0 \qquad \text{or} \qquad \Delta\vec{p}_2 = -\Delta\vec{p}_1 \qquad ...(17)$$

Dividing both sides by Δt and taking the limit as $\Delta t \to 0$, we get

$$\underset{\Delta t \to 0}{\text{Limit}} \left(\frac{\Delta \vec{p}_2}{\Delta t} \right) = - \underset{\Delta t \to 0}{\text{Limit}} \left(\frac{\Delta \vec{p}_1}{\Delta t} \right)$$

$$\frac{d\,\vec{p}_2}{dt} = - \frac{d\,\vec{p}_1}{dt} \qquad \text{...(18)}$$

i.e., Rate of change of linear momentum of B = – rate of change of linear momentum of A

or Force on B = – Force on A *i.e.*, $\boxed{\vec{F}_{BA} = -\vec{F}_{AB}}$

Hence, the forces of action and reaction are equal and opposite, which proves Newton's 3rd law of motion.

Retain in Memory

1. The terms action and reaction in third law mean nothing else but force. Therefore, a simple and clear way of stating third law is :
   Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on body B by A.
2. The terms action and reaction in the third law give us a wrong impression as if action is the cause and reaction is the effect, so that action comes always before reaction.
   Infact, there is no cause effect relationship implied in third law. The force on A by B and the force on B by A act at the same instant or simultaneously. Therefore, any one of them may be called action, and the other reaction.
3. *Forces of action and reaction act always on different bodies. Hence they never cancel each other.* Each force produces its own effect.
   If we consider a pair of bodies A and B, then according to third law, $\vec{F}_{AB} = -\vec{F}_{BA}$
   i.e., force on A by B = – force on B by A
   If action and reaction forces were to act on the same body, their resultant would be zero. As action and reaction forces never act on the same body, their resultant cannot be zero.
4. *The forces of action and reaction may appear due to actual physical contact of the two bodies or even from a distance. But they are always equal and opposite.*
5. *Newton's third law is applicable whether the bodies are at rest or they are in motion.*
6. *The third law applies to all types of forces e.g. gravitational, electric or magnetic forces, etc.*

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Sample Problem **A machine gun has a mass of 20 kg. It fires 35 g bullets at the rate of 400 bullets per minute with a speed of 400 m/s. What force must be applied to the gun to keep it in position ?**

Sol. Here, Mass of machine gun, $M = 20$ kg

mass of each bullet, $m = 35\text{ g} = 0{\cdot}035$ kg

velocity of bullet $v = 400$ m/s

Number of bullets/sec $(n) = \dfrac{400}{60} = \dfrac{20}{3}$

Force required = rate of change of linear momentum of bullets $= \dfrac{n\,(mv)}{t}$

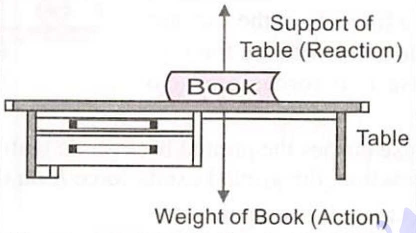
$$= \frac{20}{3} \times \frac{0{\cdot}035 \times 400}{1} = \mathbf{93{\cdot}3\ N}$$

5.18. ILLUSTRATIONS OF NEWTON'S THIRD LAW

Some of the examples of Newton's third law of motion are given below :

1. Book kept on a table. A book lying on a table exerts a force on the table, which is equal to the weight of the book. This may be called the force of action. The table supports the book, by exerting an equal force on the book. This is the force of reaction, Fig. 5.8.

FIGURE 5.8



Since the book is observed to be at rest, the net external force on it must be zero. This implies that the normal reaction must be equal and opposite to the weight of the book.

2. Walking. While walking, a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in the forward direction with an equal force (reaction). The component of reaction in the horizontal direction makes the person move forward.

Note that when the person presses the ground in the backward direction, the earth would move backwards. As mass of earth is too large, acceleration produced in earth on account of person's pushing is negligible. It cannot be detected.

3. Swimming. A swimmer pushes the water backwards (action). The water pushes the swimmer forward (reaction) with the same force. Hence the swimmer swims.

When a person jumps out of a boat, he pushes the boat backwards. According to Newton's third law, the boat exerts an equal force on the person in the forward direction. Therefore, when person jumps out of the boat, the boat moves backwards.

Similarly, a boatman pushes the river bank with a bamboo pole to take his boat into the river.

4. Firing from a gun. When a gun is fired, the bullet moves forward (action). The gun recoils backwards (reaction).

5. Flight of jet planes and rockets. The burnt fuel which appears in the form of hot and highly compressed gases escapes through the nozzle (action) in the backward direction. The escaping gases push the jet plane or rocket forward (reaction) with the same force. Hence the jet or rocket moves.

6. Rebounding of a rubber ball. When a rubber ball is struck against a wall or floor, it exerts a force on wall (action). The ball rebounds with an equal force (reaction) exerted by the wall or floor on the ball.

7. It is difficult to walk on sand or ice. This is because on pushing, sand gets displaced and reaction from sandy ground is very little. In case of ice, force of reaction is again small, because friction between our feet and ice is very little.

8. Driving a nail into a wooden block without holding the block is difficult. This is because, when the wooden block is not restir against a support, the block and nail, both move forward on being hit with a hammer. However, when the block is held firmly against a support, and the nail is hit, an equal reaction of the support drives the nail into the block.

Retain in Memory

From what we have studied so far, we find that Newton's first law defines force ; 2nd law measures force and third law gives us the nature of force.

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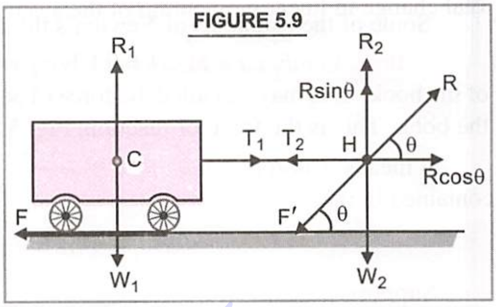
5.19. HORSE AND CART PROBLEM

FIGURE 5.9



The various forces acting on a system of horse and cart at rest are shown in Fig. 5.9.

Here, the weight (W_1) of the cart C is balanced by the reaction (R_1) of the ground on the cart. The weight W_2 of the horse H is balanced by the reaction R_2 of the ground on the horse.

The horse pulls the cart with a force T_1 in the forward direction. The cart, in turn, pulls the horse with the force T_2 in the backward direction. These two forces are also balanced.

While pulling the cart, the horse pushes the ground backwards with its feet by a force F' inclined at an angle θ with the horizontal. As a reaction, the ground exerts force R on the horse equal and opposite to F'.

R can be resolved into two rectangular components :

$R \sin \theta$ vertically upwards, and $R \cos \theta$ along the horizontal.

The component $R \cos \theta$ tends to move the cart forward. This motion is opposed by the force of friction F between the cart and the ground. The cart will move only when $\boxed{R \cos \theta > F.}$

5.20. THIRD LAW IS CONTAINED IN FIRST LAW OF MOTION

Consider an isolated system in which a book is lying on a table. The forces on the system are :

(*i*) Weight of the book acting on the table (force of action),

(*ii*) Supporting force of the table on the book (force of reaction).

As the system is at rest, the net force on the system, according to Newton's first law of motion must be zero. The resultant of the two forces of action and reaction will be zero only when they are equal and opposite. This is third law of motion. Hence the third law can be obtained from the first law.

5.21. SECOND LAW IS THE REAL LAW OF MOTION

To establish it, we shall show that (*a*) *the first law is contained in the second law, and*

(*b*) *the third law is contained in the second law.*

(*a*) First law is contained in the second law. According to Newton's second law of motion,

$$F = ma$$

If no external force is applied on a body, $F = 0 \; \therefore \; ma = 0$

As $m \neq 0$, therefore, $a = 0$

Thus there will be no acceleration in the body if no external force is applied. This means that a body at rest will remain at rest and a body in uniform motion will continue moving uniformly along the same straight line in the absence of an external force. This is what is stated by first law of motion. Hence the first law is contained in the second law.

(*b*) Third law is contained in the second law. Consider an isolated system of two bodies A and B. Let them collide. During collision, let the body A exert a force $\vec{F_1}$ (action) on body B for a time Δt. Let the body B exert a force $\vec{F_2}$ (reaction) on body A for the same time Δt.

As, change in linear momentum = Force × time

$\therefore$ change in linear momentum of $B = \vec{F_1} \times \Delta t$ and change in linear momentum of $A = \vec{F_2} \times \Delta t$

Total change in linear momentum of A and $B = \vec{F_1} \, \Delta t + \vec{F_2} \, \Delta t$

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As no external force is acting on the system, therefore according to Newton's second law of motion, the total change in linear momentum of the system is zero. Thus

$$\vec{F}_1 \Delta t + \vec{F}_2 \Delta t = 0 \quad \text{or} \quad \vec{F}_1 \Delta t = -\vec{F}_2 \Delta t$$

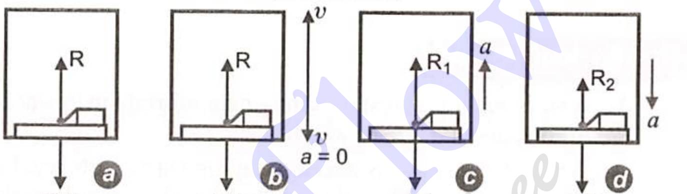
$$\text{or} \quad \vec{F}_1 = -\vec{F}_2 \quad \text{or} \quad \boxed{\text{Action} = -\text{Reaction}}$$

It means *action is equal and opposite to reaction*. This is what is stated by third law. Thus third law is contained in the second law. Hence, ***second law is the real law of motion.***

5.22. APPARENT WEIGHT OF A MAN IN A LIFT/ELEVATOR

Suppose a person of mass m is standing on a weighing machine placed in an elevator/lift. The *actual weight* of the person = mg. This acts on the weighing machine which offers a reaction R given by the reading of the weighing machine. This reaction exerted by the surface of contact on the person is the *apparent weight* of the person. We shall discuss how R is related to mg in the following different situations :

FIGURE 5.10



(a) Elevator *at rest*

(b) Elevator moving up/down with constant velocity

(c) Elevator moving up with constant acceleration

(d) Elevator moving down with constant acceleration

(i) When the elevator is at rest

Acceleration of the person = 0

$\therefore$ Net force on the person, $f = 0$, *i.e.*, $R - mg = 0$ or $\boxed{R = mg}$

i.e. apparent weight is equal to the actual weight of the person, Fig. 5.10(a).

(ii) When the elevator is moving uniformly in the upward/downward direction

Again, in uniform motion, acceleration of the person = 0. Proceeding as in case (*i*), $R = mg$, *i.e.*, apparent weight = actual weight of the person. This is also shown in Fig. 5.10(b).

(iii) When the elevator is moving up with constant acceleration

Suppose uniform upward acceleration of the person in the lift = a

$\therefore$ Net upward force on the person, $f = ma$

As is clear from Fig. 5.10(c),

$$f = R_1 - mg \quad \text{or} \quad \boxed{R_1 = mg + f = mg + ma = m(g + a)} \qquad \text{...(19)}$$

Thus $R_1 > mg$

Hence, apparent weight of the person becomes more than the actual weight, when the elevator is accelerating upwards.

(iv) When the elevator is moving down with constant acceleration

Suppose uniform downward acceleration of the person in the lift = a

$\therefore$ Net downward force on the person, $f = ma$

As is clear from Fig. 5.10(d), $f = mg - R_2$

$$\boxed{R_2 = mg - f = mg - ma = m(g - a)} \qquad \text{...(20)}$$

Thus $R_2 < mg$

Hence, apparent weight of the person becomes less than the actual weight when the elevator is accelerating downwards.

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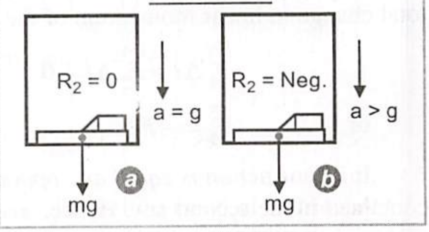
FIGURE 5.11



(v) In free fall of the lift under gravity, $a = g$

$\therefore$ From (20), $R_2 = m\,(g - g) = 0$, Fig. 5.11(*a*)

i.e., apparent weight of the person becomes zero or the person becomes weightless in free fall.

Note that weightlessness is felt only because the force of reaction between the person and the plane with which he is in contact vanishes. For details, refer to unit 6 of this book.

(vi) When downward acceleration is greater than g.

i.e., $a > g$, then from (20), $R_2 = m\,(g - a)$, R_2 becomes negative *i.e.* apparent weight of the person becomes negative, Fig. 5.11(*b*). In that event, the person will rise from the floor of the lift and crash against the ceiling of the lift.

Retain in Memory

1. If an elevator is at rest or is moving uniformly in upward/downward direction, the apparent weight of the person = actual weight.
2. When the elevator is accelerating upwards with acceleration, a, apparent weight of the person, $R_1 = m\,(g + a)$. When the elevator has downward acceleration, a, apparent weight of the person, $R_2 = m\,(g - a)$.
3. When the elevator falls freely under gravity, the apparent weight of the person,
   $R_2 = m\,(g - g)$ = Zero.

Sample Problem **The strings of a parachute can bear a maximum tension of 72 kg wt. By what minimum acceleration can a person of 90 kg descend by means of this parachute ?**

Sol. Here, $T = 72$ kg wt. $= 72 \times 9{\cdot}8$ N

$m = 90$ kg ; $a = ?$

For the person to descend, $T = m\,(g - a)$

$$72 \times 9{\cdot}8 = 90\,(9{\cdot}8 - a) \qquad \frac{72 \times 9{\cdot}8}{90} = 9{\cdot}8 - a \qquad 0{\cdot}8 \times 9{\cdot}8 = 9{\cdot}8 - a$$

$$a = 9{\cdot}8 - 0{\cdot}8 \times 9{\cdot}8 = 9{\cdot}8\,(1 - 0{\cdot}8) = \mathbf{1{\cdot}96\ m/s^2}$$

5.23. PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

> ***According to this principle, in an isolated system*, the vector sum of the linear momenta of all the bodies of the system is conserved and is not affected due to their mutual action and reaction.***

Thus, in an isolated system (*i.e.*, a system with no external force), mutual forces between pairs of particles in the system can cause changes in linear momentum of individual particles. But as the mutual forces for each pair are equal and opposite, the linear momentum changes cancel in pairs, and the total linear momentum remains unchanged. ***Hence, the total linear momentum of an isolated system of interacting particles/bodies is conserved.*** This principle is an important consequence of second and third laws of motion.

Let us consider an isolated system comprising of two bodies A and B, with initial linear momenta $\vec{p}_A$ and $\vec{p}_B$. Let them collide for a small time Δt and separate with final linear momenta $\vec{p}_A^{\,\prime}$ and $\vec{p}_B^{\,\prime}$ respectively. During collision,

If $\vec{F}_{AB}$ is force on A exerted by B, and $\vec{F}_{BA}$ is force on B exerted by A, then, according to Newton's second law,

---

*An isolated system is a system of bodies which is free from the influence of any external forces.

$$\vec{F}_{AB} \times \Delta t = \text{change in linear momentum of } A = \vec{p}_A\,' - \vec{p}_A \quad \text{...(21)}$$

$$\vec{F}_{BA} \times \Delta t = \text{change in linear momentum of } B = \vec{p}_B\,' - \vec{p}_B \quad \text{...(22)}$$

According to Newton's third law, $\vec{F}_{AB} = -\vec{F}_{BA}$

∴ From eqns. (21) and (22),

$$\vec{p}_A\,' - \vec{p}_A = -(\vec{p}_B\,' - \vec{p}_B) \quad \text{or} \quad \vec{p}_A\,' + \vec{p}_B\,' = \vec{p}_A + \vec{p}_B$$

which shows that total final linear momentum of the isolated system is equal to its total initial linear momentum. This proves the principle of conservation of linear momentum. Note that the law holds, whether the collision is elastic or inelastic.

Further, in the interaction between two bodies *A* and *B*, we find that

(*i*) total linear momentum of the system remains constant

(*ii*) change in linear momentum of the system is zero

(*iii*) change in linear momentum of *A* is equal and opposite to the change in linear momentum of *B*

(*iv*) the force that *A* exerts on *B* is equal and opposite to the force that *B* exerts on *A*.

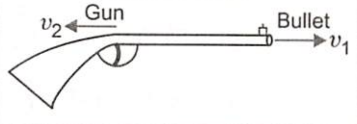
Retain in Memory

1. The law of conservation of linear momentum is *universal i.e.* it applies to both, the microscopic as well as macroscopic systems.
2. Linear momentum depends on frame of reference. Observers in different frames would find different values of linear momentum of a system. But each would observe that the value of linear momentum does not change with time, provided the system is isolated. Thus the principle of conservation of linear momentum is independent of frame of reference, though linear momentum depends on frame of reference.

5.24. PRACTICAL APPLICATIONS OF THE PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

1. Recoiling of a gun. When a bullet is fired from a gun, the gun recoils *i.e.* moves in a direction opposite to the direction of motion of the bullet, Fig. 5.12. The recoil velocity of the gun can be calculated from the principle of conservation of linear momentum.

FIGURE 5.12



Suppose m_1 = mass of bullet, m_2 = mass of gun,

$\vec{v}_1$ = velocity of the bullet,

$\vec{v}_2$ = velocity of recoil of the gun.

Before firing, the gun and the bullet both, are at rest. Therefore, total linear momentum before firing = 0. The vector sum of linear momenta on firing $= m_1 \vec{v}_1 + m_2 \vec{v}_2$. According to the principle of conservation of linear momentum, total linear momentum after firing should also be zero.

$$\therefore \quad m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0 \quad \text{...(23)}$$

$$\text{or} \quad m_2 \vec{v}_2 = -m_1 \vec{v}_1 \quad \text{or} \quad \boxed{\vec{v}_2 = -\frac{m_1 \vec{v}_1}{m_2}} \quad \text{...(24)}$$

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The negative sign shows that direction of $\vec{v}_2$ is opposite to the direction of $\vec{v}_1$ *i.e.* the gun recoils. Further, as $m_2 >> m_1$ therefore, $\vec{v}_2 << \vec{v}_1$ *i.e.* **velocity of recoil of the gun is much smaller than the velocity of the bullet.**

From (24), $\quad v_2 \propto \dfrac{1}{m_2}$

It means that a heavier gun will recoil with a smaller velocity and vice-versa.

DO YOU KNOW ?

In the case discussed above, initial K.E. of the system is zero, as both the gun and the bullet are at rest.

Final K.E. of the system $= \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right) > 0$. Thus K.E. of the system increases (and is not constant). If P.E. is assumed to be constant, mechanical energy (= K.E. + P.E.) will also increase. As M.E. is conserved, therefore, **chemical energy of gun powder must have been converted into K.E.**

2. While firing, the gun must be held tightly to the shoulder. This would save hurting the shoulder. When the gun is held tightly, the body of the shooter and the gun behave as one body. Total mass becomes large and therefore, recoil velocity of the body and the gun becomes too small.

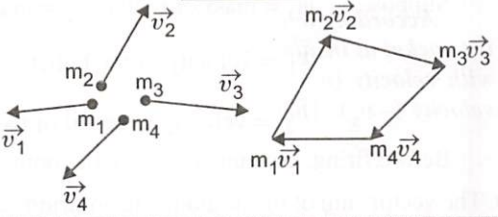
3. Flight of rockets and jet planes. In rockets and jet planes, the fuel is burnt in the presence of some oxidising agent in combustion chamber. The hot and highly compressed gases escape through the narrow opening (*i.e.*, exhaust nozzle) with large velocity. *As a result of it, the escaping gases acquire a large backward momentum.* This in turn, imparts an *equal forward momentum to the rocket in accordance with the law of conservation of linear momentum.*

4. When a man jumps out of a boat to the shore, the boat is pushed slightly away from the shore. The linear momentum of the boat is equal and opposite to that of the man in accordance with the law of conservation of linear momentum.

5. Explosion of a bomb. When a bomb falls vertically downwards, its horizontal velocity is zero and hence its horizontal momentum is zero. When bomb explodes, its pieces are scattered in different directions so that the vector sum of linear momenta of these pieces becomes zero in accordance with the law of conservation of linear momentum.

In Fig. 5.13, we have shown a bomb at rest exploding into four fragments of masses m_1, m_2, m_3, m_4 flying with respective velocities $\vec{v}_1, \vec{v}_2, \vec{v}_3$ and $\vec{v}_4$ in different directions.

FIGURE 5.13



The linear momenta of the four fragments are

$$\vec{p}_1 = m_1 \vec{v}_1 \quad ; \quad \vec{p}_2 = m_2 \vec{v}_2 \quad ;$$

$$\vec{p}_3 = m_3 \vec{v}_3 \quad \text{and} \quad \vec{p}_4 = m_4 \vec{v}_4$$

As no external force is involved in explosion, therefore according to the principle of conservation of linear momentum.

Linear momentum after explosion = linear momentum before explosion

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + m_4 \vec{v}_4 = 0 \cdot$$

The linear momenta of four fragments are represented by four sides of a polygon taken in the same order as shown in Fig. 5.13.

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6. A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

Sample Problem **A 30 kg shell is flying at 36 m/s. When the shell explodes into two parts of 12 kg and 18 kg, the lighter part stops, and heavier part flies on. What is the velocity of heavier part ?**

Sol. Here, $M = 30$ kg, $u = 36$ m/s.

$$m_1 = 12 \text{ kg}, v_1 = 0, m_2 = M - m_1 = 30 - 12 = 18 \text{ kg}, v_2 = ?$$

From the law of conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = M \times u \; ; \qquad 0 + 18 v_2 = 30 \times 36 \qquad v_2 = \frac{30 \times 36}{18} = \mathbf{60\,m/s}$$

5.25. SYSTEMS WITH VARIABLE MASS : A ROCKET

Usually, we assume that total mass of a system remains constant. Sometimes, as in case of a rocket, the mass does not remain constant. Most of the mass of a rocket on its launching pad is of the fuel. The whole of the fuel is eventually burnt and combustion products are ejected from the nozzle of the rocket engine. We handle the variable mass situation, by applying Newton's second law—not to the rocket alone, but to the rocket and its ejected combustion products taken together. The mass of the combined system does not change as the rocket accelerates. In this study, we assume that external forces *e.g.*, gravitational forces and frictional forces of earth's atmosphere are negligibly small and they do not affect the motion of the rocket.

Consider the flight of a rocket directed vertically upwards from the surface of earth. At $t = 0$, suppose

m_0 = initial mass of the rocket including that of the fuel,

v_0 = initial velocity of the rocket, Fig. 5.14(*a*).

At any time t, suppose m = mass of the rocket left,

v = velocity acquired by the rocket, Fig. 5.14(*b*).

As the exhaust gases are escaping, $m < m_0$ and $v > v_0$

In a small interval of time dt,

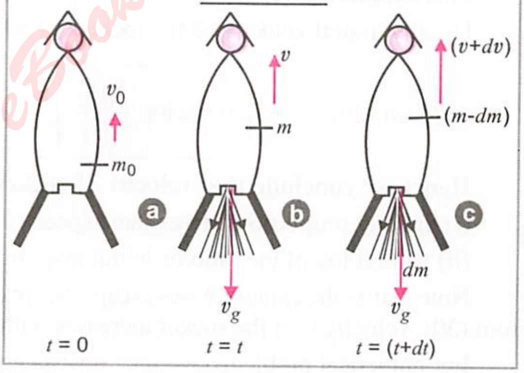
suppose dm = a small decrease in mass of the rocket = mass of the exhaust gases that escape

dv = corresponding small increase in velocity of the rocket, Fig. 5.14(*c*),

v_g = velocity of exhaust gases w.r.t. earth.

The velocity v_g of exhaust gases is taken as *negative*, because these gases move in a direction opposite to that of the rocket.

FIGURE 5.14



According to the principle of conservation of linear momentum, the linear momentum of mass m of the rocket at instant t must be equal to vector sum of linear momentum of rocket mass (m – dm) moving with velocity (v + dv) and linear momentum of mass (dm) of exhaust gases moving downwards with velocity (– v_g). Therefore,

$$mv = (m - dm)(v + dv) + dm(-v_g) \qquad \text{...(25)}$$

$$mv = mv + m(dv) - (dm)v - dm\,dv - (dm)v_g$$

As dm and dv, both are small, their product is negligibly small.

$$\therefore \qquad m\,dv = dm(v + v_g) \qquad \text{...(26)}$$

When the rocket has gone far away from earth, the velocity of exhaust gases with respect to earth becomes irrelevant. ***We take relative velocity of exhaust gases with respect to the rocket as u.*** As the gases and rocket are moving in opposite directions, therefore

$$v + v_g = -u \qquad \text{...(27)}$$

The minus sign is again for the downward direction of motion of exhaust gases, in contrast to the upward direction of motion of the rocket.

$\therefore$ From (26), $$m\,dv = -u\,dm \qquad \text{...(28)}$$

or $$dv = -u\frac{dm}{m}$$

Now, at $t = 0$; $m = m_0$ and $v = v_0$; At $t = t$; $m = m$ and $v = v$

$\therefore$ Integrating both sides of (28) within proper limits, we obtain $\displaystyle\int_{v_0}^{v} dv = \int_{m_0}^{m} -u\frac{dm}{m}$

As velocity of exhaust gases (u) is constant throughout the firing of the rocket, therefore,

$$\int_{v_0}^{v} dv = -u\int_{m_0}^{m}\frac{dm}{m} \quad \text{or} \quad [v]_{v_0}^{v} = -u\left[\log_e m\right]_{m_0}^{m}$$

or $$v - v_0 = -u\,[\log_e m - \log_e m_0] = -u\log_e\left(\frac{m}{m_0}\right) = +u\log_e\left(\frac{m_0}{m}\right)$$

or $$\boxed{v = v_0 + u\log_e\left(\frac{m_0}{m}\right)} \qquad \text{...(29)}$$

This equation gives us the velocity of the rocket at any time t, when its mass is m.

Usually, initial velocity of the rocket at $t = 0$ is zero *i.e.* $v_0 = 0$

$\therefore$ From (29), $$\boxed{v = u\log_e\left(\frac{m_0}{m}\right)} \qquad \text{...(30)}$$

Hence we conclude that velocity of rocket at any time is

(*i*) directly proportional to exhaust speed (u) of the gases and

(*ii*) natural log of the ratio of initial mass of the rocket to its mass (m) at that instant of time.

Note that as the exhaust gases escape, the residual mass (m) of the rocket decreases with time. Therefore, from (30), velocity v of the rocket increases with time. Hence motion of rocket is an ***accelerated motion.***

For numerical problems, we may rewrite eqn. (30) as

$$\boxed{v = 2{\cdot}303\,u\log_{10}\left(\frac{m_0}{m}\right)}$$

> ***Burnt out Speed of the rocket is the speed attained by the rocket when the whole of fuel of rocket has been burnt.***

Obviously, it is the *maximum velocity acquired by the rocket.* When the whole of the fuel of the rocket has been used up, the *residual mass* (m_r) of the rocket is equal to mass of empty container of the fuel. From (30),

$$\boxed{v_b = u\log_e\left(\frac{m_0}{m_r}\right) = 2{\cdot}303\,u\log_{10}\left(\frac{m_0}{m_r}\right)} \qquad \text{...(31)}$$

Upthrust on the rocket

It is the force with which the rocket moves upwards. Thus upthrust on the rocket is the upward force exerted on the rocket by ejection of exhaust gases.

Dividng both sides of (28) by small time interval dt, we get $m\frac{dv}{dt} = -u\frac{dm}{dt}$ ...**(32)**

As $\frac{dv}{dt} = a$, acceleration of the rocket at time t.

$\therefore \quad m\frac{dv}{dt} = ma = F$ = upthrust on the rocket at time t.

From (32), $\boxed{F = -u\frac{dm}{dt}}$ ...**(33)**

The negative sign indicates that thrust on the rocket is upwards, *i.e.*, in a direction opposite to the direction of escaping gases.

Ignoring negative sign, we find that thrust on the rocket at any instant is the product of velocity of escape of fuel gases w.r.t. the rocket and the rate of combustion of fuel at that instant.

We can calculate instantaneous acceleration (a) of the rocket at any time from eqn. (33), using the relation, $a = F/m$. Here, m is the mass of rocket left at time t.

Note that if effect of gravity on the motion of the rocket were taken into account, upward thrust on the rocket will decrease by (mg), the weight of the rocket. However, because of very large values of (u) and (dm/dt), **effect of gravity is negligible.**

Sample Problem 1 **A rocket motor consumes one quintal of fuel per second. The exhaust speed of gases w.r.t. rocket is 5 km/s. Calculate the force exerted on the rocket. What is the velocity acquired by the rocket, when its mass reduces to 1/100th of its initial mass ?**

Sol. Here, $\frac{dm}{dt} = 1$ quintal/sec = 100 kg/s, $u = -5$ km/s $= -5 \times 10^3$ m/s

As $\quad F = -u\left(\frac{dm}{dt}\right) \quad \therefore \quad F = -(-5 \times 10^3)(100) = \mathbf{5 \times 10^5\ N}$

Now $v = ?$, $\frac{m}{m_0} = \frac{1}{100}$ or $\frac{m_0}{m} = 100$

From $\quad v = u \log_e\left(\frac{m_0}{m}\right)$

$$v = 5 \times 10^3 \log_e 100 = (5 \times 10^3) \times 2{\cdot}303 \log_{10} 100 = 5 \times 10^3 \times 2{\cdot}303 \times 2$$

$$= \mathbf{2{\cdot}303 \times 10^4\ m/s}$$

Sample Problem 2 **Fuel is consumed in a rocket at the rate of 200 kg/s. What is the thrust experienced by the rocket if exhaust gases are ejected at a speed of 45 km/s ?**

Sol. Here, $\frac{dm}{dt} = 200$ kg/s ; $u = -45$ km/s $= -45 \times 10^3$ m/s.

Thrust on the rocket, $F = ?$

As $\quad F = -u\left(\frac{dm}{dt}\right) \quad \therefore \quad F = -(-45 \times 10^3) \times 200 = \mathbf{9 \times 10^6\ N}$

5.26. EQUILIBRIUM OF CONCURRENT FORCES

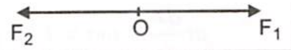
The forces which are acting at the same point are called concurrent forces.

Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.* i.e. when the particle has zero acceleration. According to Newton's 1st law, this means that the particle would be either at rest or in uniform linear motion.

Resultant of two forces $\vec{F_1}$ and $\vec{F_2}$ acting on a particle, Fig. 5.15 is

$$\vec{R} = \vec{F_1} + \vec{F_2}$$

FIGURE 5.15



Equilibrium requires

$$\therefore \quad \vec{R} = \vec{F_1} + \vec{F_2} = \vec{0} \quad \text{or} \quad \vec{F_1} = -\vec{F_2}$$

i.e., **the two forces on the particle must be equal and opposite.**

In Fig. 5.16, three concurrent forces $\vec{F_1}, \vec{F_2}, \vec{F_3}$ are shown acting at O.

Complete the parallelogram $OAC'B$. Join $\overrightarrow{OC'}$.

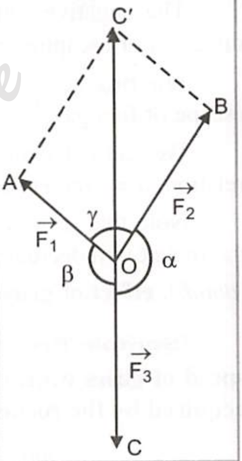
According to parallelogram law, $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC'}$

Add $\overrightarrow{OC}$ to both sides $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OC'} + \overrightarrow{OC}$

If $\overrightarrow{OC'}$ is equal and opposite to $\overrightarrow{OC}$,

then $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = -\overrightarrow{OC} + \overrightarrow{OC} = \vec{0}$, *i.e.,* $\vec{F_1} + \vec{F_2} + \vec{F_3} = \vec{0}$

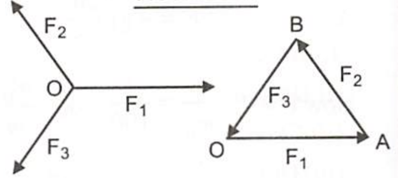
FIGURE 5.16



i.e., three concurrent forces $\vec{F_1}, \vec{F_2}, \vec{F_3}$ will be in equilibrium, when resultant of $\vec{F_1}$ and $\vec{F_2}$ is equal and opposite to the third force $\vec{F_3}$.

Also, we know from triangle law that **three concurrent forces $\vec{F_1}, \vec{F_2}, \vec{F_3}$ are in equilibrium, when they are represented by three sides of a triangle taken in the same order.** In Fig. 5.17, $\vec{F_1}$ is represented by $\overrightarrow{OA}$, $\vec{F_2}$ by $\overrightarrow{AB}$ and $\vec{F_3}$ by $\overrightarrow{BO}$. Hence their resultant is zero and they are in equilibrium. The result can be generalised to any number of forces.

FIGURE 5.17



A particle under the action of forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots \vec{F_n}$ will be in equilibrium, if these forces can be represented by the sides of a closed n sided polygon with arrows directed in the same sense, *i.e.,*

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots \vec{F_n} = 0$$

This equation implies that

$$F_{1x} + F_{2x} + F_{3x} + \ldots F_{nx} = 0 \quad \text{or} \quad \Sigma F_x = 0$$

$$F_{1y} + F_{2y} + F_{3y} + \ldots F_{ny} = 0 \quad \text{or} \quad \Sigma F_y = 0$$

$$F_{1z} + F_{2z} + F_{3z} + \ldots F_{nz} = 0 \quad \text{or} \quad \Sigma F_z = 0.$$

---

*This is the condition for translational equilibrium only. We shall study in unit 5 that a body may also have rotational equilibrium, for which net external torque on the body must be zero.

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where F_{1x}, F_{1y}, F_{1z} are the three rectangular components of $\vec{F_1}$ along x, y and z directions respectively, and so on.

Note that when concurrent forces in equilibrium, act on a body, the state of the body shall remain unchanged, *i.e.,* if the body is at rest, it will continue to be at rest. Similarly, a body will continue to move uniformly along the same straight line, if concurrent forces applied on the body are in equilibrium.

However, when the concurrent forces applied on a body are *not* in equilibrium, the state of the body will change depending on the resultant of concurrent forces.

Retain in Memory

1. Equilibrium of a particle does not mean that the particle is at rest. The particle in uniform linear motion is also said to be in equilibrium.
2. *An important result* follows when three concurrent forces are in equilibrium. This result was stated first of all by Lami and is therefore, called ***Lami's theorem.*** According to this theorem, when three concurrent forces F_1, F_2 and F_3 acting on a body are in equilibrium, then

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}} \qquad \text{...(34)}$$

where α = angle between $\vec{F_2}$ and $\vec{F_3}$,

β = angle between $\vec{F_3}$ and $\vec{F_1}$

γ = angle between $\vec{F_1}$ and $\vec{F_2}$

These forces are shown in Fig. 5.16.

5.27. SOLVING PROBLEMS IN MECHANICS

In mechanics, we often come across a system consisting of a number of bodies under the action of a number of forces, such as gravitational forces, frictional forces, forces due to connecting strings, supports etc.

To solve a typical problem in mechanics, we use the following steps :

(*i*) Draw a diagram showing various parts of the system with links, supports etc.

(*ii*) Choose any one convenient part of the system.

(*iii*) Draw a separate diagram of the chosen part showing all the forces on the chosen part by the remaining parts of the system, and also by other agencies. A diagram of this type is called ***free body diagram***

(*iv*) In free body diagram, include information about magnitude and direction of forces that are either given or you are sure of. The rest should be treated as unknown. Equate the net force acting in a particular direction to its mass times the acceleration produced in that direction.

(*v*) If necessary, we can follow the same procedure for any other part of the system. The equations of motion obtained for different parts of the system can be solved to obtain the desired results.

IMPORTANT NOTE

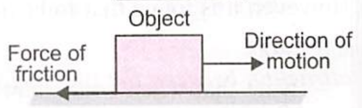
A frame of reference which is either at rest or moving with a constant velocity is called **Inertial Frame** of reference.

A frame of reference, which is accelerated is called **Non-Inertial Frame** of reference.

Newton's Laws of motion are valid in inertial frames of reference only.

5.28. FRICTION

FIGURE 5.18



According to Newton's first law of motion, a body moving uniformly along a straight line would continue to do so unless an external force is applied on it. In practice, we find things otherwise. For example, a ball rolling over the floor stops after some time. Similarly when we stop padlling our bicycle, it comes to rest after travelling a certain distance. Again, when we switch off the engine of our car, it stops after travelling some distance. All these examples show that there is some (invisible) force that opposes the motion of one body over the other, Fig. 5.18. This ***opposing force is called Friction.***

Again, when we apply a small force on a block, it does not move. The applied force must have been balanced by an opposing force (of friction). Thus force of friction comes into play even when one body tries to move over the surface of the other. Hence we define

> ***Friction as an opposing force that comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body.***

Thus force of friction is the force that developes at the surfaces of contact of two bodies and impedes (opposes) their relative motion.

Retain in Memory

A frictional force opposes even the attempted sliding/rolling of a body over a surface. When there is no attempt at sliding/rolling, there is no friction.

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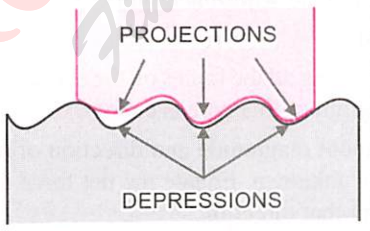
5.29. ORIGIN OF SLIDING FRICTION

(*a*) *According to Old View,* ***roughness of surfaces is the cause of friction.*** A surface which appears very smooth to the naked eye is found to have *irregularities** (roughness) when seen through a powerful microscope. This is true of every surface. When two bodies are in contact with each other, the irregularities in the surface of one body get *interlocked* in the irregularities of the other surface, Fig. 5.19. To move one body over the surface of the other, these interlockings have to be broken. Hence some force has to be applied. This applied force is a measure of friction between the two surfaces in contact.

FIGURE 5.19



Limitations of Old View

The old view of the origin of sliding friction predicts that

(*i*) Larger the area of contact between the surfaces, greater should be the force of friction between them. However, experiments show that force of friction between any two surfaces in contact is independent of the actual area of contact between them.

DO YOU KNOW ?

When two bodies touch each other, each one exerts a force on the other. These forces are called **contact forces.**

The component of contact force perpendicular to the surfaces in contact is called **Normal Reaction. And the component of contact force parallel to the surfaces in contact is the force of friction.**

Clearly, *normal reaction and force of friction are perpendicular to each other.*

Note that normal reaction and force of friction are due to intermolecular forces, which are electrical in nature.

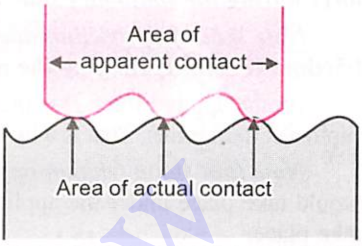
---

*The irregularities are in the form of projections and depressions.

(*ii*) When surfaces in contact are extra smooth, force of friction between them should almost vanish. However, it is found that force of friction increases when surfaces in contact are extra smooth.

(*b*) ***Modern View of friction is that it arises on account of strong atomic or molecular forces of attraction between the two surfaces at the points of actual contact.***

FIGURE 5.20



On account of roughness of surfaces, only the high points touch one another. Therefore, the actual microscopic area of contact is much less than the apparent macroscopic contact area, Fig. 5.20. These areas are roughly in the ratio 1 : 10^4. At the contact points, the molecules of the two surfaces are so close together that they exert very strong intermolecular forces of adhesion* on one another. As actual area of contact is very small, therefore, the ***adhesive pressure*** (= force/area) at the points of contact becomes too large. So much so that many of the contact points may deform plastically and become ***'Cold welded'*** together. When one body tries to move over the other, it involves rupturing of thousands of such tiny cold welds. This gives rise to opposing force of friction. Note that the tiny cold welds reform continually as new chance contacts are made.

The modern view of the origin of friction explains satisfactorily the limitations of the old view :

(*i*) Frictional force will be independent of the area of contact. This is because with increase in area of contact, force of adhesion also increases (in the same ratio). And the adhesive pressure responsible for friction, remains the same.

(*ii*) When the surfaces in contact are extra smooth, distance between the molecules of the surfaces in contact decreases, increasing the adhesive force between them. Therefore, the adhesive pressure increases, and so does the force of friction.

We may classify friction into two types :

1. External friction, which arises when two bodies in contact with each other try to move or there is an actual relative motion between the two. The external friction is also called ***contact friction.***

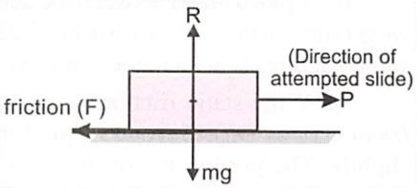
2. Internal friction, which arises on account of relative motion between every two layers of a liquid. Internal friction is also referred to as ***viscosity*** of the liquid.

Further, **external friction is of three types :**

(*i*) Static friction (*ii*) Limiting friction (*iii*) Kinetic friction.

5.30. STATIC FRICTION, LIMITING FRICTION AND KINETIC FRICTION (SLIDING FRICTION AND ROLLING FRICTION)

FIGURE 5.21



Let us consider a block of weight mg lying on a horizontal surface as shown in Fig. 5.21. When a body presses against a surface, the surface deforms even if it appears to be rigid. The deformed surface pushes on the body with a normal force R that is perpendicular to the surface. This is called ***normal reaction.*** It balances mg *i.e.*

$$\boxed{R = mg} \quad \text{...(35)}$$

Suppose a small force P is applied on the block to the right as shown. We know from experience that a small applied force may not be able to move the body practically. Clearly, the body remains at rest because some other force F comes into play in the horizontal direction and opposes the applied force P, resulting in zero net force on the body. This force F acting along the surface of the body in contact with the table is known as frictional force.

So long as the block does not move, $\boxed{F = P}$ **...(36)**

---

*Force of adhesion or adhesive force is the force of attraction between molecules of different substances. Similarly, force of cohesion or cohesive force is the force of attraction between molecules of the same substance.

This means as we increase P, friction F also increases, remaining equal to P always.

> ***The opposing force that comes into play when one body tends to move over the surface of another, but the actual motion has yet not started is called Static friction.***

We may also define *static friction as the force of friction which exactly counter balances the applied force during the stationary state of the body.*

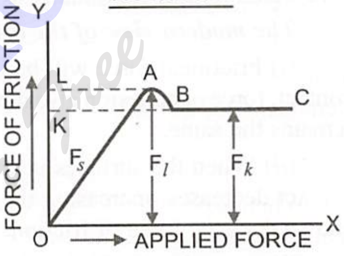
Note that static friction does not exist by itself. **When there is no applied force, there is no static friction. It comes into play the moment there is an applied force.**

As the applied force P is increased, F also increases remaining equal and opposite to the applied force (upto a certain limit). That is why the body does not move. Hence the opposing force F is called static friction.

Note that static friction opposes impending motion. The term impending motion means motion that would take place under the applied force, if friction were absent. The impending motion does not actually take place.

As we increase the applied force, a stage comes when the body is just at the verge of moving over the other. The static friction at this stage is obviously maximum. This maximum value of static friction is called **Limiting friction**. Hence

FIGURE 5.22



> ***Limiting friction is the maximum opposing force that comes into play, when one body is just at the verge of moving over the surface of the other body.***

When we increase the applied force slightly beyond limiting friction, the actual motion starts. This does not mean that friction has disappeared. It only means that the applied force is now greater than the force of limiting friction. The force of friction at this stage is called **Kinetic friction** or **Dynamic friction**. Hence,

> ***Kinetic friction or dynamic friction is the opposing force that comes into play when one body is actually moving over the surface of another body.***

It is found that the force of kinetic friction depends on normal reaction and on quality of finish of the rubbing surfaces. It does not depend upon the area of contact. But the kinetic friction does depend (though to a small degree) on the *velocity of relative motion of the bodies.*

If we plot a graph between the applied force and the force of friction, we get curve of the type shown in Fig. 5.22. The part OA of the curve represents *static friction*, F_s which goes on increasing with the applied force.

At A, the static friction is maximum. This represents the *limiting friction* F_l (= OL). Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve, therefore, represents the *kinetic friction* F_k (= OK).

We have ignored here the slight dependence of kinetic friction (F_k) on velocity of relative motion of the bodies.

Also, we find that ***kinetic friction is always slightly less than the limiting friction.***

This is because, once the motion starts actually, inertia of rest has been overcome. Also, when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

DO YOU KNOW ?

Note that it is not motion, but relative motion that the frictional force opposes.

Consider a box lying in the compartment of a train that is accelerating. If the box is stationary relative to the train, it is infact accelerating alongwith the train. The only conceivable force in the horizontal direction is the force of static friction F_s. The static friction provides the same acceleration to the box as that of the train, keeping it stationary relative to the train. If there were no friction, the floor of the train would slip by and box would remain at its initial position due to inertia. It would hit the backside of the compartment of the train.

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Further, *dynamic friction or kinetic friction may be of two types* :

1. Sliding friction 2. Rolling friction.

> ***The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction.***

For example, when a flat block is moved over the flat surface of a table, the opposing force is **sliding friction**.

> ***The opposing force that comes into play when one body is actually rolling over the surface of the other body is called rolling friction.***

For example, when a wheel, a circular disc or a ring or a sphere or a cylinder rolls over a surface, the force that opposes it is the **rolling friction**.

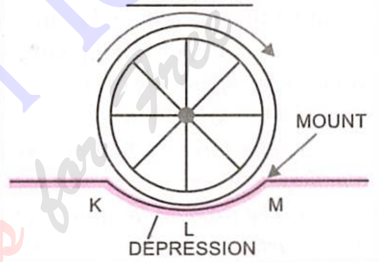
Laws of Kinetic Friction

1. The force of kinetic friction (F_k) is directly proportional to normal reaction (R) between the two surfaces in contact, *i.e.*, $F_k = \mu_k R$, where μ_k is coefficient of kinetic friction.

2. Kinetic friction does not depend upon the surface area of contact.

3. Force of kinetic friction is independent of the speed of sliding/rolling, if the heat produced does not change the condition of the surfaces in contact.

FIGURE 5.23



5.31. CAUSE OF ROLLING FRICTION

When a body rolls on a level track, the area of contact is very small. Therefore, pressure exerted which is equal to weight/area is very large. This causes a *depression* in the surface below and a *mount* or *bump* in front as shown in Fig. 5.23.

In turn, the surface of the rolling body in contact gets slightly compressed. Thus a rolling wheel (*i*) constantly pulls out of depression and goes uphill on the mount LM (*ii*) simultaneously detaches itself from the road *KL*, which is opposed by the forces of adhesion between the surfaces in contact. This causes rolling friction.

When a tyre is properly inflated, it becomes hard and gets compressed by the road to a much smaller extent. Due to it, the opposition to motion due to adhesive forces becomes less. Therefore, rolling friction reduces. ***Hence it is easier to drive a vehicle when its tyres are fully inflated.***

Note that the velocity of the point of contact of the wheel with respect to the floor remains zero all the time, although the centre of the wheel moves forward. Therefore, rolling friction is often quite small compared to the sliding friction. That is why heavy loads are transported by placing them on carts with wheels. Thus sliding friction is converted into rolling friction. For example, rolling friction of steel on steel is hardly 1% of sliding friction of steel on steel.

DO YOU KNOW ?

Experiments show that force of rolling friction (F_r) is directly proportional to the normal reaction (R) and inversely proportional to the radius (r) of the rolling cylinder or wheel. Thus

$$F_r = \mu_r \frac{R}{r}$$

where μ_r is coefficient of rolling friction. μ_r would have the dimensions of length and would be measured in metre.

5.32. STATIC FRICTION IS A SELF ADJUSTING FORCE

This means that neither the magnitude nor the direction of the force of static friction is fixed. Both these things adjust themselves according to the applied force. In Art. 3(a).3., we have seen that unless the block starts moving, $F = P$ *i.e.* force of static friction is always equal to the applied force. When we increase/decrease the applied force, the force of static friction increases/decreases accordingly. Further, the direction of the force of static friction is always opposite to the direction of the applied force. For example, when we try to move the body to the east, force of static friction is to the west and we try to move the body to the north, force of static friction is to the south and so on.

Thus static friction adjusts itself so that its magnitude is equal to the magnitude of the applied force and its direction is opposite to that of the applied force. Hence static friction is a self adjusting force.

Retain in Memory

Remember that friction arises only when body is actually sliding/rolling over the surface of another body or the body is simply trying to slide/roll over the surface of the other. Further, static friction alone is a self adjusting force, matching the growing applied force. **Limiting friction and Dynamic friction are not self-adjusting. Their values are fixed.**

5.33. LAWS OF LIMITING FRICTION

Following are the **four laws** of limiting friction :

1. *The magnitude of the force of limiting friction (F) between the surfaces of two bodies in contact is directly proportional to the normal reaction (R) between them i.e.*

$$\boxed{F \propto R}$$

When a body is heavy, normal reaction $R = mg$ increases. Therefore, F increases. Hence $P = F$ increases *i.e.* more force is required to move a heavier body than to move a lighter body.

2. *The direction of the force of limiting friction is always opposite to the direction in which one body is at the verge of moving over the other.* The force of limiting friction always acts tangentially along the surface of contact of two bodies.

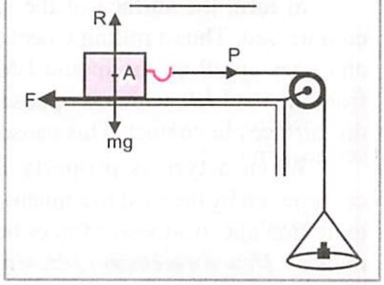
3. *The force of limiting friction is independent of the apparent area of contact, so long as normal reaction between the two surfaces in contact remains the same.*

4. *The force of limiting friction between any two surfaces in contact depends on the nature of material of the surfaces in contact (i.e. force of adhesion) and their state of polish.*

Experimental Verification of the Laws of Limiting Friction :

To verify the laws of limiting friction, we take two exactly identical rectangular blocks of wood *A* and *B*, each of same weight *mg*. Each block is provided with a hook on one side. The block is placed on a horizontal table provided with a frictionless pulley on one side, Fig. 5.24. One end of a string is attached to the hook of the block. The string is then passed over the pulley and a pan is attached to the free end of the string. Any number of weights can be added to the pan.

FIGURE 5.24

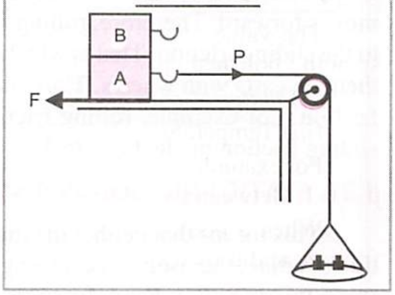


We adjust the weights in the pan till the block just begins to move. The applied force (*P*) at this stage (= wt. of pan + wts in the pan) gives us a measure of the force of limiting friction (*F*), *i.e.*,

$$F = P \quad \text{and} \quad R = mg$$

First Law. Proceed as explained above and determine the force of limiting friction in case of block *A*, Fig. 5.24. Now place the block *B* on the block *A*, Fig. 5.25. The weight is doubled. Therefore, the normal reaction also becomes double. Repeat the experiment and find the force of limiting friction in this case. It is found to be two times the force of limiting friction in case of block *A*. Thus we find that when *R* is doubled, *F* is also doubled. Hence $F \propto R$. This verifies the first law.

FIGURE 5.25

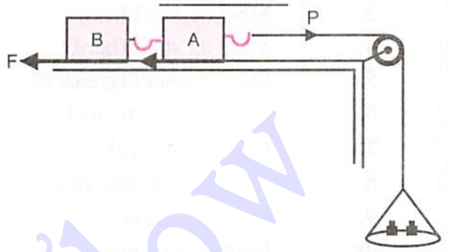


Second Law. As is clear from the above discussion, in equilibrium, $R = mg$ and $F = P$ *i.e.*, force of limiting friction is equal and opposite to the applied force *P*. Hence the direction of the force of limiting friction is always opposite to the intended direction of motion of the block. This verifies second law.

In the arrangement shown in Fig. 5.24, if pulley were on the left, weights have to be put in the pan to move the block to the left. This means force of limiting friction, in that case would be to the right. This confirms second law.

Third Law. Join the two blocks A and B and place them on the table as shown in Fig. 5.26. Repeat the experiment and determine the force of limiting friction in this case. It is found to be the same as in the case when block B is placed over the block A, Fig. 5.25. The total normal reaction in the two cases is the same, but the area of apparent contact in the case shown in Fig. 5.26 is double. Since F in both cases is the same, it shows that the force of limiting friction does not depend on the apparent area of contact between the bodies, so long as normal reaction R between them remains the same. This verifies third law.

FIGURE 5.26



Fourth Law. Take the rectangular block A. Make one surface of A smooth and well polished and the opposite surface of A very rough. Repeat the above experiment and determine the force of limiting friction (*i*) when rough surface is in contact with the table, and (*ii*) when smooth surface is in contact with the table. We observe that the force of limiting friction is less when the smooth surface is in contact with the table and more when the rough surface is in contact with the table. Hence the force of limiting friction depends on nature of surfaces in contact. This verifies fourth law.

Similarly if we take two blocks, one of wood and other of metal of the same weight, F in either case is different. This also proves that force of limiting friction depends on the *nature of material of the surfaces in contact.*

DO YOU KNOW ?

Normally, friction decreases with increase in smoothness. However, when the surfaces in contact are made too smooth by polishing, the binding force of adhesion increases and therefore, the friction increases. This is called '*cold welding*.'

5.34. COEFFICIENT OF FRICTION

According to the first law of limiting friction,

$$F \propto R$$

or $$F = \mu R \qquad \text{...(37)}$$

where μ is a constant of proportionality and is called the coefficient of limiting friction between the two surfaces in contact.

From (37), $$\boxed{\mu = \frac{F}{R}} \qquad \text{Hence,} \qquad \text{...(38)}$$

***Coefficient of limiting friction** between any two surfaces in contact is defined as the ratio of the force of limiting friction and normal reaction between them.*

The value of μ depends on (*i*) *nature of the surfaces in contact i.e.*, whether dry or wet ; rough or smooth ; polished or non polished.

(*ii*) *material of the surfaces in contact.*

(*iii*) Temperature of the surfaces in contact.

For example, when two polished metal surfaces are in contact, $\mu \approx 0{\cdot}2$, when these surfaces are lubricated. $\mu \approx 0{\cdot}1$. Between two smooth wooden surfaces, μ varies between 0·2 and 0·5. Obviously, μ has no units.

When a body is *actually moving* over the surface of another body, we replace F by F_k, the kinetic friction, and μ by μ_k.

Therefore, from (38), $$\boxed{\mu_k = \frac{F_k}{R}}$$

μ_k is then called the ***coefficient of kinetic or dynamic friction.*** As $F_k < F$, therefore μ_k is always less than μ *i.e.* coefficient of kinetic or dynamic friction is always less than the coefficient of limiting friction.

Table 5.1 gives the values of coefficient of limiting/kinetic friction between some pairs of materials :

TABLE 5.1. Coefficient of limiting friction and kinetic friction

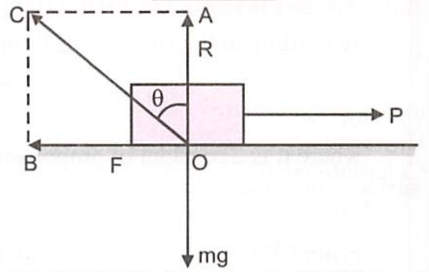
S.No.	Surfaces in contact	Coeff. of limiting friction	Coeff. of kinetic friction
1.	Wood on wood	0·70	0·40
2.	Wood on leather	0·50	0·40
3.	Steel on steel (mild)	0·74	0·57
4.	Steel on steel (hard)	0·78	0·42
5.	Steel on steel (greased)	0·10	0·05
6.	Aluminium on steel	0·61	0·47
7.	Copper on steel	0·53	0·36
8.	Cast iron on cast iron	1·10	0·15
9.	Glass on glass	0·94	0·40
10.	Rubber on concrete (dry)	0·90	0·70
11.	Teflon on steel	0·04	0·04

5.35. ANGLE OF FRICTION

> ***The angle of friction* between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction F and normal reaction R makes with the direction of normal reaction R.**

It is represented by θ.

FIGURE 5.27



In Fig. 5.27, *OA* represents the normal reaction *R* which balances the weight *mg* of the body. *OB* represents *F*, the limiting force of sliding friction, when the body tends to move to the right. Complete the parallelogram *OACB*. Join *OC*. This represents the resultant of *R* and *F*. By definition, ∠*AOC* = θ is the angle of friction between the two bodies in contact.

The value of angle of friction *depends on the nature of materials of the surfaces in contact and the nature of the surfaces.*

Relation between μ and θ

In Δ AOC, $\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{F}{R} = \mu$ ...(39)

Hence, $\boxed{\mu = \tan\theta}$ ...(40)

i.e. ***coefficient of limiting friction between any two surfaces in contact is equal to tangent of the angle of friction between them.***

5.36. ANGLE OF REPOSE OR ANGLE OF SLIDING

> ***Angle of repose* or *angle of sliding* is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down the incline.**

It is represented by α. Its value depends on material and nature of the surfaces in contact.

In Fig. 5.28, *AB* is an inclined plane such that a body placed on it just begins to slide down. ∠*BAC* = α = angle of repose.

The various forces involved are :

(*i*) weight, *mg* of the body, acting vertically downwards,

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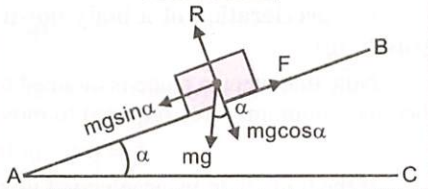
FIGURE 5.28



(*ii*) normal reaction, R, acting perpendicular to AB,

(*iii*) Force of friction F, acting up the plane AB.

Now, mg can be resolved into two rectangular components : $mg \cos \alpha$ opposite to R and $mg \sin \alpha$ opposite to F. In equilibrium,

$$F = mg \sin \alpha \qquad \text{...(41)}$$

$$R = mg \cos \alpha \qquad \text{...(42)}$$

Dividing (41) by (42), we get $\frac{F}{R} = \frac{mg \sin \alpha}{mg \cos \alpha}$, *i.e.*, $\boxed{\mu = \tan \alpha}$ ...(43)

Hence, coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

IMPORTANT NOTE

Combining (40) and (43), we obtain $\mu = \tan \theta = \tan \alpha \quad \therefore \quad \boxed{\theta = \alpha}$

i.e., angle of friction is equal to angle of repose.

Sample Problem 1 **A heavy box of mass 20 kg is placed on a horizontal surface. If coefficient of kinetic friction between the box and the horizontal surface is 0·25, calculate the force of kinetic friction. Also, calculate acceleration produced under a force of 98 N applied horizontally ?**

Sol. Here, $m = 20$ kg, $\mu = 0{\cdot}25$, $F = ?$, $P = 98$ N, $a = ?$

$$F = \mu R = \mu m g = 0{\cdot}25 \times 20 \times 9{\cdot}8 = 49 \text{ N}$$

Force that produces acceleration $f = P - F = 98 - 49 = 49$ N

$$a = \frac{f}{m} = \frac{49}{20} = \mathbf{2{\cdot}45\,m/s^2}$$

Sample Problem 2 **A wooden block is kept on a polished wooden plank whose inclination is increased gradually. The block starts slipping when the plank makes an angle of 25° with the horizontal. However, once started, the block can continue with uniform speed, if the inclination is reduced to 21°. Calculate coefficient of static and dynamic friction between the block and the plank.**

Sol. Here, angle of repose, $\alpha = 25°$

$\therefore$ coefficient of static friction, $\mu_s = \tan \alpha = \tan 25° = \mathbf{0{\cdot}4663}$

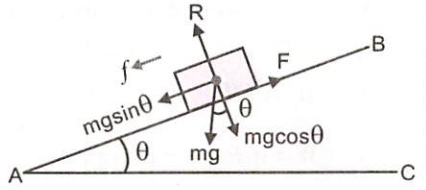
As the block continues to move uniformly, when $\theta = 21°$,

$\therefore$ Coefficient of kinetic friction, $\mu_k = \tan \theta = \tan 21° = \mathbf{0{\cdot}3839}$

5.37. ACCELERATION OF A BODY DOWN A ROUGH INCLINED PLANE

When a plane is inclined to the horizontal at an angle θ, which is greater than the angle of repose, the body placed on the inclined plane slides down with an acceleration, *a*.

FIGURE 5.29



As is clear from Fig. 5.29,

$$R = mg \cos \theta \qquad \text{...(44)}$$

Net force on the body down the inclined plane

$$f = mg \sin \theta - F \qquad \text{...(45)}$$

i.e. $f = ma = mg \sin \theta - \mu_k R \qquad \left(\because \mu = \frac{F}{R}\right)$

Using (44), we get $ma = mg \sin \theta - \mu_k\, mg \cos \theta = mg\,(\sin \theta - \mu_k \cos \theta)$

Hence $\boxed{a = g\,(\sin \theta - \mu_k \cos \theta)}$ ...(46)

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Clearly, $a < g$

i.e., **acceleration of a body down a rough inclined plane is always less than acceleration due to gravity (g).**

Note that when a plane is inclined to the horizontal at an angle θ, which is *less than the angle of repose,* then the minimum force required to move the body up the inclined plane is

$$f_1 = (mg \sin\theta + F) = mg(\sin\theta + \mu_k \cos\theta)$$

If the body is to be accelerated up the plane with acceleration a, then force required

$$= mg(\sin\theta + \mu_k \cos\theta) + ma.$$

Further, the minimum force required to push the body down the inclined plane is

$$f_2 = (F - mg\sin\theta) = mg(\mu_k \cos\theta - \sin\theta)$$

Sample Problem **A block slides down an incline of angle 30° with an acceleration of g/4. Find the coefficient of kinetic friction.**

Sol. Here, $\theta = 30^\circ$, $a = g/4$, $\mu_k = ?$

As is clear from Fig. 5.11, net force on the body down the incline,

$$f = ma = mg\sin\theta - F = mg\sin\theta - \mu_k R, \text{ where } R = mg\cos\theta$$

$$\therefore \quad \mu_k R = mg\sin\theta - ma = mg\sin 30^\circ - m\frac{g}{4} = \frac{mg}{2} - \frac{mg}{4} = \frac{mg}{4}$$

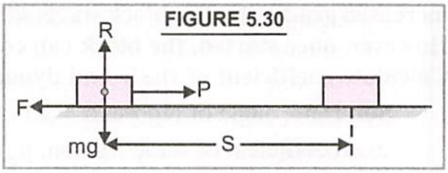
$$\mu_k = \frac{mg/4}{R} = \frac{mg}{4(mg\cos\theta)} = \frac{1}{4\cos 30^\circ} = \frac{2}{4\sqrt{3}}$$

$$\mu_k = \frac{1}{2\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{1.732}{6} = \mathbf{0.289}$$

5.38. WORK DONE IN MOVING A BODY OVER A ROUGH HORIZONTAL SURFACE

We know that friction always opposes the relative motion between any two surfaces in contact. Hence some work has to be done in moving a body over a rough horizontal surface.

FIGURE 5.30



As is clear from Fig. 5.30, on a level track, the weight (mg) of the body is balanced by the normal reaction (R)

i.e., $R = mg$ ...(47)

To just move the body on a level track, we have to apply a force (P), which is atleast equal to the force of friction (F)

i.e., $P = F$ ...(48)

Under the action of the applied force, suppose the body slides through a distance S.

As work done = force × distance

$\therefore$ $W = P \times S = F \times S$

As $\mu_k = F/R$, $\therefore$ $F = \mu_k R$

$\therefore$ $\boxed{W = \mu_k R \times S = \mu_k mg \times S}$ ...(49)

DO YOU KNOW ?

Friction is a non-conservative force, *i.e.* work done against friction is *path dependent.* In the presence of friction, some energy is always lost in the form of heat etc. Thus mechanical energy is not conserved in such cases.

Knowing mass (m) of the body, distance (S) moved and the coefficient of friction (μ), we can calculate the work done in moving the body over a rough horizontal plane. Obviously, it would depend upon (i) weight of the body, (ii) material and nature of the surfaces in contact, (iii) distance moved.

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5.39. WORK DONE IN MOVING A BODY UP A ROUGH INCLINED PLANE

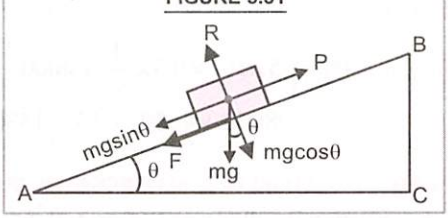
Suppose m is the mass of a body that has to be moved up a rough plane AB, inclined to the horizontal at an angle θ, shown in Fig. 5.31. The various forces involved are :

(*i*) Weight (mg) of the body, acting vertically downwards,

(*ii*) Normal reaction, (R), acting perpendicular to the plane AB.

(*iii*) Force of friction (F), acting down the plane AB, as the body moves up the plane.

FIGURE 5.31



The weight mg can be resolved into two rectangular components : $mg \cos\theta$ opposite to R and $mg \sin\theta$ down the plane AB.

In equilibrium, $$R = mg\cos\theta \qquad \text{...(50)}$$

If P is the force required to just move the body up the incline AB, then from Fig. 5.31,

$$P = mg\sin\theta + F \qquad \text{...(51)}$$

Under the action of this force, suppose the body slides through a distance S up the plane.

As Work done = force × distance

$$\therefore \quad W = P \times S = (mg\sin\theta + F)\,S \qquad \text{...using (51)}$$

$$W = (mg\sin\theta + \mu_k R)\,S$$

where μ is the coefficient of friction between the two surfaces in contact :

$$W = (mg\sin\theta + \mu_k\, mg\cos\theta)\,S \qquad \text{\{using (50)\}}$$

$$\boxed{W = mg\,(\sin\theta + \mu_k\cos\theta)\,S} \qquad \text{...(52)}$$

Knowing m, θ, S and μ, we can calculate work done in moving the body up a rough inclined plane.

Retain in Memory

Remember that in moving a body over a rough horizontal surface, work has to be done only against the force of friction, which is a function of normal reaction/weight of the body. However, in moving a body up an inclined plane, work has to be done against friction as well as component of weight down the inclined plane.

Sample Problem 1 **A cricket ball is rolled on ice with a velocity of 5·6 m/s and comes to rest after travelling 8 m. Find the coefficient of friction. Given $g = 9{\cdot}8\ \text{m/s}^2$.**

Sol. Here, $u = 5{\cdot}6$ m/s, $v = 0$, $s = 8$ m, $g = 9{\cdot}8\ \text{m/s}^2$.

From $v^2 - u^2 = 2\,a\,s$

$$0 - (5{\cdot}6)^2 = 2 \times a \times 8$$

$$\therefore \quad a = -\frac{5{\cdot}6 \times 5{\cdot}6}{16} = -1{\cdot}96\ \text{m/s}^2$$

$$\mu_k = \frac{a}{g} = \frac{1{\cdot}96}{9{\cdot}8} = \mathbf{0{\cdot}2}$$

Sample Problem 2 **Calculate the power of an engine, which can just pull a train of mass 5000 quintals up an incline of 1 in 50 at the rate of 54 km/h. The resistance due to friction is 0·8 N/quintal. Take $g = 9{\cdot}8\ \text{m/s}^2$.**

Sol. Here, power, $P' = ?$, $m = 5000$ quintals $= 5 \times 10^5$ kg.

$$\sin\theta = \frac{1}{50},\quad v = 54\ \text{km/h} = \frac{54 \times 1000}{60 \times 60}\ \text{ms}^{-1} = 15\ \text{ms}^{-1}$$

Force of friction, $F = 0{\cdot}8$ N/quintal $= 0{\cdot}8 \times 5000$ N $= 4000$ N

From (52),

$$P' = \frac{W}{t} = (mg\sin\theta + F)\times\frac{S}{t} = (mg\sin\theta + F)\times v$$

$$P' = \left(5\times10^5\times9{\cdot}8\times\frac{1}{50}+4000\right)15$$

$$P' = (98000 + 4000) \times 15 = 1530000\text{ W} = \mathbf{1530\ kW}$$

5.40. FRICTION IS A NECESSARY EVIL

Friction is called a necessary evil. It is a necessity because we cannot do without it. At the same time, it is an evil because it involves unnecessary waste of energy. The following facts make the point clear :

(*a*) Friction is a Necessity (*Advantages of friction*)

1. Walking will not be possible without friction. Our foot pressing the ground for walking will slip, when there is no friction.

2. No two surfaces will stick to each other if there is no friction.

3. Brakes of the vehicles will not work without friction.

4. Nuts and bolts for holding the parts of machinery together will not work.

5. Writing on black board or on paper will also not be possible without friction.

6. The transfer of motion from one part of a machine to the other part through belts and pulleys will not be possible without friction.

7. Adhesives will lose their purpose.

8. Cleaning with sand paper will not be possible without friction.

9. Friction between the road and tyres of a car provides the necessary force to accelerate the car. It is impossible for a car to move on a slippery road.

10. Friction between a match stick and the match box causes the match stick to catch fire.

(*b*) Friction is an evil (*Disadvantages of friction*)

1. Friction always opposes the relative motion between any two surfaces in contact. Therefore, extra energy has to be spent in overcoming friction. Thus friction involves unnecessary expense of energy. That is why output is always less than the input.

2. Friction causes wear and tear of the parts of machinery in contact. Thus their life time reduces.

3. Frictional forces result in the production of heat, which causes additional damage to the machinery and lowers its efficiency.

Hence we conclude that friction is a necessary evil.

5.41. METHODS OF CHANGING FRICTION

Some of the ways of reducing friction are :

(*i*) **By polishing.** Polishing makes the surfaces smoother. Therefore, friction reduces.

DO YOU KNOW ?

1. Though friction opposes relative motion, yet in certain cases, friction is also the cause of motion. For example,

(i) When a person pushes the ground backwards (action), the rough surface of ground reacts and exerts a forward force due to friction—which causes the motion. Without friction, person would slip and shall not be able to move.

(ii) In cycling, rear wheel moves by the force communicated to it by pedalling, while front wheel moves by itself. Therefore, like in walking, force of friction on *rear wheel* is in the *forward* direction. As front wheel moves by itself, force of friction on front wheel is in the backward direction.

However, when pedalling is stopped, both the wheels move by themselves. So the force of friction on both the wheels is in the backward direction.

Thus without friction, motion cannot be started, stopped or transferred from one body to the other.

2. About 20% of the petrol in an automobile is used up to counteract friction in the engine and in the driving. On the contrary, if friction were totally absent, we would not get on automobile to go anywhere, we would not walk or even ride a bicycle. We would not be able to hold a pencil and if we could, the pencil would not write. Nails and screws would not work. The knots in woven clothes would untie themselves.

(*ii*) **By lubrication.** Lubricants like oil, grease etc. fill up the irregularities of the surfaces, making them smoother. Due to it, the two surfaces, will not be in direct contact with eachother. One surface will slide on the oil layer present there. Hence friction decreases.

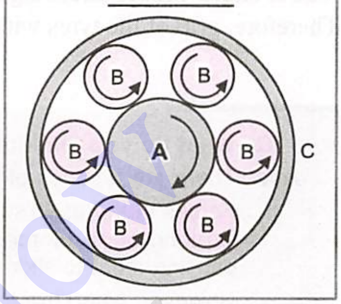
(*iii*) **By proper selection of materials.** We have studied that friction depends on nature of material of the surfaces in contact. The surfaces of moving parts of machines in contact can be lined with materials having low coefficient of friction. On the same basis, tyres are made of rubber. This is because friction between rubber and concrete is much less than friction between iron and concrete.

(*iv*) **By Streamlining.** Friction due to air is considerably reduced by streamlining the shape of the body (sharp in front) moving through air. For example, jets, aeroplanes, fast moving cars etc. are given streamline shape.

(*v*) **By using ball bearings.** As shown in Fig. 5.32, the ball bearings arrangement consists of two co-axial cylinders *A* and *C*, between which suitable number of hard steel balls *B* are arranged. The inner cylinder *A* is fitted on the axle and outer cylinder *C* is fixed to the wheel. When the wheel rotates, the balls *B* rotate in the direction shown in Fig. 5.32. The wheel thus rolls on the balls instead of sliding on the axle. In this way, sliding friction is converted into rolling friction which is much less. Ball bearings are commonly used in bicycles, etc.

Similarly, a thin cushion of air maintained between solid surfaces in relative motion is another effective way of reducing friction.

FIGURE 5.32



In many practical situations, friction is critically needed. **For example :**

(*i*) We are able to walk because of friction, All of us know how difficult it is to walk on ice, when force of static friction is small.

(*ii*) It is impossible to start a car on a very slippery road.

(*iii*) On an ordinary road, friction between tyres and road converts rotational motion of the wheels into a rolling motion, and provides the necessary external force to accelerate the car.

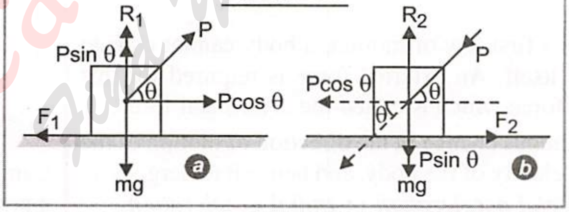
(*iv*) Kinetic friction is important for quickly stopping relative motion, though power is dissipated in the process.

DO YOU KNOW ?

It is easier to pull than to push a body

In Fig. 5.33, we have shown a block of mass *m* lying on a rough horizontal surface.

FIGURE 5.33



When *the block is pulled* with a force *P* applied at angle θ with horizontal, Fig. 5.33(*a*), we find that normal reaction, $R_1 = mg - P \sin\theta$

∴ Force of kinetic friction, $F_1 = \mu_k R_1 = \mu_k (mg - P\sin\theta)$ ...(*i*)

On the other hand, when the block is pushed with the same force *P* at same angle θ with horizontal, Fig. 5.33(*b*), we find that normal reaction, $R_2 = mg + P \sin\theta$

Force of kinetic friction, $F_2 = \mu_k R_2 = \mu_k (mg + P\sin\theta)$ ...(*ii*)

From (*i*) and (*ii*), we find that $F_1 < F_2$

i.e., force of friction against which work is done, is less in case of pulling than that in case of pushing. Hence pulling is easier than pushing.

Some of the ways of increasing friction are :

(*i*) On a rainy day, we throw some sand on the slippery ground. This increases friction between our feet and the ground. The chances of slipping reduce.

(*ii*) Similarly, sand is spread on tracks covered with snow. Force of friction between the wheels and the track increases and driving becomes safer.

(*iii*) In the manufacture of tyres, synthetic rubber is preferred because its coefficient of friction with the road is larger. Proper threading of the tyres also increases the force of friction between the tyres and the road. Therefore, grip of the tyres with the road increases.

Curiosity Question

Q. What do you think may be the cause of an earthquake ?

Ans. There are a series of fractures in earth's crust. These are called The **San Andreas Fault.** Forces in the earth's interior cause the rocks to slide past each other in a horizontal direction. So long as the forces of friction between the two sliding rocks are greater than the forces causing them to slide, the rocks only stretch and twist.

However, when the forces causing the slide exceed the force of friction, the rocks snap back like the release of a stretched rubber band. This movement of rocks with the consequent release of tremendous amounts of energy is the earthquake.

5.42. DYNAMICS OF UNIFORM CIRCULAR MOTION – CONCEPT OF CENTRIPETAL FORCE

A uniform circular motion *is the motion of an object travelling at a constant (uniform) speed along a circular path.*

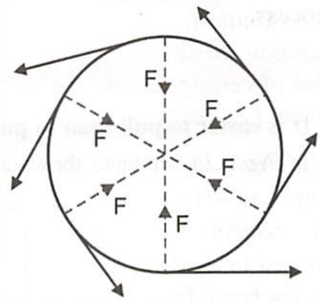
***Centripetal force** is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.*

Infact, when a body moves in a circle, its direction of motion at any instant is along the tangent to the circle at that instant. From Fig. 5.34, we find that the direction of motion of the body moving in a circle goes on changing continuously.

According to Newton's first law of motion, a body cannot change its direction of motion by itself. An external force is required for this purpose. It is this external force which is called the centripetal force.

On account of a continuous change in the direction of motion of the body, there is a change in velocity of the body, and hence it undergoes an acceleration, called ***centripetal acceleration*** or ***radial acceleration.***

FIGURE 5.34



DO YOU KNOW ?

Centripetal force does not denote a new and separate kind of force created by nature. It is simply the name given to the net force pointing towards the centre of circular path. It is the vector sum of all the force components pointing along the radial direction.

In chapter 4, we have already obtained an expression for centripetal acceleration as

$$a = \frac{v^2}{r} = r\,\omega^2 = r\,(2\,\pi\,n)^2 \qquad \text{...(53)}$$

where v is linear velocity, ω is angular velocity of the body and r is radius of the circular path, n is frequency of revolution

As $F = m\,a$, therefore,

centripetal force = mass × centripetal acceleration

i.e., $$\boxed{F = m\,v^2/r = m\,r\,\omega^2 = 4\,\pi^2\,m\,r\,n^2} \qquad \text{...(54)}$$

As stated earlier, the centripetal force acts along the radius and towards the centre of the circle. Obviously, centripetal acceleration/force changes direction continually as the body moves.

IMPORTANT NOTE

1. In uniform circular motion, speed is constant, but velocity changes because of continuous change in direction of motion. As velocity changes, the body possesses centripetal acceleration. Hence uniform circular motion is an accelerated motion.

2. The magnitude of centripetal acceleration is constant, but its direction changes continuously, though it is always directed towards the centre of the circle. Therefore, centripetal acceleration is variable.

3. The centripetal force necessary to move the earth around the sun is provided by gravitational attraction of sun on the earth.

4. The centripetal force required to move an electron around the nucleus of an atom is provided by the electrostatic force of attraction of nucleus on the electron.

5.43. CENTRIFUGAL FORCE

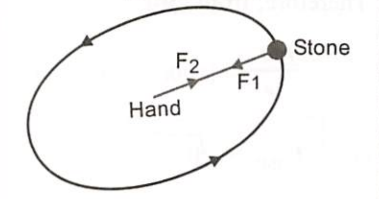
The natural tendency of a body is to move uniformly along a straight line. When we apply centripetal force on the body, it is forced to move along a circle. While moving actually along a circle, the body has a constant tendency to regain its natural straight line path. This tendency gives rise to a force called centrifugal force. Hence,

Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

Centrifugal force can be regarded as the *reaction* of centripetal force. As forces of action and reaction are always equal and opposite, therefore, magnitude of centrifugal force = $m\,v^2/r$, which is same as that of centripetal force. However, direction of centrifugal force is opposite to the direction of centripetal force, *i.e.*, ***centrifugal force acts along the radius and away from the centre of the circle.***

Note that centripetal and centrifugal forces, being the forces of action and reaction act always on different bodies. For example, when a piece of stone tied to one end of a string is rotated in a circle, *centripetal force* F_1 is applied on *the stone by the hand.* In turn, the hand is pulled outwards by *centrifugal force* F_2 acting on it, due to tendency of the stone to regain its natural straight line path. The centripetal and centrifugal forces are shown in Fig. 5.35.

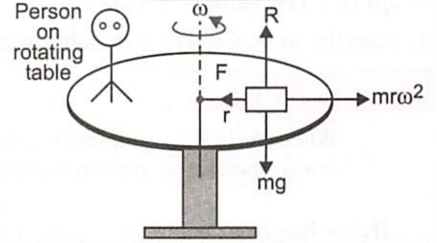
FIGURE 5.35



Alternative Argument

Sometimes, we do not treat centripetal force as the force of reaction. Instead, we say that centripetal force is a *fictitious* or *pseudo force, which comes into play only when we deal with non-inertial frame or accelerated frame of reference.* *For example,* suppose a body of mass *m* is at a distance *r* from the centre of a table rotating with uniform angular velocity ω. Let a person (the observer) be also standing on the same table, Fig. 5.36.

FIGURE 5.36



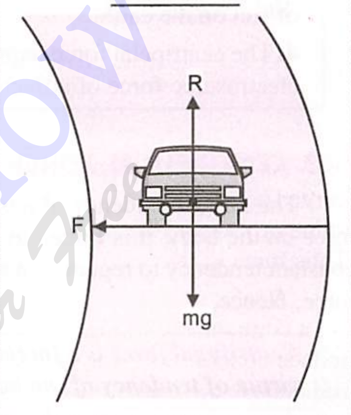
Now, the weight *mg* of the body is balanced by the normal reaction *R*. For the observer, the body is at rest, as both the body and observer are part of the rotating table. Hence the force acting on the body is zero. Now, the force of friction *F* between the body and surface of table acts towards the centre. To balance this force, the observer assumes a pseudo force equal to $\frac{mv^2}{r}$ acting on the body in radially outward direction. **This pseudo force is the centrifugal force.**

We shall now discuss some of the **applications of centripetal and centrifugal forces.**

5.44. ROUNDING A LEVEL CURVED ROAD

When a vehicle goes round a curved road, it requires some centripetal force. While rounding the curve, the wheels of the vehicle have a tendency to leave the curved path and regain the straight line path. Force of friction between the wheels and the road opposes this tendency of the wheels. This force (of friction) therefore, acts, towards the centre of the circular track and provides the necessary centripetal force.

FIGURE 5.37



Three forces are acting on the car, Fig. 5.37.

(*i*) The weight of the car, mg, acting vertically downwards,

(*ii*) Normal reaction R of the road on the car, acting vertically upwards,

(*iii*) Frictional force F, along the surface of the road, towards the centre of the turn, as explained already.

As there is no acceleration in the vertical direction,

$$R - mg = 0$$

or

$$R = mg \qquad \text{...(55)}$$

The centripetal force required for circular motion is along the surface of the road, towards the centre of the turn. As explained above, it is the force of friction that provides the necessary centripetal force. Clearly,

$$\frac{mv^2}{r} \leq F \qquad \text{...(56)}$$

where v is velocity of car while turning and r is radius of circular track.

As $\quad F = \mu R = \mu\, mg, \qquad$ [using (55)]

where μ_s is coefficient of static friction between the tyres and the road. Therefore, from (56),

$$\frac{mv^2}{r} \leq \mu\, mg \quad \text{or} \quad v \leq \sqrt{\mu\, rg}$$

$$\therefore \quad \boxed{v_{\max} = \sqrt{\mu\, rg}} \qquad \text{...(57)}$$

DO YOU KNOW ?

For a dry road, $\mu_s \simeq 0{\cdot}95$ and for icy road, $\mu_s = 0{\cdot}10$. Therefore, a dry road would allow a greater maximum speed.

Hence the *maximum velocity* with which a vehicle can go round a level curve ; without skidding is $v = \sqrt{\mu\, rg}$. The value of v depends on radius r of the curve and on coefficient of friction (μ) between the tyres and the road. Clearly, v is independent of mass of the car.

Retain in Memory

When the road is unbanked and friction is small, the vehicle has to round the curve at a much lower speed to avoid overturning.

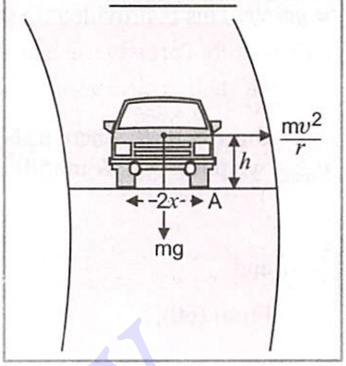
If h is height of centre of gravity of vehicle above the road, and $2\,x$ is the distance between the front wheels or the back wheels, then for no overturning, moments of forces about A must be equal and opposite. From Fig. 5.38,

$$\frac{mv^2}{r} \times h = mg \times x$$

$$\boxed{v = \sqrt{\frac{g\,r\,x}{h}}}$$

This is the safe speed for rounding a level curved road.

FIGURE 5.38



Sample Problem **A bend in a level road has a radius of 100 m. Find the maximum speed which a car turning this bend may have without skidding, if coefficient of friction between the tyres and the road is 0·8.**

Sol. Here, $r = 100$ m, $v = ?$, $\mu_s = 0{\cdot}8$.

As $\quad v = \sqrt{\mu_s\, r\, g} \quad \therefore \quad v = \sqrt{0{\cdot}8 \times 100 \times 9{\cdot}8} = \mathbf{28\ ms^{-1}}$

5.45. BANKING OF ROADS

As discussed in Art. 5.44, the maximum permissible velocity with which a vehicle can go round a level curved road without skidding depends on μ, the coefficient of friction between the tyres and the road. The value of μ decreases when road is smooth or tyres of the vehicle are worn out or the road is wet and so on. Thus force of friction is not a reliable source for providing the required centripetal force to the vehicle.

A safer course of action would be to raise outer edge of the curved road above the inner edge. By doing so, a component of normal reaction of the road shall be spared to provide the centripetal force. This would reduce considerably the wear and tear of the tyres. ***The phenomenon of raising outer edge of the curved road above the inner edge is called banking of roads.*** It facilitates vehicles to turn without slipping. We can calculate the angle of banking θ, as detailed below:

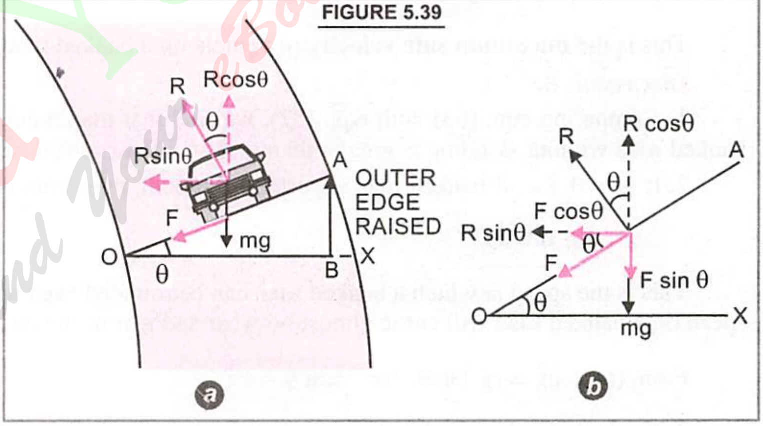
In Fig. 5.39(*a*), *OX* is a horizontal line. *OA* is the level of banked curved road whose outer edge has been raised. $\angle XOA = \theta$ = angle of banking. This is the angle which the road bed makes with the horizontal.

Three forces are acting on the vehicle as shown in Fig. 5.39(*b*).

(*i*) Weight *mg* of the vehicle acting vertically downwards.

(*ii*) Normal reaction *R* of the banked road acting upwards in a direction perpendicular to *OA*.

FIGURE 5.39



(*iii*) Force of friction *F* between the banked road and the tyres, acting along *AO*.

R can be resolved into two rectangular components : (*i*) *R* cos θ, along vertically upward direction (*ii*) *R* sin θ, along the horizontal, towards the centre of the curved road.

F can also be resolved into two rectangular components :

(*i*) *F* cos θ, along the horizontal, towards the centre of curved road

(*ii*) *F* sin θ, along vertically downward direction.

As there is no acceleration along the vertical direction, the net force along this direction must be zero. Therefore,

$$R\cos\theta = mg + F\sin\theta \qquad \text{...(58)}$$

If v is velocity of the vehicle over the banked circular road of radius r, then centripetal force required $= mv^2/r$. This is provided by the horizontal components of R and F as shown in Fig. 5.39(b).

$$\therefore \qquad R \sin\theta + F\cos\theta = \frac{mv^2}{r} \qquad \text{...(59)}$$

But $F \le \mu_s R$, where μ_s is coefficient of static friction between the banked road and the tyres. To obtain $v_{\max}$, we put $F = \mu_s R$ in (58) and (59).

$$R\cos\theta = mg + \mu_s R \sin\theta \qquad \text{...(60)}$$

and

$$R\sin\theta + \mu_s R\cos\theta = \frac{mv^2}{r} \qquad \text{...(60)}$$

From (60),

$$R(\cos\theta - \mu_s \sin\theta) = mg$$

$$R = \frac{mg}{\cos\theta - \mu_s \sin\theta} \qquad \text{...(62)}$$

From (61),

$$R(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$$

Using (62),

$$\frac{mg\,(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv^2}{r}$$

$$\therefore \qquad v^2 = \frac{rg\,(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{rg\cos\theta\,(\tan\theta + \mu_s)}{\cos\theta\,(1 - \mu_s\tan\theta)}$$

$$\boxed{v = \left[\frac{rg\,(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}\right]^{1/2}} \qquad \text{...(63)}$$

This is the **maximum safe velocity** of vehicle on a banked road.

Discussion

1. Comparing eqn. (63) with eqn. (57), we find that maximum speed with which a car can turn on a banked road without skidding is greater than that on a flat (unbanked) road.

2. If $\mu_s = 0$, *i.e.*, if banked road is perfectly smooth, then from eqn. (63),

$$\boxed{v_0 = (rg\tan\theta)^{1/2}} \qquad \text{...(64)}$$

This is the speed at which a banked road can be rounded even when there is no friction. Driving at this speed on a banked road will cause almost no wear and tear of the tyres.

From (64), $v_0^2 = rg\tan\theta$ or $\boxed{\tan\theta = v_0^2/rg}$ ...(65)

3. If speed of vehicle is less than v_0, frictional force will be up the slope. Therefore, the vehicle can be parked only if $\tan\theta \le \mu_s$.

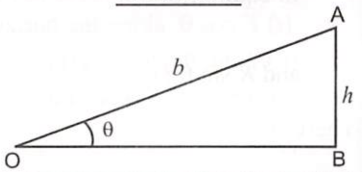
4. If h is the height AB of outer edge of the road above the inner edge and $b = OA$ is breadth of the road, then, from Fig. 5.40.

$$OB = \sqrt{OA^2 - AB^2} = \sqrt{b^2 - h^2}$$

$$\tan\theta = \frac{AB}{OB} = \frac{h}{\sqrt{b^2 - h^2}} \qquad \text{...(66)}$$

From (65) and (66), we get

FIGURE 5.40



$$\boxed{\tan\theta = \frac{v_0^2}{r\,g} = \frac{h}{\sqrt{b^2 - h^2}}} \qquad \text{...(67)}$$

From this eqn, we can calculate h. usually $h << b$. Therefore, h^2 is negligibly small compared to b^2. Eqn. (67) simplifies to

$$\boxed{\tan\theta = \frac{v_0^2}{r\,g} = \frac{h}{b}} \qquad \text{...(68)}$$

Roads are usually banked for the average speed of vehicles passing over them. However, if the speed of a vehicle is somewhat less or more than this, the self adjusting static friction will operate between the tyres and the road, and the vehicle will not skid.

The speed limit at which the curve can be negotiated safely is clearly indicated on the sign boards erected along the curved roads.

On the same basis, curved railway tracks are also banked. The level of outer rail is raised a little above the level of inner rail, while laying a curved railway track.

Note that curved portions of highways are always banked (tilted) to prevent cars from sliding off the highway. When the highway is dry, the frictional force between the tyres and road surface is often enough to prevent sliding. However, when the highway is wet, frictional force is much reduced. Therefore, banking of road/railway track is essential.

5.46. BENDING OF A CYCLIST

When a cyclist takes a turn, he also requires some centripetal force. If he keeps himself vertical while turning, his weight is balanced by the normal reaction of the ground. In that event, he has to depend upon force of friction between the tyres and the road for obtaining the necessary centripetal force. As force of friction is small and uncertain, dependence on it is not safe.

To avoid dependence on force of friction for obtaining centripetal force, the cyclist has to bend a little inwards from his vertical position, while turning. By doing so, a component of normal reaction in the horizontal direction provides the necessary centripetal force. To calculate the angle of bending with vertical, suppose

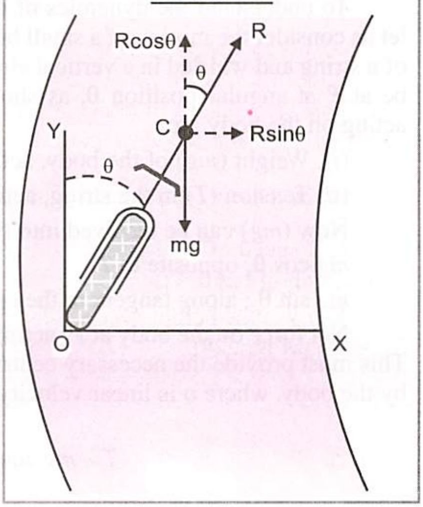
m = mass of the cyclist,

v = velocity of the cyclist while turning,

r = radius of the circular path,

θ = angle of bending with vertical.

FIGURE 5.41



In Fig. 5.41, we have shown weight of the cyclist (mg) acting vertically downwards at the centre of gravity C. R is force of reaction of the ground on the cyclist. It acts at an angle θ with the vertical.

R can be resolved into two rectangular components :

$R\cos\theta$, along the vertical upward direction,

$R\sin\theta$, along the horizontal, towards the centre of the circular track.

In equilibrium, $R\cos\theta$ balances the weight of the cyclist, *i.e.*,

$$R\cos\theta = mg \qquad \text{...(69)}$$

and $R\sin\theta$ provides the necessary centripetal force ($m\,v^2/r$)

$$\therefore \qquad R\sin\theta = \frac{m\,v^2}{r} \qquad \text{...(70)}$$

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Dividing (70) by (69), we get $\dfrac{R\sin\theta}{R\cos\theta} = \dfrac{m v^2}{r\,mg}$

$$\boxed{\tan\theta = \frac{v^2}{r\,g}} \qquad \text{...(71)}$$

Clearly, θ would depend on v and r.

For a safe turn, θ should be small, for which v should be small and r should be large *i.e.* **turning should be at a slow speed and along a track of larger radius.** This means, a safe turn should neither be fast nor sharp.

Retain in Memory

In actual practice, the value of θ will be slightly less than the calculated value, because the force of friction between the tyres and the road will contribute somewhat towards the centripetal force.

Sample Problem **An aircraft executes a horizontal loop at a speed of 720 km h^{-1}, with its wings banked at 15°. What is the radius of the loop ?**

Sol. Here, $v = 720 \text{ km h}^{-1} = \dfrac{720\times1000}{60\times60} = 200 \ m/s$

$\theta = 15°, r = ?$

As $\tan\theta = \dfrac{v^2}{r\,g}$ $\therefore$ $r = \dfrac{v^2}{g\tan\theta} = \dfrac{200\times200}{9{\cdot}8\tan15°} = \dfrac{4\times10^4}{9{\cdot}8\times0{\cdot}268}$ **= 1·52 × 10^4 m**

5.47. DYNAMICS OF NON-UNIFORM CIRCULAR MOTION

When a particle is moved in a circle under the action of a torque, it acquires an angular acceleration. Angular velocity of the particle and hence its angular momentum will change. Also, the linear velocity, linear momentum and kinetic energy of particle will change. Such a motion is therefore called non uniform circular motion.

Motion in a vertical circle

To understand the dynamics of non-uniform circular motion, let us consider the motion of a small body of mass m tied to one end of a string and whirled in a vertical circle. At any time, let the body be at P at angular position θ, as shown in Fig. 5.42. The forces acting on the body are :

(*i*), Weight (mg) of the body, acting vertically downwards,

(*ii*) Tension (T) in the string, acting along PO.

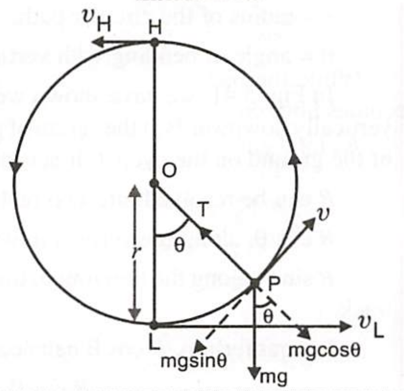
Now (mg) can be resolved into two rectangular components :

$mg\cos\theta$, opposite to T

$mg\sin\theta$; along tangent to the circle at P.

Net force on the body at P, acting along $PO = (T - mg\cos\theta)$. This must provide the necessary centripetal force ($m v^2/r$) required by the body, where v is linear velocity of the body at P.

FIGURE 5.42



$$\therefore \qquad T - mg\cos\theta = \frac{m v^2}{r}$$

$$\boxed{T = \frac{m v^2}{r} + mg\cos\theta} \qquad \text{...(72)}$$

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T will be *minimum*, when $\cos\theta = \min = -1$ *i.e.* $\theta = 180°$, the body is at the highest point H.

$$\therefore \text{ From (72), } \quad \boxed{T_{min} = T_H = \frac{m v_H^2}{r} - mg} \qquad \text{...(73)}$$

The body will move along the vertical circle only when $T_{min} \geq 0$. If $T_{min} < 0$, the string will slack and the body will fall down from H, instead of moving in the circle. Hence for completing the vertical circle *i.e.* for ***looping the loop,***

$$T_{min} = \left(\frac{m v_H^2}{r} - mg\right) \geq 0, \quad i.e., \quad \frac{m v_H^2}{r} \geq mg \quad \text{or} \quad \boxed{v_H \geq \sqrt{g\,r}} \qquad \text{...(74)}$$

Thus, ***minimum value of velocity at the highest point is*** $\sqrt{g\,r}$

Applying principle of conservation of energy, total mechanical energy at L = total mechanical energy at H

$$\frac{1}{2} m v_L^2 = \frac{1}{2} m v_H^2 + mg\,(2r)$$

using (74),

$$\frac{1}{2} m v_L^2 \geq \frac{1}{2} m\,(g\,r) + 2\,mg\,r$$

$$v_L^2 \geq 5\,g\,r, \quad i.e., \quad \boxed{v_L \geq \sqrt{5\,g\,r}} \qquad \text{...(75)}$$

Hence, ***for looping the vertical loop, the minimum velocity at the lowest point L is*** $\sqrt{5\,g\,r}$

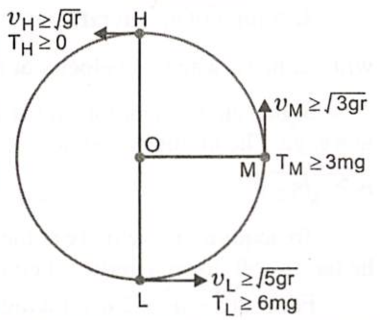
From (72), we find that tension in the string will be maximum, when $\cos\theta = \max = +1$ *i.e.* $\theta = 0°$ *i.e.* at the lowest point L.

$$\therefore \text{ from (72), } \quad \boxed{T_{max} = T_L = \frac{m v_L^2}{r} + mg} \qquad \text{...(76)}$$

when the body is looping the loop $\quad v_L \geq \sqrt{5\,g\,r}$

$$\therefore \quad T_{max} = T_L \geq \frac{m}{r}(5\,g\,r) + mg, \quad i.e., \quad \boxed{T_L \geq 6\,mg}$$

FIGURE 5.43



While the body is moving in a vertical circle and the string becomes horizontal as at M, Fig. 5.43, $\theta = 90°$

As total energy at M = total energy at L

$$\frac{1}{2} m v_M^2 + mg\,r = \frac{1}{2} m\,v_L^2$$

$$\frac{1}{2} m v_M^2 = \frac{1}{2} m\,v_L^2 - mg\,r$$

Using (75), we get

$$\frac{1}{2} m v_M^2 \geq \frac{1}{2} m\,(5\,g\,r) - mg\,r$$

$$\frac{1}{2} m v_M^2 \geq \frac{3}{2} mg\,r \quad \text{or} \quad \boxed{v_M \geq \sqrt{3\,g\,r}} \qquad \text{...(77)}$$

Using this value of v_M and $\theta = 90°$ in (72), we get $\boxed{T_M \geq 3\,mg}$

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All these results are depicted in Fig. 5.43.

Retain in Memory

From the study of motion of a body in a vertical circle, we conclude the following :

(*i*) For **looping the loop,** $v_L \geq \sqrt{5\,gr}$

(*ii*) For **oscillation over the arc** of vertical circle, $0 < \theta \leq 90°$ and $0 < v_L \leq \sqrt{2\,gr}$

This follows from energy conservation, $\frac{1}{2} m v_L^2 \leq mgr$

$$\boxed{v_L \leq \sqrt{2\,gr}}$$

Obviously, velocity becomes zero before T vanishes. That is why the body oscillates.

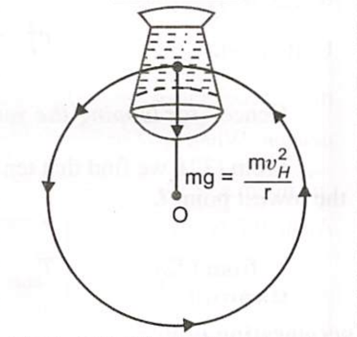
(*iii*) For **leaving the vertical circle** somewhere between $90° < \theta < 180°$, $\boxed{\sqrt{2\,gr} < v_L < \sqrt{5\,gr}}$

At the point of leaving the circle, $T = 0$, but velocity $\neq 0$.

5.48. PRACTICAL APPLICATIONS OF MOTION IN A VERTICAL CIRCLE

Following are some of the practical applications of motion in a vertical circle :

FIGURE 5.44



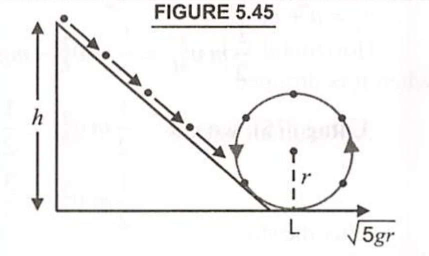
1. When a bucket containing water is rotated in a vertical circle with a velocity at the lowest point, $v_L \geq \sqrt{5\,gr}$, water shall not spil even at the highest point, when the bucket is upside down, Fig. 5.44. If the bucket is whirled slowly, so that $mg > \frac{m\,v_H^2}{r}$, then a part of the weight shall provide the necessary centripetal force $(m\,v_H^2/r)$; and the rest of the weight of water $\left(mg - \frac{m\,v_H^2}{r}\right)$ will spil. Only this much water shall leave the bucket.

2. A pilot of an aircraft can successfully loop a vertical loop without falling at the top of the loop (being without belt), when its velocity at the bottom of the loop is $\geq \sqrt{5\,g\,r}$.

3. In a circus, a motor cyclist is able to perform the feat of driving the motor cycle along a vertical circle in a cage. The motor cyclist does not fall even at the highest point, when his velocity at the bottom of the cage is $\geq \sqrt{5\,g\,r}$.

To acquire this velocity at the lowest point L of the vertical loop of radius r, he has to roll down a vertical height h, Fig. 5.45.

FIGURE 5.45



From $v^2 - u^2 = 2\,a\,s$, taking $u = 0, a = +\,g, s = h$,
we get, $v^2 - 0 = 2\,gh$

$$v = \sqrt{2\,gh}$$

For looping the loop, this velocity v acquired at L must atleast be equal to $\sqrt{5\,g\,r}$, *i.e.*, $\sqrt{2\,g\,h} = \sqrt{5\,g\,r}$

$$2\,g\,h = 5\,g\,r \quad \text{or} \quad \boxed{h = \frac{5\,r}{2}} \qquad ...(78)$$

Sample Problem **A motor cyclist loops a vertical loop of diameter 50 m, without dropping down even at uppermost point. What is the minimum speed at lowest and highest points of the loop ?**

Sol. Here, $r = 50/2 = 25$ m

$$v_L = \sqrt{5\,r\,g} = \sqrt{5 \times 25 \times 9{\cdot}8} = \mathbf{35\ m/s}$$

$$v_H = \sqrt{r\,g} = \sqrt{25 \times 9{\cdot}8} = \mathbf{15{\cdot}65\ m/s}$$

SOLVED EXAMPLES

TYPE I. LINEAR MOMENTUM AND FORCE

Formulae used. 1. Linear momentum, $\vec{p} = m\vec{v}$

2. Force = rate of change of linear momentum,

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

where $\vec{a}$ is acceleration produced in the body.

Units used. When mass m is in kg, velocity $\vec{v}$ in ms^{-1}, acceleration $\vec{a}$ in ms^{-2}, then force $\vec{F}$ is in *newton*. When m is in gram, $\vec{v}$ in cm s^{-1}, $\vec{a}$ in cm s^{-2}, then force is in *dyne*.

Also, 9.8 N = 1 kg f and 980 dyne = 1 g f

Example 1 **A bus starts from rest accelerating uniformly with 4 ms^{-2}. At $t = 10$ s, a stone is dropped from the window of the bus 2 m high. If $g = 10\ m/s^2$, what are the magnitude of velocity and acceleration of the stone at 10·2 s ?**

Solution. Horizontal velocity of bus/stone at

$t = 10$ s

$v = u + at = 0 + 4 \times 10 = 40\ ms^{-1}$,

For vertical motion of the stone,

$u = 0,\ a = g = 10\ m/s^2,\ t = 10{\cdot}2 - 10{\cdot}0 = 0{\cdot}2$ s

$v_y = u + at = 0 + 10 \times 0{\cdot}2 = 2$ m/s

Horizontal vel. of stone = velocity at $t = 10$ s when it is dropped v_x 40 m/s

$$\therefore \text{ Resultant velocity of stone } = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{40^2 + 2^2} = \mathbf{40{\cdot}04\ ms^{-1}}$$

After the stone is dropped, its acceleration along the horizontal is zero. **The stone has only vertical acceleration = 10 m/s^2.**

Example 2 **An astronaut accidently gets separated out of his small spaceship accelerating in interstellar space at a constant rate of 100 ms^{-2}. What is the acceleration of the astronaut the instant after he is outside the spaceship ? (Assume that there are no nearby stars to exert gravitational force on him).**

(NCERT Solved Example)

Solution. The moment the astronaut is out of spaceship, net force acting on the astronaut is zero. This is because there are no nearby stars to exert gravitational force on him and the small spaceship exerts negligible gravitational attraction on him.

As net force acting on the astronaut is zero, the acceleration of the astronaut is **zero.**

Example 3 **The motion of a particle of mass m is described by $y = ut + \frac{1}{2}gt^2$. Find the force acting on the particle.**

(NCERT Solved Example)

Solution. Here, $y = ut + \frac{1}{2}gt^2$

$$\text{velocity,} \quad v = \frac{dy}{dt} = u + \frac{1}{2}g\,(2t) = u + gt\ ;$$

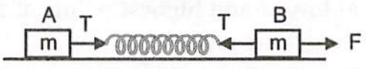
$$\text{acceleration,} \quad a = \frac{dv}{dt} = 0 + g = g$$

As $F = ma$ $\therefore$ $\mathbf{F = mg}$

Thus, the given equation describes the motion of a particle under acceleration due to gravity ; and y is the position co-ordinate in the direction of g.

Example 4 **Two bodies A and B each of mass m are fixed together by a massless spring. A force F acts on the mass B as shown in Fig. 5.46. At the instant shown, the body A has an acceleration a. What is the acceleration of B ?**

FIGURE 5.46



Solution. Let T be the tension in the string. Equation of motion of A is $T = m\,a$

If a' is acceleration of B, the equation of motion of B can be written as

$$m\,a' = F - T = F - m\,a$$

$$\therefore \quad a' = \left(\frac{F}{m} - a\right)$$

Example 5 **Astream of water flowing horizontally with a speed of 15 ms^{-1} pushes out of a tube of cross sectional area 10^{-2} m^2, and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming that it does not rebound ?**

Solution. Here, $v = 15$ ms^{-1}

Area of cross section, $a = 10^{-2}$ m^2, $F = ?$

Volume of water pushing out/sec $= a \times v$

$= 10^{-2} \times 15$ m^3 s^{-1}

As density of water is 10^3 kg/m^3, therefore, mass of water striking the wall per sec.

$$m = (15 \times 10^{-2}) \times 10^3 = 150 \text{ kg/s.}$$

$$\text{As} \quad F = \frac{\text{change in linear momentum}}{\text{time}}$$

$$\therefore \quad F = \frac{m \times v}{t} = \frac{150 \times 15}{1} = \mathbf{2250\ N}$$

TYPE II. IMPULSE

Formulae used. Impulse = force × time = change in linear momentum = $F \times t = m\,(v - u)$

Units used. The SI unit of impulse is N–s or kg ms^{-1}. Force F is in newton, time t in sec, mass m in kg and velocities u, v in ms^{-1}

Example 6 **A bullet of mass 0·04 kg moving with a speed of 90 ms^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet ?**

(NCERT Solved Example)

Solution. Here, $m = 0{\cdot}04$ kg, $u = 90$ ms^{-1},

$v = 0$, $s = 60$ cm $= 0{\cdot}6$ m, $F = ?$

From $v^2 - u^2 = 2\,as$

$$0 - 90^2 = 2\,a \times 0{\cdot}6$$

$$a = \frac{-90 \times 90}{2 \times 0{\cdot}6} = -6750 \text{ ms}^{-2}$$

From $F = ma = 0{\cdot}04 \times (-6750) = \mathbf{-270\ N}$

Negative sign is for resistive force, which is opposing.

Example 7 **A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 ms^{-1}. If the mass of the ball is 0·15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball).**

(NCERT Solved Example)

Solution. Here, $u = 12$ ms^{-1}, $v = -12$ ms^{-1},

$m = 0{\cdot}15$ kg ; Impulse = ?

As Impulse = change in linear momentum

$\therefore$ Impulse $= m\,(v - u) = 0{\cdot}15\,(-12 - 12)$

$= \mathbf{-3{\cdot}6\ Ns}$

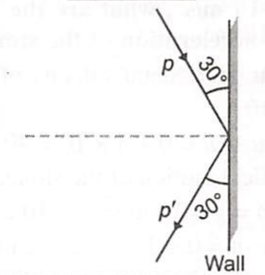
Negative sign shows that impulse is along negative x-axis, *i.e.*, from the batsman to the bowler.

Example 8 **A ball moving with a momentum of 15 kg ms^{-1} strikes against the wall at an angle of 30° and is reflected back with the same momentum at the same angle. Calculate impulse.**

Solution. Here, $p = 15$ kg ms^{-1}

$\theta = 30°$, $p' = p = 15$ kg m/s, Fig. 5.47

FIGURE 5.47



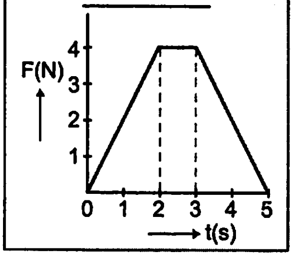
Impulse = change in momentum (along the wall)

$$= p\cos\theta + p'\cos\theta = 2\,p\cos\theta$$

$$= 2 \times 15\cos 30° = \mathbf{15\sqrt{3}\ \text{kg ms}^{-1}}$$

Example 9 **A force acting on a body of mass 2 kg varies with time as shown in Fig. 5.48. Find impulse of the force and final velocity of the body.**

FIGURE 5.48



Solution. Using Fig. 5.48.

Impulse = area under *F-t* graph

$$m(v-u)=\frac{4\times2}{2}+4\times1+\frac{4\times2}{2}=\mathbf{12\ N\text{-}s}$$

$$2(v-0)=12,\ v=\frac{12}{2}=\mathbf{6\ ms^{-1}}$$

Example 10 **A machine gun has a mass of 20 kg. It fires 20 gram bullets at the rate of 300 bullets per second at a speed of 250 m/s. What force must be applied on the gun to keep it in position ?**

Solution. Here, $M = 20$ kg

$m = 20\text{ g} = 20\times10^{-3}$ kg ; $n = 300$, $v = 250$ m/s

Force required to hold the gun in position

F = rate of change of momentum of bullets

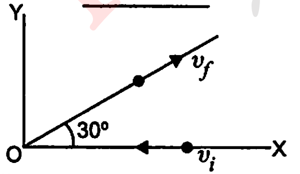
$$= n\,m\,v$$

$$F = 300\times20\times10^{-3}\times250 = \mathbf{1500\ N}$$

Example 11 **A ball of mass 0·2 kg travelling in a straight line with a speed of 15 m/s along negative *x*-axis is deflected by a bat at an angle of 30°. If the speed of the ball after deflection is 10 m/s, find the impulse on the ball.**

Solution. Here, $m = 0{\cdot}2$ kg

FIGURE 5.49



$v_{ix} = -15$ m/s ; $v_{iy} = 0$; $v_f = 10$ m/s ; $\theta = 30°$

$$v_{fx} = v_f\cos30° = 10\sqrt{3}/2 = 5\sqrt{3}\ \text{m/s}$$

$$v_{fy} = v_f\sin30° = 10\times\frac{1}{2} = 5\ \text{m/s}$$

Impulse, $I_x = m(v_{fx}-v_{ix}) = 0{\cdot}2\,(5\sqrt{3}+15)$
$= 4{\cdot}73$ Ns

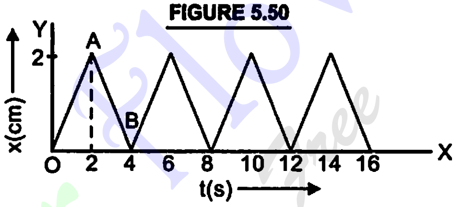
$$I_y = m(v_{fy}-v_{iy}) = 0{\cdot}2\,(5-0) = 1{\cdot}0\ \text{Ns}$$

$\therefore$ Net impulse on the ball, $I=\sqrt{I_x^2+I_y^2}$

$$=\sqrt{(4{\cdot}73)^2+1^2} = \mathbf{4{\cdot}83\ Ns}$$

Example 12 **Figure 5.50 below shows the position time graph of a particle of mass 0·04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle ? What is the magnitude of each impulse ?**

FIGURE 5.50



Solution. Here, $m = 0{\cdot}04$ kg. Position time graph shows that the particle moves from $x = 0$ at O to $x = 2$ cm at A in 2 sec.

As $x-t$ graph is a straight line, the motion is with a constant velocity

$$u=\frac{(2-0)\ \text{cm}}{(2-0)\ \text{s}} = 1\ \text{cm s}^{-1}$$

$$= 10^{-2}\ \text{ms}^{-1}$$

From $x = 2$ cm at A, particle goes to $x = 0$ at B in 2 sec.

As AB is a straight line, motion is with constant velocity,

$$v = -1\ \text{cm s}^{-1} = -10^{-2}\ \text{ms}^{-1}$$

Negative sign indicates the reversal of direction of motion. This is being repeated. We can visualise a ball moving between two walls located at $x = 0$ and $x = 2$ cm, getting rebounded repeatedly on striking against each wall. On every collision with a wall, linear momentum of the ball changes. Therefore, the ball receives impulse **after every two seconds.** Magnitude of impulse = total change in linear momentum

$$= m\,u-(m\,v) = m\,u - m\,v = m\,(u-v)$$
$$= 0{\cdot}04\,(10^{-2}+10^{-2})$$
$$= 0{\cdot}08\times10^{-2}$$
$$= \mathbf{8\times10^{-4}\ kg\ ms^{-1}}$$

TYPE III. APPARENT WEIGHT OF A PERSON IN AN ELEVATOR

Formulae used. The actual weight of a person = mg

(*i*) When the lift is at rest, his apparent weight, $R = mg$

(*ii*) When the lift is moving *uniformly* in upward/downward direction, $R = mg$

(*iii*) When the lift is *moving upwards* with an acceleration (a), $R = m\,(g + a)$

(*iv*) When the lift is moving downwards with an acceleration (a), $R = m\,(g - a)$

(*v*) In free fall of the lift under gravity, $a = g$

$\therefore \quad R = m\,(g - g) = 0$

i.e., the person becomes weightless.

Units used. a and g are in m/s^2 ; m is in kg and R is in newton (N).

Example 13 **Find the apparent weight of a man weighing 49 kg on earth, when he is standing in a lift which is (*i*) rising with an acceleration of 1·2 m/s^2 (*ii*) going down with same acceleration (*iii*) falling freely under gravity (*iv*) going up or down with uniform velocity. Take g = 9·8 m/s^2.**

Solution. Here, m = 49 kg, R = ?

(*i*) upward acceleration a = 1·2 m/s^2

$$R = m\,(g + a) = 49\,(9{\cdot}8 + 1{\cdot}2) = 539 \text{ N}$$

$$= \frac{539}{9{\cdot}8} = \mathbf{55\ kgf}$$

(*ii*) downward acceleration a = 1·2 m/s^2

$$R = m\,(g - a) = 49\,(9{\cdot}8 - 1{\cdot}2) = 49 \times 8{\cdot}6 \text{ N}$$

$$= \frac{49 \times 8{\cdot}6}{9{\cdot}8} \text{kg f} = \mathbf{43\ kgf}$$

(*iii*) free fall, $a = g$,

$$R = m\,(g - a) = m\,(g - g) = \mathbf{Zero}$$

(*iv*) when velocity is uniform, $a = 0$,

$$R = mg = \mathbf{49\ kg\ f}$$

Example 14 **A lift of mass 2000 kg is supported by thick steel ropes. If maximum upward acceleration of the lift be 1·2 m/s^2, and the breaking stress for the ropes be 2·8 × 10^8 Nm^{-2}, what should be the minimum diameter of rope ?**

Solution. Here, m = 2000 kg,

a = 1·2 m/s^2, upwards ;

$$T = m\,(g + a) = 2000\,(9{\cdot}8 + 1{\cdot}2) = 22000 \text{ N}$$

$$\text{Breaking stress} = \frac{\text{force}}{\text{area}} = \frac{T}{\pi D^2/4} = \frac{4\,T}{\pi D^2}$$

$$2{\cdot}8 \times 10^8 = \frac{4 \times 22000}{\frac{22}{7}(D^2)}$$

$$D^2 = \frac{4 \times 22000 \times 7}{22 \times 2{\cdot}8 \times 10^8} = 10^{-4}$$

$$D = 10^{-2}\ m = \mathbf{1\ cm}$$

Example 15 **A lift starts from rest with a constant upward acceleration. It moves 1·5 m in the first 0·4 s. A person standing in the lift holds a packet of 2 kg by a string. Calculate the tension in the string during the motion.**

Solution. Here, $u = 0$, s = 1·5 m, t = 0·4 s

From $s = ut + \frac{1}{2}at^2$

$$1{\cdot}5 = 0 + \frac{1}{2}\,a\,(0{\cdot}4)^2\ ;$$

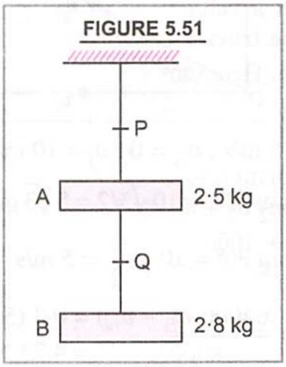
$$a = \frac{1{\cdot}5 \times 2}{0{\cdot}4 \times 0{\cdot}4} = 18{\cdot}75 \text{ m/s}^2$$

As the string is moving upwards with this acceleration

$$\therefore \quad T = m\,(g + a) = 2\,(9{\cdot}8 + 18{\cdot}75) = \mathbf{57{\cdot}1\ N}$$

Example 16 **Fig. 5.51 show two bodies A and B of masses 2·5 kg and 2·8 kg respectively suspended from a rigid support by two inextensible wires each of length 1·8 m. The upper wire is of negligible mass and lower wire is of mass 1·5 kg/m. If the entire system moves upwards with an acceleration of 2 m/s^2, find tension (*i*) at middle point P of upper wire (*ii*) at middle point Q of lower wire. Take g = 10 m/s^2.**

FIGURE 5.51



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Solution. As the system *moves upwards* with an acceleration. $a = 2\ m/s^2$

$\therefore$ tension in the wire $T = R = m\,(g + a)$

(*i*) For point *P* at mid point of upper wire

$$m = m_A + m_B + \text{mass of lower wire}$$
$$= 2{\cdot}5 + 2{\cdot}8 + 1{\cdot}8 \times 1{\cdot}5$$
$$= 8{\cdot}0 \text{ kg}$$

$\therefore$ $T = 8\,(10 + 2) = \mathbf{96\ N}$

(*ii*) For point *Q*, mid point of lower wire

$$m = m_B + \text{mass of half length of lower wire}$$
$$= 2{\cdot}8 + \frac{1{\cdot}8}{2} \times 1{\cdot}5 = 4{\cdot}15 \text{ kg}$$

$\therefore T = m\,(g + a) = 4{\cdot}15\,(10{+}2) = \mathbf{49{\cdot}8\ N}$

TYPE IV. PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM AND NEWTON'S THIRD LAW

Formulae used.

1. The vector sum of linear momenta of all the bodies in an isolated system remains constant, *i.e.*

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots . m_n \vec{v}_n = \text{constant}$$

2. Recoil velocity of gun, $\vec{v}_2 = \dfrac{-m_1 \vec{v}_1}{m_2}$

where m_2 is mass of gun, m_1 is mass of bullet and $\vec{v}_1$ is velocity of bullet.

3. The forces of action and reaction are always equal and opposite.

Units used. m_1, m_2 in kg ; v_1, v_2 in m/s ; *a* and *g* are in m/s^2

Example 17 **A car of mass 1000 kg travelling at 32 ms^{-1} dashes into the rear of a truck of mass 8000 kg, moving in the same direction with a velocity of 4 ms^{-1}. After the collision, the car bounces with a velocity of 8 ms^{-1}. What is the velocity of the truck after the impact ?**

Solution. Here, $m_1 = 1000$ kg, $u_1 = 32$ m/s,
$m_2 = 8000$ kg, $u_2 = 4$ m/s,
$v_1 = -8$ m/s, $v_2 = ?$

Using principle of conservation of linear momentum, $m_2 v_2 + m_1 v_1 = m_1 u_1 + m_2 u_2$

$$8000\, v_2 + 1000 \times (-8) = 1000 \times 32 + 8000 \times 4 = 64000$$

$$v_2 = \frac{64000 + 8000}{8000} = \mathbf{9\ m/s}$$

Example 18 **A disc of mass 10 g is kept floating horizontally by throwing 10 marbles per second against it from below. If mass of each marble is 5 g. Calculate the velocity with which marbles are striking the disc. Assume that marbles strike the disc normally and rebound downwards with the same speed.**

Solution. Here, $M = 10\ g = 10^{-2}$ kg, $n = 10$,
$m = 5\ \text{g} = 5 \times 10^{-3}$ kg, $v = ?$

To keep the disc floating horizontally.

Weight of disc = upward force on the disc
= rate of change of momentum of marbles

$$10^{-2} \times 9{\cdot}8 = n \times m \times 2\,v = 10 \times 5 \times 10^{-3} \times 2\,v$$
$$= \frac{v}{10}$$
$$v = 9{\cdot}8 \times 10^{-2} \times 10 \text{ m/s} = \mathbf{0{\cdot}98\ m/s}$$

Example 19 **A machine gun fires a bullet of mass 40 g with a velocity of 1200 m/s. The person holding it can apply a maximum force of 144 N on the gun. What is the maximum number of bullets that can be fired per second ?** (AIEEE 2004)

Solution. Here, $m = 40\ \text{g} = 40 \times 10^{-3}$ kg
$v = 1200$ m/s, $F = 144$ N, $n = ?$

As force = rate of change of momentum of bullets,

$\therefore$ $F = n \times m \times v$

$$n = \frac{F}{m \times v} = \frac{144}{40 \times 10^{-3} \times 1200} = \frac{144}{48} = 3$$

TYPE V. ROCKET PROPULSION

Formulae used. 1. Thrust on the rocket,

$$F = -u\left(\frac{dm}{dt}\right)$$

where $\dfrac{dm}{dt}$ is mass of burnt gases escaping per second, and u = exhaust speed of the burnt gases.

2. Velocity of rocket at any time *t*

$$v = u \log_e\left(\frac{m_0}{m}\right) = u\,(2{\cdot}303)\log_{10}\left(\frac{m_0}{m}\right)$$

3. Acceleration of rocket at any instant

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

Units used. u, v are in m/s ; *a* is in m/s^2, $\dfrac{dm}{dt}$ in kg/sec and *F* in newton.

Example 20 **A rocket has a mass of 2×10^4 kg, of which half is fuel. Assume that the fuel is consumed at a constant rate as the rocket is fired and there is constant thrust of 5×10^6 N. Neglecting air resistance and any possible variation of g, compute (i) the initial acceleration (ii) acceleration just when the whole fuel is consumed.**

Solution. Here, mass of rocket,

$$m = 2 \times 10^4 \text{ kg}$$

$$\text{mass of fuel, } m' = \frac{1}{2}(2 \times 10^4)\text{ kg} = 10^4 \text{ kg}$$

$$\text{upthrust} = 5 \times 10^6 \text{ N}$$

(*i*) Net force on the rocket,

$$F = \text{up thrust} - \text{weight }(mg)$$
$$= 5 \times 10^6 - 2 \times 10^4 \times 10$$
$$= 10^5 (50 - 2) = 48 \times 10^5$$

$\therefore$ Initial acceleration

$$= \frac{F}{m} = \frac{48 \times 10^5}{2 \times 10^4} = \mathbf{240\,m/s^2}$$

(*ii*) When the fuel has been completely exhausted, remaining mass of rocket,

$$m'' = m - m' = 10^4 \text{ kg}$$

Net force on the rocket,

$$F' = \text{upthrust} - \text{weight }(m'' g)$$
$$= 5 \times 10^6 - 10^4 \times 10 = 49 \times 10^5$$

$$\therefore \text{ Acceleration } = \frac{F'}{m''} = \frac{49 \times 10^5}{10^4} = \mathbf{490\,m/s^2}$$

Example 21 **Calculate the ratio m_0/m for a rocket if it is to escape from the earth. Given escape velocity = 11·2 km/s and exhaust speed of gases is 2 km/s.**

Solution. Assuming that the rocket starts from rest, velocity acquired is given by

$$v = u \log_e\left(\frac{m_0}{m}\right) = 2{\cdot}3026\, u \log_{10}\left(\frac{m_0}{m}\right)$$

Now, $v = 11{\cdot}2$ km/s and $u = 2$ km/s

$$\therefore \quad 11{\cdot}2 = 2{\cdot}3026 \times 2 \log_{10}\left(\frac{m_0}{m}\right)$$

$$\log_{10}\left(\frac{m_0}{m}\right) = \frac{11{\cdot}2}{2{\cdot}3026 \times 2} = 2{\cdot}432$$

$$\frac{m_0}{m} = \text{antilog } 2{\cdot}432 = \mathbf{270{\cdot}4}$$

Example 22 **A rocket is set for vertical firing. If the exhaust speed is 1200 ms^{-1}, how much gas must be ejected per second to supply the thrust needed**

(*i*) to overcome the weight of rocket

(*ii*) to give to the rocket an initial vertical upward acceleration of 29·6 m/s^2. Given mass of rocket = 6000 kg.

Solution. Here, $u = 1200$ m/s, $\dfrac{dm}{dt} = ?$

$$m_0 = 6000 \text{ kg}$$

(*i*) To overcome the weight of rocket,

$$\text{Thrust, } F = u\frac{dm}{dt} = m_0 g$$

$$\frac{dm}{dt} = \frac{m_0 g}{u} = \frac{6000 \times 9{\cdot}8}{1200} = \mathbf{49\ kg\ s^{-1}}$$

(*ii*) $\quad a = 29{\cdot}6 \text{ m/s}^2 \quad u\left(\dfrac{dm}{dt}\right)' = m_0(a + g)$

$$\left(\frac{dm}{dt}\right)' = \frac{m_0(a+g)}{u} = \frac{6000\,(29{\cdot}6 + 9{\cdot}8)}{1200}$$
$$= \mathbf{197\ kg/s}$$

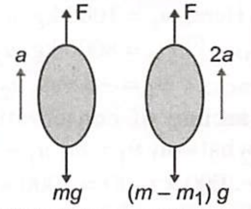
Example 23 **A balloon of mass m is rising up with an acceleration a. Show that the fraction of weight of the balloon that must be detached in order to double its acceleration is $\dfrac{ma}{(2a+g)}$. Assume that upthrust of air remains the same.**

Solution. Let F be the upthrust of air. As the balloon rises with acceleration a, Fig. 5.52.

$$\therefore \quad F - mg = ma \text{ or } F = m(g + a) \qquad ...(i)$$

When mass m_1 is removed from the balloon, acceleration becomes $(2a)$. Therefore,

FIGURE 5.52



$$F - (m - m_1)g = (m - m_1)\,2a$$

$$\text{or } F = (m - m_1)(g + 2a) \qquad ....(ii)$$

From (*i*) and (*ii*),

$$(m - m_1)(g + 2a) = m(g + a)$$

$$m - m_1 = m(g + a)/(g + 2a)$$

$$m_1 = m - \frac{m(g+a)}{g+2a} = \frac{ma}{g+2a}$$

Example 24 **A rocket motor consumes 100 kg of fuel per second, exhausting it with a speed of 6×10^3 ms^{-1}. What thrust is exerted on the rocket ? What will be the velocity of the rocket at the instant its mass is reduced to (1/40)th of its initial mass ? Take initial velocity of rocket as zero. Neglect gravity.**

Solution. (*i*) Here, $\frac{dm}{dt} = 100$ kg wt/s,

$$u = 6 \times 10^3 \text{ m/s}$$

Thrust $F = u\frac{dm}{dt} = 6 \times 10^3 \times 100$

$$= \mathbf{6 \times 10^5\ N}$$

(*ii*) $v = ?,\ m = \frac{1}{40}m_0$ or $\frac{m_0}{m} = 40$

$$v = (2{\cdot}303)\,u \log_{10}\left(\frac{m_0}{m}\right)$$

$$= 2{\cdot}303 \times 6 \times 10^3 \times \log_{10} 40$$

$$= 2{\cdot}303 \times 6 \times 10^3 \times 1{\cdot}602$$

$$= \mathbf{22{\cdot}13 \times 10^3\ m/s}$$

TYPE VI. EQUILIBRIUM OF FORCES

Formulae used

Draw *free body diagram* for each individual body or for any part of the system. Show various forces acting on the body/part. Find the net force acting on the body and equate it to its mass times the acceleration produced. The equations of motion obtained for different parts/bodies can be solved to arrive at the desired results.

Example 25 **A train is moving along a horizontal track. A pendulum suspended from the roof makes an angle of 4° with the vertical. If $g = 10$ m/s^2, what is the acceleration of the train ?**

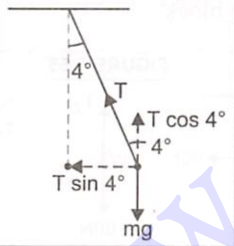
Solution. Fig. 5.53, shows equilibrium position of the pendulum. *T* is tension in the string. If *a* is acceleration of the train, then in equilibrium,

$$T\cos 4^\circ = mg \text{ and } T\sin 4^\circ = ma$$

Dividing, we get $\frac{T\sin 4^\circ}{T\cos 4^\circ} = \frac{ma}{mg} = \frac{a}{g}$

$$a = g\tan 4^\circ = 10 \times 0{\cdot}07 = \mathbf{0{\cdot}7\ m/s^2}$$

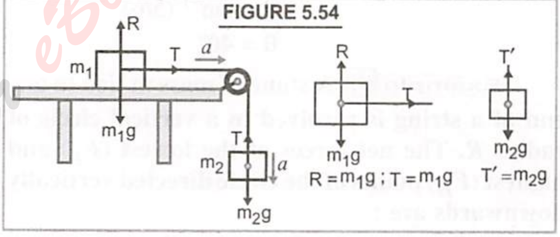
FIGURE 5.53



Example 26 **A body m_1 of mass 10 kg is placed on a smooth horizontal table. It is connected to a string which passes over a frictionless pulley and carries at the other end, a body m_2 of mass 5 kg. What acceleration will be produced in the bodies when the nail fixed on the table is removed ? What will be the tension in the string during the motion of the bodies ? What when the bodies stop ? ($g = 9{\cdot}8$ N/kg).**

Solution. When the nail fixed on the table is removed, the system of two bodies moves with an acceleration (*a*) in the direction shown in Fig. 5.54.

FIGURE 5.54



According to Newton's 2nd law,

$$(m_1 + m_2)\,a = m_2\,g$$

$$a = \frac{m_2\,g}{m_1 + m_2} = \frac{5 \times 9{\cdot}8}{(10+5)} = \mathbf{3{\cdot}27\ m/s^2}$$

Also, $T = m_1 \times a = 10 \times 3{\cdot}27 = \mathbf{32{\cdot}7\ N}$

Suppose tension in the string becomes T', when the bodies stop. In this position, acceleration, $a = 0$. Therefore, net force on each body is zero. For the body m_2, we may write

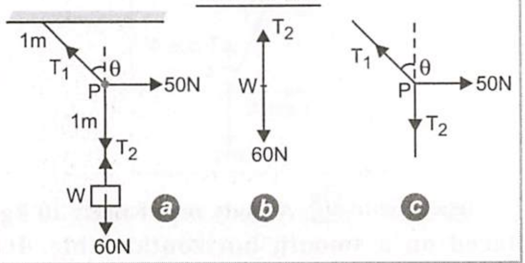
$$T' = m_2\,g = 5 \times 9{\cdot}8 = \mathbf{49\ N.}$$

Example 27 **See Fig. 5.55. A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction**

is applied at the mid-point P of the rope, as shown. What is the angle the rope makes with the vertical in equilibrium ? (Take $g = 10\ ms^{-2}$). Neglect mass of the rope. (NCERT Solved Example)

Solution. Fig. 5.55(b) represents free body diagram of 6 kg block.

FIGURE 5.55



In equilibrium, $T_2 = mg = 6 \times 10 = 60$ N

Fig. 5.55(c) represents free body diagram of forces at point P.

As the horizontal and vertical components of the resultant force must vanish separately, therefore,

$$T_1 \sin\theta = 50 \text{ N}$$

and $\quad T_1 \cos\theta = T_2 = 60$ N

Dividing, we get $\tan\theta = \dfrac{5}{6}$

$$\theta = \tan^{-1}(5/6)$$

$$\theta = \mathbf{40^\circ}$$

Example 28 **A stone of mass m tied to one end of a string is revolved in a vertical circle of radius R. The net forces at the lowest (F_L) and highest (F_H) points of the circle directed vertically downwards are :**

(a) $F_L = mg - T_1, F_H = mg + T_2$.

(b) $F_L = mg + T_1, F_H = mg - T_2$

(c) $F_L = mg + T_1 - \dfrac{mv_1^2}{R}$,

$$F_H = mg - T_2 + \frac{mv_1^2}{R}$$

(d) $F_L = mg - T_1 - \dfrac{mv_1^2}{R}$,

$$F_H = mg + T_2 + \frac{mv_1^2}{R}$$

Choose correct alternatives. T_1 , v_1 denote the tension and speed at the lowest point. T_2, v_2 denote corresponding values at the highest point.

Solution. The net force at the lowest point L is $F_L = (mg - T_1)$, and the net force at the highest point H is $F_H = mg + T_2$. Therefore, alternative (a) is correct. For details, see Art. 5.47.

Example 29 **A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of (a) force on the floor by the the crew and passengers, (b) action of the rotor of the helicopter on surrounding air, (c) force on the helicopter due to the surrounding air.**

Solution. Here, mass of helicopter,

$$m_1 = 1000 \text{ kg}$$

Mass of the crew & passengers, $m_2 = 300$ kg

upward acceleration, $a = 15\ ms^{-2}$

and $\quad g = 10\ ms^{-2}$

(a) Force on the floor of helicopter by the crew and passengers = apparent weight of crew and passengers

$$= m_2 (g + a) = 300 (10 + 15) = \mathbf{7500\ N}$$

(b) Action of rotor of helicopter on surrounding air is obviously *vertically downwards*, because helicopter rises on account of reaction to this force. Thus, force of action

$$F = (m_1 + m_2)(g + a) = (1000 + 300)(10 + 15)$$
$$= 1300 \times 25 = \mathbf{32500\ N}$$

(c) Force on the helicopter due to surrounding air is the *reaction*. As action and reaction are equal and opposite, therefore, force of reaction,

$$F' = \mathbf{32500\ N}, \textit{ vertically upwards.}$$

TYPE VII. COEFFICIENT OF FRICTION, ANGLE OF FRICTION AND MOTION ON A LEVEL ROAD

Formulae used. 1. Coefficient of friction,

$$\mu = \frac{\text{force of friction (F)}}{\text{normal reaction (R)}}$$

When a body is at rest or it is at the verge of moving horizontally over the surface of the other body, $F = P$ = applied force, and $R = mg$ = actual weight of the body.

2. If θ is the angle of friction, then $\tan\theta = \mu$

3. If the body is moving over a rough horizontal surface with a retardation a, then $\mu = a/g$.

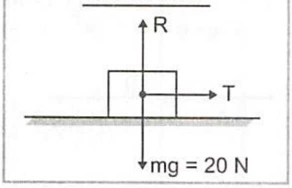
Units used. μ has no units ; θ is in radian ; F, P, R and mg are in newton ; m in kg ; g in m/s^2.

Example 30 **A block of weight 20 N is placed on a horizontal table and a tension T is applied on the block. Tension can be increased to 8 N before the block begins to slide. A force of 4 N keeps the block moving at constant speed, once it has been set in motion. Find the coefficients of static friction and kinetic friction.**

Solution. Here, $mg = 20$ N

As is clear from Fig. 5.56,

FIGURE 5.56



$$R = mg = 20 \text{ N}$$

Force of limiting friction = maximum static friction

$$= f_s = 8 \text{ N}$$

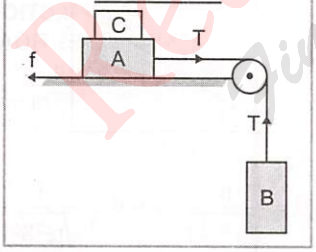
$$\therefore \quad \mu_s = \frac{f_s}{R} = \frac{8}{20} = \mathbf{0{\cdot}40}$$

Once the motion has started, force of kinetic friction, $f_k = 4$ N

$$\therefore \quad \mu_k = \frac{f_k}{R} = \frac{4}{20} = \mathbf{0{\cdot}20}$$

Example 31 **In Fig. 5.57, the masses of A and B are 10 kg and 5 kg. Calculate the minimum mass of C which may stop A from slipping. Coefficient of static friction between block A and table is 0·2.**

FIGURE 5.57



Solution. Here, $m_A = 10$ kg, $m_B = 5$ kg ,

$m_C = ?,\ \mu = 0{\cdot}2$

To avoid slipping of A,

force of friction = T = wt. of B

$$\mu\,(m_C + m_A)\,g = m_B\,g$$

$$0{\cdot}2\,(m + 10) = 5\ ;\ m + 10 = \frac{5}{0{\cdot}2} = 25$$

$$m = 25 - 10 = \mathbf{15\ kg}$$

Example 32 **Determine the maximum acceleration of the train in which a box lying on the floor will remain stationary, given that the coefficient of static friction between the box and the train's floor is 0·15. Given $g = 10$ m/s^2.**

(NCERT Solved Example)

Solution. As acceleration (a) of the box is due to static friction, therefore,

$$ma = f \le \mu_s R \le \mu_s mg$$

$$\therefore \quad a \le \mu_s g$$

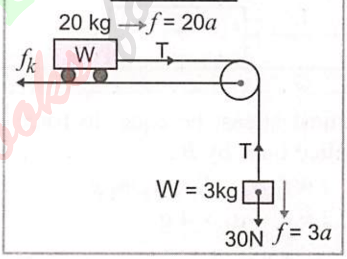
$$\text{or} \quad a_{\max} = \mu_s g = 0{\cdot}15 \times 10 = \mathbf{1{\cdot}5\ m/s^2}$$

Example 33 **What is the acceleration of the block and trolley system shown in Fig. 5.58(a), if the coefficient of kinetic friction between the trolley and the surface is 0·04 ? What is the tension in the string ? Take $g = 10$ ms^{-2}. Neglect the mass of the string.**

(Central Schools 2014)

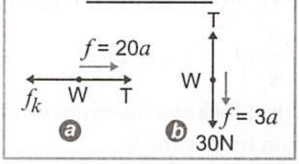
(NCERT Solved Example)

FIGURE 5.58



Solution. As the string is inextensible, and the pulley is smooth, the 3 kg block and the 20 kg trolley, both have same magnitude of acceleration (a). Applying Newton's second law of motion to free body diagram of $W = 20$ kg, Fig. 5.59(a), we get

FIGURE 5.59



$$T - f_k = 20\,a \qquad \text{...}(i)$$

$$\text{Now,} \quad f_k = \mu_k R = \mu_k\,\text{mg} = 0{\cdot}04 \times 20 \times 10 = 8 \text{ N}$$

$$\therefore \quad T - 8 = 20\,a \qquad \text{...}(ii)$$

Again applying Newton's second law of motion to free body diagram of $W = 3$ kg, Fig. 5.59(b), we get

$$30 - T = 3\,a \qquad \text{...}(iii)$$

Adding (*ii*) and (*iii*), we get

$$22 = 23\,a$$

$$a = \frac{22}{23} = \mathbf{0{\cdot}96\ ms^{-2}}$$

From (*ii*), $T = 20\,a + 8$

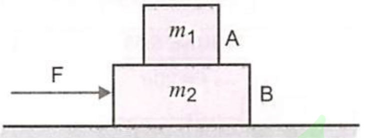
$$T = 20 \times 0{\cdot}96 + 8 = 19{\cdot}2 + 8 = \mathbf{27{\cdot}2\ N}$$

Example 34 **A block *A* of mass 4 kg is placed on another block *B* of mass 5 kg, and the block *B* rests on a smooth horizontal table. For sliding block *A* on *B*, a horizontal force of 12 N is required to be applied on *A*. How much maximum force can be applied on *B* so that both *A* and *B* move together ? Also, find out acceleration produced by this force.** **(Central Schools 2014)**

Solution. Here, $m_1 = 4$ kg, $m_2 = 5$ kg.

Force applied on A, $f = 12$ N, Fig. 5.60.

FIGURE 5.60



This must atleast be equal to force of kinetic friction applied on *A* by *B*

i.e. $f = f_k = \mu_k R = \mu_k m_1 g$

$$12 = f_k = \mu_k \times 4\,g$$

$$\therefore\ \mu_k = \frac{12}{4\,g} = \frac{3}{g}$$

As block *B* is on smooth surface, therefore to move *A* and *B* together, (maximum) force *F* required to be applied on *B* = frictional force applied on *A* by *B* plus frictional force applied on *B* by *A*

$$F = \mu_k m_1 g + \mu_k m_2 g = \mu_k (m_1 + m_2)\,g$$

$$F = \frac{3}{g}(4+5)\,g = \mathbf{27\ N}$$

As both the blocks are moving on a smooth table under the action of this force,

therefore, $a = \dfrac{F}{m_1 + m_2} = \dfrac{27}{4+5} = \mathbf{3\ ms^{-2}}$

Example 35 **A particle of mass *m* rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying minimum possible force *F*. Find its magnitude and direction.**

Solution. Let a force *F* be applied at ∠θ to move the body, as shown in Fig. 5.61. Here, *f* represents the force of friction.

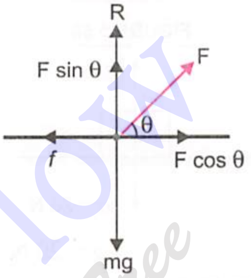
For horizontal equilibrium,

$$F\cos\theta = f = \mu R \qquad \text{...}(i)$$

For vertical equilibrium,

$$F\sin\theta + R = mg\ ;\ R = mg - F\sin\theta \qquad \text{...}(ii)$$

FIGURE 5.61



Put in (*i*), $F\cos\theta = \mu\,(mg - F\sin\theta)$

$$F(\cos\theta + \mu\sin\theta) = \mu\,mg$$

$$F = \frac{\mu\,mg}{\cos\theta + \mu\sin\theta} \qquad \text{...}(iii)$$

Now, *F* will be minimum, when denominator,

$$\cos\theta + \mu\sin\theta = \text{maximum}$$

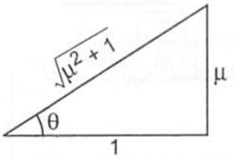
$$\therefore\ \frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$$

$$-\sin\theta + \mu\cos\theta = 0$$

or $\tan\theta = \mu$ ...(*iv*)

Using Fig. 5.62,

FIGURE 5.62



$$\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}\ ;\ \cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$

Put in (*iii*),

$$F_{\text{min}} = \frac{\mu\,mg}{\dfrac{1}{\sqrt{\mu^2+1}} + \dfrac{\mu^2}{\sqrt{\mu^2+1}}} = \frac{\mu\,mg}{\sqrt{\mu^2+1}}$$

From (*iv*), $\theta = \tan^{-1}(\mu)$

This is the direction of F_{min}.

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Example 36 **A bullet of mass 0·01 kg is fired horizontally into a 4 kg wooden block at rest, on a horizontal surface. The coefficient of kinetic friction between the block and bullet is 0·25. The bullet gets embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block ?**

Solution. Here, $m = 0{\cdot}01$ kg, $M = 4$ kg, $u = 0$,

$$u_k = 0{\cdot}25,\ s = 20\text{ m},\ v = 0$$

If f_k is kinetic friction producing retardation a in the combination, then

$$f_k = \mu_k R = \mu_k (M+m)\, g = (M+m)\, a$$

$$\therefore\quad a = \mu_k\, g = 0{\cdot}25 \times 9{\cdot}8 = 2{\cdot}45\text{ m/s}^2$$

If V is velocity of combination after the bullet is embedded in the block, then from

$$v^2 - u^2 = 2\,as$$

$$0 - V^2 = 2\,(-2{\cdot}45) \times 20 = 98$$

$$V = \sqrt{98} = 9{\cdot}8995\text{ m/s}$$

If V' is velocity of bullet, then from the principle of conservation of linear momentum,

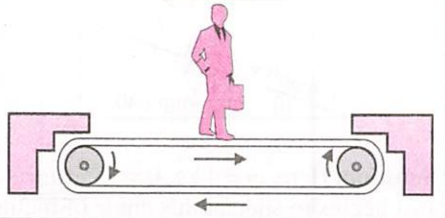
$$m\,V' = (M+m)\,V$$

$$V' = \frac{(M+m)V}{m} = \frac{(4+0{\cdot}01)}{0{\cdot}01} \times 9{\cdot}8995$$

$$= \mathbf{3969{\cdot}7\ m/s}$$

Example 37 **Figure 5.63 shows a man standing stationary with respect to a horizontal conveyer belt that is accelerating with 1 ms^{-2}. What is the net force on the man ? If the coefficient of static friction between the man's shoes and the belt is 0·2, upto what acceleration of the belt can the man continue to be stationary relative to the belt ? (Mass of man is 65 kg).**

FIGURE 5.63



Solution. Here, acceleration of conveyer belt, $a = 1$ m/s^2

As the man is standing stationary w.r.t. the belt, acceleration of the man = acceleration of belt

$$= a = 1\text{ m/s}^2$$

As $m = 65$ kg

$\therefore$ Net force on the man, $F' = ma = 65 \times 1$

$$= \mathbf{65\ N}$$

Now, $\mu = 0{\cdot}2$

Force of limiting friction $F = \mu R = \mu\, mg$

If the man remains stationary upto max. acc. a' of the belt, then $F = ma' = \mu\, mg$

$$a' = \mu\, g = 0{\cdot}2 \times 9{\cdot}8 = \mathbf{1{\cdot}96\ ms^{-2}}$$

Example 38 **A block of mass 15 kg is placed on a long trolly. The coefficient of friction between the block and the trolly is 0·18. The trolly accelerates from rest with 0·5 m s^{-2} for 20 s and then moves with a uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground (b) an observer moving with the trolly.**

Solution. Here, $m = 15$ kg, $\mu = 0{\cdot}18$,

$$a = 0{\cdot}5\text{ ms}^{-2},\ t = 20\text{ s}$$

Force on the block due to motion of the trolly $F' = ma = 15 \times 0{\cdot}5 = 7{\cdot}5$ N

It is obviously in the forward direction (of motion of the trolly)

Force of limiting friction on the block = F $= \mu\, R = \mu\, mg = 0{\cdot}18 \times 15 \times 9{\cdot}8 = 26{\cdot}46$ N

This opposes the moition of the block. The block *shall not move*. The force of static friction F will adjust itself equal and opposite to F', the applied force.

Hence to a stationary observer on the ground, **the block will appear to be at rest relative to the trolly.**

When trolly moves with uniform velocity, forward force is zero. Force of friction alone is acting on the block.

(*b*) An observer moving with the trolly has accelerated motion. The observer is, therefore non-inertial. **The law of inertia is no longer valid.**

TYPE VIII. ANGLE OF REPOSE AND MOTION OF A BODY UP AND DOWN A ROUGH INCLINED PLANE

Formulae used. **1.** If α is angle of repose, then $\tan\alpha = \mu$

2. Over an inclined plane of angle of inclination θ, normal reaction, $R = mg\cos\theta$

3. When body moves down the inclined plane, force of friction is up the plane.

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∴ Net force down the inclined plane

$$f = mg\sin\theta - F = mg(\sin\theta - \mu\cos\theta)$$

4. When body is moved up the inclined plane, force of friction is down the plane

∴ Force required,

$$f = mg\sin\theta + F = mg(\sin\theta + \mu\cos\theta)$$

Units used. In mks system, all forces F, R, mg and f are in newton ; α, θ may be in degree or radian.

Example 39 **Find the force required to move a train of mass 10^5 kg up an incline of 1 in 50 with an acceleration of 2 ms^{-2}. Coefficient of friction between the train and rails is 0·005. Take $g = 10\ ms^{-2}$.**

Solution. $F = ?$, $m = 10^5$ kg, $\sin\theta = \dfrac{1}{50}$,

$a = 2\ m/s^2$, $\mu = 0{\cdot}005$, $g = 10\ ms^{-2}$

Now,

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{50}\right)^2} = \sqrt{\frac{2499}{2500}} \simeq 1$$

$$F = mg(\sin\theta + \mu\cos\theta) + ma$$

$$= 10^5 \times 10\left(\frac{1}{50} + 0{\cdot}005 \times 1\right) + 10^5 \times 2$$

$$= 10^6 \times 0{\cdot}025 + 2 \times 10^5 = \mathbf{2{\cdot}25 \times 10^5\ N}$$

Example 40 **A railway engine weighing 40 metric ton is travelling along a level track at a speed of 54 km h^{-1}. What additional power is required to maintain the same speed up an incline of 1 in 49. Take $g = 9{\cdot}8\ m/s^2$ and $\mu = 0{\cdot}1$.**

Solution. Here, $m = 40$ metric ton $= 40 \times 10^3$ kg

$$v = 54\ km/h = \frac{54 \times 1000}{60 \times 60}\ m/s = 15\ m/s,$$

$g = 9{\cdot}8\ m/s^2$,

$\mu = 0{\cdot}1$, $\sin\theta = 1/49$; $(P_2 - P_1) = ?$

On a level track,

$$P_1 = \mu\, mg \times v = 0{\cdot}1 \times 4 \times 10^4 \times 9{\cdot}8 \times 15$$

$$= 5{\cdot}88 \times 10^5 \text{ watt}$$

Up the incline, $P_2 = mg(\sin\theta + \mu\cos\theta) \times v$

$$= 4 \times 10^4 \times 9{\cdot}8\left(\frac{1}{49} + 0{\cdot}1 \times 1\right) \times 15$$

$$= 7{\cdot}08 \times 10^5\ W$$

Additional power required

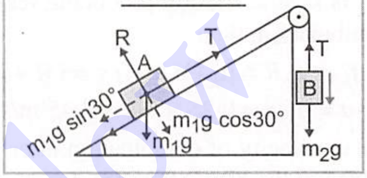
$$= P_2 - P_1 = (7{\cdot}08 - 5{\cdot}88)\,10^5\ W$$

$$= 1{\cdot}20 \times 10^5\ W = \mathbf{120\ kW}$$

Example 41 **A block A of mass 14 kg moves along an inclined plane that makes an angle of 30° with the horizontal. This block is connected to another block *B* of mass 14 kg by a taut massless string that runs around a massless frictionless pulley. The block *B* moves down with constant velocity. Calculate force of friction and coefficient of kinetic friction.**

Solution. Refer to Fig. 5.64,

FIGURE 5.64



As block *B* moves down with constant velocity, net force on it is zero.

$$\therefore\ T = m_2 g.$$

Again, block *A* will also move with constant velocity.

Therefore, net force on *A* is also zero.

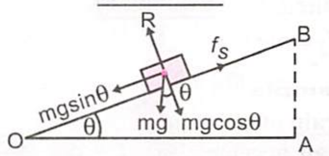
$$T - f - m_1 g \sin 30° = 0 \ \therefore f = T - m_1 g \sin 30°$$

$$f = m_2 g - m_1 g \sin 30° = \frac{1}{2} m_1 g = \mathbf{68{\cdot}6\ N}$$

$$\mu = \frac{f}{R} = \frac{68{\cdot}6}{m_1 g \cos 30°} = \frac{68{\cdot}6 \times 2}{14 \times 9{\cdot}8\sqrt{3}} = \mathbf{0{\cdot}58}$$

Example 42 **See Fig. 5.65. A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until an angle $\theta = 15°$ with the horizontal and the mass just begins to slide. What is the coefficient of static friction between the block and the surface ?** **(NCERT Solved Example)**

FIGURE 5.65



Solution. Here, $m = 4$ kg, $\theta = 15°$, $\mu_s = ?$ As the mass just begins to slide at this angle of inclination, therefore, $\theta = 15° = \alpha$, the angle of repose.

$$\mu_s = \tan\alpha = \tan 15° = \mathbf{0{\cdot}27}$$

Example 43 **A mass of 200 kg is placed on a rough inclined plane of angle 30°. If coefficient**

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of limiting friction is $1/\sqrt{3}$, find the least forces in newton, acting parallel to the plane (*i*) to keep the mass from sliding down, (*ii*) to move the mass up the plane. (Central Schools 2008)

Solution. Here, $m = 200$ kg, $\theta = 30^\circ; \mu = \dfrac{1}{\sqrt{3}}$

As is clear from Fig. 5.66(*a*) and (*b*),

$$R = mg\cos\theta$$

$\therefore$ Force of friction $F = \mu R = \mu\, mg\cos\theta$

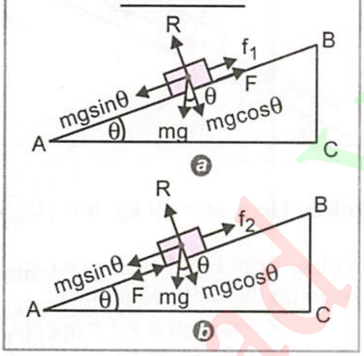
$$= \frac{1}{\sqrt{3}} \times 200 \times 9{\cdot}8 \cos 30^\circ$$

$$F = \frac{1}{\sqrt{3}} \times 200 \times 9{\cdot}8 \frac{\sqrt{3}}{2} = 980 \text{ N}$$

Also, component of weight acting down the plane $= mg\sin\theta = 200 \times 9{\cdot}8 \sin 30^\circ$

$$= 200 \times 9{\cdot}8 \times \frac{1}{2} = 980 \text{ N}$$

FIGURE 5.66



(*i*) Hence the *least force* required to keep the body from sliding down, Fig. 5.66(*a*),

$$f_1 = mg\sin\theta - F = 980 - 980 = \textbf{Zero}$$

(*ii*) To move the body up the plane, *the least force* required, Fig. 5.66(*b*),

$$f_2 = mg\sin\theta + F = 980 + 980 = \textbf{1960 N}$$

Example 44 **Find the force required to move a train of 2000 quintals up an incline of 1 in 50, with an acceleration of 2 ms^{-2}, the force of friction being 0·5 newton per quintal.**

Solution. Here, $m = 2000$ quintals

$= 2000 \times 100$ kg

$$\sin\theta = \frac{1}{50};\ a = 2 \text{ ms}^{-2}$$

Force of friction = 0·5 N per quintal

$$F = 0{\cdot}5 \times 2000 = 1000 \text{ N}$$

In moving up an inclined plane, force required against gravity

$$= mg\sin\theta = 200{,}000 \times 9{\cdot}8 \times \frac{1}{50}$$

$$= 39200 \text{ N}$$

Also, force required to produce acceleration

$$= ma = 200{,}000 \times 2 = 400{,}000 \text{ N}$$

$\therefore$ Total force required = 1000 + 39,200 + 400,000

= **440,200 N**

Example 45 **A piece of ice slides down a 45° incline in twice the time it takes to slide down a frictionless 45° incline. What is the coefficient of friction between the ice and the incline ?**

Solution. Here, $\theta = 45^\circ$; $s_1 = s_2$; $u = 0$

On the rough incline, $a_1 = g(\sin\theta - \mu\cos\theta)$

t_1 = time taken

On the frictionless incline, $a_2 = g\sin\theta$

t_2 = time taken, and $t_1 = 2\,t_2$

From $s = ut + \dfrac{1}{2}at^2$

$$s_1 = 0 + \frac{1}{2}\, g(\sin\theta - \mu\cos\theta)\ t_1^2$$

and $s_2 = 0 + \dfrac{1}{2}\, g\sin\theta\,.\ t_2^2$

As $s_1 = s_2$

$$\therefore\quad \frac{1}{2}\, g(\sin\theta - \mu\cos\theta)\ t_1^2 = \frac{1}{2}\, g\sin\theta . t_2^2$$

$$\frac{\sin\theta - \mu\cos\theta}{\sin\theta} = \frac{t_2^2}{t_1^2} = \frac{t_2^2}{(2t_2)^2} = \frac{1}{4}$$

$$1 - \mu\cot\theta = \frac{1}{4} \quad\text{or}\quad \mu\cot\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\mu = \frac{\mathbf{3}}{\mathbf{4\cot\theta}}$$

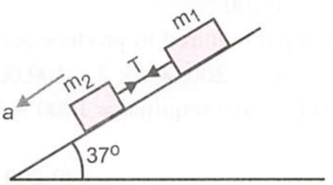
Example 46 **Two blocks m_1 = 4 kg and m_2 = 2 kg connected by a weightless rod slide down a plane having an inclination of 37°. The coefficient of dynamic friction of m_1 and m_2 with the inclined plane are μ_1 = 0·75 and μ_2 = 0·25 respectively. Find the common acceleration of the two blocks and tension in the rod. Take sin 37° = 0·6 and cos 37° = 0·8.**

Solution. Let a be the common acceleration of the blocks and T be the tension in the connecting rod,

Fig. 5.67. Components of wts of two blocks down the inclined plane $= m_1 g \sin 37^\circ + m_2 g \sin 37^\circ$

$$= (m_1 + m_2)\, g \sin 37^\circ$$

FIGURE 5.67



Frictional forces on two blocks up the inclined plane $= \mu_1 m_1 g \cos 37^\circ + \mu_2 m_2 g \cos 37^\circ$

$$= (\mu_1 m_1 + \mu_2 m_2)\, g \cos 37^\circ$$

Net force down the plane $= (m_1 + m_2)\, a$

$$\therefore \quad (m_1 + m_2)\, a = (m_1 + m_2)\, g \sin 37^\circ - (\mu_1 m_1 + \mu_2 m_2)\, g \cos 37^\circ$$

Putting the values, we get $a = \mathbf{1{\cdot}3\ m/s^2}$

To find T, we write eqn. of motion of block m_1

$$T + m_1 g \sin 37^\circ - \mu_1 m_1 g \cos 37^\circ = ma$$

on solving, we get $T = \mathbf{5{\cdot}2\ N}$

Example 47 **A particle of mass 1 gram executes an oscillatory motion on the concave surface of a spherical dish of radius 2 m, placed on a horizontal plane. If the motion of the particle starts from a point on the dish at the height of 1 cm from the horizontal plane and the coefficient of friction is 0·01, how much total distance will be moved by the particle before it comes to rest ?**

Solution. Here, $m = 1$ g, $R' = 2$ m, $h = 1$ cm,

$$\mu = 0{\cdot}01$$

Potential energy of particle $= m\,g\,h$

If d is the distance moved by the particle before coming to rest, then work done against friction is

$$W = (\mu R) \times d = \mu\, mg \times d$$

As P.E. is spent in doing this work.

$$\therefore \quad \mu\, mg \times d = m\, g\, h$$

$$d = \frac{h}{\mu} = \frac{1}{0{\cdot}01} = 100\,\text{cm} = \mathbf{1\,m}$$

Example 48 **A block of metal of mass 50 gram placed over an inclined plane at an angle of 15° slides down without acceleration. If the inclination is increased by 15°, what would be the acceleration of the block ?**

Solution. Here, $m = 50$ g $= 0{\cdot}05$ kg

Angle of repose, $\alpha = 15^\circ$, $\theta = 15^\circ + 15^\circ = 30^\circ$;

$$a = ?$$

$$\mu = \tan \alpha = \tan 15^\circ = 0{\cdot}2679$$

Net downward force on the block, down the incline,

$$F = mg \sin \theta - f$$

$$ma = mg \sin \theta - \mu\, mg \cos \theta$$

$$\therefore \quad a = g\,(\sin \theta - \mu \cos \theta)$$

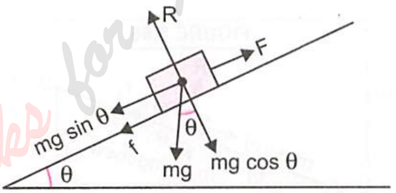
$$= 9{\cdot}8\,(\sin 30^\circ - \mu \cos 30^\circ)$$

$$= 9{\cdot}8\,(0{\cdot}5 - 0{\cdot}2679 \times \sqrt{3}/2\,)$$

$$a = \mathbf{2{\cdot}6\ m/s^2}$$

Example 49 **A body of mass 10 kg is placed on an inclined surface of angle 30°. If coefficient of limiting friction is $1/\sqrt{3}$, find the force required just to push the body up the inclined plane. Force is being exerted parallel to the inclined plane.** (Delhi 2005)

FIGURE 5.68



Solution. Here, $m = 10$ kg, $\theta = 30^\circ$, $\mu = \dfrac{1}{\sqrt{3}}$

As is clear from Fig. 5.68, force required just to push the body up the inclined plane is

$$F = mg \sin \theta + f = mg \sin \theta + \mu R$$

$$= mg \sin \theta + \mu\, mg \cos \theta$$

$$= mg\,(\sin \theta + \mu \cos \theta)$$

$$= 10 \times 9{\cdot}8 \left(\sin 30^\circ + \frac{1}{\sqrt{3}} \cos 30^\circ \right)$$

$$F = 98 \left(0{\cdot}5 + \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \right) = \mathbf{98\ N}$$

TYPE IX. MOTION ALONG A HORIZONTAL CIRCLE AND MOTION OF VEHICLE ALONG UNBANKED CIRCULAR TRACKS

Formulae used.

1. In a horizontal circle, centripetal force = centrifugal force = tension in the string

$$T = \frac{m v^2}{r} = m\, r\, \omega^2 = mr\,(2\,\pi\, n)^2$$

2. When a vehicle moves over an unbanked circular track, force of friction provides the necessary centripetal force

$$\frac{mv^2}{r} = F = \mu R = \mu mg$$

$$v = \sqrt{\mu rg}$$

3. When there is no friction, the vehicle must move at a very slow speed to avoid overturning. The safe speed $v = \sqrt{\frac{g\,rx}{h}}$

where $2x$ = distance between the front wheels or back wheels

h = height of centre of gravity of vehicle above the road

Units used. m in kg, r in metre, v in m/s, ω in rad/sec. F, T in newton ; μ has no units.

Standard Value. $g = 9{\cdot}8\ \text{m/s}^2$

Example 50 **A string breaks under a load of 4·8 kg. A mass of 0·5 kg is attached to one end of a string 2 m long and is rotated in a horizontal circle. Calculate the greatest number of revolutions that the mass can make without breaking the string.**

Solution. Here, $T = 4{\cdot}8$ kg wt. $= 4{\cdot}8 \times 9{\cdot}8$ N.

$$m = 0{\cdot}5,\ r = 2\ \text{m},\ v = ?$$

$$\text{As } T = m\,r\,\omega^2 = m\,r\,(2\,\pi\,v)^2$$

$$= 4\,\pi^2\,m\,r\,v^2$$

$$\therefore \quad v^2 = \frac{T}{4\pi^2\,m\,r} = \frac{4{\cdot}8\times 9{\cdot}8}{4\times 9{\cdot}87\times 0{\cdot}5\times 2}$$

$$= 1{\cdot}215$$

$$v^2 = \sqrt{1{\cdot}215} = 1{\cdot}102\ \text{rps}\,.$$

Greatest number of revolutions that the mass can make per minute

$$= 1{\cdot}102 \times 60$$

$$= \mathbf{66{\cdot}12\ rpm}$$

Example 51 **A car travels on a flat, circular track of radius 200 m at 30 ms^{-1} and has a centripetal acceleration = 4·5 ms^{-2}. (a) If the mass of the car is 1000 kg , what frictional force is required to provide the acceleration ? (b) If the coefficient of static friction is 0·8, what is the maximum speed at which the car can circle the track ?**

Solution. Here, $r = 200$ m, $v = 30\ \text{ms}^{-1}$

Centripetal acc., $a = 4{\cdot}5\ \text{ms}^{-2}$; $m = 1000$ kg

Frictional force required, $F = ?$

As F = Accelerating force $= m\,a = 1000 \times 4{\cdot}5$
$= \mathbf{4500\ N}$

(*b*) $\mu = 0{\cdot}8,\ v = ?$

$$v = \sqrt{\mu\,r\,g} = \sqrt{0{\cdot}8\times 200\times 9{\cdot}8} = \mathbf{39{\cdot}6\ ms^{-1}}$$

Example 52 **Two small wooden blocks are placed on a circular rotating table of radius 1 m at distances 10 cm and 60 cm from the centre of the table. The table is rotating with angular velocity 4 rad/s, about the axis of rotation. Out of these two blocks, which one continues to revolve with the table ? Given μ = 0·2.**

Solution. Here, $r = 1$ m, $\omega = 4\ \text{rad s}^{-1}$, $\mu = 0{\cdot}2$

The wooden block will continue to revolve with the table till force of friction is greater than or atleast equal to centripetal force required by it *i.e.*, $F \geq mr\,\omega^2$ or $\mu R \geq mr\,\omega^2$ or $\mu\ mg \geq mr\,\omega^2$

$$\frac{\mu\,g}{\omega^2} \geq r \quad \text{or} \quad r \leq \frac{\mu\,g}{\omega^2}$$

$$i.e. \quad r \leq \frac{0{\cdot}2\times 9{\cdot}8}{(4)^2}$$

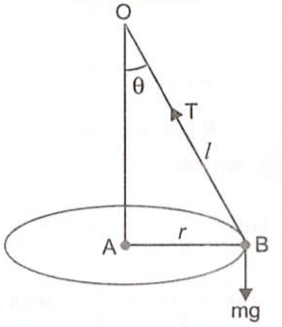
$$r \leq 0{\cdot}123\ \text{m} \qquad \text{or} \qquad r \leq 12{\cdot}3\ \text{cm}$$

∴ The block placed at 10 cm will revolve with the table.

Example 53 **A sphere of mass 200 g is attached to an inextensible string of length 130 cm whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 50 cm. Calculate the periodic time of this conical pendulum and the tension in the string.**

Solution. Refer to Fig. 5.69,

FIGURE 5.69



Here, $m = 200$ g ; $OB = l = 130$ cm ; $r = AB = 50$ cm, $t = ?$ $T = ?$

$$OA = \sqrt{l^2 - r^2} = \sqrt{(130)^2 - (50)^2} = 120 \text{ cm}.$$

As is clear from Fig. 5.69,

$$T\cos\theta = mg \qquad \text{...(i)}$$

$$T\sin\theta = \frac{mv^2}{r} = m\,r\,\omega^2 \qquad \text{...(ii)}$$

Dividing, we get

$$\tan\theta = \frac{m\,r\,\omega^2}{mg} = \frac{r\,\omega^2}{g}$$

$$\omega = \sqrt{\frac{g\tan\theta}{r}} = \frac{2\pi}{t}$$

$$t = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\times\frac{22}{7}\sqrt{\frac{0{\cdot}5}{9{\cdot}8\times 5/12}} = \mathbf{2{\cdot}19\ s}$$

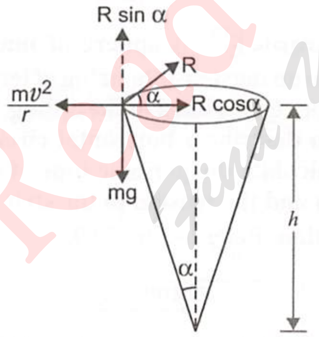
From (*i*), $T = \dfrac{mg}{\cos\theta} = \dfrac{200}{1000}\times\dfrac{9{\cdot}8}{12/13} = \mathbf{2{\cdot}12\ N}$

Example 54 **A particle describes a horizontal circle on the smooth surface of an inverted cone. The height of the plane of the circle above the vertex is 9·8 cm. Find the speed of the particle. Take $g = 9{\cdot}8$ m/s^2.**

Solution. Here, $h = 9{\cdot}8$ cm, $v = ?$

As is clear from Fig. 5.70,

FIGURE 5.70



$$R\cos\alpha = \frac{mv^2}{r}$$

$$R\sin\alpha = mg$$

Dividing, we get,

$$\tan\alpha = \frac{rg}{v^2} \quad\text{or}\quad \frac{r}{h} = \frac{rg}{v^2}$$

$$v = \sqrt{gh} = \sqrt{9{\cdot}8\times\frac{9{\cdot}8}{100}} = \mathbf{0{\cdot}98\ m/s}$$

Example 55 **A disc revolves with a speed of $33\frac{1}{3}$ rev./min. and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0·15, which of the two coins will revolve with the record ?**

Solution. The coin revolves with the record in the case when the force of friction is enough to provide the necessary centripetal force. If this force is not sufficient to provide centripetal force, the coin slips on the record.

Now, the frictional force is μR where R is the normal reaction, and $R = mg$

Hence force of friction $= \mu\, mg$ and centripetal force required is $\dfrac{mv^2}{r}$ or $m\,r\,\omega^2$

μ, ω are same for both the coins and we have different values of r for the two coins.

So to prevent slipping i.e. causing coins to rotate

$$\mu\, mg \geq m\,r\,\omega^2 \quad\text{or}\quad \mu\, g \geq r\,\omega^2 \qquad \text{...(i)}$$

For 1st coin

$$r = 4 \text{ cm} = \frac{4}{100}\text{ m}$$

$$n = 33\frac{1}{3}\text{ rev./min.} = \frac{100}{3\times 60}\text{ rev./sec}$$

$$\omega = 2\,\pi\, n = 2\,\pi\times\frac{100}{180} = 3{\cdot}49 \text{ s}^{-1}$$

$$\therefore\quad r\,\omega^2 = \frac{4}{100}\times(3{\cdot}49)^2 = 0{\cdot}49 \text{ ms}^{-2}$$

and $\mu\, g = 0{\cdot}15\times 10 = 1{\cdot}5$ ms^{-2}

As $\mu\, g > r\,\omega^2$, therefore **this coin will revolve with the record.**

For 2nd coin

$$r = 14 \text{ cm} = \frac{14}{100}\text{ m} \;;\; \omega = 3{\cdot}49 \text{ s}^{-1}$$

$$r\,\omega^2 = \frac{14}{100}\times(3{\cdot}49)^2 = 1{\cdot}705 \text{ ms}^{-2}$$

and $\mu\, g = 1{\cdot}5$ ms^{-2}

Here, $\mu\, g \geq r\,\omega^2$ is not satisfied, so **this coin will not revolve with the record.**

Note that we have nothing to do with the radius of the record (= 15 cm).

TYPE X. BENDING OF A CYCLIST AND BANKING OF ROADS

Formulae used. 1. $\tan\theta = \dfrac{v^2}{rg}$

2. $\tan\theta = \dfrac{h}{\sqrt{b^2 - h^2}} = \dfrac{v^2}{rg}$

where b is breadth of road ; h is height of outer edge above the inner edge and θ is angle of bending from the vertical in case of cyclist or it is the angle of banking in case of banked roads.

In these formulae, we ignore the role of friction.

3. When friction is also taken into account, the maximum velocity of vehicle over a banked road is

$$v_{\max} = \left[\frac{rg\,(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$$

If $\mu_s = 0$, $v_{\max} = [rg \tan\theta]^{1/2}$ or $\tan\theta = \dfrac{v^2_{\max}}{rg}$

This speed will cause no wear and tear of the tyres.

Units used. Angle θ with vertical is in degrees, velocity v in ms^{-1}, radius r in m, coefficient of friction, μ_s has no units.

Standard value. g = acc. due to gravity = $9{\cdot}8\ \text{ms}^{-2}$

Example 56 **A car is speeding on a horizontal road curving round with a radius 60 m. The coefficient of friction between the wheels and the road is 0·5. The height of centre of gravity of the car from the road level is 0·3 m and the distance between the wheels is 0·8 m. Calculate the maximum safe velocity for negotiating the curve. Will the car skid or topple if this velocity is exceeded ?**

Solution. Here, $r = 60$ m, $\mu = 0{\cdot}5$, $h = 0{\cdot}3$ m, distance between the wheels, $2x = 0{\cdot}8$ m,

$$x = 0{\cdot}4 \text{ m}$$

For no skidding, $\tan\theta = \mu = \dfrac{v^2}{rg}$

$$v = \sqrt{\mu\, rg} = \sqrt{0{\cdot}5 \times 60 \times 9{\cdot}8} = \mathbf{17{\cdot}15\ m/s}$$

For toppling, $\dfrac{mv^2}{r} \times h = mg\,x$

$$v = \sqrt{\frac{grx}{h}} = \sqrt{\frac{9{\cdot}8 \times 60 \times 0{\cdot}4}{0{\cdot}3}} = 28 \text{ m/s}$$

Hence the maximum safe velocity for negotiating the curve is **17·15 m/s.** Beyond this speed, skidding starts until **the car topples at v = 28 m/s**

Example 57 **A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction between the tyres and the road is 0·1. Will the cyclist slip while taking the turn ?**

(NCERT Solved Example)

Solution. Here, $v = 18$ km/h

$$= \frac{18 \times 1000}{60 \times 60} = 5\ \text{ms}^{-1}$$

$$r = 3 \text{ m}, \ \mu_s = 0{\cdot}1$$

On an unbanked road, frictional force alone can provide the centripetal force. Therefore, *condition for the cyclist not to slip* is that

$$\frac{mv^2}{r} \le F_s\ (= \mu_s R = \mu_s mg)$$

$$v^2 \le \mu_s rg$$

As $v^2 = 5^2 = 25$, and $\mu_s rg = 0{\cdot}1 \times 3 \times 10 = 3$

$\therefore$ the condition is not satisfied. Hence **the cyclist will slip.**

Example 58 **A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race car and the road is 0·2, what is the (*a*) optimum speed of the race car to avoid wear and tear on its tyres, and (*b*) maximum permissible speed to avoid slipping ?**

(NCERT Solved Example)

Solution. Here, $r = 300$ m, $\theta = 15°$, $\mu_s = 0{\cdot}2$

On a banked road, horizontal component of the normal reaction and the frictional force combine to provide centripetal force to keep the car moving on a circular curved track without slipping.

At the optimum speed, component of normal reaction is enough to provide the necessary centripetal force and the frictional force is not needed. Therefore, there is no wear and tear on its tyres. The optimum speed is $v_0 = (rg \tan\theta)^{1/2} = (300 \times 9{\cdot}8 \tan 15°)^{1/2}$

$$v_0 = (300 \times 9{\cdot}8 \times 0{\cdot}27)^{1/2} = \mathbf{28{\cdot}2\ m/s}$$

The maximum permissible speed to avoid slipping is given by

$$v_{\max} = \left[\frac{rg\,(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$$

$$= \left[\frac{300 \times 9{\cdot}8\,(0{\cdot}2 + 0{\cdot}27)}{(1 - 0{\cdot}2 \times 0{\cdot}27)}\right]^{1/2}$$

$$= \mathbf{38{\cdot}2\ ms^{-1}}$$

Example 59 **A circular race track of radius 400 m is banked at an angle of 10°. If the coefficient of friction between the wheels of a race car and the road is 0·2, what is the**

(i) optimum speed of the race car to avoid wear and tear on its tyres.

(ii) maximum permissible speed to avoid slipping ?

Solution. Here, $r = 400$ m, $\theta = 10°$, $\mu = 0{\cdot}2$

(*i*) Optimum speed of the car to avoid wear and tear,

$$v = \sqrt{rg \tan\theta} = \sqrt{400 \times 9{\cdot}8 \tan 10°}$$

$$= \sqrt{400 \times 9{\cdot}8 \times 0{\cdot}1763} = \sqrt{691{\cdot}1}$$

$$= \mathbf{26{\cdot}29\ m/s}$$

(*ii*) Maximum permissible speed is

$$v_{\max} = \sqrt{\frac{rg\ (\mu + \tan\theta)}{1 - \mu\tan\theta}}$$

$$= \sqrt{\frac{400 \times 9{\cdot}8\ (0{\cdot}2 + 0{\cdot}1763)}{1 - 0{\cdot}2 \times 0{\cdot}1763}}$$

$$= \mathbf{39{\cdot}10\ m/s}$$

TYPE XI. MOTION IN A VERTICAL CIRCLE

Formulae used. **1.** At any position of angular displacement θ along a vertical circle

$$T = \frac{mv^2}{r} + mg\cos\theta$$

2. At the lowest point of vertical circle, $\theta = 0°$,

$$T_L = \frac{mv_L^2}{r} + mg$$

3. At the highest point of the vertical circle,

$$\theta = 180°,\ T_H = \frac{mv_H^2}{r} - mg$$

4. Minimum velocity at the highest point for looping the loop, $v_H = \sqrt{gr}$

5. Minimum velocity at the lowest point for looping the loop, $v_L = \sqrt{5gr}$

6. When the string is horizontal, $\theta = 90°$, minimum velocity, $v = \sqrt{3gr}$

7. The body will oscillate about the lowest point of the vertical circle, when $v < \sqrt{2gr}$

8. For the body to leave the circle,

$$\sqrt{2gr} < v_L < \sqrt{5gr}$$

9. Difference in tensions $T_L - T_H = 6$ mg

10. Height through which a body should fall for looping the vertical loop, $h = 5\ r/2$

Units used

Velocities v_L, v_H, v in ms^{-1}, height h and radius r in metre, tension in the string, T, T_L, T_H in newton, acceleration due to gravity, $g = 9{\cdot}8\ ms^{-2}$ and mass m in kg.

Example 60 **A small stone of mass 200 g is tied to one end of a string of length 80 cm. Holding the other end in hand, the stone is whirled into a vertical circle. What is the minimum speed that needs to be imparted at the lowest point of the circular path, so that the stone is just able to complete the vertical circle ? What would be the tension at the lowest point of circular path ? (Take $g = 10$ m/s^2)** (NCERT Solved Example)

Solution. Here, $m = 200$ g $= 0{\cdot}2$ kg ,

$l = r = 80$ cm $= 0{\cdot}8$ m, $v_L = ?$, $T_L = ?$

From $v_L = \sqrt{5gr} = \sqrt{5 \times 10 \times 0{\cdot}8} = \mathbf{6{\cdot}32\ ms^{-1}}$

$T_L = 6$ mg $= 6 \times 0{\cdot}2 \times 10 =$ **12 N**

Example 61 **A massless string of length 1·2 m has a breaking strength of 2 kg wt. A stone of mass 0·4 kg tied to one end of the string is made to move in a vertical circle by holding the other end in hand. Can the particle describe the vertical circle ? Take $g = 10$ ms^{-2}.**

(NCERT Solved Example)

Solution. Here, $l = r = 1{\cdot}2$ m , $m = 0{\cdot}4$ kg ,

$g = 10$ ms^{-2}

Breaking strength, $F = 2$ kg wt $= 20$ N

Now, $T_{\max} = T_L = 6$ mg

$= 6 \times 0{\cdot}4 \times 10 = 24$ N,

which is more than breaking strength of string.

Therefore, **the string will break and the particle cannot describe the vertical circle.**

Example 62 **A small stone of mass 0·2 kg tied to a massless, inextensible string, is rotated in a vertical circle of radius 2 m. If the particle is just able to complete the vertical circle, what is its speed at the highest point of the circular path ?**

How would the speed get affected if the mass of the stone is increased by 50% ? Take $g = 10\ m/s^2$.
(NCERT Solved Example)

Solution. Here, $m = 0{\cdot}2$ kg, $r = 2$ m

When the particle is just able to complete the vertical circle, $v_L = \sqrt{5\,g\,r}$

and $\quad v_H = \sqrt{g\,r} = \sqrt{10\times 2} = \mathbf{4{\cdot}47\,ms^{-1}}$

As v_H does not depend upon mass, m of the stone, therefore, **value of v_H will not be affected by any change in mass of the stone.**

Example 63 **A particle of mass 150 g is attached to one end of a massless inextensible string. It is made to describe a vertical circle of radius 1 m. When the string is making an angle of 48·2° with the vertical, its instantaneous speed is 2 m/s. What is the tension in the string in this position ? Would this particle be able to complete its circular path ?** (NCERT Solved Example)

Solution. Here, $m = 150\text{ g} = 0{\cdot}150$ kg , $r = 1$ m, $\theta = 48{\cdot}2°$, $v = 2$ m/s, $T = ?$

From $\quad T = \dfrac{mv^2}{r} + mg\cos\theta$

$$T = \frac{0{\cdot}15\times 2^2}{1} + 0{\cdot}15\times 10\times \cos 48{\cdot}2°$$

$$= 0{\cdot}6 + 1{\cdot}5\times 0{\cdot}6665 = \mathbf{1{\cdot}6\ N}$$

To be able to complete the vertical circle,

$$v_H = \sqrt{gr} = \sqrt{10\times 1} = \mathbf{3{\cdot}16\ m/s}$$

As instantaneous velocity is 2 m/s, it cannot increase (to 3·16 m/s) on reaching the highest point as gravity is opposing the motion. Therefore, **the particle cannot complete its circular path.**

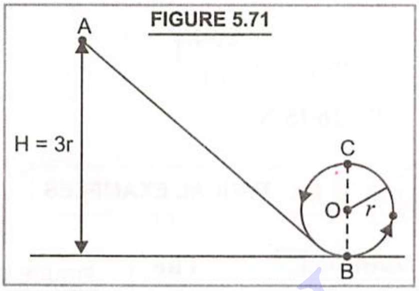
Example 64 **Fig. 5.71 shows a smooth 'looping the loop' track. A particle of mass m is released from point A, as shown. If $H = 3\,r$, would the particle loop the loop ? What is the force on the circular track when the particle is at point (*i*) B (*ii*) C ?** (NCERT Solved Example)

Solution. Here, $H = 3\,r$

When the particle of mass m is released from A, velocity acquired by it on reaching B is

$$v_B = \sqrt{2\,g\,H} = \sqrt{2\,g\times 3r} = \sqrt{6\,g\,r}$$

FIGURE 5.71



which is greater than $\sqrt{5\,g\,r}$, the minimum velocity required at B for looping the loop. Hence, **the particle will loop the loop.**

Force exerted by the circular track on the particle at B $\quad = mg + \dfrac{mv_B^2}{r}$

$$N_1 = mg + \frac{m\times 6\,gr}{r} = \mathbf{7\,mg}$$

Force exerted by circular track on the particle at C

$$N_2 = \frac{m\,v_c^2}{r} - mg,$$

where $\quad v_C^2 = v_B^2 - 4\,gr = 6\,gr - 4\,gr = 2\,gr$

$$\therefore \quad N_2 = \frac{m\times 2\,g\,r}{r} - mg = \mathbf{mg}$$

N_1 , N_2 are the forces exerted on the circular track by the particle at B, C respectively.

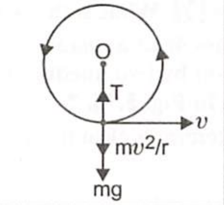
Example 65 **A child revolves a stone of mass 0·5 kg tied to the end of a string of length 40 cm in a vertical circle. The speed of the stone at the lowest point of the circle is 3 m/s. Calculate tension in the string at this point.**

Solution. Here, $m = 0{\cdot}5$ kg ; $r = 40$ cm $= 0{\cdot}4$ m

$v = 3$ m/s, $T = ?$

At the lowest point of vertical circle, Fig. 5.72.

FIGURE 5.72



$$T = mg + \frac{mv^2}{r} = m\left(g + \frac{v^2}{r}\right)$$

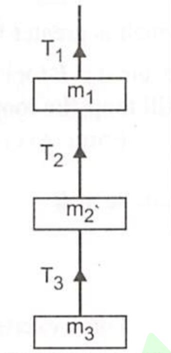
$$= 0{\cdot}5\left(9{\cdot}8 + \frac{3\times 3}{0{\cdot}4}\right)$$

$$T = 16{\cdot}15\ \text{N}$$

TYPE XII. TYPICAL EXAMPLES

Example 66 **The masses m_1, m_2 and m_3 of the three bodies shown in Fig. 5.73 are 5, 2 and 3 kg respectively. Calculate the values of tension T_1, T_2 and T_3 when (i) the whole system is going upward with an acceleration of 2 m/s^2 (ii) the whole system is stationary ($g = 9{\cdot}8$ m/s^2)**

FIGURE 5.73



Solution. All the three bodies are moving together with an upward acc. of 2 m/s^2. The force pulling the system upwards is T_1 and downward pull of gravity is $(m_1 + m_2 + m_3)\, g$

(*i*) According to Newton's 2nd law of motion,

$$T_1 - (m_1 + m_2 + m_3)\, g = (m_1 + m_2 + m_3)\, a$$

or $T_1 = (m_1 + m_2 + m_3)(a + g)$

$$= (5 + 2 + 3)(2 + 9{\cdot}8) = \mathbf{118\ N}$$

Similarly, for motion of m_2 and m_3, we write

$$T_2 = (m_2 + m_3)(a + g) = (2 + 3)(2 + 9{\cdot}8) = \mathbf{59\ N}$$

and, for motion of m_3,

$$T_3 = m_3 (a + g) = 3(2 + 9{\cdot}8) = \mathbf{35{\cdot}4\ N}$$

(*ii*) When the whole system is stationary, $a = 0$

Using the same equations as above with $a = 0$, we get

$$T_1 = (m_1 + m_2 + m_3)\, g = 10 \times 9{\cdot}8 = \mathbf{98\ N}$$

$$T_2 = (m_2 + m_3)\, g = 5 \times 9{\cdot}8 = \mathbf{49\ N}$$

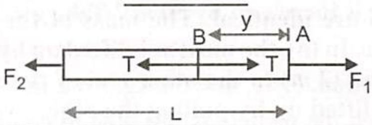
$$T_3 = m_3 \times g = 3 \times 9{\cdot}8 = \mathbf{29{\cdot}4\ N}$$

Example 67 **What is the tension in a rod of length L and mass M at a distance y from F_1, when the rod is acted on by two unequal forces F_1 and F_2 ($< F_1$) as shown in Fig. 5.74. ?**

Solution. Here, as is clear from Fig. 5.74, net force on the rod

$$f = (F_1 - F_2)$$

FIGURE 5.74



$\therefore$ Acceleration of rod, along F_1

$$= a = \frac{f}{M} = \frac{(F_1 - F_2)}{M} \qquad \text{...(i)}$$

Mass of part AB of the rod $= \left(\frac{M}{L}\right) y$

Let T be the tension in the rod at B.

Writing the equation of motion of part AB of the rod :

$$F_1 - T = ma = \frac{M}{L}\, y . \left(\frac{F_1 - F_2}{M}\right)$$

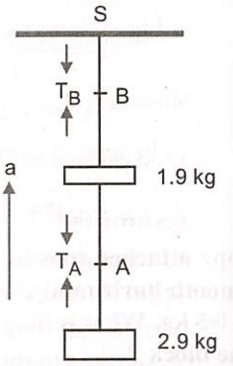
$$F_1 - T = (F_1 - F_2)\frac{y}{L}$$

or $T = F_1 - (F_1 - F_2)\dfrac{y}{L}$

$$\boldsymbol{T = F_1 (1 - y/L) + F_2 (y/L)}$$

Example 68 **Two blocks of masses 2·9 kg and 1·9 kg are suspended from a rigid support S by two inextensible wires each of length 1 m, Fig. 5.75. The upper wire has negligible mass and lower wire has uniform mass of 0·2 kg/m. The whole system of blocks, wires and support have an upward acceleration of 0·2 m/s^2. Acceleration due to gravity is 9·8 m/s^2. Find the tension at (a) the mid. pt. of lower wire (b) the mid point of upper wire.**

FIGURE 5.75



Solution. As the system is accelerated upwards, therefore, tension at any point is given by

$$T = m\,(g + a)$$

For the point A,

$$m = 2{\cdot}9 + \frac{0{\cdot}2}{2} = 3{\cdot}0$$

$$\therefore \quad T_A = m\,(g + a) = 3\,(9{\cdot}8 + 0{\cdot}2) = \mathbf{30\ N}$$

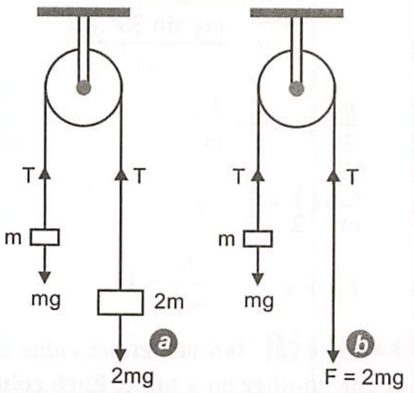
For the point B

$$m = 2{\cdot}9 + 0{\cdot}2 + 1{\cdot}9 + 0 = 5\ \text{kg}$$

$$\therefore \quad T_B = m\,(g + a) = 5\,(9{\cdot}8 + 0{\cdot}2) = \mathbf{50\ N}$$

Example 69 **The pulley arrangements of Fig. 5.76 are identical. The mass of the rope is negligible. In (a), the mass *m* is lifted up by attaching a mass (2 *m*) to the other end of the rope. In (b), *m* is lifted up by pulling the other end of the rope with a constant downward force $F = 2\,mg$. In which case, the acceleration of *m* is more ?**

FIGURE 5.76



Solution. In Fig 5.76(*a*), pulling force,

$$F = (2\,mg - T) = 2\,mg - mg = mg$$

total mass moved, $= m + 2\,m = 3\,m$

$$\therefore \text{ Acceleration, } a = \frac{\text{force}}{\text{mass}} = \frac{mg}{3m} = \frac{g}{3}$$

In Fig. 5.76(*b*) the pulling force,

$$F = (2\,mg - T) = 2\,mg - mg = mg$$

Mass moved $= m$

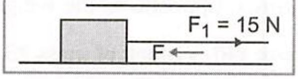
$$\therefore \text{ Acceleration, } a = \frac{F}{m} = \frac{mg}{m} = g$$

Example 70 **A pull of 15 N is applied on a rope attached to a block of mass 7 kg lying on a smooth horizontal surface. The mass of the rope is 0·5 kg. What is the force exerted on the rope by the block ?**

Solution. Here, $F_1 = 15$ N,

$m_1 = 7$ kg, $m_2 = 0{\cdot}5$ kg, $F = ?$

FIGURE 5.77



In Fig. 5.77,

Acceleration produced,

$$a = \frac{F_1}{m_1 + m_2} = \frac{15}{(7 + 0{\cdot}5)} = 2\text{ m/s}^2$$

Force exerted by rope on the black,

$$F = m_1\,a = 7 \times 2 = \mathbf{14\ N}$$

As a reaction, force exerted on the rope by the block = **14 N**

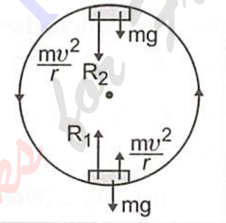
Example 71 **An aeroplane flying in the sky dives with a speed of 360 km/h in a vertical circle of radius 200 m. The weight of pilot sitting in it is 75 kg. Calculate the force with which the pilot presses his seat when the aeroplane is (*i*) at the lowest position and (*ii*) at the highest position. Take $g = 10\text{ m/s}^2$.**

Solution. In Fig. 5.78,

R_1 is normal reaction at the lowest position and R_2 is normal reaction at the highest position of vertical circle : $r = 200$ m ; $m = 75$ kg,

$v = 360$ km/h $= 100$ m/s

FIGURE 5.78



(*i*) *At the lowest position*,

$$R_1 - mg = \text{centripetal force } = \frac{mv^2}{r}$$

$$R_1 = m\left[\frac{v^2}{r} + g\right] = 75\left[\frac{100 \times 100}{200} + 10\right] = 4500\text{ N}$$

$$= \mathbf{450\ kg\ wt}$$

(*ii*) *At the highest position*, $R_2 + mg = \dfrac{mv^2}{r}$

$$R_2 = m\left(\frac{v^2}{r} - g\right) = 75\left(\frac{100 \times 100}{200} - 10\right)$$

$$= 3000\text{ N} = \mathbf{300\ kg\ wt}$$

Example 72 **A bullet of mass 0·01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0·25. The bullet remains embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block ?**

Solution. Here, mass of bullet, $m_1 = 0{\cdot}01$ kg,

mass of wooden block, $m_2 = 4$ kg

Coeff. of kinetic friction, $\mu = 0{\cdot}25$;

distance moved by the combination, $s = 20$ m

Let velocity of bullet be u_1, initial velocity of block, $u_2 = 0$

If v is the velocity with which the combination moves, then according to the principle of conservation of linear momentum,

$$(m_1 + m_2)\, v = m_1 u_1 + m_2 u_2$$

$$(0{\cdot}01 + 4)\, v = 0{\cdot}01\, u_1 + 4 \times 0$$

$$v = \frac{0{\cdot}01\, u_1}{4{\cdot}01} = \frac{u_1}{401} \quad \text{...}(i)$$

Force of dynamic friction on the combination

$$F = \mu R = \mu\,(m_1 + m_2)\, g$$

$$= 0{\cdot}25\,(0{\cdot}01 + 4) \times 9{\cdot}8 \text{ N}$$

$\therefore$ Retardation produced,

$$a = \frac{F}{m_1 + m_2} = \frac{0{\cdot}25 \times 4{\cdot}01 \times 9{\cdot}8}{4{\cdot}01} = 2{\cdot}45 \text{ ms}^{-2}$$

Using for the combination, the relation

$$v^2 - u^2 = 2\,as \quad ; \quad 0 - v^2 = 2\,(-2{\cdot}45) \times 20$$

$$v^2 = 4{\cdot}9 \times 20 = 98 \quad \text{or} \quad v = \sqrt{98}$$

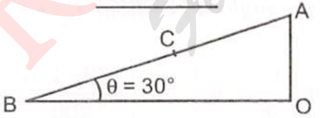
From (i), $u_1 = 401\, v$

or $\quad u_1 = 401\sqrt{98} = \mathbf{3969{\cdot}7\ ms^{-1}}$

Example 73 **A body starts rolling down an inclined plane, the top half of which is perfectly smooth and lower half is rough. Find the ratio of the force of friction and weight of the body, if the body is brought to rest just when it reaches the bottom, the angle of the plane being 30°.**

Solution. In Fig. 5.79, AB is an inclined plane of length $2l$ with centre at C.

FIGURE 5.79



$\angle ABO = \theta = 30^\circ$

The top half AC of the plane is smooth and the lower half CB is rough. Let F be the force of friction in this part.

For the top half of inclined plane,

$$u = 0\,,\ a = g \sin 30^\circ = \frac{g}{2}\,,\ s = l, v = \ ?$$

As $v^2 - u^2 = 2\,a\,s$

$$\therefore \quad v^2 - 0 = 2\left(\frac{g}{2}\right) l \quad \text{or} \quad v = \sqrt{g\,l}$$

For the bottom half of inclined plane,

$$u = \sqrt{g\,l}\,,\ v = 0,\ s = l,\ a = \ ?$$

As $\ v^2 - u^2 = 2\,a\,s$

$$\therefore \quad 0 - g\,l = 2\,a\,l,\ a = -\frac{g}{2}$$

In this part, acc. $= \dfrac{\text{net force}}{\text{mass}}$

$$a = \frac{\text{mg} \sin 30^\circ - F}{m}$$

$$-\frac{g}{2} = g \times \frac{1}{2} - \frac{F}{m}$$

or $\quad \dfrac{F}{m} = \dfrac{g}{2} + \dfrac{g}{2} = g$

or $\quad F = mg \quad \therefore \quad \mathbf{\dfrac{F}{mg} = 1}$

Example 74 **Ten one rupee coins are put on top of one another on a table. Each coin has a mass *m* kg. Give the magnitude and direction of (a) the force on the 7th coin (counted from the bottom) due to all coins above it. (b) the force on the 7th coin by the eighth coin and (c) the reaction of the sixth coin on the seventh coin.**

Solution. *(a)* The force on 7th coin is due to weight of the three coins lying above it. Therefore,

$$F = (3\text{ m})\text{ kgf} = \mathbf{(3\ mg)\ N}$$

where g is acceleration due to gravity. This force acts vertically downwards.

(b) The eighth coin is already under the weight of two coins above it and it has its own weight too. Hence force on 7th coin due to 8th coin is sum of the two forces *i.e.* $F = 2\text{ m} + \text{m} = (3\text{ m})$ kg f

$$= \mathbf{(3\ mg)\ N}$$

The force acts vertically downwards.

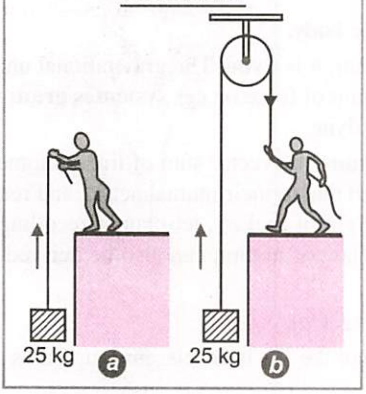
(c) The sixth coin is under the weight of four coins above it.

Reaction, $R = -F = -4$ m (kgf) $= \mathbf{-\,(4\ mg)\ N}$

Minus sign indicates that the reaction acts vertically *upwards*, opposite to the weight.

Example 75 **A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. 5.80. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding ?**

FIGURE 5.80



Solution. Here, mass of block, $m = 25$ kg

Mass of man, $M = 50$ kg

Force applied to lift the block

$F = mg = 25 \times 9{\cdot}8 = 245$ N

Weight of man $W = Mg = 50 \times 9{\cdot}8 = 490$ N.

(*a*) When block is raised by man as shown in Fig. 5.80(a), force is applied by the man in the *upward direction*. This *increases the apparent weight* of the man. Hence action on the floor.

$$W' = W + F = 490 + 245 = 735 \text{ N}$$

(*b*) When block is raised by man as shown in Fig. 5.80(b), force is applied by the man in the downward direction. This *decreases the apparent weight* of the man. Hence, action on the floor in this case would be $W' = W - F = 490 - 245 = 245$ N.

As the floor yields to a normal force of 700 N, **the mode (*b*) has to be adopted by the man to lift the block.**

Example 76 **A monkey of mass 40 kg climbs on a rope which can withstand a maximum tension of 600 N. In which of the following cases will the rope break ? The monkey (a) climbs up with an acceleration of 6 ms^{-2} (b) climbs down with an acceleration of 4 ms^{-2} (c) climbs up with a uniform speed of 5 ms^{-1} (d) falls down the rope nearly freely under gravity. Ignore the mass of the rope.**

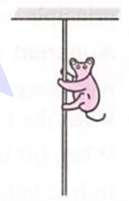
Solution. Here, mass of monkey, $m = 40$ kg

Maximum tension the rope can withstand, $T = 600$ N.

In each case, actual tension in the rope will be equal to apparent weight of monkey (R), Fig. 5.81.

The rope will break when R exceeds T.

FIGURE 5.81



(*a*) When monkey *climbs up* with $a = 6$ ms^{-2},

$$R = m(g + a) = 40(10 + 6) = 640 \text{ N}$$

(which is greater than T).

Hence the rope will break.

(*b*) When monkey *climbs down* with $a = 4$ ms^{-2},

$$R = m(g - a) = 40(10 - 4) = 240 \text{ N},$$

which is less than T

$\therefore$ The rope will not break.

(*c*) When monkey climbs up with a uniform speed $v = 5$ ms^{-1}, its acceleration $a = 0$

$\therefore$ $R = mg = 40 \times 10 = 400$ N,

which is less than T

$\therefore$ The rope will not break.

(*d*) When monkey *falls down* the rope nearly freely under gravity, $a = g$

$\therefore$ $R = m(g - a) = m(g - g) =$ Zero

Hence the rope will not break.

REVISION AT A GLANCE

1. **Newton's three laws of motion** form the basis of mechanics. According to **1st law**, a body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state. This law defines force and is also called **law of inertia.**

   According to **2nd law,** the rate of change of linear momentum of a body is directly proportional to the external force applied on the body, and this change takes place in the direction of the applied force. This law gives us a measure of force.

   According to **third law**, to every action, there is always an equal and opposite reaction. This law gives us the nature of force.
2. **Inertia** is inability of a body to change by itself, its state of rest or its state of uniform motion along a straight line. Inertia is obviously of three types : (*i*) Inertia of rest (*ii*) Inertia of motion, (*iii*) Inertia of direction.

3. **From Newton's 2nd law**, we obtain $\vec{F} = m\,\vec{a}$

   i.e. external force is the product of mass and acceleration of the body.

4. The absolute unit of force on SI is **newton** (N) and on cgs system, it is **dyne**. The gravitational unit of force on SI is **kilogram weight** or kilogram force. The gravitational unit of force on cgs system is **gram weight** or gram force. $1\text{ N} = 10^5$ dyne, $1\ g$ wt. $= 980$ dyne.

5. According to the **principle of conservation of linear momentum,** the vector sum of linear momenta of all the bodies in an isolated system is conserved, and is not affected due to their mutual action and reaction. An isolated system is that on which no external force is acting. Flight of rockets, jet planes, recoiling of a gun etc. are explained on the basis of this principle. Newton's 3rd law of motion can also be derived from this principle and vice-versa.

6. **Apparent weight** of a man in an elevator is given by $W' = m\,(g \pm a)$,

   where mg is real weight of the man. Acceleration is $(+\,a)$, when the lift is accelerating upwards and $(-\,a)$, when the lift is accelerating downwards.

   When lift is *moving uniformly* (upwards/downwards), $a = 0$, $W' = m\,g$ = real weight.

   In free fall, $a = g \quad \therefore \quad W' = m\,(g - g) = 0$

   i.e. apparent weight becomes zero.

7. When two bodies of masses m_1 and m_2 are tied at the ends of an inextensible string passing over a light frictionless pulley, **acceleration** of the system is given by

$$a = \frac{(m_1 - m_2)\,g}{(m_1 + m_2)}$$

   Tension in the string is $T = \dfrac{2\,m_1\,m_2\,g}{(m_1 + m_2)}$

8. A rocket is an example of a system with varying mass. **Rocket propulsion** has been explained in terms of the principle of conservation of linear momentum. The velocity (v) of rocket at any time t is given by

$$v = u \log_e \left(\frac{m_0}{m} \right)$$

   where u is velocity of exhaust gases relative to the rocket ; m_0 is initial mass of the rocket and m is mass of the rocket at time t.

   The thrust on the rocket is given by $F = -u \left(\dfrac{dm}{dt} \right)$

   where dm/dt is the rate of combustion of fuel at the instant t.

9. **Impulse** $\vec{I} = \vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1$

   where t is the time for which average force acts $(\vec{p}_2 - \vec{p}_1)$ is change in linear momentum of the body.

10. The forces which are acting at a point are called **concurrent forces.** They are said to be in equilibrium, when their resultant is zero.

11. **Friction** is the opposing force that comes into play when one body is actually moving over the surface of another body or one body is trying to move over the surface of the other. Two causes of friction are : roughness of surfaces in contact; Force of adhesion between the molecules of the surfaces in contact.

12. Limiting friction is the maximum value of static friction. Dynamic/Kinetic friction is somewhat less than the force of limiting friction.

13. **Static friction** is a self adjusting force.
14. **Rolling friction** is less than sliding friction.
15. **Laws of limiting friction are :**

    1st Law : $F \propto R$, where R is normal reaction.

    2nd Law : Direction of F is opposite to the intended direction of motion.

    3rd Law : F does not depend upon actual area of contact.

    4th Law : F depends upon nature of material and nature of polish of the surfaces in contact.
16. **Coefficient of friction** $\mu = F/R$

    Angle of friction (θ) is the angle which resultant of F and R makes with the direction of R. The relation between μ and θ is $\mu = \tan\theta$
17. **Angle of Repose** (α) is the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down. $\mu = \tan\alpha$
18. **Acceleration** of a body down a rough inclined plane $a = g\,(\sin\theta - \mu\cos\theta)$
19. **Work done** in moving a body over a rough horizontal surface $W = \mu R \times s = \mu\, m\, g \times s$
20. **Work done** in moving a body over a rough inclined plane $W = m\,g\,(\sin\theta + \mu\cos\theta) \times s$
21. Friction is a **necessary evil. Some of the methods of reducing friction** are polishing, lubrication ; streamlining the shape etc.
22. **Centripetal force** is the force required to move a body uniformly in a circle. Magnitude of this force is

    $F = \dfrac{mv^2}{r} = m\,r\,\omega^2$. It acts along the radius and towards the centre of the circle.
23. **Centrifugal force** is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path. Centrifugal force can be treated as the *reaction* of centripetal force. The magnitude of centrifugal force is same as that of centripetal force. The direction of centrifugal force is along the radius and *away* from the centre of the circle.
24. **While rounding a level curved road,** the necessary centripetal force is provided by the force of friction between the tyres and the road. The maximum velocity with which a vehicle can go round a level curve without skidding is $v = \sqrt{\mu\, r\, g}$
25. To avoid dependence on friction, **curved roads are usually banked** *i.e.* outer edge of the curved road is raised suitably above the inner edge. If θ is the angle of banking, then

    $$R\cos\theta = mg \qquad \text{and} \qquad R\sin\theta = \frac{mv^2}{r} \qquad \therefore \qquad \tan\theta = \frac{v^2}{rg}$$
26. When **a cyclist takes a turn**, he also requires centripetal force. For this, he bends a little inwards from his vertical position, while turning. Angle θ of bending from vertical position is given by

    $$\tan\theta = \frac{v^2}{rg}$$
27. While rounding a banked curved road, maximum permissible speed is given by

    $$v_{\max} = \left[\frac{rg\,(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}\right]^{1/2}$$

    When frictional force is ignored, the optimum speed is

    $$v_0 = (\,rg\tan\theta)^{1/2}$$

28. **Motion along a vertical circle** is a non uniform circular motion. Tension in the strong at any position is

$$T = \frac{mv^2}{r} + mg\cos\theta,$$

where θ is the angle with vertical line through lowest point of the circle.

(*i*) For looping the vertical loop, velocity of projection at lowest point L is

$$v_L \geq \sqrt{5\,g\,r}$$

(*ii*) The value of velocity at the highest point H is

$$v_H \geq \sqrt{g\,r}$$

(*iii*) Difference in tension in the string at lowest point and highest point of vertical circle is $T_L - T_H = 6$ mg

(*iv*) For oscillation over the arc of vertical circle $0 < v_L \leq \sqrt{2\,g\,r}$

(*v*) For leaving the vertical circle somewhere between $90° < \theta < 180°$, $\sqrt{2\,g\,r} < v_L < \sqrt{5\,g\,r}$

(*vi*) The minimum height h through which a motor cyclist has to descend to loop a vertical loop of radius r is

$$h = \frac{5}{2}r$$

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. For a given change in linear momentum, when time of impact increases, force

(a) decreases (b) increases

(c) remains same (d) none of the above

2. When the surfaces in contact are made too smooth by polishing, force of friction

(a) decreases (b) increases

(c) becomes zero (d) becomes infinite

3. Angle of repose for a rough inclined plane is 60°. The coefficient of friction is

(a) $\sqrt{3}$ (b) $1/\sqrt{3}$

(c) 1 (d) Zero

4. In moving a body of mass m once up and down a smooth incline of inclination θ, total work done is (S is length of the plane).

(a) $mg \sin\theta \times S$

(b) $mg \cos\theta \times S$

(c) $mg (\sin\theta - \cos\theta) S$

(d) Zero

5. In moving a body of mass m up and down a rough incline of inclination θ, work done is (S is length of the planck, and μ is coefficient of friction).

(a) $mg \sin\theta \times S$ (b) $mg \cos\theta = S$

(c) $2 \mu mg \cos\theta \times S$ (d) Zero

6. A ball rolling on ice with a velocity of 5·6 m/s stops after travelling 8 m. If $g = 9{\cdot}8$ m/s^2, coefficient of friction is :

(a) 0·1 (b) 0·2

(c) 0·3 (d) 0· 4

7. A particle will leave a vertical circle of radius r, when its velocity at the lowest point of the circle (v_L) is

(a) $\sqrt{2gr}$ (b) $\sqrt{5gr}$

(c) $\sqrt{3gr}$ (d) $\sqrt{6gr}$

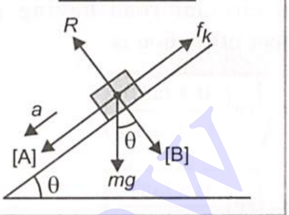
8. What is the angle of friction between two surfaces in contact if the coefficient of friction is 1/(3)½ ?

(a) 30° (b) 60°

(c) 90° (d) 0°

9. By seeing force body diagram of a body sliding down an inclined plane. Identify [A] and [B]

FIGURE 5(Q).1



(a) $mg \sin\theta$, $mg \tan\theta$ (b) $mg \cos\theta$, $mg \sin\theta$

(c) $mg \sin\theta$, $mg \cos\theta$ (d) $mg \tan\theta$, $mg \sin\theta$

10. An object of mass m lies on the floor of a lift, when lift moves with a constant speed 'v' downward, then the value of reaction force

(a) $R = 0$ (b) $R = mg$

(c) $R = mv^2/r$ (d) $R = m(g + v/t)$

11. An object of mass m lies on the floor of a lift. When the lift moves upwards with an acceleration 'a' then reaction on the object will be

(a) $m(g + a)$ (b) $m(a - g)$

(c) zero (d) mg

12. A force of 5 N changes the velocity of a body from 10 m/s to 20 m/s in 5 second. How much force is required to bring about the same change in 2 second ?

(a) 12·5 N (b) 13 N

(c) 125 N (d) 10 N

13. A cricket player catches a ball of mass 0·1 kg moving with a speed of 10 m/s in 0·1 s. Force exerted by him is :

(a) 10 N (b) 5 N

(c) 2 N (d) 1 N

14. Two bodies having same mass 40 kg are moving in opposite directions, one with a velocity of 10 m/s and other with a velocity of 7 m/s. If they collide elastically and as one body, the velocity of combination is :

(a) 10 m/s (b) 1·5 m/s

(c) 7 m/s (d) 3 m/s

15. The need to banking of road is

(a) To provide additional gravitational force for higher velocity (*b*) To provide additional centrifugal force for higher velocity

(*c*) To provide additional centripetal force for higher velocity

(*d*) To provide additional electrostatic force for higher velocity

16. Expression for maximum velocity of a car on a banked circular road having μ as limiting coefficient of friction is

(*a*) $V = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$

(*b*) $V = \sqrt{rg\left(\frac{\mu - \tan\theta}{1 + \mu\tan\theta}\right)}$

(*c*) $V = \sqrt{urg}$

(*d*) $V = urg$ $V = urg$

17. What is the angular velocity of the earth ?

(*a*) 2 π/86400 rad/s (*b*) 2 π/3600 rad/s

(*c*) 2 π/24 rad/s (*d*) 2 π/6400 rad/s

18. The safe speed of a vehicle on a horizontal circular road is independent of

(*a*) Mass of vehicle

(*b*) Coefficient of friction between road surface and tyre of vehicle

(*c*) Radius of curve

(*d*) Acceleration due to gravity

19. The ratio of angular speed of second hand to that of the minute hand of a clock is

(*a*) 60 : 1 (*b*) 1 : 60

(*c*) 1 : 1 (*d*) 1 : 6

20. A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively then

(*a*) $R_1 = R_2$ (*b*) $R_1 < R_2$

(*c*) $R_1 > R_2$ (*d*) $R_1 >= R_2$

21. A car sometimes overturns while taking a turn. When it overturns,

(*a*) The inner wheel leaves the ground first

(*b*) The outer wheel leaves the ground first

(*c*) Both the wheels leave the ground simultaneously

(*d*) Either wheel may leave the ground first

22. The linear momentum of a body changes at the rate of 10 kg m/s per second. Force acting on the body is

(*a*) 1 N (*b*) 10 N

(*c*) 1 kg f (*d*) 10 kg f

23. A force vector, applied on a mass is represented as $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$ and accelerates with 1 m/s^2. What will be the mass of the body ?

(*a*) $10\sqrt{2}$ kg (*b*) $2\sqrt{10}$ kg

(*c*) 10 kg (*d*) 20 kg

24. A ball of mass 10 gram hits a hard surface vertically with a speed of 5 m/s and rebounds with the same speed. The ball remains in contact with the surface for 0·01 second. The average force exerted by the surface on the ball is

(*a*) 100 N (*b*) 10 N

(*c*) 1 N (*d*) 0·1 N

25. A bomb at rest explodes into 3 parts of the same mass. The momentum of the 2 parts are $-2p\hat{i}$ and $p\hat{j}$. The momentum of the third part will have a magnitude of

(*a*) $\hat{p}$ (*b*) $\sqrt{3}\,\hat{p}$

(*c*) $\hat{p}\sqrt{5}$ (*d*) Zero

26. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be

(*a*) $\sqrt[3]{2} : 1$ (*b*) $1 : \sqrt{3}$

(*c*) $\sqrt{3} : 1$ (*d*) $1 : \sqrt[3]{2}$

27. A small block starts slipping down from a point *B* on an inclined plane *AB*, which is making an angle θ with the horizontal. Section *BC* is smooth and the remaining section *CA* is rough, with coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom point *A* of the inclined plane. It *BC* = 2 AC, coeff. of friction is μ = *K* tan θ, where *K* is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

28. A nucleus at rest disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be

(*a*) $1 : \sqrt{3}$ (*b*) $\sqrt{3} : 1$

(*c*) $2^{3/2} : 1$ (*d*) $1 : 2^{3/2}$

29. For a body moving with constant speed in a horizontal circle, which of the following remains constant ?

(*a*) Velocity (*b*) Acceleration

(*c*) Centropetal force (*d*) Kinetic energy

30. Two bodies having same mass 40 kg each are moving in opposite directions, one with a velocity of 10 m/s and other with a velocity of 7 m/s. If they collide-elastically, and move as one body, the velocity of the combination is

(*a*) 1·5 m/s (*b*) 3 m/s

(*c*) 10 m/s (*d*) 7 m/s

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

31. **Assertion.** Newton's second law of motion is the real law of motion.

Reason. This is because Newton's first law and third law of motion can be deduced from Newton's 2nd law of motion.

32. **Assertion.** The velocities of the bodies having equal linear momenta vary inversely as their masses.

Reason. When $p_1 = p_2$

$$m_1 v_1 = m_2 v_2$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} \quad i.e. \quad v \propto \frac{1}{m}.$$

33. **Assertion.** Inertia of a body in linear motion is measured by the mass of the body and linear velocity of the body.

Reason. Inertia of a body is equal to linear momentum of the body.

34. **Assertion.** When a horse starts suddenly, the rider tends to fall backwards due to inertia of rest.

Reason. Inertia of motion is responsible for backward fall of the rider.

35. **Assertion.** When a knife is sharpened by pressing it against a grinding stone, the sparks fly off along the tangent to the grinding stone.

Reason. This happens due to directional inertia.

36. **Assertion.** Newton's first law of motion defines force ; 2nd law gives us a measure of force and 3rd law tells us the nature of force.

Reason. The reasons for all the three laws have been explained in the text.

37. **Assertion.** When a car is being driven along a straight horizontal road, the driving force is provided by the road.

Reason. The road rather opposes the motion of the car.

38. **Assertion.** Gravitational forces between two bodies are position dependent forces. Force on a charged particle in a magnetic field is velocity dependent force.

Reason. $F_g = \frac{G m_1 m_2}{r^2}$; $F_m = Bq\,v \sin\theta$.

39. **Assertion.** Two masses 8 kg and 4 kg are connected at the two ends of an inextensible string passing over a smooth frictionless pulley. The tension in the string is 52·3 N.

Reason. $T = \frac{(2 m_1 m_2) g}{m_1 + m_2} = \frac{2 \times 8 \times 4 \times 9{\cdot}8}{(8+4)}$

$$= \frac{156{\cdot}8}{3} = 52{\cdot}3 \text{ N}$$

40. **Assertion.** Dimensions of impulse are $[MLT^{-1}]$

Reason. Impulse = force × time

$$= [MLT^{-2}] \times T = [MLT^{-1}]$$

41. **Assertion.** Force never occurs singly in nature.

Reason. Forces always occur in pairs as a result of mutual interaction between two bodies.

42. **Assertion.** Forces of action and reaction never cancel each other.

Reason. This is because forces of action and reaction act always on different bodies.

43. **Assertion.** Upthrust on a rocket fired is

$$F = -v \frac{dm}{dt}$$

where v is velocity of escaping gases w.r.t. rocket, $\frac{dm}{dt}$ is rate of change of mass (due to escaping gases).

Reason. Force is given by rate of change of linear momentum.

44. Assertion. Static friction does not exist by itself. When no force is applied, static friction is zero.

Reason. Opposition to motion appears when some force is applied, even when body is not actually moving.

45. Assertion. Out of static friction, limiting friction and dynamic friction ; static friction alone is self adjusting.

Reason. Even limiting friction and dynamic friction become self adjusting sometime.

ANSWERS

I. Multiple Choice Questions

1. (a)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (a)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (b)
21. (a)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (a)

II. Assertion-Reason Type Questions

31. (a)	**32.** (a)	**33.** (d)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (a)	**40.** (a)
41. (a)	**42.** (a)	**43.** (a)	**44.** (a)	**45.** (c)					

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. As change in linear momentum = $F \times t$,

$\therefore$ When t increases, F decreases.

2. Force of friction increases when surfaces in contact are made too smooth.

3. $\mu = \tan\theta = \tan 60^\circ = \sqrt{3}$.

4. $W = 0$, as work done against gravity = work done by gravity.

5. When inclined plane is rough, work done against friction is added.

$$W = 2\,\mu\, mg \cos\theta \times S$$

6. From $v^2 - u^2 = 2\,as$

$$0 - (5{\cdot}6)^2 = 2 \times a \times 8\,;$$

$$a = \frac{-5{\cdot}6^2}{16} = -1{\cdot}96\ \text{ms}^2$$

$$\mu = \frac{a}{g} = \frac{1{\cdot}96}{9{\cdot}8} = \mathbf{0{\cdot}2}\ \ \text{(leaving – sign)}$$

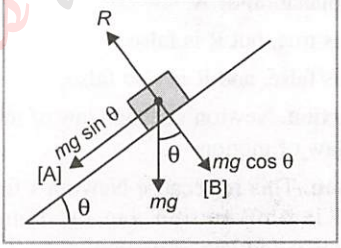
7. From the study of theory, the particle will leave a vertical circle of radius r, when its velocity at the lowest point of the circle is $\sqrt{3\,gr}$.

8. $\tan\theta = \mu = \dfrac{1}{3^{1/2}} = \dfrac{1}{\sqrt{3}}$

$\therefore\ \theta = \mathbf{30^\circ}$

9. From Fig. 5(Q).2,

FIGURE 5(Q).2



$$A = mg \sin\theta$$
$$B = mg \cos\theta$$

10. When v = constant, $a = 0$

$\therefore\quad R = mg$

11. As lift moves upwards with acceleration (a)

$$R - mg = ma$$

$\therefore\quad R = m\,(g + a)$

12. From change in momentum = $F \times t$

$$= 5 \times 5 = F \times 2$$

$$\therefore\quad F = \frac{5 \times 5}{2} = \mathbf{12{\cdot}5\ N}$$

13. $F \times t$ = Change in momentum

$F \times 0{\cdot}1 = 0{\cdot}1 \times 10$

$F = \mathbf{10\ N}$

14. Applying conservation of linear momentum,

$$(m + m)\,v = 40 \times 10 + 40\,(-7)$$

$$(40 + 40)\,v = 400 - 280 = 120$$

$$v = \frac{120}{80} = \mathbf{1{\cdot}5\ m/s}$$

15. Banking of roads is required to provide additional centripetal force for higher velocity.

16. On a banked circular road, max. velocity of vehicle

$$v = \sqrt{\frac{rg\,(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$$

17. Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60}\ \text{rad/s} = \frac{2\pi}{86400}\ \text{rad/s}$$

18. On a horizontal circular road, $v = \sqrt{\mu rg}$; does not depend upon mass of vehicle.

19. $\dfrac{\omega_1}{\omega_2} = \dfrac{2\pi/T_1}{2\pi/T_2} = \dfrac{T_2}{T_1} = \dfrac{60\text{ min}}{1\text{ min}} = \mathbf{60:1}$

20. As is known from theory, $R_1 < R_2$.

21. In case of overturning, the inner wheels leave the ground first.

22. $F = \dfrac{\Delta p}{\Delta t} = \dfrac{10\text{ kg m/s}}{\text{sec}} = 10\text{ kg m/s}^2 = \mathbf{10\ N}$

23. Here, $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$

$$F = |\vec{F}| = \sqrt{6^2 + (-8)^2 + 10^2}$$

$$= \sqrt{200} = 10\sqrt{2}$$

$$m = \frac{F}{a} = \frac{10\sqrt{2}}{1} = \mathbf{10\sqrt{2}\ kg}$$

24. Average force $= \dfrac{\text{Charge in momentum}}{\text{time of contact}}$

$$F = \frac{2(10 \times 10^{-3}) \times 5}{0{\cdot}01} = \mathbf{10\ N}$$

25. Resultant momentum of 2 parts

$$= \sqrt{(-2\,p)^2 + p^2} = p\sqrt{5}$$

As the bomb explodes at rest, momentum of third part $p\sqrt{5}$.

26. Here, on disintegration,

$$p_1 = p_2$$

$$m_1 v_1 = m_2 v_2$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{1}{2}$$

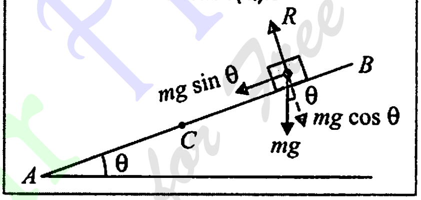
$$\frac{\frac{4}{3}\pi r_1^3 \rho}{\frac{4}{3}\pi r_2^3 \rho} = \frac{1}{2}$$

$$\frac{r_1}{r_2} = \frac{1}{2^{1/3}}$$

27. As is clear from Fig. 5(Q).3

$$f = \mu R = \mu\,mg\cos\theta$$

FIGURE 5(Q).3



As the block comes to rest on reaching A, therefore,

work done by gravity = work done against friction
(over BA) (over CA)

$$mg\sin\theta\,(BC + CA) = f \times CA$$

$$mg\sin\theta\,(2\,CA + CA) = \mu\,mg\cos\theta \times CA$$

$$mg\sin\theta \times 3 = \mu\,mg\cos\theta$$

$$\mu = \frac{3\sin\theta}{\cos\theta} = 3\tan\theta = K\tan\theta$$

Hence $\boldsymbol{K = 3}$

28. Here $\dfrac{v_1}{v_2} = 2:1$

$$\therefore \quad \frac{m_1}{m_2} = 1:2\ ; \quad \frac{r_1^3}{r_2^3} = 1:2\ ; \quad \frac{r_1}{r_2} = \frac{1}{2^{3/2}}$$

29. When a body moves in a horizontal circle with constant speed, its **KE remains constant.**

30. As the two bodies are moving in opposite directions, total linear momentum of the combination $= p = 40 \times 10 - 40 \times 7 = 120$ kg m/s.

Mass of the combination = 40 + 40 = 80 kg

Velocity of the combination $= \dfrac{120}{80} = \mathbf{1{\cdot}5\ m/s}$

II. Assertion-Reason Type Questions

31. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

32. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

33. Both, the Assertion and Reason are false.

34. Assertion is true, but the reason is false.

35. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

36. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

37. Assertion is true. But the Reason is false.

38. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

39. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

40. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

41. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

42. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

43. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

44. Both, the Assertion and Reason are true. And Reason is correct explanation of the Assertion.

45. Assertion is true, but the Reason is false.

CONCEPTUAL PROBLEMS

I. Newton's first and second Laws of Motion

1. A stone, when thrown on a glass window smashes the window pane to pieces, but a bullet from the gun passes through making a clean hole. Why ?

Sol. This is because velocity of bullet from the gun is very large. It takes very little time to cross the window pane. Particles of window pane near the hole have too little time to share the motion of the bullet. Reverse is the case when stone is thrown with hand.

2. The distance travelled by a body is directly proportional to time. Is any external force acting on it ?

Sol. Here, $s \propto t$. $\therefore s = kt$, where k is constant.

$$v = \frac{ds}{dt} = k \qquad \therefore \; a = \frac{dv}{dt} = 0$$

External force, $F = m \times a =$ Zero.

3. A particle of mass 0·3 kg is subjected to a force $F = -kx$, where $k = 15\ \text{Nm}^{-1}$. What will be its initial acceleration when particle is released from a point 20 cm away from origin ?

Sol. Here, $m = 0{\cdot}3$ kg, $k = 15$ N/m,

$$x = 20 \text{ cm} = 0{\cdot}2 \text{ m}$$

$$F = -kx = -15 \times 0{\cdot}2 = -3 \text{ N}$$

$$a = \frac{F}{m} = \frac{-3}{0{\cdot}3} = \mathbf{-10\ m/s^2}$$

4. Four blocks of the same mass m connected by cords are pulled by a force F on a smooth horizontal surface, as shown in Fig. 5(Q).4. Determine the tensions T_1, T_2, and T_3 in the cords.

Sol. Let a be the common acceleration of the whole system.

$$\therefore \; F = (m + m + m + m)\, a = 4\, ma\ ;$$

$$a = F/4\, m$$

FIGURE 5(Q).4

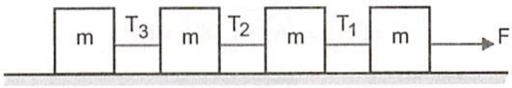
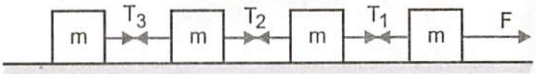


FIGURE 5(Q).5



Applying Newton's 2nd law separately for each block, Fig. 5(Q).5.

$$F - T_1 = ma,\; T_1 - T_2 = ma\ ;$$

$$T_2 - T_3 = ma,\; T_3 = ma$$

On solving these equations, we get

$$T_1 = \frac{3}{4}F, \quad T_2 = \frac{1}{2}F, \quad T_3 = \frac{1}{4}F$$

5. The speed of driving a car safely in darkness depends upon the range of headlights. Explain.

Sol. The range of headlights of car is the maximum distance (s) upto which an obstacle on the road can be seen by the driver. The car must stop before it hits the obstacle. If a is retardation produced in the car by applying brakes, then to stop the car within the distance s, the speed v of the car must be less than $\sqrt{2as}$. Hence the speed of driving a car safely depends upon s, the range of head lights.

6. A force of 5 N changes the velocity of a body from 10 ms^{-1} to 20 ms^{-1} in 5 sec. How much force is required to bring about the same change in 2 sec ?

Sol. From $F_1 = \dfrac{dp}{dt_1}$ and $F_2 = \dfrac{dp}{dt_2}$

$$\frac{F_2}{F_1} = \frac{dt_1}{dt_2} \;;$$

$$F_2 = F_1 \times \frac{dt_1}{dt_2} = \frac{5 \times 5}{2} = \mathbf{12{\cdot}5\ N}$$

7. The motion of a particle of mass m is described by $y = ut + \dfrac{1}{2}gt^2$. Find the force acting on the particle.

Sol. As $\qquad y = ut + \dfrac{1}{2}gt^2$

velocity, $\quad v = \dfrac{dy}{dt} = u + gt$

Acceleration $a = \dfrac{dv}{dt} = 0 + g$

Force, $F = ma = mg$

This is the force acting on the particle.

8. Aeroplanes having wings fly at low altitudes while jet planes fly at high altitudes. Why ?

Sol. The wings of an aeroplane push the air backwards. The aeroplane moves forward due to reaction of the pushed air. At lower altitudes, air is dense. Therefore, the plane receives sufficient reactional push to move forward.

In a jet plane, external air is sucked into the plane and compressed. Therefore, jet planes fly at high altitudes where air density is small.

II. Newton's third Law of motion and conservation of linear momentum

9. A thief jumps from the upper storey of a house with a load on his back. What is the force of the load on his back, when thief is in air ?

Sol. Zero. This is because in free fall, $g = a$.

$$F = m(g - a) = m(g - g) = \text{Zero}.$$

10. A meteorite burns in the atmosphere before it reaches the earth's surface. What happens to its momentum ?

Sol. The meteorite moves under the force of gravity, therefore, its momentum goes on changing. But total momentum of meteorite and earth is always conserved.

11. On the pan of a spring balance, is placed a beaker containing water. How will the reading of spring balance change if we dip our finger in this water ?

Sol. The reading of spring balance will increase. This is because water will exert an upthrust on the finger and the finger will exert an equal force of reaction in the downward direction.

12. A person of mass m is hanging from a rope fastened to a stationary balloon of mass M. If the person climbs the rope, then with what velocity the balloon would move and in what direction ?

Sol. Let v be the velocity of person climbing up the rope. As initial momentum of the person and balloon is zero, the balloon will move down with such a velocity u that total momentum continues to be zero. As relative velocity of person relative to earth is $(v - u)$, therefore

$$m(v - u) - Mu = 0$$

or $$mv = (M + m)\,u$$

or $$u = \frac{mv}{M + m}.$$

13. Vehicles stop on applying brakes. Does this phenomenon violate the principle of conservation of momentum ?

Sol. The vehicles stop on applying brakes. This is fully in accordance with the law of conservation of momentum. On applying brakes, we apply some opposing force on the vehicle. As a result, the vehicle comes to rest. The loss of momentum of the vehicle is exactly equal to the impulse of the applied force.

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14. Rocket can move in air free space, but jet planes cannot. Why ?

Sol. A rocket is self contained, as both the fuel and the oxygen needed to burn the fuel are available inside the rocket. Therefore, a rocket can move in air free space. A jet plane has fuel only. It needs to take oxygen from the atmosphere to burn the fuel. That is why a jet plane cannot operate in air free space.

III. Friction

15. Why is it difficult to move a bike with its brakes on ?

Sol. When the brakes of a bike are on, its wheels cannot rotate. They will simply skid. Thus rolling friction will be converted into sliding friction which is comparatively larger. Hence it becomes difficult to move the bike.

16. Sand is spread on tracks covered with snow. Why ?

Sol. Covering of snow laid tracks with sand increases the force of friction between the road and the wheels. Therefore, chances of slipping & skidding reduce.

17. When a wheel is rolling on a level road, what is the direction of frictional force between the wheel and the road ?

Sol. As the wheel is moving forward, the portion of the wheel in contact with the road moves backwards. Hence the force of friction must be acting in the forward direction, along the tangent to the surface of the road and the wheel in contact.

18. Is large brake on a bicycle wheel more effective than a small one ? Explain.

Sol. No, the large brake on a bicycle wheel is not more effective than a small one because the force of friction is independent of the surface area of contact. However, chances of failure of brake decrease when area of contact is larger. Therefore, large brakes are preferred.

19. How do we save petrol when the tyres of the motor cycle are fully inflated ?

Sol. When the tyres are fully inflated, deformation of tyres will be small. As a result of it, value of force of rolling friction will reduce. Due to it, the motorcycle will cover more distance for the given petrol consumed in it. Hence we save petrol when the tyres of the motorcycle are fully inflated.

20. A block of mass 1 kg lies on horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0·6. If the acceleration of the block is 5 ms^{-2}, calculate the frictional force acting on the block.

Sol. Limiting force of friction

$F = \mu R = \mu mg = 0{\cdot}6 \times 1 \times 9{\cdot}8 = 5{\cdot}88$ N

Applied force, $F' = ma = 1 \times 5 = 5$ N

As $F' < F$, the block would not move.

Force of friction = applied force = 5 N.

21. How does a lubricant help in reducing friction ?

Sol. When a lubricant is added to a machine, it spreads between the two surfaces rubbing each other, fills the irregularities present on the surfaces and forms a thin layer between the surfaces in contact. As a result of it, the contact between the two hard surfaces is replaced by the contact between the hard surface and lubricant layer. Due to it, the force of friction is reduced considerably.

22. When a person walks on a rough road, the frictional force exerted by the road on the person is opposite to direction of his motion. Comment.

Sol. The statement is wrong. When a person walks, he pushes the ground/road backwards with his feet. Hence, force of friction acts in the forward direction, *i.e.*, in the direction of his motion.

23. Can coefficient of friction exceed unity ?

Sol. Coefficient of friction is normally less than one. But when the surfaces in contact are very irregular having sharp minute projections and cavities, the coefficient of friction between them may exceed unity.

24. Why are ball bearings used in machinery ?

Sol. By using ball bearings between the moving parts of a machinery, the sliding friction gets converted into rolling friction. As rolling friction is much less than sliding friction, power dissipation is reduced.

IV. Dynamics of Circular Motion

25. Why has a horse to pull a cart harder during the first few steps of his motion ?

Sol. During the first few steps of his motion, the horse has to pull a cart harder, because the horse has to work against the limiting friction, whereas once the motion starts, the horse has to work against the dynamic friction which is less than the limiting friction.

26. Why does a cyclist lean to one side, while going along a curve ? In what direction does he lean ?

Sol. A cyclist leans while going along a curve. By doing so, a component of normal reaction of the ground is spared to provide him the centripetal force he requires for turning.

He has to lean *inwards* from his vertical position *i.e.* towards the centre of the circular path.

27. Why does a child in a merry-go-round press the side of his seat radially outward ?

Sol. This is because of centrifugal force acting radially outwards on the child rotating actually in the merry go round.

28. A bucket containing water is rotated in a vertical circle. Explain why water does not fall.

Sol. This is because weight of water in the bucket is spent in supplying the necessary centripetal force. For details, see Art. 5.21.

29. Why does a pilot not fall down, when his aeroplane loops a vertical loop ?

Sol. This is because at the highest point of the (vertical) loop, weight of the pilot is spent in providing the necessary centripetal force.

30. If a force is acting on a moving body in a direction perpendicular to the direction of motion, what will be its effect on speed and direction of the body ?

Sol. As force is perpendicular to the direction of motion, there will be no change in speed of the body, but the direction of motion of the body will keep on changing continuously.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Newton's first and second Laws of Motion

1. If the net force acting on a body be zero, then will the body remain necessarily in rest position ?

Ans. No, the body may be moving uniformly along a straight line.

2. The distance travelled by a moving body is directly proportional to time. Is any external force acting on it ?

Ans. When $s \propto t$, acceleration = 0. Therefore, no external force is acting on the body.

3. A person sitting in the compartment of a train moving with uniform speed throws a ball in the upward direction. What path of the ball will appear to him ? What to a person standing outside ?

Ans. Vertically downwards; parabolic to a person standing outside.

4. If force is acting on a moving body perpendicular to the direction of motion, then what will be its effect on the speed and direction of the body ?

Ans. No change in speed, but change in direction is possible.

5. A retarding force is applied to stop a motor car. If the speed of the motor car is doubled, how much more distance will it cover before stopping under the same retarding force ?

Ans. As $s \propto v^2$, therefore, motor car will cover a distance four times longer than before.

6. A body is dropped from the ceiling of a transparent cabin falling freely towards the earth. Describe the motion of the body as observed by an observer (*a*) sitting in the cabin, (*b*) standing on earth.

Ans. (*a*) The body will appear stationary in air.

(*b*) The body will appear to be falling freely under acceleration due to gravity.

7. The linear momentum of a body can change only in the direction of applied force. Comment.

Ans. The statement is correct. It is in accordance with Newton's second law of motion.

8. Calculate the net force acting on a body of mass 10 kg moving with a uniform velocity of 2 ms^{-1}.

Ans. As velocity is uniform, acceleration, $a = 0$

$F = ma = 0$

9. Calculate the mass of a body weighing 100 dyne. Take $g = 10\ \text{ms}^{-2}$.

Ans. From $W = mg$,

$$m = \frac{W}{g} = \frac{100\,\text{dyne}}{1000\,\text{cm s}^{-2}} = 10^{-1}\ \text{g}$$

10. On what factors does the thrust on a rocket depend ?

Ans. The upward thrust on a rocket depends on exhaust speed of the gases w.r.t. the rocket and rate at which mass of the exhaust gases escapes.

11. A ball is suspended by a cord from the ceiling of a motor car. What will be the effect on the position of the ball, if (*i*) the car is moving with uniform velocity (*ii*) car is accelerated (*iii*) car is turning towards left ?

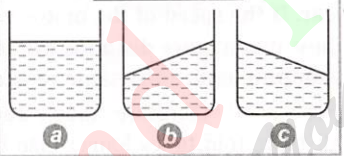
Ans. (*i*) The ball will remain suspended vertically only.

(*ii*) The ball will move in the backward direction.

(*iii*) The ball will move towards right.

12. A vessel containing water is given a constant acceleration towards the right, along the horizontal. Which of the figures 5(Q).6 correctly represents the surface of the liquid.

FIGURE 5(Q).6



Ans. Fig. 5(Q).6(*c*) represents correctly the surface of the liquid. This is because the liquid experiences an equal force towards the left. And the liquid surface has to be perpendicular to the resultant force on the liquid.

13. What is the ratio of SI to CGS units of linear momentum ?

Ans. $$\frac{\text{SI unit of } p}{\text{CGS unit of } p} = \frac{\text{kg ms}^{-1}}{\text{g cm s}^{-1}}$$

$$= \frac{10^3\,\text{g}\,10^2\,\text{cm s}^{-1}}{\text{g cm s}^{-1}} = 10^5$$

II. Newton's third Law of motion and conservation of linear momentum

14. The length of an ideal spring increases by 0·1 cm when a body of 1 kg is suspended from it. If this spring is laid on a frictionless horizontal table and bodies of 1 kg each are suspended from its ends, then what will be the increase in its length ?

Ans. Increase in length will be 0·1 cm only.

15. The two ends of a spring-balance are pulled each by a force of 10 kg. wt. What will be the reading of the balance ?

Ans. The reading of the balance will be 10 kg wt.

16. A lift is accelerated upward. Will the apparent weight of a person inside the lift increase, decrease or remain the same relative to its real weight ? If the lift is going with uniform speed, then ?

Ans. The apparent weight will increase. If the lift is going with uniform speed, the apparent weight will remain the same as the real weight.

17. A ball of 0·5 kg mass moving with a speed of 10 m/s rebounds after striking normally a perfectly elastic wall. Find the change in momentum of the ball.

Ans. Change in linear momentum $= -\,mv - mv$

$$= -\,2\,mv = -\,2 \times 0{\cdot}5 \times 10$$

$$= -\,10\ \text{kg ms}^{-1}$$

18. A body of 2 kg is suspended on a spring balance hung vertically in a lift. If the lift is falling downward under acceleration due to gravity g, then what will be the reading of the balance ? If going upward with the same acceleration, then ?

Ans. Reading of the balance = apparent weight W'

$$= m\,(g - a) = m\,(g - g) = \text{zero.,}$$

If lift is going upwards with same acc ;

$$W' = m\,(g + a) = m\,(g + g) = 2\,m\,g = 4\ \text{kg}$$

19. A 5 kg body is suspended from a spring balance, and an identical body is balanced on a pan of a physical balance. If both the balances are kept in an elevator, then what would happen in each case when the elevator is moving with an upward acceleration ?

Ans. The reading of the spring balance will increase, but there will be no effect on the equilibrium of common physical balance.

20. A thief jumps from the roof of a house with a box of weight W on his head. What will be the weight of the box as experienced by the thief during jump ?

Ans. Weight of the box during jump, $W' = m(g - a)$ $= m(g - g) = \text{zero}$

21. What is the principle of working of a rocket ?

Ans. A rocket works on the principle of conservation of linear momentum.

22. An impulsive force of 100 N acts on a body for 1 s. What is the change in its linear momentum ?

Ans. Change in linear momentum = Impulse = $F \times t$ $= 100 \times 1 = 100$ Ns.

23. What are the conditions for maximum and minimum pull of a lift on a supporting cable ?

Ans. When the lift is in free fall, pull of the lift on supporting cable is zero (minimum). When the lift is accelerating upwards, pull = $m(g + a)$ = maximum.

24. What is the principle of working of a rocket ?

Ans. A rocket is based on the principle of conservation of linear momentum.

25. A bomb explodes in mid air into two equal fragments. What is the angle between their directions of motion ?

Ans. 180°, as the two fragments would move in exactly opposite directions.

26. Can a rocket operate in free space ?

Ans. Yes, principle of conservation of linear momentum holds equally well in free space.

III. Friction

27. Why do we easily slip on a rainy day ?

Ans. Water acts as a lubricant reducing the friction between the road and our feet.

28. What are the factors on which coefficient of friction depends ?

Ans. Coefficient of friction between any two surfaces in contact depends on (*i*) material of the surfaces.

(*ii*) nature of polish of the surfaces.

29. What type of friction is involved when an axle rotates in a sleeve ?

Ans. It is sliding friction.

30. Is friction independent of actual area of contact ?

Ans. Yes, force of friction does not depend upon actual area of contact.

31. The fast moving vehicles are given streamline shape. Why ?

Ans. This reduces the force of friction due to air.

32. Rubber tyres are preferred to steel tyres. Why ?

Ans. This is because coefficient of friction between rubber tyres and road is smaller than the coefficient of friction between steel tyres and road.

33. What is the relation between coefficient of friction and angle of repose ?

Ans. $\mu = \tan \alpha$.

34. Is friction a self adjusting force ?

Ans. No, all types of friction are not self adjusting. Only static friction is a self adjusting force.

35. What is the angle between frictional force and instantaneous velocity of the body moving over a rough surface ?

Ans. The angle is 180°, because force of friction always opposes the relative motion.

36. Can we get off a frictionless horizontal surface by jumping ?

Ans. No, as frictionless surface offers no reaction.

37. What is the angle of friction between two surfaces in contact, if coefficient of friction is $1/\sqrt{3}$?

Ans. $\mu = \dfrac{1}{\sqrt{3}}$, $\theta = ?$

As $\tan \theta = \mu = \dfrac{1}{\sqrt{3}}$ $\therefore$ $\theta = 30°$

38. What is dry friction ? And what is wet friction ?

Ans. Dry friction is the force of friction between two solid surfaces in contact. Wet friction is the force of friction between a solid surface and a liquid surface.

39. What is the unit of coefficient of friction ?

Ans. Coefficient of friction has no units.

40. What is the angle of friction between two surfaces in contact if coefficient of friction is $\dfrac{1}{\sqrt{3}}$?

Ans. As $\tan \theta = \mu = \dfrac{1}{\sqrt{3}}$. $\therefore$ $\theta = 30°$.

41. It is easier to roll a barrel than to pull it along the road. Why ?

Ans. This is because rolling friction is less than the sliding friction.

IV. Dynamics of Circular Motion

42. A bucket of water is rotated in a vertical circle so that the surface of water is at a distance *r* from the axis of rotation. What is the minimum angular velocity so that the water does not spill out ?

Ans. For water not to spill, weight of water must be balanced by centrifugal force ($m\,r\,\omega^2$).

i.e., $m\,r\,\omega^2 = mg$; $\omega = \sqrt{g/r}$

43. What is the apparent weight of a body of mass *m* at (*a*) the highest and (*b*) lowest point if it is just 'looping the loop' in a vertical circle ?

Ans. Apparent weight of a body is tension in the string. While looping the loop, $T_H = 0$ and $T_L = 6$ mg.

44. The acceleration of a train travelling at 40 m/s as it goes round a curve of 160 m in radius is....... .

Ans. $a = \dfrac{v^2}{r} = \dfrac{40 \times 40}{160} = \mathbf{10\ m/s^2}$

45. A ball of 1 gm released down an inclined plane describes a circle of radius 10 cm in the vertical plane on reaching the bottom. The minimum height of the inclined plane is.........cm.

Ans. $h = \dfrac{5}{2}r = \dfrac{5}{2} \times 10 = \mathbf{25\ cm}$

46. A stone is tied to one end of a string and rotated in a vertical circle. What is the difference in tensions of the string at lowest and highest points of the vertical circle ?

Ans. $T_L - T_H = 6$ mg $= 6$ times the weight of the stone.

47. A body is moving with a uniform speed along a circle. Is there any force acting on the body ?

Ans. Yes. Centripetal force is required for changing continuously the direction of motion of the body.

48. What provides centripetal force to a car turning on a level road ? (HP Board 2006)

Ans. The force of friction between the tyres and the road provides necessary centripetal force.

49. When a body moves along a circular path, which thing experiences centrifugal force ?

Ans. Centrifugal force is experienced by the agency supplying the centripetal force needed for moving in a circle.

50. Does the angle of banking depend on mass of the vehicle ?

Ans. No, angle of banking of a road does not depend upon mass of the vehicle.

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Newton's first and second Laws of Motion

1. A body is acted upon by a number of external forces. Can it remain at rest ?

Ans. Yes, provided vector sum of all the external forces acting on the body is zero.

2. If the net force acting on a body is zero, will it remain necessarily at rest ?

Ans. No, not necessary. When net force acting on the body is zero, it may be in the state of uniform motion along a straight line.

3. Why do blades of an electric fan continue to rotate for some time, after the current is switched off ?

Ans. It happens on account of inertia of motion.

4. If the speed of a motor car is doubled, how much more distance will it cover before stopping under the same retarding force ?

Ans. As $F \times s = \dfrac{1}{2}mv^2$

$$F \times s' = \frac{1}{2}mv'^2 \quad \therefore \quad \frac{s'}{s} = \frac{v'^2}{v^2} = \frac{(2v)^2}{v^2} = 4$$

$s' = 4\,s$, *i.e.*, distance covered by motor car will become 4 times. The car will cover three times more distance than the original.

5. Two bodies of masses *M* and *m* are allowed to fall from the same height. If air resistance for each body be same, will the two bodies reach the ground simultaneously ?

Ans. No. The heavier body will reach the ground earlier. This is because, net downward force on body of mass $M = (Mg - F)$. Therefore, its acc.,

$a = \dfrac{Mg - F}{M} = g - \dfrac{F}{M}$ and acc. of body of smaller mass, $a' = g - \dfrac{F}{m}$.

As $M > m \therefore a > a'$. Hence heavier body will reach the ground earlier than the lighter body.

6. Two bodies of masses M and m are allowed to fall freely from the same height. If air resistance for each body is same, then will both the bodies reach the earth simultaneously ?

Ans. Yes, both the bodies will reach the earth together, if air resistance for each body is same.

7. A soda water bottle is falling freely. Will the bubbles of the gas rise in the water of the bottle ?

Ans. Bubbles will not rise in water. This is because water in freely falling bottle is in the state of weightlessness. No upthrust acts on the bubbles.

8. An athelete runs a certain distance before taking a long jump. Why ?

Ans. In a long jump, the horizontal range $R \propto$ (initial velocity)^2. An athelete runs a certain distance before taking a long jump because velocity acquired by running is added to the velocity of the athelete at the time of jump. Due to it, he can jump over a longer distance.

9. How do you account for the function of mud guards ?

Ans. Due to directional inertia, the rotating wheels of any vehicle throw out mud, if any, tangentially. The mud guards over the wheels stop this mud protecting the clothes etc. of the driver of the bike or motor bike.

10. Calculate the force acting on a body which changes the momentum of the body at the rate of 1 kg ms^{-2}.

Ans. As force = rate of change of momentum

$$= 1 \text{ kg ms}^{-2} = \frac{1 \text{kg m/s}}{\text{s}} = \mathbf{1N}$$

11. What is the function of shockers in a scooter ?

Ans. When a scooter moves on an uneven road, impulsive forces are exerted by the road. The function of shockers is to increase the time of impact. This would reduce the jerk experienced by the rider of the vehicle.

II. Newton's third Law of motion and conservation of linear momentum

12. Why buffers are provided between the bogies of a train ?

Ans. When a train starts or stops, buffer springs increase the time of impact between the bogies. The force acting between the bogies $F = \dfrac{\text{impulse}}{t}$ decreases. Therefore, strong jerks to the passengers are avoided.

13. When a man jumps down from a height of several storeys onto a stretched trapaulin, he receives no injury. Why ?

Ans. When a man jumps onto stretched trapaulin, the trapaulin gets depressed at the place of impact, increasing the time t of impact. Force experienced by the man F = Impulse/time decreases and the man is not hurt.

14. Air is thrown on a sail attached to a boat from an electric fan placed on the boat. Will the boat start moving ?

Ans. No, when the fan pushes the sail by air, then air also pushes the fan in the opposite direction. Since fan is placed on the boat, the vector sum of linear momenta of fan and boat is zero. The boat can move only under the force of reaction from some external agency.

15. A bird is sitting on the floor of a wire cage and the cage is in the hand of a boy. The bird starts flying in the cage. Will the boy experience any change in the weight of the cage ?

Ans. In a wire cage, air inside is in free contact with atmospheric air. Therefore, when the bird starts flying inside the cage, the weight of bird is no more experienced. Hence the cage will appear lighter than before.

III. Friction

16. The minimum force required just to move a block on a rough horizontal surface is 10 N. An applied force of 5 N fails to move the block. What are the values of static friction and dynamic friction ?

Ans. Static friction = 5 N. Dynamic friction is slightly less than 10 N.

17. Friction is a self-adjusting force. Correct the statement.

Ans. Static friction is self-adjusting force. Dynamic friction and kinetic friction are not self-adjusting.

18. Angle of repose of a rough inclined plane is 60°. What is the coefficient of friction ?

Ans. $\mu = \tan\theta = \tan 60^\circ = \sqrt{3} = \mathbf{1{\cdot}732}$

19. A ball rolling on ice with a velocity of 4·9 m/s stops after travelling 4 m. If g = 9·8 m/s^2, what is the coefficient of friction ?

Ans. Here, $u = 4{\cdot}9$ m/s, $v = 0$, $s = 4$ m,

$g = 9{\cdot}8$ m/s^2, μ = ?

$v^2 - u^2 = 2\,as = 2\,(-\mu\,g)\,s$

$$\mu = \frac{v^2 - u^2}{-2\,gs} = \frac{0-(4{\cdot}9)^2}{-2\times 9{\cdot}8\times 4} = \mathbf{0{\cdot}31}$$

20. Automobile tyres are generally provided with irregular projections over their surfaces, why ?

Ans. Irregular projections over the surface of automobile tyres increase the force of friction between the tyres and the road.

21. Why is it difficult to climb up a greasy pole ?

Ans. A greasy pole has a slippery surface having low friction. The person climbing up is not able to press it hard. As action is small, reaction of the pole is also small. Hence it becomes difficult for him to climb.

22. Why frictional force gets increased when two surfaces in contact are polished beyond a certain limit ?

Ans. When surfaces in contact are polished beyond a certain limit, number of atmos/molecules of the surfaces in contact increases and distance between them decreases. Therefore, strong adhesive forces come into play. They are responsible for increasing the friction.

IV. Dynamics of Circular Motion

23. For looping a loop of radius 4 m, through what minimum height should a particle descend ?

Ans. Here, $r = 4$ m. For looping the loop,

$$h = \frac{5}{2}r = \frac{5}{2}\times 4 = \mathbf{10\ m}$$

24. Why are the passengers thrown outwards when a car in which they are travelling suddenly takes a circular turn ?

Ans. This is because of inertia of direction of the passengers. They tend to maintain their direction of motion, while the direction of car changes on turning.

25. A motor cyclist is going in a vertical circle. What is the necessary condition so that he may not fall down ?

Ans. The necessary condition is that his velocity at the lowest point (L) must be greater than or atleast equal to $\sqrt{5\,gr}$, and velocity at the highest point should be greater than or equal to $\sqrt{gr}$. Here, r is radius of the vertical circle.

26. For oscillation of a simple pendulum of length L, what is the maximum possible velocity at the lowest position ? What happens to the motion if velocity exceeds this value ?

Ans. For oscillation of a simple pendulum max. possible velocity at the lowest point $= \sqrt{3\,g\,L}$.

When the velocity exceeds $\sqrt{3\,g\,L}$ but is less than $\sqrt{5\,g\,L}$, the bob leaves the vertical circle.

When $v = \sqrt{5\,g\,L}$, the bob will complete the vertical circle.

27. The driver of a truck travelling with a velocity v suddenly notices a brick wall in front of him at a distance d. It is better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall ? Why ?

Ans. In applying brakes, suppose F_B is the force required to stop the truck in distance (d)

$$\therefore\ F_B \times d = \frac{1}{2}mv^2 \quad \text{or} \quad F_B = \frac{mv^2}{2d}$$

In taking a turn of radius d, the force required is

$$F_T = \frac{mv^2}{d} = 2\,\mathrm{F_B} \quad \text{or} \quad F_B = \frac{1}{2}\mathrm{F_T}$$

Hence, it is better to apply brakes.

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. Explain what is meant by force, inertia and linear momentum. **[Arts. 5.2, 5.4 and 5.5]**
2. What is meant by law of inertia ? Discuss briefly the concept of inertial mass. **[Arts. 5.4 and 5.13]**
3. State second law of motion and show that second law is the real law of motion. **[Arts. 5.11 and 5.21]**
4. Define absolute and gravitational units of force. State relation between them. **[Art. 5.11]**
5. State and Explain Newton's second law of motion. Hence, deduce the relation $\vec{F} = m\vec{a}$, where the symbols have their usual meanings. **(J & K Board 2011) [Art. 5.11]**
6. Explain the term impulse. Show that impulse of a variable force is equal to area enclosed by the force-time curve. **[Art. 5.15]**

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7. Discuss the apparent weight of a man in a lift when (*i*) lift is moving upwards with a constant speed. (*ii*) lift is accelerated uniformly downwards. **[Art. 5.22]**

8. What do you understand by concurrent forces ? State the conditions for the equilibrium of a particle under the effect of concurrent forces. **[Art. 5.26]**

9. Explain static friction and kinetic friction. Can value of kinetic friction be greater than static friction ? Explain. **[Art. 5.30]**

10. Explain the origin of (*i*) sliding friction (*ii*) the rolling friction. **[Art. 5.29 and 5.31[**

11. Explain that static friction is a self adjusting force. **[Art. 5.32]**

12. Explain the terms : (*i*) angle of friction
(*ii*) coefficient of friction and
(*iii*) angle of repose. Establish a relation between them. **[Arts. 5.34, 5.35 and 5.36]**

13. Derive an expression for acceleration of a body down a rough inclined plane. **[Art. 5.37]**

14. Calculate the amount of work done in moving a body up a rough inclined plane. **[Art. 5.39]**

15. Why are the curved roads banked ? Obtain an expression for angle of banking of a curved road. **[Art. 5.45]**

16. Discuss the bending of a cyclist while describing a curved path. Find the relation for angle of bending of the cyclist. **[Art. 5.46]**

17. (*i*) How does the use of ball bearings reduce friction ?
(*ii*) How does the lubrication of a machine help in reducing friction ? **[Art. 5.41]**

18. Why does water from a bucket not spill even when it is upside down while rotating in a vertical circle ? **[Art. 5.47]**

19. Establish that friction is a necessary evil. **[Art. 5.40]**

20. Explain centripetal force and centrifugal force. Can they balance each other ? Explain. **[Art. 5.42]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Explain Newton's laws of motion and show that (*i*) Newton's first law of motion defines force and (*ii*) Newton's second law of motion gives us the measure of force. **[Arts. 5.6, 5.8 and 5.11]**

2. What are three types of inertia ? Give atleast two examples of each. **[Art. 5.10]**

3. State and prove the principle of conservation of linear momentum. Show that a gun recoils when a bullet is fired from it. **[Arts. 5.23 and 5.24]**

4. State and Explain Newton's third law of motion. Show that third law of motion is contained in first law of motion. **[Arts. 5.17 and 5.20]**

5. Name a varying mass system. Derive the expressions the (*i*) for the velocity of propulsion of a rocket at any instant. (*ii*) burnt out speed (*iii*) Thrust on the rocket. **[Arts. 5.25]**

6. Distinguish between static friction, limiting friction and kinetic friction. How do they vary with the applied force ? Which friction has least value and which one has highest value ? **[Art. 5.30]**

7. State the laws of limiting friction. How will you verify them experimentally ? **[Art. 5.33]**

8. What is friction ? Discuss the methods of reducing friction and increasing the friction. **[Art. 5.41]**

9. Discuss the motion of a body in a vertical circle. Find expressions for the minimum velocity at the lowest point while looping a loop and difference of tensions in the string at the lowest and highest points. **[Art. 5.47]**

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. According to Newton's 2nd Law of motion $\vec{F} = m\vec{a}$. Thus this law gives us a measure of force.

Again, from $F = ma = \dfrac{dp}{dt} = \dfrac{d}{dt}(mv)$

$$= m\frac{dv}{dt} + v\frac{dm}{dt}$$

If m is constant, $\dfrac{dm}{dt} = 0$, $\therefore$ $F = m\dfrac{dv}{dt}$

If v is constant, $\dfrac{dv}{dt} = 0$, $\therefore$ $F = v\dfrac{dm}{dt}$

SI unit of force is *newton* (*N*) ;
where $1\text{ N} = 1\text{ kg ms}^{-2}$

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CGS unit of force is *dyne*,

where 1 dyne = 1 g cm s^{-2}.

$$1\ N = 10^5\ dyne$$

Gravitational units of force are kg-wt or kg-*f* and gram wt. or *g-f*.

Remember that in all numerical problems, we have to use only the absolute units of force. Gravitational units are practical units only. They have to be converted into absolute units.

Some of the important consequences of Newton's 2nd Law of motion are :

(*i*) No force is required to move a body uniformly along a straight line.

(*ii*) Accelerated motion is always due to external force.

(*iii*) The second law of motion is a vector law.

(*iv*) It gives us the concept of *inertial mass* of a body as the force required to produce unit acceleration in the body.

Read the above passage carefully and answer the following very short answer and short answer questions :

1. What is the SI unit of force ?
2. How many dynes make 1 newton force ?
3. No force is required to move a body uniformly along a straight line. Is it true ?
4. Out of the following, choose the wrong statement :

   (*i*) Accelerated motion is always due to external force.

   (*ii*) Second law of motion is a vector law.

   (*iii*) 1 newton = 10 kg ms^{-2}.

CASE 2. Apparent weight of a person in a lift depends on state of motion of the lift.

(*i*) When elevator is at rest or moving uniformly in upward/downward direction

Apparent weight (R) = actual weight (mg)

(*ii*) When elevator is moving up with constant acceleration (a), apparent weight

$$R_1 = mg + f = mg + ma = m\,(g + a)$$

(*iii*) When elevator is moving down with constant acceleration (a), then apparent weight

$$R_2 = mg - f = mg - ma = m\,(g - a)$$

In free fall of lift under gravity, $a = g$

$\therefore R_2 = m\,(g - a) = m\,(g - g) =$ **Zero.**

According to the *principle of conservation of linear momentum*, in an isolated system, the vector sum of the linear momenta of all the bodies of the system is conserved and is not affected by their mutual action and reaction.

This law is universal, *i.e.* this law applies to both, the microscopic as well as macroscopic systems.

A rocket is a system with variable mass. It is also based on the principle of conservation of linear momentum. The linear momentum of mass m of the rocket at instant t is equal to vector sum of linear momentum of rocket mass $(m - dm)$ moving with velocity $(v + dv)$ and linear momentum of mass (dm) of exhaust gases moving downwards with velocity $(-v_g)$. This gives us velocity (v) of rocket at any time

$$v = u \log_e\left(\frac{m_0}{m}\right) = 2{\cdot}303u\ \log_{10}\left(\frac{m_0}{m}\right)$$

upthrust on the rocket, $F = -u\left(\frac{dm}{dt}\right)$

Read the above passage carefully and answer the following very short answer and short answer questions :

5. What is apparent weight of a person of mass m in an elevator moving uniformly in downward direction ?
6. How is apparent weight of person affected when lift is moving upwards with constant acceleration (a) ?
7. What is the apparent weight of the person when the elevator is in free fall ?
8. When a rocket is fired with a velocity u, then at any instant t, its velocity is given by ....

CASE 3. Friction is an opposing force that comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body. This force developes at the surfaces of contact of two bodies and opposes their relative motion.

The *modern view* of friction is that it arises on account of strong atomic or molecular forces of attraction between the two surfaces at the points of actual contact.

Note that static friction does not exist by itself when there is no applied force, there is no static friction. It comes into play the moment there is an applied force. Static friction opposes the impending motion. The maximum value of static friction is called **limiting friction.** When the applied force is increased slightly beyond limiting friction, the actual motion starts. The opposing force of friction at this stage is called **kinetic friction** or **dynamic friction.** The variation of force of

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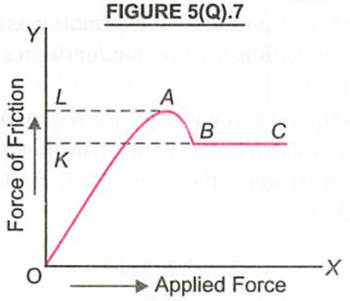
friction with applied force is as shown in Fig. 5(Q).7. Kinetic friction or Dynamic friction is always slightly less than the limiting friction. Further, static friction is a **self adjusting force.**

FIGURE 5(Q).7



The force of limiting friction (F) is directly proportional to normal reaction (R) between the surfaces in contact.

$F \propto R$ or $F = \mu R$, where μ is a constant of proportionality and is called **coefficient of friction.**

Read the above passage carefully and answer the following very short answer and short answer questions :

9. What is friction ?

10. What is modern view of friction ?

11. What is the force of friction between two surfaces in contact and at rest ?

12. How does force of friction vary with the applied force ?

ANSWERS

1. SI unit of force is netwon (N).

2. $1\text{ N} = 10^5$ dynes.

3. Yes, the statement is true.

4. The only wrong statement is (*iii*) $1\text{ N} = 10\text{ kg m/s}^2$.

5. When motion is uniform, v = constant, $a = 0$.

$\therefore$ Apparent weight of person = actual weight.

6. When lift is moving upwards with constant acceleration (a), apparent weight

$R = mg + f = mg + ma = m(g + a)$

7. In free fall, $a = g$.

$\therefore$ apparent weight of person $= m(g - a)$

$= m(g - g) = \mathbf{0}$.

8. $v = u \log_e \left(\dfrac{m_0}{m}\right) = 2{\cdot}303\, u \log_{10}\left(\dfrac{m_0}{m}\right)$

where, symbols have standard meaning.

9. Friction is the force that developes at the surface of contact of two bodies and opposes their relative motion.

10. The modern view of friction is that it arises on account of strong atomic or molecular forces of attraction between the two surfaces at the points of actual contact.

11. The force of friction when two surfaces in contact are at rest is always zero. Friction developes only when there is actual motion or even the tendency to move.

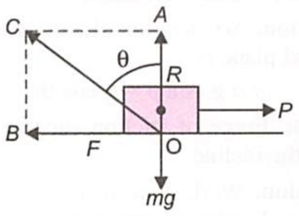
12. The variation of force of friction with applied force is shown in Fig. 5(Q).7. OL represents force of limiting friction and OK represents dynamic or kinetic friction.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. Angle of friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction (F) and normal reaction (R) makes with the direction of normal reaction. It is represented by θ.

In Fig. 5(Q).8, OA represents normal reaction R, which balances the weight (mg) of the body OB represents the force of limiting friction F, OC represents the resultant of R and F. $\angle AOC = \theta$ is the angle of friction.

FIGURE 5(Q).8



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In ΔAOC, $\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{F}{R} = \mu$

Angle of repose (α) is the minimum angle of inclination of a plane with the horizontal such that a body placed on the plane just begins to slide down the incline. We can show that $\tan\alpha = \mu$.

We can show that acceleration of a body down a rough inclined plane is

$$a = g\,(\sin\theta - \mu\cos\theta)$$

Work done in moving a body over a rough horizontal surface is

$$W = \mu_k R \times S = \mu_k\, mg \times S, \text{ and}$$

Work done in moving a body up a rough inclined plane is

$$W = mg\,(\sin\theta + \mu_k\cos\theta) \times S$$

The symbols have standard meaning. Acceleration of a body down a rough inclined plane is

$$a = g\,(\sin\theta - \mu_k\cos\theta).$$

Based on the above paragraph, answer questions no. 1 to 4 :

1. What is the relation between coefficient of friction (μ) and angle of friction (θ) ?

(*a*) $\mu = \theta$ (*b*) $\mu = \cos\theta$

(*c*) $\mu = \sin\theta$ (*d*) $\mu = \tan\theta$

2. What is acceleration of a body down a rough inclined plane ?

(*a*) $a = g\,(\sin\theta - \mu\cos\theta)$

(*b*) $a = g\,(\sin\theta + \mu\cos\theta)$

(*c*) $a = g$ (*d*) $a = g\sin\theta$

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, A and R are true and R is correct explanation of A.

(*b*) Both, A and R are true, but R is **not** a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) Both, A and R are false.

3. Assertion. Acceleration of a body down a rough inclined plane is

$$a = g\,(\sin\theta + \mu\cos\theta)$$

Reason. Force of friction supports the motion down the incline.

4. Assertion. Work done in moving a body up a rough inclined plane is

$$W = mg\,(\sin\theta + \mu_k\cos\theta) \times S$$

The symbols have usual meaning.

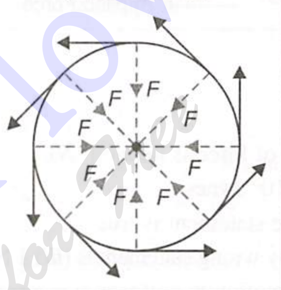
Reason. Work done in moving a body over a rough horizontal surface is

$$W = \mu_k R \times S = \mu_k\,(mg) \times S$$

CASE 2. A uniform circular motion is the motion of an object travelling at a constant/uniform speed along a circular path.

Centripetal force is the force required to move a body uniformly in a circular path. This force acts along the radius and towards the centre of the circle as shown in Fig. 5(Q).9.

FIGURE 5(Q).9



When a body moves in a circle, its direction of motion at any instant is along the tangent to the circle at that instant. As the direction of motion of the body moving in a circle goes on changing continuously, therefore, an external force is required. This force is the centripetal force.

On account of continuous change in direction of motion, the body undergoes an acceleration, called radial acceleration or centripetal acceleration

$$a = \frac{v^2}{r} = r\,\omega^2 = r\,(2\pi n)^2 = 4\pi^2 r n^2$$

centripetal force, $F = ma = m\,r\,\omega^2 = mr\,(4\,\pi^2\,n^2)$.

When we apply centripetal force on a body, it is forced to move along a circle. While moving actually, along a circle, the body has a constant tendency to regain its natural straight line path. This tendency gives rise to a force called **centrifugal force.**

Centrifugal force can be regarded as the **reaction** of centripetal force. It acts along the radius and away from the centre of the circle.

Again, when a cyclist takes a turn, he also requires some centripetal force. For this, the cyclist has to bend a little inwards from his vertical position while turning. If θ is angle of bending with vertical, we can show that

$$\tan\theta = \frac{v^2}{r\,g}$$

Similarly, we can show that it is risky rounding a level curved road.

For safety, curved roads are usually banked at angle θ by raising outer edge of the road above the inner edge, where

$$\tan\theta = \frac{v^2}{r\,g}$$

For safety, the turning should neither be **sharp** and nor it should be **fast**.

Based on the above paragraph, answer questions no. 5 to 8 :

5. What does not change in uniform circular motion ?

(*a*) speed (*b*) velocity

(*c*) direction of motion (*d*) None of these

6. Centripetal acceleration of a body can not be represented as

(*a*) $\frac{v^2}{r}$ (*b*) $r\,\omega^2$

(*c*) $4\,\pi^2\,r\,n^2$ (*d*) $\frac{\omega^2}{r}$

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, the Assertion and Reason are true, and Reason is correct explanation of Assertion.

(*b*) Both, the Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

(*c*) Assertion is true, but Reason is false.

(*d*) Both Assertion and Reason are false.

7. Assertion. Centrifugal force can be regarded as the reaction of centripetal force.

Reason. That is why, the two forces cancel eachother.

8. Assertion. While taking a turn, a cyclist has to bend through an angle θ from his vertical position where $\tan\theta = v^2/r\,g$, symbols have standard meaning.

Reason. For safe turning, centripetal force is a must.

ANSWERS

1. (*d*) **2.** (*a*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*c*) **8.** (*a*)

HINTS/EXPLANATIONS For Difficult Questions

1. The correct relation is $\mu = \tan\theta$.

2. Acceleration of a body down a rough inclined plane is
$a = g\,(\sin\theta - \mu\cos\theta)$

3. Both, the Assertion and Reason are false.

4. Both, the Assertion and Reason are true, but Reason is not a correct explanation of the Assertion.

5. In uniform circular motion, only speed of the body does not change.

6. Centripetal acceleration can not be represented as ω^2/r.

7. Assertion is true, but the Reason is false.

8. Both, the Assertion (A) and Reason (R) are true and Reason is correct explanation of the Assertion.

PROBLEMS FOR PRACTICE

TYPE I. LINEAR MOMENTUM AND FORCE

1. A 20 gram bullet moving at 300 m/s stops after penetrating 3 cm of bone. Calculate the average force exerted by the bullet. **[Ans. -3×10^4 N]**

2. A stone of mass 5 kg falls from the top of a cliff 50 m high and buries 1 m deep in sand. Find the average resistance offered by the sand and the time it takes to penetrate. **[Ans. – 2450 N ; 0·064 s]**

3. A man in a circus show, jumps from a height of 10 m and is caught by a net spread below him.

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The net sags down 2 m due to impact. Calculate average force exerted by the net on the man to stop his fall. Take mass of man = 60 kg and acc. during free fall = 10 m/s^2. **[Ans. – 3000 N]**

4. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet of releasing water at the rate of 1 kg s^{-1}, and at a speed of 5 ms^{-1}. Calculate the initial acceleration of the block. **[Ans. 2·5 m/s^2]**

5. In an X-ray machine, an electron is subjected to a force of 10^{-23} N. In how much time will the electron cover a distance of 0·1 m ? Take mass of electron = 10^{-30} kg. **[Ans. 1·4 × 10^{-4} s]**

6. A force of 10 N gives a mass m_1, an acceleration of 10 m/s^2 and a mass m_2, an acceleration of 20 m/s^2. What acceleration would it give if both the masses are tied together ? **(EAMCET 2015)**

[Ans. 6·66 m/s^2]

TYPE II. IMPULSE

7. A machine gun fires a bullet of mass 40 g with a speed of 1200 ms^{-1}. The person holding the gun can exert a maximum force of 144 N on it. What is the number of bullets that can be fired/sec from the gun ? **(AIEEE 2004)** **[Ans. 3 bullets/sec]**

8. While launching a rocket of mass 2 × 10^4 kg a force of 5 × 10^5 N is applied for 20 s. Calculate the velocity attained by the rocket at the end of 20 s. **[Ans. 500 m/s]**

9. A ball moving with a momentum of 5 kg ms^{-1} strikes against a wall at an angle of 45° and is reflected at the same angle. Calculate the impulse. **[Ans. – 7·07 kg ms^{-1}]**

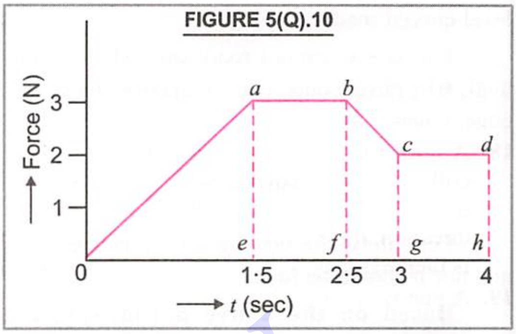
10. A ball of mass 20 gram hits a wall at an angle of 45° with a velocity of 15 m/s. If the ball rebounds at 90° to the direction of incidence, calculate the impulse received by the ball.

[Ans. $0{\cdot}3\sqrt{2}$ kg ms^{-1}]

11. A rubber ball of mass 50 g falls from a height of 100 cm and rebounds to a height of 50 cm. Find the impulse and average force between the ball and the ground, if time of contact is 0·1 s. **[Ans. 0·3778 Ns ; 3·78 N]**

12. The initial speed of a body of mass 2 kg is 5 ms^{-1}. A force acts for 4 seconds in the direction of motion of the body. The force time graph is shown in Fig. 5(Q).10. Calculate impulse of the force and also find the speed of the body.

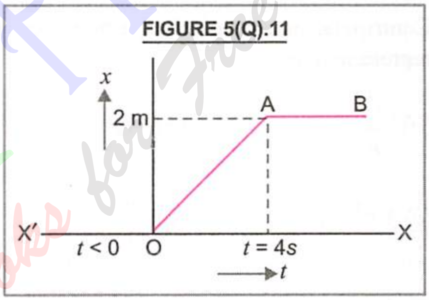
FIGURE 5(Q).10



[Ans. 8·5 Ns ; 9·25 ms^{-1}]

13. Fig. 5(Q).11 shows position time graph of a particle of mass 100 g. Find the impulse at $t = 0$ and at $t = 4$ s.

FIGURE 5(Q).11



[Ans. 0·05 kg m/s; – 0·05 kg m/s]

TYPE III. APPARENT WEIGHT OF A PERSON IN AN ELEVATOR

14. The strings of a parachute can bear a maximum tension of 72 kg wt. By what minimum acceleration can a person of 96 kg descend by means of this parachute ? **[Ans. 2·45 m/s^2]**

15. An elevator weighs 4000 kg. When the upward tension in the supporting cable is 48000 N, what is the upward acceleration ? Starting from rest, how far does it rise in 3 seconds ?

[Ans. 2·2 ms^{-2} ; 9·9 m]

16. An elevator and its load weigh a total of 800 kg. Find the tension T in the supporting cable when the elevator, originally moving downwards at 20 m/s is brought to rest with constant retardation in a distance of 50 m. **[Ans. 1·104 × 10^4 N]**

17. A man of mass 85 kg stands on a lift of mass 30 kg. When he pulls on the rope, he exerts a force of 400 N on the floor of the lift. Calculate acceleration of the lift. Given $g = 10$ m/s^2.

[Ans. 4·55 m/s^2]

TYPE IV. PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM AND NEWTON'S THIRD LAW

18. A truck of mass 2×10^4 kg travelling at 1·5 m/s collides with another truck of mass 3×10^4 kg moving with a velocity of 2·5 m/s in opposite direction. If the trucks couple up on collision, what is their velocity ? **[Ans. – 0·9 m/s]**

19. A gun weighing 9 kg fires a bullet of 30 g with a velocity of 300 m/s. What is the recoil velocity of the gun ? What is the combined momentum of gun and the bullet before and after firing ?

[Ans. 1 m/s ; Zero ; Zero]

20. A machine gun has a mass of 20 kg. It fires 25 *g* bullets at the rate of 600 bullets per minute with a speed of 200 ms^{-1}. Calculate (*i*) recoil velocity of the gun (*ii*) force required to keep the gun in position. **[Ans. 2·5 m/s, 50 N]**

21. A stream of water flowing horizontally with a speed of 15 m/s gushes out of a tube of cross sectional area 10^{-2} m^2, and hits at a vertical wall nearby. What is the force exerted by the impact of water, assuming that water rebounds with the same speed. **[Ans. 4500 N]**

22. A bomb at rest explodes into three fragments of equal masses. Two fragments fly off at right angles to each other with velocities of 9 m/s and 12 m/s. Calculate the speed of the third fragment.

[Ans. – 15 m/s]

23. A bomb at rest explodes into three parts of the same mass. The linear momenta of the two parts are $-2\,p\hat{i}$ and $p\hat{j}$. Calculate the magnitude of momentum of third part. **[Ans. $p\sqrt{5}$]**

24. A body of mass 1 kg initially at rest explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal masses fly off perpendicular to each other with a speed of 30 m/s each. What is the velocity of heavier fragment ? **(EAMCET 2016)** **[Ans. $10\sqrt{2}$ m/s]**

TYPE V. ROCKET PROPULSION

25. A balloon with mass *M* is descending down with an acceleration *a*, where $a < g$. What mass *m* of its contents must be removed so that it starts moving up with acceleration *a* ? $\left[\textbf{Ans. } \dfrac{2Ma}{g+a}\right]$

26. Fuel is consumed at the rate of 100 kg/s in a rocket. The exhaust gases are ejected at a speed of $4 \cdot 5 \times 10^4$ ms^{-1} w.r.t. the rocket. What is the thrust experienced by the rocket ? Also, calculate velocity of the rocket at the instant, when its mass is reduced to 1/10 th of its initial mass. Take initial velocity of rocket to be zero.

[Ans. $4 \cdot 5 \times 10^6$ N ; $1 \cdot 036 \times 10^5$ ms^{-1}]

27. Calculate the ratio m_0/m for a rocket to attain the escape velocity of 11·2 km s^{-1} after starting from rest, when maximum exhaust velocity of gases is 1·6 km/s. **[Ans. 1096]**

28. A rocket of initial mass 6000 kg ejects mass at a constant rate of 16 kg/s with constant relative speed of 11 km/s. What is the acceleration of the rocket one minute after blast ?**[Ans. 34·92 m/s^2]**

29. If the maximum possible exhaust velocity of a rocket be 2 km/s, calculate the ratio m_0/m for it to acquire a velocity of 11·2 km/s after starting from rest. $\left[\textbf{Ans. } \dfrac{m_0}{m} = 270 \cdot 4\right]$

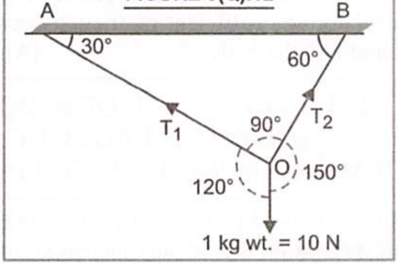
30. A rocket consumes 24 kg of fuel per second. The burnt gases escape the rocket at a speed of 6·4 km/s relative to the rocket. Calculate the upthrust received by the rocket. Also, calculate the velocity acquired by the rocket, when the mass reduces to 1/100th of its initial mass.

[Ans. $1 \cdot 536 \times 10^5$ N ; $2 \cdot 948 \times 10^4$ m/s]

TYPE VI. EQUILIBRIUM OF FORCES

31. A ball of mass 1 kg hangs in equilibrium from two strings as shown in Fig. 5(Q).12. If $g = 10$ m/s^2, what are the values of tension in strings *OA* and *OB* ?

FIGURE 5(Q).12



[Ans. 5 N ; $5\sqrt{3}$ N]

32. Two bodies of masses 11 kg and 11·5 kg are connected by a long light string passing over a smooth pulley. Calculate velocity and height ascended/descended by each body at the end of 4 s.

[Ans. 0·872 m/s, 1·744 m]

33. Two bodies of masses 4 kg and 3 kg respectively are connected by a light string passing over a smooth frictionless pulley. Calculate the acceleration of the masses and tension in the string.

[Ans. 1·4 ms^{-2}, 33·6 N]

34. Two bodies whose masses are $m_1 = 50$ kg and $m_2 = 150$ kg are tied by a light string and are placed on a frictionless horizontal surface. When m_1 is pulled by a force F, an acceleration of 5 ms^{-2} is produced in both the bodies. Calculate the value of F. What is the tension in the string 1 ?

[Ans. 1000 N, 750 N]

TYPE VII. COEFFICIENT OF FRICTION, ANGLE OF FRICTION AND MOTION ON A LEVEL ROAD

35. An engine of 100 H.P. draws a train of mass 100 metric ton with a velocity of 36 km h^{-1}. Find the coefficient of friction. **[Ans. 0·0076]**

36. A force of 3 kg wt. is just sufficient to pull a block of 4 kg over a horizontal surface. What is the angle of friction ? **[Ans. 36° 52′]**

37. A car starts with a velocity of 100 m/s on a half kilometre long bridge. The coefficient of friction between the tyres and road is 1·0. Show that one cannot drive through the bridge in less than 10 s. Take $g = 10$ m/s^2.

38. A train weighing 1000 quintals is running on a level road with a uniform speed of 72 km/h. If the frictional resistance amounts to 50 g wt. per quintal, find power in watt, take $g = 9{\cdot}8$ ms^{-2}.

[Ans. 9800 watt]

39. A body moving on the ground with a velocity of 15 m/s comes to rest after covering a distance of 25 m. If acceleration due to gravity is 10 m/s^2. Calculate the coefficient of friction between the ground and the body. **[Ans. 0·45]**

TYPE VIII. ANGLE OF REPOSE AND MOTION OF A BODY UP AND DOWN A ROUGH INCLINED PLANE

40. A box of mass 4 kg rests upon an inclined plane. This inclination is gradually increased till the box starts sliding down the plane. At this stage, slope of the plane is 1 in 3. Find coefficient of friction between the box and the plane. What force applied to the box parallel to the plane will just make the box move up the plane ? **[Ans. 0·35 ; 26·13 N]**

41. An engine of mass 6·5 metric ton is going up an incline of 5 in 13 at the rate of 9 km/h. Calculate the power of the engine if $\mu = 1/12$ and $g = 9{\cdot}8$ m/s^2. **[Ans. 73·5 kW]**

42. A body of mass m is released from the top of a rough inclined plane of length l and height h. If f is the force of friction prove that the body will reach the bottom with a velocity

$$v = \sqrt{\frac{2}{m}(mgh - f\,l)}$$

43. A block slides down an incline of angle 30° with an acceleration $g/4$. Find the coefficient of kinetic friction. $\left[\textbf{Ans. } \frac{1}{2\sqrt{3}}\right]$

44. A block of mass 2 kg rests on a plane inclined at 30° with the horizontal. The coefficient of friction between the block and the surface is 0·7. What will be the frictional force acting on the block ?

[Ans. 9·8 N]

45. Calculate the power of an engine which can pull a train of mass 25000 quintal up an incline of 1 in 100 at the rate of 10·8 km/h. Resistance due to friction is 2 N/quintal. **[Ans. 8·85 × 10^5 watt]**

46. A mass of 100 kg is resting on a rough inclined plane of angle 30°. If the coefficient of friction is $1/\sqrt{3}$, find the greatest and the least forces that acting parallel to the plane in both cases, just maintain the mass in equilibrium. **[Ans. 100 kg f, zero]**

47. A truck tows a trailor of mass 1200 kg at a speed of 10 m/s on a level road. The tension in the coupling is 1000 N.

(*i*) What is the power expended on the trailor ?

(*ii*) Find tension in the coupling when the truck ascends a road having an inclination of 1 in 6. Assume that the frictional resistance of the incline is same as that on the level road.

[Ans. (*i*) 10^4 W (*ii*) 2960 N]

48. An engine of one metric ton is going up an inclined plane of slope 1 in 2 at the rate of 36 km h^{-1}. If the coefficient of friction is $1/\sqrt{3}$, calculate the power of the engine in kW. **[Ans. 98 kW]**

TYPE IX. MOTION ALONG A HORIZONTAL CIRCLE AND MOTION OF VEHICLE ALONG UNBANKED CIRCULAR TRACKS

49. A ball of mass 0·1 kg is suspended by a string 30 cm long. Keeping the string always taut, the ball describes a horizontal circle of radius 15 cm. Calculate the angular speed of the ball.

[Ans. 6·14 rad/s]

50. What is the greatest speed at which a car having a track width of 1·5 m can turn in a circular path of radius 24·5 m without overturning ? Assume that the centre of gravity of the car is 0·49 m above the ground. Take $g = 9{\cdot}8\ \text{m/s}^2$. **[Ans. 19·2 m/s]**

51. One end of a massless spring of constant 100 N/m and natural length 0·5 m is fixed and the other end is connected to a particle of mass 0·5 kg lying on a frictionless horizontal table. The spring remains horizontal. If the mass is made to rotate at an angular velocity of 2 rad/*s*, find the elongation of the spring. **[Ans. 1 cm]**

52. A body of mass 0·5 kg is whirled in a circle with a velocity of 2 m s^{-1} using 0·5 m length of a string, which can withstand a tension of 15 N. Neglecting the force of gravity on the body, predict whether or not the string will break. Give reasons for your answer.

[Ans. The string will not break as centrifugal force is 4 N, which is less than 15 N]

53. The blades of an aeroplane propeller are 2 m long and rotate at the rate of 300 rpm. Calculate (*i*) the frequency (*ii*) the period of rotation (*iii*) the angular velocity (*iv*) the linear velocity of a point 0·5 m from the tip of the blade.

[Ans. 5 rps ; 0·2 s ; 31·43 rad s^{-1} ; 47·15 m s^{-1}]

54. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0·4 [acc. due to gravity = 10 m/s^2]. **[Ans. 11 m s^{-1}]**

TYPE X. BENDING OF A CYCLIST AND BANKING OF ROADS

55. A curve in a road forms an arc of radius 800 m. If the road is 19·6 m wide and outer edge is 1 m higher than the inner edge, calculate the speed for which it is banked. **[Ans. 20 ms^{-1}]**

56. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing his speed. Will the cyclist slip while taking the turn, if coefficient of friction between the tyres and road is 0·1 ? **[Ans. Yes]**

57. An electric bulb suspended from the roof of a railway train by a flexible wire shifts through an angle of 19° 48′, when the train goes horizontally round a curved path of 200 m radius. Find the speed of the train. **[Ans. 26·56 ms^{-1}]**

58. A 200 kg car has to go over a turn whose radius is 750 m and the angle of slope is 5°. The coefficient of friction between the car wheels and the road is 0·5. What should be the maximum speed of the car so that it may go over the turn without slipping ? Take $g = 9{\cdot}8\ \text{m/s}^2$. **[Ans. 67·2 m/s]**

59. A motor cyclist goes round a circular race course of diameter 320 m at 144 km/h. How far from the vertical must he lean inwards to keep his balance ? Take $g = 10\ \text{m s}^{-2}$ **[Ans. 45°]**

60. The radius of curvature of a railway track at a place, where the train is moving at a speed of 72 km h^{-1} is 625 m. The distance between the rails is 1·5 m. Find the angle and the elevation of the outer rail, so that there may be no side pressure on the rails. Take $g = 9{\cdot}8\ \text{ms}^{-2}$. **[Ans. 3·74° ; 9·78 cm]**

TYPE XI. MOTION IN A VERTICAL CIRCLE

61. A weightless thread can bear tension upto 3·7 kg wt. A stone of mass 500 g is tied to it and revolves in a vertical circle of radius 4 m. What will be the maximum angular velocity of the stone if $g = 10\ \text{m/s}^2$? **[Ans. 4 rad/s]**

62. The speed limit of a car over a roadways bridge in the form of a vertical arc is 9·8 m s^{-1}. Calculate the diameter of the arc. **[Ans. 19·6 m]**

63. A body weighing 0·4 kg is whirled in a vertical circle making 2 rps. If the radius of the circle is 1·2 m, find the tension in the string at (*i*) top of the circle (*ii*) bottom of the circle.

[Ans. $T_H = 71{\cdot}94$ N, $T_L = 79{\cdot}78$ N]

64. A fighter plane is pulling out for a dive at a speed of 900 km/hr. Assuming its path to be a vertical circle of radius 2000 m and its mass to be 16000 kg, find the force exerted by the air on it at the lowest point. Take $g = 9{\cdot}8\ \text{m/s}^2$.

[Ans. $6{\cdot}568 \times 10^5$ N]

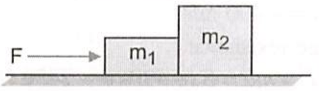
65. A stone of mass 100 g is suspended from the end of a weightless string of length 100 cm and is allowed to swing in a vertical plane. The speed of the mass is 200 cm s^{-1} when the string makes an angle of 60° with the vertical. Calculate the tension in the string at θ = 60°. Also, calculate the speed of the stone when it is in the lowest position. Given $g = 980\ \text{cm s}^{-2}$. **[Ans. $8{\cdot}9 \times 10^4$ dyne, 3·7 m s^{-1}]**

TYPE XII. TYPICAL EXAMPLES

66. Two blocks of masses m_1 and m_2 in contact lie on a horizontal smooth surface, as shown in Fig. 5(Q).13. The blocks are pushed by a force *F*. If the two blocks are always in contact, what is the force at their common interface ?

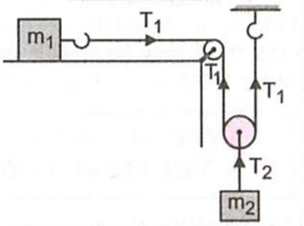
$$\left[\textbf{Ans. } f = \frac{m_2 \times F}{m_1 + m_2}\right]$$

FIGURE 5(Q).13



67. In Fig. 5(Q).14, when we ignore friction and mass of pulley, what would be the accelerations of the two blocks m_1 and m_2 ?

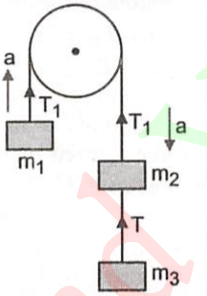
FIGURE 5(Q).14



$$\left[\textbf{Ans. } a_1 = \frac{2m_2 g}{4m_1 + m_2}\ ;\ a_2 = \frac{m_2 g}{4m_1 + m_2}\right]$$

68. In the arrangement shown in Fig. 5(Q).15, show that tension in the string between masses m_2 and m_3 is $T = \dfrac{2m_1 m_3 g}{m_1 + m_2 + m_3}$.

FIGURE 5(Q).15



69. A car of mass one metric ton travelling at 32 m/s dashes into the rear of a truck of mass 8000 kg moving in the same direction with a velocity of 4 m/s. After the collision, the car bounces backwards with a velocity of 8 m/s. What is the velocity of the truck after the impact ? **[Ans. 9 m/s]**

70. A block of mass 2·5 kg is kept on a rough horizontal surface. It is found that the block does not slide if a horizontal force less than 15 N is applied to it. Also, it is found that the block takes 5 s to slide through the first 10 m, if a horizontal force of 15 N is applied and the block is gently pushed to start the motion. Taking $g = 10\ \text{ms}^{-2}$, calculate the coefficient of static and kinetic friction between the block and the surface.

[Ans. 0·6; 0·52]

71. Find the force required to move a train of mass 5000 quintals up an incline of 1 in 50 with an acceleration of 2 m/s^2. Take force of friction = 0·2 N/quintal and $g = 10\ \text{m s}^{-2}$. **[Ans. 1·101 × 10^6 N]**

72. A small block of mass 100 g moves with uniform speed in a horizontal circular groove, with vertical side walls, of radius 25 cm. If the block takes 0·2 s to complete one round, find the normal contact force by the side wall of the groove.

[Ans. 24·65 N]

73. What is the greatest speed at which a car having a track width of 1·5 m can turn in a circular path of radius 24·5 m without overturning ? Assume that the centre of gravity of the car is 0·49 m above the ground. **[Ans. 19·17 m s^{-1}]**

74. A sphere of mass 0·1 kg is attached to an inextensible string of length 1·3 m, whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 0·5 m. Calculate time period of revolution and tension in the string. **[Ans. 2·2 s ; 1·06 N]**

75. A car of mass m moves with a constant speed v over (*a*) horizontal flat surface (*b*) convex bridge (*c*) concave bridge. What force is exerted by the car on the bridge as it passes the middle point of bridge. Radius of curvature of bridge is R

[Ans. $R_1 = mg$; $R_2 < mg$; $R_3 > mg$]

HINTS

For Difficult Problems

2. Velocity on striking the sand,

$$u = \sqrt{2gh} = \sqrt{2 \times 9{\cdot}8 \times 50} = \sqrt{980}\ \text{m/s}$$

$v = 0,\ s = 1\ \text{m},\ a = ?$

From $v^2 - u^2 = 2as$, $0 - 980 = 2 \times a \times 1$

$\therefore\quad a = -490\ \text{m/s}^2$

Average resistance,

$$F = ma = 5(-490) = \mathbf{-2450\ N}$$

From $v = u + at$

$$0 = \sqrt{980} - 490t,\ t = \frac{\sqrt{980}}{490} = \mathbf{0{\cdot}064\,s}$$

3. Let v be the vel. of man on striking the net

$u = 0,\ a = 10\ \text{m/s}^2,\ s = 10\ \text{m}$

From $v^2 - u^2 = 2as$

$v^2 - 0 = 2 \times 10 \times 10,\ v = \sqrt{200} = 10\sqrt{2}\ \text{m/s}$

For the movement of the net,

$u = 10\sqrt{2}$ m/s, $v = 0$, $s = 2$ m, $a = ?$

From $v^2 - u^2 = 2\,a\,s$, $0 - 200 = 2 \times a \times 2$,

$a = -200/4 = -50$ m/s^2

Average force exerted by net $= m \times a = 60\,(-50)$

$= \mathbf{-3000\ N}$

4. Here, $m = 2$ kg, $\dfrac{dm}{dt} = 1$ kg s^{-1},

$v = 5$ m/s, $a = ?$

$$F = ma = \frac{d}{dt}(p) = \frac{d}{dt}(mv) = v\frac{dm}{dt}$$

$$\therefore \quad a = \frac{v\,(dm/dt)}{m} = \frac{5 \times 1}{2} = \mathbf{2{\cdot}5\ m/s^2}$$

5. Here, $F = 10^{-23}$ N, $t = ?$, $s = 0{\cdot}1$ m,

$m = 10^{-30}$ kg, $u = 0$

$$a = \frac{F}{m} = \frac{10^{-23}}{10^{-30}} = 10^7\ \text{m/s}^2$$

From $s = ut + \dfrac{1}{2}at^2$

$$0{\cdot}1 = 0 + \frac{1}{2} \times 10^7\, t^2,\ t^2 = \frac{0{\cdot}2}{10^7} = 2 \times 10^{-8}$$

$$\therefore \quad t = \sqrt{2} \times 10^{-4}\ \text{s} = \mathbf{1{\cdot}4 \times 10^{-4}\ s}$$

6. Here, $m_1 = \dfrac{F}{a_1} = \dfrac{10}{10} = 1\,\text{kg}$

$$m_2 = \frac{F}{a_2} = \frac{10}{20} = 0{\cdot}5\,\text{kg}$$

If a is the acceleration in the combination of masses, then

$$a = \frac{F}{(m_1 + m_2)} = \frac{10}{(1 + 0{\cdot}5)}$$

$= \mathbf{6{\cdot}66\ m/s^2}$

7. Here, $m = 40$ g $= 40 \times 10^{-3}$ kg,

$v = 1200$ m/s, $F = 144$ N, $n = ?$

As F = rate of change of momentum of bullets

$\therefore$ $F = nmv$

$$n = \frac{F}{mv} = \frac{144}{40 \times 10^{-3} \times 1200} = \textbf{3 bullets/sec.}$$

8. Here, $m = 2 \times 10^4$ kg, $F = 5 \times 10^5$ N, $t = 20$ s

$v = ?$, $u = 0$.

As Impulse = change in momentum,

$\therefore$ $F \times t = m\,(v - u) = mv$

$$v = \frac{F \times t}{m} = \frac{5 \times 10^5 \times 20}{2 \times 10^4} = \mathbf{500\ m/s}$$

11. Velocity on striking the ground,

$$v_1 = \sqrt{2\,g\,h_1} = \sqrt{2 \times 9{\cdot}8 \times 1} = 4{\cdot}427\ \text{m/s}$$

Velocity on rebouding $v_2 = \sqrt{2\,g\,h_2}$

$$= \sqrt{2 \times 9{\cdot}8 \times \frac{1}{2}} = 3{\cdot}13\ \text{m/s}$$

Impulse = change in linear momentum

$= mv_1 - m\,(-v_2) = m\,(v_1 + v_2)$

$= 50 \times 10^{-3}\,(4{\cdot}427 + 3{\cdot}13) = \mathbf{0{\cdot}3778\ Ns}$

$$F = \frac{\text{Impulse}}{t} = \frac{0{\cdot}3778}{0{\cdot}1} = \mathbf{3{\cdot}78\,N}$$

12. Here, $m = 2$ kg. $u = 5$ ms^{-1} $v = ?$

Impulse = area under $F - t$ graph

= area of Δ $0ae$ + area of rect. $abfe$ + area of trapezium $bcgf$ + area of rect. $cdhg$

$$= \frac{3 \times 1{\cdot}5}{2} + 3 \times (2{\cdot}5 - 1{\cdot}5)$$

$$+ \left(\frac{3 + 2}{2}\right)(3 - 2{\cdot}5) + 2\,(4 - 3)$$

$= 2{\cdot}25 + 3 + 1{\cdot}25 + 2 = \mathbf{8{\cdot}5\ Ns}$

As Impulse = Change in momentum $= m\,(v - u)$

$\therefore$ $8{\cdot}5 = 2\,(v - 5)$, $\mathbf{v = 9{\cdot}25\ m/s}$

13. Fig. 5(Q).11 shows that at $t = 0$, particle is at rest and after $t = 0$, the particle has uniform velocity.

$$v = \frac{dx}{dt} = \frac{2}{4} = 0{\cdot}5\,\text{m/s}$$

$\therefore$ *Impulse at $t = 0$ is*

$= m\,(v - u)$

$= 0{\cdot}1\,(0{\cdot}5 - 0)$

$= \mathbf{0{\cdot}05\ kg\ m\text{-}s^{-1}}$

Again, Before $t = 4$ s, particle is moving with a uniform velocity $u = 0{\cdot}5$ m/s and after $t = 4$ s, particle comes to rest *i.e.* $v = 0$.

Impulse at $t = 4$ s

$= m\,(v - u) = 0{\cdot}1\,(0 - 0{\cdot}5)$

$= \mathbf{-0{\cdot}05\ kg\ ms^{-1}}$

14. Here, $R = 72$ kg wt $= 72 \times 9{\cdot}8$ N,

$a = ?$, $m = 96$ kg

$m\,(g - a) = R = 72 \times 9{\cdot}8$

$$g - a = \frac{72 \times 9{\cdot}8}{96} = \frac{3}{4}g\ ;$$

$$a = \frac{1}{4}g = \frac{9{\cdot}8}{4} = \mathbf{2{\cdot}45\ m/s^2}$$

15. Here, $m = 4000$ kg, $T = 48000$ N $a = ?$

As $T = m(g + a)$

$$\therefore \quad g + a = \frac{T}{m} = \frac{48000}{4000} = 12$$

$$a = 12 - g = 12 - 9\cdot8 = \mathbf{2\cdot2\ ms^{-2}}$$

(*ii*) $u = 0$, $s = ?$ $t = 3$ sec.

From $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2\cdot2 \times 3^2 = \mathbf{9\cdot9\ m}$

16. Here, $m = 800$ kg, $T = ?$

$u = 20$ m/s, $v = 0$, $s = 50$ m

From $v^2 - u^2 = 2as$,

$$a = \frac{v^2 - u^2}{2s} = \frac{0 - 400}{2 \times 50} = -4\ \text{m/s}^2$$

As elevator is moving downwards

$$\therefore \quad T = m(g - a) = 800\,(9\cdot8 + 4)\ \text{N}$$

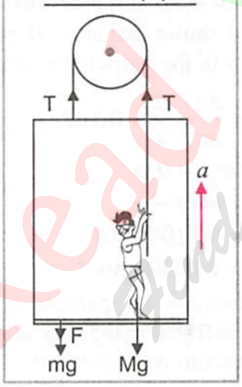
$$= \mathbf{1\cdot104 \times 10^4\ N}$$

17. Here, mass of man, $M = 85$ kg

Mass of lift, $m = 30$ kg

When the man pulls the rope with a force $F = 400$ N, let the upward acceleration of the lift be a. Let T be the tension in the rope, Fig. 5(Q).16. The equation of motion of the man is

FIGURE 5(Q).16



$$85\,a = T + F' - Mg$$

As $F' = F = 400$ N

$$\therefore \quad 85\,a = T + 400 - 85 \times 10 = T - 450 \qquad \textbf{...(}\textbf{\textit{i}}\textbf{)}$$

where 400 N is upward reaction of floor on the man.

The equation of motion of the lift is

$$30\,a = T - 400 - mg = T - 400 - 30 \times 10$$
$$= T - 700 \qquad \textbf{...(}\textbf{\textit{ii}}\textbf{)}$$

On solving (*i*) and (*ii*), we get

$$a = \mathbf{4\cdot55\ m/s^2}$$

18. Here, $m_1 = 2 \times 10^4$ kg, $v_1 = 1\cdot5$ m/s,
$m_2 = 3 \times 10^4$ kg, $v_2 = -2\cdot5$ m/s, $v = ?$

According to principle of conservation of linear momentum,

$$(m_1 + m_2)\,v = m_1 v_1 + m_2 v_2$$

$$(2 \times 10^4 + 3 \times 10^4)\,v = 2 \times 10^4 \times 1\cdot5 + 3 \times 10^4\,(-2\cdot5)$$

$$5 \times 10^4\,v = 3 \times 10^4 - 7\cdot5 \times 10^4 = -4\cdot5 \times 10^4$$

$$v = -\frac{4\cdot5}{5} = \mathbf{-0\cdot9\ m/s}$$

Minus sign indicates, resultant velocity is along v_2.

19. Here, $m_1 = 9$ kg, $m_2 = 30$ g $= 30 \times 10^{-3}$ kg
$v_2 = 300$ m/s, $v_1 = ?$ $p_1 = ?$ $p_2 = ?$

As $m_1 v_1 + m_2 v_2 = 0$

$$v_1 = -\frac{m_2 v_2}{m_1} = -\frac{30 \times 10^{-3} \times 300}{9} = \mathbf{-1\ m/s}$$

Negative sign indicates recoil velocity. Before firing, the gun and bullet, both are at rest and so is the case after firing. Therefore, **their combined momentum in either case is zero.**

20. Mass of machine gun, $M = 20$ kg

mass of each bullet, $m = 25\ g = 25 \times 10^{-3}$ kg

vel. of each bullet, $v_1 = 200$ m/s

No. of bullets/sec, $n = \frac{600}{60} = 10$

If v_2 is recoil velocity of gun, then

$M v_2 = n\,(m v_1)$

$$v_2 = \frac{n\,m\,v_1}{M} = \frac{10 \times 25 \times 10^{-3} \times 200}{20} = \mathbf{2\cdot5\ m/s}$$

$$\text{Force reqd.} = \frac{\text{Change in momentum}}{\text{time}} = \frac{n\,m\,v_1}{t}$$

$$= \frac{10 \times 25 \times 10^{-3} \times 200}{1} = \mathbf{50\ N}$$

21. Here, $v = 15$ m/s, $A = 10^{-2}$ m^2

Volume of water gushing out/sec

$= A \times v = 10^{-2} \times 15$ m^3/s

Mass of water flowing out/sec = Volume × density

$m = 15 \times 10^{-2} \times 10^3 = 150$ kg/s

As water rebounds with the same speed, therefore,

Impact of water = force on the wall

= change in linear momentum /sec

= mass/sec × change in velocity

$= 150 \times (15 + 15) = \mathbf{4500\ N}$

22. Let m be the mass of each fragment.

$$\therefore \qquad p_1 = m \times 9,\ p_2 = m \times 12$$

As $\vec{p_1}$ and $\vec{p_2}$ are perpendicular to each other, therefore, resultant momentum of the two fragments

$$p = \sqrt{p_1^2 + p_2^2} = \sqrt{(9m)^2 + (12m)^2} = 15\,\text{m}$$

According to the principle of conservation of linear momentum

$$\vec{p} + \vec{p_3} = 0$$

$$\vec{p_3} = -\vec{p} = -15\,\text{m}$$

i.e. $m \times v_3 = -15$ m

$$v_3 = \mathbf{-15\ m/s}$$

Negative sign implies that third fragment will fly in a direction opposite to the direction of resultant momentum of the two fragments.

23. Here, $\vec{p_1} = -2p\hat{i} = 2p$ along negative X-axis

$\vec{p_2} = p\hat{j} = p$ along Y-axis

Resultant momentum of the two parts

$$p' = \sqrt{p_1^2 + p_2^2} = \sqrt{(2p)^2 + p^2} = p\sqrt{5}$$

As the bomb was initially at rest, final momentum of all the three parts must be zero.

$$\therefore \quad \vec{p_3} + \vec{p'} = 0$$

$$\vec{p_3} = -\vec{p'} = -p\sqrt{5} = \boldsymbol{p\sqrt{5}} \text{ (in magnitude)}$$

24. Here, $m = 1$ kg

$$m_1 : m_2 : m_3 = 1 : 1 : 3$$

$$\therefore \quad m_1 = \frac{1}{5}\text{kg}, \quad m_2 = \frac{1}{5}\text{kg}, \quad m_3 = \frac{3}{5}\text{kg}$$

$v_1 = v_2 = 30$ m/s, $v_3 = ?$

$$p_1 = p_2 = m_1 v_1 = m_2 v_2$$

$$= \frac{1}{5} \times 30 = 6 \text{ kg m/s}$$

As these two pieces fly off perpendicular to each other, their resultant momentum

$$\sqrt{p_1^2 + p_2^2} = \sqrt{6^2 + 6^2} = 6\sqrt{2}\text{ kg m/s}$$

As the body was initially at rest, therefore from the principle of conservation of linear momentum,

$$p_3 = m_3 v_3 = 6\sqrt{2}$$

$$v_3 = \frac{6\sqrt{2}}{m_3} = \frac{6\sqrt{2}}{3/5} = \mathbf{10\sqrt{2}\,m/s}$$

25. Let F be the upthrust of air in either case.

In (*i*) case, $Mg - F = Ma$ ...(*i*)

In (*ii*) case, $F - (M - m)\,g = (M - m)\,a$ ...(*ii*)

using (*i*), $(Mg - Ma) - Mg + mg = Ma - ma$

or $m\,(g + a) = 2\,Ma$

$$m = \frac{2Ma}{g + a}$$

26. Here, $\dfrac{dm}{dt} = 100$ kg/s, $u = 4{\cdot}5 \times 10^4$ ms^{-1}

Thrust $F = u\dfrac{dm}{dt} = 4{\cdot}5 \times 10^4 \times 100$

$$= \mathbf{4.5 \times 10^6\ N}$$

Also, $v = u\log_e\left(\dfrac{m_0}{m}\right) = 2{\cdot}3026\,u\log_{10}\left(\dfrac{m_0}{m}\right)$

$v = 2{\cdot}3026 \times 4.5 \times 10^4 \log_{10} 10$

$= \mathbf{1{\cdot}036 \times 10^5\ ms^{-1}}$

27. Here, $v = 11.2$ km/s, $u = 1{\cdot}6$ km/s, $\left(\dfrac{m_0}{m}\right) = ?$

From $v = u\log_e\left(\dfrac{m_0}{m}\right)$

$$= 2{\cdot}3026\,u\log_{10}\left(\frac{m_0}{m}\right)$$

$$\log_{10}\left(\frac{m_0}{m}\right) = \frac{v}{2{\cdot}3026 \times u} = \frac{11{\cdot}2}{2{\cdot}3026 \times 1{\cdot}6} = 3{\cdot}0400$$

$$\therefore \quad \frac{m_0}{m} = \text{antilog } 3{\cdot}0400 = \mathbf{1096}$$

28. Here, $m_0 = 6000$ kg, $\dfrac{dm}{dt} = 16$ kg/s,

$u = 11$ km/s $= 11000$ m/s, $a = ?$, $t = 1$ min.$= 60$ s

Mass left after 1 minute $= m_0 - \left(\dfrac{dm}{dt}\right)t$

$= 6000 - 16 \times 60 = 5040$ kg

From $F = m\dfrac{dv}{dt} = u\dfrac{dm}{dt}$, we get

$$a = \frac{dv}{dt} = \frac{u\,(dm/dt)}{m} = \frac{11000 \times 16}{5040} = \mathbf{34{\cdot}92\ m/s^2}$$

29. Here, $u = 2$ km/s, $v = 11{\cdot}2$ km/s, $\dfrac{m_0}{m} = ?$

As rocket starts from rest, therefore,

$$v = u\log_e\left(\frac{m_0}{m}\right) = u \times 2{\cdot}303\log_{10}\left(\frac{m_0}{m}\right)$$

$$\log_{10}\frac{m_0}{m} = \frac{v}{2{\cdot}303u} = \frac{11{\cdot}2}{2{\cdot}303\times 2} = 2{\cdot}432$$

$$\frac{m_0}{m} = \text{antilog } 2{\cdot}432 = \mathbf{270{\cdot}4}$$

30. Here, $\frac{dm}{dt} = 24$ kg/s

$$u = 6{\cdot}4 \text{ km/s} = 6{\cdot}4\times 10^3 \text{ m/s}$$

$$\text{upthrust } = u\left(\frac{dm}{dt}\right) = 6{\cdot}4\times 10^3\times 24$$

$$= \mathbf{1{\cdot}536\times 10^5\ N}$$

As $\quad v = u\log_e\left(\frac{m_0}{m}\right)$

$$\therefore \quad v = 6{\cdot}4\times 10^3 \log_e\left(\frac{m_0}{m_0/100}\right)$$

$$= 6{\cdot}4\times 10^3\times 2{\cdot}303 \log_{10}(100)$$

$$v = 6{\cdot}4\times 10^3\times 2{\cdot}303\times 2$$

$$= \mathbf{2{\cdot}948\times 10^4\ m/s}$$

31. As shown in Fig 5(Q).12, the three forces T_1, T_2 and mg acting at O are in equilibrium. Applying Lami's theorem, we get

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{mg}{\sin 90^\circ}$$

$$T_1 = \frac{mg}{\sin 90^\circ}\times\sin 150^\circ = \frac{10\sin 30^\circ}{1} = \mathbf{5\ N}$$

$$T_2 = \frac{mg}{\sin 90^\circ}\times\sin 120^\circ = \frac{10\sqrt{3}/2}{1} = \mathbf{5\sqrt{3}\ N}$$

32. Here, $m_1 = 11$ kg, $m_2 = 11{\cdot}5$ kg.

$$\therefore \quad a = \frac{(m_2 - m_1)g}{m_2 + m_1} = \frac{(11{\cdot}5 - 11)}{11{\cdot}5 + 11}9{\cdot}8 = \frac{4{\cdot}9}{22{\cdot}5}$$

$$= 0{\cdot}218 \text{ m/s}^2$$

From $v = u + a\,t = 0 + 0{\cdot}218\times 4 = \mathbf{0{\cdot}872\ m/s}$

From $\quad s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 0{\cdot}218(4)^2$

$$= \mathbf{1{\cdot}744\ m}$$

34. Here, $m_1 = 50$ kg, $m_2 = 150$ kg

force appled, $F = ?$

Tension in the string, $T = ?$

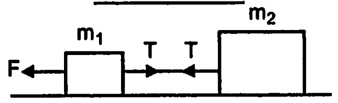
Acceleration in both the bodies,

$a = 5$ ms^{-2}, Fig. 5(Q).17.

$\therefore \quad F = (m_1 + m_2)\,a = (50 + 150)\,5 = \mathbf{1000\ N}$

Also, $T = m_2\,a = 150\times 5 = \mathbf{750\ N}$

FIGURE 5(Q).17



35. Here $P = 100$ HP $= 100\times 746$ watt,

$v = 36$ km h^{-1} $= 10$ ms^{-1}

If F is frictional force overcome by the engine, then

$$P = F\times v,\ F = \frac{P}{v} = \frac{100\times 746}{10} = 7460\,\text{N}$$

$$R = mg = 100\times 1000\times 9{\cdot}8 \text{ N} = 9{\cdot}8\times 10^5$$

$$\mu = \frac{F}{R} = \frac{7460}{9{\cdot}8\times 10^5} = \mathbf{0{\cdot}0076}$$

37. Here, $u = 100$ m/s, $s = \frac{1}{2}$ km $= 500$ m,

$\mu = 1{\cdot}0$

If a is the retardation due to friction, then

$$a = -\mu\, g = -1{\cdot}0\times 10 = -10 \text{ ms}^{-2}$$

From $\quad s = ut + \frac{1}{2}a\,t^2$

$500 = 100\,t - 5\,t^2 \quad$ or $\quad 5\,t^2 - 100\,t + 500 = 0$

or $\quad t^2 - 20\,t + 100 = 0$

or $\quad t = \frac{20\pm\sqrt{400 - 400}}{2} = 10\,\text{s}$

Clearly, **time cannot be less than 10·0 s**

38. Here, $m = 1000$ quintals

$$v = 72 \text{ km h}^{-1} = \frac{72\times 1000}{60\times 60}\text{ m s}^{-1} = 20 \text{ m s}^{-1}$$

Total frictional resistance, $F = 50\times 1000$ g wt.

$= 50000$ g wt. $= 50$ kg wt. $= 50\times 9{\cdot}8$ N

As Power $= F\times v$

$\therefore \quad P = 50\times 9{\cdot}8\times 20 = \mathbf{9800\ watt}$

39. Here, $\quad u = 15$ m/s, $v = 0$, $s = 25$ m,

$g = 10$ m/s^2 $\mu = ?$

From $v^2 - u^2 = 2\,as$.

$$a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 15^2}{2\times 25} = -4{\cdot}5\,\text{m/s}^2$$

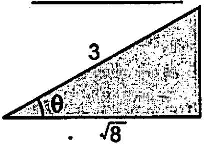
$$\mu = \frac{a}{g} = \frac{4{\cdot}5}{10} = \mathbf{0{\cdot}45}$$

40. Here, $m = 4$ kg, $\sin\theta = \frac{1}{3}$

$\mu = ?,\ F = ?$

$$\mu = \tan\theta = \frac{1}{\sqrt{8}} = \mathbf{0{\cdot}35} \quad \text{[Fig. 5(Q).18]}$$

FIGURE 5(Q).18



As the box is to move up the plane, force of friction is down the plane.

$$F = mg\sin\theta + f = mg\sin\theta + \mu\, mg\cos\theta$$
$$= mg(\sin\theta + \mu\cos\theta)$$
$$= 4\times 9{\cdot}8\left(\frac{1}{3} + 0{\cdot}35\frac{\sqrt{8}}{3}\right) = \mathbf{26{\cdot}13\ N}$$

41. Here, $m = 6{\cdot}5$ metric ton $= 6500$ kg, $\sin\theta = 5/13$

$$v = 9\ \text{km/h} = \frac{9\times 1000}{60\times 60} = 2{\cdot}5\ \text{m/s} \quad P = ? \quad \mu = \frac{1}{12}$$

When $\sin\theta = \frac{5}{13}$, $\cos\theta = \sqrt{1-(5/13)^2} = 12/13.$

$$P = mg(\sin\theta + \mu\cos\theta)\times v$$
$$= 6500\times 9{\cdot}8\left(\frac{5}{13} + \frac{1}{12}\times\frac{12}{13}\right)\times 2{\cdot}5$$
$$= 6500\times 9{\cdot}8\times\frac{6}{13}\times 2{\cdot}5 = 73500\ \text{W} = \mathbf{73{\cdot}5\ kW}$$

42. On reaching the bottom, loss in P.E. $= mgh$.

Gain in K.E. $= \frac{1}{2}mv^2$ and

work done against friction $= f\times l$.

According to the law of conservation of energy,

$$\frac{1}{2}mv^2 + f\times l = mgh$$
$$v = \sqrt{\frac{2}{m}(mgh - f\times l)}$$

which was to be proved.

43. As the block is sliding down with $a = \frac{g}{4}$,

$$\therefore\quad ma = mg\sin\theta - F$$
$$\frac{mg}{4} = mg\sin 30^\circ - F = \frac{mg}{2} - F$$
$$F = \frac{mg}{2} - \frac{mg}{4} = \frac{mg}{4}$$

Also, $R = mg\cos\theta = mg\cos 30^\circ = mg\frac{\sqrt{3}}{2}$

Now, $\mu_k = \frac{F}{R} = \frac{1}{2\sqrt{3}}$

44. As the block is at rest therefore, maximum force of static friction

$$f_s = \mu_s R \quad \text{But} \quad R = m\,g\cos\theta$$
$$\therefore\quad f_s = \mu_s m\,g\cos\theta = 0{\cdot}7\times 2\times 9{\cdot}8\cos 30^\circ$$
$$= 1{\cdot}4\times 9{\cdot}8\times 0{\cdot}866 = 11{\cdot}9\ \text{N}$$

But in this case, actual force of static friction cannot exceed the force due to gravity ($mg\sin\theta$) acting down the plane.

∴ Actual force of static friction

$$= mg\sin\theta = 2\times 9{\cdot}8\times\sin 30^\circ = \mathbf{9{\cdot}8\ N}$$

46. Here, $m = 100$ kg. $\theta = 30^\circ$; $\mu = \frac{1}{\sqrt{3}}$

Force acting on the block down the plane

$$= mg\sin\theta = 100\,g\sin 30^\circ = 50\,g\ \text{N}$$
$$= 50\ \text{kg}\,f$$

Force of friction $= f = \mu R = \mu\, mg\cos\theta$

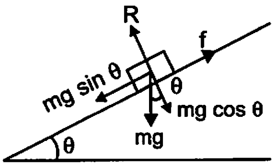
$$= \frac{1}{\sqrt{3}}mg\cos 30^\circ = \frac{1}{\sqrt{3}}100\,g\frac{\sqrt{3}}{2} = 50\,g\,\text{N}$$
$$= 50\ \text{kg}\,f$$

As the force of friction just balances the force acting on the block down the plane, therefore, mass will be in equilibrium [Fig. 5(Q).19]. The least force required is **zero**.

When we tend to move the mass up the inclined plane, force of friction acts down the plane. Therefore, greatest force required just to start upward motion (or to maintain the block in equilibrium) $= mg\sin\theta + f$

$$= 50\ \text{kg}\,f + 50\ \text{kg}\,f = \mathbf{100\ kg}\,\boldsymbol{f}$$

FIGURE 5(Q).19



47. Here, $m = 1200$ kg, $v = 10$ m/s, $T = 1000$ N

$$P = ? \quad \sin\theta = \frac{1}{6}, \quad T' = ?$$

On the level road, force applied by the truck = force of friction overcome

$$\therefore\quad f = T = 1000\ \text{N}$$

Power expended on the trailor,

$$P = f\times v = 1000\times 10 = \mathbf{10^4\ W}$$

When the truck ascends a road, it has to overcome friction and component $mg\sin\theta$ of the weight of trailor.

$\therefore \quad F = mg \sin\theta + f$

$$= 1200 \times 9{\cdot}8 \times \frac{1}{6} + 1000 = 1960 + 1000$$

$$= \mathbf{2960\ N}$$

48. Here, $m = 1$ metric ton $= 10^3$ kg

$$\sin\theta = 1/2,\ v = 36\,\text{km/h} = \frac{36 \times 1000}{60 \times 60}\ \text{m/s} = 10\ \text{m/s}$$

$$\mu = \frac{1}{\sqrt{3}} \quad P = ?$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

As is known, force required,

$$F = mg \sin\theta + f = mg \sin\theta + \mu R$$

$$= mg \sin\theta + \mu\, mg \cos\theta$$

$$= mg\,(\sin\theta + \mu \cos\theta)$$

$$= 10^3 \times 9{\cdot}8 \left(\frac{1}{2} + \frac{1}{\sqrt{3}}\,\frac{\sqrt{3}}{2}\right)$$

$$F = 9{\cdot}8 \times 10^3\ \text{N}$$

Power of engine, $P = F \times v = 9{\cdot}8 \times 10^3 \times 10$ watt

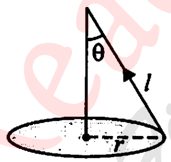
$P = \mathbf{98\ kW}$

49. Here, $m = 0{\cdot}1$ kg, $l = 30$ cm $= 0{\cdot}3$ m,

$r = 15$ cm $= 0{\cdot}15$ m, $\omega = ?$

If the string makes an angle θ with the vertical, Fig. 5(Q).20, then $\sin\theta = \frac{r}{l} = \frac{15}{30} = \frac{1}{2} \quad \therefore \quad \theta = 30°$

FIGURE 5(Q).20



From $\quad \tan\theta = \frac{r\omega^2}{g}$

$$\omega = \sqrt{\frac{g \tan\theta}{r}} = \sqrt{\frac{9{\cdot}8 \tan 30°}{0{\cdot}15}} = \mathbf{6{\cdot}14\ rad/s}$$

50. Here, $2x = 1{\cdot}5$ m, $x = \frac{1{\cdot}5}{2} = 0{\cdot}75$ m

$r = 24{\cdot}5$ m, $v = ?$

$h = 0{\cdot}49$ m, $g = 9{\cdot}8$ m/s^2

$$v = \sqrt{\frac{g\,r\,x}{h}} = \sqrt{\frac{9{\cdot}8 \times 24{\cdot}5 \times 0{\cdot}75}{0{\cdot}49}} = \mathbf{19{\cdot}2\ m/s}$$

51. Here, $k = 100$ N/m, $l_0 = 0{\cdot}5$ m, $m = 0{\cdot}5$ kg

$\omega = 2$ rad/s

Let elongation of the spring be l.

$\therefore F = kl$, which provides the necessary centripetal force $m\,\omega^2 r = m\omega^2 (l + l_0)$.

$\therefore kl = m\omega^2 (l + l_0)$

$$l = \frac{m\,\omega^2\, l_0}{k - m\omega^2} = \frac{0{\cdot}5 \times 2^2 \times 0{\cdot}5}{100 - 0{\cdot}5 \times 4} = \frac{1}{98}\,\text{m} = \mathbf{1\ cm}$$

54. $\quad v = \sqrt{\mu\, r\, g} = \sqrt{0{\cdot}4 \times 30 \times 10} = \mathbf{11\ m/s}$

55. $r = 800$ m, $b = 19{\cdot}6$ m, $h = 1$ m, $v = ?$

$$\tan\theta = \frac{v^2}{rg} = \frac{h}{\sqrt{b^2 - h^2}}$$

$$v = \left[\frac{hrg}{\sqrt{b^2 - h^2}}\right]^{1/2} = \left[\frac{1 \times 800 \times 9{\cdot}8}{\sqrt{19{\cdot}6^2 - 1^2}}\right]^{1/2}$$

$$= \mathbf{20\ m/s}$$

56. Here, $v = 18$ km/h $= \frac{18 \times 1000}{60 \times 60} = 5\,\text{m/s}$

$r = 3$ m, $\mu = 0{\cdot}1$

The maximum speed at which cyclist can take turn

$$= v = \sqrt{\mu\, rg} = \sqrt{0{\cdot}1 \times 3 \times 9{\cdot}8} = 1{\cdot}715\,\text{m/s}$$

As actual speed is 5 m/s, which is greater than the safe speed, **the cyclist will slip**

58. Here, $m = 2000$ kg, $r = 750$ m,

$\theta = 5°$, $\mu_s = 0{\cdot}5$, $v_{\max} = ?$

$$v_{\max} = \left[\frac{rg\ (\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$$

$$= \left[\frac{750 \times 9{\cdot}8\ (0{\cdot}5 + \tan 5°)}{1 - 0{\cdot}5 \tan 5°}\right]^{1/2}$$

$$= \left[\frac{750 \times 9{\cdot}8\ (0{\cdot}5 + 0{\cdot}087)}{1 - 0{\cdot}5 \times 0{\cdot}087}\right]^{1/2}$$

$v_{\max} = \mathbf{67{\cdot}2\ m/s}$

60. Here, $v = 72$ km h^{-1} $= \frac{72 \times 1000}{60 \times 60} = 20$ ms^{-1}

$r = 625$ m, $b = 1{\cdot}5$ m, $\theta = ?$ $h = ?$

As $\quad \tan\theta = \frac{v^2}{rg} \quad \therefore \tan\theta = \frac{20 \times 20}{625 \times 9{\cdot}8} = 0{\cdot}06531$;

$\theta = \tan^{-1}(0{\cdot}06531) = \mathbf{3{\cdot}74°}$

Again ; $\tan\theta = \frac{h}{\sqrt{b^2 - h^2}}$

$$h = \tan\theta\sqrt{b^2-h^2} = 0{\cdot}06531\sqrt{b^2-h^2}$$

$$h^2 = 0{\cdot}004265\,(b^2-h^2)$$

$$1{\cdot}004265\,h^2 = 0{\cdot}004265\,(1{\cdot}5)^2 = 0{\cdot}009596$$

$$h^2 = \frac{0{\cdot}009596}{1{\cdot}004265} = 0{\cdot}009555$$

$$h = 0{\cdot}09775\text{ m} = \mathbf{9{\cdot}78\ cm}$$

61. Here, $T_{max} = 3{\cdot}7$ kg wt $= 37$ N,

$m = 500\text{ g} = 0{\cdot}5\text{ kg}$

$r = 4\text{ m},\ \omega = ?\ g = 10\text{ m/s}^2$

As $$T_{max} = \frac{mv^2}{r} + mg$$

$$\therefore\ \frac{mv^2}{r} = T_{max} - mg = 37 - 0{\cdot}5\times 10 = 32$$

$$v^2 = \frac{32\,r}{m} = \frac{32\times 4}{0{\cdot}5} = 256,$$

$$v = \sqrt{256} = 16\text{ m/s}$$

$$\omega = \frac{v}{r} = \frac{16}{4} = \mathbf{4\ rad/s}$$

63. Here, $m = 0{\cdot}4$ kg , $n = 2$ *rps*, $r = 1{\cdot}2$ m

$T_H = ?\ T_L = ?$

From $T_H + m\,g = m\,r\,\omega^2 = 4\,\pi^2\,m\,r\,n^2$

$$T_H = 4\,\pi^2\,m\,r\,n^2 - m\,g$$

$$= 4\times\frac{22}{7}\times\frac{22}{7}\times 0{\cdot}4\times 1{\cdot}2\times 4 - 0{\cdot}4\times 9{\cdot}8$$

$$T_H = 75{\cdot}86 - 3{\cdot}92 = \mathbf{71{\cdot}94\ N}$$

Again, $T_L - m\,g = m\,r\,\omega^2 = 4\,\pi^2\,m\,r\,n^2$

$$T_L = 4\,\pi^2\,m\,r\,n^2 + mg$$

$$= 75{\cdot}86 + 3{\cdot}92 = \mathbf{79{\cdot}78\ N}$$

64. Here, $v = 900$ *km/h* $= \dfrac{900\times 1000}{60\times 60} = 250$ m/s.

$r = 2000$ *m*, $m = 16000$ kg, $F = ?$

At the lowest point, centripetal force ($m\,v^2/r$) is provided by the net force ($F - mg$) towards the centre of the circle.

$$\therefore\ F - mg = \frac{mv^2}{r}$$

$$F = m\,(g + v^2/r) = 16000\left(9{\cdot}8 + \frac{250\times 250}{2000}\right)$$

$$F = \mathbf{6{\cdot}568\times 10^5\ N}$$

65. Here, $m = 100$ g, $r = 100$ cm, $v = 200$ cm/s, $\theta = 60°$

$$T = \frac{m\,v^2}{r} + mg\cos\theta$$

$$= \frac{10^2\times(200)^2}{100} + 100\times 980\cos 60°$$

$$= \mathbf{8{\cdot}9\times 10^4\ dyne}$$

At $\theta = 60°$, height above the lowest point

$$h = r - r\cos 60° = r - \frac{r}{2} = \frac{r}{2}$$

As total energy at L = total energy at 60°

$$\therefore\ \frac{1}{2}m\,v_L^2 = \frac{1}{2}m\,v^2 + mg\,(r/2)$$

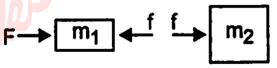
$$v_L = \sqrt{v^2 + g\,r} = \sqrt{2^2 + 9{\cdot}8\times 1} = \mathbf{3{\cdot}7\ m/s}$$

66. Let a be the common acceleration of the combination of blocks.

$$\therefore\ a = \frac{F}{m_1 + m_2} \qquad \text{...}(i)$$

If f is force exerted by m_1 on m_2 then force exerted by m_2 on m_1 is $(-)f$; as shown in Fig. 5(Q).21.

FIGURE 5(Q).21



As block of mass m_2 has acceleration, a.

$$\therefore\ f = m_2\,a = \frac{m_2\times F}{m_1 + m_2}$$

67. From Fig. 5(Q).14, it is clear that when block m_1 moves to the right through a distance x, block m_2 moves down through distance $x/2$. Therefore, $a_2 = a_1/2$.

Applying Newton's 2nd law, $T_1 = m_1\,a_1$

$\therefore\ T_2 = 2\,T_1$. Equation of motion of m_2 is

$$m_2\,g - T_2 = m_2\,a_2 \quad\text{or}\quad m_2\,g - 2T_1 = m_2\frac{a_1}{2}$$

$$\text{or}\quad m_2\,g - 2m_1\,a_1 = m_2\frac{a_1}{2} \quad\text{or}\quad a_1 = \frac{2m_2\,g}{4m_1 + m_2}$$

And $$a_2 = \frac{a_1}{2} = \frac{m_2\,g}{4m_1 + m_2}$$

68. Let T_1 be the tension in the string between m_1 and m_2 and T be the tension in the string between m_2 and m_3. If a is acceleration of the system in the direction shown, then

$$m_1\,a = T_1 - m_1\,g \qquad \text{...}(i)$$

$$(m_2 + m_3)\,a = (m_2 + m_3)\,g - T_1 \qquad \text{...}(ii)$$

and $$m_3\,a = m_3\,g - T \qquad \text{...}(iii)$$

Adding (*i*) and (*ii*),

$(m_1 + m_2 + m_3)\, a = (m_2 + m_3 - m_1)\, g$

$a = (m_2 + m_3 - m_1)\, g/(m_1 + m_2 + m_3)$

Put in (*iii*), $\dfrac{m_3(m_2 + m_3 - m_1)\, g}{m_1 + m_2 + m_3} = m_3\, g - T$

$$T = m_3\, g - \frac{m_3(m_2 + m_3 - m_1)\, g}{m_1 + m_2 + m_3}$$

$$\mathbf{T = 2\, m_1\, m_3\, g/(m_1 + m_2 + m_3)}$$

69. Here, $m_1 = 1$ metric ton $= 1000$ kg

$u_1 = 32$ m/s, $m_2 = 8000$ kg., $u_2 = 4$ m/s

$v_1 = -8$ m/s $\quad v_2 = ?$

Applying the principle of conservation of linear momentum, $m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$

$1000\,(-8) + 8000\,(v_2) = 1000 \times 32 + 8000 \times 4$

$8000\, v_2 = 64000 + 8000 = 72000$

$$v_2 = \frac{72000}{8000} = \mathbf{9\, m/s}$$

70. Here, $m = 2{\cdot}5$ kg, $F = 15$ N

$$\mu_s = \frac{F}{R} = \frac{F}{mg} = \frac{15}{2{\cdot}5 \times 10} = \mathbf{0{\cdot}6}$$

From $s = ut + \dfrac{1}{2} at^2$

$$10 = 0 + \frac{1}{2} \times a\,(5)^2,\; a = \frac{20}{25} = 0{\cdot}8 \text{ ms}^{-2}$$

Using Newton's 2nd law, $F - \mu_k\, mg = m\, a$

$$\mu_k = \frac{F - ma}{mg} = \frac{15 - 2{\cdot}5 \times 0{\cdot}8}{2{\cdot}5 \times 10} = \frac{13}{25} = \mathbf{0{\cdot}52}$$

71. Here, $m = 5000$ quintals $= 5 \times 10^5$ kg,

$\sin\theta = 1/50$, $a = 2$ m/s^2. $F = 0{\cdot}2$N/quintal

$= 0{\cdot}2 \times 5000 = 10^3$ N.

Total force required, $f = mg \sin\theta + F + ma$

$$f = 5 \times 10^5 \times 10 \times \frac{1}{50} + 10^3 + 5 \times 10^5 \times 2$$

$$= 10^5 + 10^3 + 10^6 = 10^6\,(0{\cdot}1 + 0{\cdot}001 + 1)$$

$$\mathbf{f = 1{\cdot}101 \times 10^6\ N}$$

72. Speed of the block is

$$v = \frac{2\pi r}{t} = 2 \times \frac{22}{7}\frac{(0{\cdot}25)}{0{\cdot}2}$$

$$a = \frac{v^2}{r} = \frac{(7{\cdot}85)^2}{0{\cdot}25} = 246{\cdot}49 \text{ m/s}^2$$

This acceleration is towards the centre of the circular groove. The only force in this direction is normal contact force due to side walls.

$\therefore\quad R = ma = 0{\cdot}1 \times 246{\cdot}49 = \mathbf{24{\cdot}65\ N}$

73. Here, $b = 1{\cdot}5$ m, $r = 24{\cdot}5$ m,

$h = 0{\cdot}49$ m ; $v = ?$

As the road is unbanked, therefore, for equilibrium ;

$$\frac{mv^2}{r} \times h = mg \times \left(\frac{b}{2}\right)$$

or $\quad v = \sqrt{\dfrac{g\,r\,b}{2h}} = \sqrt{\dfrac{9{\cdot}8 \times 24{\cdot}5 \times 1{\cdot}5}{2 \times 0{\cdot}49}}$

$= \mathbf{19{\cdot}17\ m/s}$

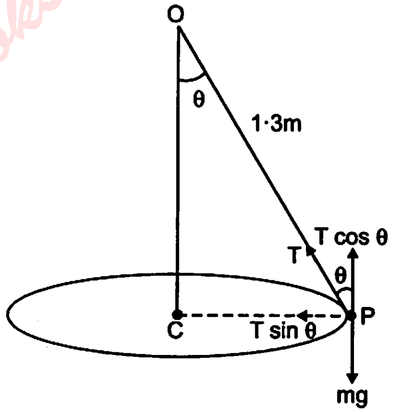
74. Here, $m = 0{\cdot}1$ kg, $OP = l = 1{\cdot}3$ m, $CP = r = 0{\cdot}5$ m time period, $t = ?$, tension $T = ?$

$$\sin\theta = \frac{r}{l} = \frac{0{\cdot}5}{1{\cdot}3} = 0{\cdot}3846$$

$$\theta = \sin^{-1}(0{\cdot}3846) = 22{\cdot}62°$$

As is clear from Fig. 5(Q).22.

FIGURE 5(Q).22



$T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{0{\cdot}1 \times 9{\cdot}8}{\cos 22{\cdot}62°} = \frac{0{\cdot}98}{0{\cdot}9231} = \mathbf{1{\cdot}06\ N}$$

Also, $T\sin\theta = mr\,\omega^2 = \dfrac{mr\, 4\pi^2}{t^2}$

$$\therefore\quad t^2 = \frac{4\pi^2\, mr}{T\sin\theta}$$

$$= 4 \times \frac{22}{7} \times \frac{22}{7} \times \frac{0{\cdot}1 \times 0{\cdot}5}{1{\cdot}06 \times 0{\cdot}3846} = 4{\cdot}846$$

$$t = \sqrt{4{\cdot}846} = \mathbf{2{\cdot}2s}$$

75. In Fig. 5(Q).23(*a*), R_1 is just equal and opposite to weight of car

i.e. $R_1 = mg$ ...(*i*)

In Fig. 5(Q).23(*b*), bridge is *convex*

$$mg - R_2 = \frac{mv^2}{r};$$

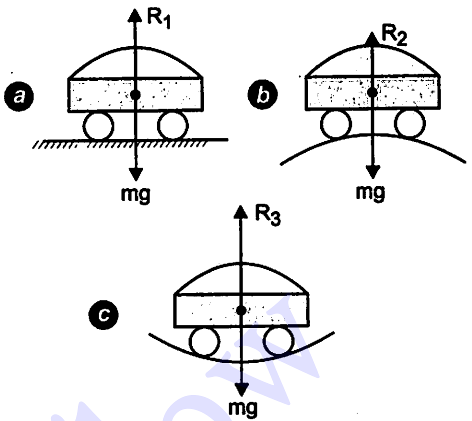
$$R_2 = mg - \frac{mv^2}{r}; \; R_2 < mg$$

In Fig. 5(Q).23(*c*), bridge is *concave*

$$R_3 - mg = \frac{mv^2}{r},$$

$$R_3 = mg + \frac{mv^2}{r}; \; R_3 > mg$$

FIGURE 5(Q).23



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NCERT EXERCISES WITH SOLUTIONS

[For simplicity in numerical calculations, take $g = 10\ \text{ms}^{-2}$]

Q. 1. Give the magnitude and direction of the net force acting on

(a) a drop of rain falling down with a constant speed

(b) a cork of mass 10 g floating on water

(c) a kite skilfully held stationary in the sky

(d) a car moving with a constant velocity of 30 km/h on a rough road.

(e) a high speed electron in space far from all gravitational objects, and free of electric and magnetic fields.

Sol. (*a*) As the rain drop is falling with a *constant* speed, its acceleration $a = 0$. Hence net force $F = ma = 0$.

(*b*) As the cork is floating on water, its weight is being balanced by the upthrust (equal to weight of water displaced). Hence net force on the cork is zero.

(*c*) As the kite is held stationary, net force on the kite is zero, in accordance with Newton's first law.

(*d*) Force is being applied to overcome the force of friction. But as velocity of the car is constant, its acc. $a = 0$. Hence net force on the car $F = ma = 0$.

(*e*) As no field (gravitational/electric/magnetic) is acting on the electron, net force on it is zero.

Q. 2. A pebble of mass 0·05 kg is thrown vertically upwards. Give the magnitude and direction of net force on the pebble, (a) during its upward motion, (b) during its downward motion, (c) at the highest point, where it is momentarily at rest.

Do your answers change if the pebble were thrown at an angle of, say 45° with the horizontal direction ?

Ignore air resistance.

Sol. Whenever a body is thrown vertically upwards or it moves vertically downwards, gravitational pull of earth gives it a uniform acceleration $a = +g = +10\ \text{ms}^{-2}$ in the *downward direction*. Therefore, the net force on the pebble in all the three cases is *vertically downwards*.

As $m = 0{\cdot}05$ kg and $a = +10\ \text{m/s}^2$.

$\therefore$ In all the three cases, $F = ma = 0{\cdot}05 \times 10 = 0{\cdot}5$ N, vertically downwards.

If the pebble were thrown at an angle of 45° with the horizontal direction, it will have horizontal and vertical components of velocity. These components do not affect the force on the pebble. Hence our answers do not alter in any case. However, in case (*c*), the pebble will not be at rest. It will have horizontal component of velocity at the highest point.

Q. 3. Give the magnitude and direction of the net force acting on a stone of mass 0·1 kg

(a) just after it is dropped from the window of a stationary train.

(b) just after it is dropped from the window of a train running at a constant velocity of 36 km/hr

(c) just after it is dropped from the window of a train accelerating with 1 ms^{-2}

(d) lying on the floor of a train, which is accelerating with 1 m s^{-2}, the stone being at rest relative to the train.

Neglect the resistance of air throughout.

Sol. (*a*) Here, $m = 0{\cdot}1$ kg, $a = +g = 10\ \text{m}/s^2$

Net force, $F = ma = 0{\cdot}1 \times 10 = 1{\cdot}0$ N

This force acts vertically downwards.

(*b*) When the train is running at a constant velocity, its acc. = 0. No force acts on the stone due to this motion. Therefore, force on the stone F = weight of stone = $mg = 0{\cdot}1 \times 10 = 1{\cdot}0$ N

This force also acts vertically downwards.

(*c*) When the train is accelerating with 1 m s^{-2}, an additional force $F' = ma = 0{\cdot}1 \times 1 = 0{\cdot}1$ N acts on the stone in the horizontal direction. But once the stone is dropped from the train, F' becomes zero and the net force on the stone is $F = mg = 0{\cdot}1 \times 10 = 1{\cdot}0$ N, acting vertically downwards.

(*d*) As the stone is lying on the floor of the train, its acceleration is same as that of the train.

$\therefore$ force acting on stone, $F = ma = 0{\cdot}1 \times 1 = 0{\cdot}1$ N

This force is along the horizontal direction of motion of the train. Note that weight of the stone in this case is being balanced by the normal reaction.

Q. 4. One end of a string of length r is connected to a particle of mass m and the other end to a small peg on a smooth horizontal surface. If the particle moves in a circle with speed v, the net force on the particle (directed towards centre) is (i) T (ii) $T - \frac{mv^2}{r}$ (iii) $T + \frac{mv^2}{r}$ (iv) 0

T is tension in the string. Choose correct alternative.

Sol. The net force on the particle directed towards the centre is T. This provides the necessary centripetal force to the particle moving in the circle.

Q. 5. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long does the body take to stop ?

Sol. Here, $F = -50$ N, $m = 20$ kg , $u = 15$ ms^{-1}, $v = 0$, t = ?

From $F = ma$, $a = \frac{F}{m} = \frac{-50}{20} = -2{\cdot}5$ ms^{-2}

From $v = u + at$

$$0 = 15 - 2{\cdot}5\,t$$

$$t = \frac{15}{2{\cdot}5} = 6\text{ s}$$

Q. 6. A constant force acting on a body of mass 3·0 kg changes its speed from 2·0 m/s to 3·5 m/s in 25s. The direction of motion of the body remains unchanged. What is the magnitude and direction of the force ?

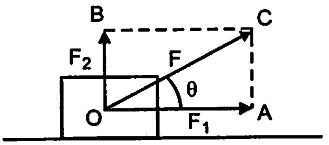
Sol. Here, $m = 3{\cdot}0$ kg, $u = 2{\cdot}0$ ms^{-1}, $v = 3{\cdot}5$ ms^{-1}, $t = 25$ s, F = ?

$$F = ma = \frac{m\,(v-u)}{t} = \frac{3{\cdot}0\,(3{\cdot}5 - 2{\cdot}0)}{25} = 0{\cdot}18\text{ N}$$

The force is along the direction of motion.

Q. 7. A body of mass 5 kg is acted upon by two perpendicular forces 8N and 6N. Give the magnitude and direction of the acceleration of the body.

FIGURE 5(N).1



Sol. Here, $m = 5$ kg, $\vec{a}$ = ?

$$\vec{F_1} = \overrightarrow{OA} = 8\text{ N}\,;\quad \vec{F_2} = \overrightarrow{OB} = 6\text{ N}$$

Resultant force, $\vec{F} = \overrightarrow{OC} = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2} = 10$ N

If $\angle AOC = \theta$, $\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{6}{8} = 0{\cdot}75 \quad \therefore \quad \theta = 36^\circ\,52'$

This is the direction of resultant force and hence the direction of acceleration of the body, Fig. 5(N).1.

Also, $$a = \frac{F}{m} = \frac{10}{5} = 2\text{ ms}^{-2}$$

Q. 8. The driver of a three wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4s just in time to save the child. What is the average retarding force on the vehicle ? The mass of the three wheeler is 400 kg and the mass of the driver is 65 kg.

Sol. Here, $u = 36 \text{ km/}h = 10 \text{ m/s}, v = 0, t = 4\text{s}, \; m = 400 + 65 = 465 \text{ kg}$

Retarding force $= F = ma = \dfrac{m(v-u)}{t} = \dfrac{465\,(0-10)}{4} = \mathbf{-\,1162{\cdot}5\ N}$

Q. 9. A rocket with a lift off mass 20000 kg is blasted upwards with a net initial acceleration of 5 ms^{-2}. Calculate the initial thrust (force) of the blast.

Sol. Here, $m = 20000 \text{ kg} = 2 \times 10^4 \text{ kg}$; Initial acc. $= 5 \text{ ms}^{-2}$; Thrust, $F = ?$

Clearly, the thrust should be such that it overcomes the force of gravity besides giving it an upward acceleration of 5 ms^{-2}. Thus the force should produce a net acceleration of $9{\cdot}8 + 5{\cdot}0 = 14{\cdot}8 \text{ ms}^{-2}$.

As thrust = force = mass × acceleration $\therefore\ F = 2 \times 10^4 \times 14{\cdot}8 = \mathbf{2{\cdot}96 \times 10^5\ N}$

Q. 10. A body of mass 0·40 kg moving initially with a constant speed of 10 m/s to the north is subjected to a constant force of 8·0 N directed towards the south for 30 s. Take the instant the force is applied to be $t = 0$, and the position of the particle at that time to be $x = 0$, predict its position at $t = -5$ s, 25 s, 100 s ?

Sol. Here, $m = 0{\cdot}40$ kg, $u = 10$ m/s due N, $F = -\,8{\cdot}0$ N (minus sign for opposite direction of force)

$$a = \frac{F}{m} = \frac{-8{\cdot}0}{0{\cdot}40} = -20 \text{ ms}^{-2} \quad \text{for } 0 \le t \le 30 \text{ s}$$

(*i*) At $t = -\,5$ s, $x = ut = 10 \times (-\,5) = \mathbf{-\,50\ m}$

(*ii*) At $t = 25$ s, $x = ut + \dfrac{1}{2}at^2 = 10 \times 25 + \dfrac{1}{2}(-\,20)\,(25)^2 = \mathbf{-\,6000\ m}$

(*iii*) At $t = 100$ s, The problem is divided into two parts. Upto 30 s, there is force/acc.

$\therefore$ from $x_1 = ut + \dfrac{1}{2}at^2 = 10 \times 30 + \dfrac{1}{2}(-\,20)\,(30)^2 = -\,8700$ m

At $t = 30$ s, $v = u + at = 10 - 20 \times 30 = -\,590$ m/s, $\therefore$ for motion from 30 s to 100 s

$x_2 = vt = -\,590 \times 70 = -\,41300$ m. $\therefore\quad x = x_1 + x_2 = -\,8700 - 41300 = -\,50000 \text{ m} = \mathbf{-\,50\ km}$

Q. 11. A truck starts from rest and accelerates uniformly with 2 ms^{-2}. At $t = 10$ s, a stone is dropped by a person standing on the top of the truck (6 m high from ground). What are the (a) velocity and (b) acceleration of the stone at $t = 11$ s ? Neglect air resistance.

Sol. Here, $u = 0, a = 2 \text{ ms}^{-2}, t = 10$ s

Let v be the velocity of the truck when the stone is dropped.

From $v = u + at$

$v = 0 + 2 \times 10 = 20 \text{ ms}^{-1}$

(*a*) Horizontal velocity of stone, when it is dropped, $v_x = v = 20 \text{ m s}^{-1}$.

As air resistance is neglected, v_x remains constant.

In the vertical direction, initial velocity of stone, $u = 0$, $a = g = 10 \text{ ms}^{-2}$

time $t = 11 - 10 = 1$ s

From $v = u + a\,t$, $v_y = 0 + 10 \times 1 = 10 \text{ ms}^{-1}$

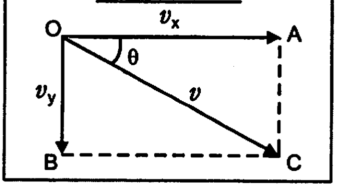


FIGURE 5(N).2

Resultant velocity of stone, OC is given by $v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 10^2} = \mathbf{22{\cdot}4\ ms^{-1}}$

If θ is the angle which the resultant velocity OC of stone makes with the horizontal direction OA, then

from Fig. 5(N).2, $\quad \tan\theta = \dfrac{v_y}{v_x} = \dfrac{10}{20} = 0{\cdot}5 \quad \therefore \quad \mathbf{\theta = 26{\cdot}6°}$

(*b*) The moment the stone is dropped from the truck, horizontal force on the stone = 0. The only acceleration of the stone is that due to gravity $a = +\,g = 10 \text{ ms}^{-2}$ in the vertically downward direction. The path followed by the stone is, however, parabolic.

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Q. 12. A bob of mass 0·1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 m/s. What is the trajectory of the bob, if the string is cut when the bob is (a) at one of its extreme positions (b) at its mean position ?

Sol. (*a*) We shall study in unit 10 that at each extreme position, velocity of the bob is zero. If the string is cut at the extreme position, bob is only under the action of '*g*'. Hence the bob will fall **vertically downwards.**

(*b*) At the mean position, velocity of the bob is 1 ms^{-1}, along the tangent to the arc, which is in the horizontal direction. If the string is cut at mean position, the bob will behave as a horizontal projectile. Hence it will **follow a parabolic path.**

Q. 13. A man of mass 70 kg stands on a weighing machine in a lift, which is moving (a) upwards with a uniform speed of 10 ms^{-1}. (b) downwards with a uniform acceleration of 5 ms^{-2}.

(c) upwards with a uniform acceleration of 5 ms^{-2}.

What would be the readings on the scale in each case ?

(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity ?

Sol. Here, $m = 70$ kg, $g = 10$ m/s^2

The weighing machine in each case measures the *reaction R i.e.* the apparent weight.

(*a*) When the lift moves upwards with a uniform speed, its *acceleration is zero.*

$\therefore$ $R = mg = 70 \times 10 = \mathbf{700\ N}$

(*b*) When the lift *moves downwards* with $a = 5$ ms^{-2} $R = m(g - a) = 70(10 - 5) = \mathbf{350\ N}$

(*c*) When the lift *moves upwards* with $a = 5$ ms^{-2} $R = m(g + a) = 70(10 + 5) = \mathbf{1050\ N}$

If the lift were to come down freely under gravity, downward acc.

$$a = g \quad \therefore \quad R = m(g - a) = m(g - g) = \textbf{Zero}$$

Q. 14. (a) Fig. 5(N).3 shows the position time graph of a particle of mass 4 kg. What is the force on the particle for $t < 0$; $0 < t < 4$ s and $t > 4$ s.

(b) The impulse at $t = 0$ and $t = 4$s. Consider one dimensional motion only.

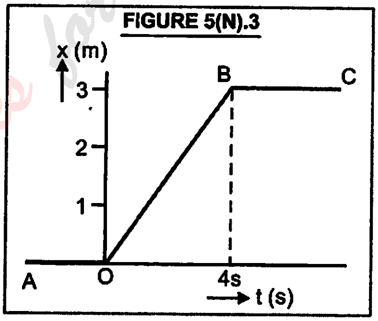


FIGURE 5(N).3

Sol. (*a*) (*i*) *For t < 0*, the position time graph is *AO* which means displacement of the particle is zero, *i.e.* particle is at rest at the origin. Hence force on the particle must be **zero.**

(*ii*) For *0 < t < 4 s*, the position time graph *OB* has a constant slope. Therefore, velocity of the particle is constant in this interval *i.e.* particle has zero acceleration. Hence force on the particle must be **zero.**

(*iii*) *For t > 4s*, the position time graph *BC* is parallel to time axis. Therefore, the particle remains at a distance of 3 m from the origin, *i.e.* it is at rest. Hence force on the particle is **zero.**

(*b*) *Impulse at t = 0*

We know, impulse = change in linear momentum. Before $t = 0$, particle is at rest *i.e.* $u = 0$

After $t = 0$, particle has a constant velocity $v = \dfrac{3}{4} = 0{\cdot}75$ m/s.

$\therefore$ Impulse $= m(v - u) = 4(0{\cdot}75 - 0) = \mathbf{3\ kg\ ms^{-1}}$

Impulse at t = 4 s

Before $t = 4$ s, particle has a constant velocity $u = 0{\cdot}75$ m/s.

After $t = 4$ s, particle is at rest *i.e.* $v = 0$

$\therefore$ Impulse $= m(v - u) = 4(0 - 0{\cdot}75) = \mathbf{-3\ kg\ ms^{-1}}$

Q. 15. Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force $F = 600$ N is applied to (i) A and (ii) B along the direction of string. What is the tension in the string in each case ?

Sol. Here, $F = 600$ N $\quad m_1 = 10$ kg, $m_2 = 20$ kg.

Let T be the tension in the string and a be the acceleration of the system, in the direction of force applied.

$$\therefore \qquad a = \frac{F}{m_1 + m_2} = \frac{600}{10 + 20} = 20\ \text{m/s}^2$$

(a) When force is applied on lighter block A, Fig. 5(N).4.

$$T = m_2 a = 20 \times 20 \text{ N} = \mathbf{400 \text{ N}}$$

FIGURE 5(N).4

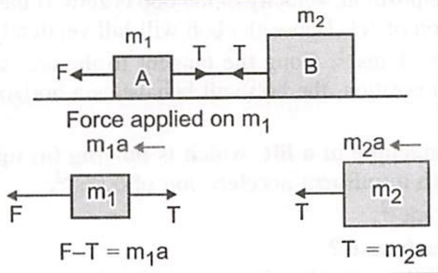
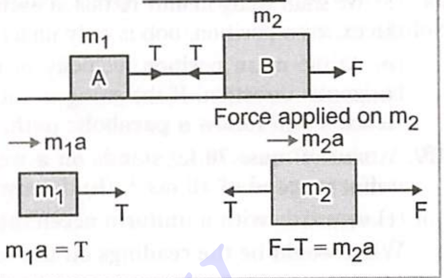


FIGURE 5(N).5



(b) When force is applied on heavier block B, Fig. 5(N).5.

$$T = m_1 a = 10 \times 20 \text{ N } T = \mathbf{200 \text{ N}}$$

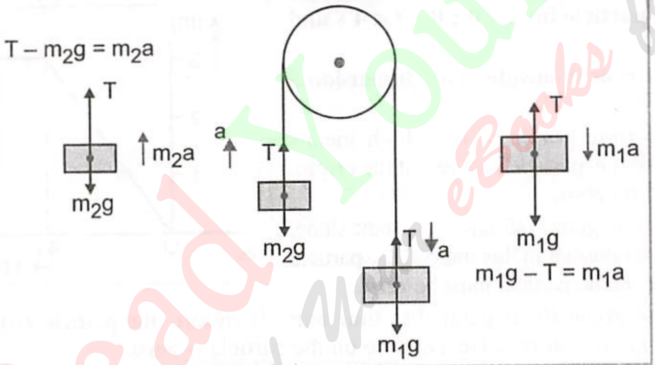
which is different from value of T in case (a). Hence our answer depends on which mass end, the force is applied.

Q. 16. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that passes over a frictionless pulley, Fig. 5(N).6. Find the acceleration of the masses and the tension in the string, when the masses are released.

DO YOU KNOW ?

From Q. 15, we learn that when two bodies of masses m and M are in contact with eachother and a force is applied on the combination, the acceleration (a) is same and does not depend on whether force is applied on m or M. However, the force of contact depends on whether force is applied on m or M.

FIGURE 5(N).6



Sol. Here, $m_2 = 8$ kg, $m_1 = 12$ kg

As $a = \dfrac{(m_1 - m_2)\, g}{m_1 + m_2}$ $\quad\therefore\quad$ $a = \dfrac{(12-8)\, 9{\cdot}8}{12+8} = \dfrac{39{\cdot}2}{20} = \mathbf{1{\cdot}96\ ms^{-2}}$

Again, $T = \dfrac{2 m_1 m_2 g}{m_1 + m_2} = \dfrac{2 \times 12 \times 8 \times 9{\cdot}8}{(12+8)} = \mathbf{94{\cdot}1\ N}$

Q. 17. A nucleus is at rest in the laboratory frame of reference. Show that if it distintegrates into two smaller nuclei, the products must be emitted in opposite directions.

Sol. Let m_1, m_2 be the masses of products and $\vec{v}_1, \vec{v}_2$ be their respective velocities. Therefore, total linear momentum after disintegration $= m_1 \vec{v}_1 + m_2 \vec{v}_2$. Before disintegration, the nucleus is at rest. Therefore, its linear momentum before disintegration is zero.

According to the principle of conservation of linear momentum,

$$= m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0 \quad \text{or} \quad \vec{v}_2 = \frac{-m_1 \vec{v}_1}{m_2}$$

Negative sign shows that $\vec{v}_1$ and $\vec{v}_2$ are **in opposite directions.**

Q. 18. Two billiard balls each of mass 0·05 kg moving in opposite directions with speed 6 ms^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other ?

Sol. Here, initial momentum of the ball $A = 0{\cdot}05\ (6) = 0{\cdot}3$ kg ms^{-1}

As the speed is reversed on collision, final momentum of the ball $A = 0{\cdot}05\ (-6) = -\ 0{\cdot}3$ kg ms^{-1}

Impulse imparted to ball A = change in momentum of ball A = final momentum – initial momentum $= -\ 0{\cdot}3 - 0{\cdot}3 = \mathbf{-\ 0{\cdot}6}$ **kg ms^{-1}.**

Q. 19. A shell of mass 0·02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms^{-1}, what is the recoil speed of the gun ?

Sol. Here, mass of shell, $m = 0{\cdot}02$ kg , mass of gun, $M = 100$ kg , muzzle speed of shell, $V = 80$ ms^{-1}

recoil speed of gun, v = ?

According to the principle of conservation of linear momentum, $mV + Mv = 0$

or $$v = -\frac{mV}{M} = \frac{-0{\cdot}02 \times 80}{100} = \mathbf{0{\cdot}016\ ms^{-1}}$$

Q. 20. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km h^{-1}. What is the impulse imparted to the ball ? Mass of the ball is 0·15 kg.

Sol. In Fig. 5(N).7, the ball hits the bat KL along AO and is deflected by the bat along OB where $\angle AOB = 45°$. ON is normal to the portion of the bat KL deflecting the ball.

$\therefore\quad \theta = \angle NOA = 45°/2 = 22{\cdot}5°$

Initial vel. along $AO = u = 54$ km h^{-1} = 15 ms^{-1},

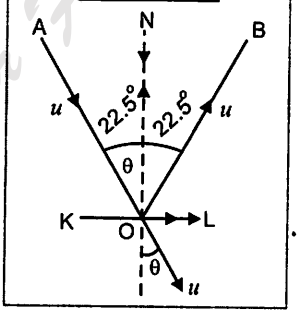
and mass of ball, $m = 0{\cdot}15$ kg.

Initial velocity along AO has two rectangular components : $u \cos\theta$ *along* NO produced and $u \sin\theta$ along the horizontal OL.

Final velocity along OB has the same magnitude = u. It is resolved into two rectangular components : $u \cos\theta$ *along* ON and $u \sin\theta$ along OL.

We observe that there is no change in velocity along the horizontal, but velocity along vertical is *just-reversed*.

FIGURE 5(N).7



$\therefore$ Impulse imparted to the ball = change in linear momentum of the ball

$= m\,u \cos\theta - (-\,m\,u \cos\theta) = 2\,m\,u \cos\theta$

$= 2 \times 0{\cdot}15 \times 15 \cos 22{\cdot}5° = 4{\cdot}5 \times 0{\cdot}9239 = \mathbf{4{\cdot}16\ kg\ ms^{-1}}$

Q. 21. A stone of mass 0·25 kg tied to the end of a string is whirled round in a circle of radius 1·5 m with a speed of 40 rev/min. in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

Sol. Here, $\quad m = 0{\cdot}25$ kg, $r = 1{\cdot}5$ m, $n = 40$ rpm $= \dfrac{40}{60}$ rps $= \dfrac{2}{3}$ rps, T = ?

$$T = m\,r\,\omega^2 = m\,r\,(2\,\pi\,n)^2 = 4\,\pi^2\,m\,r\,n^2$$

$$T = 4 \times \frac{22}{7} \times \frac{22}{7} \times 0{\cdot}25 \times 1{\cdot}5 \times \left(\frac{2}{3}\right)^2 = \mathbf{6{\cdot}6\ N}$$

If $T_{max} = 200$ N, then from

$$T_{max} = \frac{m\,v_{max}^2}{r}$$

$$v_{max}^2 = \frac{T_{max} \times r}{m} = \frac{200 \times 1{\cdot}5}{0{\cdot}25} = 1200$$

$$v_{max} = \sqrt{1200} = \mathbf{34{\cdot}6\ m/s}$$

Q. 22. If in Q. 21, the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks :

(a) the stone jerks radially outwards,

(b) the stone flies off tangentially from the instant the string breaks,

(c) the stone flies off at an angle with the tangent whose magnitude depends on the speed of the stone ?

Sol. The instant the string breaks, the stone flies off tangentially, as per Newton's first law of motion.

Q. 23. Explain why (a) a horse cannot pull a cart and run in empty space.

(b) passengers are thrown forward from their seats when a speeding bus stops suddenly.

(c) It is easier to pull a lawn mower than to push it ?

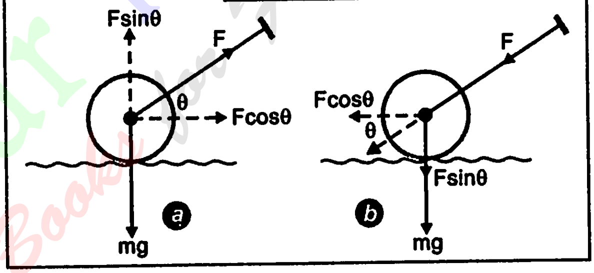
(d) A cricketer moves his hands backwards while holding a catch.

Sol. (*a*) While trying to pull a cart, a horse pushes the ground backwards with a certain force at an angle. The ground offers an equal reaction in the opposite direction, on the feet of the horse. The forward component of this reaction is responsible for motion of the cart. In empty space, there is no reaction and hence a horse cannot pull the cart and run.

(*b*) This is due to inertia of motion. When the speeding bus stops suddenly, lower part of the bodies in contact with the seats stop. The upper part of the bodies of the passengers tend to maintain the uniform motion. Hence the passengers are thrown forward.

(*c*) While pulling a lawn mower, force is applied upwards along the handle. The vertical component of this force is upwards and reduces the effective weight of the mower, Fig. 5(N).8(*a*). While pushing a lawn mower, force is applied downwards along the handle. The vertical component of this force is downwards and increases the effective weight of the mower, Fig. 5(N).8(*b*). As the effective weight is lesser in case of pulling than in case of pushing, therefore, pulling is easier than pushing.

FIGURE 5(N).8



(*d*) While holding a catch, the impulse received by the hands, *F* × *t* = change in linear momentum of the ball is constant. By moving his hands backwards, the cricketer increases the time of impact (*t*) to complete the catch. As *t* increases, *F* decreases and as a reaction, his hands are not hurt severely.

NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. A ball is travelling with uniform translatory motion. This means that

(*a*) it is at rest.

(*b*) the path can be a straight line or circular and the ball travels with uniform speed.

(*c*) all parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.

(*d*) the centre of the ball moves with constant velocity and the ball spins about its centre uniformly.

2. A metre scale is moving with uniform velocity. This implies

(*a*) the force acting on the scale is zero, but a torque about the centre of mass can act on the scale.

(*b*) the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero.

(*c*) the total force acting on it need not be zero but the torque on it is zero.

(*d*) neither the force nor the torque need to be zero.

3. A cricket ball of mass 150 g has an initial velocity $u = (3\hat{i} + 4\hat{j})\text{ms}^{-1}$ and a final velocity $v = -(3\hat{i} + 4\hat{j})\text{ms}^{-1}$ after being hit. The change in momentum (final momentum-initial momentum) is (in kg ms^{-1})

(*a*) zero (*b*) $-(0{\cdot}45\hat{i} + 0{\cdot}6\hat{j})$

(*c*) $-(0{\cdot}9\hat{i} + 1{\cdot}2\hat{j})$ (*d*) $-5(\hat{i} + \hat{j})$

4. In the previous problem 3, the magnitude of the momentum transferred during the hit is

(*a*) Zero (*b*) 0·75 kg ms^{-1}

(*c*) 1·5 kg ms^{-1} (*d*) 14 kg ms^{-1}

5. Conservation of momentum in a collision between particles can be understood from

(*a*) conservation of energy

(*b*) Newton's first law only

(*c*) Newton's second law only

(*d*) both Newton's second and third law

6. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is

(*a*) frictional force along westward

(*b*) muscle force along southward

(*c*) frictional force along south-west

(*d*) muscle force along south-west

7. A body of mass 2 kg travels according to the law $x(t) = pt + qt^2 + rt^3$ where $p = 3$ ms^{-1}, $q = 4$ ms^{-2} and $r = 5$ ms^{-3}.

The force acting on the body at $t = 2$ seconds is

(*a*) 136 N (*b*) 134 N

(*c*) 158 N (*d*) 68 N

8. A body with mass 5 kg is acted upon by a force $F = (-3\hat{i} + 4\hat{j})$N. If its initial velocity at $t = 0$ is $v = (6\hat{i} - 12\hat{j})\text{ms}^{-1}$, the time at which it will just have a velocity along the *y*-axis is

(*a*) never (*b*) 10 s

(*c*) 2 s (*d*) 15 s

9. A car of mass *m* starts from rest and acquires a velocity along east $v = v\hat{i}\ (v > 0)$ in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is

(*a*) $\frac{mv}{2}$ eastward and is exerted by the car engine.

(*b*) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road.

(*c*) more than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road.

(*d*) $\frac{mv}{2}$ exerted by the engine.

MULTIPLE CHOICE QUESTIONS-II

10. The motion of a particle of mass m is given by $x = 0$ for $t < 0$ s, $x(t) = A \sin 4\pi t$ for $0 < t$ (1/4) s ($A > 0$), and $x = 0$ for $t > (1/4)$ s. Which of the following statements is true ?

(*a*) The force at $t = (1/8)$ s on the particle is $-16\pi^2$ A m.

(*b*) The particle is acted upon by an impulse of magnitude $4\pi^2 A m$ at $t = 0$ s and $t = (1/4)$ s.

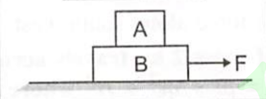
(*c*) The particle is not acted upon by any force.

(*d*) The particle is not acted upon by a constant force.

(*e*) There is no impulse acting on the particle.

11. In Fig. 5(N).9, the co-efficient of friction between the floor and the body B is 0·1. The co-efficient of friction between the bodies B and A is 0·2. A force F is applied as shown on B. The mass of A is $m/2$ and of B is m. Which of the following statements are true ?

FIGURE 5(N).9



(*a*) The bodies will move together if $F = 0{\cdot}25\,mg$.

(*b*) The body A will slip with respect to B if $F = 0{\cdot}5\,mg$.

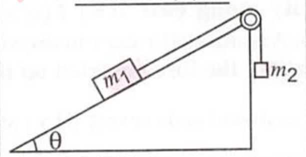
(*c*) The bodies will move together if $F = 0{\cdot}5\,mg$.

(*d*) The bodies will be at rest if $F = 0{\cdot}1\,mg$.

(*e*) The maximum value of F for which the two bodies will move together is $0{\cdot}45\,mg$.

12. Mass m_1 moves on a slope making an angle θ with the horizontal and is attached to mass m_2 by a string passing over a frictionless pulley as shown in Fig. 5(N).10. The co-efficient of friction between m_1 and the sloping surface is μ.

FIGURE 5(N).10



Which of the following statements are true ?

(*a*) If $m_2 < m_1 \sin\theta$, the body will move up the plane.

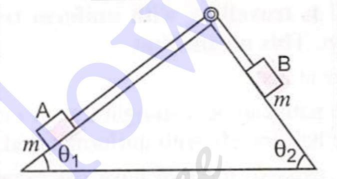
(*b*) If $m_2 > m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane.

(*c*) If $m_2 < m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane.

(*d*) If $m_2 < m_1 (\sin\theta - \mu\cos\theta)$, the body will move down the plane.

13. In Fig. 5(N).11, a body A of mass m slides on plane inclined at angle θ_1 to the horizontal and μ_1 is the coefficient of friction between A and the plane. A is connected by a light string passing over a frictionless pulley to another body B, also of mass m, sliding on a frictionless plane inclined at angle θ_2 to the horizontal. Which of the following statements are true ?

FIGURE 5(N).11



(*a*) A will never move up the plane.

(*b*) A will just start moving up the plane when

$$\mu = \frac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}.$$

(*c*) For A to move up the plane, θ_2 must always be greater than θ_1.

(*d*) B will always slide down with constant speed.

14. Two billiard balls A and B, each of mass 50 g and moving in opposite directions with speed of 5 ms^{-1} each, collide and rebound with the same speed. If the collision lasts for 10^{-3} s, which of the following statements are true ?

(*a*) The impulse imparted to each ball is 0·25 kg ms^{-1} and the force on each ball is 250 N.

(*b*) The impulse imparted to each ball is 0·25 kg ms^{-1} and the force exerted on each ball is 25×10^{-5} N.

(*c*) The impulse imparted to each ball is 0·5 Ns.

(*d*) The impulse and the force on each ball are equal in magnitude and opposite in direction.

15. A body of mass 10 kg is acted upon by two perpendicular forces, 6N and 8N. The resultant acceleration of the body is

(*a*) 1 ms^{-2} at an angle of $\tan^{-1}(4/3)$ w.r.t. 6 N force.

(*b*) 0·2 ms^{-2} at an angle of $\tan^{-1}(4/3)$ w.r.t. 6 N force.

(*c*) 1 ms^{-2} at an angle of $\tan^{-1}(3/4)$ w.r.t. 8 N force.

(*d*) 0·2 ms^{-2} at an angle of $\tan^{-1}(3/4)$ w.r.t. 8 N force.

ANSWERS

1. (*c*) **2.** (*b*) **3.** (*c*) **4.** (*c*) **5.** (*d*) **6.** (*c*) **7.** (*a*) **8.** (*b*) **9.** (*b*)
10. (*a,b,d*) **11.** (*a,b,d,e*) **12.** (*b,d*) **13.** (*b, c*) **14.** (*c, d*) **15.** (*a, c*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. In uniform translatory motion, all parts of the ball have the same velocity in magnitude and direction, and this velocity is constant.

2. When a metre scale is moving with uniform velocity, the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero.

3 Here, $m = 150 \text{ g} = 0{\cdot}15 \text{ kg}, \ \vec{u} = (3\hat{i} + 4\hat{j}) \text{ m/s}, \ \ \vec{v} = -(3\hat{i} + 4\hat{j}) \text{ m/s}$

Change in momentum $\vec{p} = m\vec{v} - m\vec{u} \ = 0{\cdot}15[-(3\hat{i}+4\hat{j}) - (3\hat{i}+4\hat{j})] \ = -0{\cdot}30[(3\hat{i}+4\hat{j})]$

$$\vec{p} = -(\mathbf{0{\cdot}9\,\hat{i} + 1{\cdot}2\,\hat{j}})\,\mathbf{kg\,m/s}$$

4. Magnitude of momentum transferred (= change in momentum), $|\vec{p}| = \sqrt{0{\cdot}9^2 + (1{\cdot}2)^2} = \mathbf{1{\cdot}5 \ kg\,ms^{-1}}$.

5. Conservation of momentum in a collision between particles can be understood from both, Newton's 2nd law and 3rd law.

6. In Fig. 5(N).12, $\overrightarrow{OA} = \vec{p}_1$ = initial momentum of player northward,

$\overrightarrow{OB} = \vec{p}_2$ = final momentum of player westward.

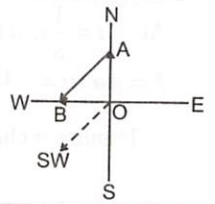
According to triangle law of vectors, $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{p}_2 - \vec{p}_1$ = change in momentum.

The change in momentum of player is along south west.

As motion is due to frictional force of reaction of the ground, therefore, force that acts on the player is frictional force along south west.

FIGURE 5(N).12



7. Here, $m = 2$ kg

$$x(t) = pt + qt^2 + rt^3$$

$$v = \frac{dx}{dt} = p + 2qt + 3rt^2$$

$$a = \frac{dv}{dt} = 0 + 2q + 6rt$$

At $t = 2$ sec, $a = 2q + 12r = 2 \times 4 + 12 \times 5 = 68 \text{ m/s}^2$

$F = ma = 2 \times 68 \text{ N} = \mathbf{136 \ N}$

8. Here, $m = 5 \text{ kg}; \ \vec{F} = (-3\hat{i} + 4\hat{j}) \text{ N}$

Initial velocity at $t = 0$, $\vec{u} = (6\hat{i} - 12\hat{j}) \text{ m/s}$, retardation, $\hat{a} = \frac{\vec{F}}{m} = \left(-\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}\right) \text{m/s}^2$

As final velocity is along Y-axis only, its *x*-component must be zero.

From $v = u + at$, for X-component only, $0 = 6\hat{i} - \frac{3\hat{i}}{5}t$

$$t = \frac{5 \times 6}{3} = \mathbf{10\,s}$$

9. Here, mass of car $= m$

As it starts from rest, $u = 0$, final velocity along east, $v = v\hat{i}$, $t = 2$ sec.

From $v = u + at$

$$v\hat{i} = 0 + \vec{a} \times 2, \quad \vec{a} = \frac{v}{2}\hat{i}$$

$$\vec{F} = m\vec{a} = \frac{mv}{2}\hat{i}, \qquad \textit{i.e.,} \text{ force of car is } \frac{mv}{2} \text{ east ward}$$

This is due to friction on the tyres exerted by the road.

Multiple Choice Questions – II

10. Here, $x = 0$ for $t < 0$ s

$x(t) = A \sin 4\pi t$ for $0 < t < \frac{1}{4}s$

$x = 0$ for $t > \frac{1}{4}s$

For $0 < t < \frac{1}{4}$ s

$$v(t) = \frac{dx}{dt} = 4\pi A \cos 4\pi t, \quad a(t) = \frac{dv}{dt} - 16\pi^2 A \sin 4\pi t$$

At $t = \frac{1}{8}$s, $a(t) = -16\pi^2 A \sin 4\pi \times \frac{1}{8} = -16\pi^2 A \times 1$

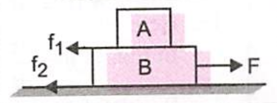
$F = ma(t) = -16\pi^2$ Am

Impulse = change in linear momentum $= F \times t = -16\pi^2 \text{ Am} \times \frac{1}{4} = -4\pi^2$ Am

The impulse (change in linear momentum) at $t = 0$ is same as at $t = \frac{1}{4}$ sec.

Further, as F depends on A which is not constant, therefore, the particle is not acted upon by a constant force.

FIGURE 5(N).13



11. Here, mass of $B = m$, mass of $A = \frac{m}{2}$

(*a*) Force of friction between A and B $\quad f_1 = 0.2\left(\frac{m}{2}\right)g = 0.1\,mg$

Force of friction between B and floor $\quad f_2 = 0.1\left(m + \frac{m}{2}\right)g = \frac{0.3}{2}mg = 0.15\,mg$

The bodies will move together, if $F = f_1 + f_2 = 0.1\ mg + 0.15\ mg = 0.25\ mg$.

(*b*) If $F = 0.5$ mg, force between A and B will exceed 0·1 mg and the body A will slip.

(*c*) The bodies cannot move together, when $F = 0.5\ mg$.

(*d*) If $F = 0.1$ mg, the bodies will be at rest as $F < (f_1 + f_2)$.

(*e*) The two bodies will move together till max. accelerating force on $A = 0.1$ mg $= f_1$

$$\text{acceleration of } A = \frac{0.1\,mg}{m/2} = 0.2\,g, \quad \text{acceleration of } B = 0.2 \text{ g}$$

max. acc. force on $B = m \times 0.2$ g $= 0.2\ mg$

As $\quad f_2 = 0.15\ mg$

∴ Maximum value of F for which A and B will move together $= 0.1\ mg + 0.2\ mg + 0.15\ mg = 0.45\ mg$

12. In Fig. 5(N).14, f is the force of friction. When the body moves up the plane, f acts down the plane.

$$f = \mu R = \mu\, m_1\, g \cos\theta$$

In that event, $m_2\, g > m_1\, g \sin\theta + f$

$$m_2\, g > m_1\, g \sin\theta + \mu\, m_1\, g \cos\theta$$

$$m_2 > m_1 (\sin\theta + \mu\cos\theta)$$

Choice (*b*) is correct.

When the body moves down the plane, f acts up the plane. In that event

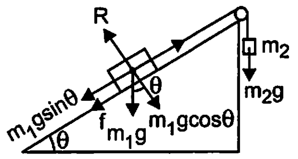
$$(m_2\, g + f) < m_1\, g \sin\theta$$

$$m_2\, g < m_1\, g \sin\theta - f$$

$$m_2\, g < m_1\, g \sin\theta - \mu\, m_1\, g \cos\theta \quad \text{or} \quad m_2 < m_1 (\sin\theta - \mu\cos\theta)$$

Choice (*d*) is correct.

FIGURE 5(N).14



13. As is clear from Fig. 5(N).15, A can move up the plane depending on values of m_1, m_2, θ_1, θ_2 and frictional force f_1.

When A just starts moving up the plane, f_1 acts down the plane.

$\therefore$ $mg\sin\theta_1 + f_1 = mg\sin\theta_2$

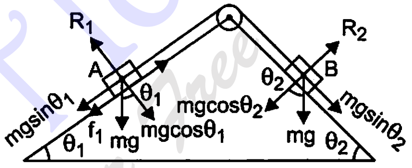
But $f_1 = \mu R_1 = \mu\, mg\cos\theta_1$

$\therefore$ $mg(\sin\theta_1 + \mu\cos\theta_1) = mg\sin\theta_2$

or $\mu\cos\theta_1 = \sin\theta_2 - \sin\theta_1$

$$\mu = \frac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}$$

FIGURE 5(N).15



As μ cannot be negative, therefore θ_2 must always be greater than θ_1.

14. Here, $m_1 = m_2 = m = 50\ g = \frac{50}{1000}\,\text{kg} = \frac{1}{20}\,\text{kg}$

initial vel. $(u) = u_1 = u_2 = 5$ m/s, final vel. $(v) = v_1 = v_2 = -5$ m/s, $t = 10^{-3}$ s

Impulse imparted to each ball = change in linear momentum of each ball

$$m(v-u) = \frac{1}{20}(-5-5) = -0{\cdot}5\,\text{Ns}$$

$$\text{Force} = \frac{\text{Impulse}}{t} = \frac{0{\cdot}5}{10^{-3}} = 500\,\text{N}$$

The impulse and force on each ball are equal in magnitude and opposite in direction.

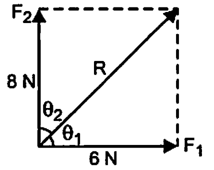
15. Here, $m = 10$ kg

In Fig. 5(N).16, $F_1 = 6$ N, $F_2 = 8$ N.

Resultant force $R = \sqrt{F_1^2 + F_2^2} = \sqrt{6^2 + 8^2} = 10\,\text{N}$

$$a = \frac{R}{m} = \frac{10}{10} = 1\,\text{m/s}^2 \text{ ; along } R.$$

FIGURE 5(N).16



Let θ_1 be angle between R and F_1

$$\tan\theta_1 = \frac{8}{6} = \frac{4}{3}$$

$$\theta_1 = \tan^{-1}(4/3) \text{ w.r.t. } F_1 = 6\text{ N force}$$

Let θ_2 be angle between R and F_2

$$\tan\theta_2 = \frac{6}{8} = \frac{3}{4}$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) \text{ w.r.t. } F_2 = 8\text{ N force}$$

VERY SHORT ANSWER QUESTIONS

16. A girl riding a bicycle along a straight road with a speed of 5 ms^{-1} throws a stone of mass 0·5 kg which has a speed of 15 ms^{-1} with respect to the ground along her direction of motion. The mass of the girl and bicycle is 50 kg. Does the speed of the bicycle change after the stone is thrown ? What is the change in speed, if so ?

Ans. Here, total mass of girl, cycle and stone $m_1 = 50{\cdot}5$ kg

initial speed, $u_1 = 5$ m/s, mass of stone, $m_2 = 0{\cdot}5$ kg, velocity of stone, $u_2 = 15$ m/s

If v is speed of girl and cycle (of mass $m = 50$ kg) after throwing the stone, then applying principle of conservation of linear momentum, we get

$$m_1 u_1 = m_2 u_2 + mv$$

$$mv = m_1 u_2 - m_2 u_2$$

$$50\, v = 50{\cdot}5 \times 5 - 0{\cdot}5 \times 15 = 252{\cdot}5 - 7{\cdot}5 = 245$$

$$v = \frac{245}{50} = 4{\cdot}9 \text{ m/s}$$

Change in speed = 5 – 4·9 = 0·1 m/s

17. A person of mass 50 kg stands on a weighing scale on a lift. If the lift is descending with a downward acceleration of 9 ms^{-2}, what would be the reading of the weighing scale ? (g = 10 ms^{-2})

Ans. Reading of weighing scale =

apparent wt. of person = $R = m\,(g - a) = 50\,(10 - 9) = 50$ N = 5 kg

18. The position time graph of a body of mass 2 kg is as given in Fig. 5(N).17. What is the impulse on the body at t = 0 s and t = 4 s ?

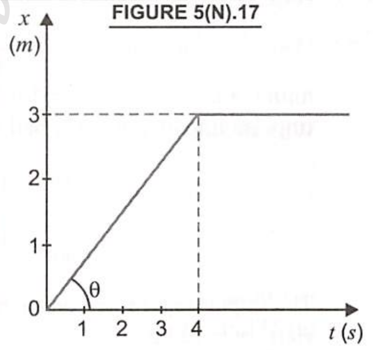


FIGURE 5(N).17

Ans. Here, m = 2 kg

As is clear from position time graph in Fig. 5(N).17,

At $t = 0$, $x = 0$ *i.e.* body is at rest. Therefore, impulse of body (at $t = 0$) = 0.

From $t = 0$ to $t = 4$ s, position time graph of body is a st. line with positive slope. The body moves with a uniform velocity.

Beyond t = 4 s, position time graph is parallel to time axis. Therefore, body comes to rest.

Now, initial vel. of body (u) = tan θ $= \frac{3}{4}$ m/s

final vel. of body (v) = 0

$$\therefore \quad \text{Impulse (at } t = 4 \text{ s)} = \text{Change in momentum} = m\,(v - u) = 2\left(0 - \frac{3}{4}\right) = -\frac{3}{2} \textbf{ kg ms}^{-1}$$

19. A person driving a car suddenly applies the brakes on seeing a child on the road ahead. If he is not wearing seat belt, he falls forward and hits his head against the steering wheel. Why ?

Ans. When the car is brought to a sudden halt, upper part of driver's body continues to move forward on account of inertia of motion. If he is not wearing seat belt, the only force that tries to stop him is frictional force exerted by the seat, which is too small. That is why, he hits his head against the steering wheel.

20. The velocity of a body of mass 2 kg as a function of t is given by $v(t) = 2t\,\hat{i} + t^2\,\hat{j}$. Find the momentum and the force acting on it, at time t = 2 s.

Ans. Here, m = 2 kg

$$\vec{v}(t) = 2t\,\hat{i} + t^2\,\hat{j}$$

At $t = 2$ s

$$\vec{v} = 4\hat{i} + 4\hat{j} = 4(\hat{i} + \hat{j})$$

$$\therefore \quad \vec{p} = m\vec{v} = 2(4\hat{i} + 4\hat{j}) = [8\hat{i} + 8\hat{j}]\ \text{kg ms}^{-1}$$

Also, $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[2t\hat{i} + t^2\hat{j}]$$

$$\vec{a} = 2\hat{i} + 2t\hat{j}$$

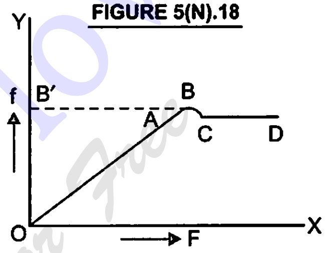
At $t = 2$ s, $\vec{a} = 2\hat{i} + 4\hat{j}$

$$\vec{F} = m\vec{a} = 2(2\hat{i} + 4\hat{j}) = (4\hat{i} + 8\hat{j})\text{N}$$

21. A block placed on a rough horizontal surface is pulled by a horizontal force *F*. Let *f* be the force applied by the rough surface on the block. Plot a graph of *f* versus *F*.

Ans. The graph of F versus f is as shown in Fig. 5(N).18. The portion OA of graph represents static friction which is self adjusting. In this portion, $f = F$. The point B corresponds to force of limiting friction (= OB'), which is the maximum value of static friction. $CD \mid\mid OX$ represents *kinetic friction,* when the body actually starts moving. The force of kinetic friction does not increase with applied force, and is slightly less than limiting friction.

FIGURE 5(N).18



22. Why are porcelain objects wrapped in paper or straw before packing for transportation ?

Ans. For transportation, porcelain objects are wrapped in paper or straw to reduce the chances of damage in case of sudden jerks or even fall. The force takes longer time to reach the porcelain objects through paper/straw. As $F \times t$ = change in momentum is constant and t increases, therefore F reduces.

23. Why does a child feel more pain when she falls down on a hard cement floor, than when she falls on the soft muddy ground in the garden ?

Ans. When a child falls on a cement floor, his body comes to rest instantly *i.e.* time taken by the body to come to stop is too small. As $F \times t$ = change in momentum = constant and t decreases, therefore F increases. The child is hurt more. The reverse is true for soft muddy ground.

24. A woman throws an object of mass 500 g with a speed of 25 ms^{-1}

(*a*) What is the impulse imparted to the object ?

(*b*) If the object hits a wall and rebounds with half the original speed, what is the change in momentum of the object ?

Ans. Here, $m = 500\text{ g} = \frac{1}{2}\text{kg}, \quad u = 25\text{ ms}^{-1}$

(*a*) Impulse imparted = change in momentum = initial momentum given $= mu = \frac{1}{2} \times 25 = 12{\cdot}5\,\text{Ns}$

(*b*) On rebounding force the wall, $v = -\frac{1}{2} \times 25 = -12{\cdot}5\,\text{m/s}$

Change in momentum = $m(v - u) = \frac{1}{2}(-12{\cdot}5 - 25) = -18{\cdot}75\text{ kg ms}^{-1}$

25. Why are mountain roads generally made winding upwards rather than going straight up ?

Ans. While going up a mountain, the opposing force of friction, $F = \mu R = \mu mg \cos\theta$, where θ is angle of slope with horizontal. To avoid skidding ; F should be large.

$\therefore$ $\cos\theta$ should be large and hence θ must be small. That is why mountain roads are generally made winding upwards. The road straight up would have larger slope.

SHORT ANSWER QUESTIONS

26. A mass of 2 kg is suspended with thread *AB*, Fig. 5(N).19. Thread *CD* of the same type is attached to the other end of 2 kg mass. Lower thread is pulled gradually, harder and harder in the downward direction so as to apply force on *AB*. Which of the threads will break and why ?

FIGURE 5(N).19

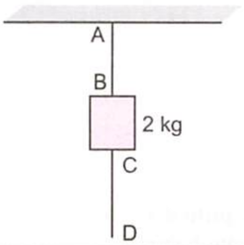
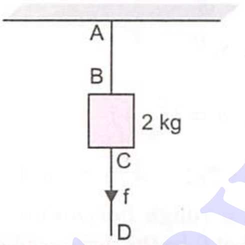


FIGURE 5(N).20



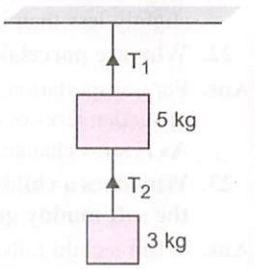
Ans. The thread *AB* will break earlier than the thread *CD*. This is becase force on *CD* = applied force *f*, and force on *AB* = *f* plus force due to 2 kg weight, as shown in Fig. 5(N).20.

27. In the above given problem if the lower thread is pulled with a jerk, what happens ?

Ans. When the lower thread *CD* is pulled with a jerk, the thread *CD* itself may break. This is because pull on *CD* may not be transmitted to *AB* instantly.

28. Two masses of 5 kg and 3 kg are suspended with help of massless inextensible strings as shown in Fig. 5(N).21. Calculate T_1 and T_2 when whole system is going upwards with acceleration = 2 ms^2 (use $g = 9{\cdot}8\ ms^{-2}$).

FIGURE 5(N).21



Ans. Here, $m_1 = 5$ kg ,

$m_2 = 3$ kg

$g = 9{\cdot}8\ m/s^2$,

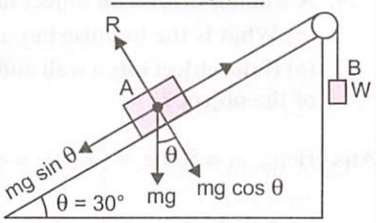
$a = 2\ m/s^2$, upwards

$$T_1 = (m_1 + m_2)(g + a) = (5 + 3)(9{\cdot}8 + 2) = 94{\cdot}4\ N$$

$$T_2 = m_2 (g + a) = 3 (9{\cdot}8 + 2) = 35{\cdot}4\ N$$

29. Block *A* of weight 100 N rests on a frictionless inclined plane of slope angle 30° Fig. 5(N).22. A flexible cord attached to *A* passes over a frictionless pulley and is connected to block *B* of weight *W*. Find the weight *W* for which the system is in equilibrium.

FIGURE 5(N).22

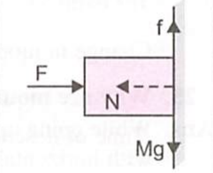


Ans. As is clear from Fig. 5(N).22, the system will be in equilibrium, when force on block *A*, up the plane is equal to force on *A* down the inclined plane *i.e.*

$$W = mg \sin\theta = mg \sin 30^\circ = 100 \times \frac{1}{2} = 50\,N$$

30. A block of mass *M* is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is μ and the acceleration due to gravity is *g*, calculate the minimum force required to be applied by the finger to hold the block against the wall ?

FIGURE 5(N).23



Ans. Let *F* be the force applied by the finger on the block. *N* is normal reaction of the wall on the block. *Mg* is weight of block acting vertically downwards. *f* is the force of friction between the wall and the block, which acts upwards as shown in Fig. 5(N).23.

The block will not fall, if $\qquad f = Mg$

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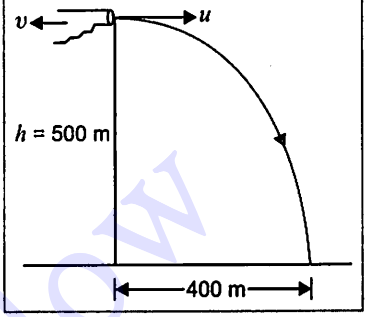
$$\mu N = Mg \quad \text{or} \quad \mu F = Mg \quad \text{or} \quad F = \frac{Mg}{\mu}$$

31. A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun. (acceleration due to gravity = 10 ms^{-2})

FIGURE 5(N).24



Ans. Here, $m_1 = 100$ kg ; $m_2 = 1$ kg ; $h = 500$ m ;
$x = 400$ m, Fig. 5(N).24

Let u be velocity of ball and t be time taken by the ball to hit the ground.

From $$h = \frac{1}{2}gt^2$$

$$500 = \frac{1}{2} \times 10t^2$$

$$t = \sqrt{100} = 10\text{s}$$

From $x = ut$, $u = \frac{x}{t} = \frac{400}{10} = 40\text{m/s}$

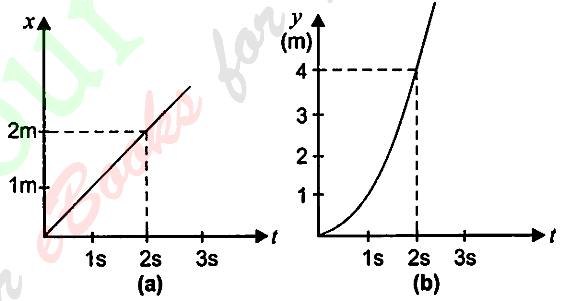
If v is recoil velocity of gun, then according to principle of conservation of linear momentum, $m_1 v = m_2 u$

$$v = \frac{m_2 u}{m_1} = \frac{1}{100} \times 40 = 0{\cdot}4\,\text{m/s}$$

32. Figure 5(N).25 shows (x, t), (y, t) diagram of a particle moving in 2-dimensions.

If the particle has a mass of 500 g, find the force (direction and magnitude) acting on the particle.

FIGURE 5(N).25



Ans. As (x, t) diagram of particle is a st. line, motion along X-axis is uniform.

From $x = ut$, $u = \frac{x}{t} = \frac{2}{2} = 1\text{m/s}$

Force along X-axis is zero.

The (y, t) diagram is a parabola. If a is uniform acceleration along Y-axis, then from $y = \frac{1}{2}at^2$

$$4 = \frac{1}{2}a \times 2^2, \ a = 2 \text{ m/s}^2$$

As $F = ma \quad \therefore \quad F = \frac{500}{1000} \times 2 = 1\text{N}$ **along Y-axis**

33. A person in an elevator accelerating upwards with an acceleration of 2 ms^{-2}, tosses a coin vertically upwards with a speed of 20 ms^{-1}. After how much time will the coin fall back into his hand ? (g = 10 ms^{-2})

Ans. Here, $u = 20$ ms^{-1} ; $v = 0$

Effective acceleration, $a' = -(g + a) = -(10 + 2) = -12$ m/s^2

If t is time of ascent of the coin, then from $v = u + at$

$$0 = 20 - 12\,t, \ t = \frac{20}{12} = \frac{5}{3}\text{s}$$

Time of descent = time of ascent $= \frac{5}{3}$s

$\therefore$ Total time after which the coin will fall black into hand $= \frac{5}{3} + \frac{5}{3} = \frac{10}{3}\text{s} = 3{\cdot}33\text{s}$

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LONG ANSWER QUESTIONS

34. There are three forces F_1, F_2 and F_3 acting on a body, all acting on a point P on the body, Fig. 5(N).26. The body is found to move with uniform speed. (*a*) Show that the forces are coplanar. (*b*) Show that the torque acting on the body about any point due to these three forces is zero.

Ans. (*a*) As the body is moving with uniform speed, its acceleration $a = 0$.

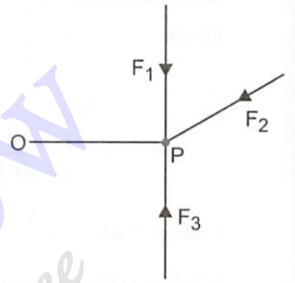
$\therefore\ \vec{F_1}+\vec{F_2}+\vec{F_3}=0.$

The three forces are passing through a point P, as shown in Fig. 5(N).26. If $\vec{F_1}, \vec{F_2}$ lie in one plane say A, their resultant lies in the same plane A. Since $\vec{F_3}=-(\vec{F_1}+\vec{F_2})$, therefore $\vec{F_3}$ must also lie in plane A. Hence $\vec{F_1}, \vec{F_2}, \vec{F_3}$ are coplanar forces.

(*b*) Since all the forces pass through same point P, therefore, torque acting on the body about P due to the these three forces is zero. About any other point, say O, torque $=\overrightarrow{OP}\times(\vec{F_1}+\vec{F_2}+\vec{F_3})$.

As $\vec{F_1}+\vec{F_2}+\vec{F_3}=0$, therefore, torque about point O = zero.

FIGURE 5(N).26



35. When a body slides down from rest along a smooth inclined plane making an angle of 45° with the horizontal, it takes time T. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time pT, where p is some number greater than 1. Calculate the co-efficient of friction between the body and the rough plane.

Ans. When a body slides down a smooth inclined plane, $a = g\sin\theta = g\sin 45^\circ = \dfrac{g}{\sqrt{2}}$

From $\quad s = ut+\frac{1}{2}at^2$

$$s = 0+\frac{1}{2}\frac{g}{\sqrt{2}}T^2 \qquad \text{...}(i)$$

When the same body slides down the rough inclined plane,

$$a = g(\sin\theta-\mu\cos\theta) = \frac{g(1-\mu)}{\sqrt{2}} \qquad \left(\because \sin 45^\circ=\cos 45^\circ=\frac{1}{\sqrt{2}}\right)$$

Again, from $\quad s = ut+\frac{1}{2}at^2$

$$s = 0+\frac{1}{2}\frac{g(1-\mu)}{\sqrt{2}}(pT)^2 \qquad \text{...}(ii)$$

From (*i*) and (*ii*) $\quad \dfrac{g(1-\mu)}{2\sqrt{2}}p^2T^2=\dfrac{gT^2}{2\sqrt{2}}$

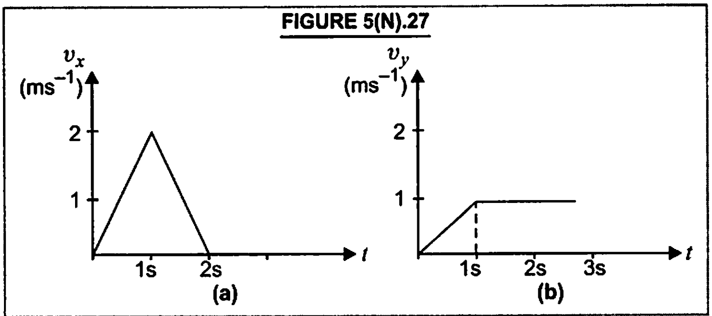
$$1-\mu=\frac{1}{p^2} \qquad \text{or} \qquad \mu=\left(1-\frac{1}{p^2}\right)$$

36. Figure 5(N).27 shows (v_x, t), and (v_y, t) diagrams for a body of unit mass. Find the force as a function of time.

Ans. As is clear from Fig. 5(N).27(*a*) and Fig. 5(N).27(*b*),

$v_x = 2t$,	for $0<t<1\ s$	$v_y = t$	for $0<t<1\ s$
$v_x = 2(2-t)$,	for $1<t<2\ s$	$v_y = 1$	for $1<t$
$v_x = 0$,	for $2<t$		

FIGURE 5(N).27



$$\therefore\ F_x = ma_x = m\frac{dv_x}{dt} \qquad F_y = ma_y = m\frac{dv_y}{dt}$$

$= 1 \times 2$ for $0 < t < 1\ s$ | $= 1 \times 1$ for $0 < t < 1\ s$

$= 1\,(-2)$ for $1 < t < 2\ s$ | $= 0$ for $1 < t$

$= 0$ for $2 < t$

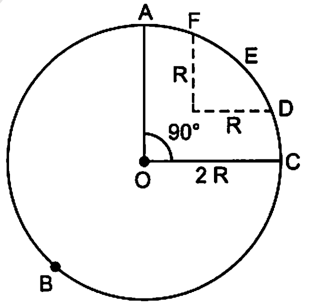
Hence $\vec{F} = 2\hat{i} + \hat{j}$ for $0 < t < 1\ s$

$\vec{F} = -2\hat{i}$ for $1 < t < 2\ s$

$\vec{F} = \vec{0}$ for $2 < t$

37. A racing car travels on a track (without banking) *ABCDEFA*, Fig. 5(N).28. *ABC* is a circular arc of radius 2 *R*. *CD* and *FA* are straight paths of length *R* and *DEF* is a circular arc of radius *R* = 100 m. The co-efficient of friction on the road is μ = 0·1. The maximum speed of the car is 50 ms^{-1}. Find the minimum time for completing one round.

FIGURE 5(N).28



Ans. As the track is unbanked, the necessary centripetal force is provided by force of friction *i.e.*

$$\frac{mv^2}{r} = F = \mu R = \mu mg \qquad \therefore \qquad v = \sqrt{\mu r g}$$

For path ABC : length $= \frac{3}{4}(2\pi 2R) = 3\pi R = 3\pi \times 100 = 300\pi$ m

$$v_1 = \sqrt{\mu 2Rg} = \sqrt{0{\cdot}1 \times 2 \times 100 \times 10} = 14{\cdot}14 \text{ m/s}$$

$$\therefore \qquad t_1 = \frac{300\pi}{14{\cdot}14} = 66{\cdot}6 \text{ s}$$

For path DEF : length $= \frac{1}{4}(2\pi R) = \frac{\pi \times 100}{2} = 50\pi$

$$v_2 = \sqrt{\mu Rg} = \sqrt{0{\cdot}1 \times 100 \times 10} = 10 \text{ m/s}$$

$$t_2 = \frac{50\pi}{10} = 5\pi \text{ sec} = 15{\cdot}7 \text{ s}$$

For paths, CD and FA

length = $R + R = 2R$ = 200 m

$$t_3 = \frac{200}{50} = 4{\cdot}0 \text{ s}$$

∴ Total time for completing one round $t = t_1 + t_2 + t_3 = 66{\cdot}6 + 15{\cdot}7 + 4{\cdot}0 =$ **86·3** s

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38. The displacement vector of a particle of mass m is given by $r(t) = \hat{i}\,A\cos\omega t + \hat{j}\,B\sin\omega t$.
(*a*) Show that the trajectory is an ellipse. (*b*) Show that $F = -m\omega^2 r$.

Ans. (*a*) Here, $\vec{r}(t) = \hat{i}\,A\cos\omega t + \hat{j}\,B\sin\omega t$ $\quad\therefore\quad x = A\cos\omega t,\ y = B\sin\omega t$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = \cos^2\omega t + \sin^2\omega t = 1$$

which is the equation of an **ellipse**.

$\therefore$ The trajectory of the particle is elliptical.

(*b*) Now, $\vec{v} = \dfrac{d\vec{r}}{dt} = -\hat{i}\,\omega A\sin\omega t + \hat{j}\,\omega B\cos\omega t$

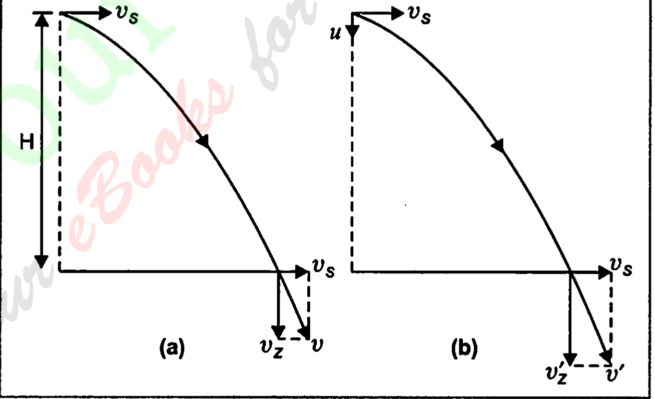
$$\vec{a} = \frac{d\vec{v}}{dt} = -\hat{i}\,\omega^2 A\cos\omega t - \hat{j}\,\omega^2 B\sin\omega t = -\omega^2[\hat{i}\,A\cos\omega t + \hat{j}\,B\sin\omega t] = -\omega^2\vec{r}$$

$\therefore \qquad \vec{F} = m\vec{a} = -m\omega^2\vec{r}$

39. A cricketer bowler releases the ball in two different ways
(*a*) giving it only horizontal velocity, and
(*b*) giving it horizontal velocity and a small downward velocity.
The speed v_s at the time of release is the same. Both are released at a height H from the ground. Which one will have greater speed when the ball hits the ground ? Neglect air resistance.

Ans. (*a*) When horizontal velocity $= v_s$ and there is no downward velocity given, As shown in Fig. 5(N).29(*a*)

FIGURE 5(N).29



$$v_z = \sqrt{2gH}$$

$\therefore$ Velocity on hitting the ground

$$v = \sqrt{v_s^2 + v_z^2} = \sqrt{v_s^2 + 2gH}$$

(*b*) When horizontal velocity $= v_s$ and downward velocity $= u$, as shown in Fig. 5(N).29(*b*).

Then $\qquad v_z' = \sqrt{u^2 + 2gH}$

velocity on hitting the ground $v' = \sqrt{v_s^2 + v_z'^2} = \sqrt{v_s^2 + u^2 + 2gH}$ Clearly $v' > v$.

Note. If u is taken as negligibly small, then $v' = v$.

40. There are four forces acting at a point O produced by strings as shown in Fig. 5(N).30, which is at rest. Find the forces F_1 and F_2.

FIGURE 5(N).30

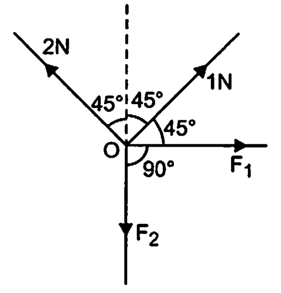
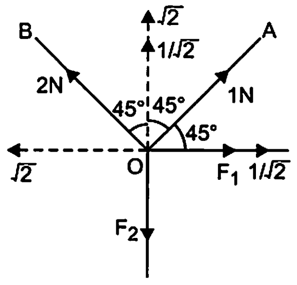


FIGURE 5(N).31



Ans. In Fig. 5(N).31, resolving force of 1 N along *OA* into two rectangular components :

$1/\sqrt{2}$ along F_1 and $1/\sqrt{2}$ opp to F_2

Similarly, two rectangular components of 2 N force along *OB* are

$\sqrt{2}$ opp. to F_2 and $\sqrt{2}$ opp. to F_1.

In equilibrium, (*i*) $F_1+\frac{1}{\sqrt{2}}=\sqrt{2}$ or $F_1=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{2-1}{\sqrt{2}}=\mathbf{0{\cdot}707\,N}$

$$(ii)\ F_2=\sqrt{2}+\frac{1}{\sqrt{2}}=\frac{2+1}{\sqrt{2}}=\frac{3}{\sqrt{2}}=\frac{3\times1{\cdot}414}{2}=\mathbf{2{\cdot}121\,N}$$

41. A rectangular box lies on a rough inclined surface. The co-efficient of friction between the surface and the box is μ. Let the mass of the box be *m*.

(*a*) At what angle of inclination θ of the plane to the horizontal will the box just start to slide down the plane ?

(*b*) What is the force acting on the box down the plane, if the angle of inclination of the plane is increased to $\alpha > \theta$?

(*c*) What is the force needed to be applied upwards along the plane to make the box either remain stationary or just move up with uniform speed ?

(*d*) What is the force needed to be applied upwards along the plane to make the box move up the plane with acceleration *a* ?

Ans. (*a*) As the box tends to slide down, force of friction will act up the plane.

As is clear from Fig. 5(N).32, box will just start sliding down, when

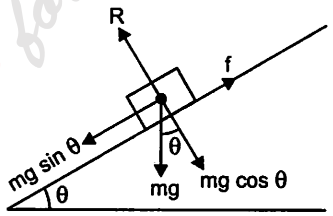
$mg\sin\theta=f=\mu R=\mu\, mg\cos\theta$

or $\tan\theta=\mu$ or $\boldsymbol{\theta=\tan^{-1}(\mu)}$.

(*b*) When angle of inclination is increased to $\alpha > \theta$, then net force acting on the box, down the plane is

$F_1=mg\sin\alpha-f=mg\sin\alpha-\mu R=\mathbf{mg\,(\sin\alpha-\mu\cos\alpha)}$.

FIGURE 5(N).32



(*c*) To keep the box either stationary or just move it up with uniform speed,

upward force needed, $F_2=mg\sin\alpha+f=\mathbf{mg\,(\sin\alpha+\mu\cos\alpha)}$

(In this case, friction would act down the plane).

(*d*) If the box is to be moved with an upward acceleration, *a*, then upward force needed,

$$F_3=\mathbf{mg\,(\sin\alpha+\mu\cos\alpha)+ma}$$

42. A helicopter of mass 2000 kg rises with a vertical acceleration of 15 ms^{-2}. The total mass of the crew and passengers is 500 kg. Give the magnitude and direction of the

(*a*) force on the floor of the helicopter by the crew and passengers.

(*b*) action of the rotor of the helicopter on the surrounding air.

(*c*) force on the helicopter due to the surrounding air, (g = 10 ms^{-2}).

Ans. Here, $m_1=2000$ kg , $a=15$ ms^{-2} ;

$m_2=500$ kg, $g=10$ ms^{-2}

(*a*) Force on the floor of the helicopter by the crew and passenger

$=m_2\,(g+a)=500\,(10+15)=12500\ \text{N}=\mathbf{12{\cdot}5\times10^3\ N}$

(*b*) Action of rotor of helicopter on the surrounding air

$=(m_1+m_2)\,(g+a)=(2000+500)\,(10+15)=2500\times25=\mathbf{6{\cdot}25\times10^4\ N}$

This force is downwards.

(*c*) Force on the helicopter due to surrounding air = reaction of force in (*b*) = **6·25 × 10⁴ N, upwards.**

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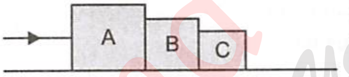
I. Multiple Choice Questions (with One Correct Answer)

I. Newton's first and second laws of motion

1. Three blocks *A*, *B* and *C* of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface, as shown in Fig. 5(CF).1. If a force of 14 N is applied on the 4 kg block, then the contact force between *A* and *B* is

FIGURE 5(CF).1



(*a*) 6 N (*b*) 8 N
(*c*) 18 N (*d*) 2 N

(AIPMT 2015)

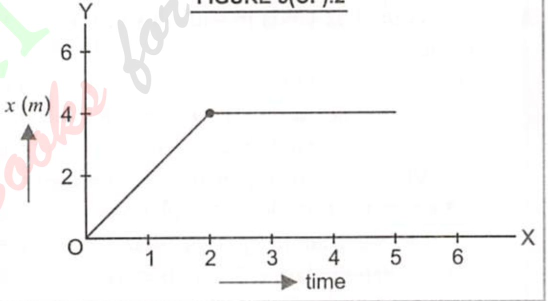
2. A balloon has 8 g of air. A small hole is pierced into it. The air escapes at a uniform rate of 7 cm/s. If the balloon shrinks in 5·6 s, then the average force acting on the balloon is

(*a*) 10^{-4} N (*b*) 10^{-2} dyne
(*c*) 56 dyne (*d*) 10^{-6} N

3. In Fig. 5(CF).2, the position time graph of a particle of mass 0·1 kg is shown. The impulse at *t* = 2 sec. is

(*a*) 0·2 kg ms^{-1} (*b*) 0·02 kg ms^{-1}
(*c*) 0·1 kg ms^{-1} (*d*) 0·4 kg ms^{-1}

FIGURE 5(CF).2

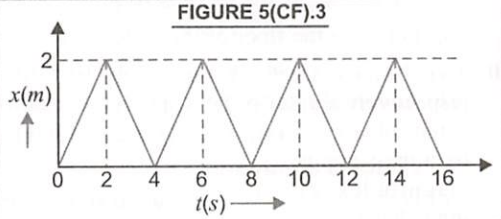


4. A particle of mass *m* is projected with velocity *v* making an angle of 45° with the horizontal. When the particle lands on the level ground, the magnitude of the change in its momentum will be

(*a*) $2\,mv$ (*b*) $mv/\sqrt{2}$
(*c*) $mv\sqrt{2}$ (*d*) zero **(AIPMT 2008)**

5. Figure 5(CF).3 shows the position–time $(x-t)$ graph of one-dimensional motion of a body of mass 0·4 kg. The magnitude of each impulse is

FIGURE 5(CF).3



ANSWERS

1. (*a*) **2.** (*a*) **3.** (*a*) **4.** (*c*)

(*a*) 0·8 Ns (*b*) 1·6 Ns
(*c*) 0·2 Ns (*d*) 0·4 Ns

(AIEEE 2010)

6. A stone is dropped from a height h. It hits the ground with a certain momentum p. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by :

(*a*) 68% (*b*) 41%
(*c*) 200% (*d*) 100%

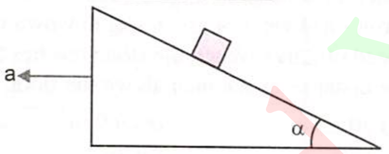
(AIPMT Main 2012)

7. A heavy iron bar of weight W is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle θ with the horizontal. What is the weight experienced by the man ?

(*a*) $W \sin \theta$ (*b*) $W \cos \theta$
(*c*) W (*d*) $W/2$

8. A block is kept on a frictionless inclined surface with angle of inclination 'α'. The incline is given an acceleration 'a' to keep the block stationary, Fig. 5(CF).4. Then a is equal to

FIGURE 5(CF).4



(*a*) $g \tan \alpha$ (*b*) g
(*c*) $g \operatorname{cosec} \alpha$ (*d*) $g/\tan \alpha$

9. A balloon with mass m is descending down with an acceleration a (where $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a ?

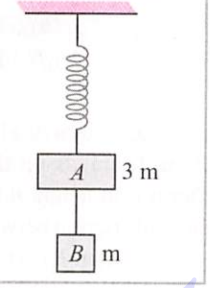
(*a*) $\dfrac{2ma}{g+a}$ (*b*) $\dfrac{2ma}{g-a}$

(*c*) $\dfrac{ma}{g+a}$ (*d*) $\dfrac{ma}{g-a}$

(AIPMT 2014)

10. Two blocks A and B of masses 3 m and m respectively are connected by a massless and inextensible string. The whole system is suspended by a mass spring as shown in Fig. 5(CF).5. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively

FIGURE 5(CF).5



(*a*) $g, \dfrac{g}{3}$ (*b*) $\dfrac{g}{3}, g$

(*c*) g, g (*d*) $\dfrac{g}{3}, \dfrac{g}{3}$

(NEET 2017)

11. An electric lift with a maximum load of 2000 kg (lift + passengers) is moving up with a constant speed of 1·5 ms^{-1}. The frictional force opposing the motion is 3000 N. the minimum power delivered by the motor to the lift in watt is ($g = 10$ m/s^2)

(*a*) 23000 (*b*) 20000
(*c*) 34500 (*d*) 23500 **(NEET 2022)**

12. A shell of mass m is at rest initially. It explodes into three fragments having mass in the ratio 2 : 2 : 1. If the fragments having equal masses fly off along mutually perpendicular directions with speed v, the speed of third fragment is

(*a*) v (*b*) $\sqrt{2}\,v$

(*c*) $2\sqrt{2}\,v$ (*d*) $3\sqrt{2}\,v$

(NEET 2022)

13. A hall of mass 0·15 kg is dropped from a height 10 m, strikes the ground and rebounds to the same height. The magnitude of impulse imparted to the ball is nearly ($g = 10$ m/s^2)

(*a*) 1·4 kg m/s (*b*) 0 kg m/s
(*c*) 4·2 kg m/s (*d*) 2·1 kg m/s

(NEET 2021)

14. Two bodies of the same mass are moving with the same speed, but in different directions in *a*-plane. They have a completely inelastic collision and move together thereafter with a final speed, which is half of the initial velocities of two bodies.

ANSWERS

5. (*a*) **6.** (*b*) **7.** (*d*) **8.** (*a*) **9.** (*a*) **10.** (*b*) **11.** (*c*) **12.** (*c*) **13.** (*c*)

The angle between their initial velocities in degrees is

(*a*) 120° (*b*) 60°
(*c*) 90° (*d*) 30°

(JEE Main 2020)

15. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch, but starts slipping after it is at height *h* from the bottom. If the coefficient of friction between the ground and the insect is 0·75, then *h* is (take $g = 10\ \text{m/s}^2$)

(*a*) 0·20 m (*b*) 0·60 m
(*c*) 0·80 m (*d*) 0·45 m

(JEE Main 2020)

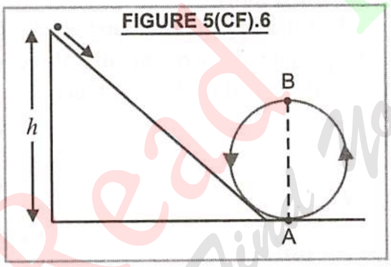
16. A mass *m* is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when

(*a*) mass is at the highest point
(*b*) wire is horizontal
(*c*) mass is at the lowest point
(*d*) inclined at an angle of 60° from vertical

(NEET 2019)

17. A body initially at rest and sliding along a frictionless track from a height *h* as shown in Fig. 5(CF).6 just completes a vertical circle of diameter *AB* = *D*. The height *h* is equal to

FIGURE 5(CF).6



(*a*) $\frac{7}{5}$ D (*b*) *D*

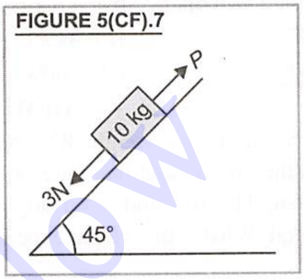
(*c*) $\frac{3}{2}$ D (*d*) $\frac{5}{4}$ D **(NEET 2018)**

18. Two particles *A* and *B* are moving in uniform circular motion in concentric circles of radii r_A and r_B with speeds V_A and V_B respectively. Their time period of rotation is the same. The ratio of angular speeds of *A* and *B* will be

(*a*) $r_A : r_B$ (*b*) $V_A : V_B$
(*c*) $r_B : r_A$ (*d*) 1 : 1 **(NEET 2019)**

19. A block of mass 10 kg is kept on a rough inclined plane as shown in Fig. 5(CF).7. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0·6. What should be the minimum value of force *P*, such that the block does not move downward ? Take $g = 10\ \text{m/s}^2$.

FIGURE 5(CF).7



(*a*) 32 N (*b*) 18 N
(*c*) 23 N (*d*) 25 N

(JEE Main 2019)

II. Newton's third law of motion and conservation of linear momentum

20. A man of mass 50 kg is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of mass 0·5 kg downwards with a speed of 2 m/s. When the stone reaches the floor, the distance of the man above the floor will be

(*a*) 20 m (*b*) 9·9 m
(*c*) 10·1 m (*d*) 10 m

21. A body of mass 1 kg, initially at rest, explodes and breaks into three parts of masses in the ratio 1 : 1 : 3. If the two pieces of equal masses fly off perpendicular to each other with a speed of 30 m/s. The speed of third piece will be

(*a*) $10\sqrt{2}$ m/s (*b*) $20\sqrt{2}$ m/s
(*c*) $5\sqrt{2}$ m/s (*d*) $40\sqrt{2}$ m/s

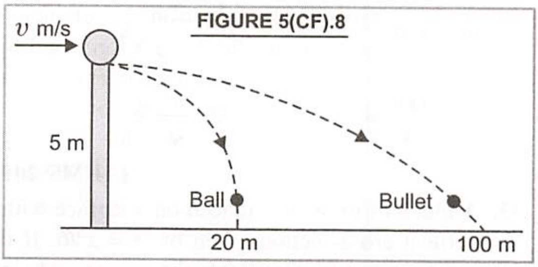
22. A ball of mass 0·2 kg rests on a vertical post of height 5 m. A bullet of mass 0·01 kg travelling with a velocity *v* m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and the bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity *v* of the bullet is :

(use $g = 10\ \text{m/s}^2$)

ANSWERS

14. (*a*) **15.** (*a*) **16.** (*c*) **17.** (*d*) **18.** (*d*) **19.** (*a*) **20.** (*c*) **21.** (*a*)

FIGURE 5(CF).8



(*a*) 250 m/s (*b*) $250\sqrt{2}$ m/s

(*c*) 400 m/s (*d*) 500 m/s

23. The line of action of the resultant of two like parallel forces shifts by one fourth of the distance between the forces when the two forces are interchanged. The ratio of the two forces is

(*a*) 3 : 4 (*b*) 1 : 2

(*c*) 3 : 5 (*d*) 2 : 3

24. A mass of 3 kg descending vertically downwards supports a mass of 2 kg by means of a light string passing over a pulley. At the end of 5 s, the string breaks. How much high from now the 2 kg mass will go ? ($g = 9{\cdot}8$ m/s^2)

(*a*) 9·8 m (*b*) 19·6 m

(*c*) 2·45 m (*d*) 4·9 m

25. A shell of mass 200 gram is ejected from a gun of mass 4 kg by an explosion that generates 1·05 kJ of energy. The initial velocity of the shell is

(*a*) 100 ms^{-1} (*b*) 80 ms^{-1}

(*c*) 40 ms^{-1} (*d*) 120 ms^{-1}

(AIPMT 2008)

26. A block *A* of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end, another block *B* of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block *A* is sliding on the table, the tension in the string is

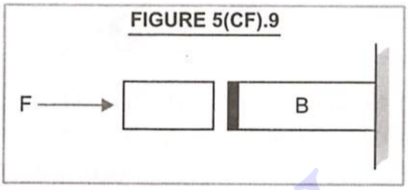
(*a*) $\dfrac{(m_2 - \mu_k m_1)\,g}{(m_1 + m_2)}$ (*b*) $\dfrac{m_1 m_2 (1 + \mu_k)\,g}{m_1 + m_2}$

(*c*) $\dfrac{m_1 m_2 (1 - \mu_k)\,g}{m_1 + m_2}$ (*d*) $\dfrac{(m_2 + \mu_k m_1)\,g}{(m_1 + m_2)}$

(AIPMT 2015)

27. Given in Fig. 5(CF). 9, are two blocks *A* and *B* of weight 20 N and 100 N respectively. These are being pressed against a wall by a force *F* as shown. If the coefficient of friction between the blocks is 0·1 and between block *B* and the wall is 0·15, the frictional force applied by the wall on the block *B* is

FIGURE 5(CF).9



(*a*) 100 N (*b*) 80 N

(*c*) 120 N (*d*) 150 N

(JEE Main 2015)

III. Friction

28. A block has been placed on an inclined plane with the slope angle θ. The block slides down the plane at constant speed. The coefficient of kinetic friction is equal to

(*a*) sin θ (*b*) cos θ

(*c*) *g* (*d*) tan θ

(AIIMS 2014)

29. A block *A* of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end, another block *B* of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block *A* is sliding on the table, the tension in the string is

(*a*) $\dfrac{(m_2 - \mu_k m_1)\,g}{(m_1 + m_2)}$ (*b*) $\dfrac{m_1 m_2 (1 + \mu_k)\,g}{m_1 + m_2}$

(*c*) $\dfrac{m_1 m_2 (1 - \mu_k)\,g}{m_1 + m_2}$ (*d*) $\dfrac{(m_2 + \mu_k m_1)\,g}{(m_1 + m_2)}$

(AIPMT 2015)

30. A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4·0 m down the plank in 4·0 s. The coefficients of static and kinetic friction between the box and the plank will be, respectively

ANSWERS

22. (*d*) **23.** (*c*) **24.** (*d*) **25.** (*a*) **26.** (*b*) **27.** (*c*) **28.** (*d*) **29.** (*b*)

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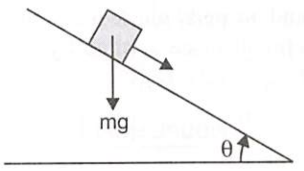
FIGURE 5(CF).10



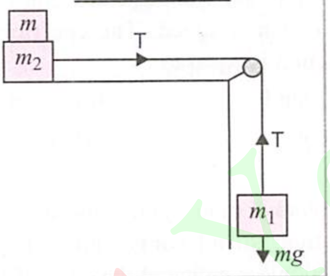
(*a*) 0·4 and 0·3 (*b*) 0·6 and 0·6

(*c*) 0·6 and 0·5 (*d*) 0·5 and 0·6

(RE-AIPMT 2015)

31. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is

FIGURE 5(CF).11



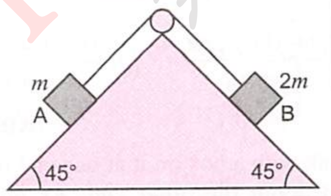
(*a*) 18.3 kg (*b*) 27.3 kg

(*c*) 43.3 kg (*d*) 10.3 kg

(JEE Main 2018)

32. Consider the Fig. 5(CF).12, shown below in which two masses of m and $2m$ are placed on a fixed triangular wedge.

FIGURE 5(CF).12



The coefficient of friction between block A and the wedge is 2/3, while that for block B and the wedge is 1/3.

If the whole system is released from rest, then acceleration of block A is

(*a*) zero (*b*) $\dfrac{2m^2}{3}g$

(*c*) $\dfrac{4m^2}{3}g$ (*d*) $\dfrac{m^2}{\sqrt{2}}g$

(AIIMS 2015)

33. A block of mass m is placed on a surface with a vertical cross-section given by, $y = x^3/6$. If the coefficient of friction is 0·5, the maximum height above the ground at which the block can be placed without slipping is

(*a*) $\dfrac{1}{3}$ m (*b*) $\dfrac{1}{2}$ m

(*c*) $\dfrac{1}{6}$ m (*d*) $\dfrac{2}{3}$ m

(JEE Main 2014)

34. A uniform wooden stick of mass 1·6 kg of length l rests in an inclined manner on a smooth vertical wall of height h ($< l$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are ($g = 10$ m/s^2)

(*a*) $\dfrac{h}{l} = \dfrac{\sqrt{3}}{16}$; $f = \dfrac{16\sqrt{3}}{3}$ N

(*b*) $\dfrac{h}{l} = \dfrac{3}{16}$; $f = \dfrac{16\sqrt{3}}{3}$ N

(*c*) $\dfrac{h}{l} = \dfrac{3\sqrt{3}}{16}$; $f = \dfrac{8\sqrt{3}}{3}$ N

(*d*) $\dfrac{h}{l} = \dfrac{3\sqrt{3}}{16}$; $f = \dfrac{16\sqrt{3}}{3}$ N

(JEE Advanced 2016)

35. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if coefficient of friction between the block and the lower half of the plane is given by

ANSWERS

30. (*c*) **31.** (*b*) **32.** (*a*) **33.** (*c*) **34.** (*d*) (*a*) $\mu = \frac{1}{\tan\theta}$ (*b*) $\mu = \frac{2}{\tan\theta}$

(*c*) $\mu = 2\tan\theta$ (*d*) $\mu = \tan\theta$

(AIPMT 2013)

36. A body takes time t to reach the bottom of an inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is $2t$. The coefficient of friction of the rough surface is

(*a*) $\frac{3}{4}\tan\theta$ (*b*) $\frac{1}{2}\tan\theta$

(*c*) $\frac{2}{3}\tan\theta$ (*d*) $\frac{1}{4}\tan\theta$

37. A given object takes n times more time to slide down 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

(*a*) $\frac{1}{2-n^2}$ (*b*) $1-\frac{1}{n^2}$

(*c*) $\sqrt{1-\frac{1}{n^2}}$ (*d*) $\sqrt{\frac{1}{1-n^2}}$

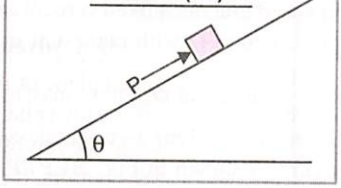
38. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end, $\mu = 0{\cdot}15$ and $g = 10\ \text{m/s}^2$. The truck starts from rest with an acceleration of $2\ \text{m/s}^2$ on a straight road. The box will fall off the truck when it is at a distance from the starting point equal to

(*a*) 14 m (*b*) 8 m

(*c*) 16 m (*d*) 4 m

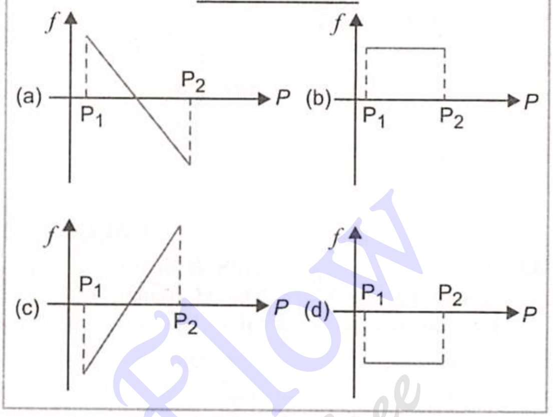
39. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane as shown in Fig. 5(CF).13.

FIGURE 5(CF).13



The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg \times (\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph in Fig. 5(CF).14 will look like

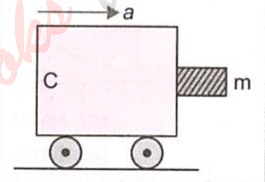
FIGURE 5(CF).14



(IIT 2010)

40. A block of mass m is in contact with the cart C as shown in Fig. 5(CF).15.

FIGURE 5(CF).15



The coefficient of static friction between the block and the cart is μ. The acceleration a of the cart that will prevent the block from falling satisfies

(*a*) $a > \frac{mg}{\mu}$ (*b*) $a > \frac{g}{\mu m}$

(*c*) $a \geq \frac{g}{\mu}$ (*d*) $a < \frac{g}{\mu}$

(AIPMT 2010)

41. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

ANSWERS

35. (*c*) **36.** (*a*) **37.** (*b*) **38.** (*c*) **39.** (*a*) **40.** (*c*) **41.** (*c*)

42. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

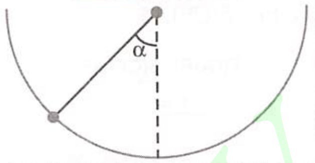
(*a*) g (*b*) $\frac{2}{3}$g

(*c*) $\frac{g}{3}$ (*d*) $\frac{3}{2}$g

(AIEEE 2011)

43. An insect crawls up a hemispherical surface very slowly, Fig. 5(CF).16. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the max. possible value of α is given by

FIGURE 5(CF).16

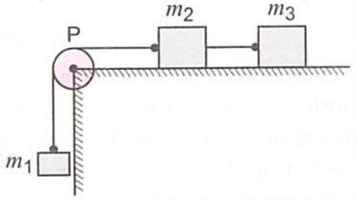


(*a*) cot α = 3 (*b*) sec α = 3

(*c*) cosec α = 3 (*d*) None

44. A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P, Fig. 5(CF).17. The mass m_1 hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction = μ). The pulley is frictionless and of negligible mass. The downward acceleration of mass m_1 is : (Assume $m_1 = m_2 = m_3 = m$)

FIGURE 5(CF).17



(*a*) $\frac{g(1-3\mu)}{9}$ (*b*) $\frac{2g\mu}{3}$

(*c*) $\frac{g(1-2\mu)}{3}$ (*d*) $\frac{g(1-2\mu)}{2}$

(AIPMT 2014)

45. A block of mass m is placed on a surface with a vertical cross-section given by, $y = x^3/6$. If the coefficient of friction is 0·5, the maximum height above the ground at which the block can be placed without slipping is

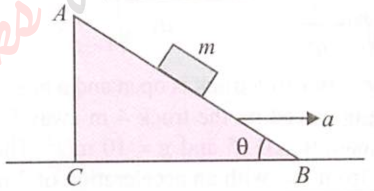
(*a*) $\frac{1}{3}$ m (*b*) $\frac{1}{2}$ m

(*c*) $\frac{1}{6}$ m (*d*) $\frac{2}{3}$ m

[JEE (Main) 2014]

46. A block of mass m is placed on a smooth inclined wedge ABC of inclination θ as shown in the Fig. 5(CF).18. The wedge is given an acceleration 'a' towards the right. The relation between a and θ for the block to remain stationary on the wedge is

FIGURE 5(CF).18



(*a*) $a = g\cos\theta$ (*b*) $a = \frac{g}{\sin\theta}$

(*c*) $a = \frac{g}{\text{cosec}\,\theta}$ (*d*) $a = g\tan\theta$

(NEET 2018)

IV. Dynamics of Circular Motion

47. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in Fig. 5(CF).19. At some instant, the angular speed of the system is $\frac{8}{9}$ω and

ANSWERS

42. (*b*) **43.** (*a*) **44.** (*c*) **45.** (*c*) **46.** (*d*)

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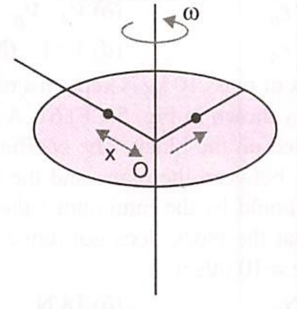
one of its masses is at a distance of $\frac{3}{5}R$ from O. At this instant, the distance of other mass from O is

FIGURE 5(CF).19



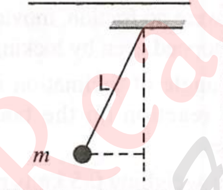
(*a*) $\frac{2}{3}R$ (*b*) $\frac{1}{3}R$

(*c*) $\frac{3}{5}R$ (*d*) $\frac{4}{5}R$

(JEE Advanced 2015)

48. A ball of mass (m) 0·5 kg is attached to the end of a string having length (L) 0·5 m. The ball is rotated on a horizontal circular path about vertical axis, Fig. 5(CF).20. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is

FIGURE 5(CF).20



(*a*) 9 (*b*) 18

(*c*) 27 (*d*) 36

(IIT 2011)

49. A car of mass m is moving on a level circular track of radius R. If μ_s represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by

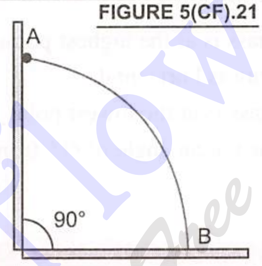
(*a*) $\sqrt{\mu_s mRg}$ (*b*) $\sqrt{Rg/\mu_s}$

(*c*) $\sqrt{mRg/\mu_s}$ (*d*) $\sqrt{\mu_s Rg}$

(AIPMT Main 2012)

50. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in Fig. 5(CF).21. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

FIGURE 5(CF).21



(*a*) always radially outwards

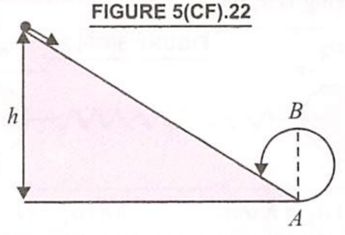
(*b*) always radially inwards

(*c*) radially outwards initially and radially inwards later

(*d*) radially inwards initially and radially outwards later. **[JEE (Advanced) 2014]**

51. A body initially at rest and sliding along a frictionless track from a height h (as shown in the Fig. 5(CF).22) just completes a vertical circle of diameter $AB = D$. The height h is equal to

FIGURE 5(CF).22



(*a*) $\frac{7}{5}D$ (*b*) D

(*c*) $\frac{3}{2}D$ (*d*) $\frac{5}{4}D$ **(NEET 2018)**

52. A block of mass 10 kg is in contact against the inner wall of a hollow cylindrical drum of radius 1 m. The coefficient of friction between the block

ANSWERS

47. (*d*) **48.** (*d*) **49.** (*d*) **50.** (*d*) **51.** (*d*) and inner wall of cylinder is 0·1. The minimum angular velocity needed for the cylinder to keep the block stationary when the cylinder is vertical and rotating about its axis will be ($g = 10$ m/s^2).

(*a*) $\sqrt{10}$ rad/s (*b*) $\dfrac{10}{2\pi}$ rad/s

(*c*) 10 rad/s (*d*) 10 π rad/s

(NEET 2019)

53. A mass m is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when

(*a*) the mass is at the highest point

(*b*) the wire is horizontal

(*c*) the mass is at the lowest point

(*d*) inclined at an angle of 60° from vertical

(NEET 2019)

54. Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speeds V_A and V_B respectively. Their time period of rotation is the same. The ratio of angular speed of A and B will be

(*a*) $r_A : r_B$ (*b*) $V_A : V_B$

(*c*) $r_B : r_A$ (*d*) 1 : 1 **(NEET 2019)**

55. A block of mass 10 kg is kept on a rough inclined plane as shown in Fig. 5(CF).61. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0·6. What should be the minimum value of force P, such that the block does not move downward ? (Take $g = 10$ m/s^2)

(*a*) 32 N (*b*) 18 N

(*c*) 23 N (*d*) 25 N

(JEE Main 2019)

II. Multiple Choice Questions (with One or More than One Correct Answers)

56. The force exerted by the floor of an elevator on the foot of a person standing there is more than the weight of the person if the elevator is

(*a*) going up and slowing down

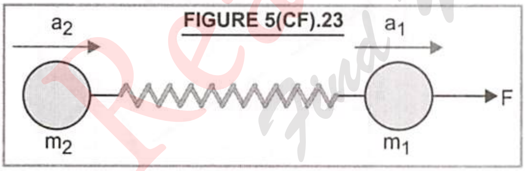
(*b*) going up and speeding up

(*c*) going down and slowing down

(*d*) going down and speeding up

57. A spring connects two particles of masses m_1 and m_2. A horizontal force F acts on m_1, Fig. 5(CF).23. Ignoring friction, when the elongation of the spring is x, then

FIGURE 5(CF).23



(*a*) $a_2 = Kx/m_2$ (*b*) $a_1 = (F - Kx)/m_1$

(*c*) $F = m_1\,a_1 + m_2\,a_2$

(*d*) $a_1 = a_2 = \dfrac{F}{(m_1 + m_2)}$

[when the spring has maximum elongation]

58. In which of the following cases, the net force is zero ?

(*a*) a drop of rain falling down with terminal velocity

(*b*) a cork of mass 20 g floating in water

(*c*) a car moving with constant speed of 60 km/h on a rough road

(*d*) in a tug of war game, if one team applies more force than other

59. Which of the following statement(s) is (are) correct ?

(*a*) If there is no friction, work needs to be done to move a body up an inclined plane.

(*b*) If there were no friction, moving vehicles could not be stopped even by locking the brakes.

(*c*) As the angle of inclination is increased, the normal reaction on the body placed on it increases.

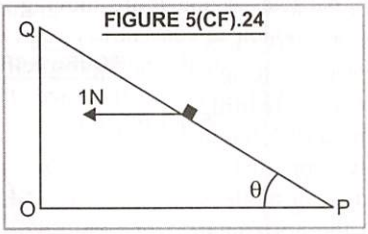
(*d*) A duster weighing 0·5 kg is pressed against a vertical board with a force of 11 N. If the coefficient of friction is 0·5, the work done in rubbing it upward through a distance of 10 cm is 0·55 J.

60. A small block of mass of 0·1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in Fig. 5(CF).24. The block remains stationary if (take $g = 10$ m/s^2)

ANSWERS

52. (*c*) **53.** (*c*) **54.** (*d*) **55.** (*a*) **56.** (*b*,*c*) **57.** (*a*,*b*,*c*,*d*)
58. (*a*,*b*,*c*) **59.** (*a*,*b*)

FIGURE 5(CF).24



(*a*) $\theta = 45°$

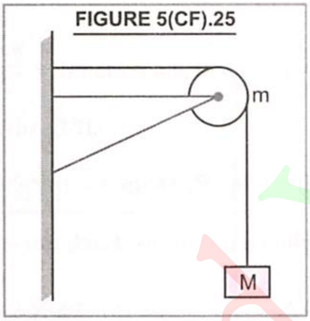
(*b*) $\theta > 45°$ and a frictional force acts on the block towards *P*

(*c*) $\theta > 45°$ and a frictional force acts on the block towards *Q*

(*d*) $\theta < 45°$ and a frictional force acts on the block towards *Q* **(IIT 2012)**

61. A string of negligible mass going over a clamped pulley of mass *m* supports a block of mass *M* as shown in Fig. 5(CF).25. The force on the pulley by the clamp is given by

FIGURE 5(CF).25



(*a*) $\sqrt{2}\,Mg$ (*b*) $\sqrt{2}\,mg$

(*c*) $[\sqrt{(M+m)^2+m^2}\,]g$

(*d*) $[\sqrt{(M+m)^2+M^2}\,]g$

62. A particle stays at rest as seen in a frame. We can conclude that

(*a*) the frame is inertial

(*b*) resultant force on the particle is zero

(*c*) the frame may be inertial but resultant force on the particle is zero

(*d*) the frame may be non-inertial but there is a non zero resultant force.

63. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of frictional force acting on the cylinder are

(*a*) up the incline while ascending and down the incline while descending,

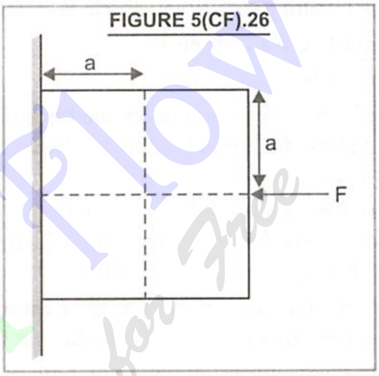
(*b*) up the incline while ascending as well as descending,

(*c*) down the incline while ascending and upto the incline while descending,

(*d*) down the incline while ascending as well as descending.

64. A block of mass *m* is at rest under the action of force *F* against a wall as shown in Fig. 5(CF).26. Which of the following statements is incorrect ?

FIGURE 5(CF).26



(*a*) $f = mg$ [where *f* is the friction force]

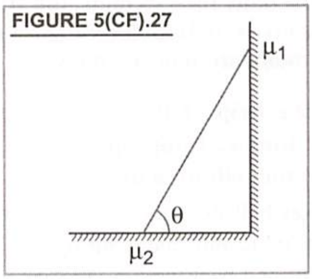
(*b*) $F = N$ [where *N* is the normal force]

(*c*) *F* will not produce torque

(*d*) *N* will not produce torque **(IIT 2005)**

65. In Fig. 5(CF).27, a ladder of mass *m* is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then

FIGURE 5(CF).27



(*a*) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_2 \tan\theta = mg/2$

(*b*) $\mu_1 \neq 0$, $\mu_2 = 0$ and $N_1 \tan\theta = mg/2$

ANSWERS

60. (*a,c*) **61.** (*c*) **62.** (*c,d*) **63.** (*b*) **64.** (*c,d*) (*c*) $\mu_1 \neq 0$, $\mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(*d*) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

[JEE (Advanced) 2014]

66. Consider a vehicle going on a horizontal road towards east. Neglect any force by the air. The frictional force on the vehicle by the road

(*a*) is zero if the vehicle is moving with a uniform velocity

(*b*) is towards east if the vehicle is accelerating

(*c*) must be towards east

(*d*) must be towards west.

67. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts

(*a*) In the backward direction on the front wheel and in the forward direction on the rear wheel when cycle is being pedalled

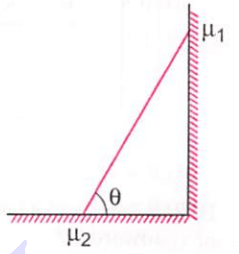
(*b*) In the forward direction on the front wheel and in the backward direction on the rear wheel

(*c*) In the backward direction on both the wheels when pedalling is stopped

(*d*) In the forward direction on both the wheels.

68. In the Fig. 5(CF).28, a ladder of mass *m* is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then

FIGURE 5(CF).28



(*a*) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_2 \tan\theta = mg/2$

(*b*) $\mu_1 \neq 0$, $\mu_2 = 0$ and $N_1 \tan\theta = mg/2$

(*c*) $\mu_1 \neq 0$, $\mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(*d*) $\mu_1 = 0$, $\mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

(JEE Advanced 2014)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1 **According to Newton's second law of motion, *F* = *ma*, where *F* is the force required to produce an acceleration *a* in a body of mass *m*. If *a* = 0 then *F* = 0, *i.e.*, no external force is required to move a body uniformly along a straight line. If a force *F* acts on a body for *t* seconds, the effect of the force is given by Impulse = *F* × *t* = change in linear momentum of the body.**

With the help of the passage given above, choose the most appropriate alternative for each of the following questions :

69. A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that the ball is turned back with a velocity of 20 m/s. If duration of contact between the ball and the bat is 0·01 sec. The impulse of the force is

(*a*) 7·4 N-s (*b*) 4·8 N-s

(*c*) 1·2 N-s (*d*) 4·7 N-s

70. Average force exerted by the bat is

(*a*) 480 N (*b*) 120 N

(*c*) 1200 N (*d*) 840 N

Comprehension 2 **Friction between any two surfaces in contact is the force that opposes the relative motion between them. The force of limiting friction (*F*) between any two surfaces in contact is directly proportional to the normal reaction (*R*) between them *i.e.*, *F* ∝ *R* or *F* = μ*R*, where μ is coefficient of limiting friction. If θ is angle of friction, then μ = tan θ.**

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions :

ANSWERS

65. (*c*,*d*) **66.** (*b*,*d*) **67.** (*a*,*c*) **68.** (*c*,*d*) **69.** (*b*) **70.** (*a*)

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71. A force of 49 N is just able to move a block of wood weighing 10 kg on a rough horizontal surface. The coefficient of friction is

(*a*) 0·5 (*b*) 4·9
(*c*) 10/49 (*d*) 49/9·8

72. The acceleration produced in the block in the above question is

(*a*) 9·8 ms^{-2} (*b*) 0·3 ms^{-2}
(*c*) 1·5 ms^{-2} (*d*) 4·9 ms^{-2}

IV. Matching Type Questions

DIRECTIONS. In the following questions, match column I and column II. Select the correct match out of the four choies given at the end.

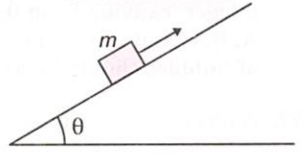
73.

	Column I		Column II
(A)	Body lying on a horizontal surface	(*p*)	is a self adjusting force
(B)	Static friction	(*q*)	is maximum value of static friction
(C)	Limiting friction	(*r*)	is less than limiting friction
(D)	Dynamic friction	(*s*)	force of friction = 0

(*a*) A-*s*, B-*p*, C-*q*, D-*r* (*b*) A-*p*, B-*q*, C-*r*, D-*s* (*c*) A-*s*, B-*r*, C-*q*, D-*p* (*d*) A-*r*, B-*q*, C-*p*, D-*s*

74. A block of mass *m* is thrown upwards with some initial velocity up a rough inclined plane as shown in Fig. 5(CF).29 on the block :

FIGURE 5(CF).29

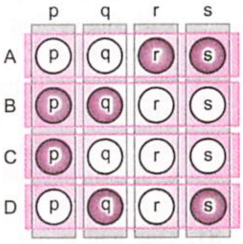


	Column I		Column II
(A)	Net force along the plane	(*p*)	$m(g \sin^2\theta + \mu g \sin\theta\cos\theta)$
(B)	Net force perpendicular to plane	(*q*)	$m(g \sin\theta + \mu g \cos\theta)$
(C)	Net force in horizontal direction	(*r*)	Zero
(D)	Net force in vertical direction	(*s*)	$m(g \sin\theta\cos\theta + \mu g \cos^2\theta)$

(*a*) A-*s*, B-*r*, C-*q*, D-*p* (*b*) A-*p*, B-*q*, C-*r*, D-*s* (*c*) A-*p*, B-*q*, C-*s*, D-*r* (*d*) A-*q*, B-*r*, C-*s*, D-*p*

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*r*, *s* ; B-*p*, *q* ; C-*p* ; D-*q*, *s*, then the correctly bubbled matrix will look like the one shown here :



	p	q	r	s
A	○	○	●	●
B	●	●	○	○
C	●	○	○	○
D	○	●	○	●

75.

	Column I		Column II
(A)	centripetal force	(*p*)	$\tan\theta = v^2/rg$
(B)	centrifugal force	(*q*)	$v_{max} = \sqrt{\mu rg}$
(C)	bending of cyclist	(*r*)	mv^2/r
(D)	rounding a level curve	(*s*)	$m\,r\,\omega^2$

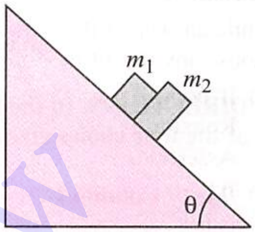
ANSWERS

71. (*a*) **72.** (*d*) **73.** (*a*) **74.** (*d*) **75.** A-*r*, *s* ; B-*r*, *s* ; C-*p* ; D-*q*

76. A block of mass $m_1 = 1$ kg and another mass $m_2 = 2$ kg, are placed together (see Fig. 5(CF).30) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0{\cdot}3$. In List II, expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.

[**Useful information :** $\tan(5{\cdot}5°) \approx 0{\cdot}1$:
$\tan(11{\cdot}5°) = 0{\cdot}2$; $\tan(16{\cdot}5°) \approx 0{\cdot}3$]

FIGURE 5(CF).30



List-I	List-II
(A) θ = 5°	(*p*) $m_2\, g \sin\theta$
(B) θ = 10°	(*q*) $(m_1 + m_2)\, g \sin\theta$
(C) θ = 15°	(*r*) $\mu\, m_2\, g \cos\theta$
(D) θ = 20°	(*s*) $\mu\,(m_1 + m_2)\, g \sin\theta$

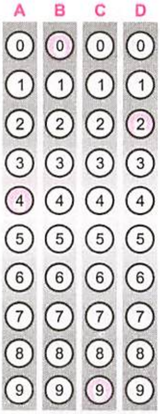
Code :

(*a*) A-*p*, B-*p*, C-*p*, D-*r*
(*b*) A-*q*, B-*q*, C-*q*, D-*r*
(*c*) A-*q*, B-*q*, C-*q*, D-*s*
(*d*) A-*q*, B-*q*, C-*r*, D-*r*

(JEE Advanced 2014)

VI. Integer Type Questions

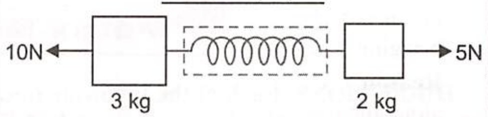
DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :



77. A block is released from top of a smooth inclined plane. It reaches the bottom of the plane in $\sqrt{2}\,$s. The time taken (in second) by the body to cover 1st half of inclined plane is :

78. Two bodies of masses 3 kg and 2 kg are connected by a spring balance, Fig. 5(CF).31. Two forces of 10 N and 5 N are applied on the blocks as shown. The reading of the spring balance (in newton) would be :

FIGURE 5(CF).31



79. A block weighing 4 N is supported by two ropes. One rope is horizontal and the other makes an angle of 30° with the ceiling. The tension (in newton) in the rope attached to the ceiling is :

80. A chain consisting of 5 links each of mass 0·1 kg is lifted vertically with a constant acceleration of 2 m/s^2 as shown in Fig. 5(CF).32. The force of interaction (in newton) between the top link and the link immediately below it will be : Take $g = 10$ m/s^2.

81. One end of massless rope, which passes over a massless and frictionless pulley *P* is tied to a hoosk *C*, while the other end is free, Fig. 5(CF).33. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in m/s^2) can a man of 60 kg climb down the rope ?

FIGURE 5(CF).32

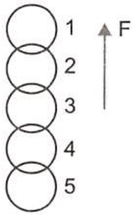
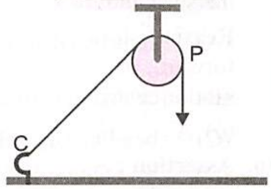


FIGURE 5(CF).33



ANSWERS

76. A-*q* ; B-*q* ; C-*r* ; D-*r* **77.** (1) **78.** (7) **79.** (8) **80.** (3) **81.** (4)

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. The following questions consist of two statements each, printed as **Assertion** and **Reason.**

While answering these questions you are required to choose any one of the following four responses.

(A) If both, Assertion and Reason are true and the Reason is the correct explanation of the Assertion.

(B) If both, Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If both, Assertion and Reason are false.

82. Assertion : A person receives more injury when he falls from a height on a concrete pavement than when he falls from the same height on a bed of sand.

Reason : The force exerted on the person by the concrete pavement is more than that exerted by the bed of sand.

(*a*) A (*b*) B (*c*) C (*d*) D

83. Assertion : Slope of momentum-time graph gives acceleration.

Reason : Acceleration is given by rate of change of momentum.

(*a*) A (*b*) B (*c*) C (*d*) D

84. Assertion : The work done in bringing a body down from the top to the base along a frictionless inclined plane is the same as the work done in bringing it down along the vertical side.

Reason : The gravitational force on the body along the inclined plane is the same as that along the vertical side.

(*a*) A (*b*) B (*c*) C (*d*) D

85. Assertion : A rocket moves forward by pushing the surrounding air backwards.

Reason : It drives the necessary thrust to move forward, according to Newton's second law of motion.

(*a*) A (*b*) B (*c*) C (*d*) D

86. Assertion : A rocket in flight is an illustration of projectile.

Reason : Because rocket is not projected with some initial velocity.

(*a*) A (*b*) B (*c*) C (*d*) D

87. Assertion : A body dropped from a given height and another body projected horizontally from the same height strike the ground simultaneously.

Reason : Because horizontal velocity has no effect in the vertical direction.

(*a*) A (*b*) B (*c*) C (*d*) D

88. Assertion : Friction is a self adjusting force.

Reason : Because friction does not depend upon mass of the body.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. Each of the following questions contains two statements. Check if

(A) Statement-1 is true ; Statement-2 is true and Statement-2 is correct explanation of Statement-1.

(B) Statement-1 is true ; Statement-2 is true but Statement-2 is not a correct explanation of Statement-1.

(C) Statement-1 is true but Statement-2 is false.

(D) Statement-1 is false, but Statement-2 is true.

89. Statement-1 : Force of friction depends on the actual area of contact.

Statement-2 : Smoother the surfaces of contact, smaller is opposition to motion.

(*a*) A (*b*) B (*c*) C (*d*) D

90. Statement-1 : Thrust on a rocket depends only on velocity of exhaust gases. Rate of decrease of mass is irrelevant.

Statement-2 : Larger the velocity, greater is the thrust.

(*a*) A (*b*) B (*c*) C (*d*) D

91. Statement-1 : A horse cannot run a cart in empty space.

Statement-2 : A cart runs only on account of reaction of the ground on the feet of the horse.

(*a*) A (*b*) B (*c*) C (*d*) D

92. Statement-1 : A force of 1 kg *f* produces an acceleration of 1 m/s^2 in a body of mass 1 kg.

Statement-2 : It follows from $a = F/m$

(*a*) A (*b*) B (*c*) C (*d*) D

ANSWERS

82. (*a*) **83.** (*d*) **84.** (*c*) **85.** (*d*) **86.** (*d*) **87.** (*a*) **88.** (*d*) **89.** (*d*) **90.** (*d*)
91. (*a*) **92.** (*d*)

93. Statement-1 : A ball of mass 100 g hits a bat with a speed of 72 km/hr, and bounces back with the same speed, in one second. The force exerted by the bat on the ball is 4 N.

Statement-2 : It follows from $F = ma$

(*a*) A (*b*) B (*c*) C (*d*) D

94. Statement-1 : The maximum speed with which a vehicle can go round a level curve of diameter 20 m without skidding is $\sqrt{10}$ m/s , given $\mu = 0{\cdot}1$.

Statement-2 : It follows from $v \le \sqrt{\mu rg}$.

(*a*) A (*b*) B (*c*) C (*d*) D

VIII. Multiple Choice Questions (Based on Experimental Skills)

95. The force of limiting friction between a body and the surface of contact is 5 N. A force of 7 N is applied on the body and the actual motion starts. The effective force of friction now is

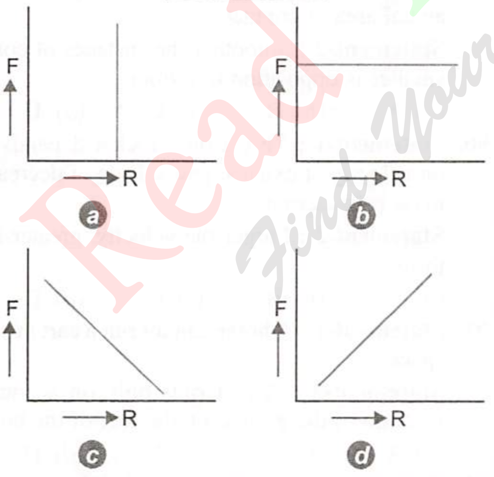
(*a*) zero (*b*) 5 N
(*c*) 7 N (*d*) < 5 N.

96. When a wheel is rolling on a level road, the direction of frictional force between the wheel and road is in

(*a*) backward direction (*b*) forward direction
(*c*) depends on speed (*d*) cannot say

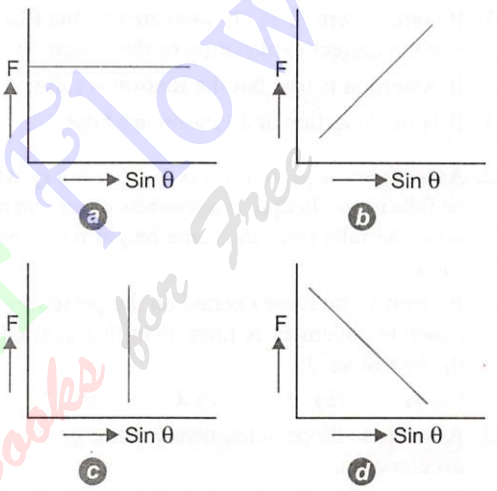
97. In a laboratory experiment, four students plotted graphs between force of limiting friction (*F*) and normal reaction (*R*), Fig. 5(CF).34. Which one is correct ?

FIGURE 5(CF).34



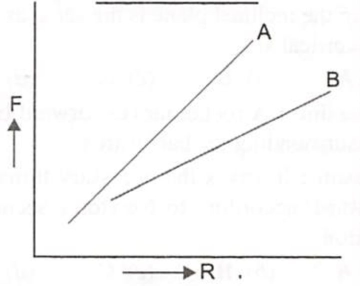
98. The variation of net downward force *F* on a body on a rough inclined plane versus sine of angle of inclination (θ) is shown by the four graphs, Fig. 5(CF).35. The correct one is

FIGURE 5(CF).35



99. For two bodies *A* and *B* of same material held on a horizontal plane, force of limiting friction *F* versus normal reaction *R* graphs are as shown in Fig. 5(CF).36. Which one has smoother surface in contact with the plane ?

FIGURE 5(CF).36



(*a*) *A* (*b*) *B*
(*c*) Both *A* and *B* (*d*) Neither *A* nor *B*.

ANSWERS

93. (*b*) **94.** (*a*) **95.** (*d*) **96.** (*b*) **97.** (*d*) **98.** (*b*) **99.** (*b*)

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For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Here $F = 14\text{ N}$,

$$m_A = 4\text{ kg}, m_B = 2\text{ kg}, m_C = 1\text{ kg}$$

Total mass $m = m_A + m_B + m_C$

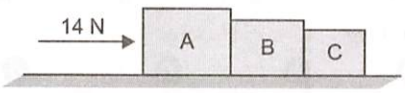
$$= 4 + 2 + 1 = 7\text{ kg}$$

Acceleration of the system

$$a = \frac{F}{m} = \frac{14}{7} = 2\text{ m/s}^2$$

The contact force between 4 kg and 2 kg block will move 2 kg and 1 kg block with the same acceleration. Let F_1 be the contact force, Fig. 5(CF).37

FIGURE 5(CF).37



$\therefore$ $F_1 = (m_B + m_C)\, a = (2 + 1) = \mathbf{6\ N}$

2. Here, $m = 8\,g = \frac{8}{1000}\text{ kg}$,

$$v = 7\text{ cm/s} = \frac{7}{100}\text{ m/s}$$

$$t = 5{\cdot}6\text{ sec}, \quad F = ?$$

According to Newton's 2nd Law :

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = \frac{v\,dm}{dt} \quad [\because v \text{ is constant}]$$

$$F = \frac{7}{100} \times \frac{8}{1000 \times 5{\cdot}6} = \mathbf{10^{-4}\ N}$$

3. From the graph, Fig. 5(CF).2, it is clear that upto $t = 2{\cdot}0$ s, the body moves with a constant velocity $=$ slope of position time graph $= \frac{4}{2} = 2$ m/s. After $t = 2{\cdot}0$ s, position time graph is parallel to time axis, which means that the body comes to rest.

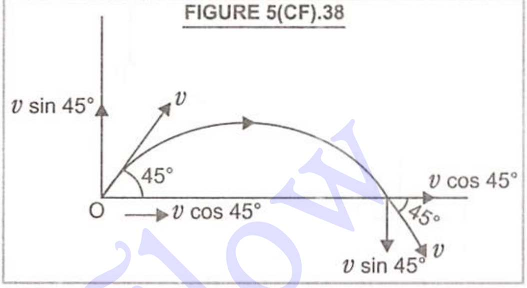
$\therefore$ Change in velocity $= dv = 2$ m/s.

Impulse = change in momentum

$$= dp = m\,dv = 0{\cdot}1 \times 2 = \mathbf{0{\cdot}2\ kg\ ms^{-1}}$$

4. As is clear from Fig. 5(CF).38, when the particle lands on the level ground at A, there is no change in linear momentum along the horizontal.

FIGURE 5(CF).38



However, the vertical component of velocity is reversed.

$\therefore$ change in momentum in vertical direction

$$= mv \sin 45° - (-mv \sin 45°)$$

$$= 2\,mv \sin 45° = 2\,mv \times \frac{1}{\sqrt{2}} = \sqrt{2}\,mv$$

5. Here, $m = 0{\cdot}4$ kg ; From Fig. 5(CF).3, when $s = 2$ m, $t = 2$ s and $u = 0$,

From $s = ut + \frac{1}{2}at^2$

$$2 = 0 + \frac{1}{2}a \times 2^2 \quad \text{or} \quad a = 1\text{ m/s}^2$$

From $v = u + at = 0 + 1 \times 2 = 2$ m/s

Impulse = change in momentum = mv

$$= 0{\cdot}4 \times 2 = \mathbf{0{\cdot}8\ N\text{-}s}$$

6. Velocity of stone hitting the ground, when dropped from a height h is $v = \sqrt{2gh}$. Therefore,

$$p = mv = m\sqrt{2gh}\,.$$

New height $= h + h = 2\,h$.

$\therefore$ New velocity, $v' = \sqrt{2g\,(2h)} = \sqrt{2}\,v$

New momentum, $p' = mv' = m\sqrt{2}\,v$

Percentage increase in momentum

$$= \frac{(p' - p)100}{p} = \frac{(\sqrt{2} - 1)\,mv \times 100}{mv}$$

$$= \mathbf{41{\cdot}4\%}$$

7. Let F_1 be the weight experienced by man and F_2 be the reaction of the ground on the other end.

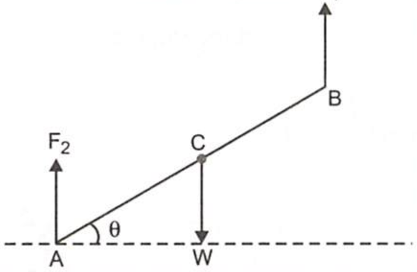
HINTS/EXPLANATIONS

For Difficult Questions

$\therefore \qquad W = F_1 + F_2 \qquad$ ...(*i*)

Taking moments about *A*, Fig. 5(CF).39

FIGURE 5(CF).39

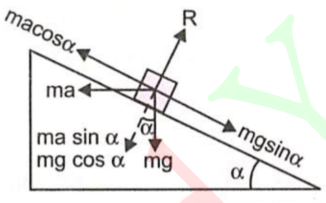


$$F_1 (AD) - W (AE) = 0$$

$$F_1 = \frac{W (AE)}{AD} = \frac{W [(l/2)\cos\theta]}{[l \cos\theta]} = \frac{W}{2}$$

8. The block will remain stationary, when as shown in Fig. 5(CF).40,

FIGURE 5(CF).40



$$ma \cos\alpha = mg \sin\alpha$$

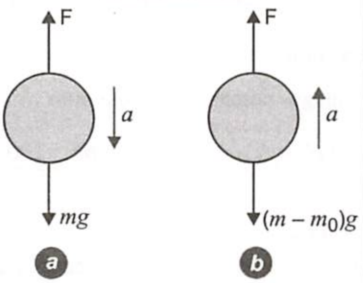
$$a = \frac{g \sin\alpha}{\cos\alpha} = \boldsymbol{g \tan\alpha}$$

9. Let *F* be the upthrust on balloon due to air.

Case (*i*) When balloon is descending down with acceleration *a*, then as shown in Fig. 5(CF).41(*a*),

$mg - F = ma \qquad$ ...(*i*)

FIGURE 5(CF).41



Case (*ii*) Let m_0 be the mass removed from the balloon, then the balloon moves upwards with acceleration *a*, Fig. 5(CF).41(*b*)

Now $\qquad F - (m - m_0)\, g = (m - m_0)\, a \qquad$ ...(*ii*)

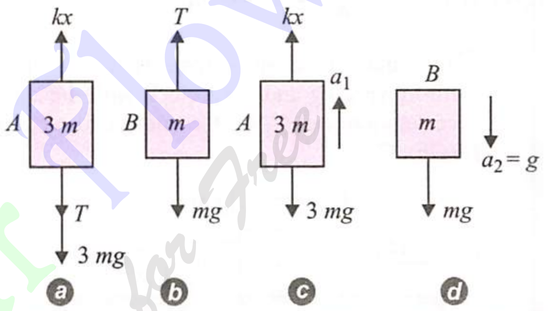
Adding (*i*) and (*ii*), we get

$$mg - (m - m_0)\, g = ma + (m - m_0)\, a$$

Solving it, we get, $\qquad m_0 = \dfrac{2ma}{g + a}$

10. Before the string is cut, let *x* be the extension in spring as shown in Fig. 5(CF).42(*a*) and *T* be the tension in string. Then

FIGURE 5(CF).42



$T + 3\, mg = kx \qquad$ ...(*i*)

In 5(CF).42(*b*), $T = mg \qquad$ ...(*ii*)

From (*i*) and (*ii*) ; $mg + 3\, mg = kx$

or $\qquad kx = 4\, mg \qquad$ ...(*iii*)

After the string is cut, $T = 0$

Let a_1 and a_2 be the accelerations of masses *A* and *B*, then

Refer to Fig. 5(CF).42(*c*),

$$kx - 3\, mg = 3\, m \times a_1$$

or $\qquad a_1 = \dfrac{kx - 3mg}{3m} = \dfrac{4\, mg - 3\, mg}{3m} = \boldsymbol{\dfrac{g}{3}}$

Refer to Fig. 5(CF).42(*d*)

$$ma_2 = mg \quad \text{or} \quad a_2 = \boldsymbol{g}$$

11. Maximum load, $M = 2000$ kg

$v = 1{\cdot}5 \text{ ms}^{-1}$

Force of friction $f = 3000$ N

Upward force required, $F = Mg + f$

$= 2000 \times 10 + 3000 = 23000$ N

Minimum power, $P = \vec{F} \cdot \vec{v} = F\, v \cos 0°$

$= 23000 \times 1{\cdot}5$

$= \mathbf{34500\ W}$

12. Suppose, masses of three fragments are 2 *m*, 2 *m* and *m*.

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Momentum of fragments along X-axis $= 2mv\,\hat{i}$

Momentum of fragments along Y-axis $= 2mv\,\hat{j}$

Net momentum $= \sqrt{(2mv)^2 + (2mv)^2}$

$$= 2\sqrt{2}\,mv$$

If v' is velocity of third fragment, then

$$2\sqrt{2}\,mv = mv' \quad \text{or} \quad v' = \mathbf{2\sqrt{2}\,v}$$

13. Here, $m = 0{\cdot}15$ kg, $h = 10$ m.

Velocity of ball at ground, $v = \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 10} = 10\sqrt{2} \text{ m/s}$$

Linear momentum of ball on hitting the ground

$$\vec{p}_i = mv(-\hat{j})$$

Linear momentum of the ball while going away

$$\vec{p}_f = mv(\hat{j})$$

Impulse of ball = Change in linear momentum

$$I = mv\,(\hat{j}) - (-mv\,\hat{j}) = 2mv\,\hat{j}$$

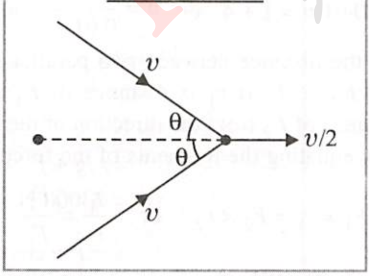
$$= 2 \times 0{\cdot}15 \times 10\sqrt{2}\,\hat{j}$$

$$I = (4{\cdot}2 \text{ kg ms}^{-1})\,\hat{j}$$

$$|I| = 4{\cdot}2 \text{ kg m/s}$$

14. Let m be the mass of each body moving with the same speed (v) along different directions in a plane as shown in Fig. 5(CF).43

FIGURE 5(CF).43



Final speed of the combination of mass ($2\,m$) is $v/2$.

If θ is the angle each ball is making with the horizonal, then applying the principle of conservation of linear momentum, along the horizonal, we get

$$2\,mv \cos\theta = (2\,m)\,v/2$$

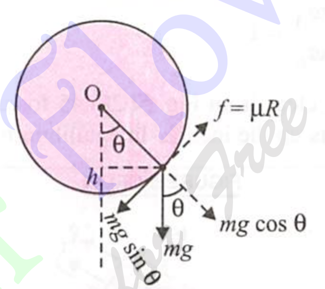
$$\cos\theta = 1/2$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Hence angle between the initial velocities of the two balls $= 2\,\theta = 2 \times 60^\circ = \mathbf{120^\circ}$.

15. In Fig. 5(CF).44, the insect starts slipping at a height h from the bottom of the ditch. This would happen when

FIGURE 5(CF).44



$$mg \sin\theta = f = \mu\,R = \mu\,mg\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \mu = 0{\cdot}75$$

$$\theta = \tan^{-1}(0{\cdot}75) = 37^\circ$$

From Fig. 5(CF).44,

$$\cos\theta = \frac{R-h}{R}$$

$$\cos 37^\circ = 1 - \frac{h}{R} = 1 - h \qquad (\because R = 1)$$

$$h = 1 - \cos 37^\circ$$

$$= 1 - 0{\cdot}7986$$

$$= \mathbf{0{\cdot}2014}$$

16. The wire is most likely to break when mass is at the lowest point. This is because at lowest point, tension in the wire is maximum.

17. As the track is frictionless, total mechanical energy of the body remains constant, *i.e.*,

$$(\text{ME})_{\text{at top}} = (\text{ME})_{\text{at bottom}}$$

$$0 + mgh = \frac{1}{2}mv_L^2$$

$$h = \frac{v_L^2}{2}$$

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For completing the vertical circle of diameter D,

$$v_L = \sqrt{5gR}$$

$$\therefore \qquad h = \frac{5gR}{2g} = \frac{5}{2}R = \frac{5}{4}D$$

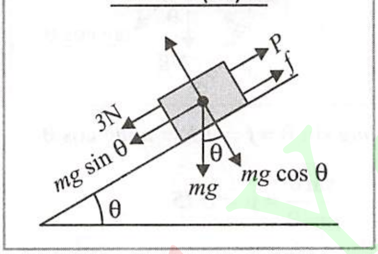
18. Here, $T_A = T_B$

$$\frac{2\pi}{\omega_A} = \frac{2\pi}{\omega_B}$$

$$\text{or} \qquad \frac{\omega_A}{\omega_B} = \mathbf{1}$$

19. As is clear from Fig. 5(CF).45 force of friction (f) acts up the incline. In equilibrium,

FIGURE 5(CF).45



$$f + P = mg\sin\theta + 3$$
$$P = mg\sin\theta - f + 3$$
$$P = mg\sin\theta - \mu\, mg\cos\theta + 3$$
$$= mg(\sin\theta - \mu\cos\theta) + 3$$
$$= 10 \times 10\,(\sin 45^\circ - 0{\cdot}6\cos 45^\circ) + 3$$
$$= \frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}} + 3$$
$$P = \frac{40}{1{\cdot}414} + 3 = \mathbf{32\ N}$$

20. Here, $m_1 = 50$ kg, $m_2 = 0{\cdot}5$ kg, $v_2 = 2$ m/s
$S = 10$ m, $v_1 = ?$

From the conservation of linear momentum,
$m_1 v_1 = m_2 v_2$ Therefore, $50 \times v_1 = 0{\cdot}5 \times 2$

$$v_1 = \frac{1}{50}\ \text{m/s}$$

Time taken by the stone to cover the distance of 10 m and reach the floor (in gravity free space)

i.e., $$t = \frac{S}{v_2} = \frac{10}{2} = 5\,\text{s}$$

distance covered by the man (in the opposite direction) in 5 sec $= v_1 \times t = \frac{1}{50} \times 5 = 0{\cdot}1\,\text{m}$

Distance of the man above the floor
$= 10 + 0{\cdot}1 = \mathbf{10{\cdot}1\ m}$

21. Here, total mass = 1 kg

$$m_1 : m_2 : m_3 = 1 : 1 : 3$$
$$\therefore\ m_1 = m_2 = \frac{1}{1+1+3} = 0{\cdot}2\,\text{kg},$$
$$m_3 = 1 - 0{\cdot}4 = 0{\cdot}6\ \text{kg}$$
$$v_1 = v_2 = 30\ \text{m/s},$$
$$p_1 = p_2 = 0{\cdot}2 \times 30 = 6\ \text{kg m/s}$$

As p_1, p_2 are perpendicular to each other, so according to law of conservation of linear momentum, p_3 should be the resultat of p_1 and p_2.

$$\therefore\ p_3 = \sqrt{p_1^2 + p_2^2} = \sqrt{(6)^2 + (6)^2} = 6\sqrt{2}\ \text{kg m/s}$$

$$p_3 = m_3 v_3 \ \therefore\ v_3 = \frac{p_3}{m_3} = \frac{6\sqrt{2}}{0{\cdot}6} = \mathbf{10\sqrt{2}\ m/s}$$

22. Here, $h = 5$ m, $m_1 = 0{\cdot}2$ kg, $m_2 = 0{\cdot}01$ kg,
$d_1 = 20$ m ; $d_2 = 100$ m

As $$h = ut + \frac{1}{2}gt^2 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$5 = 5\,t^2 \quad \text{or} \quad t = 1\ \text{s}$$

Further as $v_1 \times t = 20$ m or $v_1 \times 1 = 20$
$$v_1 = 20\ \text{m/s}$$
$$v_2 \times t = 100\ \text{m} \quad \text{or} \quad v_2 \times 1 = 100$$
$$v_2 = 100\ \text{m/s}$$

Applying law of conservation of linear momentum

$(0{\cdot}01)\,v = (0{\cdot}01)(100) + (0{\cdot}2)(20)$

$$0{\cdot}01\,v = 1 + 4 \quad \text{or} \quad v = \frac{5}{0{\cdot}01} = \mathbf{500\ m/s}$$

23. Let the distance between two parallel forces F_1 and F_2 be L. If r_1 is distance of F_1 and r_2 is distance of F_2 from the direction of the resultant, then equating the moments of the forces

$$F_1 \times r_1 = F_2 \times r_2 \quad \text{or} \quad \frac{r_1}{r_2} = \frac{F_2}{F_1}$$

Also, $$r_1 = \frac{F_2 . L}{F_1 + F_2}$$

and $$r_2 = \frac{F_1\,L}{F_1 + F_2}$$

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As $\quad r_1 = r_2 + \frac{L}{4}$

$$\therefore \quad \frac{F_2 L}{F_1 + F_2} = \frac{F_1 L}{F_1 + F_2} + \frac{L}{4}$$

$$\frac{F_2}{F_1 + F_2} = \frac{5F_1 + F_2}{4(F_1 + F_2)}$$

or $\quad 5F_1 + F_2 = 4F_2$

$$5F_1 = 3F_2 \quad \text{or} \quad \frac{F_1}{F_2} = \mathbf{\frac{3}{5}}$$

24. Acceleration of the system before the string breaks,

$$a = \frac{\text{net pulling force}}{\text{total mass}} = \frac{3g - 2g}{(3+2)} = \frac{g}{5}$$

After 5 s, velocity of the system,

$$v = at = \frac{g}{5} \times 5 = g \text{ m/s}$$

Required height, $h = \frac{v^2}{2g} = \frac{g \times g}{2g} = \frac{g}{2}$

$$= \frac{9{\cdot}8}{2} = \mathbf{4{\cdot}9\,m}$$

25. Here, $m_1 = 200 \text{ g} = \frac{200}{1000} \text{kg} = \frac{1}{5} \text{kg}$

$m_2 = 4$ kg, total energy,

$E = 1{\cdot}05 \text{ kJ} = 1050 \text{ J}.$

Let u_1 be initial velocity of the shell and u_2 be the initial velocity of the gun.

According to principle of conservation of linear momentum, $\quad m_1 u_1 + m_2 u_2 = 0$

$$u_2 = -\frac{m_1}{m_2} u_1 = \frac{-1}{5 \times 4} u_1 = -\frac{u_1}{20}$$

Now, $\quad E = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

$$1050 = \frac{1}{2} \times \frac{1}{5} u_1^2 + \frac{1}{2} \times 4 \left(-\frac{u_1}{20}\right)^2$$

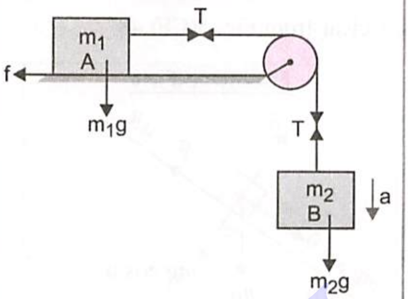
$$= \frac{u_1^2}{10} + \frac{u_1^2}{200} = \frac{21}{200} u_1^2$$

$$u_1^2 = \frac{200 \times 1050}{21} = 10000 \;;$$

$$u_1 = \sqrt{10000} = \mathbf{100\,m/s}$$

26.

FIGURE 5(CF).46



As is clear from Fig. 5(CF).46,

force of friction, $f = \mu_k R = \mu_k m_1 g$

Equations of motion of two blocks are

$$m_2 g - T = m_2 a \quad \text{...(i)}$$

$$T - \mu_k m_1 g = m_1 a \quad \text{...(ii)}$$

From (i) $\quad m_2 g - m_2 a = T$

Putting the value of T in (ii)

$$m_2 g - m_2 a - \mu_k m_1 g = m_1 a$$

$$a = \frac{(m_2 - \mu_k m_1) g}{m_1 + m_2}$$

Putting in (i)

$$m_2 g - T = m_2 \left[\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right] g$$

$$T_2 = m_2 g - m_2 \left[\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right] g$$

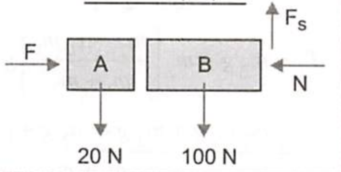
$$= \frac{m_2 m_1 g + m_2^2 g - m_2^2 g + \mu_k m_1 m_2 g}{m_1 + m_2}$$

$$\therefore \quad T = \mathbf{\frac{m_1 m_2 (1 + \mu_k) g}{m_1 + m_2}}$$

27. As is clear from Fig. 5(CF).47, for the systm to be in vertical equilibrium.

$$F_s = 100 + 20 = \mathbf{120\ N}$$

FIGURE 5(CF).47



28. When the block slides down the inclined plane with constant speed, the inclination of the plane with horizontal is equal to the angle of repose (θ).

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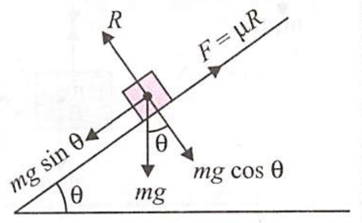
As is clear from Fig. 5(CF).48

FIGURE 5(CF).48



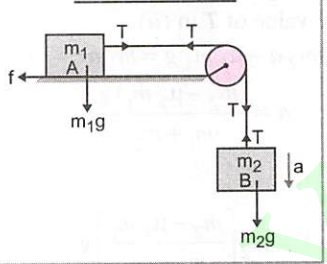
$$R = mg \cos \theta \qquad \text{...(i)}$$

$$F = \mu R = mg \sin \theta \qquad \text{...(ii)}$$

Dividing (*ii*) by (*i*), we get

$$\boldsymbol{\mu = \tan \theta}$$

29. FIGURE 5(CF).49



As is clear from Fig. 5(CF).49,

force of friction, $f = \mu_k R = \mu_k m_1 g$

Equations of motion of two blocks are

$$m_2 g - T = m_2 a \qquad \text{...(i)}$$

$$T - \mu_k m_1 g = m_1 a \qquad \text{...(ii)}$$

From (*i*) $\quad m_2 g - m_2 a = T$

Putting the value of T in (*ii*)

$$m_2 g - m_2 a - \mu_k m_1 g = m_1 a$$

$$a = \frac{(m_2 - \mu_k m_1) g}{m_1 + m_2}$$

Putting in (*i*),

$$m_2 g - T = m_2 \left[\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right] g$$

$$T_2 = m_2 g - m_2 \left[\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right] g$$

$$= \frac{m_2 m_1 g + m_2^2 g - m_2^2 g + \mu_k m_1 m_2 g}{m_1 + m_2}$$

$$\therefore \quad \boldsymbol{T = \frac{m_1 m_2 (1 + \mu_k) g}{m_1 + m_2}}$$

30. When box just slips, then

Coefficient of static friction,

$$\mu_s = \tan 30^\circ = \frac{1}{\sqrt{3}} \approx \mathbf{0{\cdot}6}$$

When box slides down the plane, coefficient of kinetic friction will be involved.

Let *a* be the acceleration of the box sliding down the plane, then using

$$S = ut + \frac{1}{2} at^2, \text{ we have}$$

$$4 = 0 + \frac{1}{2} \times a \times 4^2 \quad \text{or} \quad a = \frac{2}{4} = 0{\cdot}5 \,\text{ms}^{-2}$$

Here, $\quad a = g \sin \theta - \mu_k g \cos \theta$

$$0{\cdot}5 = 9{\cdot}8 \times \sin 30^\circ - \mu_k \times 9{\cdot}8 \cos 30^\circ$$

$$0{\cdot}5 = 9{\cdot}8 \times \frac{1}{2} - \mu_k \times 9{\cdot}8 \times \frac{\sqrt{3}}{2}$$

$$0{\cdot}5 = 4{\cdot}9 - \mu_k \times 4{\cdot}9 \sqrt{3}$$

$$\mu_k = \frac{4{\cdot}9 - 0{\cdot}5}{4{\cdot}9 \sqrt{3}} = \frac{4{\cdot}4}{4{\cdot}9 \times 1{\cdot}73} \approx \mathbf{0{\cdot}5}$$

31. Here, $m_1 = 5$ kg ; $m_2 = 10$ kg

$$\mu = 0{\cdot}15, \quad m = ?$$

To stop the moving block m_2, force applied ($= m_1 g$) must be less than the force of friction,

$$f = \mu (m + m_2) g$$

i.e., $\quad m_1 g < \mu (m + m_2) g$

or $\quad 5 < 0{\cdot}15 (m + 10)$

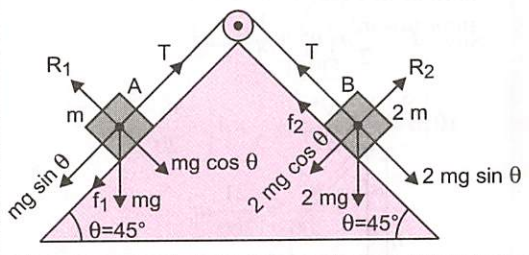
or $\quad 5 < 0{\cdot}15 + 1{\cdot}5 \quad$ or $\quad 3{\cdot}5 < 0{\cdot}15$ m

or $\quad m > \frac{3{\cdot}5}{0{\cdot}15} \quad$ or $\quad \boldsymbol{m > 27{\cdot}3}$ **kg**

Minimum weight = **27·3 kg**

32. Refer to Fig. 5(CF).50,

FIGURE 5(CF).50



Effective pulling force on masses along the plane when mass 2 *m* moves down and mass *m* moves up the plane is

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$$F_{\text{eff}} = 2\,mg \sin 45^\circ - mg \sin 45^\circ$$

$$= mg/\sqrt{2} = \sqrt{2}\,mg/2$$

Effective force of friction on two masses is

$$F_{\text{friction}} = f_1 + f_2 = \mu_1\, mg \cos\theta + \mu_2\, 2mg \cos\theta$$

$$= \frac{2}{3} mg \frac{1}{\sqrt{2}} + \frac{1}{3} \times 2mg \times \frac{1}{\sqrt{2}}$$

$$= mg\left[\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{3}\right] = \frac{2\sqrt{2}}{3} mg$$

As $F_{\text{eff}} < F_{\text{friction}}$, so the masses will not move.

Hence the **acceleration of the system will be zero.**

33. Given $y = x^3/6$ ...(*i*)

$$\frac{dy}{dx} = \frac{x^2}{2}$$

The block is just placed on the surface at height *y*, where it does not slip, see Fig. 5(CF).51. Then

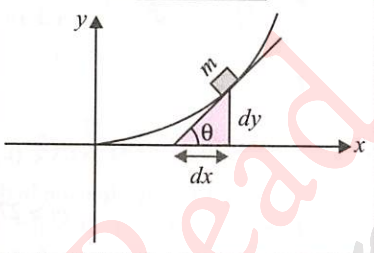
$$\tan\theta = \frac{dy}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, $\mu = \tan\theta = 0{\cdot}5$

or $0{\cdot}5 = x^2/2$ or $x^2 = 1$ or $x = \pm 1$

From (*i*), $y = (1)^3/6 =$ **(1/6) m**

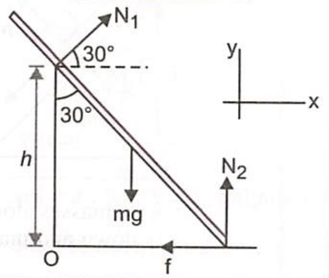
FIGURE 5(CF).51



34. Here, $m = 1{\cdot}6$ kg length $= l\ (> h)$

In Fig. 5(CF).52, N_1 is reaction of wall perpendicular to rod, N_2 is reaction of ground normal to floor. *f* is force of friction along the floor towards the wall.

FIGURE 5(CF).52



In equilibrium, $\Sigma F_x = 0$

$N_1 \cos 30^\circ - f = 0$ ...(*i*)

$\Sigma F_y = 0$

$N_1 \sin 30^\circ + N_2 - mg = 0$ ...(*ii*)

Also, $\tau_0 = 0$

$$\therefore\ mg \times \frac{l}{2}\cos 60^\circ - N_1\left(\frac{h}{\cos 30^\circ}\right) = 0 \quad \text{...(iii)}$$

Also, $N_1 = N_2$ ...(*iv*)

From (*i*), $f = N_1 \cos 30^\circ = N_1 \dfrac{\sqrt{3}}{2}$

From (*ii*) and (*iv*),

$$N_1 \times \frac{1}{2} + N_1 = mg$$

or $$\frac{3}{2} N_1 = mg$$

From (*iii*), $mg\dfrac{l}{2} \times \dfrac{1}{2} = N_1 \dfrac{h.2}{\sqrt{3}} = \dfrac{2}{3} mg.\dfrac{2h}{\sqrt{3}}$

$$\frac{l}{4} = \frac{4h}{3\sqrt{3}} \quad \therefore \quad \frac{h}{l} = \mathbf{\frac{3\sqrt{3}}{16}}$$

We can show that $f = \mathbf{\dfrac{16\sqrt{3}}{3}}$

35. For the upper half of inclined plane,

$v^2 = 2\,a_1\,s$, where $a_1 = g \sin\theta$

For the lower half of inclined plane,

$0 = v^2 - 2\,a_2\,s$, where $a_2 = \mu\, g \cos\theta - g \sin\theta$

From these two equations, $a_1 = a_2$

$g \sin\theta = \mu\, g \cos\theta - g \sin\theta$

$2\,g \sin\theta = \mu\, g \cos\theta$

$\mathbf{2 \tan\theta = \mu}$

36. When the plane is smooth,

$$a = g \sin\theta,\ t_1 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g \sin\theta}}$$

When plane is rough, $a = g\,(\sin\theta - \mu \cos\theta)$

$$\therefore\ t_2 = \sqrt{\frac{2s}{g\,(\sin\theta - \mu\cos\theta)}}$$

As $t_2 = 2\,t_1$

$$\therefore\ \sqrt{\frac{2s}{g\,(\sin\theta - \mu\cos\theta)}} = 2\sqrt{\frac{2s}{g \sin\theta}}$$

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Squaring, we get

$$\frac{1}{\sin\theta - \mu\cos\theta} = \frac{4}{\sin\theta}$$

$$\sin\theta = 4\sin\theta - 4\mu\cos\theta$$

$$4\mu\cos\theta = 3\sin\theta$$

or $$\mu = \frac{3}{4}\tan\theta$$

37. From $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$,

$$t = \sqrt{\frac{2s}{a}}$$

For smooth plane, $a = g\sin\theta$

For rough plane, $a' = g(\sin\theta - \mu\cos\theta)$.

$$\therefore\ t' = \sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}} = nt = n\sqrt{\frac{2s}{g\sin\theta}}$$

$$\therefore\ n^2 g(\sin\theta - \mu\cos\theta) = g\sin\theta$$

When $\theta = 45°$, $\sin\theta = \cos\theta = 1/\sqrt{2}$

Solving, we get $\mu = \left(1 - \frac{1}{n^2}\right)$

38. Max. acc. of the block $= \mu g = 0{\cdot}15 \times 10$
$= 1{\cdot}5\ \text{m/s}^2$

Acc. of truck $= 2\ \text{m/s}^2$

$\therefore$ Relative acc. of box w.r.t. truck truck $a_r = 1{\cdot}5 - 2{\cdot}0 = -0{\cdot}5\ \text{m/s}^2$ (backwards). It will fall off the truck in a time $t = \sqrt{\frac{2l}{a_r}} = \sqrt{\frac{2\times4}{0{\cdot}5}} = 4\,\text{s}$

Displacement of truck during this time

$$s = \frac{1}{2}at^2 = \frac{1}{2}\times2\times4^2 = \mathbf{16\,m}$$

39. Initially, the frictional force (which is limiting force of friction) is upwards and is maximum. When force applied (P) is increased, the body starts moving. The dynamic frictional force decreases and its direction is reversed. Thus, **graph (*a*) is correct.**

40. When acceleration of the cart is a, then as shown in Fig. 5(CF).53,

$$R = ma$$

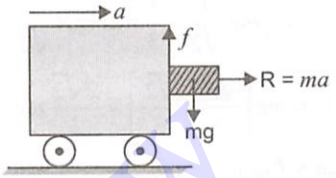
The block will not fall till force of friction,

$$f \geq mg$$

$$\mu R \geq mg$$

$$\mu\, ma \geq mg \quad \text{or} \quad a \geq \frac{g}{\mu}$$

FIGURE 5(CF).53



41. Force required to just push the block up the inclined plane of inclination θ is

$$F_{up} = mg\sin\theta + \mu\, mg\cos\theta$$

Force required to just prevent the block from sliding down is

$$F_{down} = mg\sin\theta - \mu\, mg\cos\theta$$

According to question, $F_{up} = 3F_{down}$

$\therefore\ mg\sin\theta + \mu\, mg\cos\theta = 3(mg\sin\theta - \mu\, mg\cos\theta)$

or $\sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

As $\theta = 45°$, therefore,

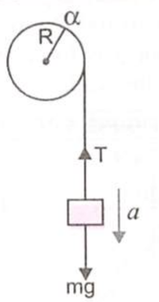
$$\frac{1}{\sqrt{2}} + \mu\times\frac{1}{\sqrt{2}} = 3\left[\frac{1}{\sqrt{2}} - \mu\times\frac{1}{\sqrt{2}}\right]$$

On solving, $\mu = \frac{1}{2}$

Given, $N = 10\mu = 10\times\frac{1}{2} = \mathbf{5}$

42. In Fig. 5(CF).54, let T be the tension in the string, α be the angular acceleration of pulley and a be the linear downward acceleration of mass m.

FIGURE 5(CF).54



Torque on pulley, $\tau = T\times R$; $\tau = I\alpha$.

where I = moment of inertia of pulley $= \frac{mR^2}{2}$.

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$$\therefore \quad TR = I\alpha = \frac{mR^2}{2}\frac{a}{R} \qquad (\because\ a = \alpha R)$$

$$\text{or} \quad T = \frac{ma}{2}$$

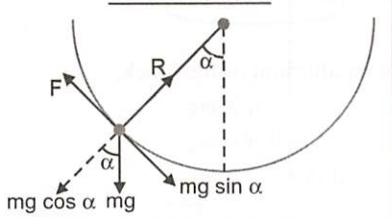
Taking equation of motion of mass m, we have

$$mg - T = ma$$

$$\text{or} \quad mg - \frac{ma}{2} = ma \quad \text{or} \quad a = \frac{2}{3}\mathbf{g}$$

43. As is clear from Fig. 5(CF).55,

FIGURE 5(CF).55



$$F = mg\sin\alpha$$

$$R = mg\cos\alpha$$

$$\frac{F}{R} = \tan\alpha, \quad i.e., \quad \mu = \tan\alpha = \frac{1}{3}$$

$$\therefore \quad \mathbf{\cot\alpha = 3}$$

44. Let a be the downward acceleration of mass m.

Pulling force on all the three masses,

$$F = (m_1 + m_2 + m_3)\, a$$

force of friction on masses m_2 and m_3,

$$f = \mu\,(m_2 + m_3)\, g.$$

Equation of motion of mass m_1 is

$$(m_1 + m_2 + m_3)\, a = m_1 g - \mu\,(m_2 + m_3)\, g$$

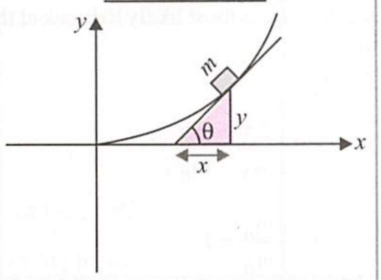
$$\text{or} \quad (m + m + m)\, a = mg - \mu\,(m + m)\, g$$

$$\text{or} \quad a = \frac{mg\,(1 - 2\mu)}{3m} = \frac{\mathbf{g\,(1 - 2\mu)}}{\mathbf{3}}$$

45. Given $\quad y = x^3/6 \qquad ...(i)$

$$\frac{dy}{dx} = \frac{x^2}{2}$$

FIGURE 5(CF).56



The block is just placed on the surface at height y, where it does not slip, Fig. 5(CF).56. Then

$$\tan\theta = \frac{dy}{dx} = \frac{x^2}{2}$$

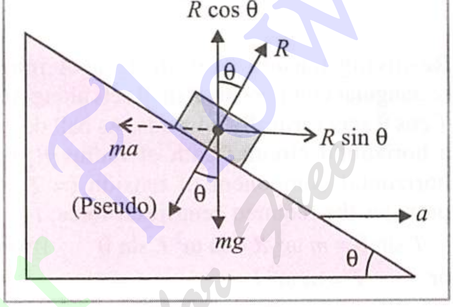
At limiting equilibrium, $\mu = \tan\theta$

$$\text{or} \quad 0{\cdot}5 = x^2/2 \quad \text{or} \quad x^2 = 1 \quad \text{or} \quad x = \pm 1$$

From (i), $\quad y = (1)^3/6 = \mathbf{(1/6)\ m}$

46. Let R be the reactional force acting on the block due to inclined plane.

FIGURE 5(CF).57



The various forces acting on the block are shown in Fig. 5(CF).57. When the block is just stationary on the inclined plane, then

$$R\sin\theta = ma \qquad ...(i)$$

$$\text{and} \quad R\cos\theta = mg \qquad ...(ii)$$

Dividing (i) by (ii), we get

$$\tan\theta = \frac{a}{g} \quad \text{or} \quad \mathbf{a = g\tan\theta}$$

47. Let the distance of other mass from the centre O be x.

Applying Law of conservation of angular momentum.

Initial Angular momentum = final angular momentum

$$MR^2\omega = \left[MR^2 + \frac{M}{8}\left(\frac{3}{5}R\right)^2 + \frac{M}{8}x^2\right]\frac{8}{9}\omega$$

$$R^2 = \left[\frac{200R^2 + 9R^2 + 25x^2}{200}\right]\frac{8}{9}$$

on solving, we get $\quad x = \dfrac{\mathbf{4}}{\mathbf{5}}\,\boldsymbol{R}$

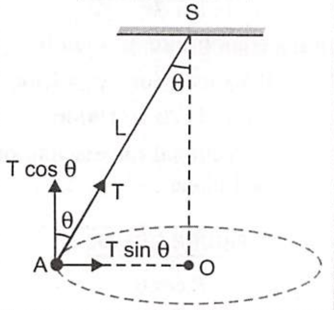
48. As shown in Fig. 5(CF).58,

$$R = AO = L\sin\theta \qquad ...(i)$$

HINTS/EXPLANATIONS

For Difficult Questions

FIGURE 5(CF).58



Resolving tension T of string at A into two rectangular components $T \sin\theta$ acts along AO and $T\cos\theta$ acts vertically upwards. As ball describes a horizontal circular path of radius R, so the horizontal component of tension ($= T\sin\theta$) provides the required centripetal force, *i.e.*,

$$T\sin\theta = m\,\omega^2 R = m\,\omega^2 L\sin\theta \qquad \text{[from (}i\text{)]}$$

or $$T = m\,\omega^2 L$$

or $$\omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0{\cdot}5\times 0{\cdot}5}} = \mathbf{36\ rad/s}$$

49. For smooth driving of the car,
force of friction = centripetal force

$$\mu_s R = \mu_s (mg) = \frac{mv^2}{R}$$

$$v = \sqrt{\mu_s Rg}$$

50. When bead is at A, the force it applies on the wire is **radially inwards.** When it reaches at B, it strikes the surface and pushed outwards. Thus, force is **radially outwards.**

51. As the track is frictionless, the total mechanical energy of the body remains constant, *i.e.*,

$$\text{M.E.}_{\text{(at top)}} = \text{M.E.}_{\text{(at bottom)}}$$

$$0 + mgh = \frac{1}{2}mv_L^2 + 0$$

$$mgh = \frac{1}{2}mv_L^2$$

or $$h = \frac{v_L^2}{2g}$$

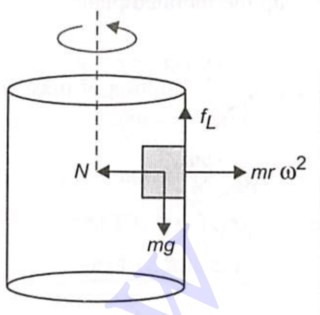
For completing the vertical circle of diameter D,

$$v_L \geq \sqrt{5gR}$$

$$\therefore \quad h = \frac{5gR}{2g} = \frac{5}{2}R = \frac{\mathbf{5}}{\mathbf{4}}\boldsymbol{D}$$

52. Here, $m = 10$ kg, $r = 1$ m, $\mu = 0{\cdot}1$, $\omega = ?$
$g = 10\ \text{m/s}^2$.

FIGURE 5(CF).59



For equilibrium of the block,

$$f_L \geq mg$$

$$\mu N \geq mg$$

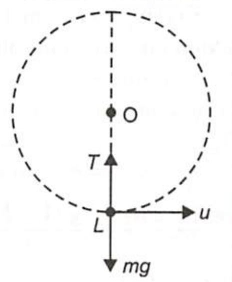
$$\mu\,(mr\,\omega^2) \geq mg$$

$$\omega \geq \sqrt{\frac{g}{\mu r}}$$

$$\omega_{\min.} = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{10}{0{\cdot}1\times 1}} = \mathbf{10\ rad/s}$$

53. As is clear from Fig. 5(CF).60.

FIGURE 5(CF).60



$$T = mg + \frac{mu^2}{l}$$

The tension at the lowest point is maximum. Hence the wire is most likely to break at the lowest point.

54. As $T_A : T_B$

$$\frac{2\pi}{\omega_A} = \frac{2\pi}{\omega_B}$$

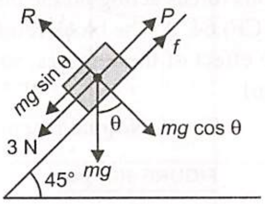
$$\therefore \quad \frac{\omega_A}{\omega_B} = 1$$

HINTS/EXPLANATIONS

For Difficult Questions

55. As is clear from Fig. 5(CF).61 force of friction (f) acts up the inclined plane.

FIGURE 5(CF).61



In equilibrium,

$$f + P = mg\sin\theta + 3$$

$$P = mg\sin\theta - f + 3$$

$$= mg\sin\theta - \mu\, mg\cos\theta + 3$$

$$= mg\,(\sin\theta - \mu\cos\theta) + 3$$

$$P = 10 \times 10\,(\sin 45^\circ - 0{\cdot}6\cos 45^\circ) + 3$$

$$= \left(\frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}}\right) + 3$$

$$= \mathbf{32\ N}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

56. If elevator is slowing down, acceleration = $-a$

If elevator is speeding up, acceleration = $+a$

(a) going up and slowing down

$$R - mg = m(-a) \qquad \therefore\ R = m(g-a)$$

(b) going up and speeding up

$$\therefore\ R - mg = ma,\ \boldsymbol{R = m\,(g+a)}$$

(c) going down and slowing down

$$mg - R = m(-a),\ \text{or}\ \boldsymbol{R = m\,(g+a)}$$

(d) going down and speeding up

$$mg - R = ma, \quad \text{or} \quad R = m\,(g-a)$$

$\therefore$ **Option (b) and (c) are correct.**

57. Here, (a) Restoring force on spring = $-Kx$

for mass m_2

$$m_2 a_2 - Kx = 0$$

$$m_2 a_2 = Kx \qquad \text{...}(i)$$

$$\boldsymbol{a_2 = \frac{Kx}{m_2}}$$

(b) For mass m_1, $F - Kx = m_1 a_1$ ...(ii)

$$\therefore \quad \boldsymbol{a_1 = \frac{F - Kx}{m_1}}$$

(c) Adding (i) and (ii),

$$F - Kx + Kx = m_1 a_1 + m_2 a_2$$

$$\therefore \quad \boldsymbol{F = m_1 a_1 + m_2 a_2}$$

(d) If $a_1 = a_2$

$$F = (m_1 + m_2)\, a$$

$$\text{or} \quad a = \boldsymbol{\frac{F}{m_1 + m_2}}$$

58. (a) If a rain drop falls with terminal velocity

weight = upthrust + viscous drog,

$$\therefore \quad \boldsymbol{F_{net} = 0}$$

(b) For a cork floating in water

weight = upthrust, $\therefore\ \boldsymbol{F_{net} = 0}$

(c) In case of constant speed of a car, $a = 0$

$$\therefore \quad \boldsymbol{F = ma = 0}$$

(d) In a tug of war game, if one team applies more force than other, then net force will be the resultant of two forces. Which will not be zero.

$$\therefore\ \boldsymbol{F_{net} \neq 0.}$$

59. (a) If there is no friction, the work needs to be done to move a body up a rough inclined plane, but work has to be done against gravity. Therefore, this is correct.

(b) If there is no friction, moving vehicles could not be stopped because vehicles are stopped by friction. Therefore, this is correct.

(c) As $R = mg\cos\alpha$,

$\therefore$ R decreases as α increases.

(d) Here $F = 0{\cdot}5$ kg f + μR

$$F = (0{\cdot}5 \times 10 + 0{\cdot}5 \times 11)\ \text{N} = 10{\cdot}5\ \text{N}$$

Work done, $W = F \times S = 10{\cdot}5 \times 0{\cdot}1 = \mathbf{1{\cdot}05\ J}$

60. Here, $m = 0{\cdot}1$ kg, $F = 1$ N, $g = 10$ m/s^2.

In Fig. 5(CF).62, we have resolved mg into two rectangular components : $mg\cos\theta \perp$ to PQ and $mg\sin\theta$ along QP. Again, two rectangular components of F are $F\cos\theta$ along PQ and $F\sin\theta \perp PQ$.

The block will remain stationary, when forces along PQ have zero resultant

$$i.e., \qquad mg\sin\theta = F\cos\theta$$

HINTS/EXPLANATIONS

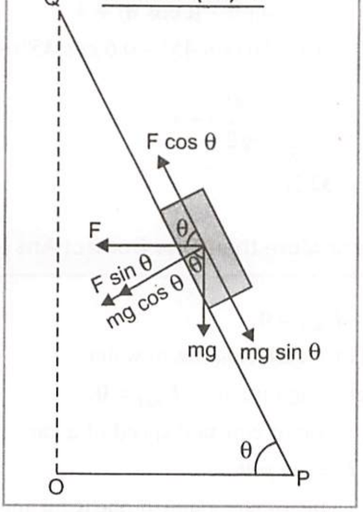
For Difficult Questions

$$0.1 \times 10 \sin\theta = 1 \cos\theta$$

or $\tan\theta = 1$ or $\theta = 45^\circ$

Choice (*a*) is correct.

FIGURE 5(CF).62



If $\theta > 45^\circ$, $mg \sin\theta > F \cos\theta$

the block tends to move down the incline *QP*. Frictional force must act on the block towards *Q*. **Choice (*c*) is correct.** With the same argument, we can show that **choices (*b*) and (*d*) are incorrect.**

61. Force on the pulley by the clamp = resultant of $T = (M + m)\,g$ and mg acting along horizontal and vertical directions respectively, Fig. 5(CF).25

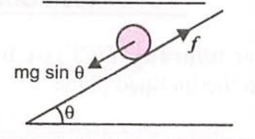
$$= \sqrt{[(M+m)\,g]^2 + (mg)^2}$$

$$= [\sqrt{(M+m)^2 + m^2}\,]\,g$$

62. Particle will be seen at rest only when frame is inertial and resultant force on particle is zero. Also, if frame is non inertial (*i.e.*, accelerated), the particle must also possess the same acc. in magnitude and direction *i.e.* resultant force on the particle must be non zero.

63. As shown in Fig. 5(CF).63, component of weight ($mg \sin\theta$) is always down the inclined plane, whether the cylinder is rolling up or it is rolling down. Therefore, for no slipping, sense of angular acceleration must be the same in both the cases. Therefore, force of friction (f) acts up the inclined plane in both the cases.

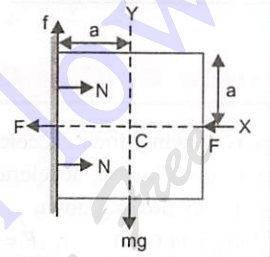
FIGURE 5(CF).63



64. The various forces acting on the block are shown in Fig. 5(CF).64. As the block remains stationary under the effect of these forces, so $mg = f$ (force of friction)

$$F = N \text{ (Normal reaction)}$$

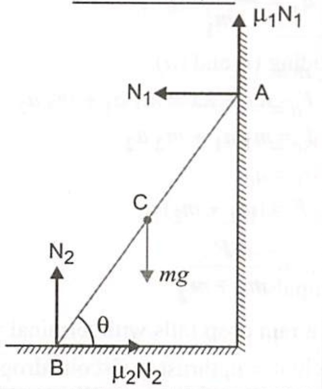
FIGURE 5(CF).64



Torque due to F or mg is zero as the line of action of F or mg passes through centre C. Since the body is in equilibrium, so the sum of torque due to force of friction ($\vec{\tau}_f$) and torque due to normal reaction ($\vec{\tau}_N$) must be zero, *i.e.* $\vec{\tau}_f + \vec{\tau}_N = 0$. As line of action of $\vec{f}$ and $\vec{N}$ may not pass through centre C of block, so $\vec{\tau}_f$ may not be zero, and $\vec{\tau}_N$ may also not be zero. Therefore, $\vec{f}$ does not produce any torque is wrong and $\vec{N}$ does not produce a torque is also wrong.

65. Refer to Fig. 5(CF).65, for maximum equilibrium, μ_2 **can never be zero.** When $\mu_1 = 0$, we have

FIGURE 5(CF).65



HINTS/EXPLANATIONS

For Difficult Questions

$$N_1 = \mu_2 N_2 \quad \text{...(i)}$$

and $$N_2 = mg \quad \text{...(ii)}$$

As ladder is in equilibrium, so taking moments of forces about point B, we have

$$mg\frac{L}{2}\cos\theta = N_1 L\sin\theta$$

or $$N_1 = \frac{mg}{2\tan\theta} \quad \text{or} \quad N_1 \tan\theta = \frac{mg}{2}$$

When $\mu_1 \neq 0$, and there is equilibrium of ladder,

then $$\mu_1 N_1 + N_2 = mg \quad \text{...(iii)}$$

and $$\mu_2 N_2 = N_1 \quad \text{...(iv)}$$

Putting (*iv*) in (*iii*), we get,

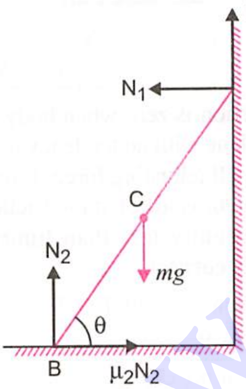
$$\mu_1\mu_2 N_2 + N_2 = mg \quad \text{or} \quad N_2 = \frac{mg}{1+\mu_1\mu_2}$$

66. When vehicle is moving with uniform velocity, its acc. = 0, external force applied = 0. Therefore, force of friction = 0. When vehicle is accelerating towards east, force exerted by rear wheels on the ground makes force of friction act in the forward direction *i.e.* towards east.

67. In cycling, rear wheel moves by the force communicated to it by pedalling and front wheel moves on its own. Hence while pedalling, force exerted by rear wheel on ground, makes force of friction act in forward direction. Front wheel moving by itself experiences force of friction in backward direction. When pedalling is stopped, both the wheels moving themselves experience friction in backward direction.

68. Refer to Fig. 5(CF).66 shown here. For static equilibrium, μ_2 **can never be zero.** When $\mu_1 = 0$, then

FIGURE 5(CF).66



$$N_1 = \mu_2 N_2 \quad \text{...(i)}$$

and $$N_2 = mg \quad \text{...(ii)}$$

As ladder is in equilibrium, so taking moments of forces about point B, we have

$$mg\frac{L}{2}\cos\theta = N_1 L\sin\theta$$

or $$N_1 = \frac{mg}{2\tan\theta}$$

or $$N_1 \tan\theta = \frac{mg}{2}$$

When $\mu_1 \neq 0$, and there is equilibrium of ladder,

then $$\mu_1 N_1 + N_2 = mg \quad \text{...(iii)}$$

and $$\mu_2 N_2 = N_1 \quad \text{...(iv)}$$

Putting (*iv*) in (*iii*), we get,

$$\mu_1\mu_2 N_2 + N_2 = mg \quad \text{or} \quad N_2 = \frac{mg}{1+\mu_1\mu_2}$$

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

69. Here, $m = 150 \text{ g} = 150 \times 10^{-3}$ kg

$u = -12$ m/s, $v = 20$ m/s, $t = 0{\cdot}01$ s

Impulse = change in linear momentum

$= m(v-u)$

$= 150 \times 10^{-3}\,[20 - (-12)]$

$= \mathbf{4{\cdot}8}$ **N-s**

70. As Impulse = $F \times t$

$$\therefore \quad F = \frac{\text{Impulse}}{t} = \frac{4{\cdot}8}{0{\cdot}01} = \mathbf{480\,N}$$

71. Here, $P = F = 49$ N, $m = 10$ kg

$$\mu = \frac{F}{R} = \frac{F}{mg} = \frac{49}{10\times 9{\cdot}8} = \mathbf{0{\cdot}5}$$

72. Force producing acceleration,

f = applied force − force of friction

$= 1{\cdot}2 - 0{\cdot}45 = 0{\cdot}75$ kg f

$= 0{\cdot}75 \times 9{\cdot}8$ N

$$a = \frac{f}{m} = \frac{0{\cdot}75\times 9{\cdot}8}{1{\cdot}5} = \mathbf{4{\cdot}9\ ms^{-2}}$$

HINTS/EXPLANATIONS

For Difficult Questions

IV. Matching Type Questions

73. Force of friction is zero when body is lying on a horizontal plane with no tendency to move. Static friction is a self adjusting force. Limiting friction is the maximum value of static friction. Dynamic friction is slightly less than limiting friction. **Choice (*a*) is correct.**

74. As is clear from Fig. 5(CF).67

$$R = mg\cos\theta$$

$$f = \mu R = \mu\, mg\cos\theta$$

∴ Net force along the plane

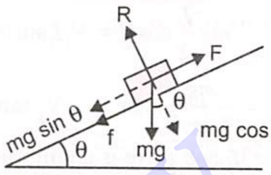
$$F = mg\sin\theta + f = mg\sin\theta + \mu\, mg\cos\theta$$

$$= m(g\sin\theta + \mu\, g\cos\theta)$$

Net force perpendicular to plane $= F\cos 90^\circ$

= Zero

FIGURE 5(CF).67



Net force in horizontal direction

$$= F\cos\theta = m(g\sin\theta\cos\theta + \mu\, g\cos^2\theta)$$

Net force in vertical direction

$$= F\sin\theta = m(g\sin^2\theta + \mu\, g\sin\theta\cos\theta)$$

Choice (*d*) is correct.

V. Matrix-Match Type Questions

75. Centripetal force $= \dfrac{mv^2}{r} = m\,r\,\omega^2$

Centrifugal force is reaction of centripetal force having the same magnitude.

Bending of cyclist is governed by the relation $\tan\theta = v^2/rg$. While rounding a level curve,

$$v_{max} = \sqrt{\mu\, rg}$$

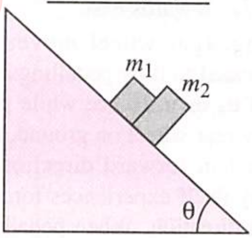
76. Here, coefficient of friction between block m_1 and the plane = 0 and for block m_2, and plane $\mu_s = \mu_k = \mu = 0{\cdot}3$.

For equilibrium,

$$\mu\, m_2\, g\cos\theta = (m_1 + m_2)\, g\sin\theta$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\mu\, m_2}{m_1 + m_2} = \frac{0{\cdot}3\times 2}{1+2} = 0{\cdot}2$$

FIGURE 5(CF).68



As $\tan(11{\cdot}5^\circ) = 0{\cdot}2$,

∴ Block m_2 starts sliding at $\theta > 11{\cdot}5^\circ$. When $\theta = 5^\circ$ and $\theta = 10^\circ$, the block will be at rest.

∴ Force of friction $= (m_1 + m_2)\, g\sin\theta$

For $\theta = 15^\circ$ and $\theta = 20^\circ$, block starts sliding.

∴ Force of friction $= \mu\, m_2\, g\cos\theta$

∴ **Option (*d*) is correct.**

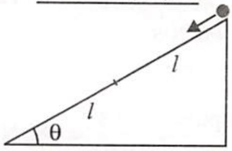
Integer Type Questions

77. Let $2\,l$ be the length of inclined plane and θ be the angle of inclination as shown in Fig. 5(CF).69.

From $\quad s = ut + \dfrac{1}{2}at^2$

$$2l = 0 + \frac{1}{2}(g\sin\theta)(\sqrt{2})^2 \qquad \text{...}(i)$$

FIGURE 5(CF).69



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If t is time taken to cover the first half of inclined plane, then

$$l = 0 + \frac{1}{2}(g\sin\theta)t^2 \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$2 = \frac{2}{t^2},\ \ t^2 = \frac{2}{2} = 1\ ;\ t = \mathbf{1\ s}$$

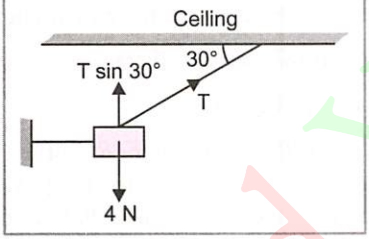
78. Total mass, $m = m_1 + m_2 = 3 + 2 = 5$ kg
Net force $F = F_1 - F_2 = 10 - 5 = 5$ N

Acceleration, $a = \frac{F}{m} = \frac{5}{5} = 1\,\text{m/s}^2$, along 10 N force

Reading of spring balance $= F_1 - m_1 a$
$= 10 - 3 \times 1$
$= \mathbf{7\ N}$

79. As is clear from Fig. 5(CF).70, the block will be in equilibrium, when

FIGURE 5(CF).70



$T \sin 30° = W = 4$

$$T \times \frac{1}{2} = 4$$

or $\quad T = \mathbf{8\ N}$

80. If F is the upward force applied, then
$F - 5\,mg = 5\,ma$
$F = 5\,m\,(g + a)$

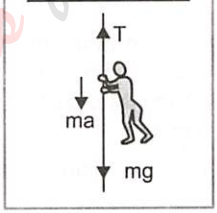
If T is force of interaction between the top link and link immediately below it, then

$$ma = F - mg - T$$
$$T = F - mg - ma$$
$$= 5\,m\,(g + a) - m\,(g + a)$$
$$T = 4\,m\,(g + a)$$
$$= 4 \times 0{\cdot}1\,(10 + 2) = \mathbf{3\ N}$$

81. If a is acceleration of man climbing down the rope, then as is clear from Fig. 5(CF).71.
$ma = mg - T$
$60\,a = 60 \times 10 - 360 = 240$

$$a = \frac{240}{60} = \mathbf{4\ m/s^2}$$

FIGURE 5(CF).71



VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

82. Change its momentum is given by $dP = F\,dt$.

In case of concrete pavement, man is brought to rest almost immediately and as such Δt is small, therefore F is large. So person receives more injury on concrete pavement.

83. Both, assertion and reason are false because slope of momentum time graph gives $\frac{dp}{dt}$

$$\frac{dp}{dt} = \frac{d}{dt}(mv)$$

$$\therefore \qquad m\frac{dv}{dt} = ma = \textbf{force.}$$

84. Work done is same, because gravitational forces are conservative forces. Gravitational force on the body along the inclined plane is not same as that along the vertical. The reason is false.

85. A rocket moves forward on account of escaping of exhaust gases in the backward direction. Both, assertion and reason are false.

86. Rocket is propelled by combustion of fuel and does not move under the action of gravity alone. Therefore, rocket is not an example of projectile. We do give initial velocity to rocket.

87. Component of horizontal velocity (u) along the vertical $= u \cos 90° = 0$. Therefore, vertical motion is not affected. Time taken by the two bodies to strike the ground is the same.

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For Difficult Questions

88. Only static friction is a self adjusting force. This is because force of static friction is equal and opposite to applied force (so long as actual motion does not start). Friction does depend upon weight of the body.

FOR ENGINEERING STUDENTS

89. Force of friction is independent of the actual area of contact. Statement-1 is false, but Statement-2 is true.

90. Thrust on a rocket depends both, on rate of decrease of mass and also on velocity of exhaust gases. Statement-1 is false. However, Statement-2 is true.

91. Both the statements are true and statement-2 is correct explanation of the statement-1.

92. Statement-1 is false. A force of 1 N produces an acceleration of 1 m/s^2 in a body of mass 1 kg. However, Statement-2 is true.

93. Here, $m = 100 \text{ g} = 10^{-1}$ kg,

$$u = 72 \text{ km/hr} = 20 \text{ m/s}$$

$$= \frac{10^{-1}(20+20)}{1} = 4 \text{ N}$$

The statement-1 is true. Statement-2 is also true, but it does not explain the assertion properly.

94. $v = \sqrt{\mu rg} = \sqrt{0{\cdot}1 \times 10 \times 10} = \sqrt{10}$ m/s .

Both the statements are true, and statement-2 is correct explanation of statement-1.

VIII. Multiple Choice Questions (Based on Experimental Skills)

95. Effective force of friction = dynamic friction which is slightly less than force of limiting friction (= 5 N).

96. The motion of the point of contact of wheel with the level road is directed backwards tangentially. Therefore, frictional force must be in forward direction.

97. As $F \propto R$, therefore, **choice (*d*) is the correct graph** between F and R.

98. When a body of mass m is held on a rough inclined plane of inclination θ, the net downward force on the body is

$$F = mg \sin\theta - f,$$

where f is the constant force of friction.

As $\sin\theta$ increases, F increases.

Choice (*b*) is correct.

99. As $\mu = F/R$ = slope of F–R. curve, which is greater for A, therefore, for body A, force of friction is larger and for body B, force of friction is smaller.

∴ Surface of body *B* in contact with the plane must be smoother.

UNIT-4

WORK, ENERGY AND POWER

CHAPTER 6.

WORK, ENERGY AND POWER

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6

WORK, ENERGY AND POWER

6.1. INTRODUCTION

In everyday language, we often use the terms 'Work', 'Energy' and 'Power'. A teacher teaching a class, a student preparing for examination, mom cooking food, a farmer ploughing the field, *all are said to be working. A person who can put in long hours of work is said to have large stamina or more energy. In karate or boxing, we talk of powerful punches that are delivered at a great speed.* We shall see in this unit that the exact definitions of the three terms are related loosely to the physiological pictures these terms generate in our mind.

6.2. WORK

In Physics, work and energy have precise meanings, which must not be confused with their everyday meanings. For example, when we lift boxes of books onto library shelves, we get tired and hungry.* We need to eat food to get more energy. We define, work *W* as the energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work and energy transferred from the object is negative work.

Thus, basically, work is transferred energy. 'Doing work' is nothing but, the act of energy transfer. Quantitatively,

> ***Work** is said to be done by a force when the body is displaced actually through some distance in the direction of the applied force.*

However, when there is *no displacement* in the direction of the applied force, no work is said to be done, *i.e., work done is zero,* when displacement of the body in the direction of the force is zero.

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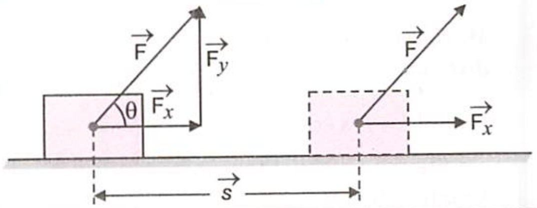
*It is interesting to note that hunger is not an indication of work done. Even when we are resting, we experience the sensation of hunger after a few hours. Infact, hunger is an indication of the need for more fuel. Work done by the body cannot be defined in terms of hunger.

Suppose a constant force $\vec{F}$ acting on a body produces a displacement $\vec{s}$ in the body along the positive x-direction, Fig. 6.1.

FIGURE 6.1



If θ is the angle which $\vec{F}$ makes with the positive x-direction of the displacement, then the two rectangular components of $\vec{F}$ are

(*i*) $\vec{F}_x$ in the direction of $\vec{s}$ where $F_x = F\cos\theta$

(*ii*) $\vec{F}_y$ perpendicular to $\vec{s}$ where $F_y = F\sin\theta$

The component $\vec{F}_y$ does no work as the body does not move up or down. All the work is done by the component $\vec{F}_x$

i.e.,
$$W = F_x \times s = F\cos\theta \times s$$

$$\boxed{W = (F\cos\theta)\, s} \qquad ...(1)$$

Hence, work done by the force is the product of component of force in the direction of displacement and the magnitude of displacement.

Retain in Memory

To calculate work done by a force, we use only the force component along the object's displacement. The force component perpendicular to the object's displacement does no work.

Eqn. (1) can be rewritten as

$$\boxed{W = \vec{F}\,.\,\vec{s}} \qquad ...(2)$$

Thus, **work done by a force is the dot product of force and displacement.**

In terms of rectangular components, $\vec{F}$ and $\vec{s}$, may be written as

$$\vec{F} = \hat{i}\,F_x + \hat{j}\,F_y + \hat{k}\,F_z \quad \text{and} \quad \vec{s} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$$

From (2), $W = \vec{F}\,.\,\vec{s} = (\hat{i}\,F_x + \hat{j}\,F_y + \hat{k}\,F_z)\cdot(\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)$

$$\boxed{W = x\,F_x + y\,F_y + z\,F_z} \qquad ...(3)$$

Obviously, work is a scalar quantity, *i.e.*, it has magnitude only and no direction. However, work done by a force can be positive or negative or zero, as detailed in Art. 6.4.

6.3. DIMENSIONS AND UNITS OF WORK

As work = force × distance $\therefore\ W = (M^1\,L^1\,T^{-2}) \times L$

$$\boxed{W = [M^1\,L^2\,T^{-2}]}$$

This is the dimensional formula of work.

The **units** of work are of two types : 1. Absolute units 2. Gravitational units

***(a)* Absolute units**

1. Joule. It is the absolute unit of work on SI.

*Work done is said to be **one joule,** when a force of one newton actually moves a body through a distance of one metre in the direction of applied force.*

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From $W = F\,s\cos\theta$

$1\text{ joule} = 1\text{ newton} \times 1\text{ metre} \times \cos 0° = 1\text{ N–}m$

2. Erg. It is the absolute unit of work on cgs system.

Work done is said to be one erg, when a force of one dyne actually moves a body through a distance of one cm. in the direction of applied force.

From $W = F\,s\cos\theta$

$1\text{ erg} = 1\text{ dyne} \times 1\text{ cm} \times \cos 0°$

Relation between joule and erg

As $1\text{ joule} = 1\text{ N} \times 1\text{ m} \times \cos 0°$ $\therefore$ $1\text{ J} = 10^5\text{ dyne} \times 10^2\text{ cm} \times 1 = 10^7\text{ ergs}$

$\boxed{1\text{ joule} = 10^7\text{ ergs}}$

(*b*) Gravitational units

These are also called the *practical units* of work.

1. Kilogram-metre (kg-m). It is the gravitational unit of work on MKS system.

Work done is said to be one kg-m, when a force of 1 kg f moves a body through a distance of 1 m in the direction of the applied force.

From $W = F\,s\cos\theta$

$1\text{ kg–}m = 1\text{ kg}f \times 1\text{ m} \times \cos 0° = 9{\cdot}8\text{ N} \times 1\text{ m} = 9{\cdot}8\text{ joule}$, *i.e.*, $\boxed{1\text{ kg} - \text{m} = 9{\cdot}8\text{ J}}$

2. Gram-centimetre (g-cm). It is the gravitational unit of work on cgs system.

Work done is said to be one g-cm, when a force of 1 g f moves a body through a distance of 1 cm. in the direction of the applied force.

From

$$W = F\,s\cos\theta$$
$$1\text{ g-cm} = 1\text{ g}f \times 1\text{ cm} \times \cos 0°$$
$$1\text{ g-cm} = 980\text{ dyne} \times 1\text{ cm} \times 1$$
$$\boxed{1\text{ g-cm} = 980\text{ ergs}}$$

Relation between kg-m and g-cm

$$1\text{ kg-m} = 10^3\text{ g} \times 10^2\text{ cm}$$
$$\boxed{1\text{ kg-m} = 10^5\text{ g-cm}}$$

6.4. NATURE OF WORK DONE

Although work done is a scalar quantity, its value may be positive, negative or even zero, as detailed below :

(*a*) POSITIVE WORK

As $W = \vec{F}\cdot\vec{s} = F\,s\cos\theta$ $\therefore$ when θ is acute ($< 90°$), $\cos\theta$ is positive. Hence, work done is positive.

For example : (*i*) When a body falls freely under the action of gravity, $\theta = 0°$, $\cos\theta = \cos 0° = +1$. Therefore, *work done by gravity on a body falling freely is positive,* Fig. 6.2.

(*ii*) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.

(*iii*) When a gas filled in a cylinder fitted with a movable piston is allowed to expand, work done by the gas is positive. This, is because force due to gaseous pressure and displacement of piston are in the same direction.

(*iv*) When a spring is stretched, work done by the stretching force is positive.

(b) NEGATIVE WORK

As $W = \vec{F} \cdot \vec{s} = F\, s \cos\theta$

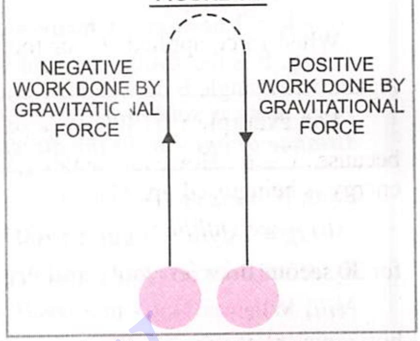
$\therefore$ When θ is obtuse (> 90°), cos θ is negative. Hence, work done is negative.

For example :

(*i*) When a body is thrown up, its motion is opposed by gravity. The angle θ between gravitatonal force $\vec{F}$ and the displacement $\vec{s}$ is 180°. As $\cos\theta = \cos 180° = -1$, therefore, *work done by gravity on a body moving upwards is negative.*

This is shown in Fig. 6.2.

FIGURE 6.2


NEGATIVE WORK DONE BY GRAVITATIONAL FORCE
POSITIVE WORK DONE BY GRAVITATIONAL FORCE

(*ii*) When a body is moved over a rough horizontal surface, the motion is opposed by the force of friction. Hence, *work done by frictional force is negative.* Note that work done by the applied force is *not* negative. It is positive.

(*iii*) When brakes are applied on a moving vehicle, work done by the braking force is negative.

(*iv*) When a positive charge is moved closer to another positive charge, work done by electrostatic force of repulsion between the charges is negative.

Retain in Memory

1. Work is positive when force $(\vec{F})$ and displacement $(\vec{s})$ are parallel to each other that is they are in the same direction. And work is negative when force $(\vec{F})$ and displacement $(\vec{s})$ are antiparallel to each other.
   Thus work done by the gravitational force is positive, when the object falls and work done by the gravitational force is negative, when the object is thrown upwards, Fig. 6.2.
2. Again, when we lift an object by applying a vertical force on it, work done by applied force is positive and work done by gravitational pull is negative.
3. Positive work accelerates a body, *i.e.*, velocity of the body increases. Negative work retards the body, *i.e.*, velocity of the body decreases.

DO YOU KNOW ?

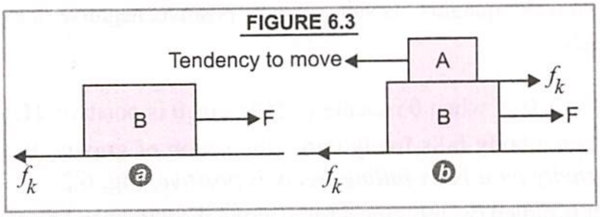
Work done by kinetic friction is not always negative. It may also be positive in some cases.

For example

When a force F is applied on a block B to move it on a rough horizontal surface, force of friction f_k opposes the motion, Fig. 6.3 (*a*). Work done by force of friction on the block B is negative, *i.e.*,

$$W = f_k\,(s)\cos 180° = -f_k\,(s).$$

FIGURE 6.3



Imagine another block A kept on rough top of block B. When force F is applied on block B, then due to inertia of rest, block A tends to move in the backward direction. The force of friction (f_k) on block A is in the forward direction (*i.e.*, to the right). Therefore, work done by force of friction on block A is

$$W = f_k\,(s)\cos 0° = +f_k\,(s)$$

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(c) ZERO WORK

When force applied $\vec{F}$ or the displacement $\vec{s}$ or both are zero, work done $W = F\,s\cos\theta$ is zero. Again, when angle θ between $\vec{F}$ and $\vec{s}$ is 90°, $\cos\theta = \cos 90° = 0$. Therefore, work done is zero.

For example : (*i*) *When we push hard against a wall,* the force we exert on the wall does no work, because $\vec{s} = 0$. However, in this process, our muscles are contracting and relaxing alternately and internal energy is being used up. That is why we do get tired.

(*ii*) *A weightlifter holding a 100 kg mass steadily on his shoulder* for 30 second does no work on the load during this time, because $\vec{s} = 0$.

(*iii*) When a coolie carrying some load on his head moves on a horizontal platform, θ = 90°, *i.e.*, motion is at 90° to the force applied. Therefore, work done by the coolie is zero. Similarly, when *a person carrying a brief case moves on a horizontal road*, work done by the person is zero.

(*iv*) A block moving on a perfectly smooth horizontal table is not being acted by any external force (as there is no friction). Therefore, no work is done even when the block undergoes a large displacement.

(*v*) *When a body tied to one end of a string is rotated uniformly in a circle*, Fig. 6.4, work done by the centripetal force applied along the string is zero. This is because θ = 90° and $W = F\,s\cos 90° = 0$.

(*vi*) If we assume that the *moon's orbit around the earth is perfectly circular, then the earth's gravitational force does no work.* This is because earth's gravitational force is radially inwards and moon's instantaneous displacement is tangential, so that θ = 90° and $\cos\theta = \cos 90° = 0$.

(*vii*) Tension in the string of a simple pendulum is always perpendicular to displacement of the bob, Fig. 6.5. Therefore, *work done by tension is always zero.*

FIGURE 6.4

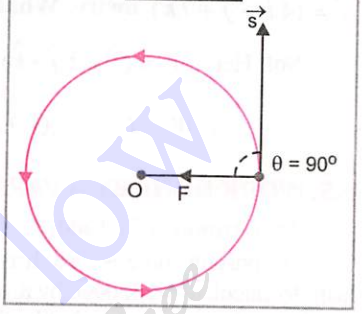
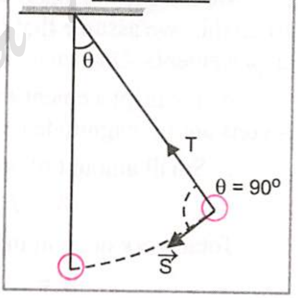


FIGURE 6.5



DO YOU KNOW ?

1. A weight lifter does work in lifting the weight off the ground, but does no work in holding the weight up for any length of time.
2. When circular motion is *not uniform,* a tangential force is acting on the body in addition to radial force. Work done by radial force is zero, but work done by tangential force is not zero. Obviously, K.E. of body would change.

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1. It should be clearly understood that work done in example (*iv*) to (*vii*) is zero, only under ideal/perfect conditions, *i.e.,* when there is neither any opposing force of friction nor any resistance due to air etc. In actual practice, both the opposing forces are present, and therefore, some work is certainly done and some energy is spent.
2. Work done is zero when (*i*) force acting on the body is zero, or (*ii*) displacement of the body in the direction of applied force is zero, or (*iii*) force and displacement are perpendicular to each other.
3. When two or more forces act on an object, net work done on the object is
   (*i*) algebraic sum of the work done by each force or
   (*ii*) work done by the net force of these forces in moving the object : $W = F_{net}\,(s)\,(\cos\theta)$
4. When the body is in equilibrium (static or dynamic), resultant force on the body = 0
   Therefore, $W = 0$

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Sample Problem 1 **A box is pushed through 4·0 m across a floor offering 100 N resistance. How much work is done by the (i) applied force (ii) resisting force ?**

Sol. Here, $s = 4{\cdot}0$ m, $F = 100$ N, $W = ?$

Work done by applied force $W = F\,s \cos 0° = F\,s \cos 0° = 100 \times 4 \times 1 = \mathbf{400\ J}$

Work done by resisting force $W = F\,s \cos 180° = 100 \times 4\,(-1) = \mathbf{-400\ J}$

Sample Problem 2 **A force $\vec{F} = (\hat{i} + 2\hat{j} - \hat{k})$ N moves a particle along a vector $\vec{s} = (4\hat{i} + \hat{j} + 7\hat{k})$ metre. What is the work done ?**

Sol. Here, $\vec{F} = (\hat{i} + 2\hat{j} - \hat{k})$ N and $\vec{s} = (4\hat{i} + \hat{j} + 7\hat{k})$ metre, $W = ?$

$$W = \vec{F}\cdot\vec{s} = (\hat{i} + 2\hat{j} - \hat{k})\cdot(4\hat{i} + \hat{j} + 7\hat{k}) = 4(\hat{i}\cdot\hat{i}) + 2(\hat{j}\cdot\hat{j}) - 7(\hat{k}\cdot\hat{k}) = 4 + 2 - 7 = \mathbf{-1\ J}$$

6.5. WORK DONE BY A VARIABLE FORCE

(*a*) Graphical Method.

A constant force is rare. It is the variable force which is encountered more commonly. We can, therefore, learn to calculate work done by a variable force. For the sake of simplicity, let us consider a force acting along the fixed direction, say *x*-axis, but having a variable magnitude, as shown in Fig. 6.6.

We have to calculate work done in moving the body from *A* to *B* under the action of this variable force. To do this, we assume that the entire displacement from *A* to *B* is made up of a large number of infinitesimal displacements. One such displacement shown in Fig. 6.6(*a*) is from *P* to *Q*.

As the displacement $PQ = dx$ is infinitesimally small, we consider that all along this displacement, force is constant in magnitude (= *PS*) as well as in direction.

∴ Small amount of work done in moving the body from *P* to *Q* is

$$dW = F \times dx = (PS)\,(PQ) = \text{area of strip } PQRS \qquad ...(4)$$

Total work done in moving the body from *A* to *B* is given by

$$W = \Sigma\, dW = \Sigma\, F \times dx$$

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit. And the sum approaches a definite value equal to the area under the curve *CD* as shown in Fig. 6.6(*b*).

Hence, we may rewrite,

$$W = \underset{dx \to 0}{\text{Limit}}\ \Sigma\, F\,(dx)$$

In the language of integral calculus, we may write it as

$$W = \int_{x_A}^{x_B} F\,(dx),$$

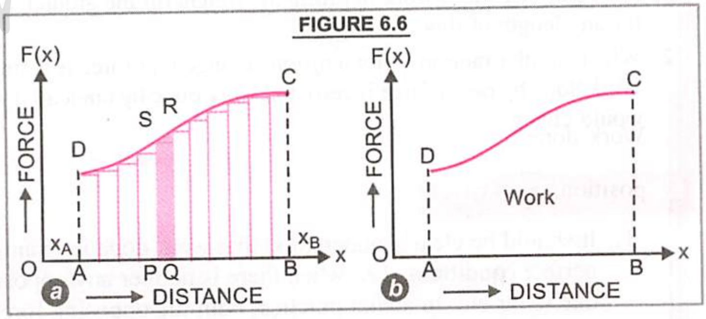
where $x_A = OA$ and $x_B = OB$

$$W = \int_{x_A}^{x_B} \text{area of the strip PQRS}$$

= total area under the curve between *F* and *x*-axis from $x = x_A$ to $x = x_B$

$$\boxed{W = \text{Area } ABCDA}$$

FIGURE 6.6



Hence, work done by a variable force is numerically equal to the area under the force curve and the displacement axis.

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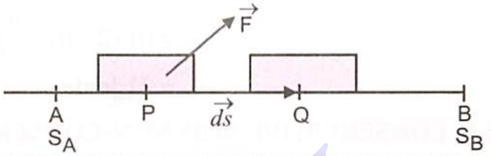
Note that area under force-displacement curve must be added with proper sign to obtain net work done by the force varying in magnitude and direction.

(*b*) Mathematical Treatment (of work done by a variable force).

Suppose we have to calculate work done in moving a body from a point A (S_A) to a point B (S_B) under the action of a varying force, Fig. 6.7. Here, S_A and S_B are the distances of the points *A* and *B* *w.r.t.* some reference point.

FIGURE 6.7



At any stage, suppose the body is at P, where force on the body is $\vec{F}$. Under the action of this force, let the body undergo an infinitesimally small displacement $\overrightarrow{PQ} = \vec{ds}$. During such a small displacement, if we assume that the force remains constant, then small amount of work done in moving the body from *P* to *Q* is

$$dW = \vec{F}.\vec{ds} \qquad \text{...(5)}$$

When $\vec{ds} \to 0$, total work done in moving the body from *A* to *B* can be obtained by integrating the above expression between S_A and S_B.

$$\therefore \qquad \boxed{W = \int_{S_A}^{S_B} \vec{F}.\vec{ds}} \qquad \text{...(6)}$$

IMPORTANT NOTE

In three dimensional analysis, we take

$$\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}$$

and

$$\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$dW = \vec{F}.\vec{ds} = (F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) = F_x\,dx + F_y\,dy + F_z\,dz$$

Work done by force $\vec{F}$ while the particle moves from an initial position, $\vec{r_i}$ (x_i, y_i, z_i) to a final position, $\vec{r_f}$ (x_f, y_f, z_f) is

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

Sample Problem **A force $F = (10 + 0{\cdot}50\,x)$ acts on a particle in *x* direction, where *F* is in newton and *x* is in metre. Find the work done by this force during a displacement from $x = 0$ to $x = 2$ m.**

Sol. Here, $F = (10 + 0{\cdot}50\,x)$

Small amount of work done in moving the particle through a small distance *dx* is

$$dW = \vec{F}.\vec{dx} = (10 + 0{\cdot}5x)\;dx$$

Total work done,
$$W = \int_{x=0}^{x=2} (10 + 0{\cdot}5x)\, dx$$

$$W = \left[10x + 0{\cdot}5\frac{x^2}{2}\right]_0^2$$

$$= 10\,(2-0) + \frac{0{\cdot}5}{2}\,(2^2 - 0) = 20 + 1$$

= 21 joule

DO YOU KNOW ?

1. When a body moves with a constant velocity, work done on/by the body is zero. This is because, when velocity is constant, net force on the body is zero. Therefore, work done = Zero.

2. When a ball hits a bat, the ball exerts a force $\vec{F_1}$ on the bat. According to Newton's third law of motion, the bat exerts a force $\vec{F_2}$ on the ball equal in magnitude to $\vec{F_1}$, but opposite in direction. Therefore, $\vec{F_1} + \vec{F_2} = 0$. But the algebraic sum of work done on the ball and work done on the bat in this process is **not zero.**

6.6. CONSERVATIVE AND NON-CONSERVATIVE FORCES

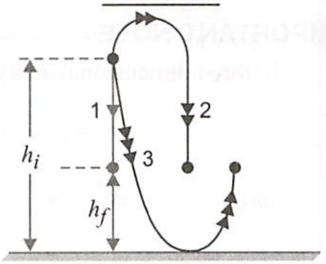
(*a*) Conservative forces

> ***A force is said to be conservative, if work done by or against the force in moving a body depends only on the initial and final positions of the body, and not on the nature of path followed between the initial and the final positions.***

This means, work done by or against a conservative force in moving a body over any path between fixed initial and final positions will be the same.

In Fig. 6.8, we have shown a ball going from an initial height h_i to a final height h_f along three different paths (1, 2, 3). In each case, work done by the gravitational force is the same = $mg\,(h_i - h_f)$ as the change in vertical distance $(h_i - h_f)$ is the same.

FIGURE 6.8



Thus, ***gravitational force is a conservative force.*** To prove this, let us calculate amount of work done against the gravitational force in moving a body of mass *m* through a height *AB* = *h* over different paths from *A* to *B*.

Fig. 6.9(*a*) shows the body being raised vertically upwards. Force applied $F = mg$.

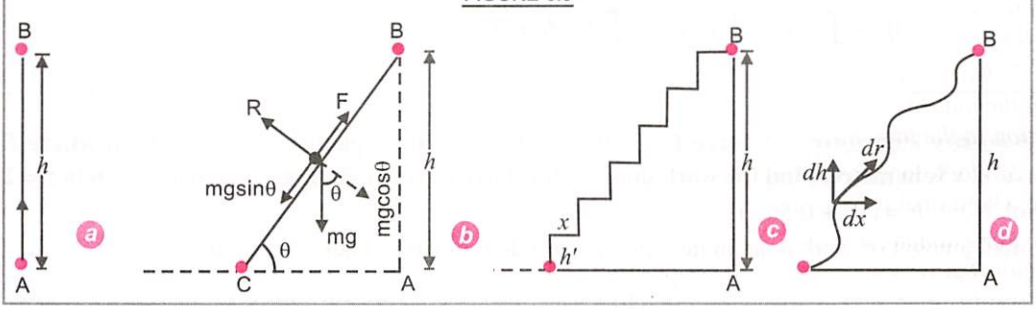
As work done $= \vec{F}\cdot\vec{s} = Fs\cos\theta$

$$\therefore \quad W_1 = (mg)\,h\cos 0^\circ = mgh \qquad ...(7)$$

Fig. 6.9(*b*) shows the body being taken along a *smooth inclined plane CB* of height *AB* = *h* and inclination θ.

As is clear from Fig. 6.9(*b*), force applied is $F = mg\sin\theta$ and the distance moved along the direction of force is *CB*.

FIGURE 6.9



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$\therefore$ Work done $= \vec{F} \cdot \overrightarrow{CB} = F\,(CB)\cos 0^\circ$

$$W_2 = mg\sin\theta \times CB = mg \times \frac{AB}{CB} \times CB$$

$$W_2 = mgh \qquad \textbf{...(8)}$$

Fig. 6.9(*c*) shows that body is being taken through the same height $AB = h$ over a staircase. Let there be n steps, each of vertical height h' and horizontal width x.

$\therefore$ Work done, $W_3 = n\,[mgh' \cos 0^\circ + mg\,x \cos 90^\circ]$. As $n \times h' = h$, therefore,

$$W_3 = n \times mgh' = mgh \qquad \textbf{...(9)}$$

Fig. 6.9(*d*) shows the body being carried through the same height $AB = h$ over any arbitrary zig zag path. This path can be supposed to be made up of a large number of infinitesimally small horizontal displacements (say, dx) and vertical displacements (say, dh).

$\therefore$ Work done $= \Sigma\, mg\,(dh)\cos 0^\circ + \Sigma\, mg\,(dx)\cos 90^\circ$

$$W_4 = mgh \qquad \textbf{...(10)}$$

From the above discussion, we find that $W_1 = W_2 = W_3 = W_4 = mgh$, *i.e.*, work done is the same, whatever be the path followed, between the given initial position A and final position B. This establishes that *gravitational force is a conservative force.*

Other examples of conservative forces are : force in an elastic spring ; electrostatic force between two electric charges, magnetic force between two magnetic poles. The last two forces are called *central forces* as they act along the line joining the centres of two charged/magnetied bodies. Thus, all central forces are conservative forces.

IMPORTANT NOTE

A roller coaster track is an example of a closed path. Gravitational force is the only force that does work on the car, assuming that there is no friction and no air resistance. Normal reaction of the track does no work as it is always perpendicular to the motion of the car. Over the entire trip, gravitational force does as much positive work as negative work, so that net work done over the closed path is zero.

Retain in Memory

Remember that work done in moving a body over a smooth inclined plane does not depend upon slope of inclined plane. As $W = m\,g\,h$, therefore, work done depends only on height h of the inclined plane. For a given height, work done in moving a body over any smooth inclined plane of any inclination will be the same.

Properties of conservative forces :

1. *Work done by or against a conservative force, in moving a body from one position to the other depends only on the initial position and final position of the body.*

2. *Work done by or against a conservative force does not depend upon the nature of the path followed by the body in going from initial position to the final position.*

DO YOU KNOW ?

For a conservative force, any choice of path between two given points gives the same amount of work. This result is very powerful, as it allows us to calculate work done by substituting some convenient path between two given points, in place of any inconvenient path.

3. Work done by or against a conservative force in moving a body through any round trip (*i.e.*, closed path, where final position coincides with the initial position of the body) is always zero.

For example, in case of gravitational force, Fig. 6.7, if we take work done in moving the body from A to B, *against* gravity as negative, then work done in moving the body from B to A, *by* gravity has to be taken as positive, *i.e.*, $W_{AB} = -W_{BA}$

$$\therefore \qquad W_{AB} + W_{BA} = 0 \qquad \text{...(11)}$$

Infact, work done in taking the body from A to B is stored in the body in the form of potential energy. It is this energy which is spent in moving the body from B to A. Thus, over the round trip ($A \to B \to A$), net work done is zero.

(b) Non-conservative Forces

A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions.

In Fig. 6.10(a), suppose W_1, W_2, W_3 denote the amounts of work done in moving a body from A to B along three different paths 1, 2, 3 respectively. If the force is non-conservative,

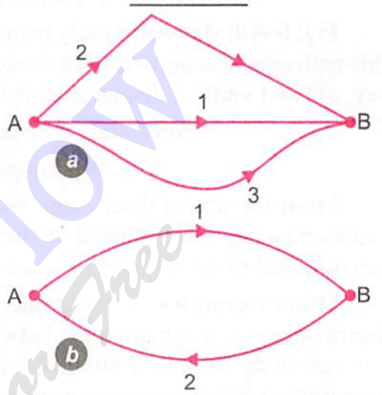
$$W_1 \neq W_2 \neq W_3$$

Fig. 6.10(b) shows a particle moving along a *closed path* : $A \to 1 \to B \to 2 \to A$. If W_1 is work done in moving the particle from $A \to 1 \to B$ and W_2 is work done in moving the particle from $B \to 2 \to A$, then for a non-conservative force,

$$|W_1| \neq |W_2|$$

$\therefore$ *Net work done along the closed path, $A \to B \to A$ is not zero, i.e.* $\oint \vec{F} . \vec{ds} \neq 0$

FIGURE 6.10



Common examples of non-conservative force are :

(i) Force of friction (ii) Viscous force (iii) Air resistance (iv) Tension, normal force.

For example, if a body is moved from a position A to another position B on a *rough table*, work done against frictional force (W) shall depend on the length of the path between A and B and also on the positions A and B.

Further, if the body is brought back to its initial position A, on the same path, the same work (W) has to be done against the frictional forces, which oppose the motion always. Hence, net work done against the frictional forces in moving a body over a round trip is not zero. It is numerically equal to 2 W.

If $\quad E_i$ = total value of initial energy, and $\quad E_f$ = total value of final energy, then

$$\boxed{E_i - E_f = 2\,W}$$

The work done by/against non-conservative forces appears in some other forms of energy like heat, sound, light energy etc.

IMPORTANT NOTE

When all forms of energy are taken into account, law of conservation of energy is true, both for conservative and non-conservative forces.

6.7. POWER

Often, we say a person is physically fit and powerful, if he not only climbs up four floors of a tall building, but also *climbs them fast*.

Power of a person or machine is defined as the time rate at which work is done by it.

$$i.e., \quad \text{Power} = \text{Rate of doing work} = \frac{\text{work done}}{\text{time taken}} \qquad \text{...(12)}$$

Thus, power of a body measures how fast it can do the work. When a body takes lesser time to do a particular amount of work, its power is said to be greater and vice-versa.

The power at a particular instant of time t is the ratio of small work done (dW) to the small time interval (dt) around t, *i.e.*, $P = \dfrac{dW}{dt}$

Now, $dW = \vec{F}.\vec{ds}$, where $\vec{F}$ is the force applied and $\vec{ds}$ is the small displacement.

$$\therefore \quad P = \frac{\vec{F}.\vec{ds}}{dt}$$

But $\frac{\vec{ds}}{dt} = \vec{v}$, the instantaneous velocity. $\therefore \quad \boxed{P = \vec{F}.\vec{v}}$ ...(13)

Thus, ***power can be expressed as the dot product of force and velocity.***

If θ is angle between $\vec{F}$ and $\vec{v}$, then $P = \vec{F}.\vec{v} = F v \cos\theta$

However, when $\vec{v}$ is along $\vec{F}$, $\theta = 0°$, $\therefore \; P = F v \cos 0° = F v$

As power is the ratio of two scalar quantities W and t, therefore, **power is a scalar.** That is why it is expressed as *dot product* of $\vec{F}$ and $\vec{v}$.

Dimensions of power can be deduced as :

$$\boxed{P = \frac{W}{t} = \frac{M^1L^2T^{-2}}{T^1} = [M^1L^2T^{-3}]}$$

Units of power

The absolute unit of power in SI is **watt,** which is denoted by W.

From $\quad P = W/t$

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ sec}}, \quad i.e., \quad \boxed{1\ W = 1\ J\ s^{-1}}$$

Hence,

Power of a body is said to be one watt, if it can do one joule of work in one second.

The **bigger units** of power are 1 kilo watt = 1000 watt, *i.e.,* 1 kW = 10^3 W

and 1 mega watt = 1,000,000 watt, *i.e.,* 1 MW = 10^6 W

The absolute unit of power in cgs system is 1 erg s^{-1}

Obviously, 1 W = 1 Js^{-1} = 10^7 erg s^{-1}.

The gravitational unit of power on MKS system is **(kg f) m s^{-1}** and in cgs system, it is **(g f) cm s^{-1}**.

Another popular unit of power (used mostly in engineering) is **horse power** (h.p.), where

$$\boxed{1 \text{ h.p.} = 746 \text{ W}}$$

This unit is still used to describe the output of automobiles, motorbikes etc.

Table 6.1 gives us average power consumption by an adult in some of the common processes/ activities.

TABLE 6.1. Average power consumption in some common activities

S. No.	Activity	Power (watt)
1.	Heart beat	1·2
2.	Sleeping	75
3.	Slow walking	200

Sample Problem 1 **An elevator weighing 500 kg is to be lifted up at a constant velocity of 0·4 m/s. What should be the minimum horse power of the motor to be used ?**

Sol. Here, $m = 500$ kg, $v = 0{\cdot}4$ m/s, $P = ?$

$$P = F\,v = (mg) \times v = 500 \times 9{\cdot}8 \times 0{\cdot}4 = \mathbf{1960\ watt}$$

If we assume that there is no loss against friction etc. in the motor, then minimum horse power of motor,

$$P = \frac{1960}{746}\ \text{h.p} = \mathbf{2{\cdot}62\ h.p}$$

Sample Problem 2 **A person weighing 70 kg runs up a flight of 30 steps in 35 seconds. What is the power of the person if each step is 20 cm high ?**

Sol. Here, $M = 70$ kg, $t = 35$ s

$$h = 30 \times 20\ \text{cm} = 6\ \text{m},\ P = ?\quad P = \frac{W}{t} = \frac{Mgh}{t} = \frac{70 \times 9{\cdot}8 \times 6}{35} = \mathbf{117{\cdot}6\ watt}$$

6.8. ENERGY

In day to day life, we use the term 'energy' in many ways. For example, a child who runs and plays long after adults are tired is said to be full of energy. When a person has lot of energy, he can run faster or farther ; he can jump higher and so on.

The concept of energy allows us to keep track of the changes that take place in a system. The system could be our body, our car, our home, the earth or even the universe. **Some of the *characteristics of energy* are :**

(*i*) The entire matter possesses energy. (*ii*) Energy menifests itself in so many different forms. (*iii*) Energy can neither be created nor be destroyed. Total quantity of energy in the universe is constant. However, no body knows how much is this total energy. (*iv*) Energy can be stored, and it can be transferred to other matter. (*v*) Energy can be transformed from one form to the other. (*vi*) The amount of energy transferred/transformed can be measured. (*vii*) Some energy transfers/transformations can be seen, heard, felt or even tasted.

Basically, we define

Energy of a body as the capacity or ability of the body to do the work.

When a body is capable of doing more work, it is said to possess more energy. The reverse is also true. For example, we admire a long distance runner for her stamina or energy. Similarly, a person who can work for 14-16 hours a day is said to have a larger stamina or energy or greater capacity to work.

Like work, energy is a scalar quantity, having magnitude only and no direction. The *dimensions* of energy are the same as the dimensions of work, *i.e.*, $[M^1 L^2 T^{-2}]$. The *units* of measurement of energy are also the same as the units of work, *i.e.*, In SI, absolute unit of energy is **joule** and in cgs system, the absolute unit of energy is **erg, which have already been defined.**

Some practical units of energy and their equivalence to joule are given in Table 6.2.

TABLE 6.2. Some Practical units of energy

S. No.	Unit	Symbol	Equivalence in (J)
1.	erg	erg	10^{-7} J
2.	calorie	cal.	$4{\cdot}2$ J
3.	kilowatt hour	kWh	$3{\cdot}6 \times 10^6$ J
4.	electron volt*	eV	$1{\cdot}6 \times 10^{-19}$ J
5.	Mega electron volt	MeV	$1{\cdot}6 \times 10^{-13}$ J

---

*One electron volt (eV) is the energy acquired by an electron on passing through a potential difference of 1 volt, *i.e.*, $1\ \text{eV} = (1{\cdot}6 \times 10^{-19}\ \text{C}) \times 1\ \text{volt} = 1{\cdot}6 \times 10^{-19}$ J

Retain in Memory

It should be clearly understood that energy is different from power. Whereas energy refers to the total amount of work a body can do, power determines the rate of doing work. Thus, in power, time taken to complete the work is significant. But in energy, time is irrelevant.

6.9. KINETIC ENERGY

The kinetic energy of a body is the energy possessed by the body by virtue of its motion.

For example, (*i*) A bullet fired from a gun can pierce through a target on account of kinetic energy of the bullet.

(*ii*) Wind mills work on the kinetic energy of air. For example, sailing ships use the kinetic energy of wind.

(*iii*) Water mills work on the kinetic energy of water. For example, fast flowing stream has been used to grind corn.

(*iv*) A nail is driven into a wooden block on account of kinetic energy of the hammer striking the nail.

Formula for Kinetic Energy

Kinetic Energy of a body can be obtained either from

(*i*) the amount of work done in stopping the moving body, or from

(*ii*) the amount of work done in giving the present velocity to the body from the state of rest.

Let us use the second method :

Suppose m = mass of a body at rest (*i.e.*, $u = 0$), F = Force applied on the body

a = acceleration produced in the body in the direction of force applied.

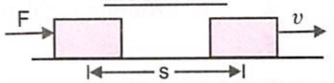
v = velocity acquired by the body in moving through a distance s, Fig. 6.11.

From $v^2 - u^2 = 2as$

$v^2 - 0 = 2as$

$$a = \frac{v^2}{2s} \qquad \text{...(14)}$$

FIGURE 6.11



As $F = ma$ $\quad\therefore$ using (14), $F = m\left(\dfrac{v^2}{2s}\right)$

Work done on the body, W = Force × distance

$$W = m\,\frac{v^2}{2s} \times s = \frac{1}{2}\,mv^2$$

This work done on the body is a measure of kinetic energy (K.E.) acquired by the body,

$$\therefore \quad \boxed{\text{K.E. of body} = W = \frac{1}{2}\,mv^2} \qquad \text{...(15)}$$

Alternative Method

The formula for kinetic energy of a body is also obtained by the method of calculus :

Let m = mass of a body, which is initially at rest (*i.e.*, $u = 0$), $\vec{F}$ = Force applied on the body,

$\vec{ds}$ = small displacement produced in the body in the direction of the force applied.

$\therefore$ Small amount of work done by the force, $dW = \vec{F}\cdot\vec{ds} = F\,ds\cos 0° = F\,ds$ ...(16)

If a is acceleration produced by the force, then from

$$F = ma = m\,\frac{dv}{dt}$$

From (16), $dW = \left(m\,\dfrac{dv}{dt}\right) ds = m\left(\dfrac{ds}{dt}\right) dv$

$$dW = m\,v\,d\,v \qquad \left(\because \frac{ds}{dt} = v\right)$$

∴ Total work done by the force in increasing the velocity of the body from zero to v is

$$W = \int_0^v m\,v\,d\,v = m\int_0^v v\,d\,v = m\left[\frac{v^2}{2}\right]_0^v$$

$$W = \frac{1}{2}m\,v^2 \qquad \therefore \quad \text{K.E. of the body} = W = \frac{1}{2}m\,v^2$$

DO YOU KNOW ?

'Speed thrills but kills' is so true a slogan. When speed of car is made twice, its $KE\left(=\frac{1}{2}mv^2\right)$ becomes 4 times. Therefore, work that needs to be done on the car to stop it during a crash becomes four fold. That really kills.

Thus, kinetic energy of a body is half the product of mass of the body and square of velocity of the body.

We observe that K.E. $\propto m$ and also K.E. $\propto v^2$, *i.e.*, a heavier body and a body moving faster possesses greater kinetic energy. The reverse is also true.

The absolute unit of K.E. on SI is **joule** and on cgs system, it is **erg.**

Some typical kinetic energy values are given in Table 6.3.

TABLE 6.3. Some Typical Kinetic Energy values

S. No.	Object	Mass (kg)	Speed (ms^{-1})	K.E. (J)
1.	Stone dropped from 10 m	0·5	14	≈ 50
2.	Rain drop at terminal speed	$3{\cdot}5 \times 10^{-5}$	9	$\approx 1{\cdot}4 \times 10^{-3}$
3.	Air molecule	10^{-26}	500	$\approx 10^{-21}$
4.	Running athlete	70	10	$3{\cdot}5 \times 10^{3}$
5.	Bullet	5×10^{-2}	200	10^{3}
6.	Car	2000	20	4×10^{5}

Retain in Memory

1. The expression K.E. $= \frac{1}{2}m\,v^2$ holds even when the force applied varies in magnitude or in direction or in both. Thus, the expression is valid irrespective of how the body acquires the velocity v.
2. K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}\,.\,\vec{v})$. Therefore K.E. is a **scalar quantity.**
3. K.E. of a body depends on the speed of the body and not on its velocity, *i.e.*, direction of velocity does not affect the K.E. of the body.
4. As mass (m) and square of velocity (v^2) are always positive, therefore, K.E. of the body is always positive. It is never negative.
5. K.E. of a body having non-uniform circular motion is not constant. For example, when a body is moving in a vertical circle, its K.E. is maximum at the lowest point and minimum at the highest point.
6. **Kinetic energy of a body depends upon the frame of reference.** For example, K.E. of a person of mass m sitting in a train moving with vel. v is $\left(\frac{1}{2}m\,v^2\right)$ in the frame of earth, and K.E. of the same person = 0, in the frame of the train.

Sample Problem 1 **The linear momentum of a body is increased by 10%. What is the percentage change in its KE ?**

Sol. Here, $p_2 = \frac{110}{100}\, p_1$,

i.e., $m v_2 = \frac{11}{10}\, m v_1$ or $v_2 = \frac{11}{10}\, v_1$

$$\text{Now,}\quad \frac{E_2}{E_1} = \frac{\frac{1}{2} m v_2^2}{\frac{1}{2} m v_1^2} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{11}{10}\right)^2 = \frac{121}{100}$$

Clearly, $E_2 > E_1$

$$\text{Percentage increase in K.E.} = \frac{(E_2 - E_1)}{E_1} \times 100 = \left(\frac{121}{100} - 1\right) \times 100 = \mathbf{21\%}$$

Sample Problem 2 **A body of mass 5 kg initially at rest, is subjected to a force of 20 N. What is the kinetic energy acquired by the body at the end of half minute ?**

Sol. Here, $m = 5$ kg, $u = 0$, $F = 20$ N

$$\text{KE} = ?\ ;\quad t = \frac{1}{2}\ \text{min} = 30\ \text{s}\ ;\quad a = \frac{F}{m} = \frac{20}{5} = 4\ \text{m/s}^2$$

$$v = u + at = 0 + 4 \times 30 = 120\ \text{m/s}$$

$$\text{KE} = \frac{1}{2} m v^2 = \frac{1}{2} \times 5\,(120)^2 = \mathbf{3{\cdot}6 \times 10^4\ J}$$

6.10. RELATION BETWEEN KINETIC ENERGY AND LINEAR MOMENTUM

Let m = mass of a body, v = velocity of the body.

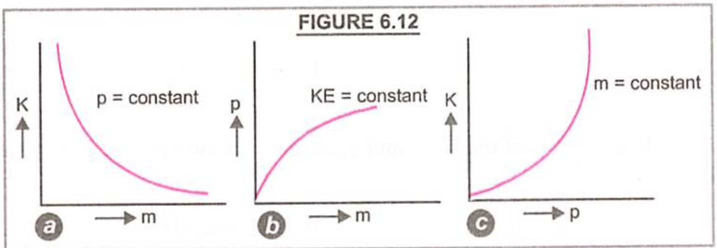
$\therefore$ Linear momentum of the body, $p = m v$

and K.E. of the body $= \frac{1}{2} m v^2 = \frac{1}{2m}(m^2 v^2)$

$$\therefore \quad \boxed{\text{K.E.} = \frac{p^2}{2m}} \qquad \text{...(17)}$$

This is an important relation. It shows that a body cannot have K.E. without having linear momentum. The reverse is also true.

FIGURE 6.12



Further, if p = constant, K.E. $\propto \frac{1}{m}$

This is shown in Fig. 6.12(a).

If K.E. = constant, $p^2 \propto m$ or $p \propto \sqrt{m}$

This is shown in Fig. 6.12(b).

If m = constant, $p^2 \propto$ K.E. or $p \propto \sqrt{\text{K.E.}}$

This is shown in Fig. 6.12(c).

6.11. WORK ENERGY THEOREM OR WORK ENERGY PRINCIPLE

> ***According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body.***

Thus, when a force does some work on a body, the kinetic energy of the body increases by the same amount. Conversely, when an opposing (retarding) force is applied on a body, its kinetic energy decreases. The decrease in kinetic energy of the body is equal to the work done by the body against the retarding force. Thus, according to work energy principle, *work and kinetic energy are equivalent quantities.*

Proof. To prove the work-energy theorem, we confine ourselves to motion in one dimension.

Suppose m = mass of a body, u = initial velocity of the body, F = force applied on the body along its direction of motion, a = acceleration produced in the body, v = final velocity of the body after t seconds

Small amount of work done by the applied force on the body, $dW = F\,(ds)$, where ds is the small distance moved by the body in the direction of the force applied.

Now, $$F = ma = m\left(\frac{dv}{dt}\right)$$

$$\therefore \quad dW = F\,(ds) = m\left(\frac{dv}{dt}\right)ds = m\left(\frac{ds}{dt}\right)dv = mv\,dv \qquad \left(\because \frac{ds}{dt} = v\right)$$

Total work done by the applied force on the body in increasing its velocity from u to v is

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$$W = \frac{1}{2}\,m\,(v^2 - u^2) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\frac{1}{2}mv^2 = K_f$ = final K.E. of the body and $\frac{1}{2}mu^2 = K_i$ = initial K.E. of the body

$$\therefore \quad W = K_f - K_i = \text{Increase in K.E. of body}$$

i.e., work done on the body = increase in K.E. of body.

Similarly, we can show that if some work is drawn from the body, K.E. of the body decreases by the same amount. This proves work energy theorem or work energy principle.

Work Energy Theorem for a variable force

As is known, work done on a body by a variable force F in displacing the body from initial position x_i to the final position x_f is given by

$$W = \int_{x_i}^{x_f} F(x)\,dx \qquad \text{...}(i)$$

If m is mass of the body and a_x is acceleration produced in the body along x-axis, then $F(x) = ma_x$ Put in (i),

$$W = \int_{x_i}^{x_f} ma_x\,dx \qquad \text{...}(ii)$$

Now, $$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\cdot\frac{dx}{dt}$$

But $$\frac{dx}{dt} = v_x \qquad \therefore \quad a_x = v_x\frac{dv_x}{dx} \qquad \text{...}(iii)$$

Putting in (*ii*) and changing the limits of integration in terms of v, *i.e.*,

$$W = \int_{x_i}^{x_f} mv_x \frac{dv_x}{dx} dx$$

$$W = \int_{v_i}^{v_f} mv_x . dv_x = \left[m\frac{v_x^2}{2} \right]_{v_i}^{v_f} = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

i.e., work done = increase in K.E. of the body

which is the work energy theorem.

IMPORTANT NOTE

Relation between force and K.E.

From work energy theorem, $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$F \times s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\boxed{F = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mu^2}{s}}$$

Clearly, to cause the same change in K.E. of two bodies, smaller force has to be applied through larger distance and vice-versa.

When a car and a bus having same K.E. undergo same retarding force, they would stop on travelling the same distance.

Again, when a car and a bus moving with same speed, undergo same retarding force, the bus will travel larger distance than the car, before coming to a stop.

Retain in Memory

1. Whereas Newton's second law is a relation between acceleration and force at any instant of time, work energy theorem involves an integral over an interval of time.
2. Newton's second law in two or three dimensions is in vector form, but the work-energy theorem is in scalar form.
3. In the above discussion, we have assumed that the work done by the force is effective only in changing the kinetic energy of the body. It should, however, be remembered that *work done on a body may also be stored in the body in the form of potential energy.*

   In that event, we shall say that work done on the body = increase in P.E. of the body.

   If both the PE and KE of the body change, then according to work energy theorem,

   work done on the body = increase in P.E. and K.E. of the body.

   Similarly, work done by the body = decrease in P.E. and K.E. of the body.
4. Conversely, when work done on the body is positive, its K.E. will increase and when work done on the body is negative, its K.E. will decrease.
5. Work energy principle/theorem is true for any system of particles in the presence of all types of forces (external as well as internal). These forces may be conservative or non conservative.
6. When we consider a system of particles and the particles exert forces on one another, then change in K.E. of the system is equal to work done on the system by the external as well as internal forces.

Sample Problem **Find the average frictional force that would stop a car weighing 500 kg in a distance of 25 m, if the initial speed is 72 km/h.**

Sol. Here, $F = ?$, $m = 500$ kg, $s = 25$ m, $v = 0$, $u = 72$ km/h $= \dfrac{72\times1000}{60\times60}$ m/s = 20 m/s

According to work energy principle, W = change in K.E. of the body

$$Fs = \frac{1}{2}\, m\,(v^2 - u^2)$$

$$F\times25 = \frac{1}{2}\times500\,(0-20^2) = -\frac{500\times400}{2}$$

$$F = -\frac{500\times200}{25} = -4000\text{ N} \quad \therefore \text{ Average frictional force} = \mathbf{4000\ N}$$

6.12. POTENTIAL ENERGY

> ***The potential energy** of a body is defined as the energy possessed by the body by virtue of its position or configuration in some field.*

Thus, potential energy is the energy that can be associated with the configuration (or arrangement) of a system of objects that exert forces on one another. Obviously, if configuration of the system changes, then its potential energy changes.

Potential energy takes many forms. For example :

(*i*) Our body transforms potential energy in food into internal energy to maintain body temperature. Some of the energy from food is converted into KE of blood, muscles and bones. Some of the potential energy of food is converted into electrochemical energy to operate the nervous system of the body.

(*ii*) Batteries also contain potential energy by separating charges that are attracted towards each other. This potential energy can be transformed into other forms of energy by completing the circuit.

(*iii*) Because of gravitational attraction of masses towards each other, arises the *gravitational potential energy*. This type of energy is hidden until the object is allowed to fall.

(*iv*) Potential energy can be stored in objects by compressing them, stretching them, bending them or twisting them in shape. This is called *elastic potential energy*. It can be converted into other forms of energy by allowing the object to regain its original shape.

When we wind the spring of our watch, potential energy is stored in the spring on account of configuration of the turns of the spring. As the spring unwinds, it works to move the hands of the watch. Thus, the wound spring has the potential to do the work.

Again, it is due to potential energy of the compressed spring in a loaded pistol that the bullet is released with a large velocity on firing the pistol.

Similarly, when a stretched bow is released, the arrow goes forward with a large velocity on account of potential energy of the streched bow.

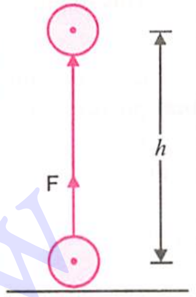
Gravitational Potential Energy

> ***Gravitational potential energy** of a body is the energy possessed by the body by virtue of its position above the surface of the earth.*

For example, in the 1996 Olympics, when Andrey lifted the record breaking weights above his head, he increased the separation between the weights and earth. The work his force did changed the configuration (*i.e.*, relative location of weights and earth) and hence changed the gravitational potential energy of the weights-earth system.

Infact, the amount of work done in carrying the body from surface of earth to its present position against the gravitational force of earth, is stored in the body in the form of its gravitational potential energy. When the body is allowed to fall from that position, it can do the amount of work equal to its gravitational potential energy. For example, gravitational potential energy of water stored to great heights in dams is used to run turbines and produce electric energy at hydroelectric power stations.

FIGURE 6.13



To calculate gravitational potential energy, suppose

m = mass of a body, g = acceleration due to gravity on the surface of earth.

h = height through which the body is raised, Fig. 6.13.

If we assume that height h is not too large and the value of g is practically constant over this height, then the force applied just to overcome gravitational attraction is

$$F = mg \qquad \textbf{...(18)}$$

As the distance moved is in the direction of the force applied, therefore,

$$\text{Work done} = \text{force} \times \text{distance}$$

$$W = F \times h = mgh$$

Note that we have taken the upward direction to be positive. Therefore, work done by applied force $= +\,mgh$. However, work done by gravitational force $= -\,mgh$.

This work gets stored as potential energy. The gravitational potential energy of a body, as a function of height (h) is denoted by $V(h)$, and it is *negative of work done by the gravitational force in raising the body to that height.*

$$\boxed{\text{Gravitational P.E.} = V(h) = mgh} \qquad \textbf{...(19)}$$

Relation between conservative force and potential energy of the system

When a force F_x acting on a body displaces it through a distance Δx in the direction of the force, work done by the force on the body is

$$\Delta W = F_x\,\Delta x$$

As gravitational P.E., $\Delta U = -\,\Delta W$

$$\therefore \qquad \Delta U = -F_x\,\Delta x \quad \text{or} \quad F_x = -\frac{\Delta U}{\Delta x}$$

When $\Delta x \to 0$,

$$F_x = \underset{\Delta x \to 0}{\text{Lt}} \left(\frac{\Delta U}{\Delta x}\right) = -\frac{\partial U}{\partial x}$$

Similarly,

$$F_y = -\frac{\partial U}{\partial y} \quad \text{and} \quad F_z = -\frac{\partial U}{\partial z}$$

Now,

$$\vec{F} = \hat{i}\,F_x + \hat{j}\,F_y + \hat{k}\,F_z$$

$$= -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k} = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)U$$

$$\boxed{\vec{F} = -\vec{\nabla}\,U}$$

where $\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$ is called the **del operator.**

Mathematically, the potential energy $V(x)$ is defined if the force $F(x)$ can be written as

$$F(x) = -\frac{dV}{dx}$$

This implies that $$\int_{x_i}^{x_f} F(x)\,dx = -\int_{V_i}^{V_f} dV = V_i - V_f \qquad ...(21)$$

i.e., **work done by a conservative force like gravity in taking the body from initial position (x_i) to final position (x_f) is equal to difference between initial and final P.E. of the body.**

When the body is released from height h, it comes down with an increasing speed. The velocity v with which the body hits the ground is calculated from the fact that the *gravitational P.E. of the body at height h manifests itself as K.E. of the body on reaching the ground, i.e.,* $\frac{1}{2}mv^2 = mgh$ or $\boxed{v = \sqrt{2gh}}$

The following points should be clearly understood regarding potential energy.

1. *Potential Energy is defined only for conservative forces. It does not exist for non conservative forces.* For example, when we move a body up a smooth inclined plane of height h, work done on the body is (mgh). This is the P.E. stored in the body. If the inclined plane were rough, work will be done against friction too in moving the body up the plane, but that part of work is not retrievable and potential energy acquired by the body is (mgh) only.

2. Potential energy depends upon frame of reference. It may be positive or negative.

By convention, in a conservative field, P.E. is equal to negative of work done in moving a body from some reference position to given position.

3. A body in motion may or may not have potential energy.

4. The formula of P.E. = mgh is infact, the gravitational potential energy of earth body system. The height h is taken above some reference level, which is defined in the given problem. If reference level is taken as surface of earth, then variation of gravitational P.E. with height is obviously, linear in nature.

By convention, gravitational P.E. of a body is said to be *positive*, when work is done *by* the body in returning to its reference position. Similarly, gravitational P.E. of the body is said to be *negative*, when work is done *on* the body to bring it back to the reference position.

5. Work done in assembling a body from infinitesimally small fragments distributed initially at infinite separations is also stored in the body itself in the form of ***(self) potential energy.***

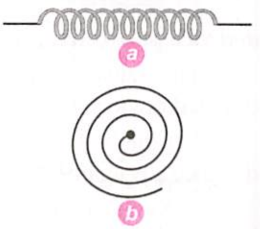
Sample Problem **How high must a body be lifted so that it gains P.E. equal to its KE while moving with a velocity of 30 m/s ? Take $g = 10\text{ m/s}^2$.**

Sol. Here, $h = ?$ $v = 30$ m/s, $g = 10\text{ m/s}^2$.

If m is mass of the body, then as P.E. = K.E. $\therefore\ mgh = \frac{1}{2}mv^2$; $h = \frac{v^2}{2g} = \frac{30\times30}{2\times10} = \mathbf{45\ m}$

6.13. POTENTIAL ENERGY OF A SPRING

FIGURE 6.14



Springs can be of many types. Important among these are *helical springs* as shown in Fig. 6.14(a) and *spiral springs*, as shown in Fig. 6.14(b).

Usually, we assume that the springs are massless. Therefore, restoring elastic force in a spring is same everywhere.

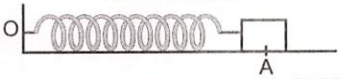
To produce compression or extension in a spring, two equal and opposite forces are to be applied on the spring. In equilibrium, restoring force developed due to elasticity of the spring is equal to applied force ; and is always opposite to the applied force.

A definite amount of work has to be done against the restoring force, both in compressing or in stretching the spring. This work done is stored in the spring in the form of *elastic potential energy of the spring.*

Thus, potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

To calculate it, consider an elastic spring *OA* of negligible mass. The end *O* of the spring is fixed to a rigid support and a body of mass *m* is attached to the free end *A*. Let the spring be oriented along *x*-axis and the body of mass *m* lie on a perfectly frictionless horizontal table, Fig. 6.15.

FIGURE 6.15



The position of the body A, when spring is unstretched is chosen as the *origin.*

When the spring is *compressed* or *elongated*, it tends to recover to its original length, on account of elasticity. The force trying to bring the spring back to its original configuration is called ***restoring force*** or ***spring force.***

For a small stretch or compression, spring obeys ***Hook's law, i.e.,*** for a spring,

Restoring Force ∝ stretch or compression

$$-F \propto x \qquad \text{or} \qquad -F = kx \qquad \text{...(22)}$$

where k is a constant of the spring and is called ***spring constant.***

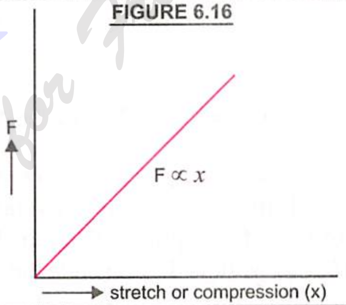
We may define spring constant as the restoring force per unit displacement of the spring, *i.e.,* $k = -\dfrac{F}{x}$.

Units of k are N/m or dyne/cm.

The dimensional formula of $k = \dfrac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$

The variation of restoring force ($-F$) with stretch or compression (x) is shown in Fig. 6.16. Clearly, greater the stretch or compression, greater will be the restoring force and vice-versa.

FIGURE 6.16



Some more information about force constant or spring constant. *From (22), k = – F/x. Therefore, force constant of a spring is equal to slope of force versus stretch curve. Its units are N/m and dimensions are* $[MT^{-2}]$. *Greater the force constant of a spring, smaller will be the stretch or compression of the spring for a given force.*

Therefore, ***a spring with greater force constant is said to be stiffer.*** *The spring with smaller force constant is soft.*

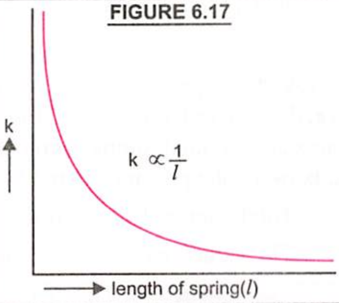
If $x = 1$, $F = -k \times 1$ *or* $k = -F$

Hence, force constant of a spring is numerically equal to the force required to produce unit displacement (compression or stretch) in the spring. The value of k depends on length of spring l, radius of the spring R and nature of material of the spring wire.

It is established that for a spring, $k \propto \dfrac{1}{l}$

i.e., smaller the length of the spring, greater will be the force constant and vice-versa. This is shown in Fig. 6.17.

FIGURE 6.17



The negative sign in eqn. (22) indicates that the restoring force is directed always towards the equilibrium position.

Let the body be displaced further through an infinitesimally small distance dx, against the restoring force.

∴ Small amount of work done in increasing the length of the spring by dx is $$dW = -F\,dx = kx\,dx \qquad \text{...(23)}$$

Total work done in giving displacement x to the body can be obtained by integrating (23) from $x = 0$ to $x = x$, *i.e.,*

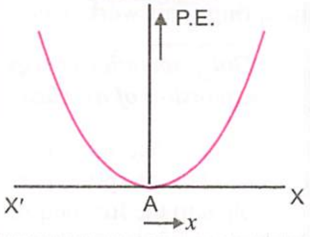
$$W = \int_{x=0}^{x=x} k\,x\,dx = k\left[\frac{x^2}{2}\right]_{x=0}^{x=x} = k\left[\frac{x^2}{2} - 0\right] = \frac{1}{2}kx^2$$

This work done is stored in the spring as *potential energy* of the spring.

$$\therefore \quad \boxed{\text{P.E.} = W = \frac{1}{2}kx^2} \qquad \text{...(24)}$$

The variation of potential energy with distance x is shown in Fig. 6.18.

FIGURE 6.18



Alternate Method. The same formula for P.E. of a spring can also be obtained *graphically*.

The variation of restoring force (F) with displacement (x) within elastic limits, is shown in Fig. 6.19.

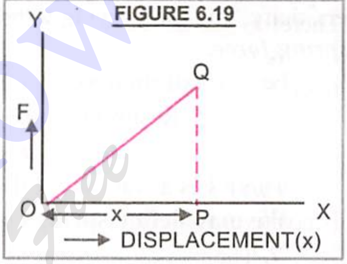
Taking the potential energy of the spring in unstretched position as zero, the potential energy of the spring having displacement x is numerically equal to area of ΔOPQ.

i.e., P.E. of spring = area of ΔOPQ

$$= \frac{\text{base (OP)} \times \text{height (PQ)}}{2} = \frac{x \times F}{2}$$

But $F = kx$ $\therefore$ P.E. of the spring at B $= \dfrac{x \times kx}{2} = \dfrac{1}{2}kx^2$

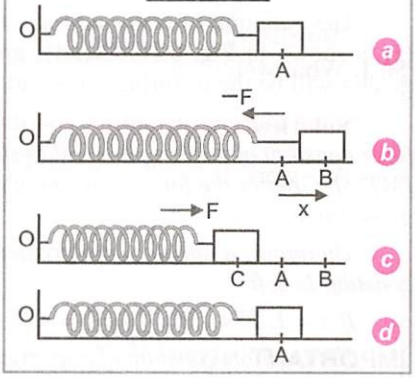
FIGURE 6.19



Regarding potential energy of a spring, the following points should be clearly understood :

1. In Fig. 6.20(a), A is normal position of the body in unstretched position of the spring. When the body is pulled from A to B, where $AB = x$, work is done against the restoring force F. Potential energy stored in the spring (at B) $= \dfrac{1}{2}kx^2$

FIGURE 6.20



2. If we were to leave the body at B after stretching the spring, it would move to A on account of potential energy stored in the spring.

At A, $x = 0$,

$\therefore$ P.E. of the spring $= \dfrac{1}{2}kx^2 = 0$

The entire P.E. of the spring is converted into K.E. of the body at A. That is why the body cannot stop at A. It goes over to the position C, where $AC = -x$, Fig. 6.20(c).

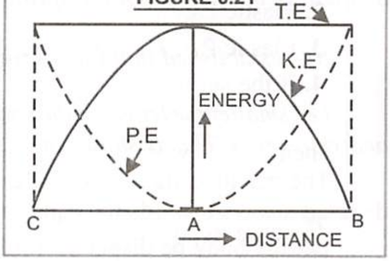
At C, the P.E. of the spring $= \dfrac{1}{2}k(-x)^2 = \dfrac{1}{2}kx^2$

Thus, at C, the K.E. of the body is converted fully into potential energy of the spring. From C, the body moves to A, when whole of P.E. stored is converted into K.E., Fig. 6.20(d). This is repeated. The body thus starts vibrating simple harmonically* about the equilibrium position A, between the positions B and C.

Total energy T.E. in all the positions remains the same.

The variation of P.E., K.E. and T.E. with distance is shown in Fig. 6.21.

FIGURE 6.21



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*For details, see Art. 14.15.

3. If v_m is the maximum speed and x_m is the maximum displacement, then

$$\frac{1}{2} m v_m^2 = \frac{1}{2} k x_m^2$$

$$\boxed{v_m = \sqrt{\frac{k}{m}} . x_m} \qquad \text{...(25)}$$

As k/m has the dimensions of $[T^{-2}]$, therefore, this equation is dimensionally correct.

4. Work done by the external pulling force is positive, as it overcomes the spring force. However, work done by the spring force or restoring force is equally negative.

5. We have said above that work done by the spring force in a cyclic process is zero. It is position dependent ($F = -kx$). The spring force does work which depends only on the initial and final positions. Therefore, the *spring force is a conservative force.*

6. In Fig. 6.20, we find that (*i*) When the displacement x from the equilibrium position A is zero, restoring force F = 0, Fig. 6.20(*a*) and (*d*).

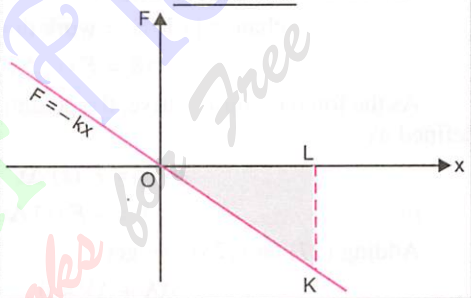
(*ii*) For the stretched spring $x >> 0$, $F << 0$, *i.e.,* force is negative, Fig. 6.20(*b*).

(*iii*) For the compressed spring $x << 0$, $F >> 0$, *i.e.,* force is positive, Fig. 6.20(*c*).

7. The plot of restoring force F versus displacement x is represented in Fig. 6.22. The area of the shaded triangle OKL represents the work done by the force. Because of opposing signs of F and x, this work done is negative,

i.e., $$W = \frac{-kx^2}{2}$$

FIGURE 6.22



Sample Problem **The potential energy of a spring, when stretched through a distance x is 50 J. What would be the work done in stretching it further through the same distance ?**

Sol. Here, $x_1 = x$, $E_1 = \frac{1}{2} kx^2 = 50$ J, $W = ?$ $x_2 = x + x = 2x$

As work done = increase in P.E. of the spring

$$\therefore \qquad W = E_2 - E_1 = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2 = \frac{1}{2} k[(2x)^2 - x^2]$$

$$W = 3 \times \frac{1}{2} kx^2 = 3 \times 50 = \mathbf{150\ J}$$

IMPORTANT NOTES

1. Elastic P.E. of a spring in relaxed position is zero.

2. Elastic P.E. of a spring is always positive, being proportional to square of displacement.

3. Elastic P.E. appears when the spring is stretched or compressed.

4. If the spring is already stretched by a distance x_1, and then it is stretched further to a distance x_2,

then $$\text{work done, } \Delta W = \int_{x_1}^{x_2} kx\,dx = \frac{1}{2} k(x_2^2 - x_1^2).$$

Increase in potential energy of already stretched spring,

$$\Delta V = -\Delta W = -\frac{1}{2} k(x_2^2 - x_1^2).$$

6.14. MECHANICAL ENERGY AND ITS CONSERVATION

The mechanical energy (E) of a body is the sum of kinetic energy (K) and potential energy (V) of the body

i.e.,
$$\boxed{E = K + V} \qquad \text{...(26)}$$

Obviously, mechanical energy of a body is a scalar quantity measured in joule.

We can show that the total mechanical energy of a system is conserved if the forces, doing work on the system are conservative. (or when net work done by external non-conservative forces is zero).

This is called the principle of conservation of total mechanical energy.

For simplicity, we assume the motion to be one dimensional only. Suppose a body undergoes a small displacement Δx under the action of a conservative force F. According to work energy theorem,

change in K.E. = work done

$$\Delta K = F(x)\,\Delta x \qquad \text{...(27)}$$

As the force is conservative, the potential energy function V (x) is defined as

$$-\Delta V = F(x)\,\Delta x$$

or
$$\Delta V = -F(x)\,\Delta x \qquad \text{...(28)}$$

Adding (27) and (28), we get

$$\Delta K + \Delta V = 0$$

or
$$\Delta(K + V) = 0, \text{ which means}$$

$$\boxed{(K + V) = E = \text{constant}} \qquad \text{...(29)}$$

i.e., sum of the kinetic energy and potential energy of the body is a constant over the whole path, x_i to x_f. It implies that

$$K_i + V(x_i) = K_f + V(x_f) \quad \text{or} \quad K + V(x) = \text{constant}$$

i.e., total mechanical energy, E = constant

It means that individually, kinetic energy K and the potential energy $V(x)$ may vary from point to point, but their sum is constant throughout.

The following points should be clearly understood as regards mechanical energy E.

1. Value of mechanical energy of a body depends upon frame of reference.

2. In certain cases, mechanical energy may only be potential energy. In certain other cases, mechanical energy may only be kinetic energy. When $K = 0$, $V = 0$, only then $E = 0$. However, when $E = 0$, it is not necessary that $K = 0$ and $V = 0$. Infact, K is always positive. When V is equally negative, $E = 0$.

3. As $E - V = K$ and K is always positive, therefore, for a system to exist, $E - V \geq 0$ or $E \geq V$.

4. Negative value of mechanical energy indicates a *bound state, e.g.,* electron in an atom; and a satellite revolving around a planet, both are in bound state having negative mechanical energy.

DO YOU KNOW ?

1. The earth's crust is not uniform. It has discontinuities and dislocations that are called **fault lines.** These fault lines in the earth's crust are like **'compressed springs'** and possess a large amount of potential energy. When these fault lines readjust, an **earthquake results.** The stored potential energy is released in the form of kinetic energy.

2. The energy associated with state of separation of charged particles that interact electrically is called **electric potential energy.**

Electric Potential Energy (V) of two point charges q_1 and q_2 separated by a distance r in vacuum/air is $V = \dfrac{q_1 q_2}{4\pi\epsilon_0 r}$

where $\dfrac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{Nm}^2\,\text{C}^{-2}$,

ϵ_0 is called absolute electrical permittivity of free space (vacuum/air).

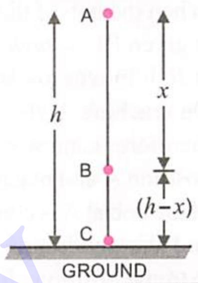
DO YOU KNOW ?

Mechanical energy of a body falling freely in air towards earth, is **NOT conserved.** This is because a non-conservative force (*i.e.*, resistive force due to air) acts on the body in addition to a conservative force (*i.e.*, gravitational force).

6.15. ILLUSTRATION OF THE LAW OF CONSERVATION OF MECHANICAL ENERGY

To illustrate the law further, let us calculate kinetic energy K.E., potential energy P.E. and total energy T.E. of a body falling freely under gravity.

FIGURE 6.23



Let m be the mass of the body held at A, at a height h above the ground, Fig. 6.23.

As the body is at rest at A, therefore,

At A : K.E. of the body = 0

P.E. of the body = mgh, where g is acceleration due to gravity at A.

T.E. of the body = K.E. + P.E. = $0 + mgh$

$$E_1 = mgh \qquad ...(30)$$

Let the body be allowed to fall freely under gravity, when it strikes the ground at C with a velocity v.

From $v^2 - u^2 = 2\,a\,s$

$$v^2 - 0 = 2\,(g)\,h$$

$$v^2 = 2\,gh \qquad ...(31)$$

$\therefore$ **At C :** K.E. of the body $= \frac{1}{2} m v^2 = \frac{1}{2} m (2\,gh) = mgh$

P.E. of the body = $mgh = mg\,(0) = 0$

Total energy of the body = K.E. + P.E.

$$E_2 = mgh + 0 = mgh \qquad ...(32)$$

In free fall, let the body cross any point B with a velocity v_1, where $AB = x$

From $v^2 - u^2 = 2\,a\,s$

$$v_1^2 - 0 = 2\,(g)\,x \qquad ...(33)$$

$$v_1^2 = 2\,gx$$

At B : K.E. of the body $= \frac{1}{2} m v_1^2 = \frac{1}{2} m (2gx) = mgx$

Height of the body at B above the ground = CB = $(h - x)$ $\therefore$ P.E. of the body at B = $mg\,(h - x)$

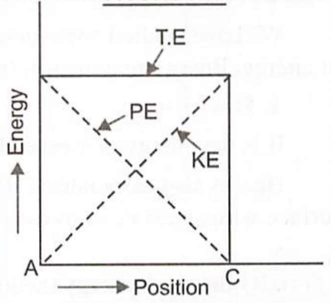
Total energy of the body at B = K.E. + P.E.

$$E_3 = mgx + mg\,(h - x) = mgx + mgh - mgx = mgh \qquad ...(34)$$

From (30), (32), (34), we find that $\boxed{E_1 = E_2 = E_3 = mgh}$

i.e., total energy of the body during free fall remains constant at all positions. The form of energy, however keeps on changing. At A, energy is entirely potential energy and at C, it is entirely K.E. In between A and C, energy is partially potential and partially kinetic. This variation of energy is shown in Fig. 6.24. Total mechanical energy stays constant at mgh, throughout.

FIGURE 6.24



Thus, in an isolated system, where only conservative forces cause energy changes, the kinetic energy and potential energy may change, but the mechanical energy of the system (which is sum of K.E. and P.E.) cannot change.

We can, therefore, equate the sum of K.E. and P.E. at one instant to the sum of K.E. and P.E. at another instant – without considering intermediate states.

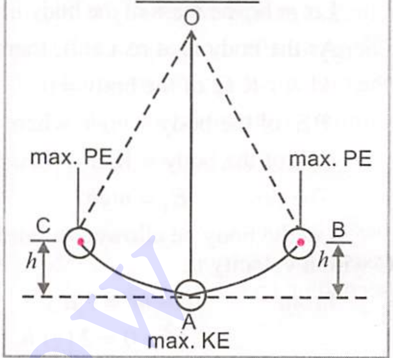
This law has been found to be valid in every situation. No violation, whatsoever, of this law has ever been observed. Hence, *what remains constant is the mechanical energy of the system.*

Some other examples of conservation of mechanical energy are :

1. Vibrations of a simple pendulum

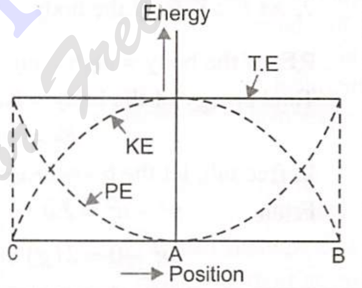
In Fig. 6.25, OA is normal position of rest of a simple pendulum. When the bob of the pendulum is displaced to B, through a height h, it is given P.E. = mgh, where m is mass of the bob. On releasing the bob at B, it moves towards A. P.E. of the bob is being converted into K.E. On reaching A, the entire P.E. has been converted into K.E. The bob, therefore, cannot stop at A. On account of inertia, it overshoots the position A and reaches C at the same height h above A. The entire K.E. of the bob at A is converted into P.E. at C. The whole process is repeated and the pendulum vibrates about the equilibrium position OA. At extreme positions B and C, the bob is momentarily at rest. Therefore, its K.E. = 0. The entire energy at B and C is potential energy. At A, there is no height and hence no potential energy. The entire energy at A is kinetic energy.

FIGURE 6.25



The variation of energy with position of the bob is shown in Fig. 6.26.

FIGURE 6.26

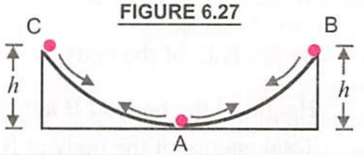


2. Motion of a small spherical ball over a watch glass

Fig. 6.27 shows a watch glass or a large concave mirror embedded in clay. Consider a tiny spherical ball placed at the edge B at a height h above the centre A. If m is mass of the ball, P.E. of the ball held at $B = mgh$.

When the ball is released at C, it starts rolling down the mirror. P.E. of ball is being converted into K.E. At the bottom A, velocity of the ball is maximum as the entire P.E. has been converted into K.E. The ball cannot stop at A on account of inertia. It goes over to the other edge B. The velocity of the ball goes on decreasing and so does its K.E. At C, K.E. = 0, and P.E. is maximum. The entire process is repeated, and thus the ball keeps on rolling over the mirror about A.

FIGURE 6.27



3. Vibrations of an elastic spring

As discussed already in Art. 6.13, total energy is conserved in case of an elastic spring.

6.16. DIFFERENT FORMS OF ENERGY

We have studied some details of potential energy and kinetic energy. These are not the only two forms of energy. Energy may manifest itself in several other forms. Some of the examples are:

1. Heat Energy

It is the energy possessed by a body by virtue of random motion of the molecules of the body.

Heat is also associated with the force of friction. When a block of mass m sliding on a rough horizontal surface with speed v, stops over a distance x, work done by the force of kinetic friction f over a distance x is $-f(x)$. By the work energy theorem, $\frac{1}{2}mv^2 = f(x)$. We often say that K.E. of the block is **lost** due to frictional force. However, when we examine the block and the horizontal surface carefully, we detect a slight increase in their temperatures. Thus, work done by friction is **not lost,** but it is transferred as **heat energy** of the system.

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2. Internal Energy

It is the total energy possessed by the body by virtue of particular configuration of its molecules and also their random motion. Thus, internal energy of a body is the sum of potential energy and kinetic energy of the molecules of the body. P.E. is due to configuration of the molecules against intermolecular forces and K.E. is due to random motion of the molecules.

DO YOU KNOW ?

Internal energy of a body is sometimes called the *microscopic mechanical energy* of the body.

3. Electrical Energy

The flow of electric current causes bulbs to glow, fans to rotate and bells to ring. A definite amount of work has to be done in moving the free charge carriers in a particular direction through all the electrical appliances. This causes the expenditure of electrical energy. An urban Indian household consumes about 200 joule of energy per second on an average.

4. Chemical Energy

Chemical energy arises from the fact that the molecules participating in the chemical reaction have different binding energies. A stable chemical compound has less energy than the separated parts. A chemical reaction is basically a rearrangement of atoms. For example, coal consists of carbon and a kilogram of it, when burnt releases 3×10^7 J of energy. Chemical energy is associated with the forces that give rise to the stability of the substances. These forces bind atoms into molecules, molecules into polymeric chains etc. The chemical energy obtainable from the combustion of coal, cooking gas, wood and petroleum have become indispensable for us.

5. Nuclear Energy

It is the energy obtainable from an atomic nucleus. Two distinct modes of obtaining nuclear energy are (*i*) Nuclear fission (*ii*) Nuclear fusion.

Nuclear fission involves splitting of a heavy nucleus into two or more lighter nuclei , whereas nuclear fusion involves fusing of two or more lighter nuclei to form a heavy nucleus.

In both, the nuclear fission as well as nuclear fusion, a certain mass disappears, which appears in the form of nuclear energy in accordance with Einstein mass energy relation, $E = mc^2$. For details, see Art. 6.17.

Further, the explanation of huge amounts of energy released by the sun is also based on Einstein mass energy relation. Four light hydrogen nuclei fuse to form a helium nucleus, whose mass is somewhat less than the sum of the masses of the four hydrogen nuclei. This mass difference (Δm) is called the **mass defect.** It is the source of solar energy = $(\Delta m)\, c^2$.

Table 6.4 gives approximate energy associated with some of the important phenomena.

TABLE 6.4. Energy associated with some important phenomena

S. No.	Phenomenon	Energy (J)
1.	Energy required to break one bond in DNA	$\approx 10^{-20}$
2.	Energy spent in turning a page	$\approx 10^{-3}$
3.	Work done by each human heart beat	$\approx 0{\cdot}5$
4.	Daily food intake of a human adult	$\approx 10^7$
5.	Energy released in burning 1 litre of gasoline	$\approx 3 \times 10^7$
6.	K.E. of a jet aircraft	$\approx 10^9$
7.	Energy released in burning 1000 kg of coal	$\approx 3 \times 10^{10}$
8.	Energy release of 15 megaton fusion bomb	$\approx 10^{17}$
9.	Annual solar energy incident on earth	$\approx 5 \times 10^{24}$
10.	Rotational energy of earth	$\approx 10^{29}$
11.	Energy released in a supernova explosion	10^{44}
12.	Rotational energy of Milky way	$\approx 10^{52}$
13.	Big Bang	$\approx 10^{68}$

6.17. MASS ENERGY EQUIVALENCE

In Classical (old) Physics, mass and energy are two separate physical quantities. In an isolated system, mass is constant, and energy too remains constant. (Science of chemistry was developed on the assumption that in all chemical reactions, mass and energy are conserved separately). In the year 1905, Einstein made an incredible discovery that *energy can be transformed into mass and mass can be transformed into energy.* One can be obtained at the cost of the other. The mass energy equivalence relation as put forth by Einstein is

$$\boxed{E = mc^2} \qquad \text{...(35)}$$

where m = mass that disappears, E = energy that appears, c = velocity of light in vacuum.

This is when mass is being converted into energy. Conversely, when an amount of energy E is converted into mass, the mass that appears is $m = E/c^2$

Thus, according to (modern) Quantum Physics, ***mass and energy are not conserved separately, but are conserved as a single entity called 'mass-energy'. E is the energy that the object possesses even when it is at rest, simply because it has mass.***

Thus, the law of conservation of mass and law of conservation of energy have been unified by this relation into a single law of conservation of mass energy. This relation is of great importance in Physics. It has solved many hitherto unsolved problems. For example, the origin of solar energy, stellar energy, nuclear energy from fission, fusion ; binding energy of nuclei etc. have been traced using this relation.

As speed of light in vacuum, $c = 3 \times 10^8$ m/s is very large, a staggering amount of energy is associated with a mere one kilogram of matter. Thus, for $m = 1$ kg, $E = mc^2 = 1\,(3 \times 10^8)^2 = 9 \times 10^{16}$ J

This is equivalent to the annual electrical output of a large (3000 MW) power generating station.

The energy equivalent of some typical objects are given in Table 6.5.

TABLE 6.5. Energy Equivalent of some objects

S. No.	Object	Rest Mass (kg)	Energy Equivalent (E)
1.	Electron	$9{\cdot}11 \times 10^{-31}$	$8{\cdot}19 \times 10^{-14}$ J $\simeq$ 511 keV
2.	Proton	$1{\cdot}67 \times 10^{-27}$	$1{\cdot}50 \times 10^{-10}$ J $\simeq$ 938 MeV
3.	Uranium atom	$3{\cdot}95 \times 10^{-25}$	$3{\cdot}55 \times 10^{-8}$ J $\simeq$ 225 GeV
4.	Dust Particle	1×10^{-13}	1×10^{4} J $\simeq$ 2 kcal

Illustrations of inter conversion of mass and energy

(*a*) Annhilation of matter. Under suitable conditions, an electron (e^-) and a positron (e^+)* annhilate (or destroy) each other resulting in the release of two γ-rays. Thus mass is converted into energy according to Einstein equation, $E = mc^2$. This process is represented as $\boxed{e^- + e^+ = \gamma + \gamma}$

As is known,

mass of electron = mass of positron = $9{\cdot}1 \times 10^{-31}$ kg

Total mass of e^- and e^+ , $m = 2 \times 9{\cdot}1 \times 10^{-31}$ kg

According to Einstein equation, energy released

$$E = mc^2 = 2 \times 9{\cdot}1 \times 10^{-31}\,(3 \times 10^8)^2 \text{ J}$$

$$= 1{\cdot}638 \times 10^{-13} \text{ J} = \frac{1{\cdot}638 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}} \text{ MeV} = 1{\cdot}02 \text{ MeV}$$

$$\text{Energy of each photon } = \frac{1{\cdot}02}{2} \text{ MeV } = \mathbf{0{\cdot}51 \text{ MeV}}$$

---

*A positron (e^+) is a particle whose mass is same as that of electron, but it carries unit positive charge instead of an equal negative charge on electron. Positron is also called **antiparticle** of electron.

(*b*) **Pair Production.** It is the reverse of the phenomenon of annhilation of matter. In this case, a γ-ray photon of energy 1·02 MeV interacts with a nucleus under suitable conditions and gives rise to an electron (e^-) and a positron (e^+) pair of particles. Thus energy is converted into mass according to Einstein equation.

$$E = mc^2$$

or
$$m = \frac{E}{c^2} = \frac{1{\cdot}02\,\text{MeV}}{(3\times10^8\ \text{ms}^{-1})^2} = \frac{1{\cdot}02\times1{\cdot}6\times10^{-13}\ \text{J}}{9\times10^{16}\ (\text{ms}^{-1})^2} = 18{\cdot}13\times10^{-31}\ \text{kg}$$

This is the combined mass of electron and positron.

$$\therefore \text{ Mass of each particle } = \frac{18{\cdot}13\times10^{-31}}{2}\ \text{kg} = 9{\cdot}065\times10^{-31}\ \text{kg}$$

This is the rest mass of an electron/positron.

Variation of Mass with velocity

According to Einstein, the relativistic variation of mass with velocity is given by the relation

$$\boxed{m = \frac{m_0}{\sqrt{1 - v^2/c^2}}}$$

where m_0 = rest mass of the particle, *i.e.*, mass of particle when at rest

m = effective mass or relativistic mass of the particle moving with velocity v

c = velocity of light in vacuum

The expression indicates that mass of particle/body increases with increase in its velocity. When $v = c$, $m = \infty$, *i.e.*, when velocity of particle/body becomes equal to velocity of light in vacuum/air, its mass becomes infinite.

Sample Problem 1 **In a nuclear reaction, the mass defect is 1 atomic mass unit. What is the energy released ?**

Sol. Here, $\Delta m = 1\ amu = 1{\cdot}66\times10^{-27}$ kg ; $E = ?$

$$E = (\Delta m)\,c^2 = 1{\cdot}66\times10^{-27}\times(3\times10^8)^2 = \mathbf{1{\cdot}494\times10^{-10}\ J}$$

Sample Problem 2 **If 2000 kg of water is heated from 0°C to 100°C, what will be the corresponding increase in mass of water ? Take specific heat of water = 1 cal. $g^{-1}\ {}^\circ C^{-1}$.**

Sol. Here, $m = 2000\ \text{kg} = 2\times10^6$ g, $\Delta T = 100 - 0 = 100$°C, $c = 1$ cal. $\text{g}^{-1}\,{}^\circ\text{C}^{-1}$.

$\therefore$ Heat energy supplied/gained by water, $\Delta Q = c.m\,(\Delta T) = 1\times2\times10^6\times100 = 2\times10^8$ cals

or $\Delta Q = E = 2\times10^8\times4{\cdot}2\ \text{J} = 8{\cdot}4\times10^8$ J.

Hence, increase in mass, $$\Delta m = \frac{E}{c^2} = \frac{8{\cdot}4\times10^8}{(3\times10^8)^2} = \mathbf{9{\cdot}3\times10^{-9}\ kg}$$

6.18. TRANSFORMATION OF ENERGY

It is the phenomenon of change of energy from one form to the other. We come across such changes in day-to-day life. For example :

(*i*) In an electric bulb, electric energy is converted into light energy and heat energy,

(*ii*) In an electric iron, electric heater, geyser etc., electric energy is converted into heat energy.

(*iii*) In an electric fan, electric motor, electric energy is converted into mechanical energy.

(*iv*) In a hydroelectric power station, potential energy of water is converted ultimately into electric energy.

(*v*) In a heat engine, chemical energy from coal/oil is converted into mechanical energy.

(*vi*) In a nuclear reactor, mass is being converted into energy.

(*vii*) In the sun and other stars, mass is being converted into energy, and so on.

(*viii*) The explosion of an atom bomb gives out heat and light as a result of transformation of atomic energy.

6.19. THE PRINCIPLE OF CONSERVATION OF ENERGY

In Art. 6.14, we have established the principle of conservation of mechanical energy, *i.e.*, total mechanical energy of a system is conserved if the forces doing work on the system are conservative.

When some of the forces involved are non-conservative, a part of mechanical energy may be changed into other forms of energy such as heat, light and sound energy. Therefore, according to the principle of conservation of energy,

> ***If we account for all forms of energy, the total energy of an isolated system does not change.***

Thus, energy may be transformed from one form to another, but the total energy of an isolated system shall remain constant.

As the universe as a whole can be treated as an isolated system, the sum total of energy of all kinds in the universe remains constant at all times. Thus, energy can neither be created nor be destroyed. If one part of the universe loses energy, some other part of the universe would gain an equal amount of energy.

The principle of conservation of energy cannot be proved as such. However, no violation of this principle has ever been observed.

The concept of transformation of energy into various forms and its ultimate conservation is of vital importance in all branches of science, including engineering and industry.

Sample Problem **A rubber ball falls on a floor from a height of 19·6 m. Calculate the velocity with which it strikes the ground. To what height will the ball rebounce if it loses 25% of its energy on striking the ground ?**

Sol. Here, $h_1 = 19{\cdot}6$ m, $v_1 = ?$ Loss of energy = 25% $h_2 = ?$

On striking the ground, K.E. = P.E.

$$\frac{1}{2} m v_1^2 = m g h_1$$

$$v_1 = \sqrt{2 g h_1} = \sqrt{2 \times 9{\cdot}8 \times 19{\cdot}6} = \mathbf{19{\cdot}6\ m/s}$$

As the body loses 25% of its energy, final energy $E_2 = \frac{75}{100}$ of initial energy

i.e., $$m g h_2 = \frac{75}{100}(m g h_1) \qquad h_2 = \frac{3}{4} h_1 = \frac{3}{4} \times 19{\cdot}6 = \mathbf{14{\cdot}7\ m}$$

6.20. COLLISIONS

In common language, a collision is said to occur when objects crash into eachother. For example, collision between automobiles, a hammer striking a nail, collision between billiard balls, carrom coins etc.

Collisions range from the microscopic scale of subatomic particles to astronomical scale of colliding stars and colliding galaxies. Often, the collisions are too brief to be visible, though they involve significant distortion of the colliding bodies.

In general, we define

> ***a collision as an isolated event in which two or more colliding bodies exert relatively strong forces on eachother for a relatively short time.***

Thus, actual physical contact between two bodies is not necessary for a collision. For example, an alpha particle speeding towards nucleus of an atom gets deflected by the electrostatic force of repulsion without actual physical contact with the nucleus. The alpha particle is said to have undergone collision with the nucleus.

The major object of the study of collisions is to find out as much as possible, about the forces that act during a collision from the knowledge of the state of particles before and after the collision. Infact, all our understanding of the subatomic world–electrons, protons, neutrons, muons, quarks etc. comes from experiments involving collisions. **Two key rules of the collision game are :**

(*i*) Law of conservation of linear momentum, and (*ii*) Law of conservation of energy.

Types of collision

Collisions between particles have been divided broadly into two types :

1. *Elastic collisions*
2. *Inelastic collisions*

(*a*) Elastic collisions

A collision in which there is absolutely no loss of kinetic energy is called an elastic collision.

For example, collision between atomic and sub atomic particles are elastic collisions. Practically, collision between two ivory balls can also be taken as an elastic collision.

The **basic characteristics of an elastic collision are :**

(*i*) *The linear momentum is conserved,*

(*ii*) *Total energy of the system is conserved,*

(*iii*) *The kinetic energy is conserved.*

Obviously, the kinetic energy in such collisions is not converted into any other form of energy like heat energy, sound energy etc.

(*iv*) The forces involved during elastic collisions must be conservative forces.

(*b*) Inelastic collisions

A collision in which there occurs some loss of kinetic energy is called an inelastic collision.

As there is always some loss of kinetic energy in most of the collisions, therefore, collisions we come across in daily life are *generally inelastic*.

If two bodies stick to each other, after colliding, the collision is said to be perfectly inelastic. For example, mud thrown on a wall sticks to the wall. The loss of K.E. of mud is complete. The collision is perfectly inelastic.

Similarly, when an arrow gets stuck in a target and the two move together, the collision is **perfectly inelastic.** Likewise, if a meteorite collides head on with earth, it often gets buried in earth. The collision is **perfectly inelastic.**

The ***basic characteristics of an inelastic collision*** are :

(*i*) *The linear momentum is conserved.*

(*ii*) *Total energy is conserved.*

(*iii*) *Kinetic energy is NOT conserved.* Obviously, a part of kinetic energy is converted into some other form of energy, *e.g.*, heat energy, sound energy etc.

(*iv*) *Some or all the forces involved in an inelastic collision may be non-conservative in nature.*

For example. When we drop a rubber ball onto a hard floor, and the collision between the ball and the floor is elastic, the ball would lose no kinetic energy because of the collision, and would rebound to its original height. However, the actual rebound height may be somewhat shorter, showing some loss of kinetic energy in the collision. Therefore, the collision is somewhat inelastic. However, if we neglect the small loss of K.E., we may approximate the collision as elastic.

When we drop a golf ball on the ground, it rebounds to nearly 60% of its original height. Therefore, this collision cannot be approximated as elastic. It is an inelastic collision. Again, when we drop a ball of wet putty on to the floor, it sticks to the floor and does not rebound at all. As the putty sticks, the collision is a perfectly inelastic collision.

DO YOU KNOW ?

We normally think of 'elastic' as referreing to something that always returns to or keeps its original shape. Thus, in an elastic collision, objects maintain their original shapes and are not deformed by the action of forces. On the other hand, objects in an inelastic collision are deformed during the collision and lose some kinetic energy in the form of heat energy, sound energy, internal elastic potential energy etc.

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The main characteristics of collisions as discussed above are summarised in **Table 6.6.**

TABLE 6.6. Characteristics of Collisions

S. No.	Type of Collision	Linear Momentum	Total Energy	Kinetic Energy
1.	Perfectly elastic collision	Conserved	Conserved	Conserved
2.	Inelastic collision	Conserved	Conserved	Not-conserved
3.	Perfectly Inelastic collision	Conserved	Conserved	Not-conserved

Retain in Memory

It should be clearly understood that total energy and linear momentum, both are conserved in elastic as well as inelastic collisions. However, K.E. is not conserved in an inelastic collision. In an elastic collision, K.E. is also conserved.

6.21. COEFFICIENT OF RESTITUTION OR COEFFICIENT OF RESILIENCE (e)

In actual practice, collisions between all real objects are neither perfectly elastic nor they are perfectly inelastic. They are called *imperfect* or *semi elastic collisions.*

The degree of elasticity of a collision is determined by a quantity, called coefficient of restitution or coefficient of resilience.

Coefficient of restitution **or** ***coefficient of resilience of a collision is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision.***

It is represented by 'e'.

$$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of approach (before collision)}}$$

If u_1, u_2 are velocities of two bodies before collision, and v_1, v_2 are their respective velocities after collision, then

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}} \qquad \text{...(36)}$$

For a perfectly elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision

$$\therefore \quad \boxed{e = 1} \qquad \text{...(37)}$$

For a perfectly inelastic collision, rel. vel. of separation after collision = 0, as the two bodies, after collision, move together

$$\therefore \quad \boxed{e = 0} \qquad \text{...(38)}$$

For all other collisions, e lies between 0 and 1, *i.e.,* $\boxed{0 < e < 1.}$

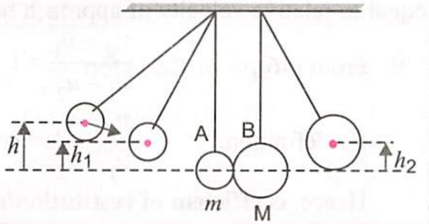
Note that we shall discuss about collisions in systems that are **closed,** *i.e.,* no mass enters or leaves the system. Further, the systems are **isolated,** *i.e.,* no net external forces act on the bodies within the system. Infact, external forces, *e.g.,* gravity and friction are negligibly small compared to the strong impulsive forces during collision (which are internal forces).

Determination of e

Let us take two spheres A and B of masses m and M ($>> m$) respectively suspended as simple pendulums from a common support. In the normal position of rest, the two spheres are in light contact with each other as shown in Fig. 6.28.

Let the sphere A be displaced to a height h and released from there. The velocity acquired by sphere A just before striking sphere B is $u_1 = \sqrt{2\,gh}$

FIGURE 6.28



As sphere B is initially at rest, $u_2 = 0$.

On collision, let B rise to a height h_2 and A recoil back to a height h_1.

$$\therefore \quad v_1 = \sqrt{2gh_1} \quad \text{and} \quad v_2 = \sqrt{2gh_2}$$

$$\text{As} \quad e = \frac{v_2 - v_1}{u_1 - u_2} \quad \therefore \; e = \frac{\sqrt{2gh_2} - \sqrt{2gh_1}}{\sqrt{2gh} - 0} = \frac{\sqrt{h_2} - \sqrt{h_1}}{\sqrt{h}}$$

Knowing h, h_1 and h_2, we can calculate the coefficient of restitution of the collision.

6.22. ELASTIC COLLISION IN ONE DIMENSION

It involves two bodies moving initially along the same straight line, striking against each other without loss of kinetic energy and continuing to move along the same straight line after collision.

Suppose two balls A and B of masses m_1 and m_2 are moving initially along the same straight line with velocities u_1 and u_2 respectively, Fig. 6.29(*a*).

When $u_1 > u_2$, relative velocity of approach before collision,

$$= u_1 - u_2 \qquad \text{...(39)}$$

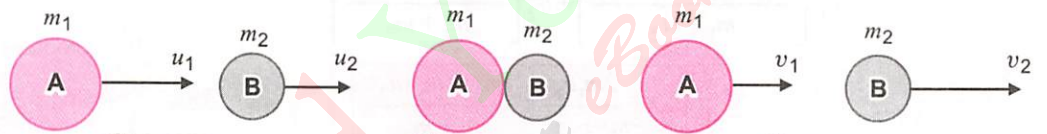
Therefore, the two balls collide, Fig. 6.29(*b*). Let the collision be perfectly elastic. After collision, suppose v_1 is velocity of A and v_2 is velocity of B along the same straight line, Fig. 6.29(*c*). When $v_2 > v_1$, the bodies separate after collision.

Relative velocity of separation after collision $= v_2 - v_1$ ...(40)

Linear momentum of the two balls before collision $= m_1 u_1 + m_2 u_2$

Linear momentum of the two balls after collision $= m_1 v_1 + m_2 v_2$

FIGURE 6.29



(a) BEFORE COLLISION (b) DURING COLLISION (c) AFTER COLLISION

As linear momentum is conserved in an elastic collision, therefore

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \qquad \text{...(41)}$$

$$\text{or} \qquad m_2 (v_2 - u_2) = m_1 (u_1 - v_1) \qquad \text{...(42)}$$

$$\text{Total K.E. of the two balls before collision} = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \qquad \text{...(43)}$$

$$\text{Total K.E. of the two balls after collision} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad \text{...(44)}$$

As K.E. is also conserved in an elastic collision, therefore from (43) & (44),

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \quad \text{or} \quad \frac{1}{2} m_2 (v_2^2 - u_2^2) = \frac{1}{2} m_1 (u_1^2 - v_1^2)$$

$$\text{or} \qquad m_2 (v_2^2 - u_2^2) = m_1 (u_1^2 - v_1^2) \qquad \text{...(45)}$$

Dividing (45) by (42), we get

$$\frac{m_2 (v_2^2 - u_2^2)}{m_2 (v_2 - u_2)} = \frac{m_1 (u_1^2 - v_1^2)}{m_1 (u_1 - v_1)} \quad \text{or} \quad \frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)} = \frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)}$$

$$\text{or} \qquad v_2 + u_2 = u_1 + v_1 \quad \text{or} \quad \boxed{v_2 - v_1 = u_1 - u_2} \qquad \text{...(46)}$$

Hence, in one dimensional perfectly elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

From (46), $$\frac{v_2 - v_1}{u_1 - u_2} = 1$$

By definition, $$\frac{v_2 - v_1}{u_1 - u_2} = e = 1$$

Hence, **coefficient of restitution/resilience of a perfectly elastic collision in one dimension is unity.**

Calculation of velocities after collision :

Velocity of A :

From (46), $$v_2 = u_1 - u_2 + v_1 \qquad ...(47)$$

Putting in (41), we get $$m_1 v_1 + m_2 (u_1 - u_2 + v_1) = m_1 u_1 + m_2 u_2$$

$$m_1 v_1 + m_2 u_1 - m_2 u_2 + m_2 v_1 = m_1 u_1 + m_2 u_2$$

$$v_1 (m_1 + m_2) = (m_1 - m_2) u_1 + 2 m_2 u_2$$

$$\boxed{v_1 = \frac{(m_1 - m_2) u_1}{m_1 + m_2} + \frac{2 m_2 u_2}{m_1 + m_2}} \qquad ...(48)$$

Velocity of B :

Put this value of v_1 from (48) in (47)

$$v_2 = u_1 - u_2 + \frac{(m_1 - m_2) u_1}{m_1 + m_2} + \frac{2 m_2 u_2}{m_1 + m_2} = u_1 \left[1 + \frac{m_1 - m_2}{m_1 + m_2}\right] + u_2 \left[\frac{2m_2}{m_1 + m_2} - 1\right]$$

$$= u_1 \left[\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right] + u_2 \left[\frac{2m_2 - m_1 - m_2}{m_1 + m_2}\right]$$

$$\boxed{v_2 = \frac{2 m_1 u_1}{m_1 + m_2} + \frac{(m_2 - m_1) u_2}{m_1 + m_2}} \qquad ...(49)$$

Retain in Memory

The expression for v_2 can be obtained from (48), the expression for v_1, by replacing m_1 by m_2 and u_1 by u_2. The reverse is also true, *i.e.*, (48) can also be obtained from (49), in the same way

Particular cases :

1. **When masses of two bodies are equal,** *i.e.*, $m_1 = m_2 = m$, say

From (48), $v_1 = \dfrac{2 m u_2}{2 m} = u_2$, *i.e.*, velocity of A after collision = velocity of B before collision.

From (49), $v_2 = \dfrac{2 m_1 u_1}{2 m_1} = u_1$, *i.e.*, velocity of B after collision = velocity of A before collision.

Hence, when two bodies of equal masses undergo perfectly elastic collision in one dimension, their velocities are just interchanged. This is shown in Fig. 6.31 (*a*).

This result has an important application in a nuclear reactor. Fast moving neutrons in a nuclear reactor are slowed down by making them collide against the nuclei of a *moderator*. As nuclei of a moderator, *i.e.*, hydrogen atoms (*i.e.*, protons) have roughly the same mass as that of neutron, on collision, fast moving neutrons are slowed down transferring their energy to protons. Therefore, hydrogenic materials (*e.g.*, heavy water, paraffin etc.) are best suited as moderators in nuclear reactors.

2. **When the target body B is initially at rest,** *i.e.,* $u_2 = 0$

From (48), $$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \qquad ...(50)$$

From (49), $$v_2 = \frac{2m_1 u_1}{m_1 + m_2} \qquad ...(51)$$

Three cases arise further : (*a*) **When masses of two bodies are equal,** *i.e.,* $m_1 = m_2$

From (50), $v_1 = 0$

From (51), $$v_2 = \frac{2m_1 u_1}{2m_1} = u_1$$

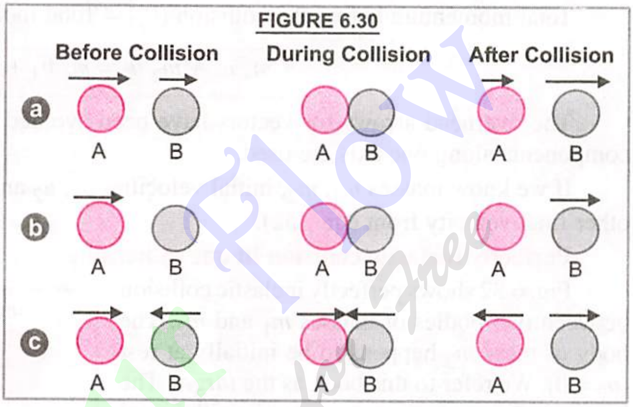


FIGURE 6.30

i.e., body A comes to rest and body B starts moving with the initial velocity of A. Obviously, in such a collision, **100 % K.E. of A is transferred to the body B.** This is shown in Fig. 6.30(*b*).

(*b*) **When target body B at rest is very heavy,** *i.e.,* $m_2 >> m_1$, *i.e.,* m_1 can be ignored compared to m_2

Putting $m_1 = 0$ in (50) and (51), we obtain

$$v_1 = -\frac{m_2}{m_2}u_1 = -u_1 \ ; v_2 = 0$$

Hence, when a light projectile body A collides against a heavy target body B at rest ; **A rebounds with its own velocity and B continues to be at rest.** This is what happens when a ball rebounds to the same height from which it was thrown, on striking a floor.

(*c*) **When target body B at rest has negligible mass,** *i.e.,* $m_2 << m_1$, *i.e.,* m_2 can be ignored compared to m_1

Putting $m_2 = 0$ in (50) and (51), we get $v_1 = \frac{m_1}{m_1}u_1 = u_1$; $v_2 = \frac{2m_1 u_1}{m_1} = 2u_1$

Hence, when a heavy projectile body A undergoes an elastic collision with a light target body B at rest, **the body A keeps on moving with the same velocity of its own and the body B starts moving with double the initial velocity of A.**

A head on elastic collision between two balls moving in opposite directions is shown in Fig. 6.30(c).

Sample Problem **A body of mass 2 kg makes an elastic collision with another body at rest and continues to move in the original direction with a speed equal to one third of its original speed. Find the mass of the second body.**

Sol. Here, $m_1 = 2$ kg, $u_2 = 0$, $v_1 = \frac{u_1}{3}$, $m_2 = ?$

In a perfectly elastic collision in one dimension, $v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2 u_2}{m_1 + m_2}$

$$\frac{u_1}{3} = \frac{(2 - m_2)}{2 + m_2}u_1 + 0$$

$$2 + m_2 = 6 - 3m_2$$
$$4m_2 = 4, \ m_2 = \mathbf{1\ kg}$$

6.23. INELASTIC COLLISION IN ONE DIMENSION

FIGURE 6.31

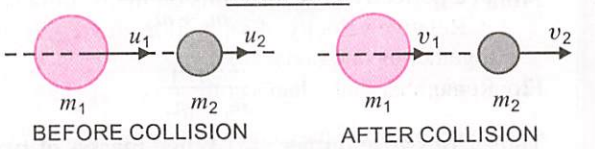


Fig. 6.32 shows two bodies of masses m_1 and m_2 moving with velocities, u_1 and u_2 respectively, along a single axis. They collide involving some *loss of kientic energy*. Therefore, the *collision is inelastic*. Let v_1 and v_2 be the velocities of the two bodies after collision.

As the two bodies form one system, which is closed and isolated, we can write the law of conservation of linear momentum for the two body system as :

Total momentum before the collision (P_i) = Total momentum after the collision (P_f)

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2} \quad \text{...(52)}$$

The overhead arrows for vectors have been avoided as the collision is one dimensional and velocity components along one axis are used.

If we know masses m_1, m_2, initial velocities u_1, u_2 and one of the final velocities, we can calculate the other final velocity from eqn. (52).

Perfectly inelastic collision in one dimension

FIGURE 6.32

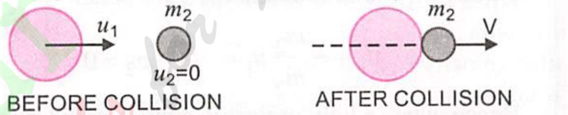


Fig. 6.32 shows perfectly inelastic collision between two bodies of masses m_1 and m_2. The body of mass m_2 happens to be initially at rest ($u_2 = 0$). We refer to this body as the *target*. The incoming body of mass m_1, moving with initial velocity u_1 is referred to as the *projectile*. After the collision, the two bodies move together with a common velocity V. The collision is perfectly inelastic.

As the total linear momentum of the system remains constant, therefore $P_i = P_f$

i.e., $$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$$

or $$m_1 u_1 = (m_1 + m_2) V \qquad (\because u_2 = 0)$$

or $$\boxed{V = \frac{m_1 u_1}{m_1 + m_2}} \quad \text{...(53)}$$

DO YOU KNOW ?

In a completely inelastic collision, the two bodies stick together, *i.e.*, they do not separate at all. Linear momentum alone is conserved in such a collision.

Knowing m_1, m_2 and u_1, we can calculate the final velocity V. As the mass ratio $\left(\frac{m_1}{m_1 + m_2}\right) < 1$, therefore, V < u_1.

We can calculate **loss of K.E. in this collision.**

Total K.E. before collision $= E_1 = \frac{1}{2} m_1 u_1^2$

Total K.E. after collision $= E_2 = \frac{1}{2}(m_1 + m_2) V^2 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u_1}{m_1 + m_2}\right)^2$ ...using (53)

$$= \frac{m_1^2 u_1^2}{2(m_1 + m_2)}$$

Loss of K.E. $= E_1 - E_2 = \frac{1}{2} m_1 u_1^2 - \frac{m_1^2 u_1^2}{2(m_1 + m_2)} = \frac{m_1^2 u_1^2 + m_1 m_2 u_1^2 - m_1^2 u_1^2}{2(m_1 + m_2)} = \frac{m_1 m_2 u_1^2}{2(m_1 + m_2)}$, ...(54)

which is positive.

Therefore, some K.E. is always lost in an inelastic collision.

Retain in Memory

1. In a perfectly inelastic collision in one dimension, coefficient of restitution, $e = 0$.
   ∴ Relative velocity of separation after collision is zero, *i.e.*, the two bodies, after collision, move together as one body.
2. Remember that when a collision occurs in a closed isolated system, the linear momentum of each colliding body may change, but the total linear momentum $\vec{P}$ of the system remains constant, whether the collision is elastic or inelastic.

6.24. ELASTIC COLLISION IN TWO DIMENSIONS OR OBLIQUE COLLISION

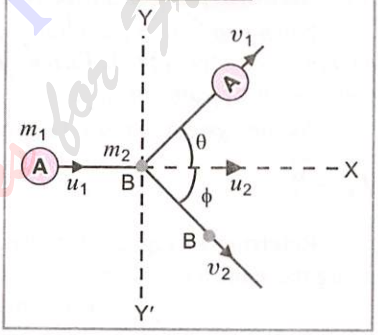
If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a **one-dimensional collision or head on collision.** In the case of small spherical bodies, this is possible if the direction of travel of body 1 passes through the centre of body 2.

In general, the collisions are two dimensional, where the initial velocities and the final velocities may lie in a plane. So, *when two bodies travelling initially along the same straight line collide without loss of kinetic energy and move along different directions in a plane after collision, the collision is said to be elastic collision in two dimensions.*

Suppose m_1, m_2 are the masses of two bodies A and B moving initially along X-axis with velocities u_1 and u_2 respectively.

When $u_1 > u_2$, the two bodies collide. After collision, let the body A move with a velocity v_1 at an angle θ with X-axis. Let the body B move with a velocity v_2 at an angle ϕ with X-axis as shown in Fig. 6.33. Here, θ is known as **angle of scattering** and ϕ is known as **the angle of recoil.**

FIGURE 6.33



As the collision is elastic, kinetic energy is conserved.

∴ Total K.E. after collision = Total K.E. before collision

$$\text{or} \quad \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 \qquad \text{...(55)}$$

$$\text{or} \quad m_1 v_1^2 + m_2 v_2^2 = m_1 u_1^2 + m_2 u_2^2 \qquad \text{...(56)}$$

As linear momentum is conserved in elastic collision, therefore, *along X-axis*, total linear momentum after collision = total linear momentum before collision.

$$m_1 v_1 \cos\theta + m_2 v_2 \cos\phi = m_1 u_1 + m_2 u_2 \qquad \text{...(57)}$$

Now, *along Y-axis*, linear momentum before collision is zero (as both the bodies are moving along X-axis). And after collision, total linear momentum along Y-axis is $(m_1 v_1 \sin\theta - m_2 v_2 \sin\phi)$

$$\therefore \quad m_1 v_1 \sin\theta - m_2 v_2 \sin\phi = 0 \qquad \text{...(58)}$$

From three eqns. (56), (57) and (58), we have to calculate four variables v_1, v_2, θ and ϕ, which is not possible. We have, therefore, to measure experimentally any one parameter, *i.e.*, final velocities v_1 of A, v_2 of B or their directions θ and ϕ. The rest of the three parameters can then be calculated from the three equations, (56), (57) and (58).

SPECIAL CASE.

When two particles of same mass undergo perfectly elastic collision in two dimensions, i.e., $m_1 = m_2$

For simplicity, let us take $u_1 = u$ and $u_2 = 0$.

From (56), $$v_1^2 + v_2^2 = u^2 \qquad \text{...(59)}$$ From (57), $$v_1 \cos\theta + v_2 \cos\phi = u \qquad \text{...(60)}$$

From (58), $$v_1 \sin\theta - v_2 \sin\phi = 0 \qquad \text{...(61)}$$

Using (60), we obtain from (59),

$$v_1^2 + v_2^2 = (v_1 \cos\theta + v_2 \cos\phi)^2 = v_1^2 \cos^2\theta + v_2^2 \cos^2\phi + 2\, v_1 v_2 \cos\theta \cos\phi$$

$$v_1^2 (1 - \cos^2\theta) + v_2^2 (1 - \cos^2\phi) = 2\, v_1 v_2 \cos\theta \cos\phi$$

or $$v_1^2 \sin^2\theta + v_2^2 \sin^2\phi = 2 v_1 v_2 \cos\theta \cos\phi \qquad \text{...(62)}$$

From (61), $v_2 \sin\phi = v_1 \sin\theta$

Put in (62), $2 v_1^2 \sin^2\theta = 2 v_1 v_2 \cos\theta \cos\phi$

or $$\cos\theta = \frac{2 v_1^2 \sin^2\theta}{2 v_1 v_2 \cos\phi} = \left(\frac{v_1}{v_2}\right)\frac{\sin^2\theta}{\cos\phi} \qquad \text{...(63)}$$

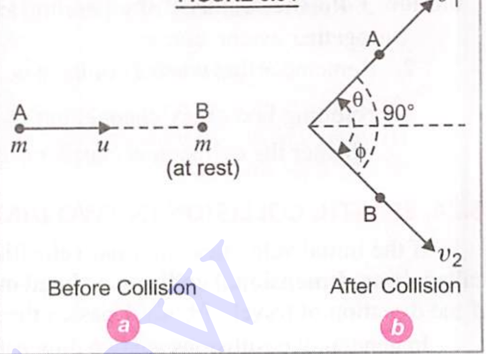
Now, $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

Using (63) and (61), we get

$$\cos(\theta+\phi) = \frac{v_1}{v_2}\frac{\sin^2\theta}{\cos\phi}\cos\phi - \frac{v_1}{v_2}\sin^2\theta = 0$$

$$\therefore \quad \boxed{\theta+\phi = 90^\circ} \qquad \text{...(64)}$$

FIGURE 6.34



Hence, we conclude that ***in a perfectly elastic collision in two dimensions between two particles of same mass, the two particles move along mutually perpendicular directions after the collision.*** This is shown in Fig. 6.34.

6.25. INELASTIC COLLISION IN TWO DIMENSIONS

When two bodies travelling initially along the same straight line collide involving some loss of kinetic energy, and move after collision, along different directions in a plane, the collision is said to be inelastic collision in two dimensions.

As the system is closed and isolated, the total linear momentum of the system remains constant, *i.e.*,

$$\vec{P}_f = \vec{P}_i$$

Referring to Fig. 6.31 and equating final momentum $(\vec{P}_f)$ along X-axis to initial momentum $(\vec{P}_i)$ along the same axis, we get

$$m_1 v_1 \cos\theta + m_2 v_2 \cos\phi = m_1 u_1 + m_2 u_2 \qquad \text{...(65)}$$

As initial momentum of the two bodies along Y-axis is zero, therefore applying the law of conservation of linear momentum along Y-axis, we get

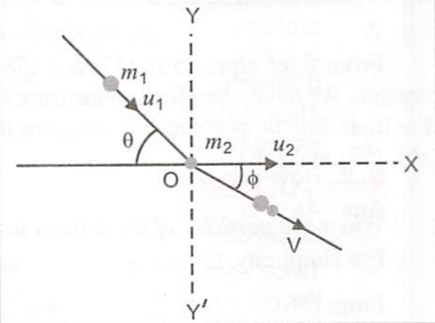
$$0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\phi \qquad \text{...(66)}$$

Knowing m_1, m_2 ; u_1, u_2 ; θ and φ, we can calculate v_1 and v_2 from eqns. (65) and (66).

Perfectly inelastic collision in two dimensions

Fig. 6.35 shows perfectly inelastic collision between two bodies of masses m_1 and m_2. The body of mass m_2 is moving initially with velocity u_2 along X-axis. The body of mass m_1 is moving with velocity u_1 at an angle θ with X-axis as shown. After the collision at O, the two bodies stick to eachother and move with a common velocity V at an angle φ with X-axis as shown in Fig. 6.35.

FIGURE 6.35



As the system is closed and isolated, the total linear momentum of the system remains constant.

Referring to Fig. 6.35, and equating initial momentum along X-axis to final momentum along the same axis, we get

$$m_1 u_1 \cos\theta + m_2 u_2 = (m_1 + m_2) V \cos\phi \qquad \text{...(67)}$$

Again, applying the law of conservation of linear momentum along Y-axis, we get

$$m_1 u_1 \sin\theta + 0 = (m_1 + m_2) V \sin\phi \qquad \text{...(68)}$$

Knowing m_1, m_2 ; u_1, u_2 and θ, we can calculate final velocity V and its direction, *i.e.*, ∠φ from eqns. (67) and (68).

Sample Problem **A ball moving with a speed of 9 m/s strikes an identical ball at rest, such that after the collision, the direction of each ball makes an angle of 30° with the original line of motion. Find the speeds of the two balls after collision.**

Sol. Here, $m_1 = m_2 = m$, $u_1 = 9$ m/s, $u_2 = 0$;
$\theta_1 = \theta_2 = 30°$, $v_1 = ?$ $v_2 = ?$

Applying principle of conservation of linear momentum

(*i*) *along the direction of motion* (X-axis)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

$$m \times 9 + 0 = m v_1 \cos 30° + m v_2 \cos 30°$$

or $$9 = v_1\sqrt{3}/2 + v_2\sqrt{3}/2 = \frac{(v_1+v_2)\sqrt{3}}{2}$$

$$v_1 + v_2 = \frac{18}{\sqrt{3}} \qquad \text{...}(i)$$

(*ii*) *along a direction perpendicular to the direction of motion* (Y-axis)

$$0 + 0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$$

$$\therefore \quad m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

$$m v_1 \sin 30° = m v_2 \sin 30°$$

$$\therefore \quad v_1 = v_2$$

Put in (*i*), $$2v_1 = \frac{18}{\sqrt{3}}$$

$$v_1 = \frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{3}\sqrt{3}} = 3\sqrt{3} \text{ m/s}$$

Hence, the two balls move with the same velocity = **$3\sqrt{3}$ m/s after collision.**

DO YOU KNOW ?

In our everyday world, collisions take place only when two bodies strike each other. Let us consider an alpha particle coming towards a nucleus and getting away in some direction. Similarly, a comet coming from a distance towards the sun and getting deflected. In both the cases, we deal with **forces involving action at a distance. Such an event is called scattering or collision from a distance.** The velocities and directions of the two bodies involved depend on their initial velocities, their masses, shapes, sizes and also on the type of interaction involved.

Curiosity Questions

Q. 1. Can you visualize how a car reaches high speeds and how a person scales high cliffs ?

Ans. A car reaching high speeds acquires kinetic energy and a person scaling high cliffs acquires potential energy.

In the case of car, the source of K.E. is petrol/diesel and oxygen from air. In case of humans, the source of P.E. is food and oxygen. In both the cases, fuel and oxygen are combined chemically, and the process is called *combustion* or *burning.* We may visualize the combustion process as removing some of the energy stored in the bonds between the atoms. This energy is called chemical energy, which is essentially a form of potential energy as it depends on the position of atoms.

Thus K.E. of car and P.E. of person, both are coming from chemical energy/potential energy stored in the bonds between the atoms of fuel and oxygen.

Q. 2. How can tidal energy be harnessed usefully ?

Ans. As is known, hydropower is the least expensive way to generate electricity, and that too with least pollution problems caused by burning coal and oil to produce electricity.

Twice each day, the Atlantic Ocean pours into the Bay of Fundy (in eastern Canada) producing the highest tides in the world ranging from 12 m to an extreme of 17 m. The huge amount of kinetic energy available in these tides can be converted into electrical energy by building a dam and hydroelectric power plant at the entrance to the bay.

Such a dam would obstruct the natural flow of tides. Every twelve hours, the incoming tides will rush through the open flood gates in the dam and be impounded behind the dam.

When the tides recede, water would exit through turbines generating hydroelectric power.

SOLVED EXAMPLES

TYPE I. WORK

Formulae used.

1. Work = $\vec{F}\cdot\vec{s} = F\,s\cos\theta$

= algebraic sum of areas under ($F - s$) graph.

2. Work = $\vec{F}\cdot\vec{s}$

$$= (\hat{i}F_x + \hat{j}F_y + \hat{k}F_z)\cdot(\hat{i}x + \hat{j}y + \hat{k}z)$$

$$W = (x F_x + y F_y + z F_z)$$

Units used. Force in newton (N), distance in metre (m), work in joule (J)

Example 1 **Calculate work done in raising a stone of mass 5 kg and specific gravity 3 lying at the bed of a lake through a height of 4 metre.**

Solution. Here, mass of stone in air, $m = 5$ kg

specific gravity of stone = 3

As sp. gravity

$$= \frac{\text{mass of stone in air}}{\text{mass of an equal volume of water}}$$

$$\therefore \quad 3 = \frac{5\text{ kg}}{\text{mass of an equal volume of water}}$$

Mass of an equal volume of water $= \frac{5}{3}$ kg. This is weight of water displaced.

∴ Apparent weight of stone in water

$$= \left(5 - \frac{5}{3}\right) = \frac{10}{3}\text{ kg}$$

∴ force applied, $F = \frac{10}{3} \times 9{\cdot}8$ N

Work done in raising the stone through, $s = 4$ m is

$$W = F \times s = \left(\frac{10}{3} \times 9{\cdot}8\right) \times 4 = \mathbf{130{\cdot}66\ J}$$

Example 2 **A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ? Take $g = 10\ \text{m/s}^2$.**

Solution. Here, $L = 2$ m, $M = 4$ kg,

$l = 60$ cm $= 0{\cdot}6$ m, $W = ?$

Mass of hanging part of chain $= m = \frac{4}{2} \times 0{\cdot}6$

$= 1{\cdot}2$ kg

Height of c.g. of hanging part from the table

$$h = \frac{0{\cdot}60}{2} = 0{\cdot}30\text{ m}$$

Work done = force × distance = mgh

$= 1{\cdot}2 \times 10 \times 0{\cdot}3 =$ **3·6 J**

Example 3 **A particle moves along the X-axis from $x = 0$ to $x = 5$ m under the influence of a force given by $F = (7 - 2x + 3x^2)$. Find the work done in the process.**

Solution. Here, $F = 7 - 2x + 3x^2$.

From $x = 0$ to $x = 5$ m, work done by the force,

$$W = \int_0^5 F\,dx = \int_0^5 (7 - 2x + 3x^2)\,dx$$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5$$

$$= 7(5 - 0) - (5^2 - 0^2) + (5^3 - 0)$$

$= \mathbf{135\ J}$

Example 4 **A body moves from point A to B under the action of a force, varying in magnitude as shown in Fig. 6.36. Obtain the work done. Force is expressed in newton and displacement is in metre.**

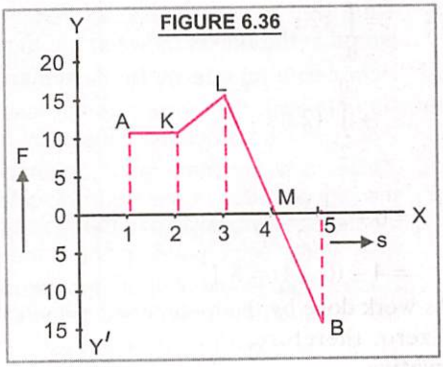


FIGURE 6.36

Solution. As work done W is equal to area under the F-s curve, therefore,

$$W_{AB} = W_{12} + W_{23} + W_{34} + W_{45}$$

= area under AK + area under KL

+ area under LM – area enclosed by MB

$$= 10\,(1) + \left\{10\,(1) + \frac{5\times1}{2}\right\} + \frac{15\times1}{2} - \frac{15\times1}{2}$$

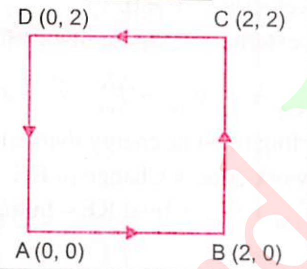
$$= 10 + 10 + 2\cdot5 + 7\cdot5 - 7\cdot5 = \mathbf{22\cdot5\ J}$$

Retain in Memory

Note that work done is taken as negative when direction of force is reversed. Total work done is the algebraic sum of the works done in parts.

Example 5 **A force $\vec{F} = 2x\,\hat{j}$ newton acts in a region where a particle moves anticlockwise in a square loop of 2 m in x–y plane as shown in Fig. 6.37. Calculate the total amount of work done. Is this force a conservative force or a non-conservative force ?**

FIGURE 6.37



Solution. Here, $\vec{F} = 2x\,\hat{j}$ newton

Total work done in moving the particle

$$W = \int_{ABCDA} \vec{F}\cdot d\vec{x}$$

$$= \int_0^2 (2x\,\hat{j})\cdot(dx)\,\hat{i} + \int_0^2 (2x\,\hat{j})\cdot(dx)\,\hat{j}$$

$$+ \int_2^0 (2x\,\hat{j})\cdot dx(-\hat{i}) + \int_2^0 (2x\,\hat{j})\cdot dx(-\hat{j})$$

$$= 0 + 2\left[\frac{x^2}{2}\right]_0^2 + 0 - 2\left[\frac{x^2}{2}\right]_2^0$$

$$= 4 - (0 - 4) = \mathbf{8\ J}$$

As work done by the force around a round trip is not zero, therefore, the force must be **non-conservative.**

Example 6 **The displacement x of a particle of mass 5 g moving in one dimension under the action of a constant force is related to the time by the equation $t = \sqrt{x} + 3$, where x is in metre and t is in second. Find (i) the displacement of the particle when its velocity is zero, and (ii) work done by the force in first six seconds.**

Solution. From $t = \sqrt{x} + 3$, $\sqrt{x} = t - 3$

Squaring both sides, $x = t^2 - 6t + 9$ ...(*i*)

Now, $v = \dfrac{dx}{dt} = 2t - 6$

and $a = \dfrac{dv}{dt} = 2$

When $v = 0$, $2t - 6 = 0$, $2t = 6$, $t = 3$ s

From (*i*), $x = 3^2 - 6\,(3) + 9 = 0$, *i.e.*, **displacement is zero.**

(*ii*) From (*i*), at $t = 0$, $x = 0 - 0 + 9 = 9$ m

and at $t = 6$; $x = 6^2 - 6 \times 6 + 9 = 9$ m

∴ In the first six seconds, net displacement of the particle is zero.

As work done = force × displacement.

∴ Work done in first six seconds = **zero**

Example 7 **A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (*a*) How much work does the road do on the cycle ? (*b*) How much work does the cycle do on the road ?**

(NCERT Solved Example)

Solution. (*a*) Work done by the road on the cycle is work done by the stopping frictional force on the cycle.

$$W_1 = F\,s\cos\theta = (200)\,(10)\cos 180^\circ$$

$$= \mathbf{-\,200\ J}$$

This is obviously negative.

(*b*) From Newton's third law, an equal and opposite force acts on the road due to the cycle.

∴ Force on the road, $F = 200$ N.

But the road undergoes no displacement. Therefore, work done by the cycle on the road is **zero.**

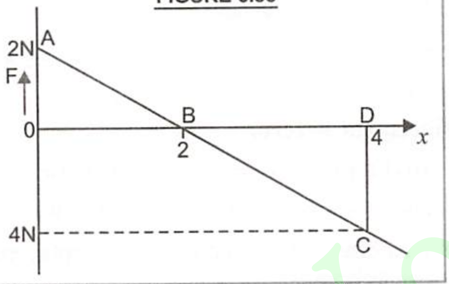
Retain in Memory

From the above question, we learn that though force on body A exerted by body B is always equal and opposite to the force on body B exerted by A ; yet the work done on A by B is not necessarily equal and opposite to the work done on B by A.

Example 8 **The variation of force acting on a body with the displacement of body is shown in Fig. 6.38. Calculate work done by the force in the interval**

(*i*) $0 \le x \le 2\,m$ **(*ii*) $2\,m \le x \le 4\,m$.**

FIGURE 6.38



Solution. Work done by the force = area under F x graph

(*i*) in interval $0 \le x \le 2$ m

work done = area of ΔOAB

$$= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 2 \times 2 = \mathbf{2\ J}$$

(*ii*) in interval $2\,m \le x \le 4\,m$

work done = – area of ΔBCD

$$= -\frac{1}{2} \times BD \times DC = -\frac{1}{2} \times 2 \times 4 = \mathbf{-4\ J}$$

TYPE II. KINETIC ENERGY AND WORK ENERGY THEOREM

Formulae used.

1. Kinetic energy $= \frac{1}{2}mv^2 = p^2/2m$

2. Work done *on* the body = final KE – initial KE

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Similarly, work done *by* the body = initial KE – final KE

Units used. m in kg, u, v in m/s, work and energy in joule, linear momentum p in kg ms^{-1}.

Example 9 **If KE of a body increases by 300%, by what % will the linear momentum of the body increase ?** **(Delhi 2001)**

Solution. $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $\therefore$ $p = \sqrt{2mK}$

When KE increases by 300%,

new KE $K' = K + 3K = 4K$.

$$p' = \sqrt{2mK'} = \sqrt{2m(4K)} = 2\sqrt{2mK} = 2p$$

% age increase in momentum

$$= \frac{(p'-p)\times 100}{p} = \frac{(2p-p)100\%}{p} = \mathbf{100\%}$$

Example 10 **Two identical 5 kg blocks are moving with same speed of 2 ms^{-1} towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Calculate work done by (*i*) external forces and (*ii*) internal forces.** **(CBSE 2001)**

Solution. Here, $m_1 = m_2 = 5$ kg,

$v_1 = 2$ m/s, $v_2 = -2$ m/s

Final velocity $v = 0$.

As no external forces are involved, therefore,

$$\vec{F}_{ext} = 0,\ W_{ext} = \vec{F}_{ext} \cdot \vec{s} = \textbf{Zero}$$

According to work energy theorem,

Total work done = Change in KE

$$W_{ext} + W_{int} = \text{final KE} - \text{initial KE}$$

$$= 0 - \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right)$$

$$0 + W_{int} = -\frac{1}{2}[5(2)^2 + 5(-2)^2] = -20\ \text{J}$$

$$\therefore \quad W_{int} = \mathbf{-20\ J}$$

Negative sign shows that internal forces of action and reaction act on the two blocks in a direction opposite to their motion.

Example 11 **A particle of mass 0·5 kg travels in a straight line with a velocity $v = (5\,x^{5/2})$ m/s. How much work is done by the net force during the displacement from $x = 0$ to $x = 2$ m ?**

Solution. Here, $m = 0{\cdot}5$ kg

$v = 5\,x^{5/2}$; $W = ?$

When $x = 0$, $v_1 = 0$

When $x = 2$ m, $v_2 = 5\,(2)^{5/2}$

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Using work energy theorem,

$$W = \text{change in K.E.} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = \frac{1}{2}m\,(v_2^2 - v_1^2) = \frac{1}{2}\times 0\cdot 5\,[\{5\,(2)^{5/2}\}^2 - 0]$$

$$= \frac{1}{2}\times\frac{1}{2}\times 25\times 2^5 = \mathbf{200\ J}$$

Example 12 **It is well known that a rain drop falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop, but is otherwise undetermined. Consider a drop of mass 1·00 g falling from a height of 1·00 km. It hits the ground with a speed of 50·0 ms^{-1}. (*a*) What is the work done by the gravitational force ? (*b*) What is the work done by the unknown resistive force ?**

(NCERT Solved Example)

Solution. Here, $m = 1\cdot 00\ \text{g} = 10^{-3}\ \text{kg}$,

$h = 1\cdot 00\ \text{km} = 10^3\ \text{m},\ v = 50\cdot 0\ \text{ms}^{-1}$

(*a*) Work done by the gravitational force

$$W_1 = mgh = 10^{-3}\times 10\times 10^3 = \mathbf{10\ J}$$

(*b*) The change in K.E. of the drop is

$$\Delta K = \frac{1}{2}mv^2 - 0 = \frac{1}{2}\times 10^{-3}\,(50)^2 = 1\cdot 25\ \text{J}$$

If W_2 is work done by the resistive force, then according to work energy theorem,

$$\Delta K = W_1 + W_2$$

$$\therefore\quad W_2 = \Delta K - W_1$$

$$W_2 = 1\cdot 25 - 10\cdot 00 = \mathbf{-\,8\cdot 75\ J}$$

Clearly, work done by resistive force is negative.

Example 13 **When an automobile moving with a speed of 36 km/h reaches an upward inclined road of angle 30°, its engine is switched off. If the coefficient of friction is 0·1, how much distance will the automobile move before coming to rest ? Take g = 10 ms^{-2}.**

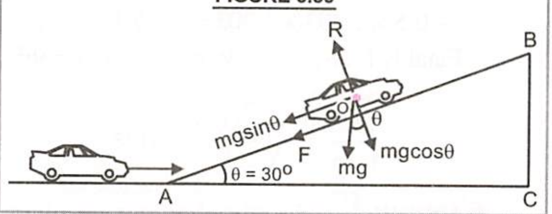
Solution. Here, initial speed,

$$u = 36\ \text{km/h} = \frac{36\times 1000}{60\times 60}\ \text{ms}^{-1} = 10\ \text{ms}^{-1}$$

$$\theta = 30^\circ,\ \mu = 0\cdot 1,\ s = ?$$

Final velocity, $v = 0$

FIGURE 6.39



Work done in moving up the inclined road = K.E. of the vehicle, Fig. 6.39,

$$(mg\sin\theta + F)\times s = \frac{1}{2}mu^2$$

$$(mg\sin\theta + \mu R)\times s = \frac{1}{2}mu^2$$

$$(mg\sin\theta + \mu\,mg\cos\theta)\,s = \frac{1}{2}mu^2$$

$$s = \frac{\frac{1}{2}mu^2}{mg\,(\sin\theta + \mu\cos\theta)} = \frac{u^2}{2\,g\,(\sin\theta + \mu\cos\theta)}$$

$$= \frac{10\times 10}{2\times 10\,(\sin 30^\circ + 0\cdot 1\cos 30^\circ)} = \mathbf{8\cdot 53\ m}$$

Example 14 **A block of mass m = 1 kg moving on a horizontal surface with speed v_i = 2 ms^{-1} enters a rough patch ranging from *x* = 0·10 m to *x* = 2·01 m. The retarding force F_r on the block in this range is inversely proportional to x over this range**

$$F_r = -\frac{k}{x}\quad \text{for } 0\cdot 1 < x < 2\cdot 01\ \text{m}$$

$$= 0\quad \text{for } x < 0\cdot 1\ \text{m and } x > 2\cdot 01\ \text{m}$$

where k = 0·5 J. What is the final K.E. and speed v_f of the block as it crosses the patch ?

(NCERT Solved Example)

Solution. Here, $m = 1\ \text{kg},\ v_i = 2\ \text{ms}^{-1};\ k = 0\cdot 5\ \text{J}$

$$\text{Initial K.E., } K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\times 1\,(2)^2 = 2\,\text{J}$$

Work done against friction

$$W = \int_{x=0\cdot 1m}^{x=2\cdot 01m} F_r\,dx = \int_{x=0\cdot 1m}^{x=2\cdot 01m} -\frac{k}{x}\,dx$$

$$= -0\cdot 5\,\left[\log_e x\right]_{x=0\cdot 1\text{m}}^{x=2\cdot 01\text{m}}$$

$$= -0\cdot 5\,\log_e\frac{2\cdot 01}{0\cdot 1}$$

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$$W = -0{\cdot}5 \times 2{\cdot}303 \log_{10} 20{\cdot}10$$
$$= -0{\cdot}5 \times 2{\cdot}303 \times 1{\cdot}303 = -1{\cdot}5 \text{ J}$$

$\therefore$ Final K.E., $K_f = K_i + W = 2{\cdot}0 - 1{\cdot}5 \text{ J} = \mathbf{0{\cdot}5 \text{ J}}$

$$v_f = \sqrt{\frac{2k_f}{m}} = \sqrt{\frac{2 \times 0{\cdot}5}{1}} = 1 \text{ ms}^{-1}$$

Example 15 **To stimulate car accidents, the auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed of 18·0 km/*h* on a smooth road and colliding with a horizontally mounted spring of spring constant $6{\cdot}25 \times 10^3$ Nm^{-1}. What is the maximum compression of the spring ?**

(NCERT Solved Example)

Solution. Here, $m = 1000$ kg,

$v = 18{\cdot}0$ km/h $= 5$ m/s, $k = 6{\cdot}25 \times 10^3$ Nm^{-1},

$x_m = ?$

At maximum compression of the spring, the K.E. of the car is converted entirely into P.E. of the spring, *i.e.*,

$$\frac{1}{2} mv^2 = \frac{1}{2} k\, x_m^2$$

$$x_m = \sqrt{\frac{m}{k}} \times v = \sqrt{\frac{1000}{6{\cdot}25 \times 10^3}} \times 5$$
$$= 0{\cdot}4 \times 5 = \mathbf{2{\cdot}00 \text{ m}}$$

Example 16 **In the above Example 15, taking the coefficient of friction, μ to be 0·5, calculate the maximum compression of the spring.**

(NCERT Solved Example)

Solution. In the presence of friction, both the spring force and the frictional force f act so as to oppose the compression of the spring.

Work done by the two forces

$$W = -\frac{1}{2} k\, x_m^2 - f\,(x_m)$$

$$W = -\frac{1}{2} k\, x_m^2 - \mu\, mg\,(x_m)$$

$$(\because\ f = \mu R = \mu\, mg)$$

and change in K.E.,

$$\Delta K = K_f - K_i = 0 - \frac{1}{2} mv^2$$

Applying work energy theorem, $\Delta K = W$

$$\therefore\quad -\frac{1}{2} mv^2 = -\frac{1}{2} kx_m^2 - \mu\, mg\,(x_m)$$

$$\therefore\quad k\, x_m^2 + 2\mu\, mg\,(x_m) - mv^2 = 0$$

It is a quadratic equation in x_m

$$\therefore\quad x_m = \frac{-2\mu\, mg + [4\mu^2 m^2 g^2 + 4mk\, v^2]^{1/2}}{2k}$$

Putting the numerical values and solving, we get $x_m = \mathbf{1{\cdot}35 \text{ m}}$

Example 17 **Express (*a*) the energy required to break one bond in DNA (10^{-20} J) in *e*V.**

(*b*) the kinetic energy of an air molecule (10^{-21} J) in *e*V.

(*c*) the daily intake of a human adult (10^7 J) in kilocalories. (NCERT Solved Example)

Solution. (*a*) Energy required to break one bond in DNA is

$$= 10^{-20} \text{ J} = \frac{10^{-20}}{1{\cdot}6 \times 10^{-19}} \text{ eV} \approx \mathbf{0{\cdot}06 \text{ eV}}$$

(*b*) K.E. of an air molecule is

$$= 10^{-21} \text{ J} = \frac{10^{-21}}{1{\cdot}6 \times 10^{-19}} \text{ eV} \approx \mathbf{0{\cdot}0062 \text{ eV}}$$

(*c*) The daily intake of a human adult

$$= 10^7 \text{ J} = \frac{10^7}{4{\cdot}2 \times 10^3} \text{ kcal} = \mathbf{2400 \text{ kcal}}$$

TYPE III. POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Formulae used

1. Gravitational P.E., $U = mgh$.
2. For a conservative force, $F = -\dfrac{dU}{dx}$
3. Change in P.E. $= \Delta U = U_f - U_i = -W = -\displaystyle\int_{x_i}^{x_f} F\,dx$.
4. When work is done by conservative forces only mechanical energy is conserved, *i.e.*, $K + U$ = constant.

Units. used. m in kg, h in metre ; U, W, K in joule.

Example 18 **In a ballistics demonstration, a police officer fires a bullet of mass 50·0 g with speed 200 ms^{-1} on soft plywood of thickness 2·00 cm. The bullet emerges only with 10% of its initial kinetic energy. What is the emergent speed of the bullet ?** (NCERT Solved Example)

Solution. Here, $m = 50{\cdot}0$ g

$$= \frac{50}{1000}\text{ kg} = \frac{1}{20}\text{ kg}$$

$$v_i = 200\text{ ms}^{-1}$$

$\therefore$ Initial K.E., $K_i = \frac{1}{2}mv_i^2$

$$= \frac{1}{2}\times\frac{1}{20}(200)^2 = 1000\text{ J}$$

Final K.E., $K_f = 10\%\,(K_i)$

$$= \frac{10}{100}\times 1000\text{ J} = 100\text{ J}$$

If v_f is emergent speed of the bullet, then

$$\frac{1}{2}mv_f^2 = K_f$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2\times 100}{1/20}} = \mathbf{63{\cdot}2\ m/s}$$

Note that K.E. is reduced by 90% but speed is reduced by nearly 68%.

Example 19 **A force is acting on a body moving along x-axis in the direction of motion of the body. If this force produces a potential energy $U = A\,x^4$, where $A = 1{\cdot}2\ \text{Jm}^{-4}$, then what is the force acting on the body when the body is at $x = -\,0{\cdot}8$ m ?**

Solution. Here, $U = A\,x^4$, $A = 1{\cdot}2\ \text{Jm}^{-4}$, $F_x = ?$

$$x = -\,0{\cdot}8\text{ m}$$

As $\quad F_x = -\frac{dU}{dx} = -\frac{d}{dx}(A\,x^4)$

$\therefore \quad F_x = -A\,(4\,x^3) = -\,4\,A\,x^3$

$$= -\,4\times 1{\cdot}2\,(-\,0{\cdot}8)^3 = \mathbf{2{\cdot}46\ N}$$

Example 20 **A 3·0 kg block has a speed of 2 m/s at A and 6 m/s at B. If the distance from A to B along the curve is 12 m, how large a frictional force acts on it ? Assuming the same friction, how far from B will it stop ?**

Solution. Here, $m = 3{\cdot}0$ kg

$$h = 4\text{ m (at } A)$$

$$x = 1\text{ m (at } B)$$

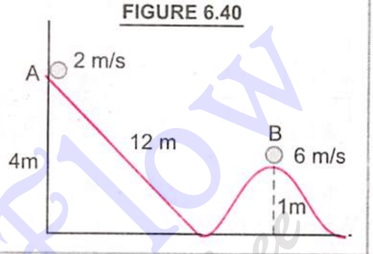
$$v_A = 2\text{ m/s},\ v_B = 6\text{ m/s}$$

$$s_{AB}\text{ (along the curve)} = 12\text{ m}$$

Let the force of friction be f.

As change in total energy of block = Work done against friction,

FIGURE 6.40



$$\therefore\ mg\,(h - x) + \frac{1}{2}m\,(v_A^2 - v_B^2) = f\times s_{AB}$$

$$3\times 9{\cdot}8\,(4-1) + \frac{1}{2}\times 3\,(2^2 - 6^2) = f\times 12$$

$$88{\cdot}2 - 48 = f\times 12$$

$$f = \frac{40{\cdot}2}{12} = \mathbf{3{\cdot}35\ N}$$

Total energy at $B = mgx + \frac{1}{2}m\,v_B^2$

$$= 3\times 9{\cdot}8\times 1 + \frac{1}{2}\times 3\times 6^2$$

$$= 29{\cdot}4 + 54 = 83{\cdot}4\text{ J}$$

If s' is distance travelled from B before stopping, then $\quad f\times s' = 83{\cdot}4$

$$s' = \frac{83{\cdot}4}{f} = \frac{83{\cdot}4}{3{\cdot}35} = \mathbf{24{\cdot}5\ m}$$

TYPE IV. POTENTIAL ENERGY OF A SPRING

Formulae used

1. When a spring is stretched or compressed through a distance x, restoring force $F = -\,Kx$, where K is force constant of spring.
2. Work done on spring = P.E. of spring,

$$W = U = \frac{1}{2}Kx^2$$

Units used. x is in metre, F in newton, K in newton/metre : W, U in joule.

Example 21 **Two springs have force constants K_1 and K_2, where $K_1 > K_2$. On which spring, more work is done if**

(*i*) they are stretched by the same force ?

(*ii*) they are stretched by the same amount ?

Solution. (*i*) Under the action of same force F, let the springs be stretched by x_1 and x_2 respectively.

$$\therefore F = K_1 x_1 = K_2 x_2, \quad \therefore \frac{x_1}{x_2} = \frac{K_2}{K_1}$$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}K_1 x_1^2}{\frac{1}{2}K_2 x_2^2} = \frac{K_1}{K_2}\left(\frac{x_1}{x_2}\right)^2 = \frac{K_1}{K_2}\left(\frac{K_2}{K_1}\right)^2 = \frac{K_2}{K_1}$$

As $\quad K_1 > K_2 \qquad \therefore W_2 > W_1$

(*ii*) When the two springs are stretched by the same distance, say x,

then $$\frac{W_1}{W_2} = \frac{\frac{1}{2}K_1 x^2}{\frac{1}{2}K_2 x^2} = \frac{K_1}{K_2}$$

As $\quad K_1 > K_2$, therefore $W_1 > W_2$

Example 22 **A block of mass m, initially at rest is dropped from a height h onto a spring whose force constant is K. Find the maximum distance x through which the spring will be compressed.**

Solution. When the spring is compressed through distance x ; the mass m drops through a distance $(h + x)$

$\therefore$ Potential energy of spring

$$= \frac{1}{2}Kx^2 = mg\,(h + x)$$

or $$\frac{1}{2}Kx^2 - mgx - mgh = 0$$

or $$x^2 - \frac{2mg}{K}x - \frac{2mgh}{K} = 0$$

$$\therefore x = \frac{2mg}{K} \pm \sqrt{\left[\left(\frac{2mg}{K}\right)^2 + \frac{8mgh}{K}\right]}$$

Example 23 **A spring of force constant 24 N/m is resting on a frictionless horizontal surface. A force of 10 N is applied on the block of mass 4 kg at one end of the spring. What is the speed of the block when it has been moved through a distance of 0·5 m ?**

Solution. Here, $K = 24$ N/m, $F = 10$ N, $v = ?$
$x = 0{\cdot}5$ m

By the law of conservation of energy,

Work done on the spring = Gain in KE + Gain in P.E

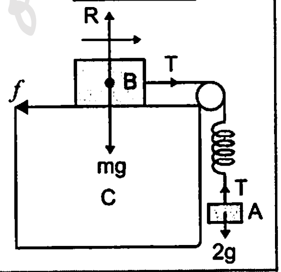
$$F \times s = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

$$10 \times 0{\cdot}5 = \frac{1}{2} \times 4\,v^2 + \frac{1}{2} \times 24\,(0{\cdot}5)^2$$

or $\quad 2\,v^2 = 5 - 3 = 2,\ \ v = \mathbf{1\ m/s}$

Example 24 **Two blocks A and B are connected to each other as shown in Fig. 6.41. The pulley is frictionless and the string and spring are massless. Block B slides over the horizontal top surface of stationary block C and block A slides along vertical side of C, both with same uniform speed. Spring constant of spring is 1960 Nm^{-1} and coefficient of friction between the blocks B and C is 0·2. If mass of block A is 2 kg, calculate mass of block B and energy stored in the spring.**

FIGURE 6.41



Solution. As is clear from Fig. 6.41,

for block A, $T = 2\,g$;

for block B, $T = f = \mu\,R = \mu\,mg$

$\therefore\ T = \mu\,mg$ or $2\,g = 0{\cdot}2\,mg$, or $m = \mathbf{10\ kg}$

Also, $T = 2\,g = 2 \times 9{\cdot}8 = \mathbf{19{\cdot}6\ N}$

If x is extension produced in the spring,

$$T = Kx \quad \text{or} \quad x = \frac{T}{K} = \frac{19{\cdot}6}{1960} = \mathbf{0{\cdot}01\ m}$$

Energy stored in the spring,

$$U = \frac{1}{2}Kx^2 = \frac{1}{2} \times 1960\,(0{\cdot}01)^2 = \mathbf{0{\cdot}098\ J}$$

Example 25 **If a body of mass m suspended by a spring comes to rest after a downward displacement y_0, calculate (a) force constant of the spring, (b) loss in gravitational potential energy, (c) gain in elastic potential energy of spring.**

Solution. (*a*) In equilibrium ; $mg = K\,y_0$

$$\therefore \qquad K = \frac{mg}{y_0}$$

(*b*) As mass descends through a distance y_0,

$\therefore$ loss in gravitational P.E. $= mg\,y_0$.

(*c*) As the spring is stretched by y_0, therefore, gain in elastic potential energy

$$\text{P.E.} = \frac{1}{2}K\,y_0^2 = \frac{1}{2}\left(\frac{mg}{y_0}\right)y_0^2 = \frac{1}{2}mg\;y_0$$

IMPORTANT NOTE

In the above problem, loss in gravitational P.E. $= mg\,y_0$

and gain in elastic P.E. of spring $= \frac{1}{2}\;mg\,y_0$

However, the law of conservation of energy is not contradicted as the difference $\frac{1}{2}\;mg\;y_0$ is converted first into kinetic energy of mass (due to which it oscillates), and finally lost as heat to the surrounding air.

TYPE V. MASS ENERGY EQUIVALENCE

Formulae used.

1. $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$

where m is effective mass of the body moving with velocity v ; m_0 is rest mass of the body ; c is velocity of light in vacuum.

2. $E = mc^2$, where m is the mass that disappears and E is the energy that appears.

Units used. m_0 and m in kg, c in m/s and E in joule. When mass m is in atomic mass unit, energy E is in MeV, where 1 amu = 931 MeV.

Standard value. $c = 3 \times 10^8$ m/s

Example 26 **Estimate the amount of energy released in the nuclear fusion reaction :**

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1$$

Given that

$M\,(_1H^2) = 2{\cdot}0141$ u, $M\,(_2He^3) = 3{\cdot}0160$ u

$M_n = 1{\cdot}0087$ u ;

where 1 u $= 1{\cdot}661 \times 10^{-27}$ kg.

Express your answer in units of MeV

Solution. Here, mass of two deutrons

$= 2 \times 2{\cdot}0141 = 4{\cdot}0282$ u

Mass of $_2He^3$ and neutron

$= 3{\cdot}0160 + 1{\cdot}0087$

$= 4{\cdot}0247$ u

Loss of mass $= \Delta m = 4{\cdot}0282 - 4{\cdot}0247$

$= 0{\cdot}0035$ u $= 0{\cdot}0035 \times 1{\cdot}661 \times 10^{-27}$ kg

Energy released $E = (\Delta m)\,c^2$

$= 0{\cdot}0035 \times 1{\cdot}661 \times 10^{-27}\,(3 \times 10^8)^2$ joule

$= 52{\cdot}2 \times 10^{-14}$ joule

As $\qquad 1\text{ MeV} = 1{\cdot}602 \times 10^{-13}$ joule

$$\therefore \text{ Energy released } = \frac{52{\cdot}2\times10^{-14}}{1{\cdot}602\times10^{-13}} = \mathbf{3{\cdot}26\ MeV}$$

Example 27 **A particle of rest mass m_0 moves with a speed $c/2$. Calculate its**

(*i*) mass **(*ii*) momentum**

(*iii*) total energy **(*iv*) K.E.**

Solution. (*i*) From

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{m_0}{\sqrt{1 - 1/4}} = \frac{2m_0}{\sqrt{3}}$$

$$(ii)\ \text{momentum} = mc/2 = \frac{2\,m_0}{\sqrt{3}}\cdot\frac{c}{2} = \frac{m_0 c}{\sqrt{3}}$$

$$(iii)\ \text{total energy} = mc^2 = \frac{2\,m_0}{\sqrt{3}}\,c^2$$

(*iv*) K.E. = total energy – rest mass energy

$$= \frac{2\,m_0}{\sqrt{3}}\,c^2 - m_0\,c^2$$

$$= m_0\,c^2\left[\frac{2}{\sqrt{3}} - 1\right]$$

$$= \frac{m_0 c^2}{\sqrt{3}}(2 - \sqrt{3})$$

TYPE VI. POWER

Formulae used

1. Power $= \dfrac{\text{Work}}{\text{time}} = \dfrac{\text{energy}}{\text{time}}$ **2.** $P = \vec{F}\cdot\vec{v} = F\,v\cos\theta$

Units used. Work and energy are in joule ; time in second and power in watt.

1 kW = 10^3 W, 1 MW = 10^6 W and 1 H.P. = 746 W

Example 28 **An electric motor is used to lift an elevator and its load (total mass 1500 kg) to a height of 20 m. The time taken for the job is 20 s. What is work done ? What is the rate at which work is done. If efficiency of the motor is 75%, at which rate is the energy supplied to the motor ?**

Solution. Here, $m = 1500$ kg, $h = 20$ m, $t = 20$ s.

Work done, $W = mgh = 1500 \times 9{\cdot}8 \times 20$

$$= \mathbf{2{\cdot}94 \times 10^5\ J}$$

$$\text{Rate of doing work} = \frac{\text{work}}{\text{time}} = \frac{2{\cdot}94 \times 10^5}{20}$$

$$= \mathbf{1{\cdot}47 \times 10^4\ W}$$

$$\text{As} \quad \eta = \frac{\text{output power}}{\text{input power}}$$

$\therefore$ Input power (= rate at which energy is supplied)

$$= \frac{1{\cdot}47 \times 10^4}{75\%} = \frac{1{\cdot}47 \times 10^4 \times 100}{75}$$

$$= \mathbf{1{\cdot}96 \times 10^4\ Js^{-1}}$$

Example 29 **The human heart discharges 75 ml of blood at each beat against a pressure of 0·1 m of Hg. Calculate the power of the heart assuming that the pulse frequency is 80 beats per minute. Given, density of mercury = 13·6 × 10^3 kg/m^3.**

Solution. Here, Volume of blood discharged/beat, $V = 75 \text{ ml} = 75 \times 10^{-6}\ \text{m}^3$

pressure, $P = 0{\cdot}1$ m of Hg

$$P = 0{\cdot}1 \times (13{\cdot}6 \times 10^3) \times 9{\cdot}8\ \text{Nm}^{-2}$$

[from $P = h\rho g$]

Work done/beat = PV

Total work done in 80 beats = PV × 80

$$\text{As} \quad \text{Power} = \frac{\text{work}}{\text{time}}$$

$$\therefore \quad \text{Power} = \frac{\text{PV} \times 80}{t}$$

$$= \frac{0{\cdot}1 \times (13{\cdot}6 \times 10^3) \times 9{\cdot}8 \times (75 \times 10^{-6}) \times 80}{60}$$

$$= \mathbf{1{\cdot}33\ watt}$$

Example 30 **An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watt and in horse power.**

(NCERT Solved Example)

Solution. Here, $m = 1800$ kg

frictional force, $f = 4000$ N ;

uniform speed, $v = 2{\cdot}0$ m/s

Downward force on elevator is

$$F = mg + f = 1800 \times 10 + 4000 = 22000 \text{ N}$$

The motor must supply enough power to balance this force. As

$$P = F \times v$$

$$\therefore \quad P = 22000 \times 2 = \mathbf{44000\ watt}$$

$$P = \frac{44000}{746} \text{ h.p.} = \mathbf{59\ h.p}$$

Example 31 **The turbine pits at Niagra Falls are 50 m deep. The average horse power developed is 5000. If the efficiency of the generator is 85%, how much water passes through the turbine per minute ? Take $g = 10$ m/s^2.**

Solution. Here, $h = 50$ m, H.P. = 5000, $\eta = 85\%$, m_2 = ? useful power = 5000 HP = 5000 × 746 watt.

Total power generated

$$= \frac{5000 \times 746}{85\%} = \frac{500 \times 746 \times 100}{85}$$

$$= 4{\cdot}388 \times 10^6 \text{ watt.}$$

Total work done by the falling water in 1 minute

$$mg \times h = 4{\cdot}388 \times 10^6 \times 60 \text{ J}$$

$$m = \frac{4{\cdot}388 \times 10^6 \times 60}{gh}$$

$$= \frac{4{\cdot}388 \times 10^6 \times 60}{10 \times 50}$$

$$m = \mathbf{5{\cdot}39 \times 10^5\ kg}$$

TYPE VII. COLLISIONS

Formulae used

1. In both elastic collisions and inelastic collisions, linear momentum is conserved.

i.e., $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

2. In elastic collisions, kinetic energy is also conserved

i.e., $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

3. In one dimensional perfectly elastic collision, velocities after collision are given by :

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2 u_2}{m_1 + m_2}$$

$$v_2 = \frac{2m_1 u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2}$$

4. The degree of elasticity of a collision is measured by coefficient of restitution (e),

$$e = \frac{\text{rel. vel. of separation after collision}}{\text{rel. vel. of approach before collision}} = \frac{v_2 - v_1}{u_1 - u_2}$$

For a perfectly elastic collision, $e = 1$, and for a perfectly inelastic collision, $e = 0$

5. In a perfectly inelastic collision in one dimension $m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

Units used. m_1, m_2 are in kg, velocities u_1, u_2, v_1, v_2 are in m/s, linear momentum in kg ms^{-1} and energy in joule.

Example 32 **A ball is dropped from a height h. It rebounds from the ground a number of times. If coefficient of restitution is e, to what height does it go after nth rebounding ?**

Solution. Let v_0 = velocity with which ball strikes the ground first time, after v_n is velocity after nth rebound.

$$\therefore \quad e = \frac{v_1}{v_0} = \frac{v_2}{v_1} = \frac{v_3}{v_2} = ... = \frac{v_n}{v_{n-1}}$$

$$e^n = \frac{v_1}{v_0} \times \frac{v_2}{v_1} \times \frac{v_3}{v_2} \times .... \frac{v_n}{v_{n-1}} = \frac{v_n}{v_0} \qquad ...(i)$$

But $v_0 = \sqrt{2gh}$ and $v_n = \sqrt{2gH}$.

Here, H is the height to which the ball rises after nth rebounding.

From (*i*),

$$e^n = \frac{v_n}{v_0} = \frac{\sqrt{2gH}}{\sqrt{2gh}} = \sqrt{\frac{H}{h}}$$

$$\therefore \qquad \mathbf{H = e^{2n} \times h}$$

Example 33 **A plastic ball is dropped from a height of 1 m and rebounds several times from the floor. If 1·3 s elapses from the moment it is dropped to the second impact with the floor, what is the coefficient of restitution ?**

Solution. Here, $h = 1$ m, $u = 0$.

Velocity with which the ball strikes the floor for the first time,

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 9{\cdot}8 \times 1} = 4{\cdot}427 \text{ m/s}$$

Time taken by the ball to strike the floor for the first time,

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1}{9{\cdot}8}} = \mathbf{0{\cdot}452\,s}$$

Time taken by the ball between first and second impact with the floor is

$$t_2 = 1{\cdot}3 - t_1 = 1{\cdot}3 - 0{\cdot}452 = 0{\cdot}848 \text{ s}$$

Time taken by the ball for upward journey after first impact

$$= \frac{t_2}{2} = \frac{0{\cdot}848}{2} = 0{\cdot}424 \text{ s}$$

$\therefore$ upward velocity after first impact

$$v_2 = g \times \frac{t_2}{2} = 9{\cdot}8 \times 0{\cdot}424 = 4{\cdot}155 \text{ m/s}$$

$\therefore$ Coefficient of restitution,

$$e = \frac{v_2}{v_1} = \frac{4{\cdot}155}{4{\cdot}427} = \mathbf{0{\cdot}939}$$

Example 34 **In a nuclear reactor, a neutron of high speed ($\approx 10^7$ ms^{-1}) must be slowed down to 10^3 ms^{-1} so that it can have a high probability of interacting with isotope $_{92}U^{235}$ and causing it to fission. Show that a neutron can lose most of its K.E. in an elastic collision with a light nuclei like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei usually heavy water (D_2O) or graphite is called a moderator.**

(NCERT Solved Example)

Solution. Initial K.E. of neutron is

$$K_1 = \frac{1}{2} m_1 u_1^2$$

Velocity of neutron after collision with deuterium,

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2}$$

Final K.E. of neutron is

$$K_2 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$$

$\therefore$ Fractional K.E. retained by neutron is

$$f_1 = \frac{K_2}{K_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

For deuterium, $m_2 = 2\,m_1 \quad \therefore \quad f_1 = \frac{1}{9}$,

Fractional K.E. lost by neutron

$$f_2 = 1 - f_1 = 1 - \frac{1}{9} = \frac{8}{9} \approx \mathbf{90\%}$$

This is the fractional K.E. gained by moderating nuclei.

Therefore, almost 90% of neutron energy is transferred to deuterium. Similarly, in case of carbon, we can show that $f_1 = \mathbf{28{\cdot}4\%}$ and $f_2 = \mathbf{71{\cdot}6\%}$

Example 35 **Consider the collision depicted in Fig. 6.42 to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue and the second ball is called the target. The billiard player wants to sink the target ball in a corner pocket, which is at an angle $\theta_2 = \phi = 37^\circ$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain $\theta_1 = \theta$.** (NCERT Solved Example)

Solution. If $\vec{u}_1$ is initial velocity of cue, and $\vec{u}_2 = 0$ is initial velocity of target

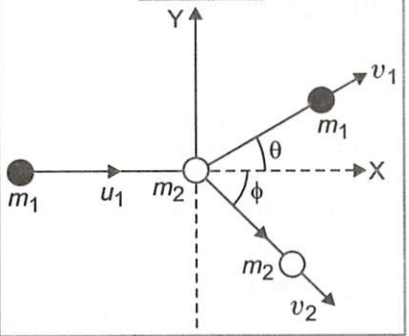
$\vec{v}_1$ = final velocity of cue, and $\vec{v}_2$ = final velocity of target, then from momentum conservation,

$$m\vec{u}_1 = m\vec{v}_1 + m\vec{v}_2 \quad \text{or} \quad \vec{u}_1 = \vec{v}_1 + \vec{v}_2$$

$$u_1^2 = \vec{u}_1 . \vec{u}_1 = (\vec{v}_1 + \vec{v}_2).(\vec{v}_1 + \vec{v}_2)$$

$$u_1^2 = v_1^2 + v_2^2 + 2\vec{v}_1 . \vec{v}_2 \qquad \text{...}(i)$$

FIGURE 6.42



According to conservation of K.E.

$$\frac{1}{2}mu_1^2 = \frac{1}{2}m(v_1^2 + v_2^2) \quad \therefore \ u_1^2 = v_1^2 + v_2^2$$

Put in (*i*), $v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2\vec{v}_1 . \vec{v}_2$

$$\therefore \qquad 2\vec{v}_1 . \vec{v}_2 = 0$$

$$2\,v_1\,v_2 \cos(\theta + \phi) = 0$$

$$\cos(\theta_1 + 37^\circ) = 0 = \cos 90^\circ$$

$$\theta_1 + 37^\circ = 90^\circ$$

$$\theta_1 = 90^\circ - 37^\circ = \mathbf{53^\circ}$$

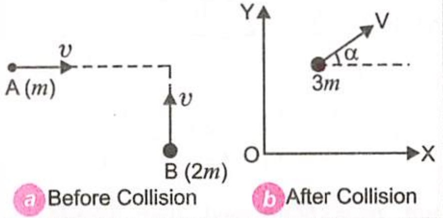
Retain in Memory

When two equal masses undergo a glancing elastic collision with one of them at rest, then after the collision, they will move at 90° to eachother.

Example 36 **Two particles *A* and *B* of masses m and 2 m are moving along the *X*-axis and *Y*-axis respectively with the same speed v. They collide at the origin and coalesce into one body, after the collision. What is the velocity of this coalesced mass ? What is the loss of energy during this collision ?** (NCERT Solved Example)

Solution. Fig. 6.43(*a*) shows particles *A* and *B* before collision and Fig. 6.43(*b*) shows the coalesced mass after collision.

FIGURE 6.43



(a) Before Collision (b) After Collision

In Fig. 6.43(*b*), we have taken velocity of combined mass (3 m) as $\vec{V}$ making an angle α with positive *X*-axis.

Using law of conservation of linear momentum

(*i*) *along X-axis*

$$mv = 3\,m\,V\cos\alpha \qquad \text{...}(i)$$

(*ii*) *along Y-axis*

$$2\,mv = 3\,m\,V\sin\alpha \qquad \text{...}(ii)$$

Dividing (*ii*) by (*i*), we get

$$2 = \tan\alpha \text{ or } \alpha = \tan^{-1}(2) = \mathbf{63{\cdot}4^\circ}$$

Squaring (*i*) and (*ii*) and adding, we get

$$(3\,m\,V)^2(\cos^2\alpha + \sin^2\alpha) = m^2v^2 + 4\,m^2v^2$$
$$= 5\,m^2v^2$$

$$V^2 = \frac{5}{9}v^2 \quad \text{or} \quad V = \frac{\sqrt{5}}{3}v$$

Now, total K.E. before collision,

$$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 = \frac{3}{2}mv^2$$

total K.E. after collision,

$$K_2 = \frac{1}{2}(3m)V^2 = \frac{1}{2}(3m)\frac{5}{9}v^2 = \frac{5}{6}mv^2$$

Loss of K.E. during the collision,

$$\Delta K = K_1 - K_2$$

$$\Delta K = \frac{3}{2}mv^2 - \frac{5}{6}mv^2 = \frac{2}{3}mv^2$$

Example 37 **Two particles of masses 0·5 kg and 0·25 kg moving with velocities 4·0 m/s and – 3·0 m/s collide head on in a perfectly inelastic collision. Find the velocity of the composite particle after collision and KE lost in the collision.**

Solution. Here, $m_1 = 0{\cdot}5$ kg, $m_2 = 0{\cdot}25$ kg

$u_1 = 4{\cdot}0$ m/s, $u_2 = -3{\cdot}0$ m/s

If v is velocity of composite particle, then from the principle of conservation of linear momentum,

$$(m_1 + m_2)\,v = m_1u_1 + m_2u_2$$

$$(0{\cdot}5 + 0{\cdot}25)\,v = 0{\cdot}5 \times 4 + 0{\cdot}25\,(-3{\cdot}0)$$

$$v = \frac{2 - 0{\cdot}75}{0{\cdot}75} = \frac{1{\cdot}25}{0{\cdot}75} = \mathbf{1{\cdot}67\ m/s}$$

$$\text{KE lost} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)\,v^2$$

$$= \frac{1}{2}\times 0{\cdot}5 \times 4^2 + \frac{1}{2}\times 0{\cdot}25 \times (-3{\cdot}0)^2 - \frac{1}{2}(0{\cdot}5 + 0{\cdot}25)\left(\frac{5}{3}\right)^2$$

$$= 4 + \frac{9}{8} - \frac{25}{24} = \frac{98}{24} = \mathbf{4{\cdot}1\ J}$$

TYPE VIII. TYPICAL EXAMPLES

Example 38 **A toy rocket of mass 0·1 kg has a small fuel of mass 0·02 kg, which it burns out in 3s. Starting from rest on a horizontal smooth track, it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket ? What is the energy content per unit mass of the fuel ? (Ignore the small mass variation of the rocket during fuel burning).**

Solution. Here,

mass of toy rocket, $m_1 = 0{\cdot}1$ kg ;

mass of fuel, $m_2 = 0{\cdot}02$ kg ; $t = 3\ s$

initial velocity $u = 0$

final velocity $v = 20$ m/s

From $v = u + at$

$$20 = 0 + a \times 3,\ a = \frac{20}{3}\ \text{ms}^{-2}$$

Thrust on the rocket, $F = m_1a$

$$= 0{\cdot}1 \times \frac{20}{3} = \mathbf{\frac{2}{3}\ N}$$

If s is the distance moved by the rocket in 3 sec, then from

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times\frac{20}{3}\times 3 \times 3 = 30\ \text{m}.$$

Energy content of the fuel

= Work done on rocket

= Thrust × distance

$$= \frac{2}{3}\times 30 = 20\ \text{J}$$

Energy content per unit mass of the fuel

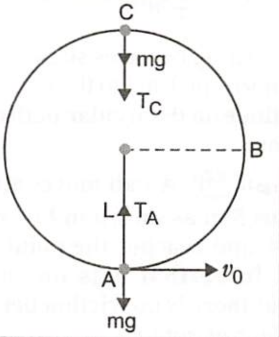
$$= \frac{20}{0{\cdot}02} = \mathbf{1000\ joule/kg}$$

Example 39 **A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack on reaching the topmost point C, Fig. 6.44. Obtain an expression for (*i*) v_0 (*ii*) the speeds at points B and C. (*iii*) the ratio of kinetic energies (K_B/K_C) at B and C.**

Comment on the nature of the trajectory of the bob after it reaches the point C.

(NCERT Solved Example)

FIGURE 6.44



Solution. There are two external forces on the bob : gravity (mg) and tension (T) in the string. The tension does no work as displacement is always perpendicular to the string. Total mechanical energy (E) of the system is conserved.

If we take P.E. of the system to be zero at the lowest point A, then

At A, $$E = \frac{1}{2} m v_0^2 \qquad ...(i)$$

The necessary centripetal force mv_0^2 / L is provided by $(T_A - mg)$.

$$\therefore \qquad T_A - mg = \frac{mv_0^2}{L} \qquad ...(ii)$$

At the highest point C, the string slackens as the tension in the string (T_c) becomes zero.

$$\therefore \text{ At C,} \qquad mg = \frac{mv_C^2}{L} \qquad ...(iii)$$

and total energy, $$E = \frac{1}{2} m v_C^2 + mg\,(2L) \; ...(iv)$$

From (*iii*), $$v_C = \sqrt{gL} \qquad ...(v)$$

From (*iv*), $$E = \frac{1}{2} m (gL) + 2mgL = \frac{5}{2} mgL$$

Using (*i*), $$\frac{1}{2} m v_0^2 = \frac{5}{2} mg\,L, \quad v_0 = \sqrt{5gL} \qquad ...(vi)$$

At B, the energy is $$E = \frac{1}{2} m v_B^2 + mg\,(L)$$

or $$\frac{1}{2} m v_B^2 = E - mg\,(L) = \frac{5}{2} mg\,L - mg\,L$$

$$= \frac{3}{2} mg\,L$$

$$v_B = \sqrt{3gL}$$

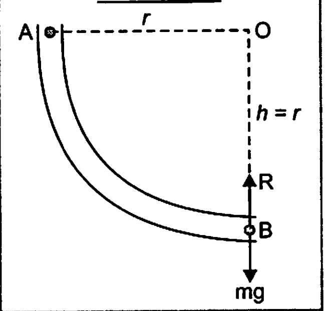
$$\frac{K_B}{K_C} = \frac{\frac{1}{2} m v_B^2}{\frac{1}{2} m v_C^2} = \frac{3gL}{gL} = \frac{3}{1}$$

At C, the string becomes slack and the velocity of the bob is horizontal and to the left. Therefore, the **bob will continue on its circular path and complete the revolution.**

Example 40 **A ball moves along a curved path of radius 5 m as shown in Fig. 6.45. It starts from point *A* and reaches the point *B*. Calculate the normal force that acts on the ball at *B*, assuming that there is no friction between the ball and the surface of contact.**

Solution.

FIGURE 6.45



As is clear from Fig. 6.45,

P.E. of ball at A = K.E. of ball at B

$$mgh = \frac{1}{2} mv^2$$

$$v^2 = 2\,gh = 2\,gr$$

centripetal force on the ball at B

$$f = \frac{mv^2}{r} = \frac{m}{r}(2\,gr) = 2\,mg$$

If R is normal force of reaction acting on the ball in the upward direction at B, then

$$f = R - W \text{ or } R = f + W = 2\,mg + mg = 3\,mg$$

Example 41 **A bullet of mass 0·012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0·4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by thin wire. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.**

Solution. Here, $m_1 = 0{\cdot}012$ kg, $u_1 = 70$ m/s ;

$$m_2 = 0{\cdot}4 \text{ kg}, \; u_2 = 0$$

As the bullet comes to rest with respect to the block, the two behave as one body. Let v be the velocity acquired by the combination.

Applying principle of conservation of linear momentum, $(m_1 + m_2)\,v = m_1 u_1 + m_2 u_2 = m_1 u_1$

$$v = \frac{m_1 u_1}{m_1 + m_2} = \frac{0{\cdot}012 \times 70}{0{\cdot}012 + 0{\cdot}4} = \frac{0{\cdot}84}{0{\cdot}412} = 2{\cdot}04\,\text{ms}^{-1}$$

Let the block rise to a height h.

P.E. of the combination = K.E. of the combination

$$(m_1 + m_2)\,gh = \frac{1}{2}\,(m_1 + m_2)\,v^2$$

$$\therefore \qquad h = \frac{v^2}{2g} = \frac{2{\cdot}04 \times 2{\cdot}04}{2 \times 9{\cdot}8} = 0{\cdot}212\text{m}$$

For calculating heat produced, we calculate energy lost (W), where

W = initial K.E. of bullet – final K.E. of combination

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) v^2$$

$$= \frac{1}{2} \times 0{\cdot}012 (70)^2 - \frac{1}{2}(0{\cdot}412)(2{\cdot}04)^2$$

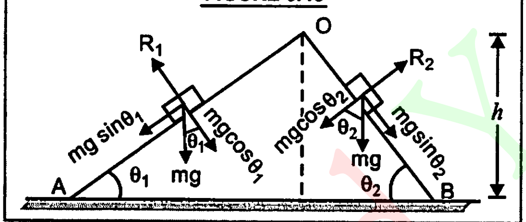
$$W = 29{\cdot}4 - 0{\cdot}86 = 28{\cdot}54 \text{ joule}$$

$$\therefore \text{ Heat produced, } H = \frac{W}{J} = \frac{28{\cdot}54}{4{\cdot}2} = \mathbf{6{\cdot}8\,cal.}$$

Example 42 **Two inclined frictionless tracks, one gradual and the other steep meet at O, from where two stones are allowed to slide down from rest, one on each track. Will the stones reach the bottom at the same time ? Will they reach there with the same speed ? Explain.**

Solution. In Fig. 6.46, OA and OB are two smooth planes inclined to the horizontal at $\angle\theta_1$ and $\angle\theta_2$ respectively.

FIGURE 6.46



As height of both the planes is the same, therefore, both the stones will reach the bottom with *same speed*.

As P.E. at O = K.E. at A = K.E. at B

$$\therefore \quad mgh = \frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 \quad \therefore \quad v_1 = v_2$$

As is clear from Fig. 6(N).8, acceleration of the two blocks are

$$a_1 = g \sin\theta_1 \quad \text{and} \quad a_2 = g \sin\theta_2$$

As $\theta_2 > \theta_1 \quad \therefore \quad a_2 > a_1$

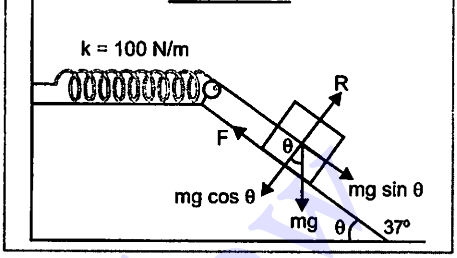
From $v = u + a\,t = 0 + at$ or $t = \dfrac{v}{a}$

As $t \propto \dfrac{1}{a}$, and $a_2 > a_1 \quad \therefore \quad t_2 < t_1$

i.e., **Second stone will take lesser time and reach the bottom earlier than the first stone.**

Example 43 **A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 Nm^{-1} as shown in Fig. 6.47. The block is released from rest with the spring in the unstretched position.**

FIGURE 6.47



The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is frictionless.

Solution. As is clear from Fig. 6.47.

$$R = mg \cos\theta,$$

$$F = \mu R = \mu\, mg \cos\theta$$

Net force on the block down the incline

$$= mg \sin\theta - F$$

$$= mg \sin\theta - \mu\, mg \cos\theta$$

$$= mg (\sin\theta - \mu \cos\theta)$$

distance moved, $x = 10$ cm $= 0{\cdot}1$ m.

In equilibrium,

work done = P.E. of stretched spring

$$mg(\sin\theta - \mu\cos\theta)\, x = \frac{1}{2} K x^2$$

$$2\, mg(\sin\theta - \mu\cos\theta) = K x$$

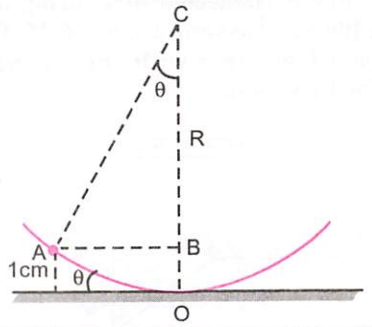
$$2 \times 1 \times 10 (\sin 37^\circ - \mu \cos 37^\circ) = 100 \times 0{\cdot}1$$

$$20 (0{\cdot}601 - \mu \,.\, 0{\cdot}798) = 10$$

$$\therefore \quad \mu = \mathbf{0{\cdot}126}$$

Example 44 **A particle of mass 1 g executes an oscillatory motion on the concave surface of a spherical dish of radius 2 m placed on a horizontal plane, Fig. 6.48. If the motion of the particle begins from a point on the dish at a height of 1 cm. from the horizontal plane and coefficient of friction is 0·01, find the total distance covered by the particle before coming to rest.**

FIGURE 6.48



Solution. In Fig. 6.48, $CO = CA = 200$ cm,

$OB = 1$ cm.

$\therefore \quad BC = CO - OB = 200 - 1 = 199$ cm.

In ΔABC, $\quad \cos\theta = \frac{BC}{AC} = \frac{199}{200}$

Treating the dish as an inclined plane, and using the principle of energy conservation, *i.e.*,

Loss in P.E. = work done against friction

$$m\,g\,h = \mu\,R \times s = \mu\,mg\cos\theta \times s$$

$$s = \frac{m\,g\,h}{\mu\,mg\cos\theta} = \frac{h}{\mu\cos\theta}$$

$$s = \frac{1/100}{0{\cdot}01 \times 199/200} = \frac{200}{199} = \mathbf{1.005\ m}$$

Example 45 **A particle of mass 0·1 kg has an initial speed of 4 ms^{-1} at a point *A* on a rough horizontal road. The coefficient of friction between the object and road is 0·15. The particle moves to a point *B* at a distance of 2 m from *A*. What is the speed of particle at *B* ? Take $g = 10$ ms^{-2}** (NCERT Solved Example)

Solution. Here,

$m = 0{\cdot}1$ kg, $u = 4$ ms^{-1},

$\mu = 0{\cdot}15$, $v = ?$

$s = 2$ m, $a = -\mu\,g = -0{\cdot}15 \times 10$

$= -1{\cdot}5$ m/s^2

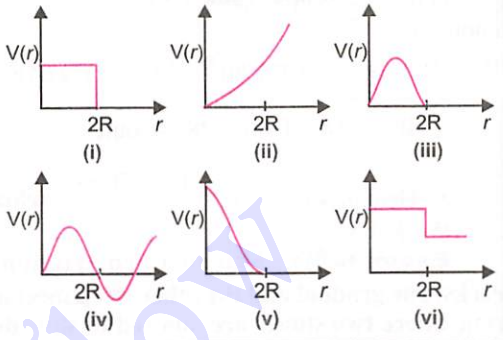
From $\quad v^2 - u^2 = 2\,as$

$$v^2 = u^2 + 2\,as = 4^2 + 2\,(-1{\cdot}5) \times 2 = 10$$

$$v = \sqrt{10} = \mathbf{3{\cdot}16\ m/s}$$

Example 46 **Which of the following potential energy curves in Fig. 6.49 cannot possibly describe the elastic collision of two billiard balls ? Here *r* is distance between centres of the balls.**

FIGURE 6.49

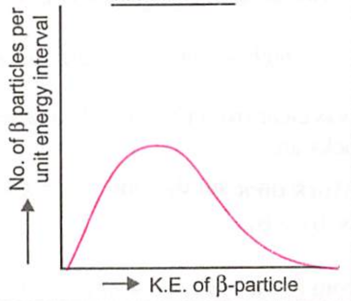


Solution. The potential energy of a system of two masses varies inversely as the distance (r) between them, *i.e.*, $V(r) \propto \frac{1}{r}$. When the two billiard balls touch eachother, P.E. becomes zero, *i.e.*, at $r = R + R = 2R$; $V(r) = 0$. Out of the given graphs, curve (v) only satisfies these two conditions. Therefore, all other curves cannot possibly describe the elastic collision of two biliard balls.

Example 47 **Consider the decay of a free neutron at rest : $n \longrightarrow p + e^-$.**

Show that the two body decay of this type must necessarily give an electron of fixed energy, and therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron in a nucleus, Fig. 6.50.

FIGURE 6.50



Solution. In the decay process, $n \rightarrow p + e$ energy of electron is equal to $(\Delta\,m)\,c^2$

where $\Delta\,m$ = mass defect = mass of neutron − mass of proton and electron ;

which is fixed. Therefore, two body decay of this type cannot explain the observed continuous energy distribution in the β-decay of a neutron in a nucleus.

Note. The simple result of this exercise was one among the several arguments advanced by W. Pauli to predict the existence of a third particle in the decay products of β-decay. This particle is known as neutrino. We now know that it is a particle of intrinsic spin 1/2 (like e^-, p or n), but is neutral, and either massless or having an extremely small mass (compared to electron's mass). This new particle interacts very weakly with matter. The correct decay process of neutron is : $n \rightarrow p + e^- + \bar{\nu}$].

REVISION AT A GLANCE

1. **Work done** (*W*) by a constant force $\vec{F}$ in producing a displacement $\vec{s}$ in a body is

$$W = \vec{F}.\vec{s} = Fs\cos\theta\,,$$

where θ is smaller angle between $\vec{F}$ and $\vec{s}$.

However, if the force is not constant,

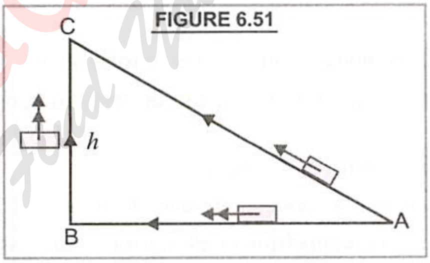
$$W = \int \vec{F}.d\vec{s} \text{ = area under the force displacement graph, added with proper sign.}$$

Work done is a *scalar* quantity.

However, work done is *positive* when θ lies between 0 and π/2. Work done is *negative* when θ lies between $\frac{\pi}{2}$ and π. Work done is *zero*, when $\theta = \frac{\pi}{2}$

2. **SI unit of work** is *joule* and the cgs unit of work is *erg.*, where 1 joule = 10^7 ergs.
3. Work done by a body *does not depend upon the time taken* to complete the work.
4. **In a conservative field,** work done is independent of the path followed by the body. For example, in moving a body of mass *m* through a height *h* in a gravitational field is *same*, via the path *AC* or the path *ABC*, each being equal to *mgh*, Fig. 6.51.

FIGURE 6.51



5. **Work done by spring force**

$$W = -\frac{1}{2}k\,x^2$$

where *k* is spring constant, being equal to restoring force per unit displacement and *x* is the displacement from normal position of rest. When one end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal table from $x = x_1$ to $x = x_2$, then

$$W = \frac{-1}{2}k(x_2^2 - x_1^2)$$

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6. **Power** of a body is defined as the *time rate of doing work* by the body. Thus, in power, time taken by the body to complete that work is significant.

$$P = \frac{dW}{dt} = \frac{\vec{F}.\vec{ds}}{dt} = \vec{F}.\vec{v} \qquad \left(\because \vec{v} = \frac{\vec{ds}}{dt}\right)$$

Thus, $P = \vec{F}.\vec{v} = F\,v\cos\theta$

Here, θ is smaller angle between force $\vec{F}$ and velocity $\vec{v}$ of the body.

Power is a *scalar* quantity. SI unit of power is watt (W), where 1 W = 1 J/s

The practical unit of power is horse power (*h.p*), where 1 *h.p* = 746 W

7. **Energy** of a body is defined as the *capacity of the body* to do the work. Energy is a *scalar* quantity. It has the same units and dimensions as those of work.

Some practical units of energy and their relation with SI unit of enery (joule) are :

(*i*) 1 calorie = 4·2 J.

(*ii*) 1 kilowatt hour (kWh) = $3{\cdot}6 \times 10^6$ J.

(*iii*) 1 electron volt (1 eV) = $1{\cdot}6 \times 10^{-19}$ J

Energy can appear in many forms *e.g.*, mechanical, chemical, thermal (heat), optical (light), acoustical (sound), molecular, atomic, nuclear, solar, stellar and so on. Energy can be changed from one form to the other. The principle governing this change is the "*Principle of conservation of energy*", according to which sum total of energy in this universe remains constant. The amount of energy disappearing in one form is exactly equal to the amount of energy appearing in any other form.

8. **Kinetic Energy** of a body is the energy possessed by the body by virtue of its motion. A bullet fired from a gun has *K.E. of translation* and a rotating wheel on a machine has *K.E. of rotation.*

K.E. of translation $= \frac{1}{2}mv^2$ where m is mass and v is velocity of the body.

K.E. of rotation $= \frac{1}{2}I\,\omega^2$

where I is moment of inertia of the body about the axis of rotation and ω is angular velocity of the body.

9. The energy stored in a body or system by virtue of its configuration/position (in a field) is called *potential energy.*

$$\text{Gravitational P.E.} = mgh$$

where m is mass of a body at a height h above a reference level, where acceleration due to gravity is g.

The energy associated with state of compression or expansion of an elastic spring is called *elastic potential energy.*

10. **Mechanical Energy**

Mechanical energy E of a particle, object or system is defined as the sum of K.E. and P.E. of the system. Note that K.E. is always positive, but P.E. may be positive or negative. Infact, when forces involved are repulsive, P.E. is positive. When forces involved are attractive, P.E. is negative. The mechanical energy may be zero, positive or negative. Negative mechanical energy represents a *bound state, e.g.,* an electron in an atom or a satellite moving around a planet.

11. According to **work energy theorem**, work and energy are equivalent. Work done by a force on a body results in the net increase in K.E. of the body by the same amount. Work drawn from a body results in the decrease of K.E. of the body by the same amount.

12. According to Einstein, mass can be converted into energy and energy can be converted into mass. One can be obtained at the cost of the other. The equivalence between mass and energy is expressed in terms of *Einstein relation.* $E = mc^2$

where m is the mass that disappears and E is the energy that appears, c is velocity of light in vacuum

$$c = 3 \times 10^8 \text{ m/s.}$$

13. **Conservative and non-conservative forces**

A force is said to be conservative, when work done by or against the force in moving a body from one point to another is same for all paths between the two points. In addition, work done by a conservative force along a closed path is always zero. A force, which does not obey the above two characteristics is said to be a *non-conservative force.* For example, gravitational forces and electrostatic forces are conservative forces. On the contrary, force of friction is a non-conservative force.

14. **Collisions**

When a body strikes against another body or one body influences the other from a distance, the two are said to collide. Collisons are of two types :

(*a*) *Perfectly elastic collision* is that in which there is *no change in kinetic energy of the system, i.e.,*

total K.E. before collision = total K.E. after collision. For example, collisions between atomic and subatomic particles are perfectly elastic collisions.

(*b*) *Perfectly inelastic collision* is that in which *K.E. is NOT conserved.* Here, the bodies stick together after impact.

Note that *linear momentum* is conserved in every collision, elastic as well as inelastic. Further, total energy is also conserved in all such collisions. *K.E. alone is not conserved in inelastic collisions.*

15. If the initial and final velocities of colliding masses lie along the same line (along the common normal) then it is known as **head on collision.**

From the law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

From the principle of conservation of K.E.,

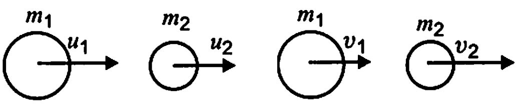
$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

We can show that $$v_1 = \frac{(m_1 - m_2)}{m_1 + m_2} u_1 + \frac{2 m_2 u_2}{m_1 + m_2}$$

and $$v_2 = \frac{2 m_1 u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2} u_2$$

16. In a head on collision, relative velocity after impact is in a constant ratio to the relative velocity of the bodies before impact, and is in the opposite direction. From Fig. 6.52,

FIGURE 6.52



$$\frac{v_1 - v_2}{u_1 - u_2} = -e$$

The constant e is known as *coefficient of restitution* or *resilience of the two bodies.*

For a perfectly elastic collision, $e = 1$ and for *a perfectly inelastic collision,* $e = 0$. Thus, $0 \leq e \leq 1$.

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. How high must a body be lifted so that it gains PE equal to KE while moving with a velocity of 30 m/s ? Take $g = 10$ m/s^2

(*a*) 45 m (*b*) 30 m

(*c*) 60 m (*d*) 20 m

2. Elastic potential energy of a spring in relaxed position is

(*a*) positive (*b*) negative

(*c*) infinite (*d*) zero

3. Energy required to break one bond in DNA is

(*a*) 10^{20} J (*b*) 10^{12} J

(*c*) 10^{7} J (*d*) 10^{-13} J

4. Energy associated with Big Bang is of the order of

(*a*) 10^{40} J (*b*) 10^{60} J

(*c*) 10^{68} J (*d*) 10^{-68} J

5. An inelastic collision is that in which

(*a*) no loss of potential energy

(*b*) no less of kinetic energy

(*c*) no loss of total energy

(*d*) some loss of kinetic energy

6. When a body is thrown up, work done by gravity on the body is

(*a*) positive (*b*) zero

(*c*) negative (*d*) cannot say

7. A person holds on a weight of 10 kg at a height of 5 m above the ground for 5 minutes. Work done by him is

(*a*) zero (*b*) 50 J

(*c*) 250 J (*d*) 300 J

8. A body is undergoing non-uniform circular motion. Work done by radial force on the body is

(*a*) zero (*b*) positive

(*c*) negative (*d*) none of these

9. Which one of the following is a non-conservative force ?

(*a*) gravitational force (*b*) electrostatic force

(*c*) magnetic force (*d*) force of friction

10. The linear momentum of a body is increased by 10%. What is the percentage increase in its K.E. ?

(*a*) 10% (*b*) 20%

(*c*) 21% (*d*) none of these

11. Electric potential energy V between two charges varies with distance (r) between them as

(*a*) $V \propto r$ (*b*) $V \propto \frac{1}{r}$

(*c*) $V \propto r^2$ (*d*) $V \propto \frac{1}{r^2}$

12. In any kind of collision,

(*a*) linear momentum is always conserved

(*b*) kinetic energy is always conserved

(*c*) both (*a*) and (*b*)

(*d*) neither (*a*) nor (*b*)

13. For a perfectly elastic collision and a perfectly inelastic collision, values of coefficient of restitution are respectively

(*a*) 0, 0 (*b*) 0, 1

(*c*) 1, 1 (*d*) 1, 0

14. A light body and a heavy body have equal kinetic energy. Which has greater momentum ?

(*a*) The heavy body

(*b*) The light body

(*c*) Both have equal momentum

(*d*) Data given is incomplete

15. A boy pushes the wall and fails to displace it. Therefore he does :

(*a*) positive work

(*b*) negative work

(*c*) positive but not maximum work

(*d*) no work at all

16. Out of a pair of identical Spring Constant 240 Newton per metre, one is compressed by 10 cm and the other is stretched by 10 cm. The difference in potential energy stored in the two springs is :

(*a*) zero (*b*) 4 J

(*c*) 12 J (*d*) 1·2 J

17. Which of the following is unit of energy ?

(*a*) watt second (*b*) watt

(*c*) kilowatt (*d*) Joules per second

18. The work performed on an object does not depend on

(*a*) the force applied (*b*) the displacement

(*c*) the initial velocity of object

(*d*) the angle at which the force is inclined to the displacement

19. The force between a pair of protons is repulsive, what change is observed in potential energy, as the two protons are brought closer :

(*a*) increases (*b*) decreases

(*c*) remains unchanged

(*d*) may increase or decrease

20. How high must a body be lifted to gain an amount of potential energy equal to the kinetic energy it has, when moving at speed of 20 metre per second. (take value of acceleration due to gravity at that place equal to 10 metre per second square)

(*a*) 200 m (*b*) 20 m

(*c*) 2 m (*d*) 2000 m

21. Work done by a body against frictional force always results in a loss of :

(*a*) kinetic energy (*b*) potential energy

(*c*) both of them (*d*) none of them

22. A body of mass 50 kg slides over a horizontal distance of 1 m. If coefficient of friction between their surfaces is 0·2, then work done against friction is (Take $g = 9{\cdot}8\ \text{ms}^{-2}$)

(*a*) 98 J (*b*) 72 J

(*c*) 56 J (*d*) 34 J

23. A block of mass 1 kg slides down an inclined plane of inclination 60° starting from its top. If coefficient of kinetic friction is 0·5 and length of the plane is 1 m, then work done against friction is

(*a*) 19·6 J (*b*) 9·8 J

(*c*) 4·9 J (*d*) 2·45 J

24. One-fourth chain is hanging down from a table. The work done to bring the hanging part of the chain on the table is

(Mass of chain = *M* and length = *L*)

(*a*) $\dfrac{MgL}{32}$ (*b*) $\dfrac{MgL}{16}$

(*c*) $\dfrac{MgL}{8}$ (*d*) $\dfrac{MgL}{4}$

25. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle, which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle is

(*a*) – 7 J (*b*) + 7 J

(*c*) + 10 J (*d*) + 13 J

26. A constant force $(\vec{F}) = 3\hat{i} + c\hat{j} + 6\hat{k}$ acting on a particle causes a displacement : $(\vec{s}) = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If the work done is 6 J, then value of '*c*' is

(*a*) 0 (*b*) 1

(*c*) – 6 (*d*) 12

27. A particle moves from position $(\vec{r_1}) = 3\hat{i} + 2\hat{j} - 6\hat{k}$ to position $(\vec{r_2}) = 14\hat{i} + 13\hat{j} + 9\hat{k}$ under the action of a force $(\vec{F}) = 4\hat{i} + \hat{j} - 3\hat{k}$. Work done by the force is

(*a*) 50 units (*b*) 75 units

(*c*) 10 units (*d*) 200 units

28. The force constant of a spring is *k*. And that of another spring of the same material is 2 *k*. If both the springs are stretched through the same distance, then ratio between the works done by the springs is

(*a*) 1 : 1 (*b*) 2 : 1

(*c*) 1 : 2 (*d*) 1 : 4

29. The work done in stretching a certain spring, through a distance '*d*' is 20 J. The amount of work done to stretch this spring through an additional distance 2 *d* will be

(*a*) 160 J (*b*) 120 J

(*c*) 100 J (*d*) 200J

30. If speed of a vehicle increases by $2\ \text{ms}^{-1}$, its kinetic energy is doubled, then original speed of the vehicle is

(*a*) $(\sqrt{2}+1)\ \text{m-s}^{-1}$ (*b*) $2(\sqrt{2}-1)\ \text{m-s}^{-1}$

(*c*) $2(1 \pm \sqrt{2})\ \text{m-s}^{-1}$ (*d*) $2(\sqrt{2}+1)\ \text{m-s}^{-1}$

31. A particle is projected at an angle of 45° with the horizontal. The kinetic energy of the particle, at the highest point in its flight, is (where *K* is initial kinetic energy of the particle)

(*a*) zero (*b*) $\dfrac{K}{4}$

(*c*) $\dfrac{K}{2}$ (*d*) $\dfrac{3K}{4}$

32. The potential energy of a body is given by : $A - Bx^2$ (where *x* is the displacement). The magnitude of force acting on the particle is

(*a*) constant

(*b*) proportional to *x*

(*c*) proportional to x^2

(*d*) inversely proportional to *x*

33. The potential energy between two atoms in a molecule, is given by :

$$U_x = \frac{a}{x^{12}} - \frac{b}{x^6}$$

where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium, when

(*a*) $x = 0$ (*b*) $x = \left(\frac{a}{2b}\right)^{1/6}$

(*c*) $x = \left(\frac{2a}{b}\right)^{1/6}$ (*d*) $x = \left(\frac{11a}{5b}\right)^{1/6}$

34. A body whose mass is 120 g has a velocity $\vec{v} = (2\hat{i} + 5\hat{j})$ ms^{-1} at a certain instant. Its kinetic energy is

(*a*) 1·74 J (*b*) 2·75 J

(*c*) 3·75 J (*d*) 4·75 J

35. Two springs have their force constant k_1 and k_2. Both are stretched till their stored elastic potential energies are equal. The ratio of stretching force $F_1 : F_2$ is equal to

(*a*) $k_1 : k_2$ (*b*) $k_1^2 : k_2^2$

(*c*) $\sqrt{k_1} : \sqrt{k_2}$ (*d*) $\sqrt{k_2} : \sqrt{k_1}$

36. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, then the shortest distance in which the car can be stopped is

(*a*) $\frac{v_0^2}{\mu}$ (*b*) $\frac{v_0^2}{\mu g}$

(*c*) $\frac{v_0^2}{2\mu g}$ (*d*) $\frac{v_0}{\mu g}$

37. An open knife of mass '*m*' is dropped from a height '*h*' on a wooden floor. If the knife edge penetrates upto the depth '*d*' into the wood, the average resistance offered by the wood to the knife edge is

(*a*) mg (*b*) $mg\ 1 - \frac{h}{d}$

(*c*) $mg\left(1 + \frac{h}{d}\right)$ (*d*) $mg\ 1 + \frac{h}{d}$

38. A machine delivering constant power moves a body along a straight line. The distance moved by the body in time t is proportional to

(*a*) t (*b*) $\sqrt{t}$

(*c*) $t^{-\frac{3}{2}}$ (*d*) $t^{\frac{3}{4}}$

39. A body of mass m, accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time 't' is

(*a*) $\frac{mv_1 t}{t_1}$ (*b*) $\frac{mv_1^2 t}{t_1}$

(*c*) $\frac{mv_1 t^2}{t_1}$ (*d*) $\frac{mv_1^2 t^2}{t_1^2}$

40. A force of $(2\hat{i} + 3\hat{j} + 4\hat{k})$ N acts on a body for 4 s and produces a displacement of $(3\hat{i} + 4\hat{j} + 5\hat{k})$ m. The power used is

(*a*) 4·5 W (*b*) 6·5 W

(*c*) 7·5 W (*d*) 9·5 W

41. A body of mass 5 kg moving with a velocity 10 ms^{-1} collides with another body of mass 20 kg at rest and comes to rest. The velocity of second body, due to collision, is

(*a*) 2·5 ms^{-1} (*b*) 5 ms^{-1}

(*c*) 7·5 ms^{-1} (*d*) 10 ms^{-1}

42. A stationary particle explodes into two pieces of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies $\frac{K_1}{K_2}$ is

(*a*) 1 (*b*) $\frac{m_1}{m_2}$

(*c*) $\frac{m_2}{m_1}$ (*d*) $\frac{m_1 v_2}{m_2 v_1}$

43. A body of mass 40 kg and having velocity 4 ms^{-1} collides with another body of mass 60 kg having velocity 2 ms^{-1}. If after the collision, both the bodies move together, then loss in kinetic energy when the collision is perfectly inelastic is

(*a*) 385 J (*b*) 192 J

(*c*) 48 J (*d*) 12 J

44. Which of the following is a non-conservative force ?

(*a*) Gravitational force

(*b*) Force in elastic spring

(*c*) Electrostatic force between two charges

(*d*) Viscous force.

45. Choose the wrong statement out of the following.

(*i*) Air resistance is a non-conservative force

(*ii*) Time has nothing to do with energy (*iii*) 1 Mega electron volt = $1{\cdot}6 \times 10^{-13}$ J

(*iv*) Dimensional formula of power is $[ML^2T^{-2}]$

(*a*) (*iv*) only (*b*) (*iii*) only

(*c*) (*ii*) only (*d*) (*i*) only

46. For a given height, work done in moving a body over any smooth inclined plane of any inclination will be the same.

(*a*) True always

(*b*) False always

(*c*) depends on several other factors

(*d*) cannot predict

47. When a ball hits a bat, with a force F_1, the bat exerts a force F_2 on the ball equal in magnitude to F_1, but opposite in direction. Therefore $\vec{F_1} + \vec{F_2} = 0$.

The algebraic sum of work done on the ball and work done on the bat in this process will be

(*a*) never zero (*b*) zero always

(*c*) zero sometimes (*d*) cannot predict

48. Out of the following units of energy, select the largest one.

(*a*) electron volt (*b*) calorie

(*c*) erg (*d*) kilowatt hour

49. Which of the following is a vector ?

(*a*) Potential energy

(*b*) Kinetic energy

(*c*) Mechanical energy

(*d*) Force

50. How is force constant (k) of a spring related to length (l) of the spring ?

(*a*) $k \propto l$ (*b*) $k \propto \dfrac{1}{l}$

(*c*) $k \propto l^2$ (*d*) $k \propto 1/l^2$

51. If the forces doing work on a system are conservative, then what is conserved ?

(*a*) total mechanical energy

(*b*) potential energy

(*c*) kinetic energy

(*d*) none of the above

52. To cause the same change in K.E., smaller force has to be applied through

(*a*) smaller distance (*b*) larger distance

(*c*) same distance (*d*) none of the above

53. The energy equivalent of an electron of rest mass $9{\cdot}11 \times 10^{-31}$ kg is

(*a*) 512 keV (*b*) 512 MeV

(*c*) 512 eV (*d*) 931 MeV

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

54. Assertion. Energy and power represent the same thing.

Reason. More the energy, more the power and vice-versa.

55. Assertion. When a force $\vec{F}$ acting on a body moves it with a velocity $\vec{v}$, the power spent is $P = \vec{F} \cdot \vec{v}$.

Reason. Power is a scalar given by dot product of $\vec{F}$ and $\vec{v}$.

56. Assertion. Kilowatt hour is the unit of energy.

Reason. 1 kwh = $3{\cdot}6 \times 10^{13}$ ergs.

57. Assertion. Work is a scalar quantity. It has magnitude only and no direction.

Reason. Work done by a force can be positive, negative or zero.

58. Assertion. Work done by kinetic friction is not always negative.

Reason. Friction always supports relative motion.

59. Assertion. When a body is in static or dynamic equilibrium, work done on or by the body is zero.

Reason. In static or dynamic equilibrium, resultant force on the body is zero.

60. Assertion. The dimensional formula of power is $[M^1 L^2 T^{-3}]$.

Reason. Power is equal to rate at which work is done.

61. Assertion. Kinetic energy of a body depends upon the frame of reference.

Reason. Velocity of person is the same, whatever be the frame of reference.

62. Assertion. When linear momentum of a body is increased by 10%, its K.E. increases by 21%.

Reason. KE $\propto v^2$, irrespective of any other factor.

63. Assertion. When a body is moving with a constant linear momentum, its K.E varies inversely as its mass.

Reason. KE $= \dfrac{p^2}{2m}$, when p is constant, KE $\propto \dfrac{1}{m}$.

64. **Assertion.** In a nuclear reaction, the mass defect is 1 *amu*. The energy released is $1{\cdot}494 \times 10^{-10}$ J.

Reason. $E = (\Delta m)\, c^2$

$$= (1\ amu)\,(3 \times 10^8)^2$$
$$= 1{\cdot}66 \times 10^{-27}\,(3 \times 10^8)^2 \text{ J}$$
$$= 1{\cdot}494 \times 10^{-10} \text{ J}$$

65. **Assertion.** If we account for all forms of energy, the total energy of an isolated system does not change.

Reason. When the system is isolated, there is no question of gain or loss of energy-from any where. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

ANSWERS

I. Multiple Choice Questions

1. (*a*) **2.** (*d*) **3.** (*a*) **4.** (*c*) **5.** (*d*) **6.** (*c*) **7.** (*a*) **8.** (*a*) **9.** (*d*) **10.** (*c*)
11. (*b*) **12.** (*a*) **13.** (*d*) **14.** (*a*) **15.** (*d*) **16.** (*a*) **17.** (*a*) **18.** (*c*) **19.** (*a*) **20.** (*b*)
21. (*a*) **22.** (*a*) **23.** (*d*) **24.** (*a*) **25.** (*b*) **26.** (*c*) **27.** (*c*) **28.** (*c*) **29.** (*a*) **30.** (*c*)
31. (*c*) **32.** (*b*) **33.** (*c*) **34.** (*a*) **35.** (*c*) **36.** (*c*) **37.** (*c*) **38.** (*c*) **39.** (*d*) **40.** (*d*)
41. (*a*) **42.** (*c*) **43.** (*b*) **44.** (*d*) **45.** (*a*) **46.** (*a*) **47.** (*a*) **48.** (*d*) **49.** (*d*) **50.** (*b*)
51. (*a*) **52.** (*b*) **53.** (*a*)

II. Assertion-Reason Type Questions

54. (*d*) **55.** (*a*) **56.** (*b*) **57.** (*b*) **58.** (*c*) **59.** (*a*) **60.** (*a*) **61.** (*c*) **62.** (*c*) **63.** (*a*)
64. (*a*) **65.** (*a*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. Solution from Hand written page.

2. In relaxed position extension/compression $x = 0$. Elastic P.E = mg x = zero.

3. From the knowledge of theory, Energy required to break one bond in DNA is 10^{20} J.

4. Again, from the knowledge of theory, energy associated with Big Bang is ~ 10^{68} J.

5. In an inelastic collision, there is always some loss of kinetic energy.

6. Work done by gravity is negative because it acts downwards and motion is upwards.

7. $W = 0$, because distance moved, $s = 0$.

8. $W = 0$, because in this case, $\theta = 90°$, $\cos\theta = \cos 90° = 0$.

9. Force of friction is non-conservative.

10. $E = \dfrac{p^2}{2m}$

When p is increased by 10% $p' = \dfrac{11}{10}\,p$

$$E' = \frac{p'^2}{2m} = \left(\frac{11}{10}\right)^2 \frac{p^2}{2m} = \frac{121}{100}\,E$$

∴ Percentage increase in KE = 21%.

11. From theory, $V \propto \dfrac{1}{r}$

12. Linear momentum is conserved in all collisions.

13. For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision, $e = 0$.

14. From $p = \sqrt{2mK}$, $p \propto \sqrt{m}$.

∴ The heavy body has greater momentum.

15. $W = F\,s\cos\theta = 0\ (\because\ s = 0)$

16. As P.E. $= \dfrac{1}{2}Kx^2$, and in both cases, K and x are same, therefore, P.E. is same. Difference in P.E stored is zero.

17. Watt second is the unit of energy.

18. Work performed does not depend upon initial velocity of object.

19. P.E. will increase as work is done against the force of repulsion between two protons.

20. As $m\,gh = \dfrac{1}{2}\,mv^2$

$$\therefore\ h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = \mathbf{20\ m}$$

21. Work done against friction results in loss of kinetic energy.

22. Here, $m = 50$ kg, $s = 1$ m, $\mu = 0{\cdot}2$

$$W = \mu\, mg \times s = 0{\cdot}2 \times 50 \times 9{\cdot}8 \times 1$$
$$= \mathbf{98\ J}$$

23. Work done against friction

$$= \mu\, mg \cos\theta \times s = 0{\cdot}5 \times 1 \times 9{\cdot}8 \cos 60^\circ \times 1$$
$$= 4{\cdot}9 \times \frac{1}{2} \times 1 = \mathbf{2{\cdot}45\ J}$$

24. Weight of one fourth chain $= \dfrac{Mg}{4}$

Distance moved = distance of centre of gravity of hanging part from the table = $L/8$

$$\therefore\quad W = \frac{Mg}{4} \times \frac{L}{8} = \frac{\mathbf{MgL}}{\mathbf{32}}$$

25. Here, $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N

$$\vec{r} = (2\hat{i} - \hat{j})$$
$$W = \vec{F} \cdot \vec{r} = (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$$
$$= 10 - 3 = \mathbf{7\ J}$$

26. Here, $\vec{F} = 3\hat{i} + c\hat{j} + 6\hat{k}$

$$\vec{s} = -4\hat{i} + 2\hat{j} + 3\hat{k}\ ;\ W = -6\text{ J}$$
$$W = \vec{F} \cdot \vec{s} = (3\hat{i} + c\hat{j} + 6\hat{k}) \cdot (-4\hat{i} + 2\hat{j} + 3\hat{k})$$
$$-6 = -12 + 2c + 18 = 2c + 6$$
$$2c = -6 - 6 = -12,\ \mathbf{c = -6}$$

27. Here, $\vec{r} = \vec{r}_2 - \vec{r}_1$

$$= (14\hat{i} + 13\hat{j} + 9\hat{k}) - (3\hat{i} + 2\hat{j} - 6\hat{k})$$
$$= (11\hat{i} + 11\hat{j} + 15\hat{k})$$
$$W = \vec{F} \cdot \vec{r} = (4\hat{i} + \hat{j} - 3\hat{k}) \cdot (11\hat{i} + 11\hat{j} + 15\hat{k})$$
$$W = 44 + 11 - 45 = \mathbf{10\ units}$$

28. $W_1 = \dfrac{1}{2} k x^2$. $W_2 = \dfrac{1}{2}(2k)x^2 \ \therefore\ \dfrac{W_1}{W_2} = \dfrac{1}{2}$

29. $W_1 = \dfrac{1}{2} k d^2 = 20$ J,

$$W_2 = \frac{1}{2} k (d + 2d)^2 = \frac{1}{2} k\, (9d^2)$$
$$W = W_2 - W_1 = \frac{1}{2} k\, (9d^2) - \frac{1}{2} k d^2$$
$$= 8 \times \frac{1}{2} k d^2 = 8 \times 20\text{ J} = \mathbf{160\ J}$$

30. Let original speed of vehicle be v.

$$\text{KE} = E_1 = \frac{1}{2} m v^2$$
$$E_2 = \frac{1}{2} m\, (v + 2)^2 = 2E_1 = 2 \times \frac{1}{2} m v^2$$
$$(v + 2)^2 = 2v^2$$
$$v^2 + 4 + 4v = 2v^2$$
$$\text{or}\quad v^2 - 4v - 4 = 0$$
$$v = \frac{4 \pm \sqrt{16 + 16}}{2}$$
$$v = 2 \pm \frac{4\sqrt{2}}{2} = \mathbf{2(1 \pm \sqrt{2})}$$

31. If v is velocity of projection, then

$$K = \frac{1}{2} m v^2$$

At the highest point, $v' = v \cos 45^\circ = \dfrac{v}{\sqrt{2}}$.

$$\therefore\ \text{K.E.} = \frac{1}{2} m v'^2 = \frac{1}{2} m \left(\frac{v}{\sqrt{2}}\right)^2$$
$$= \frac{1}{2}\left(\frac{1}{2} m v^2\right) = \frac{\mathbf{K}}{\mathbf{2}}$$

32. Here, $U = A - Bx^2$

$$F = -\frac{dU}{dx} = -\frac{d}{dx}(A - Bx^2) = 2Bx$$
$$\therefore\quad F \propto x$$

33. Here, $U_x = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

$$-F = \frac{dU_x}{dx} = a(-12)\, x^{-13} - b\,(-6)\, x^{-7}$$

For stable equilibrium,

U_x = minimum, $F = 0$.

$$\frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$$
$$\frac{x^{13}}{x^7} = \frac{+12a}{6b} = \frac{2a}{b}\quad \text{or}\quad x = \left(\frac{2a}{b}\right)^{1/6}$$

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34. Here, $m = 120\text{ g} = 0{\cdot}120\text{ kg}$

$$v = (2\hat{i} + 5\hat{j})\text{ m/s} = \sqrt{2^2 + 5^2} = \sqrt{29}\text{ m/s}$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0{\cdot}120\,(\sqrt{29})^2$$

$$= \frac{29 \times 0{\cdot}12}{2} = \mathbf{1{\cdot}74\ J}$$

35. $F_1 = k_1 x_1,\ F_2 = k_2 x_2$

$$\frac{1}{2}k_1 x_1^2 = \frac{1}{2}k_2 x_2^2$$

$$x_1 (k_1 x_1) = x_2 (k_2 x_2)$$

$$x_1 F_1 = x_2 F_2 \qquad \therefore \quad \frac{F_1}{F_2} = \frac{x_2}{x_1} = \frac{\sqrt{k_1}}{\sqrt{k_2}}$$

36. From $v^2 - u^2 = 2\,as$

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - v_0^2}{-2\mu g} = \frac{v_0^2}{2\mu g}$$

37. If F is the average resistance offered by the wood, then

$$F \times d = mg\,(h + d)$$

$$F = \frac{mg\,(h+d)}{d} = mg\left(1 + \frac{h}{d}\right)$$

38. Power $= ML^2\,T^{-3}$ = constant

$$L^2 = \frac{\text{const.}}{MT^3} = \text{const. } T^{-3}$$

$$L \propto T^{-3/2} \text{ or } t^{-3/2}$$

39. From $v = u + at$

$$v_1 = 0 + at_1 \quad \therefore \quad a = \frac{v_1}{t_1}$$

Again, from $v = u + at = 0 + \left(\frac{v_1}{t_1}\right)t$

Power $= F \times v = ma \times v = m\left(\frac{v_1}{t_1}\right)\left(\frac{v_1}{t_1}\right)t$

$$P = m\,\frac{v_1^2}{t_1^2}\,t$$

40. $P = \frac{W}{t} = \frac{\vec{F} \cdot \vec{r}}{t}$

$$= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})}{4}$$

$$= \frac{6 + 12 + 20}{4} = \mathbf{9{\cdot}5\ W}$$

41. Here, $m = 5\text{ kg},\ u_1 = 10\text{ m/s},$

$m_2 = 20\text{ kg},\ u_2 = 0$

$v_1 = 0,\ v_2 = ?$

From $m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$

$5 \times 0 + 20\,v_2 = 5 \times 10 + 20 \times 0$

$$v_2 = \frac{50}{20} = \mathbf{2{\cdot}5\ m/s}$$

42. As the particle was stationary,

$\therefore\ m_1 v_1 - m_2 v_2 = 0$

or $\frac{v_1}{v_2} = \frac{m_2}{m_1}$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{m_1}{m_2}\left(\frac{m_2}{m_1}\right)^2 = \frac{\mathbf{m_2}}{\mathbf{m_1}}$$

43. Here, $m_1 = 40\text{ kg},\ u_1 = 4\text{ m/s}$

$m_2 = 60\text{ kg},\ u_2 = 2\text{ m/s}$

If v is combined velocity after collision, then

$(m_1 + m_2)\,v = m_1 u_1 + m_2 u_2$

$(40 + 60)\,v = 40 \times 4 + 60 \times 2 = 280$

$$v = \frac{280}{100} = 2{\cdot}8\text{ m/s}$$

$$\text{Loss of energy} = \frac{m_1 m_2 u_1^2}{2(m_1 + m_2)}$$

$$= \frac{40 \times 60 \times 4^2}{2(40 + 60)} = \frac{2400 \times 16}{2 \times 100}$$

$$= \mathbf{192\ J}$$

44. Viscous force is a non-conservative force.

45. Statement (*iv*) is wrong as $P = [ML^2T^{-3}]$

46. The statement is true always.

47. Yes, the algebraic sum of the work done on the ball and work done on the bat in this process is never zero.

48. Largest unit of energy is kilowatt hour ($= 3{\cdot}6 \times 10^6$ J)

49. Force is a vector.

50. As is known, $k \propto \frac{1}{l}$.

51. When forces doing work on a system are conservative, total mechanical energy is conserved.

52. As KE = work done = force × distance

$\therefore$ smaller force has to be applied through larger distance.

53. From $E = mc^2 = (9{\cdot}11 \times 10^{-31})\,(3 \times 10^8)^2$ J

$= 9{\cdot}11 \times 9 \times 10^{-15}$ J

$$E = \frac{9{\cdot}11 \times 9 \times 10^{-15}}{1{\cdot}6 \times 10^{-16}} \text{ keV} = 512 \text{ keV}.$$

II. Assertion-Reason Type Questions

54. Both, the Assertion and Reason are false.

55. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

56. Both, the Assertion and Reason are true, but Reason is not a correct explanation of the Assertion.

57. Both, the Assertion and Reason are true, but Reason is not a correct explanation of the Assertion.

58. Assertion is true, but Reason is false.

59. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

60. $P = \frac{W}{t} = \frac{M\,L^2\,T^{-2}}{T} = [M\,L^2\,T^{-3}]$

Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

61. Assertion is true, but the Reason is false. In the frame of reference of the train, velocity of person is zero.

62. Assertion is true, but Reason is false.

63. Both, the Assertion and Reason are true, and Reason is correct explanation of the Assertion.

64. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

65. Both, the Assertion and Reason are true, and Reason is correct explanation of the Assertion.

CONCEPTUAL PROBLEMS

I. Work, Energy and Power

1. A body is in accelerated motion. Is it possible that no work is done ? Give some example.

Sol. Yes, it is possible that work done is zero even when a body is in accelerated motion. This happens when force is acting in a direction perpendicular to the direction of motion. For example, earth moving around the sun in a circular orbit under the gravitational force perpendicular to its path. Hence work done is zero, though the motion is accelerated.

2. A light body and a heavy body have same kinetic energy. Which one has greater linear momentum ?

Sol. Here, $E_1 = E_2$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$$

or $$\frac{v_2^2}{v_1^2} = \frac{m_1}{m_2}$$

or $$\frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}}$$

As $p_2 = m_2 v_2$ and $p_1 = m_1 v_1$

$$\therefore \quad \frac{p_2}{p_1} = \frac{m_2 v_2}{m_1 v_1} = \frac{m_2}{m_1} \cdot \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m_2^2\, m_1}{m_1^2\, m_2}}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{m_2}{m_1}}\ ; \text{ If } m_2 > m_1, \text{ then } p_2 > p_1,$$

i.e., a heavier body has greater linear momentum.

3. A truck and a car moving with the same K.E. are stopped by applying same retarding force by means of brakes. Which one will stop at a smaller distance ?

Sol. Both the truck and the car will stop at the same distance. This follows from work energy principle

K.E. = work done in stopping

= force × distance.

As K.E. of both and retarding force applied on both are equal, therefore, the distances covered must by equal.

4. A truck and a car are moving with the same K.E. on a straight road. Their engines are simultaneously switched off. Which one will stop at a lesser distance ?

Sol. The vehicle stops when its K.E. is spent in working against the force of friction between the tyres and the road. This force of friction varies directly with weight of the vehicle.

As K.E. = work done

= force of friction × distance.

$$\therefore \quad E = F \times s \text{ or } s = \frac{E}{F}$$

For given K.E ; distance s will be smaller, where F is larger, such as in case of a truck.

Therefore, the truck will stop at a lesser distance.

5. The velocity of an aeroplane is made twice (a) What will happen to the momentum ? Will the momentum remain conserved ? (b) What will happen to the K.E ? Will the energy remain conserved ?

Sol. When velocity of aeroplane is made twice, its momentum will become twice.

The momentum of plane and that of air will remain conserved. This is because when momentum of plane becomes twice, the momentum of air in the opposite direction will also become twice.

(*b*) K.E. of the plane will become four times. The additional energy will be obtained from the burning of the fuel of the plane. Total energy shall remain conserved.

6. In a thermal station, coal is used for the generation of electricity. Mention how energy changes from one form to the other before it is transformed into electrical energy ?

Sol. When coal is burnt, heat energy is produced. This converts water into steam.

This steam rotates the turbine and thus heat energy is converted into mechanical energy of rotation. The generator converts this mechanical energy into electrical energy.

7. The heart rate (number of heart beats/minute) scales as $1/L$, where L is the characteristic length of the mammal. (a) Can you explain this ? (b) The scale factor of a human relative to a monkey is about 2·5. What is the monkey's heart rate ?

Sol. Human heart rate is about 70 beats/minute. More the characteristic length of the mammal, smaller is the heart beat rate.

As scale factor of a human relative to a monkey is about 2·5, therefore, monkey's heart rate = 70 × 2·5 = **175 beats/minute.**

8. The absolute value of potential energy (and therefore total energy) has no physical significance. It is the difference of potential energies that matters. One can, therefore, add or subtract a constant to the potential energy (provided we do it to potential energy at every position for a given force) without any change in the physical situation. By convention, for forces which fall off to zero at large distances, the potential energy at infinity is taken to be zero. With this choice, is the potential energy positive or negative for (a) electron-positron bound state, (b) planet-satellite system, (c) electron-electron system ?

Sol. By convention, for forces which fall off to zero at large distances, the potential energy at infinity is taken to be zero. These forces may be attractive or repulsive. Calculations show that if the forces are *attractive*, then potential energy at finite distance is *Negative* and vice-versa. On this basis, we conclude that

(*a*) for electron positron bound state, Potential Energy is *negative*

(*b*) for planet satellite system, potential energy is again *negative*.

(*c*) for electron-electron system, potential energy is *positive*.

9. A spring of force constant k is cut into two pieces of lengths l_1 and l_2. Calculate force constant of each part.

Sol. For a spring $k \propto \frac{1}{l}$.

$$\therefore \quad k_1 \propto \frac{1}{l_1}, \; k_2 \propto \frac{1}{l_2} \quad \text{and} \quad k \propto \frac{1}{l_1 + l_2}$$

$$\therefore \quad \frac{k_1}{k} = \frac{l_1 + l_2}{l_1} = 1 + \frac{l_2}{l_1}$$

$$\text{or} \quad k_1 = k\left(1 + \frac{l_2}{l_1}\right)$$

$$\text{and} \quad \frac{k_2}{k} = \frac{l_1 + l_2}{l_2} = \frac{l_1}{l_2} + 1$$

$$\text{or} \quad k_2 = k\left(1 + \frac{l_1}{l_2}\right)$$

IMPORTANT NOTES

1. If $l_1 = l_2, k_1 = k_2 = 2k$, *i.e.*, when length is halved, spring constant of each half becomes twice.
2. If $\frac{l_1}{l_2} = \frac{1}{n}$, then $k_1 = k(1 + n)$;

$$k_2 = k\left(1 + \frac{1}{n}\right)$$

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10. If stretch in a spring of force constant k is doubled, calculate

(a) ratio of final to initial force in the spring.

(b) ratio of elastic energies stored in the two cases.

(c) work done in changing to the state of double stretch.

Sol. (*a*) For a given spring, $F = kx$

$$\therefore \quad \frac{F_2}{F_1} = \frac{kx_2}{kx_1} = \frac{2\,x}{x} = \mathbf{2}$$

(*b*) For a given spring, $U = \frac{1}{2}kx^2$;

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}kx_2^2}{\frac{1}{2}kx_1^2} = \frac{(2\,x)^2}{x^2} = \mathbf{4}$$

(*c*) As work done in stretching the spring is stored in the spring in the form of elastic potential energy of the spring, therefore,

$$W = U_2 - U_1 = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

$$= \frac{1}{2}k\,[(2x)^2 - x^2\,] = \frac{3}{2}\boldsymbol{kx^2}$$

11. The potential energy of two atoms separated by a distance x is given by $U = -A/x^6$, where A is a positive constant. What is the force exerted by one atom on another atom ?

Sol. Here, $U = -A/x^6$, $F = ?$

From $$F_x = -\frac{\partial}{\partial x}(U)$$

$$= -\frac{\partial}{\partial x}\left(\frac{-A}{x^6}\right) = A\left(\frac{-6}{x^7}\right)$$

$\therefore$ force exerted by one atom on another atom

$$= \mathbf{-\,6\,A/x^7}$$

12. A force is acting on a body moving along x-axis in the direction of motion of the body. If this force produces a potential energy $U = A\,x^4$, where $A = 1{\cdot}2$ J m^{-4}, then what is the force acting on the body when the body is at $x = -\,0{\cdot}8$ m.

Sol. Here, $U = A\,x^4$

$$F_x = -\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}(A\,x^4) = -4\,A\,x^3$$

When $x = -\,0{\cdot}8$ m, then

$$F_x = -\,4 \times 1{\cdot}2\,(-\,0{\cdot}8)^3 = \mathbf{2{\cdot}46\ N}$$

13. Chemical, gravitational and nuclear energies are nothing but potential energies for different types of forces in nature. Explain this statement clearly with examples.

Sol. A system of particles has potential energy when these particles are held certain distance apart against some force.

For example, chemical energy is due to chemical bonding between the atoms. Gravitational energy arises when objects are held at some distance against the gravitational attraction. Similarly, nuclear energy arises on account of nuclear forces.

II. Collisions

14. When is the exchange of energy maximum during an elastic collision ?

Sol. Exchange of energy during an elastic collision is maximum (= 100%), when the two colliding bodies have equal masses.

15. A body of mass m moving with speed v collides elastically head on with another body of mass m, initially at rest. Show that the moving body will come to a stop after collision.

Sol. Let v_1 be velocity of first ball and v_2 be the velocity of 2nd ball after collision.

As linear momentum is conserved.

$$\therefore \quad m \times v + m \times 0 = mv_1 + mv_2$$

or $$v = v_1 + v_2 \qquad ...(i)$$

As KE is conserved.

$$\therefore \quad \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$v^2 = v_1^2 + v_2^2$$

or $$v^2 - v_1^2 = v_2^2$$

or $$(v + v_1)(v - v_1) = v_2^2$$

From (*i*), $v - v_1 = v_2$

$$\therefore \quad (v + v_1)\,v_2 = v_2^2 \quad \text{or} \quad v + v_1 = v_2$$

or $$v = v_2 - v_1 \qquad ...(ii)$$

Add (*i*) and (*ii*), $2\,v = 2\,v_2$ or $v_2 = v$

From (*i*), $v_2 = v_1 + v_2 \quad \therefore \ \boldsymbol{v_1 = 0}$

which was to be proved.

16. Explain how fast moving neutrons can be quickly slowed down by passing them through water or heavy water.

Or

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Hydrogenic materials are used as moderators in nuclear reactors to slow down the neutrons. Why ?

Sol. We have studied that when two bodies of same mass undergo an elastic collision, their velocities are interchanged after collision. Water and heavy water are hydrogenic materials containing protons having approximately the same mass as that of a neutron. When fast moving neutrons collide with protons, the neutrons come to rest and protons move with the velocity of neutrons.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Work, Energy and Power

1. State the factors on which work done by a force depends.

Ans. Work done by a force depends on (*i*) magnitude of force (*ii*) magnitude of displacement (*iii*) angle between force applied and displacement.

2. What is work done by the force of tension in the string of a simple pendulum ?

Ans. Work done is zero, as displacement of bob of simple pendulum is always perpendicular to the string of the pendulum.

3. What is represented by area under the force displacement curve ?

Ans. It represents the work done by the force over the given displacement.

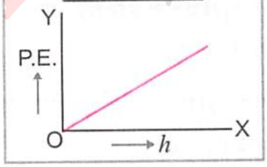
4. Is power a scalar or vector ?

Ans. Power is a scalar.

5. Show graphically the variation of potential energy of an object thrown vertically upwards w.r.t. its height.

Ans. The P.E. versus height graph is a straight line with slope = mg as shown in Fig. 6(Q).1.

FIGURE 6(Q).1



6. What is spring constant ? What are its SI units ?

Ans. Spring constant of a spring is restoring force developed in the spring per unit elongation/ compression. SI units of spring constant are N/m.

7. Name the process in which momentum changes but KE does not.

Ans. In uniform circular motion, momentum changes but KE does not change.

8. What is the source of KE of falling rain drops ?

Ans. Potential energy of falling rain drops goes on decreasing and their KE goes on increasing.

9. What is work done in holding a 15kg suitcase while waiting for a bus for 15 minutes ?

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Ans. Work done is zero, because displacement is zero.

10. Is it practically possible to have situations where $(E - V) < 0$?

Ans. No, because $(E - V)$ represents K.E., which cannot be negative.

11. Can K.E. of a system be increased or decreased without applying any external force on the system ?

Ans. Yes, by doing work through internal forces *e.g.*, in explosion of a bomb.

12. Does K.E. depend upon the direction of motion involved ? Can it be negative ? Does its value depend on frame of refrence ?

Ans. No, K.E. does not depend on the direction of motion. No, K.E cannot be negative. Yes, value of K.E. depends upon the frame of reference.

13. Name the largest and smallest practical unit of energy.

Ans. 'Kilowatt hour' is the largest practical unit of energy and 'electron volt' is the smallest practical unit of energy.

14. Is work done by a non conservative force always negative ? Comment.

Ans. No. For example, work done by a non conservative force like friction is zero, so long as the body does not start moving. Again when friction causes motion, work done by friction is positive.

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15. Does potential energy of a spring decrease/ increase when it is compressed or stretched ?

Ans. When a spring is compressed or stretched, potential energy of the spring increases in both the cases. This is because work is done by us in compression as well as stretching.

16. When an air bubble rises in water, what happens to its potential energy ?

Ans. Potential energy of air bubble decreases, because work is done by upthrust on the bubble.

17. In which motion, momentum changes but K.E. does not ?

Ans. In uniform circular motion.

18. A shot fired from a cannon explodes in the air. What will be the changes in momentum and K.E. ?

Ans. There will be no change in linear momentum, but the K.E. will increase. This is because chemical potential energy of the explosives in the shot is converted into K.E.

19. The momentum of an object is doubled. How does its K.E. change ?

Ans. K.E. becomes four times.

20. Can P.E. of an object be negative ?

Ans. Yes, it can be negative, when forces involved are attractive.

21. Does the work done in moving a body depend on how fast or how slow the body is moved ?

Ans. No, time is not involved, in work or energy. Time is involved only in power.

22. A spring is cut into two equal halves. How is the spring constant of each half affected ?

Ans. Spring constant of each half becomes twice the spring constant of the original spring.

23. Is it possible to exert a force which does work on a body without changing its kinetic energy ? If so, give example.

Ans. Yes, when a spring is compressed or when a body is pulled with a constant velocity on a rough horizontal surface.

24. In case of a moving body, as force of friction is $\mu\, mg$, can we regard $\mu\, mg\, x$ as potential energy similar to mgh ?

Ans. No, potential energy is defined only for conservative forces.

II. Collision

25. Is it possible to have a collision in which the whole of KE is lost ?

Ans. Yes. For example, in perfectly inelastic collision of two bodies moving towards eachother with equal linear momenta.

26. If two objects collide and one is initially at rest (*a*) is it possible for both to be at rest after collision ? (*b*) is it possible for any one to be at rest after collision ?

Ans. (*a*) No, because momentum will not be conserved in that case.

(*b*) Yes, when masses of two objects are equal and collision is perfectly elastic.

27. In which, elastic or inelastic collision, the momentum is conserved ? What about K.E. ?

Ans. Momentum is conserved in both the types of collisions, but K.E. is conserved only in elastic collisions.

28. Which physical quantites are conserved in an elastic collision ?

Ans. Linear momentum and KE.

29. Which physical quantity is conserved in both, elastic and inelastic collision ?

Ans. Linear momentum is conserved in both elastic and inelastic collisions.

30. If two bodies stick together after collision. Will the collision be elastic or inelastic ?

Ans. Inelastic collision.

31. A bullet gets embedded in a wooden block. Where does its KE go ?

Ans. KE of bullet gets converted into heat and sound energy.

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Work, Energy and Power

1. Does the work done in raising a body through a certain height depend upon how fast it is raised ?

Ans. No. Work done depends only on weight of body and height through which it is raised. It does not depend upon the time taken.

2. In a tug of war, one team is giving way to other. What work is being done and by whom ?

Ans. Work is being done by the winning team. Work done = resultant force applied by the two teams × displacement of the loosing team.

3. What sort of energy is associated with a bird flying in air ?

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Ans. A bird flying in air has mechanical energy, which is the sum of potential energy and kinetic energy of the bird.

4. Momentum of a body is doubled. By what percent does its KE change ?

Ans. $K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

When p is doubled kinetic energy K becomes 4 times. % age increase in KE $= \frac{(4K - K)100}{K}$

$= \mathbf{300\%}$

5. When a spring is compressed or stretched, what happens to its potential energy ?

Ans. In either case, work has to be done. Therefore, potential energy of the spring increases.

6. Out of kilowatt hour and electron volt, which is bigger unit of energy and by what factor ?

Ans. Kilowatt hour is bigger unit of energy than electron volt.

$$\frac{1\,\text{kWh}}{1\,\text{eV}} = \frac{10^3 \times 60 \times 60\,\text{J}}{1{\cdot}6 \times 10^{-19}\,\text{J}} = \mathbf{2{\cdot}25 \times 10^{25}}$$

7. 'Work done in moving a body over a closed loop is zero'. Is this statement true for all forces ?

Ans. No, the statement is true only when motion is under conservative forces.

8. A rocket explodes in mid air. What happens to its total momentum and total KE ?

Ans. Explosion is under the internal forces. As no external force is involved, the total momentum is conserved. However, total KE increases because chemical energy of the fuel changes into KE.

9. A particle is moving in a circular path of given radius with number of rotations per second (*i*) constant (*ii*) decreasing (*iii*) increasing.

What happens to work done in the three cases ?

Ans. As is known, work done = change in KE

$$\left(= \frac{1}{2}mr^2\omega^2 = 2\pi^2 mr^2 x^2\right).$$

(*i*) When n is constant, KE is constant. Work done is zero.

(*ii*) When n is decreasing, KE decreases ; work is done by the body.

(*iii*) When n is increasing, KE increases. Work is done on the body.

10. Can you associate potential energy with a non conservative force ?

Ans. No. Potential energy can be associated only with a conservative force.

11. A spark is produced when two stones strike against each other. Why ?

Ans. Work done in striking the two stones against each other is converted into heat. This appears as a spark.

12. In an exothermic chemical reaction, is mass being converted into energy ?

Ans. Yes, mass is converted into heat energy, though this mass is too little.

13. A hydrogen bomb is more powerful than an atom bomb. Why ?

Ans. A hydrogen bomb based on nuclear fusion is more powerful than an atom bomb based on nuclear fission. This is because, energy released per unit mass of the fuel is much larger in nuclear fusion than in nuclear fission.

14. A shot fired from a cannon explodes in air. What will be the changes in the momentum and the kinetic energy ?

Ans. The linear momentum will be conserved, because explosion occurs from within, *i.e.*, under internal forces only. However, KE will increase due to (chemical) potential energy of the explosives.

15. Explain with reason whether the potential energy in the following cases increases or decreases :

(*a*) a spring is compressed,

(*b*) a spring is stretched,

(*c*) two dissimilar charges are brought near each other,

(*d*) a body is taken away against the gravitational force.

Ans. (*a*) When a spring is compressed, work is done on the spring in compressing it. Therefore, P.E. increases.

(*b*) When a spring is stretched, work is done on the spring in stretching it. Therefore, P.E. increases.

(*c*) Two dissimilar charges attract each other and come closer. Work is done by the field. Therefore, P.E. decreases.

(*d*) Work is done by us in taking the body away against the gravitational force. Therefore, P.E. increases.

(*e*) Air bubble rises up in water because of upthrust. Therefore, P.E. decreases.

16. **Two springs A and B are identical but A is harder than B ($K_A > K_B$). On which spring more work will be done if : (*a*) they are stretched through the same distance, (*b*) they are stretched by same force ?**

Ans. (*a*) As $W = \frac{1}{2}Kx^2$, and $K_A > K_B$

$\therefore$ work done on spring A will be more than work done on spring B (for same x)

(*b*) As $F = Kx$, therefore, $x = F/K$

As $K_A > K_B \; \therefore \; x_A < x_B$

As $W = \frac{1}{2}Kx^2 = \frac{1}{2}\left(\frac{F}{x}\right)x^2 = \frac{1}{2}Fx$

As $x_A < x_B \; \therefore \; W_A < W_B$ or $W_B > W_A$

i.e., more work is required in case of spring B than in the case of spring A.

17. **A man rowing a boat upstream is at rest with respect to shore. (*a*) Is he doing any work ? (*b*) If he stops rowing and moves down with the stream, is any work being done on him ?**

Ans. (*a*) No work is being done by the net force because displacement of boat relative to the shore is zero.

(*b*) When he stops rowing, force of water flow will produce displacement w.r.t the shore. Therefore, work is done by force of flowing water. KE of the person will increase.

18. **A spring is compressed by tieing its ends together tightly. It is then placed in acid and dissolves. What happens to its stored potential energy.**

Ans. When the spring dissolves in acid, the elastic potential energy stored in the spring passes to the acid. The internal energy of the acid increases and hence its temp. rises.

19. **(*a*) Two masses one *n* times heavier than the other are dropped from same height. How do their momentum compare just before they hit the ground ?**

(*b*) Two masses one *n* times heavy as the other have equal kinetic energy. How do their momentum compare ?

Ans. (*a*) When dropped from same height, the two masses hit the ground with *same velocity*

$= \sqrt{2gh} \quad \therefore \quad \frac{p_2}{p_1} = \frac{m_2}{m_1} = n$

(*b*) We know that K.E., $K = \frac{p^2}{2m}$

$\therefore \quad p = \sqrt{2mK}$

As K is same, $\frac{p_2}{p_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{n}$

20. **(*a*) Can kinetic energy of a system be changed without changing its momentum ?**

(*b*) Can momentum of a system be changed without changing its kinetic energy ?

Ans. (*a*) Yes. For example, when a bomb explodes linear momentum is conserved, but KE changes.

(*b*) Yes. For example, in case of uniform circular motion. KE remains unchanged, but linear momentum changes because of change in the direction of motion.

21. **A meteorite burns in the atmosphere before it reaches earth's surface. What happens to its momentum ?**

Ans. The momentum of the meteorite is transferred to the air molecules. Thus, momentum conservation principle is not violated.

22. **Can a body have momentum without energy ?**

Ans. Yes. When $E = K + V = 0$, either both are zero or $K = -V$. Thus, K.E. may or may not be zero.

As $p = \sqrt{2mK}$

$\therefore \quad p = 0$, only when $K = 0$ and $p \neq 0$, when $K = -V$

23. **Can a body have energy without momentum ?**

Ans. Yes, when $p = 0$, $K = \frac{p^2}{2m} = 0$

But $E = K + V = V$ (Pot. energy), which may or may not be zero.

24. **Can a body have momentum when its energy is negative ?**

Ans. Yes, when $K < V$, total energy $E = K + V$ is negative. The body has the momentum ($\because$ $K \neq 0$). For example, in an atom, electron has momentum, though its energy is negative.

25. **The potential energy function for a particle executing simple harmonic motion is given by $V(x) = \frac{1}{2}kx^2$, where k is the force constant of the oscillator. For $k = \frac{1}{2}$ Nm^{-1}, show that a particle of total energy 1 joule moving under this potential must turn back when it reaches $x = \pm 2$ m.**

Ans. As is known, the particle will turn back when whole of its energy is converted into potential energy, *i.e.*, $V(x) = \frac{1}{2}kx^2 = 1\,(\text{joule})$

$$\frac{1}{2} \times \frac{1}{2} x^2 = 1 \text{ or } x^2 = 4$$

$$x = \pm 2 \text{ m}$$

26. A stone is dropped from the top of a high tower. Will the mechanical energy of the stone be conserved or not if the force of friction due to air is not neglected ?

Ans. Mechanical energy is conserved only when forces involved are conservative. As force of friction due to air is non-conservative, therefore, mechanical energy of the stone is not conserved.

27. A man can jump higher on moon than on earth. With same effort, can a runner improve his timing for 100 m race on moon as compared to that on earth ?

Ans. The man can jump higher on moon than on earth, because the acceleration due to gravity on moon is less than acceleration due to gravity on earth. However, acceleration due to gravity has no effect on horizontal motion. Therefore, a runner cannot improve his timing on the moon for 100 metre race.

II. Collisions

28. In any perfectly inelastic collision, is whole of KE lost ?

Ans. No. Only that much KE is lost as is necessary for the conservation of linear momentum.

29. Throwing mud on a wall is an example of perfectly inelastic collision. Comment.

Ans. True. When mud is thrown on a wall, it sticks to the wall. KE of mud reduces to zero leading to non-conservation of KE. There is absolutely no tendency of mud to separate from wall. So, the collision is perfectly inelastic.

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. What is meant by zero work ? State the conditions under which a force does no work. Give any one example. **[Art. 6.4]**
2. Two bodies of unequal masses have same linear momentum. Which one has greater K.E ? **[Conceptual Problem 6, Page 6/64]**
3. Two bodies of unequal masses have same K.E. Which one has greater linear momentum ? **[Conceptual Problem 7, Page 6/65]**
4. How do potential energy and K.E. of a spring vary with displacement ? Is this variation different from variation in potential energy and K.E. of a body in free fall ? **[Arts. 6.13 and 6.14]**
5. What is meant by mass energy equivalence ? Discuss its significance in Physics. **[Art. 6.17]**
6. Hydrogenic materials are used as moderators in nuclear reactors to slow down the neutrons. Why ? **[Conceptual Problem 22, Page 6/67]**
7. How do you justify energy conservation from the vibrations of a simple pendulum ? **[Art. 6.15]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Explain what is meant by work. Obtain an expression for work done by a constant force. **[Art 6.2]**
2. Discuss the absolute and gravitational units of work on mks and cgs systems. **[Art. 6.3]**
3. What is meant by positive work, negative work and zero work ? Illustrate your answer with two examples of each type. **[Art. 6.4]**
4. Obtain graphically and mathematically work done by a variable force. **[Art. 6.5]**
5. What are conservative and non-conservative forces, explain with examples. Mention some of their properties. **[Art. 6.6]**
6. What is meant by power and energy ? Give their units. **[Arts. 6.7 and 6.8]**
7. Explain the meaning of K.E. with examples. Obtain an expression for K.E of a body moving uniformly ? **[Art. 6.9]**
8. State and explain work energy principle. **[Art. 6.11]**
9. What do you mean by potential energy ? Give any two examples of potential energy other than that of the gravitational potential energy. **[Art. 6.12]**
10. Obtain an expression for gravitational potential energy of a body. **[Art. 6.12]**

11. Explain what is meant by potential energy of a spring ? Obtain an expression for it and discuss the nature of its variation. **[Art. 6.13]**

12. Mention some of the different forms of energy and discuss them briefly. **[Art. 6.16]**

13. Give a brief account of mass energy equivalence. What is its significance in Physics ? **[Art. 6.17]**

14. State and establish principle of conservation of energy. **(J & K Board 2011) [Art. 6.14]**

15. How do vibrations of a simple pendulum and motion of a ball over a watch glass illustrate the principle of energy conservation ? **[Art. 6.15]**

16. What is meant by a collision ? Discuss two types of collision with their essential characteristics. **[Art. 6.20]**

17. Discuss elastic collision in one dimension. Obtain expressions for velocities of the two bodies after such a collision. **(J & K Board 2011) [Art. 6.22]**

18. Discuss briefly inelastic collisions in one dimension. **[Art. 6.23]**

19. Give a brief account of elastic collisions in two dimensions. **[Art. 6.24]**

20. Briefly analyse inelastic collision in two dimensions. **[Art. 6.25]**

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. Work is said to be done by a force, when the body is displaced through some distance in the direction of the applied force.

Work done by the force is the product of the component of force in the direction of displacement and the magnitude of displacement.

Work done by a force is the dot product of force $(\vec{F})$ and displacement $(\vec{s})$ *i.e.,* $W = \vec{F} \cdot \vec{s}$

$$W = (MLT^{-2}) \times L = (M^1 L^2 T^{-2})$$

Absolute unit of work on *MKS* system is **joule** and on *CGS* system, the absolute unit of work is **erg**. We can show that 1 joule = 10^7 erg

Gravitational units of work are **kilogram meter** and **gram cm**, where 1 kg m = 10^5 gm cm

Although, work done is a scalar quantity, its value may be positive (when $\theta < 90°$), negative (when $\theta > 90°$) or zero (when $\theta = 90°$)

Work done by a variable force is numerically equal to the **area** under the force curve and the displacement axis.

Read the above passage carefully and answer the following very short answer and short answer questions :

1. What is work done by a force $\vec{F}$ displacing a body through $\vec{s}$?
2. Is work a scalar or vector ?
3. Name SI unit and *cgs* unit of work. What is the relation between them ?
4. What is work done by a variable force ?

CASE 2. Potential energy of a spring is the energy associated with the state of compression or expansion of the elastic spring. For a small stretch or compression the spring obeys Hooks Law, *i.e.,*

Restoring force ∝ stretch or compression

i.e., $-F \propto x$ or $-F = kx$, where k is called the **spring constant**. It is measured in N/m.

A spring with greater force constant is said to be **stiffer**. Further, for a spring, $k \propto \frac{1}{l}$ *i.e.,* smaller the length of the spring, greater will be the force constant.

Small amount of work done in increasing the length of the spring by dx is

$$dW = -F\,dx = kx\,dx$$

Total work done in giving displacement x to the spring is

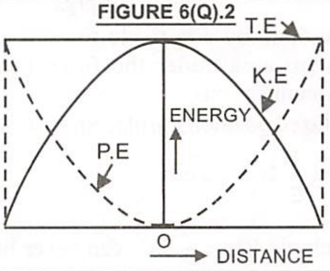
$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2 .$$

∴ Potential energy of the spring

$$\text{P.E} = W = \frac{1}{2}kx^2$$

When the spring is released, it regains its original configuration. The entire P.E of the spring is converted into kinetic energy. The body can not stop at the mean position and goes over to extreme position on the other side. Total energy in all the positions remains the same. The variation of P.E ; K.E and T. E with distance/ displacement x is shown in Fig. 6(Q).2.

FIGURE 6(Q).2



If v_m is maximum speed and x_m is maximum displacement, then

$$\frac{1}{2} m\, v_m^2 = \frac{1}{2} k\, x_m^2$$

$$v_m = \sqrt{\frac{k}{m}}\, x_m$$

Note that elastic P.E of a spring in relaxed position is zero. Further, elastic P.E of a spring is always positive. Elastic P.E appears only when the spring is compressed or stretched.

Read the above paragraph carefully and answer the following very short answer and short answer questions :

5. What is elastic P.E of a spring in relaxed position ?

6. Can elastic P.E of a spring be negative ?

7. How does spring constant (k) of a spring vary with length of the spring ?

8. A body of mass m is attached to the free end of a spring, how are maximum displacement (x_m) and maximum speed (v_m) of the body related ?

CASE 3. Power of a person or machine is defined as the time rate at which work is done by it.

$$\text{Power} = \frac{\text{Work done}}{\text{time taken}}$$

$$P = \frac{dW}{dt} = \frac{\vec{F}.\vec{ds}}{dt} = \vec{F}.\vec{v}$$

Thus power can be expressed as dot product of force and velocity. It is a **scalar quantity.** SI unit of power is **watt**, where

1 watt = 1 joule/sec, and

1 kilowatt (kw) = 10^3 watt

1 Mega watt (MW) = 10^6 watt

Another popular unit of power is horse power (h . p) where 1 h.p = 746 watt

Average power consumption in heart beat is 1·2 watt, in sleeping 75 watt and in slow walking is 200 watt.

Energy of a body is the capacity or ability of the body to do the work. When a body of mass m is moving with a velocity v, kinetic energy of the body $= \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{p^2}{2m}$. KE of a body depends upon the frame of reference.

According to **Work Energy Theorem** or **Work energy principle**. Work done by a net force in displacing a body is equal to change in KE of the body. Thus according to work energy principle, work and kinetic energy are equivalent quantities. This principle applies to both, the constant force as well as variable force.

Read the above passage carefully and answer the following very short answer and short answer questions :

9. Name a popular unit of power. How is it related to SI unit of power ?

10. What is the relation between force applied ($\vec{F}$), velocity attained ($\vec{v}$) and power (P) of the body ?

11. How is KE of a body related to its linear momentum ?

12. State work energy theorem/principle.

ANSWERS

1. $W = \vec{F}.\vec{s} = F\, s \cos\theta$,

where θ is angle between $\vec{F}$ and $\vec{s}$.

2. Work done is a scalar quantity.

3. SI unit of work is *joule* and *CGS* unit of work is *erg* 1 joule = 10^7 ergs

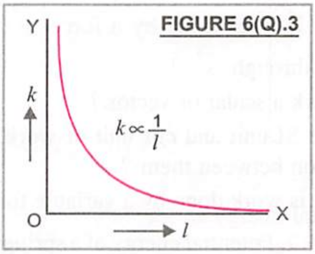
4. Work done by a variable force is numerically, equal to area under the force curve and the displacement axis.

5. In relaxed position, displacement $x = 0$, ∴ Elastic P.E. $= \frac{1}{2} kx^2$ = Zero.

6. No, elastic P.E $= \frac{1}{2} kx^2$ can never be negative.

7. As $k \propto \frac{1}{l}$, therefore, variation of k with l is as shown in Fig. 6(Q).3.

FIGURE 6(Q).3



8. Using the principle of conservation of energy,

KE of body = P.E of spring

$$\frac{1}{2} m v_m^2 = \frac{1}{2} k x_m^2$$

$$v_m = \sqrt{\frac{k}{m}}\, x_m$$

9. Horse power ($H.P$) is a popular is unit of power.

$$1\, H.P = 746 \text{ watt}$$

where watt is SI unit of power.

10. $P = \vec{F}.\vec{v} = F v \cos\theta$

11. $\text{KE} = \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{p^2}{2m}$,

where $p = mv$ is linear momentum.

12. According to Work Energy principle, work done by a net force in displacing body is equal to change in K.E. of the body. Thus work and kinetic energy are equivalent quantities.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. **Potential energy** of a body is defined as the energy possessed by the body by virtue of its position or configuration in some field.

Gravitational P.E. of a body is the energy possessed by the body by virtue of its position above the surface of earth.

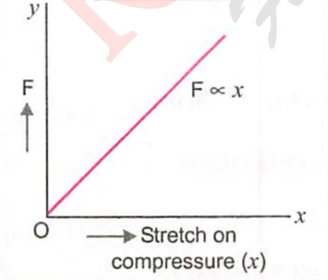
Gravitational P.E $= mgh$

The **potential energy of a spring** is the energy associated with the state of compression or expansion of an elastic spring.

For a small stretch or compression, the spring obeys Hook's law, *i.e.*, $-F \propto x$ or $-F = kx$ where k is a constant, called **spring constant**.

We may define spring constant as the restoring force per unit displacement of the spring. For a spring $k \propto \frac{1}{l}$. We can show that potential energy stored in the spring is P.E $= W = \frac{1}{2} kx^2$. In relaxed position, $x = 0$; P.E = zero. P.E. appears only when the spring is compressed or stretched.

FIGURE 6(Q).4



As total energy in all the positions remains the same, therefore,

$$\frac{1}{2} mv_m^2 = \frac{1}{2} k x_m^2 \quad \text{or} \quad v_m = \sqrt{\frac{k}{m}}\, x_m$$

The mechanical energy (E) of a body is the sum of KE (K) and potential energy (V) of the body.

$$E = K + V$$

According to the principle of conservation of total mechanical energy, if the forces working on the system are conservative, total ME of the system remains constant.

Vibrations of a simple pendulum, vibrations of an elastic spring, motion of a small spherical ball over a watch glass etc. all are based in the principle of conservation of mechanical energy.

Based on the above paragraph, answer questions no. 1 to 4 :

1. Gravitational potential energy of a body of mass m at a height h above the surface of earth is

(*a*) mgh (*b*) $-mgh$

(*c*) $2\, mgh$ (*d*) $\frac{mh}{g}$

2. Potential energy stored in a spring is

(*a*) $\frac{kx^2}{2}$ (*b*) $2\, kx^2$

(*c*) $\sqrt{2}\, kx^2$ (*d*) none of these

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

3. **Assertion.** For a small stretch or compression, a spring obeys Hook's law.

   Reason. For a spring, force constant, $k \propto \frac{1}{l}$ where l is length of the spring.

4. **Assertion.** The mechanical energy (E) of a body is the sum of K.E (K) and P.E (V) of the body.

   Reason. Total mechanical energy of a system is constant under all conditions.

CASE 2. **Einstein** made an incredible discovery that **energy can be transformed into mass** and mass can be transformed into energy. One can be obtained at the cost of the other. The equivalence relation is $\boxed{E = mc^2}$, where m is the mass that disappears and E is the energy that appears, c is velocity of light in vacuum = 3×10^8 m/s.

According to modern quantum theory, mass and energy are not conserved separately but are conserved as single entity called **mass-energy**. For example, energy equivalent of an electron is 511 keV, a proton is 938 MeV, uranium atom is 225 G eV and so on. The relativistic variation of mass with velocity is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

where m_0 is rest mass of particle, m is effective mass or relativistic mass of particle moving with velocity v.

If we account for all forms of energy, the total energy of an isolated system does not change. **This is the principle of conservation of energy**.

A **collision** is an isolated event in which two or more colliding bodies exert relatively strong forces on one another for a relatively short time.

Two **key rules** of the collision game are :

(*i*) Law of conservation of linear momentum.

(*ii*) Law of conservation of energy.

Based on the above paragraph, answer questions no. 5 to 8 :

5. The equivalence relation between mass (m) and energy (E) is

   (*a*) $E = mc$ (*b*) $E = m\sqrt{c}$

   (*c*) $E = mc^2$ (*d*) $E = \sqrt{m}\,.\,c^2$

6. The energy equivalent of a nucleon is

   (*a*) 938 MeV (*b*) 983 MeV

   (*c*) 983 eV (*d*) 938 keV

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

7. **Assertion.** Mass and energy are not conserved separately, but are conserved as a single entity called mass energy.

   Reason. This is because energy can be transformed into mass and mass can be transformed into energy.

8. **Assertion.** The relativistic variation of mass with velocity is given by $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$. Symbols have usual meaning.

   Reason. Law of conservation of linear momentum and Law of conservation of energy, are two key rules of collisions.

ANSWERS

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*c*) **6.** (*a*) **7.** (*a*) **8.** (*b*)

HINTS/EXPLANATIONS For Difficult Questions

1. Gravitational PE = mgh
2. P.E stored in a spring $= \frac{1}{2} kx^2$
3. Both, A and R are true, but R is not a correct explanation of A.
4. A is true, but R is false.
5. $E = mc^2$
6. 1 nucleon = 938 MeV
7. Both, A and R are true and R is correct explanation of A.
8. Both, A and R are true, but R is not a correct explanation of A.

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PROBLEMS FOR PRACTICE

TYPE I. WORK

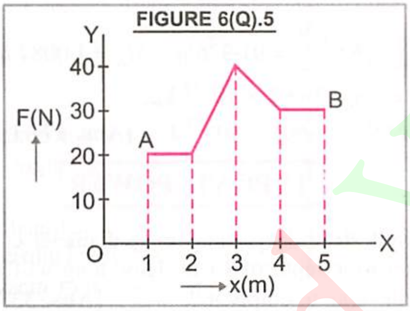
1. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, calculate work required to pull the hanging part on the table.

$$\left[\textbf{Ans. } \frac{MgL}{18}\right]$$

2. A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he covers a distance of 40 m with an acceleration of 1 ms^{-2}, find the work done by the man. **[Ans. 80 J]**

3. A body moves from a point *A* to *B* under the action of a force shown in Fig. 6(Q).5. What is the amount of work done ?

FIGURE 6(Q).5



[Ans. 115 J]

4. A body of mass 0·3 kg is taken up an inclined plane of length 10 m and height 5 m, and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0·15. What is the

(*a*) work done by the gravitational force over the round trip ?

(*b*) work done by the applied force on the upward journey ?

(*c*) work done by the frictional force over the round trip,

(*d*) kinetic energy of the body at the end of the trip ?

[Ans. (*a*) Zero, (*b*) 18·5 J (*c*) –7·6 J (*d*) 10·9 J]

5. A force exerts an impulse J on a body changing its speed from *u* to *v*. The force and object's motion are along the same line. Show that the work done by the force is ***J* (*u* + *v*)/2.**

6. A particle moves from a point $\vec{r_1} = (2\hat{i} + 3\hat{j})$ to another point $\vec{r_2} = (3\hat{i} + 2\hat{j})$ during which a certain force $\vec{F} = (5\hat{i} + 5\hat{j})$ acts on it. Calculate work done by the force on the particle during this displacement. **[Ans. Zero]**

TYPE II. KINETIC ENERGY AND WORK ENERGY THEOREM

7. In a ballistic demonstration, a police officer fires a bullet of mass 50 g with a speed of 220 m/s on a wooden target. The bullet emerges with 20% of its initial K.E. What is the emergent speed of the bullet ? **[Ans. 98·38 m/s]**

8. A toy rocket of mass 0·1 kg has a small fuel of mass 0·02 kg, which it burns out in 3 second. Starting from rest on a horizontal smooth track, it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the thrust of the rocket ? What is energy content of the fuel per unit mass. You may ignore the small variation in mass of rocket during fuel burning.

$$\left[\textbf{Ans. } \frac{2}{3}\textbf{N; 1000 J kg}^{-1}\right]$$

9. While catching a cricket ball of mass 200 g moving with a velocity of 20 ms^{-1}, the player draws his hands backwards through 20 cm. Find the work done in catching the ball and the average force exerted by the ball on the hand.

[Ans. 40 J, 200 N]

10. A body of mass 2 kg is resting on rough horizontal surface. A force of 20 N is applied to it for 10 s parallel to the surface. If coefficient of kinetic friction between the surfaces in contact is 0·2. Calculate (*i*) work done by applied force in 10 s. (*ii*) change in KE of object in 10 s.
Take $g = 10$ m/s^2. **(Delhi 2004)**

[Ans. 8000 J ; 6400 J]

11. A horizontal force of 5 N acts on a body of mass 2 kg initially at rest. It starts moving on a table having coefficient of friction = 0·2. Calculate

(*i*) work done by the applied force in 5 s

(*ii*) work done by force of friction in 5 s

(*iii*) work done by net force in 5 s

(*iv*) change in K.E. of the body in 5 s.

What do you conclude from this ?

[Ans. 33·75 J ; – 26·46 J, 7·29 J ; Work Energy Theorem is verified]

TYPE III. POTENTIAL ENERGY AND CONSERVATION OF ENERGY

12. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10 g moving with a speed of 200 m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back. Take $g = 10$ m/s^2. **[Ans. 0·2 m]**

13. A body dropped from a height H reaches the ground with a speed of $1{\cdot}2\sqrt{gH}$. Calculate the work done by air-friction. **[Ans. – 0·28 mg H]**

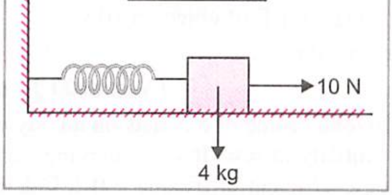
14. A pendulum bob has a speed of 3 m/s while passing through its lowest position. What is its speed when it makes an angle of 60° with the vertical ? The length of the pendulum is 0·5 m. Take $g = 10$ m/s^2. **[Ans. 2 m/s]**

TYPE IV. POTENTIAL ENERGY OF A SPRING

15. A ball of mass m is dropped from a height h on a platform fixed at the top of a vertical spring. The platform is depressed by a distance x. What is the spring constant K ? $\left[\textbf{Ans. } K = \dfrac{2mg\,(h+x)}{x^2}\right]$

16. A block of mass 4 kg is attached to a spring of force constant 24 N/m. To start with, the spring is unstretched and block is at rest on a frictionless horizontal surface. When a constant horizontal force of 10 N is applied on the block to move the block through a distance of 0·5 m, what is the speed of the block ?

FIGURE 6(Q).6



[Ans. 1 m/s]

17. An object is attached to a vertical spring and lowered slowly to its equilibrium position. This stretches the spring by a distance d. If the same object is attached to the same vertical spring, but permitted to fall freely, through what distance does it stretch the spring ? **[Ans. 2 *d*]**

TYPE V. MASS ENERGY EQUIVALENCE

18. What is the minimum amount of energy released in the annhilation of an electron positron pair ? Take rest mass of electron/positron $= 9{\cdot}11 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m/s.

[Ans. 1·023 MeV]

19. When slow neutrons are incident on a target containing ${}_{92}U^{235}$, a possible fission reaction is

$${}_{92}U^{235} + n \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,n + Q$$

Estimate the amount of energy released using the following data :

$$M\left[{}_{92}U^{235}\right] = 235{\cdot}04\ u\ ;$$

$$M\left({}_{56}Ba^{141}\right) = 140{\cdot}91\ u\ ;$$

$$M\left[{}_{36}Kr^{92}\right] = 91{\cdot}926\ u\ ;\ M_n = 1{\cdot}0087\ u.$$

Take $1\ u = 1{\cdot}661 \times 10^{-27}$ kg,
1 MeV $= 1{\cdot}602 \times 10^{-13}$ J **[Ans. 174·1 MeV]**

TYPE VI. POWER

20. A well 20 m deep and 3 m in diameter contains water to a depth of 14 m. How long will a 5 hp engine take to empty it ? **[Ans. 3381·6 s]**

21. An elevator which can carry a maximum load of 1800 kg is moving with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 5000 N. Calculate the minimum power delivered by the motor to the elevator in watt and hp. Take $g = 10$ m/s^2. **[Ans. 46 kW ; 61·66 hp]**

22. An engine pumps up 100 kg of water through a height of 10 m in 5 second. If the efficiency of the engine is 60 %, calculate the power of the engine in kW. Take $g = 10$ ms^{-2}. **[Ans. 3·33 kW]**

23. A pump can throw up 10 quintals of coal per hour from a coal mine 120 m deep. Calculate the power of the engine in watt assuming that efficiency is 80%. **[Ans. 408·3 watt]**

24. A car of mass 1000 kg accelerates uniformly from rest to a velocity of 54 km/h in 5 second. Calculate (*a*) its acceleration (*b*) its gain in K.E. (*c*) average power of the engine during this period, neglect friciton.

[Ans. (*a*) 3 ms^{-2} (*b*) 1·125 × 10^5 J (*c*) 22500 W]

25. A car weighing 1120 kg is going up an incline of 1 in 56 at the rate of 20 m in 2 s. Find the power of the engine if frictional force is 64 N.

[Ans. 2600 watt]

26. A train weighing 100 metric ton is running on a level track with a uniform speed of 72 km h^{-1}. If the frictional resistance amounts to 0·5 kg per metric ton, find the power of the engine.

[Ans. 9·8 kW]

27. An engine of weight 6·5 metric ton is going up an inclined plane of 5 in 13 at the rate of 9 km/h. If the coefficient of friction is $\frac{1}{12}$, calculate the power of the engine. Take $g = 9{\cdot}8$ m/s^2.

[Ans. 73·5 kW]

28. The human heart forces 4000 cm^3 of blood per minute through the arteries under pressure of 130 mm. The density of blood is 1·03 g/cc. What is the horse power of the heart ? **[Ans. $1{\cdot}17 \times 10^{-4}$ hp]**

TYPE VII. COLLISIONS

29. A ball moving with a speed of 9 m/s strikes an identical ball at rest such that after the collision, the direction of each ball makes 30° angle with the original line of motion. Find the speeds of the two balls after collision. Is KE conserved here ?

[Ans. $v_1 = v_2 = 3\sqrt{3}$ m/s ; No]

30. A nucleus of radium ($_{88}Ra^{226}$) decays to $_{86}Rn^{222}$ by the emission of an alpha particle $_2He^4$ of energy 4·8 MeV. If mass of $_{86}Rn^{222}$ is 222·0 amu and mass of $_2He^4$ is 4·003 amu, calculate the recoil energy of daughter nucleus $_{86}Rn^{222}$.

[Ans. 0·0866 MeV]

31. What percentage of K.E. of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 4 times its mass ?

[Ans. 64%]

32. A bomb at rest explodes into two fragments of masses 3·0 kg and 1·0 kg. The total kinetic energy of the fragments is 6×10^4 J. Calculate (*i*), kinetic energy of the bigger fragment (*ii*) momentum of the smaller fragment.

[Ans. $1{\cdot}5 \times 10^4$ J (ii) – 300 kg ms^{-1}]

33. A stationary U^{238} nucleus emits an α-particle ($_2He^4$) with velocity $1{\cdot}4 \times 10^7$ m/s and kinetic energy 4·1 MeV. Calculate the velocity of the residual nucleus. **[Ans. $-2{\cdot}4 \times 10^5$ m/s]**

34. A sphere of mass m moving with velocity u hits another stationary sphere of same mass. If e is coefficient of restitution, what is the ratio of velocities of two spheres after collision ?

$$\left[\textbf{Ans. } \frac{v_1}{v_2} = \frac{1-e}{1+e}\right]$$

35. What percentage of K.E. of a moving particle is transferred to stationary particle, when it strikes the stationary particle of (*a*) 100 times its mass (*b*) equal mass ? **[Ans. (*a*) 3·92 % (*b*) 100 %]**

36. What percentage of K.E. of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (*a*) 19 times in mass (*b*) equal in mass (*c*) $\frac{1}{19}$th of its mass ?

[Ans. (*a*) 19 % (*b*) 100 % (*c*) 99·8 %]

37. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg. moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

[Ans. 0·2 m]

38. A moving ball of mass m undergoes a head on collision with another stationary ball of mass 2 m. Show that the colliding ball loses $\frac{8}{9}$th of its energy after collision.

39. A particle of mass m strikes on ground with angle of incidence 45°. If coefficient of restitution, $e = \frac{1}{\sqrt{2}}$, find the velocity after impact and angle of reflection. $\left[\textbf{Ans. } \frac{\sqrt{3}}{2}v \; ; \theta = \tan^{-1}\sqrt{2}\right]$

40. A body of mass m falls from a height h and collides with another body of same mass at rest. After collision, the two bodies combine and move through distance (d) till they come to rest. Find the work done against the resistive force.

$$\left[\textbf{Ans. } \frac{mgh}{2}\right]$$

41. A 10 kg ball and a 20 kg ball approach each other with velocties 20 m/s and 10 m/s respectively. What are their velocities after collision, if the collision is perfectly elastic ?

[Ans. – 20 m/s ; 10 m/s]

42. A sphere of mass *m* collides against a pendulum of mass *m* with horizontal velocity v_0. (*i*) Calculate the maximum height reached by the pendulum if the masses scatter elastically along the line of initial motion, (*ii*) What is the maximum height reached by the pendulum if the two masses stick together ?

$$\left[\textbf{Ans. } (i)\frac{v_0^2}{2g}\ (ii)\frac{v_0^2}{8g}\right]$$

43. A ball moving on a horizontal frictionless plane hits an identical ball at rest with a velocity of 0·5 m/s. If the collision is elastic, calculate the speed imparted to the target ball, if the speed of projectile after the collision is 30 cm/s. Show that the two balls will move at right angles to eachother, after the collision. **[Ans. 0·4 m/s]**

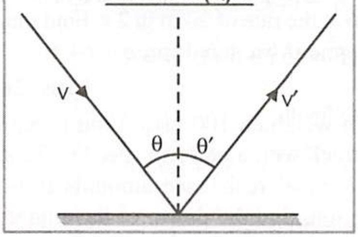
44. A ball moving with velocity of 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0·5, find their velocities after collision. **(AIPMT 2010)**

[Ans. 0 ; 1 m/s]

TYPE VIII. TYPICAL PROBLEMS

45. A ball of mass *m* hits a floor with a speed *v* making an angle of incidence θ with the normal. The coefficient of restitution is *e*. Find the speed of the reflected ball and the angle of reflection of the ball, [Fig. 6(Q).7].

FIGURE 6(Q).7



$$\left[\textbf{Ans. } v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}\right.$$
$$\left.\theta' = \tan^{-1}(\tan\theta/e)\right]$$

46. A body falling on the ground from a height of 10 m rebounds to a height of 2·5 m. Calculate (*i*) the percentage loss of kinetic energy of the body during its collision with the ground, (*ii*) ratio of the velocities of the body just before and just after the collision. **[Ans. 75%, 2]**

47. Fe^{57} nucleus emits a γ-ray photon of energy 14·4 keV. If the mass of nucleus is 56·935 amu, calculate the recoil energy of the nucleus.

Given 1 amu = $1{\cdot}66 \times 10^{-27}$ kg.

[Ans. $1{\cdot}95 \times 10^{-6}$ keV]

48. A particle moves in a straight line with retardation proportional to its displacement. Calculate the loss of K.E. for any displacement *x*. **(AIEEE 2004)**

$$\left[\textbf{Ans. } -\frac{mKx^2}{2}\right]$$

HINTS

For Difficult Problems

1. Here, Weight of length L of the chain = Mg

Weight of length $\frac{L}{3}$ of the chain which is hanging

$$= \frac{1}{3}\,\text{Mg}$$

The centre of gravity of the hanging part lies at its middle point, *i.e.*, at a distance = L/6 below the edge of the table.

Work required to pull the hanging part on the table,

$$W = \text{force} \times \text{distance} = \left(\frac{1}{3}Mg\right)\times\frac{L}{6} = \frac{MgL}{18}$$

2. Here, $m = 2$ kg, $s = 40$ m, $a = 1\ \text{ms}^{-2}$, $W = ?$

$$W = Fs\cos 0° = mas = 2 \times 1 \times 40 = \mathbf{80\ J}$$

3. Work done = area under *F*–*x* graph, Fig. 6(Q).2

$$= 20\times1+\left(\frac{20+40}{2}\right)\times1+\left(\frac{40+30}{2}\right)\times1+30\times1$$

$$= 20 + 30 + 35 + 30 = \mathbf{115\ J}$$

5. As impulse = change in momentum

$$J = m(v-u) \qquad \text{...}(i)$$

According to work energy theorem,

$$W = \text{increase in K.E.} = \frac{1}{2}\,m(v^2-u^2)$$

$$W = \frac{1}{2}\,m(v-u)(v+u)$$

Using (*i*), $W = J(v+u)/2$

6. Here, $\vec{r_1} = (2\hat{i} + 3\hat{j})$ and $\vec{r_2} = (3\hat{i} + 2\hat{j})$

$\vec{F} = (5\hat{i} + 5\hat{j})$, $W = ?$

Displacement, $\vec{s} = \vec{r_2} - \vec{r_1} = (\hat{i} - \hat{j})$

$$W = \vec{F}.\vec{s} = (5\hat{i} + 5\hat{j}).(\hat{i} - \hat{j}) = 5 - 5 = \mathbf{0}$$

7. Here, $m = 50 \text{ g} = \frac{50}{1000} \text{ kg} = \frac{1}{20} \text{ kg}$

$u = 220$ m/s

Initial KE of bullet, $E_1 = \frac{1}{2} mu^2$

$$E_1 = \frac{1}{2} \times \frac{1}{20} \times (220)^2 = 1{\cdot}21 \times 10^3 \text{ J}$$

Final KE of bullet, $E_2 = 20\% \ E_1$

$$= \frac{20}{100} \times 1{\cdot}21 \times 10^3 \text{ J} = 0{\cdot}242 \times 10^3 \text{ J}$$

From $E_2 = \frac{1}{2} mv^2$

$$v = \sqrt{\frac{2E_2}{m}} = \sqrt{\frac{2 \times 0{\cdot}242 \times 10^3}{1/20}}$$

$v = \mathbf{98{\cdot}38 \text{ m/s}}$

8. Here, $M = 0{\cdot}1$ kg, $m = 0{\cdot}02$ kg, $t = 3$ s, $u = 0$, $v = 20$ m/s, Thrust = ?, energy/mass = ?

Thrust of rocket,

$$F = Ma = \frac{M(v-u)}{t} = \frac{0{\cdot}1(20-0)}{3} = \frac{2}{3}\text{N}$$

KE gained by rocket $= \frac{1}{2} Mv^2 = \frac{1}{2} \times 0{\cdot}1 (20)^2$

$= 20$ J

Energy/mass of fuel $= \frac{20}{0{\cdot}02} = \mathbf{1000 \text{ J kg}^{-1}}$

9. Here, $m = 200 \text{ g} = \frac{1}{5}$ kg, $u = 20$ m/s, $v = 0$,

$s = 20 \text{cm} = \frac{1}{5} \text{m}$, $W = ?$, $F = ?$

Work done = Loss of KE

$$= 0 - \frac{1}{2} mu^2 = -\frac{1}{2} \times \frac{1}{5} (20)^2 = \mathbf{-40 \text{ J}}$$

As $W = F \times s$, $F = \frac{W}{s} = \frac{40}{1/5} = \mathbf{200 \text{ N}}$

10. Here, $m = 2$ kg, $u = 0$, $F = 20$ N, $t = 10$ s,

$\mu_k = 0{\cdot}2$, $f_k = \mu_k mg = 0{\cdot}2 \times 2 \times 10 = 4$ N

Net force, $F - f_k = 20 - 4 = 16$ N

Acceleration, $a = \frac{16}{2} = 8 \text{ m/s}^2$

$$s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 8 \times 10^2 = 400 \text{ m}$$

Work done by applied force $= F \times s$

$= 20 \times 400 = \mathbf{8000 \text{ J}}$

Change in KE = work done by net force

$= 16 \times 400 = \mathbf{6400 \text{ J}}$

11. Here, $F = 5$ N, $m = 2$ kg, $u = 0$, $\mu = 0{\cdot}2$

Force of friction, $f = \mu R = \mu mg = 0{\cdot}2 \times 2 \times 9{\cdot}8$

$= 3{\cdot}92$ N

Net force on the body, $F' = F - f = 5 - 3{\cdot}92 = 1{\cdot}08$ N

Acceleration of body $= \frac{F'}{m} = \frac{1{\cdot}08}{2} = 0{\cdot}54 \text{ ms}^{-2}$

Distance travelled in 5 s is

$$s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} (0{\cdot}54) \times 5^2 = 6{\cdot}75 \text{ m}$$

(*i*) work done by applied force

$W_1 = F \times s = 5 \times 6{\cdot}75 \text{ J} = \mathbf{33{\cdot}75 \text{ J}}$

(*ii*) work done by force of friction

$W_2 = -f \times s = -3{\cdot}92 \times 6{\cdot}75 = \mathbf{-26{\cdot}46 \text{ J}}$

(*iii*) work done by net force

$W_3 = F' \times s = 1{\cdot}08 \times 6{\cdot}75 = \mathbf{7{\cdot}29 \text{ J}}$

(*iv*) velocity of body after 5 s

$v = u + at = 0 + 0{\cdot}54 \times 5 = 2{\cdot}7$ m/s

Increase in K.E. $= \frac{1}{2} mv^2 - \frac{1}{2} mu^2$

$$= \frac{1}{2} \times 2 (2{\cdot}7)^2 - 0 = \mathbf{7{\cdot}29 \text{ J}}$$

Since increase in K.E. of the body = work done by the net force on the body, therefore **work energy theorem stands verified.**

12. Here, $l = 1$ m, $M = 1$ kg, $m = 10$ g, $u = 200$ m/s,

$h = ?$ $g = 10 \text{ m/s}^2$

If v is velocity of wooden bob with bullet inside, then from conservation of linear momentum

$(M + m)v = m \times u$,

$$v = \frac{mu}{(M+m)} = \frac{10^{-2} \times 200}{(1 + 10^{-2})} = \frac{2}{1{\cdot}01} \text{m/s}$$

Now, $$h = \frac{v^2}{2g} = \left(\frac{2}{1{\cdot}01}\right)^2 \times \frac{1}{2 \times 10} = \mathbf{0{\cdot}2\ m}$$

13. The forces acting on the body are

(*i*) force of gravity (*ii*) air friction.

According to work energy theorem, total work done on the body = Gain in K.E.

$$W = \frac{1}{2}mv^2 = \frac{1}{2}m(1{\cdot}2\sqrt{gH})^2 = 0{\cdot}72\ mgH$$

As work done by gravity $W_1 = mgH$.

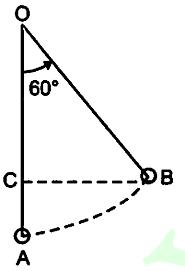
$\therefore$ Work done by friction, $W_2 = W - W_1$

$= 0{\cdot}72\ mgH - mgH$

$W_2 = \mathbf{-\ 0{\cdot}28\ mgH}$

14. Refer to Fig. 6(Q).8. In moving from A to B, the bob rises through a distance

FIGURE 6(Q).8



$AC = OA - OC = l - l\cos\theta = l(1 - \cos\theta)$

If v_1 is velocity at B, then applying principle of conservation of energy,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - mgl(1 - \cos\theta)$$

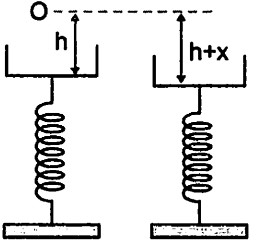
$$v_1^2 = v^2 - 2\,gl\,(1 - \cos\theta)$$

$$= 3^2 - 2 \times 10 \times 0{\cdot}5\ (1 - \cos 60°) = 9 - 5 = 4$$

$v_1 = \sqrt{4} = \mathbf{2\ m/s}$

15. As is clear from Fig. 6(Q).9, the ball falls through a total distance $(h + x)$ and the spring is compressed through distance x.

FIGURE 6(Q).9



$\therefore$ P.E. lost by the ball = $mg\,(h + x)$

Work done on the spring $= \frac{1}{2}Kx^2$

From energy conservation principle,

$$\frac{1}{2}Kx^2 = mg\,(h + x)$$

$$\mathbf{K = \frac{2mg\,(h + x)}{x^2}}$$

16. Here, $K = 24$ N/m, $m = 4$ kg

$F = 10$ N, $x = 0{\cdot}5$ m

Work done on the spring = gain in KE + gain in PE

$$\therefore \quad Fx = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

$$10 \times 0{\cdot}5 = \frac{1}{2} \times 4\,v^2 + \frac{1}{2} \times 24\,(0{\cdot}5)^2$$

$$2\,v^2 = 5 - 3 = 2\ ;\ v^2 = 1,\ v = \mathbf{1\ m/s}$$

17. As is known, in equilibrium position,

$F = mg = Kd$

or $$K = \frac{mg}{d} \qquad \text{...}(i)$$

When the object falls freely, its gravitational P.E. is converted into elastic P.E. of the spring

i.e., $$mgx = \frac{1}{2}Kx^2 = \frac{1}{2}\left(\frac{mg}{d}\right)x^2$$

or $1 = \frac{x}{2d}$ or $\mathbf{x = 2\,d}$

19. Mass defect, $\Delta m = 235{\cdot}04 + 1{\cdot}0087 - 140{\cdot}91 - 91{\cdot}926 - 3 \times 1{\cdot}0087$

$\Delta m = 0{\cdot}1866\ \text{u} = 0{\cdot}1866 \times 1{\cdot}661 \times 10^{-27}$ kg

Energy released, $E = (\Delta m)\,c^2$

$= 0{\cdot}1866 \times 1{\cdot}661 \times 10^{-27}\,(3 \times 10^8)^2$ J

$$= \frac{0{\cdot}1866 \times 1{\cdot}661 \times 9 \times 10^{-11}}{1{\cdot}602 \times 10^{-13}}\ \text{MeV}$$

$E = \mathbf{174{\cdot}1\ MeV}$

20. Here, depth of well = 20 m, radius $r = \frac{3}{2}$ m

depth of water = 14 m, t = ?

$P = 5\ hp = 5 \times 746$ W

Area of cross section of well,

$$A = \pi r^2 = \frac{22}{7}(3/2)^2 = \frac{99}{14}\,\text{m}^2$$

Volume of water in well,

$$V = A \times d = \frac{99}{14} \times 14 = 99\,\text{m}^3$$

Mass of water in well, $m = V \times \rho = 99 \times 10^3$ kg

When the well is emptied, depth of water changes from (20 – 14) m in the beginning to (20 – 0) m at the end.

$$\therefore \text{ Average height raised, } h = \frac{6+20}{2} = 13\,\text{m}$$

Work done, $W = mgh = 99 \times 10^3 \times 9{\cdot}8 \times 13$

$$\text{As } P = \frac{W}{t}, \quad t = \frac{W}{P} = \frac{99 \times 10^3 \times 9{\cdot}8 \times 13}{5 \times 746}$$

$$= \mathbf{3381{\cdot}6 \text{ second}}$$

21. Here, $m = 1800$ kg, $v = 2\,\text{ms}^{-1}$,

$f = 5000$ N, $P = ?$

Minimum force required to be applied,

$F = mg + f = 1800 \times 10 + 5000 = 23000$ N

$P = F \times v = 23000 \times 2 = 46000$ W = **46 kW**

$$= \frac{46000}{746} hp = \mathbf{61{\cdot}66 \text{ hp}}$$

22. Here, $m = 100$ kg, $h = 10$ m, $t = 5$ s

$g = 10\,\text{m/s}^2$, $\eta = 60\%$

$$\text{output power, } P_o = \frac{mgh}{l} = \frac{100 \times 10 \times 10}{5}$$

$$= 2000 \text{ watt} = 2 \text{ kW}$$

Input power $P_i = ?$

$$\text{As } \eta = \frac{P_o}{P_i} \quad \therefore \quad P_i = \frac{P_o}{\eta} = \frac{2}{60/100} = \mathbf{3{\cdot}33 \text{ kW}}$$

24. Here, $m = 1000$ kg

$$u = 0, \; v = 54 \text{ km/h} = \frac{54 \times 1000}{60 \times 60} \text{m/s} = 15 \text{ m/s}$$

$t = 5$ s

$a = ?$ gain in K.E. = ? Average power = ?

From $v = u + at$

$$15 = 0 + a \times 5\,; \; a = \frac{15}{5} = \mathbf{3\,m/s^2}$$

$$\text{Gain in K.E.} = \frac{1}{2} m (v^2 - u^2)$$

$$= \frac{1}{2} \times 1000 (15^2 - 0)$$

$$= \mathbf{1{\cdot}125 \times 10^5 \text{ J}}$$

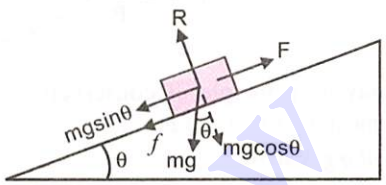
Average power spent

$$= \frac{W}{t} = \frac{\text{gain in K.E.}}{t} = \frac{1{\cdot}125 \times 10^5}{5}$$

P = **22500 W**

27. Here, $m = 6{\cdot}5$ metric ton $= 6{\cdot}5 \times 1000 = 6500$ kg

FIGURE 6(Q).10



$$\sin\theta = \frac{5}{13}, \quad v = 9 \text{ km/h} = \frac{9 \times 1000}{60 \times 60} = 2{\cdot}5\,\text{m/s}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - (5/13)^2} = \frac{12}{13}\,;$$

$$\mu = \frac{1}{12}$$

Force applied by the engine, Fig. 6(Q).8,

$$F = mg \sin\theta + f = mg \sin\theta + \mu\, mg \cos\theta$$

$$= mg (\sin\theta + \mu \cos\theta)$$

$$= 6500 \times 9{\cdot}8 \left(\frac{5}{13} + \frac{1}{12} \times \frac{12}{13} \right)$$

$$= 6500 \times 9{\cdot}8 \times \frac{6}{13} = 29400\,\text{N}$$

Power of engine, $P = F \times v$

$= 29400 \times 2{\cdot}5 = 73500$ W

P = **73·5 kW**

28. Here, $V = 4000$ c.c, $t = 1$ min. $= 60$ s

Pressure, $P = h \rho g = 130 \times 10^{-1} \times 1{\cdot}03 \times 980$ dyne/cm^2

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{P \times V}{t}$$

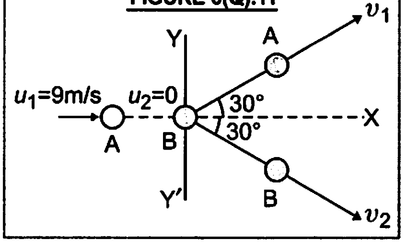
$$= \frac{13 \times 1{\cdot}03 \times 980 \times 4000}{60} \text{ erg/sec.}$$

$$= 218{\cdot}7 \times 4 \times 10^{-4} \text{ watt}$$

$$\text{Horse power} = \frac{218{\cdot}7 \times 4 \times 10^{-4}}{746} = \mathbf{1{\cdot}17 \times 10^{-4}}$$

29. Here, $u_1 = 9$ m/s, $u_2 = 0$.

FIGURE 6(Q).11



Applying principle of conservation of linear momentum, Fig. 6(Q).11,

(*i*) *along X-axis*

$$m \times 9 + m \times 0 = mv_1 \cos 30° + mv_2 \cos 30°$$

$$m \times 9 = m\frac{\sqrt{3}}{2}(v_1 + v_2)$$

$$\therefore \quad v_1 + v_2 = \frac{18}{\sqrt{3}} = 6\sqrt{3} \qquad ...(i)$$

(*ii*) *along Y-axis*

$$m \times 0 + m \times 0 = mv_1 \sin 30° - mv_2 \sin 30°$$

$$0 = m(v_1 - v_2) \times \frac{1}{2}, v_1 - v_2 = 0, v_1 = v_2.$$

From (*i*), $v_1 = v_2 = \mathbf{3\sqrt{3}}$ **m/s**

Total KE before collision

$$= \frac{1}{2}m(3\sqrt{3})^2 + \frac{1}{2}m(3\sqrt{3})^2 = 27\,m$$

which is less than total KE before collision.

$\therefore$ **K.E. is not conserved.**

30. The nuclear decay is represented as

$${}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_2He^4$$

The KE of a particle is given by $K = \frac{p^2}{2m}$

$$\therefore \quad p = \sqrt{2mK}$$

As momentum is conserved in nuclear decay.

$\therefore \quad mK$ = constant

$$m_{Rn} K_{Rn} = m_\alpha K_\alpha$$

or $K_{Rn} = \frac{m_\alpha K_\alpha}{m_{Rn}} = \frac{4{\cdot}003 \times 4{\cdot}8}{222} = \mathbf{0{\cdot}0866\ MeV}$

31. Here, $m_1 = m$, $u_1 = v$, say, $m_2 = 4\,m$
$u_2 = 0$, $v_1 = ?$

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$$

$$v_1 = \frac{(m - 4m)v + 0}{m + 4m} = -\frac{3}{5}v$$

initial K.E., $E_1 = \frac{1}{2}m_1 u_1^2 = \frac{1}{2}mv^2$

final K.E., $E_2 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m\left(-\frac{3}{5}v\right)^2$

$$= \frac{1}{2} \times \frac{9}{25}mv^2$$

% K.E. transferred $= \frac{(E_1 - E_2)}{E_1} \times 100$

$$= \frac{\left(1 - \frac{9}{25}\right)\frac{1}{2}mv^2 \times 100}{\frac{1}{2}mv^2} = \mathbf{64\%}$$

32. If v_1, v_2 are velocities of fragments of masses $m_1 = 3$ kg and $m_2 = 1$ kg, then according to the principle of conservaton of linear momentum,

$$m_1 v_1 + m_2 v_2 = 0$$

$$3\,v_1 + 1\,v_2 = 0 \text{ or } \frac{v_1}{v_2} = -\frac{1}{3}$$

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{3}{1}\left(-\frac{1}{3}\right)^2 = \frac{1}{3}$$

As total K.E. $= E = E_1 + E_2 = 6 \times 10^4$ J

$$\therefore \quad E_1 = \frac{6 \times 10^4 \times 1}{(1+3)} = \mathbf{1{\cdot}5 \times 10^4\ J}$$

$$E_2 = 6 \times 10^4 - 1{\cdot}5 \times 10^4 = 4{\cdot}5 \times 10^4 \text{ J}$$

Linear momentum of smaller fragment

$$p_2 = \sqrt{2mE_2} = \sqrt{2 \times 1 \times 4{\cdot}5 \times 10^4}$$

$$= 3 \times 10^2 \text{ kg ms}^{-1}$$

As the direction is opposite

$\therefore \quad p_2 = \mathbf{-300\ kg\ ms^{-1}}$

33. Applying principle of conservation of linear momentum

$$m_\alpha v_\alpha + m_r v_r = 0 \quad \therefore \quad v_r = -\frac{m_\alpha}{m_r} \times v_\alpha$$

When ${}_2He^4$ is emitted from U^{238}, residual nucleus has a mass $m_r = 238 - 4 = 234$

$$\therefore \quad v_r = -\frac{4}{234} \times 1{\cdot}4 \times 10^7 \text{ m/s}$$

$$= -\mathbf{2{\cdot}4 \times 10^5 \text{ m/s}}$$

34. Here, $m_1 = m_2 = m$, $u_1 = u$, $u_2 = 0$

Let v_1 and v_2 be their respective velocities after collision.

As $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\therefore \quad mu + 0 = m(v_1 + v_2)$ or $u = v_1 + v_2$ ...(*i*)

Again, by definition, $e = \dfrac{v_2 - v_1}{u - 0}$

$\therefore \quad v_2 - v_1 = u\,e$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$$v_2 = \frac{(1+e)u}{2}$$

$$v_1 = \frac{(1-e)u}{2} \qquad \therefore \quad \frac{v_1}{v_2} = \frac{\mathbf{1-e}}{\mathbf{1+e}}$$

37. Applying principle of conservation of linear momentum, we get $mv = (M+m)V$

$10^{-2} \times (2 \times 10^2) = (1 + 0{\cdot}01)V$

$$V = \frac{2}{1{\cdot}01}$$

K.E. of the block with bullet in it, is converted into P.E., as it rises through a height *h*.

$$\therefore \quad \frac{1}{2}(M+m)V^2 = (M+m)\,g\,h$$

$$V^2 = 2\,g\,h \quad \text{or} \quad h = \frac{V^2}{2g}$$

$$h = \left(\frac{2}{1{\cdot}01}\right)^2 \times \frac{1}{2 \times 9{\cdot}8} = \mathbf{0{\cdot}2\,m}$$

39. Refer to Fig. 6(Q).10,

Before collision, $v_x = \dfrac{v}{\sqrt{2}}, \quad v_y = \dfrac{v}{\sqrt{2}}$

After collision ; $v_x{}' = \dfrac{v}{\sqrt{2}}(= v_x)$

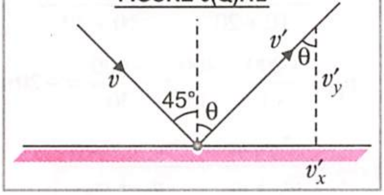
$$\text{As} \quad e = \frac{\text{normal compt. of vel. of separation}}{\text{normal compt. of vel. of approach}}$$

$$e = \frac{v_y{}'}{v_y}$$

$$v_y{}' = e\,v_y = \frac{1}{\sqrt{2}} \cdot \frac{v}{\sqrt{2}} = \frac{v}{2}$$

As is clear from Fig. 6(Q).12,

FIGURE 6(Q).12



Velocity after impact,

$$v' = \sqrt{v_x'^2 + v_y'^2} = \sqrt{\frac{v^2}{2} + \frac{v^2}{4}} = \frac{\sqrt{3}}{2}v$$

If θ is angle of reflection,

$$\tan\theta = \frac{v_x{}'}{v_y{}'} = \frac{v}{\sqrt{2}} \cdot \frac{2}{v} = \sqrt{2}$$

$$\theta = \tan^{-1}\sqrt{2}$$

40. Refer to Fig. 6(Q).13, Here, $m_1 = m$, $v_1 = \sqrt{2gh}$

$m_2 = m$, $v_2 = 0$

If *v* is the combined velocity after collision, using the principle of conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\,v$$

$$m\sqrt{2gh} + 0 = 2mv$$

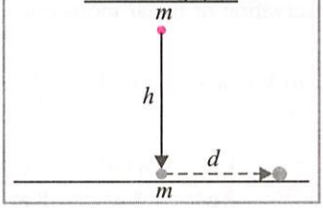
$$v = \sqrt{\frac{gh}{2}} \qquad \text{...(i)}$$

Work done against resistive forces

= energy lost by the combination of bodies

$$= \frac{1}{2}(2m)v^2 = m\left(\sqrt{\frac{gh}{2}}\right)^2 = \frac{\mathbf{mgh}}{\mathbf{2}}$$

FIGURE 6(Q).13



41. Here, $m_1 = 10$ kg, $m_2 = 20$ kg

$u_1 = 20$ m/s, $u_2 = -10$ m/s, $v_1 = ?$, $v_2 = ?$

As the collision is perfectly elastic,

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2 u_2}{m_1 + m_2}$$

$$= \frac{(10-20)\,20}{10+20} + \frac{2\times 20(-10)}{10+20}$$

$$v_1 = \frac{-200}{30} - \frac{400}{30} = \frac{-600}{30} = \mathbf{-20\,m/s}$$

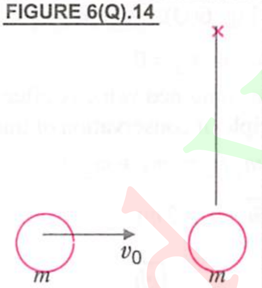
and $$v_2 = \frac{2m_1 u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2}$$

$$= \frac{2\times 10\times 20}{10+20} + \frac{(20-10)(-10)}{20+10}$$

$$= \frac{300}{30} = \mathbf{10\,m/s}$$

42. As the collision is perfectly elastic, the total K.E. is transferred to the ball of the pendulum, Fig. 6(Q).14. If h is maximum height reached by the pendulum, then as P.E. = K.E.

FIGURE 6(Q).14



$$mgh = \frac{1}{2}mv_0^2$$

$$h = \frac{\mathbf{v_0^2}}{\mathbf{2g}}$$

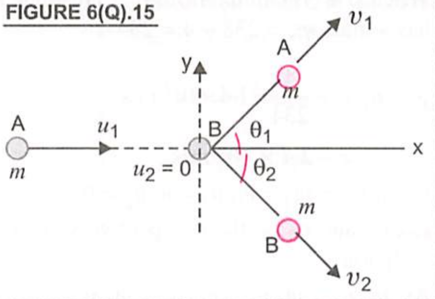
(*ii*) When the two masses stick together, velocity of the combination (V) is obtained from principle of conservation of linear momentum, *i.e.*,

$$(2\,m)\,V = m \times v_0 + 0,\; V = \frac{v_0}{2}$$

$$\therefore \quad h = \frac{V^2}{2g} = \frac{(v_0/2)^2}{2g} = \frac{\mathbf{v_0^2}}{\mathbf{8g}}$$

43. Let m be the mass of each ball. The situation is shown in Fig. 6(Q).15.

FIGURE 6(Q).15



As the collision is elastic, KE is conserved.

$$\therefore\; \frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$u_1^2 + u_2^2 = v_1^2 + v_2^2$$

$$(0{\cdot}5)^2 + 0 = (0{\cdot}3)^2 + v_2^2$$

$$v_2 = \sqrt{(0{\cdot}5)^2 - (0{\cdot}3)^2} = \mathbf{0{\cdot}4\,m/s}$$

According to law of conservation of linear momentum along X-axis,

$$0{\cdot}5\,m + 0 = 0{\cdot}3\,m\cos\theta_1 + 0{\cdot}4\,m\cos\theta_2$$

or $$5 = 3\cos\theta_1 + 4\cos\theta_2$$

or $$3\cos\theta_1 = 5 - 4\cos\theta_2 \quad \textbf{....(}\textbf{\textit{i}}\textbf{)}$$

Again, applying law of conservation of linear momentum along Y-axis,

$$0{\cdot}3\,m\sin\theta_1 = 0{\cdot}4\,m\sin\theta_2$$

or $$3\sin\theta_1 = 4\sin\theta_2 \quad \textbf{...(}\textbf{\textit{ii}}\textbf{)}$$

Squaring (*i*) and (*ii*) and adding, we get

$$9(\cos^2\theta_1 + \sin^2\theta_1) = (5 - 4\cos\theta_2)^2 + (4\sin\theta_2)^2$$

$$9 \times 1 = 25 + 16\cos^2\theta_2 - 40\cos\theta_2 + 16\sin^2\theta_2$$

$$= 16(\cos^2\theta_2 + \sin^2\theta_2) - 40\cos\theta_2 + 25$$

$$40\cos\theta_2 = 16 + 25 - 9 = 32$$

$$\cos\theta_2 = \frac{32}{40} = \frac{4}{5}$$

From (*i*), $$\cos\theta_1 = \frac{5}{3} - \frac{4}{3}\times\frac{4}{5} = \frac{3}{5}$$

$$\therefore \quad \sin\theta_1 = \sqrt{1-\cos^2\theta_1} = 4/5$$

$$\sin\theta_2 = \sqrt{1-\cos^2\theta_2} = 3/5$$

$$\sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$$

$$= \frac{4}{5}\times\frac{4}{5} + \frac{3}{5}\times\frac{3}{5} = 1$$

$$\therefore \quad \boldsymbol{\theta_1 + \theta_2 = 90^\circ}$$

44. Here, $u_1 = 2$ m/s, $u_2 = 0$, $m_1 = m$, $m_2 = 2$ m

$e = 0{\cdot}5$, $v_1 = ?$ $v_2 = ?$

As $$e = \frac{v_2 - v_1}{u_1 - u_2} = 0{\cdot}5$$

$$\therefore\ v_2 - v_1 = 0{\cdot}5\,(u_1 - u_2) = 0{\cdot}5\,(2-0) = 1 \qquad \text{...}(i)$$

According to the law of conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$$
$$m\,v_1 + 2\,m\,v_2 = m \times 2 + 0$$
$$v_1 + 2\,v_2 = 2 \qquad \text{...}(ii)$$

Add (i) and (ii) : $3\,v_2 = 3$, $v_2 = \mathbf{1\ m/s}$

From (i), $v_1 = v_2 - 1 = 1 - 1 = \mathbf{0}$

45. Suppose v' is speed after collision and θ' is angle of reflection, Fig. 6(Q).7.

During collision, the floor exerts a force on the ball along the normal. There is no force parallel to the surface. Therefore, velocity component of ball parallel to the floor remains constant, *i.e.,*

$$v' \sin\theta' = v \sin\theta \qquad \text{...}(i)$$

For components normal to the floor,

velocity of separation = $v' \cos\theta'$

velocity of approach = $v \cos\theta$

$$\therefore \quad \frac{v'\cos\theta}{v\cos\theta} = e$$

$$v' \cos\theta' = e\,v \cos\theta \qquad \text{...}(ii)$$

From (i) and (ii)

$$v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

and $$\tan\theta' = \frac{\tan\theta}{e}$$

$$\therefore \quad \theta' = \tan^{-1}\left(\frac{\tan\theta}{e}\right)$$

Note. When collision is perfectly elastic,

$e = 1$, $\therefore\ v' = v$ and $\theta' = \theta$.

46. Here, $h_1 = 10$ m, $h_2 = 2{\cdot}5$ m

Let v_1 be the velocity of the body just before collision with the ground and v_2 be the velocity of the body just after collision.

$$\therefore \quad \frac{1}{2}mv_1^2 = m\,g\,h_1 \quad \text{and} \quad \frac{1}{2}mv_2^2 = m\,g\,h_2$$

Dividing, we get

$$\frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2{\cdot}5} = 4\ ; \quad \therefore\ \frac{v_1}{v_2} = 2$$

$$\%\text{ age loss in KE } = \frac{(K_1 - K_2)}{K_1} \times 100$$
$$= \frac{mg\,(h_1 - h_2)}{mgh_1} \times 100$$
$$= \frac{(10 - 2{\cdot}5)}{10} \times 100 = \mathbf{75\%}$$

47. The nuclear decay is represented as

$$\underset{\substack{\text{(excited)}\\ \text{state}}}{\mathrm{Fe}^{57}} \rightarrow \underset{\substack{\text{(ground)}\\ \text{state}}}{\mathrm{Fe}^{57}} + \underset{(14{\cdot}4\,\text{keV})}{\gamma\text{-ray photon}}$$

According to de Broglie hypothesis, linear momentum of photon,

$$p = \frac{E}{c} = \frac{14{\cdot}4 \times 1{\cdot}6 \times 10^{-16}\ \text{J}}{3 \times 10^8\ \text{m/s}}$$
$$= 7{\cdot}68 \times 10^{-24}\ \text{kg ms}^{-1}$$

From conservation of linear momentum,

momentum of daughter nucleus = momentum of photon

$$p = 7{\cdot}68 \times 10^{-24}\ \text{kg ms}^{-1}$$

$\therefore$ Recoil energy of nucleus

$$K = \frac{p^2}{2m} = \frac{(7{\cdot}68 \times 10^{-24})^2}{2 \times 56{\cdot}935 \times 1{\cdot}66 \times 10^{-27}}$$
$$= 0{\cdot}312 \times 10^{-21}\ \text{J}$$

$$K = \frac{0{\cdot}312 \times 10^{-21}}{1{\cdot}6 \times 10^{-16}}\,\text{keV} = \mathbf{1{\cdot}95 \times 10^{-6}\ keV}$$

48. Here, retardation $\propto$ displacement

$$-a \propto x \quad \text{or} \quad -a = K\,x \qquad \text{...}(i)$$

where K is constant of proportionality.

From (i), $$-a = -\frac{dv}{dt} = K\,x \quad \text{or} \quad \frac{dv}{dt}\frac{dx}{dx} = -K\,x$$

As $\dfrac{dx}{dt} = v$, therefore, $v\,dv = -K\,x\,dx$

$$\int_0^v v\,dv = -K\int_0^x x\,dx$$

or $$\left[\frac{v^2}{2}\right]_u^v = -K\left[\frac{x^2}{2}\right]_0^x$$

or $$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -\frac{K\,x^2 \times m}{2}$$

Thus loss in K.E., $$\Delta K = -\frac{\mathbf{K\,m\,x^2}}{\mathbf{2}}$$

NCERT EXERCISES

WITH SOLUTIONS

Q. 1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative :

(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) Work done by gravitational force in the above case.

(c) Work done by friction on a body sliding down an inclined plane.

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Sol. Work done $W = \vec{F} . \vec{s} = F s \cos \theta$ where θ is smaller angle between force $\vec{F}$ and displacement $\vec{s}$.

(*a*) To lift the bucket, force equal to weight of bucket has to be applied vertically upwards and the bucket moves along the same direction, *i.e.*, $\theta = 0°$, therefore $W = F s \cos 0° = F s$. It is ***positive.***

(*b*) The bucket moves in a direction opposite to the gravitational force which acts vertically downwards.

$\therefore$ $\theta = 180°$, $W = F s \cos 180° = - F s$. It is ***negative.***

(*c*) Friction always opposes the relative motion.

$\therefore$ $\theta = 180°$, $W = F s \cos 180° = - F s$. It is ***negative.***

(*d*) As the body moves along the direction of applied force $\theta = 0°$,

$\therefore$ $W = F s \cos 0° = F s$. It is ***positive.***

(*e*) The direction of resistive force is opposite to the direction of motion of the bob, *i.e.*, $\theta = 180°$. Hence, work done, in this case, is ***negative.***

Q. 2. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0·1. Calculate the

(a) work done by applied force in 10 s.

(b) work done by friction in 10 s.

(c) work done by the net force on the body in 10 s.

(d) change in K.E. of the body in 10 s ; and interpret your result.

Sol. Here, $m = 2$ kg, $u = 0$, $F = 7$ N ; $\mu = 0{\cdot}1$, $W = ?$; $t = 10$ s

Acceleration produced by applied force ; $a_1 = F/m = 7/2 = 3{\cdot}5$ m/s^2

Force of friction, $f = \mu R = \mu mg = 0{\cdot}1 \times 2 \times 9{\cdot}8 = 1{\cdot}96$ N

Retardation produced by friction, $a_2 = \frac{-f}{m} = -\frac{1{\cdot}96}{2} = -0{\cdot}98 \text{ ms}^{-2}$

Net acceleration with which body moves, $a = a_1 + a_2 = 3{\cdot}5 - 0{\cdot}98 = 2{\cdot}52 \text{ ms}^{-2}$

Distance moved by the body in 10 second, from $s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 2{\cdot}52 \times (10)^2 = 126$ m.

(*a*) Work done by the applied force $= F \times s$

$$W_1 = 7 \times 126 = \mathbf{882\ J}$$

(*b*) Work done by the force of friction $W_2 = - f \times s = - 1{\cdot}96 \times 126 = \mathbf{-\ 246{\cdot}9\ J}$

(*c*) Work done by the net force $W_3 =$ Net force $\times$ distance $= (F - f)\, s = (7 - 1{\cdot}96)\, 126 = \mathbf{635\ J}$

(*d*) From $v = u + at$

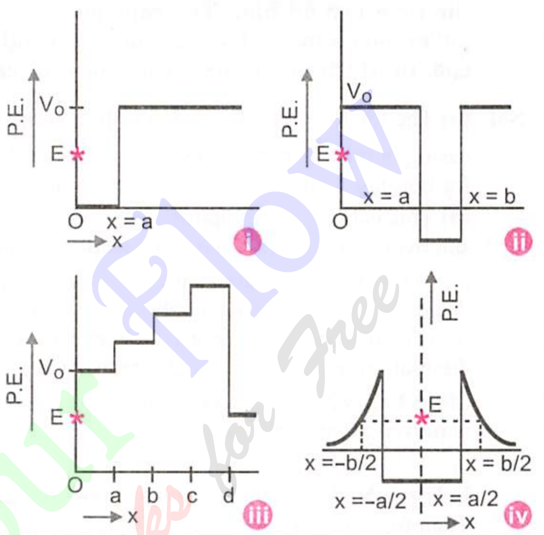
$v = 0 + 2{\cdot}52 \times 10 = 25{\cdot}2\ \text{ms}^{-1}$ $\quad\therefore$ Final K.E. $= \frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times (25{\cdot}2)^2 = 635$ J.

Initial K.E. $= \frac{1}{2} mu^2 = 0.$ $\quad\therefore$ Change in K.E. $= 635 - 0 =$ **635 J**

This shows that change in K.E. of the body is equal to work done by the net force on the body.

Q. 3. Given in Fig. 6(N).1 are examples of some potential energy functions in one dimension (i) to (iv). The total energy *E* of the particle is indicated by the cross mark on energy axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

FIGURE 6(N).1



Sol. We know that total energy $E =$ K.E. + P.E. or K.E. $= E -$ P.E. and K.E. can never be negative. The object cannot exist in the region, where its K.E. would become negative.

(*i*) For $x > a$, P.E. $(V_0) > E$

$\therefore$ K.E. becomes negative. Hence, the object cannot exist in the region $x > a$.

(*ii*) For $x < a$ and $x > b$, P.E. $(V_0) > E$.

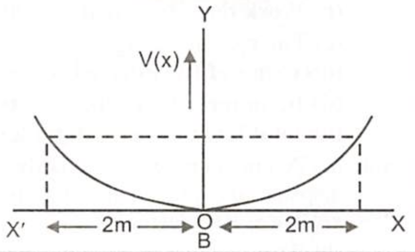
$\therefore$ K.E. becomes negative. Hence the object cannot be present in the region $x < a$ and $x > b$.

(*iii*) Object cannot exist in any region because P.E. $(V_0) > E$ in every region.

(*iv*) On the same basis, the object cannot exist in the region $-b/2 < x < -a/2$ and $a/2 < x < b/2$.

Q. 4. The potential energy function for a particle executing simple harmonic motion is given by $V(x) = 1/2\, kx^2$, where *k* is the force constant of the oscillator. For $k = 0{\cdot}5$ N m^{-1}, the graph of $V(x)$ versus *x* is shown in Fig. 6(N).2. Show that a particle of total energy 1 joule moving under this potential must turn back, when it reaches $x = \pm 2$ m.

FIGURE 6(N).2



Sol. At any instant, the total energy of an oscillator is the sum of K.E. and P.E.

i.e. $$E = \text{K.E.} + \text{P.E.} = \frac{1}{2} mu^2 + \frac{1}{2} kx^2$$

The particle turns back at the instant, when its velocity becomes zero *i.e.* $u = 0$.

$$\therefore \qquad E = 0 + \frac{1}{2} kx^2$$

As $E = 1$ joule and $k = \frac{1}{2}$ N/m $\quad \therefore \quad 1 = \frac{1}{2} \times \frac{1}{2} x^2$ or $x^2 = 4$, $x = \pm$ **2 m**

Q. 5. Answer the following : (a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere ?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?

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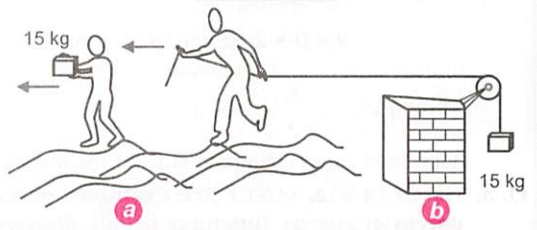
(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?

(d) In Fig. 6(N).3(*a*), the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 6(N).3(*b*), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?

FIGURE 6(N).3



Sol. (*a*) The total energy of a rocket in flight depends on its mass *i.e.* P.E. + K.E. = $mgh + \frac{1}{2}mv^2$. When the casing burns up, its mass decreases. The total energy of the rocket decreases. Hence, heat energy required for burning is obtained from the rocket itself and not from the atmosphere.

(*b*) This is because gravitational force is a conservative force. Work done by the gravitational force of the sun over a closed path in every complete orbit of the comet is zero.

(*c*) When the artificial satellite orbiting the earth comes closer and closer to earth, its potential energy decreases. As sum of potential energy and kinetic energy is constant, therefore, K.E. of satellite and hence its velocity goes on increasing. However, total energy of the satellite decreases a little on account of dissipation against atmospheric resistance.

(*d*) In Fig. 6(N).3(*a*), force is applied on the mass, by the man in vertically upward direction but distance is moved along the horizontal

$\therefore \quad \theta = 90°$. $W = F\,s\cos 90° =$ Zero.

In Fig. 6(N).3(*b*), force is applied along the horizontal and the distance moved is also along the horizontal. Therefore, $\theta = 0°$.

$$W = F\,s\cos\theta = mg \times s\cos 0°$$

$$W = 15 \times 9{\cdot}8 \times 2 \times 1 = 294 \text{ joule.}$$

$\therefore$ **Work done in 2nd case is greater.**

Q. 6. Point out the correct alternative : (a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(c) The rate of change of total momentum of a many particle system is proportional to the external force/sum of the internal forces of the system.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum /total energy of the system of two bodies.

Sol. (*a*) Potential energy of the body decreases. The conservative force does positive work on a body, when it displaces the body in the direction of force. The body, therefore, approaches the centre of force, decreasing *x*. Hence, P.E. decreases.

(*b*) Work is done by a body against friction at the expense of its kinetic energy. Hence *K.E. of the body decreases.*

(*c*) Internal forces cannot change the total or net momentum of a system. Hence the rate of change of total momentum of many particle system is *proportional to the external force* on the system.

(*d*) In an inelastic collision of two bodies, the quantities which do not change after the collision are total linear momentum and total energy of the system of two bodies. The total K.E changes as some energy appears in other forms.

Q. 7. State if each of the following statements is true or false. Give reasons for your answer.

(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Sol. (*a*) *False*. The total momentum and total energy of the system are conserved and not of each body.

(*b*) *False*. The external forces on the body may change the total energy of the body.

(*c*) *False*. Work done in the motion of a body over a closed loop is zero only when the body is moving under the action of conservative forces (like gravitational or electrostatic forces). It is not zero when the forces are non-conservative e.g. frictional forces etc.

(*d*) *True*, because in an inelastic collision, some kinetic energy usually changes into some other form of energy.

Q. 8. Answer carefully, with reasons : (a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact) ?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls ?

(c) What are the answers to (a) and (b) for an inelastic collision ?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Sol. (*a*) No, K.E. is not conserved during the given elastic collision. K.E. before and after collision is the same. Infact, during collision, K.E. of the balls gets converted into potential energy.

(*b*) Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

(*c*) In an inelastic collision, total K.E. is not conserved during collision and even after collision. The total linear momentum is however, conserved during as well as after collision.

(*d*) The collision is elastic, because the forces involved are conservative.

Q. 9. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to (i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol. From $v = u + at$

$v = 0 + at = at$

As power, $P = F \times v \quad \therefore \quad P = (ma) \times at = ma^2 t$

As m and a are constants, therefore, $P \propto t$

Q. 10. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to (i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol. As power, P = force × velocity $\quad \therefore \quad P = [MLT^{-2}]\,[LT^{-1}] = [ML^2 T^{-3}]$

As $\quad P = [ML^2 T^{-3}] = \text{constant} \;\therefore\; L^2 T^{-3} = \text{constant or } \dfrac{L^2}{T^3} = \text{constant} \;\therefore\; L^2 \propto T^3 \text{ or } L \propto T^{3/2}$

Q. 11. A body constrained to move along the z-axis of a co-ordinate system is subject to a constant force $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})$ newton. What is the work done by this force in moving the body over a distance of 4 m along the z-axis ?

Sol. Here, $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})\,\text{N}$

$\vec{s} = 4\hat{k}$ (∵ 4 m distance is along z-axis).

$W = ?$

As $W = \vec{F}.\vec{s} \quad \therefore \quad W = (-\hat{i} + 2\hat{j} + 3\hat{k}).(4\hat{k}) = 12\hat{k}.\hat{k} = \mathbf{12\,J}$

Q. 12. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV and the second with 100 keV. Which is faster, the electron or proton ? Obtain the ratio of their speeds. Take mass of electron = $9{\cdot}11 \times 10^{-31}$ kg, mass of proton = $1{\cdot}67 \times 10^{-27}$ kg and 1 eV = $1{\cdot}6 \times 10^{-19}$ J.

Sol.
$$E_1 = \frac{1}{2} m_e v_e^2 = 10 \times 1{\cdot}6 \times 10^{-16} \qquad \text{...(}i\text{)}$$

$$E_2 = \frac{1}{2} m_p v_p^2 = 100 \times 1{\cdot}6 \times 10^{-16} \qquad \text{...(}ii\text{)}$$

Divide (*i*) by (*ii*) $\dfrac{m_e v_e^2}{m_p v_p^2} = \dfrac{1}{10}$

$$\frac{v_e}{v_p} = \sqrt{\frac{1}{10}\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}}{10\times9.11\times10^{-31}}} = 10^2\sqrt{\frac{1.67}{91.1}} = \mathbf{13.53}$$

Q. 13. A rain drop of radius 2 mm. falls from a height of 500 m above the ground. It falls with decreasing acceleration due to viscous resistance of air until half its original height. It attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first half and second half of its journey ? Take density of water = 10^3 kg/m^3. What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1} ?

Sol. Here, $r = 2 \text{ mm} = 2 \times 10^{-3}$ m.

Distance moved in each half of the journey, $s = \dfrac{500}{2} = 250\,\text{m}$.

Density of water, $\rho = 10^3$ kg/m^3.

Mass of rain drop = volume of drop × density

$$m = \frac{4}{3}\pi r^3 \times \rho = \frac{4}{3}\times\frac{22}{7}\ (2\times10^{-3})^3\times10^3 = 3.35\times10^{-5}\text{ kg}$$

$$\therefore \quad W = mg \times s = 3.35\times10^{-5}\times9.8\times250 = \mathbf{0.082\ J}$$

It should be clearly understood that whether the drop moves with decreasing acceleration or with uniform speed, work done by the gravitational force on the drop remains the same.

If there were no resistive forces, energy of drop on reaching the ground.

$$E_1 = mgh = 3.35\times10^{-5}\times9.8\times500 = 0.164\text{ J}$$

Actual energy, $E_2 = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\times3.35\times10^{-5}\,(10)^2 = 1.675\times10^{-3}$ J

∴ Work done by the resistive forces, $W = E_1 - E_2 = 0.164 - 1.675\times10^{-3}$ $W = \mathbf{0.1623\ joule}$

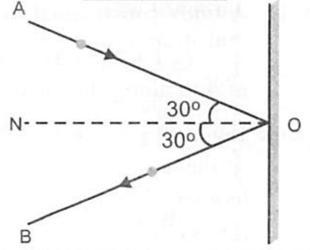
Q. 14. A molecule in a gas container hits the wall with speed 200 m/s at an angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?

Sol. We know that momentum is conserved in all collisions, elastic as well as inelastic. Let us now check if K.E. is conserved or not.

As the wall is too heavy, the recoiling molecule produces no velocity in the wall, Fig. 6(N).4

If m is mass of the gas molecule and M is mass of wall, then total K.E after collision.

FIGURE 6(N).4



$$E_2 = \frac{1}{2}m(200)^2 + \frac{1}{2}M\,(0)^2 = 2\times10^4\text{ m J}$$

which is the K.E. of the molecule before collision. $[E_1 = \dfrac{1}{2}\ m\ (200)^2 = 2\times10^4\text{ m J}]$

Hence, the **collision is *elastic*.**

Q. 15. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground and the efficiency of the pump is 30 %, how much electric power is consumed by the pump ?

Sol. Here, volume of water = 30 m^3 ; t = 15 min. = 15 × 60 = 900 s.

Height, h = 40 m ; efficiency, $\eta = 30\%$.

As density of water = $\rho = 10^3$ kg/m^3, ∴ Mass of water pumped, m = volume × density = 30×10^3 kg

Actual power consumed or output power $P_0 = \frac{W}{t} = \frac{mgh}{t} = \frac{30\times10^3\times9{\cdot}8\times40}{900} = 13070$ watt

If P_i is input power (required), then as $\eta = \frac{P_0}{P_i}$ $\therefore$ $P_i = \frac{P_0}{\eta} = \frac{13070}{30/100} = 43567\ W = \mathbf{43{\cdot}567\ kW}$

Q. 16. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head on by another ball bearing of the same mass moving initially with a speed v, Fig. 6(N).5. If the collision is elastic, which of the following is a possible result after collision ?

Sol. Let m be the mass of each ball bearing.

Before collision, total K.E. of the system $= \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$

After collision. K.E. of the system is

Case I, $E_1 = \frac{1}{2}(2m)(v/2)^2 = \frac{1}{4}mv^2$

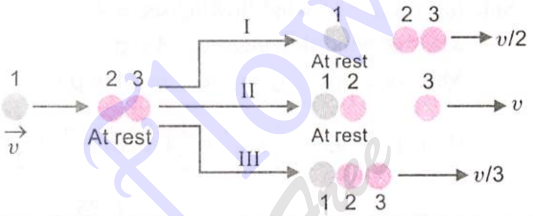
Case II, $E_2 = \frac{1}{2}mv^2$

Case III, $E_3 = \frac{1}{2}(3m)(v/3)^2 = \frac{1}{6}mv^2$

We observe that K.E. is conserved only in case II.

Hence **case II is the only possibility.**

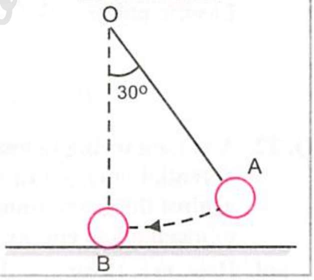
FIGURE 6(N).5



Q. 17. The bob A of a simple pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6(N).6. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.

Sol. The bob A shall not rise. This is because when two bodies of same mass undergo an elastic collision, their velocities are interchanged. After collision, ball A will come to rest and the ball B would move with the velocity of A, Fig. 6(N).6. Thus the bob A will not rise after the collision.

FIGURE 6(N).6



Q. 18. The bob of a pendulum is released from a horizontal position A as shown in Fig. 6(N).7. If the length of the pendulum is 1·5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipates 5% of its initial energy against air resistance ?

Sol. Here, $h = 1{\cdot}5$ m, $v = ?$

Energy dissipated = 5 %

Taking B as the lowest position of the bob, its potential energy at B is zero.

At the horizontal position A, total potential energy of the bob is mgh

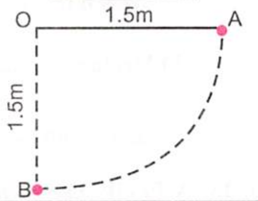
In going from A to B, P.E. of the bob is converted into K.E.

Energy converted = 95 % (mgh)

It v is velocity acquired at B, then K.E. $= \frac{1}{2}mv^2 = \frac{95}{100}mgh$

$$v = \sqrt{\frac{95}{100}\times2gh} = \sqrt{\frac{19}{20}\times2\times9{\cdot}8\times1{\cdot}5} = \mathbf{5{\cdot}28\,ms^{-1}}$$

FIGURE 6(N).7



Q. 19. A trolley of mass 300 kg carrying a sand bag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0·05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?

Sol. As the trolley carrying the sand bag is moving uniformly, therefore, external force on the system = Zero. When the sand leaks out, it does not lead to the application of any external force on the trolley. Hence, **the speed of the trolley shall not change.**

Q. 20. A body of mass 0·5 kg travels in a straight line with velocity $v = ax^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$ m ?

Sol. Here, $m = 0{\cdot}5$ kg ; $v = ax^{3/2}$, $a = 5\ m^{-1/2}\ s^{-1}$, $W = ?$

Initial vel. at $x = 0$, $v_1 = a \times 0 = 0$, Final vel. at $x = 2$, $v_2 = a\ 2^{3/2} = 5 \times 2^{3/2}$

$$\text{Work done} = \text{increase in K.E.} = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$W = \frac{1}{2} \times 0{\cdot}5\ [(5 \times 2^{3/2})^2 - 0] = \mathbf{50\ J}$$

Q. 21. The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of air passing through it in time t ? (b) What is K.E. of the air ? (c) Assume that the wind mill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, $v = 36$ km/h and the density of air is 1·2 kg/m³. What is electrical power produced ?

Sol. (*a*) Volume of wind flowing/sec $= A\ v$

Mass of wind flowing/sec $= A\ v\ \rho$

Mass of air passing in t sec, $m = A\ v\ \rho\ t$

$$(b)\ \text{K.E. of air} = \frac{1}{2}mv^2 = \frac{1}{2}(A v \rho t)v^2 = \frac{1}{2}A v^3 \rho t$$

$$(c)\ \text{Electrical energy produced} = \left(\frac{25}{100}\right) \times \text{K.E. of air} = \frac{1}{4} \times \frac{1}{2} A v^3 \rho t$$

$$\text{Electric power} = \frac{1}{8}\frac{A v^3 \rho t}{t} = \frac{1}{8} A v^3 \rho$$

$$P = \frac{1}{8} \times 30 \times (10)^3 \times 1{\cdot}2 = 4500 \text{ watt} = \mathbf{4{\cdot}5\ kW}$$

Q. 22. A person trying to lose weight (dieter) lifts a 10 kg mass through 0·5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated (a) How much work does she do against the gravitational force ? (b) Fat supplies $3{\cdot}8 \times 10^7$ J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?

Sol. Here, $m = 10$ kg, $h = 0{\cdot}5$ m, $n = 1000$

(*a*) Work done against gravitational force

$$W = n\ (mgh) = 1000 \times (10 \times 9{\cdot}8 \times 0{\cdot}5) = 49000 \text{ J.}$$

$$(b)\ \text{Mechanical energy supplied by 1 kg of fat} = 3{\cdot}8 \times 10^7 \times \frac{20}{100} = 0{\cdot}76 \times 10^7 \text{ J/kg}$$

$$\therefore \text{ Fat used up by the dieter} = \frac{1\text{kg}}{0{\cdot}76 \times 10^7} \times 49000 = \mathbf{6{\cdot}45 \times 10^{-3}\ kg}$$

Q. 23. A family uses 8 kW of power. (*a*) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ? (*a*) Compare this area to that of the roof of a typical house.

Sol. Let the area be A square metre.

$\therefore$ Total power = 200 A.

$$\text{Useful electrical energy produced/sec} = \frac{20}{100}(200\,A) = 40\ A = 8000 \text{ (watt)}$$

$$\text{Therefore,} \quad A = \frac{8000}{40} = \mathbf{200\,sq.m}$$

This area is comparable to the roof of a large house of 250 sq. metre.

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NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because,

(*a*) the two magnetic forces are equal and opposite, so they produce no net effect.

(*b*) the magnetic forces do not work on each particle.

(*c*) the magnetic forces do equal and opposite (but non-zero) work on each particle.

(*d*) the magnetic forces are necessarily negligible.

2. A proton is kept at rest. A positively charged particle is released from rest at a distance *d* in its field. Consider two experiments ; one in which the charged particle is also a proton and in another, a positron. In the same time *t*, the work done on the two moving charged particles is

(*a*) same as the same force law is involved in the two experiments.

(*b*) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.

(*c*) more for the case of a positron, as the positron moves away a larger distance.

(*d*) same as the work done by charged particle on the stationary proton.

3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is

(*a*) constant and equal to *mg* in magnitude.

(*b*) constant and greater than *mg* in magnitude.

(*c*) variable but always greater than *mg*.

(*d*) at first greater than *mg*, and later becomes equal to *mg*.

4. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is

(*a*) + 2000 J (*b*) – 200 J

(*c*) zero (*d*) – 20,000 J

5. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall ?

(*a*) Kinetic energy (*b*) Potential energy

(*c*) Total mechanical energy

(*d*) Total linear momentum.

6. During inelastic collision between two bodies, which of the following quantities always remain conserved ?

(*a*) Total kinetic energy

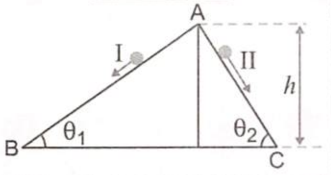
(*b*) Total mechanical energy

(*c*) Total linear momentum

(*d*) Speed of each body.

7. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in Fig. 6(N).8.

FIGURE 6(N).8



Which of the following statement is correct ?

(*a*) Both the stones reach the bottom at the same time but not with the same speed.

(*b*) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.

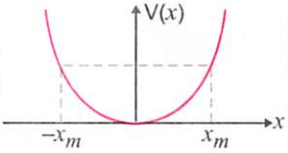
(*c*) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.

(*d*) Both the stones reach the bottom at different times and with different speeds.

8. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$

where k is the force constant of the oscillator [Fig. 6(N).9]. For $k = 0.5$ N/m, the graph of $V(x)$ versus x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the P.E. and K.E., respectively of the particle at $x = + x_m$, then which of the following is correct ?

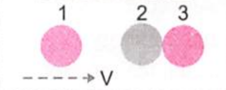
FIGURE 6(N).9



(*a*) $V = O, K = E$ (*b*) $V = E, K = O$
(*c*) $V < E, K = O$ (*d*) $V = O, K < E$.

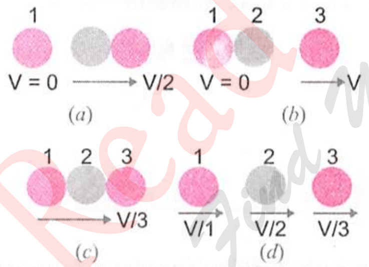
9. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V as shown in Fig. 6(N).10.

FIGURE 6(N).10



If the collision is elastic, which of the following [Fig. 6(N).11] is a possible result after collision ?

FIGURE 6(N).11

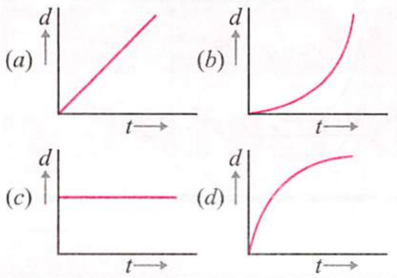


(*a*) (*b*) (*c*) (*d*)

10. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is
(*a*) 1.5 J (*b*) 50 J
(*c*) 10 J (*d*) 100 J

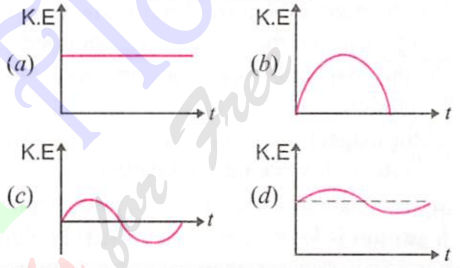
11. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in Fig. 6(N).12 correctly shows the displacement-time curve for its motion ?

FIGURE 6(N).12



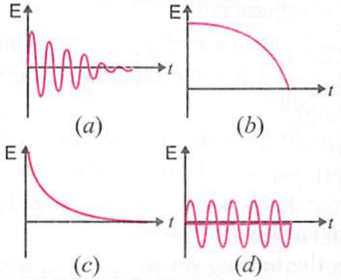
12. Which of the diagrams shown in Fig. 6(N).13 most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit ?

FIGURE 6(N).13



13. Which of the diagrams shown in Fig. 6(N).14 represents variation of total mechanical energy of a pendulum oscillating in air as function of time ?

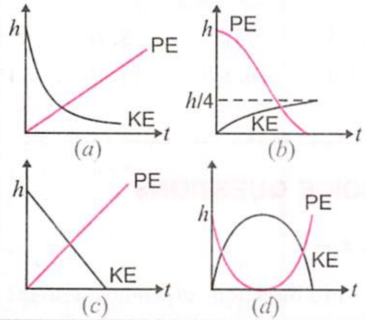
FIGURE 6(N).14



14. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be
(*a*) $250\ \pi^2$ (*b*) $100\ \pi^2$
(*c*) $5\ \pi^2$ (*d*) 0

15. A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in Fig. 6(N).15 correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground ?

FIGURE 6(N).15



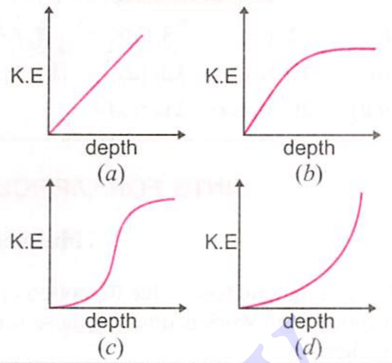
16. **In a shotput event, an athlete throws the shotput of mass 10 kg with an initial speed of 1 ms^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 ms^{-2}, the kinetic energy of the shotput when it just reaches the ground will be**

(*a*) 2·5 J (*b*) 5·0 J

(*c*) 52·5 J (*d*) 155·0J

17. **Which of the diagrams in Fig. 6(N).16 correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity ?**

FIGURE 6(N).16



18. **A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0·001s, the force that the batsman had to apply to hold the bat firmly at its place would be**

(*a*) 10.5 N (*b*) 21 N

(*c*) 1·05 × 10^4 N (*d*) 2·1 × 10^4 n

MULTIPLE CHOICE QUESTIONS-II

19. **A man of mass *m*, standing at the bottom of the staircase of height *L* climbs it and stands at its top.**

(*a*) Work done by all forces on man is equal to the rise in potential energy *mgL*.

(*b*) Work done by *all* forces on man is zero.

(*c*) Work done by the gravitational force on man is *mgL*.

(*d*) The reaction force from a step does not do work because the point of application of the force does not move while the force exists.

20. **A bullet of mass *m* fired at 30° to the horizontal leaves the barrel of the gun with a velocity *v*. The bullet hits a soft target at a height *h* above the ground while it is moving downward and emerges out with half the kinetic energy it had before hitting the target.**

Which of the following statements are correct in respect of bullet after emerges out of the target ?

(*a*) The velocity of the bullet will be reduced to half its initial value.

(*b*) The velocity of the bullet will be more than half of its earlier velocity.

(*c*) The bullet will continue to move along the same parabolic path.

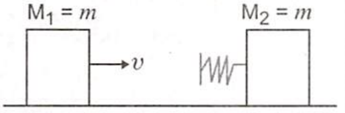
(*d*) The bullet will move in a different parabolic path.

(*e*) The bullet will fall vertically downward after hitting the target.

(*f*) The internal energy of the particles of the target will increase.

21. **Two blocks M_1 and M_2 having equal mass are free to move on a horizontal frictionless surface. M_2 is attached to a massless spring as shown in Fig. 6(N).17. Initially M_2 is at rest and M_1 is moving toward M_2 with speed *v* and collides head-on with M_2.**

FIGURE 6(N).17



(*a*) While spring is fully compressed, all the KE of M_1 is stored as PE of spring.

(*b*) While spring is fully compressed, the system momentum is not conserved, though final momentum is equal to initial momentum.

(*c*) If spring is massless, the final state of M_1 is state of rest.

(*d*) If the surface on which blocks are moving has friction, then collision cannot be elastic.

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ANSWERS

1. (*b*) **2.** (*c*) **3.** (*d*) **4.** (*c*) **5.** (*c*) **6.** (*c*) **7.** (*c*) **8.** (*b*) **9.** (*b*)
10. (*b*) **11.** (*b*) **12.** (*d*) **13.** (*c*) **14.** (*a*) **15.** (*b*) **16.** (*d*) **17.** (*b*) **18.** (*c*)
19. (*b,d*) **20.** (*b,d,f*) **21.** (*c,d*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. As the magnetic forces due to motion of electron and proton act in a direction perpendicular to the direction of motion, no work is done by these forces. That is why one ignores the magnetic force of one particle on another.

2. Force between two protons = force between a proton and a positron. As positron is much lighter than proton, it moves away through much larger distance compared to proton. As work done = force × distance, therefore in the same time t, work done in case of positron is more than that in case of proton.

3. When the man gets straight up and stand, reaction of ground on the man = mg. However, when he is squatting on the ground, reaction of ground is more than mg, as the man is to exert some extra force on the ground to stand up.

4. As the road does not move at all, therefore, work done by the cycle on the road must be zero.

5. In free fall under the action of gravity alone in vacuum, the total mechanical energy of the body remains constant.

6. In inelastic collision between two bodies, total linear momentum remains conserved.

7. As both surfaces I and II are frictionless and two stones slide from rest from the same height, therefore, both the stones reach the bottom with same speed $\left(\frac{1}{2}mv^2 = mgh\right)$. As acceleration down plane II is larger ($a_2 = g \sin\theta_2$ is greater than $a_1 = g\sin\theta_1$), therefore, stone II reaches the bottom earlier than stone I.

8. At $x = +x_m$, the particle turns back. Therefore, its velocity at this point is zero. Therefore, kinetic energy $K = 0$. The total energy E is in the form of potential energy *i.e.* $V = E$.

9. As the ball bearings are identical, their masses are equal. In elastic collision, their velocities are interchanged. In collision between 1 and 2 ; vel. of 1 becomes zero and vel. of 2 becomes V. In collision between 2 and 3, vel. of 2 becomes zero and vel. of 3 becomes V. **Choice (*b*) is correct.**

10. Here, $m = 0{\cdot}5$ kg, $v = ax^{3/2}$, where $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$, $W = ?$

$$\text{Acceleration, } A = \frac{dv}{dt} = \frac{d}{dt}(ax^{3/2}) = a \times \frac{3}{2}x^{1/2}\frac{dx}{dt} = \frac{3a}{2}x^{1/2}(ax^{3/2})$$

$$A = \frac{3a^2}{2}x^2$$

$$F = mA = 0{\cdot}5 \times \frac{3a^2}{2}x^2 = \frac{3a^2}{4}x^2$$

$$W = \int_0^2 F.dx = \int_0^2 \frac{3a^2}{4}x^2\,dx = \frac{3a^2}{4}\left[\frac{x^3}{3}\right]_0^2 = \frac{3}{4\times 3}(5)^2[2^3 - 0] = \mathbf{50\ J}$$

11. Here, $P = [ML^2T^{-3}]$ = constant

As mass M of body is fixed, $\therefore\ L^2T^{-3}$ = constant

$$\frac{L^2}{T^3} = \text{constant} \quad \text{or} \quad L \propto T^{3/2} \quad \text{or} \quad \text{displacement } (d) \propto t^{3/2}$$

Choice (*b*) is correct.

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12. As the earth moves once around the sun in its elliptical orbit, its K.E. is maximum when it is closest to the sun and minimum when it is farthest from the sun. As K.E. of earth is never zero during its motion, **choice (*d*) is correct.**

13. When a pendulum oscillates in air, it loses energy continuously in overcoming resistance due to air. Therefore, total mechanical energy of the pendulum decreases continuously with time. The variation of total mechanical energy E with time t is shown correctly by **curve (*c*).**

14. Here, $m = 5$ kg, $r = 1$ m

$$\omega = \frac{300}{60}\text{ rps} = 5 \text{ rps} = 5 \times 2\pi \text{ rad s}^{-1}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}\times 5(1\times 10\pi)^2 = \mathbf{250\pi^2\ J}$$

15. At a height h above the ground, P.E. of raindrop is maximum and K.E. = 0. As the raindrop falls, its P.E. goes on decreasing and K.E. goes on increasing upto a point $h/4$ above the ground. At this stage, rain drop has acquired near terminal velocity (= constant). Therefore, at this stage, K.E. tends to be constant. P.E. becomes zero when raindrop falls to the ground. Choice (*b*) is most appropriate.

16. As the shotput reaches the ground, its K.E. = P.E. of shotput when it is thrown + K.E. given

$$= mgh + \frac{1}{2}mv^2 = 10\times 10\times 1{\cdot}5 + \frac{1}{2}\times 10(1)^2 = 150 + 5 = \mathbf{155\ J}$$

17. When an iron sphere falls freely in a lake, its motion is accelerated due to gravity and retarded due to viscous force. The overall effect is increase in velocity and hence increase in K.E. till the sphere acquires terminal velocity, which is constant. Hence K.E. of sphere beyond this depth of lake becomes constant. **Choice (*b*) is most appropriate.**

18. Here, $m = 150\text{ g} = \frac{150}{1000}\text{kg} = \frac{3}{20}\text{kg}$, $u = 126\text{ km/h} = \frac{126\times 1000}{60\times 60}\text{m/s} = 35\text{ m/s}$

$v = -126$ km/h $= -35$ m/s, $t = 0{\cdot}001$ s

As $F \times t$ = impulse = change in momentum of ball

$$\therefore\ F \times 0{\cdot}001 = m(v-u) = \frac{3}{20}(-35-35) = -\frac{21}{2}$$

$$F = -\frac{21}{2\times 0{\cdot}001} = -1{\cdot}05\times 10^4\text{ N}$$

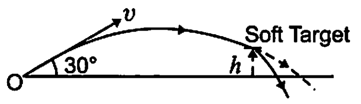
Negative sign indicates force applied in a direction opposite to the direction of motion of ball.

Multiple Choice Questions – II

19. When a man of mass m climbs up the staircase of height L, work done by the gravitational force on man is (–) mgL, and work done by muscular force is mgL. If we ignore air resistance and friction, then the work done by all forces on man is equal to $-mgL + mgL$ = zero. Further, reaction force from a step does not do work because the point of application of force does not move while the force exists.

20. Assuming air resistance to be zero.

FIGURE 6(N).18



K.E. of bullet emerging from soft target $= \frac{1}{2}mv'^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$$v' = \frac{v}{\sqrt{2}} = 0{\cdot}707\,v$$

i.e. velocity is more than half of its earlier velocity.

As the bullet loses some of its vertical velocity component, therefore, velocity on emerging from the soft target changes. The bullet will move in a different parabolic path.

The energy lost in passing through the target is transferred to particles of the target. Therefore, their internal energy increases. **Choices (*b*), (*d*), (*f*) are correct.**

21. While spring is fully compressed, the entire K.E. of M_1 is not stored as P.E. of spring as M_2 may move. If spring is massless, then as $M_1 = M_2$, velocities of M_1 and M_2 are interchanged on collision. M_1 comes to rest, instead of M_2. Choice (*c*) is correct.

If surface on which blocks are moving has friction, loss of energy is involved. Collision cannot be elastic. Choice (*d*) is correct.

VERY SHORT ANSWER QUESTIONS

22. A rough inclined plane is placed on a cart moving with a constant velocity *u* on horizontal ground. A block of mass *M* rests on the incline. Is any work done by force of friction between the block and incline ? Is there then a dissipation of energy ?

Ans. The force of friction between the block and the incline prevents the block from sliding down. As the block does not move, therefore, **no** work is done by the force of friction.

As **no** work is done by the force of friction, therefore, there is **no** dissipation of energy.

23. Why is electrical power required at all when the elevator is descending ? Why should there be a limit on the number of passengers in this case ?

Ans. Electric power is required to prevent the elevator from falling freely under gravity. Limit on the number of passengers in this case is needed again to prevent the elevator from descending with large velocity.

24. A body is being raised to a height *h* from the surface of earth. What is the sign of work done by

(*a*) applied force (*b*) gravitational force ?

Ans. From $W = Fs \cos\theta$,

(*a*) for applied force $\theta = 0°$, $\cos\theta = \cos 0° = 1$. $\therefore$ W = positive, and

(*b*) for gravitational force, $\theta = 180°$, $\cos\theta = \cos 180° = -1$

W = negative.

25. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2m.

Ans. In this case, work is done only against friction between the road and tyres. Against gravity,

$$W = Fs \cos 90° = \text{Zero}.$$

26. A body falls towards earth in air. Will its total mechanical energy be conserved during the fall ? Justify.

Ans. No. This is because some work has to be done against resistive force of air which is a non-conservative force. Therefore, gain in K.E. < loss in P.E. Hence mechanical energy is not conserved.

27. A body is moved along a closed loop. Is the work done in moving the body necessarily zero ? If not, state the condition under which work done over a closed path is always zero.

Ans. No, not necessarily. Work done in moving a body over a closed path will be zero only when all the forces acting on the system are conservative forces.

28. In an elastic collision of two billiard balls, which of the following quantities remain conserved during the short time of collision of the balls (*i.e.*, when they are in contact).

(*a*) Kinetic energy. (*b*) Total linear momentum ?

Give reason for your answer in each case.

Ans. In such a collision, total linear momentum is always conserved. However, when the balls are in contact, there may occur some deformation. So part of K.E. may be converted into elastic potential energy. Therefore, K.E. may not be conserved.

29. Calculate the power of a crane in watts, which lifts a mass of 100 kg to a height of 10 m in 20s.

Ans. Here, $m = 100$ kg, $h = 10$ m, $t = 20$ s

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{100 \times 9{\cdot}8 \times 10}{20} = 490\text{ W}$$

30. The average work done by a human heart while it beats once is 0·5 J. Calculate the power used by heart if it beats 72 times in a minute.

Ans. $P = \frac{W}{t} = \frac{72 \times 0{\cdot}5}{60}\text{ W} = 0{\cdot}6\text{ W}$

31. Give example of a situation in which an applied force does not result in a change in kinetic energy.

Ans. In an atom, an electron revolves around the nucleus with a constant speed in a given orbit. The force applied is electrostatic force of attraction between the nucleus and electron. As speed v is constant,

$$\text{K.E.} = \frac{1}{2}mv^2 = \text{constant}$$

32. Two bodies of unequal mass are moving in the same direction with equal kinetic energy. The two bodies are brought to rest by applying retarding force of same magnitude. How would the distance moved by them before coming to rest compare ?

Ans. Work done in stopping the body = force × distance = K.E. of body, which is same for two bodies.

As retarding force applied is the same, therefore, distance moved by both the bodies before coming to rest must be the same.

33. A bob of mass *m* suspended by a light string of length *L* is whirled into a vertical circle as shown in Fig. 6(N).19. What will be the trajectory of the particle if the string is cut at

(*a*) Point B ? (*b*) Point C ? (*c*) Point X ?

FIGURE 6(N).19

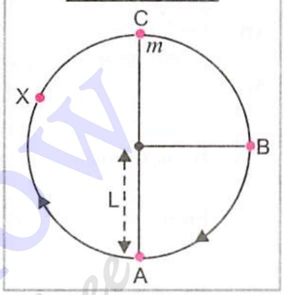
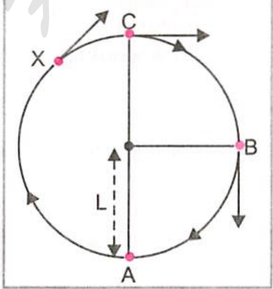


FIGURE 6(N).20



Ans. When the string is cut, centripetal force is no longer provided. The bob flies along the tangent to the arc at that point.

(*a*) When string is cut at *B*, the bob moves vertically downwards.

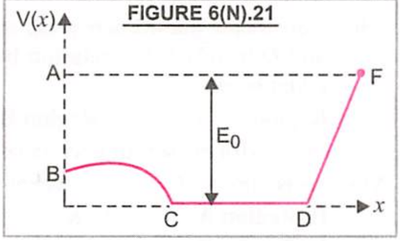
(*b*) When string is cut at *C*, initial vel. of bob is along the horizontal to the right as shown in Fig. 6(N).20. Therefore, the bob follows a parabolic path with vertex at *C*.

(*c*) When string is cut at X, initial velocity is along tangent to the circle at X. The bob will follow a parabolic path with vertex higher than *C*.

SHORT ANSWER QUESTIONS

34. A graph of potential energy *V* (*x*) verses *x* is shown in Fig. 6(N).21. A particle of energy E_0 is executing motion in it. Draw graph of velocity and kinetic energy versus *x* for one complete cycle AFA.

FIGURE 6(N).21



Ans. Kinetic energy versus *x* graph

As $KE = E_0 - V(x)$

$\therefore$ *at A*, $V(x) = E_0$

$KE = 0$

at B $V(x) < E_0$

$\therefore$ KE is + but not max.

at C and at D, $V(x) = 0$

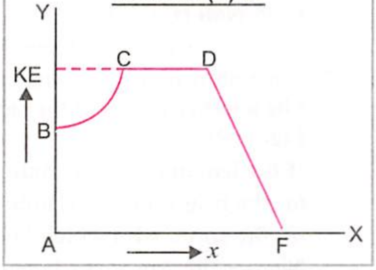
$\therefore$ KE = max.

At F, $V(x) = E_0$

$\therefore$ KE = 0

The variation of KE with *x* is as shown in Fig. 6(N).22.

FIGURE 6(N).22



Velocity versus *x* graph

As $KE = \frac{1}{2}mv^2$ $\therefore$ At *A* and *F*, where KE = 0, $v = 0$.

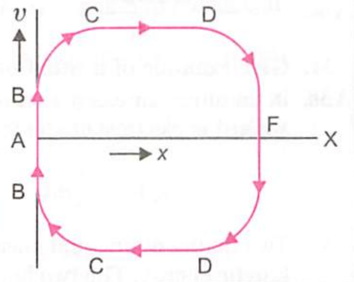
At C and D, KE = max. Therefore, $v = \pm$ max.

At B, KE is + but not maximum.

Therefore, $v = \pm$ some value (< max.)

The variation of velocity v with x is as shown in Fig. 6(N).23.

FIGURE 6(N).23



35. A ball of mass m, moving with a speed $2\,v_0$, collides inelastically ($e > 0$) with an identical ball at rest. Show that (*a*) For head-on collision, both the balls move forward.

(*b*) For a general collision, the angle between the two velocities of scattered balls is less than 90°.

Ans. (*a*) If v_1, v_2 are velocities of two balls after collision, then according to the principle of conservation of linear momentum.

$$m\,(2\,v_0) = mv_1 + mv_2 \qquad \text{or} \qquad 2\,v_0 = v_1 + v_2 \qquad \text{...(i)}$$

By definition, $$e = \frac{v_2 - v_1}{v_2 + v_1} = \frac{v_2 - v_1}{2\,v_0} \qquad \therefore \qquad v_2 = v_1 + 2\,v_0\,e \qquad \text{...(ii)}$$

From (*i*), $$2\,v_0 = v_1 + (v_1 + 2\,v_0\,e)$$

or $$= 2\,(v_1 + v_0\,e) \qquad \text{or} \qquad v_1 = v_0\,(1 - e)$$

As $e < 1$, v_1 has the same sign as v_0. $\therefore$ the ball moves forward after collision.

From (*ii*), $$v_2 = v_0\,(1 - e) + 2\,v_0\,e = v_0\,(1 + e)$$

Again, as $e < 1$, v_2 has the same sign as v_0. Hence the other ball also moves forward after collision.

(*b*) According to principle of conservation of linear momentum,

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

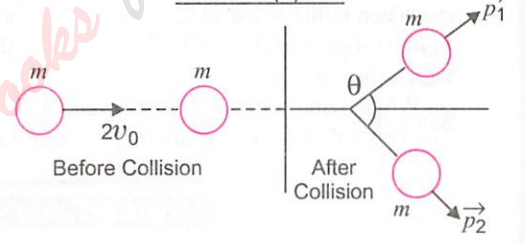
For a general collision, some KE is lost.

$$\therefore \qquad \left(\frac{p^2}{2m}\right) > \left(\frac{p_1^2}{2m}\right) + \left(\frac{p_2^2}{2m}\right)$$

or $$p^2 > (p_1^2 + p_2^2),$$

which is possible only when $\theta < 90°$, Fig. 6(N).24 ; which was to be proved.

FIGURE 6(N).24



36. Consider a one-dimensional motion of a particle with total energy E. There are four regions A, B, C and D in which the relation between potential energy V, kinetic energy (K) and total energy E is as given below :

Region A : $V > E$, Region B : $V < E$, Region C : $K > E$, Region D : $V > K$

State with reason in each case whether a particle can be found in the given region or not.

Ans. As is known, $E = K + V$, where K is always positive but V can be positive or negative.

In Region A. $V > E$ As $K = E - V$ $\therefore$ K is negative, which is not permissible.

In Region B. $V < E$ or $E > V$ *i.e.* total energy is greater than PE. $\therefore$ Particle can be found in region B.

In Region C. $K > E$, which is possible when V is negative. $\therefore$ Particle can be found in region C.

In Region D. $V > K$ *i.e.* PE > KE which is allowed.

Hence particle can be found in region D.

37. The bob A of a pendulum released from horizontal to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6(N).25.

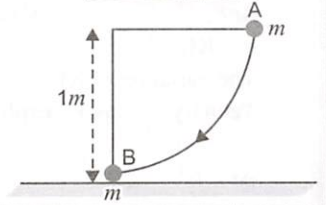
If the length of the pendulum is 1 m, calculate

(*a*) the height to which bob A will rise after collision.

(*b*) the speed with which bob B starts moving.

Neglect the size of the bobs and assume the collision to be elastic.

FIGURE 6(N).25



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Ans. (*a*) As the collision is elastic and two balls have same mass, therefore, ball A transfers its entire linear momentum to B and does not rise at all.

(*b*) Speed with which B starts moving = speed with which A hits B = $\sqrt{2gh} = \sqrt{2\times 9{\cdot}8\times 1} = 4{\cdot}42$ m/s.

38. A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 ms^{-1}. Calculate (*a*) the loss of P.E. of the drop. (*b*) the gain in K.E. of the drop. (*c*) Is the gain in K.E. equal to loss of P.E. ? If not why, Take g = 10 ms^{-2}

Ans. Here, $m = 1{\cdot}00$ g $= 10^{-3}$ kg, $h = 1$ km $= 10^3$ m, $v = 50$ m/s ; $g = 10$ ms^{-2}

(*a*) Loss of P.E. of drop = $mgh = 10^{-3}\,(10)\,10^3 = 10$ J

(*b*) Gain in K.E. of drop $= \frac{1}{2}mv^2 = \frac{1}{2}\times 10^{-3}\,(50)^2 = 1{\cdot}25\,\text{J}$

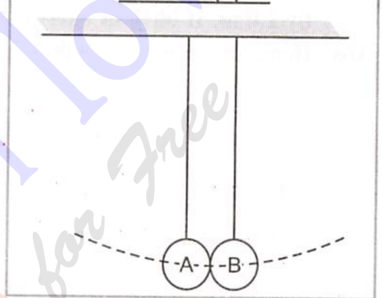
(*c*) No. This is because a part of P.E. lost is used up in doing work against the viscous drag of air.

39. Two pendulums with identical bobs and lengths are suspended from a common support such that in rest position, the two bobs are in contact, Fig. 6(N).26. One of the bobs is released after being displaced by 10° so that it collides elastically head-on with the other bob.

(*a*) Describe the motion of two bobs.

(*b*) Draw a graph showing variation in energy of either pendulum with time, for $0 \le t \le 2T$, where T is the period of each pendulum.

FIGURE 6(N).26



Ans. As two pendulums have identical bobs and lengths, they will execute simple harmonic motions of same time period, say T.

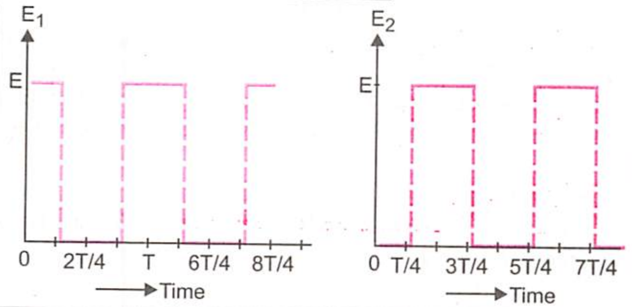
At $t = 0$, suppose bob A is displaced by 10° to the left. It is given potential energy $E_1 = E$. Energy of B, $E_2 = 0$.

When A is released, it strikes B at $t = T/4$. In the head on elastic collision between A and B, A comes to rest and B gets velocity of A. Therefore, $E_1 = 0$ and $E_2 = E$. At $t = 2\,T/4$, B reaches its extreme right position when K.E. of B is converted into P.E. $= E_2 = E$. Energy of A, $E_1 = 0$. At $t = 3\,T/4$, B reaches its mean position, when its P.E. is converted into KE. $= E_2 = E$. It collides elastically with A and transfers whole of its energy to A. Thus, $E_2 = 0$ and $E_1 = E$. The entire process is repeated.

The values of energies of A and B at different time intervals are tabulated here. The plot of energy with time $0 \le t \le 2\,T$ is shown separately for A and B in Fig. 6(N).27.

time (t)	Energy of A E_1	Energy of B E_2
0	E	0
$T/4$	0	E
$2\,T/4$	0	E
$3\,T/4$	E	0
$4\,T/4$	E	0
$5\,T/4$	0	E
$6\,T/4$	0	E
$7\,T/4$	E	0
$8\,T/4$	E	0

FIGURE 6(N).27



40. Suppose the average mass of raindrops is $3{\cdot}0 \times 10^{-5}$ kg and their average terminal velocity 9 ms^{-1}. Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.

Ans. Here, average mass of rain drops $= 3{\cdot}0 \times 10^{-5}$ kg

$$v = 9 \text{ ms}^{-1},\ A = 1 \text{ sq. metre},\ h = 100 \text{ cm} = 1 \text{ m.}$$

$\therefore$ Volume of water due to rain, $V = A \times h = 1 \times 1 = 1 \text{ m}^3$

As. density of water, $\rho = 10^3 \text{ kg/m}^3$

$\therefore$ mass of water due to rain $M = V\rho = 1 \times 10^3$ kg

$$\text{Energy transferred} = \text{KE} = \frac{1}{2} M v^2 = \frac{1}{2} \times 10^3 (9)^2 = \mathbf{4{\cdot}05 \times 10^4\ J}$$

41. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 50,000 kg is moving with a speed of 36 km h^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, calculate the spring constant.

Ans. Here, $l = 1{\cdot}5$ m, $m = 50000 \text{ kg} = 5 \times 10^4$ kg

$$v = 36 \text{ km h}^{-1} = \frac{36 \times 1000}{60 \times 60} \text{ ms}^{-1} = 10 \text{ ms}^{-1}, \quad x = 1{\cdot}0 \text{ m},\ k = ?$$

$$\text{Total KE} = \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times 10^4 (10)^2 = 2{\cdot}5 \times 10^6 \text{ J}$$

As 90% of energy of wagon is lost due to friction, therefore, energy transferred to shock absorber,

$$E = \frac{10}{100} \times 2{\cdot}5 \times 10^6 = 2{\cdot}5 \times 10^5 \text{ J}$$

From $$E = \frac{1}{2} kx^2$$

$$k = \frac{2E}{x^2} = \frac{2 \times 2{\cdot}5 \times 10^5}{1^2} = \mathbf{5 \times 10^5\ N/m}$$

42. An adult weighing 600 N raises the centre of gravity of his body by 0.25 m while taking each step of 1 m length in jogging. If he jogs for 6 km, calculate the energy utilised by him in jogging assuming that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food, calculate the energy equivalents of food that would be required to compensate energy utilised for jogging.

Ans. Here, $mg = 600$ N, $h = 0{\cdot}25$ m per step

$$\text{Number of steps} = n = \frac{6 \times 10^3}{1} = 6 \times 10^3$$

$$\text{Energy utilised in jogging} = n\,(mg)\,h = 6 \times 10^3 \times 600 \times 0{\cdot}25 \text{ J}$$

$$E = 9 \times 10^5 \text{ J}$$

This is 10% of intake. $\therefore$ Intake energy $= 10\,E = \mathbf{9 \times 10^6\ J}$

43. On complete combustion, a litre of petrol gives off heat equivalent to 3×10^7 J. In a test drive, a car weighing 1200 kg, including the mass of driver, runs 15 km per litre while moving with a uniform speed on a straight track. Assuming that friction offered by the road surface and air to be uniform, calculate the force of friction acting on the car during the test drive. If the efficiency of the car engine were 0.5.

Ans. Here, 1 litre of petrol gives off 3×10^7 J of energy.

As efficiency of engine is 0·5, energy used by car $E = 0{\cdot}5 \times 3 \times 10^7 \text{ J} = 1{\cdot}5 \times 10^7$ J.

If F is force of friction, then $F \times s = E$

$$F \times (15 \times 10^3) = 1{\cdot}5 \times 10^7$$

$$F = \mathbf{10^3\ N}$$

LONG ANSWER QUESTIONS

44. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined surface [Fig. 6(N).28]. The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline, calculate

(*a*) work done against gravity

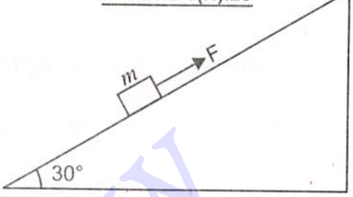
(*b*) work done against force of friction

(*c*) increase in potential energy

(*d*) increase in kinetic energy

(*e*) work done by applied force.

FIGURE 6(N).28



Ans. In Fig. 6(N).29, f is the force of friction acting down the incline.

Here, $m = 1$ kg, $\theta = 30°$

$F = 10$ N, $\mu = 0{\cdot}1$, $s = 10$ m.

(*a*) Work done against gravity

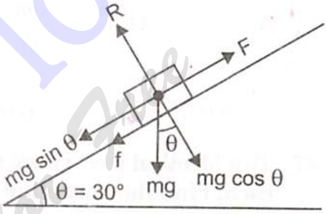
$$W_g = mg \sin\theta \times s = 1 \times 10 \sin 30° \times 10 = \frac{100}{2} = \mathbf{50\ J}$$

(*b*) Work done against friction

$$W_f = f \times s = \mu R \times s = \mu\, mg \cos\theta \times s$$

$$= 0{\cdot}1 \times 1 \times 10 \cos 30° \times 10 = 10 \times 0{\cdot}866 = \mathbf{8{\cdot}66\ J}$$

FIGURE 6(N).29



(*c*) Increase in P.E. $= mgh = mg\,(s \sin\theta) = 1 \times 10 \times 10 \sin 30° = \mathbf{50\ J}$

(*d*) Increase in K.E. $= \Delta K = ?$

Acceleration of block up the incline

$$a = \frac{F - (mg \sin\theta + \mu\, mg \cos\theta)}{m} = \frac{10 - \left(1 \times 10 \times \frac{1}{2} + 0{\cdot}1 \times 10 \times 0{\cdot}866\right)}{1} = 4{\cdot}13\ \text{m/s}^2$$

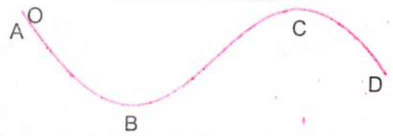
From $v^2 - u^2 = 2\,as$

$$\Delta K = \frac{1}{2} m (v^2 - u^2) = \frac{1}{2} m \times 2\,as = mas = 1 \times 4{\cdot}13 \times 10 = \mathbf{41{\cdot}3\ J}$$

45. A curved surface is shown in Fig. 6(N).30. The portion BCD is free of friction. There are three spherical balls of identical radii and masses. Balls are released from rest one by one from A which is at a slightly greater height than C.

With the surface AB, ball 1 has large enough friction to cause rolling down without slipping ; ball 2 has a small friction and ball 3 has a negligible friction.

FIGURE 6(N).30



(*a*) For which balls is total mechanical energy conserved ?

(*b*) Which ball (s) can reach D ?

(*c*) For balls which do not reach D, which of the balls can reach back A ?

Ans. (*a*) As ball 1 rolls down *AB* without slipping, friction is large enough to cause only rotational motion. However, no energy is lost due to friction. Ball 3 has negligible friction. Therefore, total mechanical energy is conserved in case of balls 1 and 3. Small friction in case of ball 2 involves some dissipation of energy.

(*b*) Ball 3 alone can cross over *C* (lower than *A*) and reach *D* as it has negligible friction. Ball 1 acquires some rotational energy and ball 2 loses some energy overcoming friction. They cannot reach *D*.

(*c*) Balls 1 and 2 turn back before reaching *C*. Ball 2 cannot reach back to *A* on account of frictional losses. Ball 1 cannot roll back to *A* because of kinetic friction.

46. A rocket accelerates straight up by ejecting gas downwards. In a small time interval Δt, it ejects a gas of mass Δm at a relative speed u. Calculate KE of the entire system at $t + \Delta t$ and t and show that the device that ejects gas does work $= \left(\frac{1}{2}\right) \Delta m \cdot u^2$ in this time interval (neglect gravity).

Ans. Let M be the mass of rocket at any time t and

v, the velocity of rocket at this time

Δm = mass of gas ejected in time interval Δt with relative vel. u.

$\therefore$ At time t; (KE) $= E_1 = \frac{1}{2}Mv^2$, and At time $(t + \Delta t)$, (KE) $= E_2$ = KE of rocket + KE of gas

$$= \frac{1}{2}(M - \Delta m)(v + \Delta v)^2 + \frac{1}{2}\Delta m(v - u)^2$$

Expanding and neglecting terms which are too small, we get

$$E_2 = \frac{1}{2}Mv^2 + Mv\Delta v - (\Delta m)vu + \frac{1}{2}(\Delta m)u^2$$

$$E_2 - E_1 = [M\Delta v - (\Delta m).u]v + \frac{1}{2}(\Delta m)u^2 \qquad ...(i)$$

As $M\frac{dv}{dt} = \left(\frac{dm}{dt}\right)|u| \quad \therefore \quad M\Delta v - (\Delta m)u = 0$

From (i), $E_2 - E_1 = \frac{1}{2}(\Delta m)u^2 = W$, using work energy theorem.

47. Two identical steel cubes (masses 50 g, side 1 cm) collide head on face to face with a speed of 10 cm/s each. Find the maximum compression of each. Young's modulus for steel = $Y = 2 \times 10^{11}$ N/m^2.

Ans. Here, $m = 50$ g $= 50 \times 10^{-3}$ kg

side, $L = 1$ cm $= 0{\cdot}01$ m, $v = 10$ cm/s $= 0{\cdot}1$ m/s

$Y = 2 \times 10^{11}$ N/m^2

Max. compression, ΔL = ?

According to Hook's law,

$$Y = \frac{F.L}{A\Delta L} \quad \text{or} \quad F = \frac{YA\Delta L}{L} \qquad ...(i)$$

If k is compression constant of the cube, then $F = k\,\Delta L$

Using (i), $YA\frac{\Delta L}{L} = k\,\Delta L$

$$k = \frac{YA}{L} = \frac{YL^2}{L} = YL \qquad ...(ii)$$

$$\text{Initial KE} = 2 \times \frac{1}{2}mv^2 = (50 \times 10^{-3})(0{\cdot}1)^2 = 5 \times 10^{-4}\text{ J}$$

$$\text{Final PE} = 2 \times \frac{1}{2}F(\Delta L) = 2 \times \frac{1}{2}(k\Delta L)(\Delta L) = 2 \times \frac{1}{2}k(\Delta L)^2 = \text{Initial KE} = 5 \times 10^{-4}$$

$$\therefore \quad \Delta L = \sqrt{\frac{5 \times 10^{-4}}{k}} = \sqrt{\frac{5 \times 10^{-4}}{YL}} = \sqrt{\frac{5 \times 10^{-4}}{2 \times 10^{11} \times 0{\cdot}01}} = \sqrt{2{\cdot}5 \times 10^{-13}} = 5 \times 10^{-7}\text{ m}$$

48. A baloon filled with helium rises against gravity increasing its potential energy. The speed of the baloon also increases as it rises. How do you reconcile this with the law of conservation of mechanical energy ? You can neglect viscous drag of air and assume that density of air is constant.

Ans. Let V = volume of helium (baloon), m = mass of baloon

ρ_{He} = density of helium, ρ_{air} = density of air

Volume V of baloon displaces volume V of air.

∴ The baloon experiences an upthrust $= V(\rho_{air} - \rho_{He})\,g = ma = m\frac{dv}{dt}$ ...(*i*)

Integrating both sides w.r.t. t, we get

$$V(\rho_{air} - \rho_{He})\,gt = mv \qquad \text{...(ii)}$$

From (*i*), $$a = \frac{V(\rho_{air} - \rho_{He})\,g}{m}$$

If the baloon rises to a height h, then from

$$s = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}\,\frac{V(\rho_{air} - \rho_{He})\,gt^2}{\text{m}} \qquad \text{...(iii)}$$

$$\text{K.E. of baloon } = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{(\rho_{air} - \rho_{He})^2 V^2 g^2 t^2}{2m} \qquad \text{...using (ii)}$$

$$= \frac{1}{2}mv^2 = V(\rho_{air} - \rho_{He})\,g \times \left[\frac{(\rho_{air} - \rho_{He})V}{2m}\,gt^2\right]$$

using (*iii*), we get

$$\frac{1}{2}mv^2 = V(\rho_{air} - \rho_{He})\,gh$$

$\frac{1}{2}mv^2 + V\rho_{He}\,gh = V\rho_{air}\,gh$, *i.e.*, K.E. of baloon + P.E. of baloon = change in P.E. of air.

Hence we conclude that as the Helium baloon goes up, an equal volume of air comes down. Increase in P.E. and K.E. of baloon is at the cost of P.E. of air (that comes down).

Competition FOCUS

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I. Multiple Choice Questions (with One Correct Answer)

I. Work, Energy and Power

1. A block of mass 10 kg, moving in x-direction with a constant speed of 10 m/s, is subjected to a retarding force $F = 0{\cdot}1\,x$ J/m during its travel from $x = 20$ m to 30 m. Its final KE will be

(*a*) 450 J (*b*) 275 J
(*c*) 250 J (*d*) 475 J **(AIPMT 2015)**

2. A particle of mass m is driven by a machine that delivers a constant power K watts. If the particle starts from rest, the force on the particle at time t is

(*a*) $\sqrt{mK}\,t^{-1/2}$ (*b*) $\sqrt{2mK}\,t^{-1/2}$
(*c*) $\frac{1}{2}\sqrt{mK}\,t^{-1/2}$ (*d*) $\sqrt{\frac{mK}{2}}\,t^{-1/2}$

(AIPMT 2015)

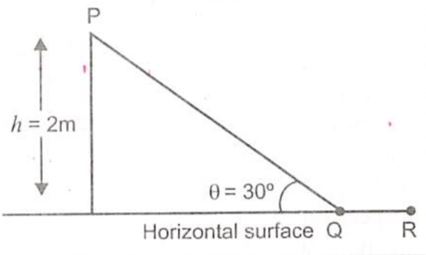
3. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\,\hat{i} + 3t^2\,\hat{j})$ N, where $\hat{i}$ and $\hat{j}$ are unit vectors along X and Y axis. What power will be developed by the source at time t ?

(*a*) $(2\,t^2 + 4\,t^4)$ W (*b*) $(2\,t^3 + 3\,t^4)$ W
(*c*) $(2\,t^3 + 3\,t^5)$ W (*d*) $(2\,t + 3\,t^3)$ W

(NEET 2016)

4. A point particle of mass m moves along the uniformly rough track PQR as shown in Fig. 6(CF).1. The coefficient of friction, between the particle and rough track equals μ. The particle is released from rest from the point P and it comes to rest at a point R. The energies lost by the ball, over the parts PQ and QR of the track are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and distance $x\ (= QR)$ are respectively close to

FIGURE 6(CF).1



(*a*) 0·2 and 6·5 m (*b*) 0·2 and 3·5 m
(*c*) 0·29 and 3·5 m (*d*) 0·29 and 6·5 m

(JEE Main 2016)

5. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the P.E. lost each time he lowers the

ANSWERS

1. (*d*) 2. (*d*) 3. (*c*) 4. (*c*)

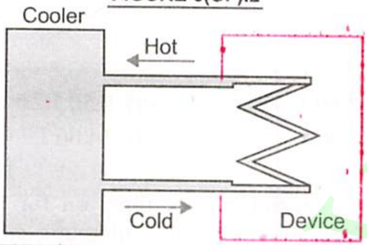
mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up ? Fat supplies $3{\cdot}8 \times 10^7$ J of energy per kg which is converted to mechanical energy with 20% efficiency rate. Take $g = 9{\cdot}8$ m/s^2.

(*a*) $2{\cdot}45 \times 10^{-3}$ kg (*b*) $6{\cdot}45 \times 10^{-3}$ kg
(*c*) $9{\cdot}89 \times 10^{-3}$ kg (*d*) $12{\cdot}89 \times 10^{-3}$ kg

(JEE Main 2016)

6. A water cooler of storage capacity 120 litre can cool water at a constant rate of P watt. In a closed circulation system, as shown in Fig. 6(CF).2, water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litre of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watt) for which the device can be operated for 3 hours is

FIGURE 6(CF).2



(*a*) 1600 (*b*) 2067
(*c*) 2533 (*d*) 3933

Take specific heat of water = 4·2 kJ kg^{-1} K^{-1} and density of water is 1000 kg/m^3.

(JEE Advanced 2016)

7. Two similar springs P and Q have spring constants K_P and K_Q, such that $K_P > K_Q$. They are stretched first by the same amount (case *a*), then by the same force (case *b*). The work done by the springs W_P and W_Q are related as, in case (*a*) and case (*b*), respectively

(*a*) $W_P = W_Q$; $W_P = W_Q$
(*b*) $W_P < W_Q$; $W_Q = W_P$
(*c*) $W_P > W_Q$; $W_Q > W_P$
(*d*) $W_P = W_Q$; $W_P > W_Q$ **(AIPMT 2015)**

8. A particle of mass m is driven by a machine that delivers a constant power K watts. If the particle starts from rest, the force on the particle at time t is

(*a*) $\sqrt{mK}\, t^{-1/2}$ (*b*) $\sqrt{2mK}\, t^{-1/2}$
(*c*) $\frac{1}{2}\sqrt{mK}\, t^{-1/2}$ (*d*) $\sqrt{\frac{mK}{2}}\, t^{-1/2}$

9. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8} mv_0^2$, the value of k will be

(*a*) 10^{-4} kg m^{-1} (*b*) 10^{-1} kg m^{-1} s^{-1}
(*c*) 10^{-3} kg m^{-1} (*d*) 10^{-3} kg s^{-1}

(JEE Main 2017)

10. A time dependent force $F = 6\,t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be

(*a*) 9 J (*b*) 18 J
(*c*) 4·5 J (*d*) 22 J

(JEE Main 2017)

11. A long spring when stretched by x cm, has a potential energy U. On increasing the stretching to nx cm, the potential energy stored in spring will be

(*a*) U/n (*b*) $n\,U$
(*c*) $n^2\,U$ (*d*) U/n^2

12. Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take 'g' constant with a value 10 m/s^2. The work done by the (*i*) gravitational force and the (*ii*) resistive force of air is

(*a*) (*i*) – 10 J (*ii*) – 8·25 J
(*b*) (*i*) 1·25 J (*ii*) – 8·25 J
(*c*) (*i*) 100 J (*ii*) 8·75 J
(*d*) (*i*) 10 J (*ii*) – 8·75 J

(NEET 2017)

13. The velocity of a particle at which the kinetic energy is equal to its rest mass energy is

(*a*) $\left(\frac{3c}{2}\right)$ (*b*) $3\frac{c}{\sqrt{2}}$
(*c*) $\frac{(3c)^{1/2}}{2}$ (*d*) $\frac{c\sqrt{3}}{2}$

14. A running man has the same kinetic energy as that of a boy of half his mass. The man speeds up by 2 ms^{-1} and the boy changes his speed by

ANSWERS

5. (*d*) 6. (*b*) 7. (*c*) 8. (*d*) 9. (*a*) 10. (*c*) 11. (*c*) 12. (*d*) 13. (*d*)

x ms^{-1} so that the kinetic energies of the boy and the man are again equal. Then x in ms^{-1} is

(*a*) $-2\sqrt{2}$ (*b*) $2\sqrt{2}$

(*c*) $\sqrt{2}$ (*d*) 2

15. A moving block having mass m, collides with another stationary block having mass 4 m. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be

(*a*) 0.8 (*b*) 0.25

(*c*) 0.5 (*d*) 0.4 **(NEET 2018)**

16. If the potential energy of a gas molecule is

$U = \frac{M}{r^6} - \frac{N}{r^{12}}$, M and N being positive constants, then the potential energy at equilibrium must be

(*a*) zero (*b*) $M^2/4$ N

(*c*) $N^2/4$ M (*d*) $MN^2/4$

(*e*) $NM^2/4$

17. A body of mass 5 kg is moving with a momentum of 10 kg *m/s*. A force of 0·2 N acts on it in the direction of motion of the body for 10 sec. The increase in its kinetic energy is

(*a*) 2·8 J (*b*) 3·2 J

(*c*) 3·8 J (*d*) 4·4 J

18. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion ?

(*a*) 1.0 cm (*b*) 1.5 cm

(*c*) 2.0 cm (*d*) 3.0 cm

19. A vertical spring with force constant K is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring so that the spring is compressed by a distance d. The net work done in the process is

(*a*) $mg\,(h + d) - \frac{1}{2}\,\text{K}\,d^2$

(*b*) $mg\,(h - d) - \frac{1}{2}\,\text{K}\,d^2$

(*c*) $mg\,(h - d) + \frac{1}{2}\,\text{K}\,d^2$

(*d*) $mg\,(h + d) + \frac{1}{2}\,\text{K}\,d^2$ **(AIPMT 2007)**

20. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring, and compresses it till the block is motionless. The kinetic frictional force is 15 N and spring constant is 10,000 N/m. The spring compresses by

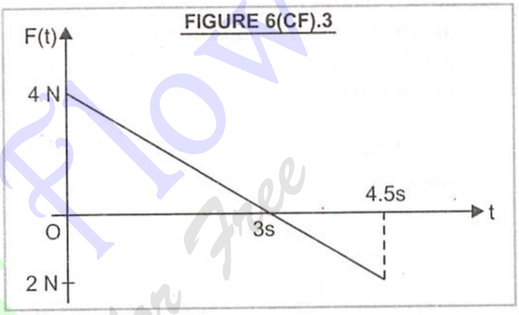
(*a*) 8·5 cm (*b*) 5·5 cm

(*c*) 2·5 cm (*d*) 11·0 cm

(AIEEE 2007)

21. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards, it is subjected to a time-dependent force $F(t)$ in the x direction. Theforce, $F(t)$ varies with t as shown in Fig. 6(CF).3. The kinetic energy of the block after 4.5 seconds is

FIGURE 6(CF).3



(*a*) 4.50 J (*b*) 7.50 J

(*c*) 5.06 J (*d*) 14.06 J

(IIT 2010)

22. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is

$D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(*a*) $\frac{b^2}{12a}$ (*b*) $\frac{b^2}{4a}$

(*c*) $\frac{b^2}{6a}$ (*d*) $\frac{b^2}{2a}$ **(AIEEE 2010)**

23. A man of 50 kg mass is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0·5 kg mass downwards with a speed 2 m/s. When the stone reaches the floor, the distance of the man above the floor will be

(*a*) 9·9 m (*b*) 10·1 m

(*c*) 1·0 m (*d*) 20 m

(AIPMT 2010)

ANSWERS

14. (*b*) **15.** (*b*) **16.** (*b*) **17.** (*d*) **18.** (*a*) **19.** (*a*) **20.** (*b*) **21.** (*c*) **22.** (*b*) **23.** (*b*)

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24. Energy required to break one bond in DNA is

(*a*) 10^{-10} J (*b*) 10^{-18} J

(*c*) 10^{-7} J (*d*) 10^{-3} J

(*e*) 10^{-20} J **(Kerala CEE 2010)**

25. If a spring of stiffness 'k' is cut into two parts 'A' and 'B' of length $l_A : l_B = 2 : 3$, then the stiffness of spring 'A' is given by

(*a*) $\frac{5}{2}k$ (*b*) $\frac{3k}{5}$

(*c*) $\frac{2k}{5}$ (*d*) k **(AIEEE 2011)**

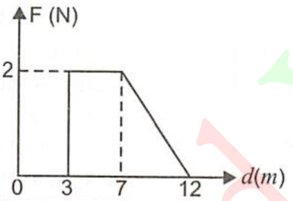
26. At time $t = 0$ s, particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to

(*a*) $\sqrt{t}$ (*b*) constant

(*c*) t (*d*) $\frac{1}{\sqrt{t}}$ **(AIEEE 2011)**

27. Force F on a particle moving in a straight line varies with distance d as shown in Fig. 6(CF).4. The work done on the particle during its displacement of 12 m is

FIGURE 6(CF).4



(*a*) 18 J (*b*) 21 J

(*c*) 26 J (*d*) 13 J

[AIPMT (Prelim) 2011]

28. The potential energy of particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is :

(*a*) $B/2A$ (*b*) $2A/B$

(*c*) A/B (*d*) B/A

[AIPMT (Prelim) 2012]

29. A car of mass m starts from rest and accelerates so that the instantaneous power delivered to the car has a constant magnitude P_0. The instantaneous velocity of this car is proportional to :

(*a*) $t^2 P_0$ (*b*) $t^{1/2}$

(*c*) $t^{-1/2}$ (*d*) $\frac{t}{\sqrt{m}}$

[AIPMT (Prelim) 2012]

30. A particle is moving in a circular path of radius a under the action of an attractive potential

$U = -\frac{k}{2r^2}$. Its total energy is

(*a*) $-\frac{k}{4a^2}$ (*b*) $\frac{k}{2a^2}$

(*c*) Zero (*d*) $-\frac{3}{2}\frac{k}{a^2}$

(JEE Main 2018)

31. A force $F = (20 + 10\,y)$ acts on a particle in y direction, where F is in newton and y is in metre. Work done by this force to move the particle from $y = 0$ to $y = 1$ m is

(*a*) 30J (*b*) 5 J

(*c*) 25 J (*d*) 20 J **(NEET 2019)**

32. When an object is shot from the bottom of a long smooth inclined plane kept at an angle of 60° with horizontal, it can travel a distance x_1 along the plane ? But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance · x_1/x_2 will be

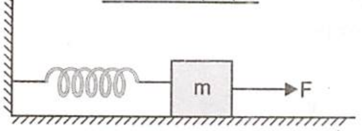
(*a*) $1 : \sqrt{2}$ (*b*) $\sqrt{2} : 1$

(*c*) $1 : \sqrt{3}$ (*d*) $1 : 2\sqrt{3}$

(NEET 2019)

33. A block of mass m, lying on a smooth horizontal surface is attached to a spring of negligible mass and spring constant K. The other end of the spring is fixed, The block is initially at rest in equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is

FIGURE 6(CF).5



(*a*) $\frac{2F}{\sqrt{mK}}$ (*b*) $\frac{F}{\pi\sqrt{mK}}$

(*c*) $\frac{\pi F}{\sqrt{mK}}$ (*d*) $\frac{F}{\sqrt{mK}}$

(JEE Main 2019)

ANSWERS

24. (*e*) 25. (*a*) 26. (*d*) 27. (*d*) 28. (*b*) 29. (*b*) 30. (*c*) 31. (*c*) 32. (*c*) 33. (*d*)

34. The energy required to break one bond in DNA is 10^{-20} J. This value in eV is nearly

(*a*) 0·6 (*b*) 0·06

(*c*) 0·006 (*d*) 6 **(NEET 2020)**

35. The energy equivalent of 0·5 g of a substance is

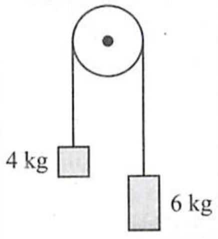
(*a*) $4 \cdot 5 \times 10^{13}$ J (*b*) $1 \cdot 5 \times 10^{13}$ J

(*c*) $0 \cdot 5 \times 10^{13}$ J (*d*) $4 \cdot 5 \times 10^{16}$ J

(NEET 2020)

36. Two bodies of masses 4 kg and 6 kg are tied to the ends of a massless string, passing over a pulley, which is frictionless. The acceleration of the system in terms of acc. due to gravity (g) is

FIGURE 6(CF).6



(*a*) $g/2$ (*b*) $g/5$

(*c*) $g/10$ (*d*) g **(NEET 2020)**

37. Water falls from a height of 60 m at the rate of 15 kg/sec to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine ?

Take $g = 10$ m/s^2

(*a*) 7·0 kW (*b*) 10·2 kW

(*c*) 8·1 kW (*d*) 12·3 kW

(NEET 2021)

38. A particle is released from height S from the surface of earth. At a certain height, its K.E is three times its P.E. The height from the surface of earth and the speed of the particle at that instant are respectively

(*a*) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$ (*b*) $\frac{S}{4}, \frac{3gS}{2}$

(*c*) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$ (*d*) $\frac{S}{2}, \frac{\sqrt{3gS}}{2}$

(NEET 2021)

39. A 60 HP electric motor lifts an elevator having a maximum load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (1 HP = 746 W and $g = 10$ m/s^2)

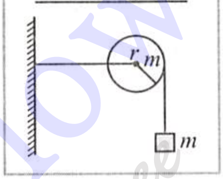
(*a*) 1·7 m/s (*b*) 2·0 m/s

(*c*) 1·9 m/s (*d*) 1·5 m/s

(JEE Main 2020)

40. A bob of mass m is tied by a massless sting whose other end portion is wound on a flywheel (disc) of radius r and mass m. When released from rest, the bob starts falling vertically. When it has covered a distance h, the angular speed of the wheel will be

FIGURE 6(CF).7



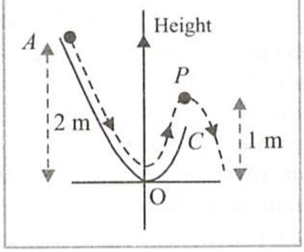
(*a*) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (*b*) $r\sqrt{\frac{3}{4gh}}$

(*c*) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (*d*) $r\sqrt{\frac{3}{2gh}}$

(JEE Main 2020)

41. A particle of mass $m = 1$ kg slides down a frictionless track AOC, starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P (height 1 m), the K.E of the particle in joule is ...

FIGURE 6(CF).8



(*a*) 10 J (*b*) 15 J

(*c*) 20 J (*d*) 30 J

(JEE Main 2020)

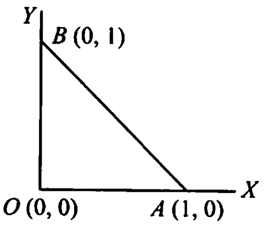
42. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. Work done by this force in moving a particle from point A (1, 0)

ANSWERS

34. (*b*) 35. (*a*) 36. (*b*) 37. (*c*) 38. (*a*) 39. (*c*) 40. (*a*) 41. (*a*)

to B (0, 1) along the line segment shown in Fig. 6(CF).9 in joule is

FIGURE 6(CF).9



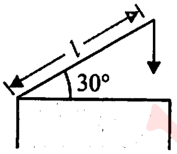
(*a*) 3/2 (*b*) 1

(*c*) 2 (*d*) 1/2

(JEE Main 2020)

43. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle of 30° with horizontal, Fig. 6(CF).10. Its angular speed, when it hits the table is $\sqrt{n}\,\text{s}^{-1}$, where n is an integer. Its value is

FIGURE 6(CF).10



(*a*) 10 (*b*) 20

(*c*) 15 (*d*) 25

(JEE Main 2020)

44. A cricket ball of mass 0·15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball, and moves horizontally a distance of 0·2 m while launching the ball, value of F is

(*a*) 150 N (*b*) 100 N

(*c*) 125 N (*d*) 200 N

(JEE Main 2020)

45. A bullet of mass 5 gram travelling with a speed of 210 m/s strikes a fixed wooden target. One half of its K.E is converted into heat in the bullet, while the other half is converted into heat in the wood. If the specific heat of material of the bullet is 0·030 cal/g/°C, the rise in temp. of the bullet is

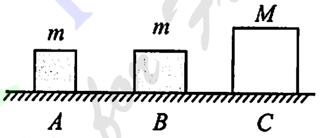
(*a*) 83·3°C (*b*) 87·5°C

(*c*) 119·2°C (*d*) 38·4°C

(JEE Main 2020)

46. Three blocks A, B and C are lying on a smooth horizontal surface as shown in Fig. 6(CF).11. A and B have equal masses, m each, while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C also perfectly inelastically. If $\frac{5}{6}$th of initial KE is lost in whole process. What is the value of M/m ?

FIGURE 6(CF).11



(*a*) 5 (*b*) 4

(*c*) 3 (*d*) 2 **(JEE Main 2019)**

47. The energy that will be ideally radiated by a 100 kW transmitter in 1 hour is

(*a*) 36×10^7 J (*b*) 36×10^9 J

(*c*) 36×10^5 J (*d*) 1×10^5 J

(NEET 2022)

48. Water falls from a height of 60 m at the rate of 15 kg/sec to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine ? Take $g = 10\ \text{m/s}^2$

(*a*) 7·0 kW (*b*) 10·2 kW

(*c*) 8·1 kW (*d*) 12·3 kW

(NEET 2021)

49. Body A of mass 4 m moving with a speed u collides with another body B of mass 2 m at rest. The collision is head on elastic in nature. After the collision, the fraction of energy lost by the colliding body is

(*a*) 1/9 (*b*) 8/9

(*c*) 4/9 (*d*) 5/9 **(NEET 2019)**

50. Two particles A and B move with constant velocities $\vec{v}_1$ and $\vec{v}_2$. At the initial moment their position vectors are $\vec{r}_1$ and $\vec{r}_2$ respectively. The condition for particles A and B for their collision is

(*a*) $\vec{r}_1 \times \vec{v}_1 = \vec{r}_2 \times \vec{v}_2$

(*b*) $\dfrac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \dfrac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$

(*c*) $\vec{r}_1 \cdot \vec{v}_1 = \vec{r}_2 \cdot \vec{v}_2$

(*d*) $\vec{r}_1 \times \vec{v}_2 = \vec{r}_2 \times \vec{v}_1$ **(RE-AIPMT 2015)**

51. On a frictionless surfaces, a block of mass M moving at speed v collides elastically with another block of mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $v/3$. The second block's speed after the collision is

(*a*) $\sqrt{3}\,v/2$ (*b*) $2\sqrt{2}\,v/3$

(*c*) $3\,v/4$ (*d*) $3v/\sqrt{2}$

(RE-AIPMT 2015)

52. A particle of mass m moving in the x-direction with speed $2\,v$ is hit by another particle of mass $2\,m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(*a*) 44% (*b*) 50%

(*c*) 56% (*d*) 62%

(JEE Main 2015)

53. A block of mass 0·50 kg is moving with a speed of 2·00 m/s on a smooth surface. It strikes another mass of 1 kg at rest and they move as a single body. The energy loss during the collision is

(*a*) 0·16 J (*b*) 1·00 J

(*c*) 0·67 J (*d*) 0·34 J

54. A body of mass m_1 collides elastically with another body of mass m_2 at rest. If the velocity of m_1 after collision is $\dfrac{2}{3}$ times its initial velocity, the ratio of their masses is :

(*a*) 1 : 5 (*b*) 5 : 1

(*c*) 5 : 2 (*d*) 2 : 5

55. A bullet weighing 5 g and moving with a velocity 600 m/s strikes a 5 kg block of ice resting on a frictionless surface. The speed of the block after the collision is

(*a*) 6 cm/s (*b*) 60 cm/s

(*c*) 6 m/s (*d*) 0·6 cm/s

56. A ball moving with velocity of 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0·5, then their velocities (in m/s) after collision will be

(*a*) 0, 2 (*b*) 0, 1

(*c*) 1, 1 (*d*) 1, 0·5

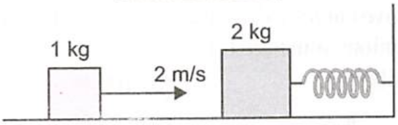
57. A ball of mass 1 kg moving with velocity 3 m/s, collides with spring of natural length 2 m and force constant 144 N/m. What will be length of compressed spring ?

(*a*) 2 m (*b*) 1·5 m

(*c*) 1 m (*d*) 0·5 m

58. A spring block system is resting on a frictionless floor as shown in Fig. 6(CF).12. The spring constant is 2·0 N m^{-1} and mass of the block is 2·0 kg. Ignore the mass of the spring. Initially, the spring is in an unstretched condition. Another block of mass 1·0 kg moving with a speed of 2·0 ms^{-1} collides elastically with the first block. The collision is such that the 2·0 kg block does not hit the wall. The distance in metres between the two blocks when the spring returns to its unstretched position for the first time after the collision is ....

FIGURE 6(CF).12



(*a*) 2 m (*b*) 3 m

(*c*) 1 m (*d*) 4 m

(JEE Advanced 2018)

ANSWERS

49. (*b*) 50. (*b*) 51. (*b*) 52. (*c*) 53. (*c*) 54. (*b*) 55. (*b*) 56. (*b*) 57. (*b*) 58. (*a*)

59. A shell is fired from a cannon with velocity v m/s at an angle θ with the horizontal direction. At the highest point in its path, it explodes into 2 pieces of equal mass. One of the pieces retraces its path to the cannon. The speed in m/s of the other piece immediately after the explosion is

(*a*) $3\,v\cos\theta$ (*b*) $2\,v\cos\theta$

(*c*) $(3/2)\,v\cos\theta$ (*d*) $(\sqrt{3}/2)\,v\cos\theta$

60. An object of mass 40 kg and having velocity 4 m/s collides with another object of mass 60 kg having velocity 2 m/s. The loss of energy when the collision is perfectly inelastic is

(*a*) 392 J (*b*) 440 J

(*c*) 48 J (*d*) 110 J

61. Particle A makes a perfectly elastic collision with another particle B at rest. They fly apart in opposite direction with equal speeds. The ratio of their masses $\frac{m_A}{m_B}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{1}{\sqrt{3}}$

62. The first ball of mass m moving with the velocity v collides head on with the second ball of mass m at rest. If the coefficient of restitution is e, then the ratio of the velocities of the first and the second ball after the collision is

(*a*) $\frac{1-e}{1+e}$ (*b*) $\frac{1+e}{1-e}$

(*c*) $\frac{1+e}{2}$ (*d*) $\frac{1-e}{2}$

63. A ball moving with a velocity of 6 m/s strikes an identical stationary ball. After collision, each ball moves at an angle of 30° with the original line of motion. What are the speeds of the balls after the collision ?

(*a*) $\frac{\sqrt{3}}{2}$ m/sec (*b*) 3 m/sec

(c) $2\sqrt{3}$ m/sec (*d*) $\sqrt{3}$ m/sec

64. A wooden block of mass 10 gm is dropped from the top of a tower 100 m high. Simultaneously, a bullet of mass 10 gm is fired from the foot of the tower vertically upwards with a velocity of 100 m/sec, Fig. 6(CF).13. If the bullet is embedded in it, how high will it rise above the tower before it starts falling ?

(Consider g = 10 m/sec^2)

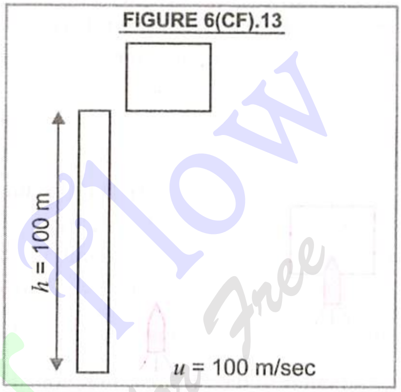


FIGURE 6(CF).13

(*a*) 80 m (*b*) 85 m

(*c*) 75 m (*d*) 10 m

65. A car weighing 2×10^3 kg and moving at 20 m/s along a main road collides with a lorry of mass 8×10^3 kg which emerges at 5 m/s from a cross road at right angles to the main road. If the two vehicles lock, what will be their velocity after the collision ?

(*a*) $4/\sqrt{2}$ m/s, 45° with cross road

(*b*) $4/\sqrt{2}$ m/s, 60° with cross road

(*c*) $4/\sqrt{2}$ m/s, 60° with main road

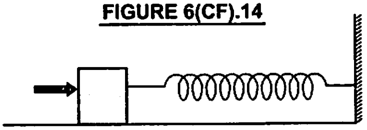
(*d*) $4\sqrt{2}$ m/s, 45° with main road

66. A block of mass 0·18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0·1. Initially, the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the Fig. 6(CF).14. The block slides a distance of 0·06 m and comes to rest for the first time. The initial velocity of the block in m/s is $V = N/10$. Then N is

ANSWERS

59. (*a*) 60. (*c*) 61. (*b*) 62. (*a*) 63. (*c*) 64. (*c*) 65. (*d*)

FIGURE 6(CF).14



(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 2

67. A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity 4 ms^{-1}. It collides with a horizontal spring of force constant 200 Nm^{-1}. The maximum compression produced in the spring will be :

(*a*) 0·5 m (*b*) 0·6 m

(*c*) 0·7 m (*d*) 0·2 m

[AIPMT (Prelim) 2012]

68. Two spheres *A* and *B* of masses m_1 and m_2 respectively collide. *A* is at rest initially and *B* is moving with velocity v along *x*-axis. After collision *B* has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass *A* moves after collision in the direction.

(*a*) same as that of *B*

(*b*) Opposite to that of *B*

(*c*) $\theta = \tan^{-1}(1/2)$ to the *x*-axis

(*d*) $\theta = \tan^{-1}(-1/2)$ to the *x*-axis

[AIPMT (Prelim) 2012]

69. A body of mass (4 *m*) is lying in *xy* plane at rest. It suddenly explodes into three pieces. Two pieces each of mass (*m*) move perpendicular to eachother with equal speeds (v). The total kinetic energy generated due to explosion is

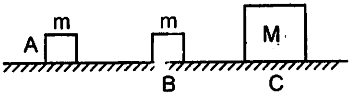
(*a*) mv^2 (*b*) $\frac{3}{2}mv^2$

(*c*) $2\,mv^2$ (*d*) $4\,mv^2$

(AIPMT 2014)

70. There blocks *A*, *B* and *C* are lying on a smooth horizontal surface, as shown in Fig. 6(CF).15. *A* and *B* have equal masses *m*, while *C* has mass *M*. Block *A* is given an initial speed v towards *B* due to which it collides with *B* perfectly inelastically. The combined mass collides with *C*, also perfectly inelastically. If $\frac{5}{6}$th of initial KE is lost in whole process, what is the value of *M*/*m* ?

FIGURE 6(CF).15



(*a*) 5 (*b*) 2

(*c*) 4 (*d*) 3 **(JEE Main 2019)**

71. When a rubber band is stretched by a distance *x*, it exerts a restoring force of magnitude $F = ax + bx^2$, where *a* and *b* are constants. The work done in stretching the unstretched rubber band by *L* is

(*a*) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (*b*) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

(*c*) $aL^2 + bL^3$ (*d*) $\frac{1}{2}(aL^2 + bL^3)$

(JEE Main 2014)

72. Hailstorms are observed to strike the surface of a frozen lake at an angle of 30° with the vertical and rebound at an angle of 60° with vertical. Assuming the contact to be smooth, the coefficient of restitution is

(*a*) 1/4 (*b*) 1/3

(*c*) 1/5 (*d*) 3/4

73. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(*a*) $\frac{v_0}{4}$ (*b*) $\sqrt{2}v_0$

(*c*) $\frac{v_0}{2}$ (*d*) $\frac{v_0}{\sqrt{2}}$

(JEE Main 2018)

74. Body *A* of mass 4 m moving with a spread *u* collides with another body *B* of mass 2 m at rest. The collision is head on and elastic in nature. After the collision, the fraction of energy lost by the colliding body *A* is

(*a*) 1/9 (*b*) 8/9

(*c*) 4/9 (*d*) 5/9 **(NEET 2019)**

III. Multiple Choice Questions (with One or More than One Correct Answers)

75. Two balls having linear momenta $\vec{p}_1 = p\,\hat{i}$ and $\vec{p}_2 = -p\,\hat{i}$ undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option(s) is (are) not allowed for any non-zero value of p, a_1, a_2, b_1, b_2, c_1, c_2

(*a*) $\vec{p}_1' = a_1\,\hat{i} + b_1\,\hat{j} + c_1\,\hat{k}$, $\vec{p}_2' = a_2\,\hat{i} + b_2\,\hat{j}$

(*b*) $\vec{p}_1' = c_1\,\hat{k}$, $\vec{p}_2' = c_2\,\hat{k}$

(*c*) $\vec{p}_1' = a_1\,\hat{i} + b_1\,\hat{j} + c_1\,\hat{k}$, $\vec{p}_2' = a_2\,\hat{i} + b_2\,\hat{j} - c_1\,\hat{k}$

(*d*) $\vec{p}_1' = a_1\,\hat{i} + b_1\,\hat{j}$, $\vec{p}_2' = a_2\,\hat{i} + b_1\,\hat{j}$

76. When work done on a particle is positive, then its

(*a*) KE increases (*b*) KE decreases

(*c*) KE remains constant

(*d*) momentum increases

77. A particle is acted upon a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of particle takes place in a plane. It follow that

(*a*) its velocity is constant

(*b*) its acceleration is constant

(*c*) its KE is constant

(*d*) it moves in a circular path

78. If you lift a suitcase from the ground and keep it on the table, the work done by you does not depend on

(*a*) the path taken by suitcase

(*b*) the time taken by you in doing so

(*c*) weight of suitcase

(*d*) frame of reference

79. If the force is always perpendicular to motion, then KE remains constant

(*a*) KE remains constant

(*b*) Work done is zero

(*c*) velocity is constant

(*d*) speed is constant

80. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ?

(*a*) Total momentum of the system is 3 kg ms^{-1}

(*b*) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

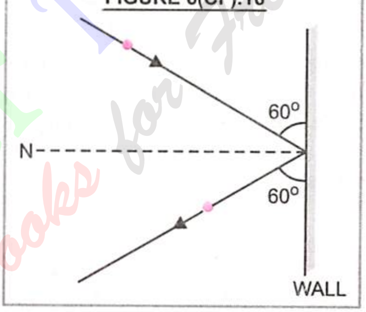
(*c*) Kinetic energy of the centre of mass is 0·75 J

(*d*) Total kinetic energy of the system is 4 J

(IIT JEE 2010)

81. A 3 kg ball strikes a heavy rigid wall with a speed of 10 m/s at an angle of 60° with the wall. It gets reflected with the same speed at 60° as shown in Fig. 6(CF).16. If the ball is in contact with wall for 0·2 s what is average force exerted on the ball by the wall ?

FIGURE 6(CF).16



(*a*) 300 N (*b*) zero

(*c*) $150\sqrt{3}$ N (*d*) 150 N

82. A force F acting on a body depends on its displacement S as $F \propto S^{-1/3}$. The power delivered by F will depend on displacement as

(*a*) $S^{2/3}$ (*b*) $S^{1/2}$

(*c*) S (*d*) S^0

83. A ball falls vertically on to a floor, with momentum p, and then bounces repeatedly, the coefficient of restitution is e. The total momentum imparted by the ball to the floor is

(*a*) $p\,(1 + e)$ (*b*) $\dfrac{p}{1-e}$

(*c*) $p\dfrac{(1+e)}{(1-e)}$ (*d*) $p\left(1-\dfrac{1}{e}\right)$

84. No work is done by a force on an object if

(*a*) the object is stationary but the point of application of the force moves on the object

ANSWERS

75. (*a,d*) 76. (*a,d*) 77. (*c,d*) 78. (*a,b*) 79. (*a,b,d*) 80. (*a,c*) 81. (*d*) 82. (*d*) 83. (*c*)

(*b*) the object moves in such a way that the point of application of the force remains fixed

(*c*) the force is always perpendicular to its velocity

(*d*) the force is always perpendicular to its acceleration.

85. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(*a*) it moves in a circular path

(*b*) its velocity is constant

(*c*) its acceleration is constant

(*d*) its kinetic energy is constant.

86. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

(*a*) $\tan\phi$ (*b*) $2\tan\phi$

(*c*) $2\cos\phi$ (*d*) $2\sin\phi$

87. A smooth block is released at rest on a 45° incline and then slides a distance '*d*'. The time taken to slide is '*n*' times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(*a*) $\mu_s = \sqrt{1-\frac{1}{n^2}}$ (*b*) $\mu_s = 1-\frac{1}{n^2}$

(*c*) $\mu_k = \sqrt{1-\frac{1}{n^2}}$ (*d*) $\mu_k = 1-\frac{1}{n^2}$

88. A body moving towards a finite body at rest collides with it. It is possible that

(*a*) both the bodies move after collision

(*b*) both the bodies come to rest

(*c*) the stationary body remains stationary, the moving body changes its velocity

(*d*) the moving body comes to rest and the stationary body starts moving.

89. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

(*a*) force applied on particle is constant,

(*b*) speed of the particle is proportional to time

(*c*) the distance of the particle from the origin increases linearly with time

(*d*) force is conservative.

(JEE Advanced 2018)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1 **Work is said to be done by a force acting on a body, provided the body is displaced actually in any direction except in a direction perpendicular to the direction of the force. Mathematically, $W = \vec{F}\cdot\vec{s} = Fs\cos\theta$. Whereas energy is capacity of a body to do the work ; power is the rate at which the body can do the work.**

$$P = \frac{W}{t} = \vec{F}\cdot\frac{\vec{s}}{t} = \vec{F}\cdot\vec{v}$$

Both, work and energy are measured in joule and power is measured in watt.

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions :

90. In the above question, work done by the resisting force is

(*a*) 400 J

(*b*) – 400 J

(*c*) 25 J

(*d*) – 25 J

91. A truck draws a tractor of mass 1000 kg at a steady rate of 20 m s^{-1} on a level road. The tension in the coupling is 2000 N. Power spent on the tractor is

(*a*) 40 W

(*b*) 20 W

(*c*) 20 kW

(*d*) 40 kW

ANSWERS

84. (*a,b,d*) **85.** (*a,d*) **86.** (*b*) **87.** (*d*) **88.** (*a,d*) **89.** (*a,b,d*) **90.** (*b*) **91.** (*d*)

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Comprehension 2 **Potential energy of a body is the energy possessed by the body by virtue of its position. P.E. = *m g h* where the symbols have their usual meaning. Kinetic energy of a body is the energy possessed by the body by virtue of its velocity.**

$$\text{K.E.} = \frac{1}{2} m v^2$$

Energy can neither be created nor be destroyed. However energy can be changed from one form to the other, such that energy appearing in one form is equal to the energy disappearing in the other form.

With the help of the passage given above, choose the most appropriate alternative for each of the following questions :

92. A body of mass 1 kg is allowed to fall freely under gravity. The momentum of the body 5 second after it starts falling is

(*a*) 100 kg m s^{-1}

(*b*) 50 kg m s^{-1}

(*c*) 150 kg m s^{-1}

(*d*) 200 kg m s^{-1}

93. The body will attain this K.E. when it falls freely from a height of

(*a*) 125 m

(*b*) 250 m

(*c*) 1250 m

(*d*) 2500 m

94. The ratio of potential energy to kinetic energy at a height of 62·5 m above the ground is

(*a*) 2

(*b*) 1

(*c*) 3

(*d*) 4

IV. Matching Type Questions

DIRECTIONS. In the following questions, match column I and column II. Select the correct match out of the four choices given at the end.

95.

	Column I		Column II
(A)	Work done is maximum	(*p*)	potential energy is stored
(B)	Velocity of a body is doubled	(*q*)	spring constant becomes twice
(C)	Length of a spring is halved	(*r*)	K.E. becomes four times
(D)	On compressing or stretching a spring	(*s*)	Angle between force and displacement is 0°

(*a*) A-*s* ; B-*r* ; C-*q* ; D-*p* (*b*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*c*) A-*s* ; B-*r* ; C-*p* ; D-*q* (*d*) A-*p* ; B-*r* ; C-*s* ; D-*q*

96.

	Column I		Column II
(A)	K.E. of a body cannot be negative	(*p*)	K.E. increases by 300%
(B)	In uniform circular motion	(*q*)	K.E. increases
(C)	A shot fired from a gun explodes in air	(*r*)	K.E. remains constant
(D)	Momentum of a body is increased by 100%	(*s*)	True

(*a*) A-s, B-r, C-q, D-p (*b*) A-p, B-q, C-r, D-s (*c*) A-p, B-r, C-q, D-s (*d*) A-s, B-r, C-q, D-p

ANSWERS

92. (*b*) **93.** (*a*) **94.** (*b*) **95.** (*a*) **96.** (*d*)

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*r*, *s*, B-*p*, *q*, C-*p*, D-*q*, *s*, then the correctly bubbled matrix will look like the one shown here :

	p	q	r	s
A	○	○	●	●
B	●	●	○	○
C	●	○	○	○
D	○	●	○	●

97.

Column I	Column II
(A) Magnetic potential energy	(*p*) negative
(B) electrostatic potential energy	(*q*) positive
(C) elastic potential energy	(*r*) zero
(D) gravitational potential energy	(*s*) not defined

98.

Column I	Column II
(A) In a perfectly elastic collision	(*p*) some K.E. is lost
(B) In a perfectly inelastic collision	(*q*) $0 < e < 1$
(C) In a partially elastic collision	(*r*) $e = 1$
(D) In a partially inelastic collision	(*s*) $e = 0$

99. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (α and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]^2$	(*p*) The force acting on the particle is zero at $x = a$.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(*q*) The force acting on the particle is zero at $x = 0$.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(*r*) The force acting on the particle is zero at $x = -a$
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(*s*) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x \rvert < a$.
	(*t*) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

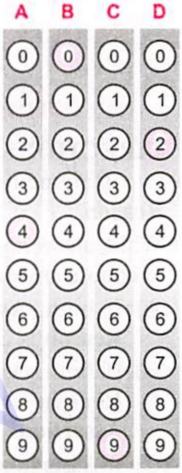
(JEE Advanced 2015)

ANSWERS

97. A-*p*, *q*, *r* ; B-*p*, *q*, *r* ; C-*q*, *r* ; D-*p*, *r*
98. A-*r* ; B-*p*, *s* ; C-*p*, *q* ; D-*p*, *q*
99. A-*p*,*q*,*r*,*t* ; B-*q*,*s* ; C-*p*,*q*,*r*,*s* ; D-*p*,*r*,*s*

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :



100. A bullet fired into a fixed target loses half of its velocity after penetrating through 3 cm. How much further (in cm) will it penetrate before coming to rest, assuming that it faces constant resistance to motion ?

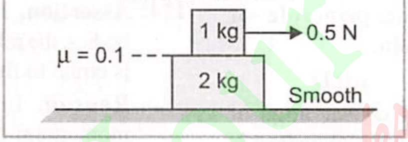
101. The potential energy of a 2 kg particle free to move along the *x*-axis is given by

$$V(x) = \left[\frac{x^4}{4} - \frac{x^2}{2}\right] \text{ joule}$$

The total mechanical energy of the particle is 0·75 J. The maximum speed of the particle (in m/s) is :

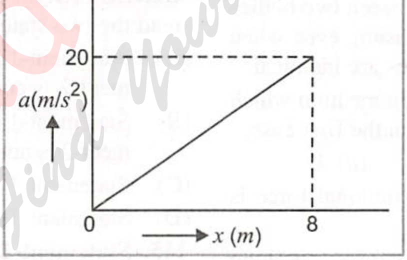
102. A force of 0·5 N is applied on upper block as shown in Fig. 6(CF).17.The work done (in joule) by upper block on lower block for a displacement of 3 m is :

FIGURE 6(CF).17



103. A 100 g block moves along *X*-axis. Its acceleration as a function of displacement is shown in Fig. 6(CF).18. Calculate work done (in joule) on the block by the force applied as the block moves from $x = 0$ to $x = 8$ m.

FIGURE 6(CF).18



104. Two bodies of masses *m* and 4 m are moving with equal linear momenta. The ratio of their kinetic energies is :

105. Consider an elliptically shaped rail *PQ* in the vertical plane with $OQ = 4$ m and $OP = 3$ m. A block of mass 1 kg is pulled along the rail from *P* to *Q* with a force of 18 N, which is always parallel to line *PQ*. Assuming no frictional losses, the KE of the block when it reaches $Q = n \times 10$ joule. Take $g = 10$ m/s^2. What is the value of *n* ? **(JEE Advanced Paper 1, 2014)**

ANSWERS

100. (1) 101. (1) 102. (1) 103. (8) 104. (4) 105. (5)

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. The following questions consist of two statements each, printed as **Assertion** and **Reason.**

While answering these questions, you are required to choose any one of the following four responses :

(A) If both, **Assertion** and **Reason** are true and **Reason** is correct explanation of the **Assertion.**

(B) If both, **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

(C) If **Assertion** is true but the **Reason** is false.

(D) If both, **Assertion** and **Reason** are false.

106. Assertion. When two equal masses undergo a glancing elastic collision with one of them at rest then after the collision, they will move at 90° to eachother.

Reason. It follows from the principle of conservation of linear momentum.

(*a*) A (*b*) B (*c*) C (*d*) D

107. Assertion. According to law of conservation of mechanical energy, change in potential energy is equal and opposite to the change in kinetic energy.

Reason. Mechanical energy is not a conserved quantity.

(*a*) A (*b*) B (*c*) C (*d*) D

108. Assertion. A quick collision between two bodies is more violent than a slow collision, even when the initial and the final velocities are identical.

Reason. The rate of change of momentum which determines the force is greater in the first case.

(*a*) A (*b*) B (*c*) C (*d*) D

109. Assertion. Work done by the frictional force is negative.

Reason. This is because frictional force acts along the direction of motion.

(*a*) A (*b*) B (*c*) C (*d*) D

110. Assertion. Work done by or against gravitational force in moving a body from one point to another is independent of the actual path followed between the two points.

Reason. This is because gravitational forces are conservative forces.

(*a*) A (*b*) B (*c*) C (*d*) D

111. Assertion. Time taken by a body to complete a given work has nothing to do with energy of the body.

Reason. Because power of a body is the rate of doing work.

(*a*) A (*b*) B (*c*) C (*d*) D

112. Assertion. A spring has potential energy, both, when it is compressed or stretched.

Reason. This is because in compressing or stretching, work is done on the spring against the restoring force.

(*a*) A (*b*) B (*c*) C (*d*) D

113. Assertion. When current is drawn from a cell, chemical energy is converted into heat energy.

Reason. This is because wire through which current flows gets heated.

(*a*) A (*b*) B (*c*) C (*d*) D

114. Assertion. In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

Reason. In an elastic collision, the linear momentum of the system is conserved.

(*a*) A (*b*) B (*c*) C (*d*) D

(RE-AIPMT 2015)

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions, read the two statements and check if

(A) Statement-1 is true ; statement-2 is true, and statement-2 is correct explanation of statement-1.

(B) Statement-1 is true ; statement-2 is true, but statement-2 is not a correct explanation of statement-1.

(C) Statement-1 is true, but statement-2 is false.

(D) Statement-1 is false, but statement-2 is true.

115. Statement-1. Kinetic energy is conserved in both, perfectly elastic and perfectly inelastic collisions.

Statement-2. Because linear momentum is conserved in both.

(*a*) A (*b*) B (*c*) C (*d*) D

116. Statement-1. For any collision, coefficient of restitution (*e*) lies between 0 and 1.

Statement-2. This is because no collision may be 100% elastic or 100% inelastic.

(*a*) A (*b*) B (*c*) C (*d*) D

ANSWERS

106. (*a*) **107.** (*c*) **108.** (*a*) **109.** (*c*) **110.** (*a*) **111.** (*b*) **112.** (*a*) **113.** (*d*) **114.** (*d*) **115.** (*d*) **116.** (*a*)

117. **Statement-1.** A body with negative energy cannot have momentum.

Statement-2. This is because momentum can be positive only.

(*a*) A (*b*) B (*c*) C (*d*) D

118. **Statement-1.** Energy released when a mass of one microgram disappears in a process is 9×10^7 J

Statement-2 : It follows from $E = \frac{1}{2} m v^2$

(*a*) A (*b*) B (*c*) C (*d*) D

119. If two springs S_1 and S_2 of force constants K_1 and K_2 respectively are stretched by the same force it is found that more work is done on spring S_1 than on spring S_2.

Statement-1. If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement-2. $K_1 < K_2$.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIEEE 2012)

120. **Statement-1.** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2. Principle of conservation of momentum holds true for all kinds of collisions.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIEEE 2010)

121. **Statement-1.** A point particle of mass *m* moving with speed *v* collides with stationary point particle of mass *M*. The maximum energy loss possible is given as $f\left(\frac{1}{2} mv^2\right)$, where $f = \frac{m}{M+m}$

Statement-2. Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(*a*) A (*b*) B (*c*) C (*d*) D

(JEE Main 2013)

HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Here, $m = 10$ kg, $u = 10$ m/s, $F = -0{\cdot}1\,x$ J/m [as force is retarding]

$x_1 = 20$ cm, $x_2 = 30$ cm

As $$W = \int_{x_1}^{x_2} F\,dx = \int_{20}^{30} -0{\cdot}1\,x\,dx = -0{\cdot}1\left[\frac{x^2}{2}\right]_{20}^{30}$$

$$= -\frac{0{\cdot}1}{2}[(30)^2 - (20)^2] = -\frac{0{\cdot}1}{2}[900-400]$$

$$= -\frac{0{\cdot}1}{2}[500] = -25\,\text{J}$$

$$K.E_i = \frac{1}{2}mu^2 = \frac{1}{2} \times 10 \times (10)^2 = 500\,\text{J}$$

According to work-energy theorem,

$$W = K.E_f - K.E_i \quad \text{or} \quad -25 = K.E_f - 500$$

$$\boldsymbol{K.E_f = 475\ \text{J}}$$

2. Here, power = *K* watt, time = *t*

as $$P = F v = mav,\ a = \frac{dv}{dt}$$

$$K = mv\frac{dv}{dt} \quad \text{or} \quad v\,dv = \frac{K\,dt}{m} \qquad \text{...(i)}$$

Integrating, we get

$$\int v\,dv = \int \frac{K\,dt}{m} \quad \text{or} \quad \frac{v^2}{2} = \frac{K}{m}\int dt$$

or $$\frac{v^2}{2} = \frac{K}{m}t \quad \text{or} \quad v = \sqrt{\frac{2Kt}{m}}$$

Now, $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\sqrt{\frac{2Kt}{m}}\right) = \sqrt{\frac{2K}{m}}\left(\frac{1}{2}t^{-1/2}\right)$$

$$= \frac{1}{2}\sqrt{\frac{2K}{mt}}$$

As $$F = ma = m \times \frac{1}{2}\sqrt{\frac{2K}{mt}} \quad \text{or} \quad F = \sqrt{\frac{mK}{2t}}$$

or $$\boldsymbol{F = \sqrt{\frac{mK}{2}}\,t^{-1/2}}$$

ANSWERS

117. (*c*) **118.** (*c*) **119.** (*d*) **120.** (*b*) **121.** (*d*)

HINTS/EXPLANATIONS

For Difficult Questions

3. Here, $m = 1$ kg

$$\vec{F} = (2t\,\hat{i} + 3t^2\,\hat{j})\,\text{N},\ P = ?$$

From $F = ma = m\dfrac{dv}{dt}$

$$\frac{dv}{dt} = \frac{F}{m} = \frac{(2t\,\hat{i} + 3t^2\,\hat{j})}{1} = (2t\,\hat{i} + 3t^2\,\hat{j})\,\text{m/s}^2$$

$$dv = (2t\,\hat{i} + 3t^2\,\hat{j})\,dt$$

$$v = \int (2t\,\hat{i} + 3t^2\,\hat{j})\,dt$$

$$= \frac{2t^2}{2}\hat{i} + \frac{3t^3}{3}\hat{j} = (t^2\,\hat{i} + t^3\,\hat{j})$$

Now, $P = \vec{F}\cdot\vec{v} = (2t\,\hat{i} + 3t^2\,\hat{j}).(t^2\,\hat{i} + t^3\,\hat{j})$

$$\mathbf{P = (2\,t^3 + 3\,t^5)\ watt}$$

4. As energy lost over PQ = energy lost over QR

$\therefore\ (\mu\, mg \cos\theta)\, PQ = (\mu\, mg)\, QR$

or $\qquad QR = PQ\cos\theta \qquad$ ...(*i*)

From Fig. 6(CF).1, $\sin\theta = \sin 30^\circ = \dfrac{2}{PQ} = \dfrac{1}{2}$

$\therefore \qquad PQ = 4$ m

From (*i*), $QR = 4\cos 30^\circ = 4\dfrac{\sqrt{3}}{2} = 2\sqrt{3}$ m

$= \mathbf{3{\cdot}5\ m}$

Again, decrease in P.E. = loss of energy due to friction in PQ and QR

$$mgh = (\mu\, mg\cos\theta)\, PQ + \mu\, mg \times QR$$

$$h = \mu\cos\theta \times PQ + \mu \times QR$$

$$2 = \mu\cos 30^\circ \times 4 + \mu \times 2\sqrt{3}$$

$$= \mu\left(4\times\frac{\sqrt{3}}{2} + 2\sqrt{3}\right) = \mu\times 4\sqrt{3}$$

$$\mu = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0{\cdot}288 = \mathbf{0{\cdot}29}$$

5. Here, $m = 10$ kg $\quad h = 1$ m $\quad n = 1000$

Work done in lifting mass $= n \times mgh$

$= 1000 \times 10 \times 9{\cdot}8 \times 1 \qquad$ ...(*i*)

If M is mass of fat burnt, then energy gained

$= M \times 3{\cdot}8 \times 10^7 \times \dfrac{20}{100} \qquad$ ...(*ii*)

If there is no loss of energy, then from (*i*) and (*ii*)

$$M \times 3{\cdot}8\times 10^7 \times \frac{20}{100} = 1000 \times 10 \times 9{\cdot}8$$

$$M = \frac{9{\cdot}8\times 10^4 \times 5}{3{\cdot}8\times 10^7} = \mathbf{12{\cdot}89 \times 10^{-3}\ kg}$$

6. Heat generated in device in 3 hours

$= P' \times t' = 3 \times 10^3 \times 3 \times 60 \times 60 = 324 \times 10^5$ J

Heat used in heating water

$= ms\,\Delta\theta = 120 \times 1 \times 4{\cdot}2 \times 10^3 \times (30 - 10)$

$= 100{\cdot}8 \times 10^5$ J

Heat absorbed by coolant

$P \times t = P' \times t - ms\,\Delta\theta$

$= 324 \times 10^5 - 100{\cdot}8 \times 10^5 = 223{\cdot}2 \times 10^5$ J

$$P = \frac{223{\cdot}2\times 10^5}{60\times 60} = \mathbf{2067\ watt}$$

7. Here, $K_P > K_Q$ (Given)

In case (*a*) : $x_1 = x_2 = x$,

$$\therefore\ \frac{W_P}{W_Q} = \frac{\frac{1}{2}K_P\,x^2}{\frac{1}{2}K_Q\,x^2} = \frac{K_P}{K_Q} \qquad \therefore\ \ \mathbf{W_P > W_Q}$$

In case (*b*) : $F_1 = F_2 = F$,

for constant force, $W = \dfrac{F^2}{2K}$

$$\therefore\ \frac{W_P}{W_Q} = \frac{\dfrac{F^2}{2K_P}}{\dfrac{F^2}{2K_Q}} = \frac{K_Q}{K_P},\ \ \therefore\ \mathbf{W_Q > W_P}$$

8. Here, power = K watt, time = t

as $\quad P = F\,v = mav,\ a = \dfrac{dv}{dt}$

$K = mv\dfrac{dv}{dt}\ $ or $\ v\,dv = \dfrac{K\,dt}{m} \qquad$ ...(*i*)

Integrating, we get

$$\int v\,dv = \int \frac{K\,dt}{m}\ \text{ or }\ \frac{v^2}{2} = \frac{K}{m}\int dt$$

$$\text{or}\quad \frac{v^2}{2} = \frac{K}{m}t\ \text{ or }\ v = \sqrt{\frac{2Kt}{m}}$$

$$\text{as}\quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\sqrt{\frac{2Kt}{m}}\right) = \sqrt{\frac{2K}{m}}\left(\frac{1}{2}t^{-1/2}\right)$$

$$= \frac{1}{2}\sqrt{\frac{2K}{mt}}$$

$$\text{As}\quad F = ma = m\times\frac{1}{2}\sqrt{\frac{2K}{mt}}\ \text{ or }\ F = \sqrt{\frac{mK}{2t}}$$

$$\text{or}\quad \mathbf{F = \sqrt{\frac{mK}{2}}\,t^{-1/2}}$$

HINTS/EXPLANATIONS

For Difficult Questions

9. Here, $m = 10^{-2}$ kg, $v_0 = 10$ m/s, $t = 10$ s,
$F = -k\,v^2$; $k = ?$

If v_t is velocity of body after 10 s, then

$$\frac{1}{2}\,m v_t^2 = \frac{1}{8}\,m\,v_0^2 \;\therefore\; v_t = \frac{v_0}{2} = \frac{10}{2} = 5 \text{ m/s}$$

From $F = -k\,v^2$

$$m\,\frac{dv}{dt} = -k\,v^2$$

$$\frac{dv}{dt} = \frac{-k}{m}\,v^2 = -100\,k\,v^2$$

or $$\frac{dv}{v^2} = -100\,k\,dt$$

$$\int_{10}^{5} \frac{dv}{v^2} = -100\,k \int_{0}^{10} dt$$

$$\frac{1}{5} - \frac{1}{10} = 100\,k\,(10-0)$$

$$\frac{1}{10} = 100 \times 10\,k\,dt$$

$$\mathbf{k = 10^{-4}\ kg\ m^{-1}}$$

10. Here, $F = 6\,t$, $m = 1$ kg, $u = 0$, $W = ?$

Now, $$F = m\,a = m\,\frac{dv}{dt} = 1 \times \frac{dv}{dt}$$

$$\therefore \quad \frac{dv}{dt} = F = 6\,t$$

$$dv = 6\,t\,dt$$

$$\int_0^v dv = \int_0^1 6t\,dt$$

$$v = \left[6\,\frac{t^2}{2}\right]_0^1 = 3\,(1^2 - 0) = 3 \text{ m/s}$$

From work energy theorem,

$$W = \Delta\,\text{KE} = \frac{1}{2}\,m\,(v^2 - u^2)$$

$$= \frac{1}{2} \times 1\,(3^2 - 0^2) = \mathbf{4{\cdot}5\ J}$$

11. Here, $U = \frac{1}{2}Kx^2$. Let P.E. be U' when spring is stretched to nx cm.

$$\therefore \quad U' = \frac{1}{2}K\,(nx)^2 = \frac{1}{2}K\,n^2\,x^2$$

Now $$\frac{U'}{U} = \frac{\frac{1}{2}K\,n^2\,x^2}{\frac{1}{2}Kx^2} = n^2 \quad \text{or} \quad \mathbf{U' = n^2\,U}$$

12. Work done by gravity $= mgh = (10^{-3}) \times 10 \times 10^3$
$= \mathbf{10\ J}$

Work done by resistive force = final energy − initial energy

$$= \left(0 + \frac{1}{2}mv^2\right) - (mgh + 0) = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2} \times 10^{-3} \times 50^2 - 10^{-3} \times 10 \times 10^3 \quad = \mathbf{-8{\cdot}75\ J}$$

13. The relativistic K.E. of a particle of rest mass m_0 is given by

$$K = (m - m_0)\,c^2, \text{ where } m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Here, m is mass of particle moving with velocity v.
As K.E. = rest mass energy

$$\therefore\; (m - m_0)\,c^2 = m_0\,c^2 \;\text{ or }\; mc^2 = 2\,m_0\,c^2$$

$$\frac{m_0}{\sqrt{1 - v^2/c^2}} = 2\,m_0$$

$$1 - v^2/c^2 = \frac{1}{4}$$

$$v^2/c^2 = 3/4, \quad v = \frac{\sqrt{3}}{2}\,c$$

14. If m is mass of boy, the mass of man = 2 m
As K.E. of man = K.E. of boy

$$\therefore \quad \frac{1}{2}(2\,m)\,v^2 = \frac{1}{2}mv'^2$$

$$v' = \sqrt{2}\,v$$

In the second case,

$$\frac{1}{2}(2\,m)\,(v+2)^2 = \frac{1}{2}m\,(v' + x)^2$$

$$2\,(v+2)^2 = (\sqrt{2}\,v + x)^2$$

$$\sqrt{2}\,v + x = \sqrt{2}\,(v+2) = \sqrt{2}\,v + 2\sqrt{2}$$

$$\mathbf{x = 2\sqrt{2}\ m/s}$$

15. Here, $m_1 = m$, $m_2 = 4$ m
$v_1 = v$, $v_2 = 0$, $e = ?$

If V' is final velocity of heavier block, then according to the law of conservation of linear momentum,

$$m_1\,v_1 + m_2\,v_2 = m_2\,V'$$

$$m \times v + 4\,m \times 0 = 4\,m\,V' \quad \therefore \quad V' = v/4$$

HINTS/EXPLANATIONS

For Difficult Questions

Coefficient of restitution,

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$e = \frac{v/4}{v} = \frac{1}{4} = \mathbf{0{\cdot}25}$$

16. Here, $U = \dfrac{M}{r^6} - \dfrac{N}{r^{12}}$;

$$F = -\frac{dU}{dr} = \frac{-d}{dr}\left(\frac{M}{r^6} - \frac{N}{r^{12}}\right)$$

$$= \left(\frac{-6M}{r^7} - \frac{12N}{r^{13}}\right)$$

In equilibrium position, $F = 0$

$$\therefore \quad \frac{6M}{r^7} = \frac{12N}{r^{13}} \quad \text{or} \quad r^6 = \frac{2N}{M}$$

$$\text{Hence,} \quad U = \frac{M}{(2N/M)} - \frac{N}{(2N/M)^2} = \frac{\mathbf{M^2}}{\mathbf{4N}}$$

17. Initial K.E. $= \dfrac{P_1^2}{2\text{m}} = \dfrac{10\times10}{2\times5} = 10$ J

Impulse $= p_2 - p_1 = F \times t$

$p_2 - 10 = 0{\cdot}2 \times 10$; $p_2 = 12$ kg m/s

Final K.E. $= \dfrac{P_2^2}{2\text{m}} = \dfrac{12\times12}{2\times5} = 14{\cdot}4$ J

Increase in K.E. $= 14{\cdot}4 - 10 = \mathbf{4{\cdot}4\ J}$

18. Using law of conservation of total energy

Case (*i*), $\dfrac{1}{2}mu^2 = \dfrac{1}{2}m\left(\dfrac{u}{2}\right)^2 + F\times 3$

or $\dfrac{3}{4}\left(\dfrac{1}{2}mu^2\right) = F \times 3$ **...(*i*)**

Case (*ii*), $\dfrac{1}{2}mu^2 = F \times S$ **...(*ii*)**

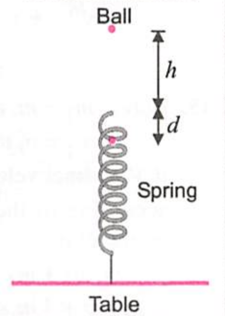
or $\dfrac{3}{4} = \dfrac{3}{S}$ or $S = 4$ cm

$\therefore$ Further distance travelled

$= 4 - 3 = \mathbf{1\ cm}$

19. As the spring is compressed through a distance (*d*), therefore, total energy supplied by the ball $= mg\ (h + d)$, being the potential energy of the ball, Fig. 6(CF).19.

FIGURE 6(CF).19



Now, P.E. of compressed spring $= \dfrac{1}{2}\text{K}\,d^2$

$\therefore$ Net work done in the process

$$W = mg\ (h + d) - \frac{1}{2}\mathbf{K}\,\mathbf{d^2}$$

20. Here, $m = 2$ kg, $v = 4$ m/s

Force of kinetic friction, $F = 15$ N

spring constant, $K = 10000$ N/m

Let the spring be compressed by x metre.

K.E. supplied = Work done against friction + P.E. of spring

$$\frac{1}{2}mv^2 = Fx + \frac{1}{2}Kx^2$$

$$\frac{1}{2}\times2\times4^2 = 15x + \frac{10000}{2}x^2$$

or $5000\,x^2 + 15\,x - 16 = 0$

On solving, we get $x = 0{\cdot}055$ m $= \mathbf{5{\cdot}5\ cm}$

21. If v is the final velocity, then change in momentum = impulse = area enclosed by graph with time axis

$$2\,(v - 0) = \frac{4\times3}{2} - \frac{1}{2}\times\frac{3}{2}\times2 = 4.5$$

$\therefore \quad v = 2.25$ m/s

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times2\times(2.25)^2$$

$$= \mathbf{5.06\ J}$$

22. $U_{(x)} = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$ ...(*i*)

$$F = -\frac{dU(x)}{dx} = -\frac{12a}{x^{13}} + \frac{6b}{x^7}$$

At equilibrium distance between the molecules, $F = 0$

$$\therefore \ -\frac{12a}{x^{13}} + \frac{6b}{x^7} = 0 \ \text{ or } \ x = \left(\frac{2a}{b}\right)^{1/6}$$

From (*i*) $\quad U_{(\infty)} = \dfrac{a}{\infty^{12}} - \dfrac{b}{\infty^6} = 0$

Dissociation energy of molecule,

$D = U_{(x=\infty)} - U_{\text{at equilibrium}}$

$$= 0 - \left(\frac{a}{x^{12}} - \frac{b}{x^6}\right) \qquad \text{where, } x = \left(\frac{2a}{b}\right)^{1/6}$$

$$D = \frac{-a}{x^{12}} + \frac{b}{x^6} = -\frac{a}{(2a/b)^{12/6}} + \frac{b}{(2a/b)^{6/6}}$$

$$= -\frac{b^2}{4a} + \frac{b^2}{2a} = \frac{\mathbf{b^2}}{\mathbf{4a}}$$

HINTS/EXPLANATIONS

For Difficult Questions

23. Here, $m_1 = 50$ kg, $m_2 = 0{\cdot}5$ kg

$u_1 = ?,\ u_2 = 2$ m/s

By momentum conservation,

$m_1 u_1 + m_2 u_2 = 0$

$$u_1 = \frac{-m_2 u_2}{m_1} = \frac{-0{\cdot}5\times 2}{50} = -\frac{1}{50}\ \text{m/s}$$

Negative sign is for upward motion of man. In gravity free space, time taken by stone to reach the floor, $t = \dfrac{s}{u_2} = \dfrac{10}{2} = 5\,\text{s}$

Upward distance moved by the man $= u_1 \times t$

$$= \frac{1}{50}\times 5 = 0{\cdot}1\ \text{m}.$$

∴ Distance of man above the floor

$= (10 + 0{\cdot}1)$ m $=$ **10·1 m**

24. From the knowledge of theory, energy required to break one bond in DNA is $\mathbf{10^{-20}\ J}$

25. If l is the total length of spring, then

$$l_A = \frac{2l}{5} \quad \text{and} \quad l_B = \frac{3l}{5}$$

If k_A, k_B are the stiffness of A and B, then

$$K\,l = k_A\, l_A = k_B\, l_B \ \text{ or } \ k_A = \frac{kl}{l_A} = \frac{kl}{(2l/5)} = \mathbf{\frac{5}{2}k}$$

26. Let kinetic energy $E = ct$. But $E = \dfrac{p^2}{2m}$

where c is a constant, p is linear momentum and m is the mass of particle.

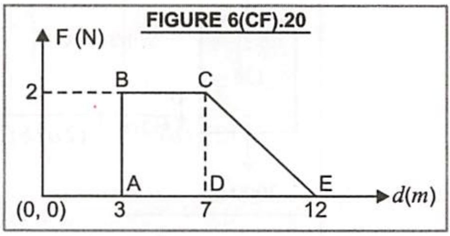
$$\therefore \quad p = \sqrt{2Em} = \sqrt{2ctm}$$

$$\text{Force, } F = \frac{dp}{dt} = \frac{d}{dt}(2cmt)^{\frac{1}{2}} = \sqrt{2cm}\times\frac{1}{2}t^{-\frac{1}{2}}$$

$$\text{or} \quad F \propto \frac{1}{\sqrt{t}}$$

27. Refer to Fig. 6(CF).20,

FIGURE 6(CF).20



Work done = area under F-d graph

= area of rectangle ABCD + area of triangle DCE

$$= 2\times(7-3) + \frac{1}{2}\times 2\times(12-7) = 8 + 5 = \mathbf{13\ J}$$

28. The potential energy of particle in a force field is

$$U = \frac{A}{r^2} - \frac{B}{r}$$

$$\frac{dU}{dr} = -\frac{2A}{r^3} + \frac{B}{r^2}$$

For equilibrium, $\dfrac{dU}{dr} = 0$ or $-\dfrac{2A}{r^3} + \dfrac{B}{r^2} = 0$

$$\text{or} \qquad r = \frac{2A}{B}$$

The equilibrium will be stable, if $\dfrac{d^2U}{dr^2} = +$ for this value of r.

$$\text{Now, } \frac{d^2U}{dr^2} = \frac{d}{dr}\left(-\frac{2A}{r^3} + \frac{B}{r^2}\right) = \left(\frac{6A}{r^4} - \frac{2B}{r^3}\right),$$

which is positive for $r = 2A/B$.

Hence **choice (*b*) is correct.**

29. Constant power of car, $P_0 = F(v)$

$$= ma(v) = m\frac{dv}{dt}(v) \quad \text{or} \quad P_0\, dt = mv\, dv$$

Integrating both sides, we get

$$\int_0^t P_0\, dt = \int_0^v mv\, dv = \frac{mv^2}{2}, \quad i.e., \quad P_0(t) = \frac{mv^2}{2}$$

$$v = \sqrt{\frac{2P_0(t)}{m}} \quad \text{or} \quad v \propto t^{1/2}$$

30. Here, $U = -\dfrac{K}{2r^2}$. Total energy = ?

$$\text{As} \quad F = -\frac{dU}{dr}$$

$$F = -\frac{d}{dr}\left(-\frac{K}{2r^2}\right) = +\frac{K}{2}\frac{(-2)}{r^3} = -\frac{K}{r^3}$$

This provides the necessary centripetal force $\left(\dfrac{mv^2}{r}\right)$

$$\therefore \quad \frac{mv^2}{r} = \frac{K}{r^3} \quad \text{or} \quad mv^2 = \frac{K}{r^2}$$

$$\text{KE} = \frac{mv^2}{2} = \frac{K}{2r^2}$$

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HINTS/EXPLANATIONS

For Difficult Questions

$$\text{PE} = -\frac{K}{2r^2}$$

$$\text{Total energy} = \text{KE} + \text{PE} = \frac{K}{2r^2} - \frac{K}{2r^2} = \textbf{Zero}$$

31. Work done by the variable force

$$W = \int_{y_1}^{y_2} F\,dy = \int_{y=0}^{y=1} (20+10y)\,dy$$

$$\left[20y + \frac{10y^2}{2}\right]_0^1 = 20(1-0) + 5(1^2 - 0^2)$$

$$= \mathbf{25\ J}$$

32. As is known from theory ($v^2 - u^2 = 2as$)

$$\text{Stopping distance, } x_1 = \frac{u^2}{2g\sin 60^\circ}$$

$$\text{and} \quad x_2 = \frac{u^2}{2g\sin 30^\circ}$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1/2}{\sqrt{3}/2} = \mathbf{1 : \sqrt{3}}$$

33. When speed is max. (constant), $a = 0$.

$\therefore \quad x = F/K.$

Applying work energy theorem,

$$\Delta\text{KE} = -\frac{1}{2}Kx^2 + Fx$$

$$\frac{1}{2}mU_{\max}^2 = -\frac{1}{2}K\left(\frac{F^2}{K^2}\right) + \frac{F^2}{K} = \frac{F^2}{2K}$$

$$U_{\max}^2 = \frac{F^2}{mK} \quad \text{or} \quad U_{\max} = \frac{F}{\sqrt{mK}}$$

34. Here, $E = 10^{-20}$ J

As $1\ \text{eV} = 1{\cdot}6 \times 10^{-19}$ J

$$\therefore \quad E = \frac{10^{-20}}{1{\cdot}6 \times 10^{-19}}\ \text{J} = \frac{10^{-1}}{1{\cdot}6} = \mathbf{0{\cdot}06\ eV}$$

35. Here, $m = 0{\cdot}5$ g, $E = ?$

From $E = mc^2 = (0{\cdot}5 \times 10^{-3})(3 \times 10^8)^2$ J

$$= \mathbf{4{\cdot}5 \times 10^{13}\ J}$$

36. Here, $m_1 = 4$ kg, $m_2 = 6$ kg, $a = ?$

$$a = \frac{(m_2 - m_1)g}{m_2 + m_1} = \frac{(6-4)g}{6+4} = \mathbf{g/5}$$

37. Here, $h = 60$ m $\quad \dfrac{dm}{dt} = 15\,\text{kg/s}$

$$\text{Energy produced/sec} = \left(\frac{dm}{dt}\right)gh = 15 \times 10 \times 60$$

Wastage = 10%

$$\therefore \quad \text{Power generated} = \frac{90}{100} \times 15 \times 10 \times 60$$

$$= 8100\ \text{W} = \mathbf{8{\cdot}1\ kW}$$

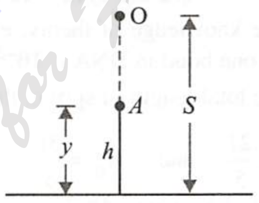
38. In falling from O to A through a height $(S - y)$, velocity acquired

$$v = \sqrt{2g(S-y)}$$

At A, P.E. of particle $= mgy = u$

$$\text{K.E. of particle} = \frac{1}{2}mv^2 = K$$

FIGURE 6(CF).21



As per question, $K = 3u$

$$\frac{1}{2}mv^2 = 3mgy$$

$$\text{or} \quad \frac{1}{2}m\,2g(S-y) = 3mgy$$

$$S - y = 3y \quad \text{or} \quad \mathbf{y = S/4}$$

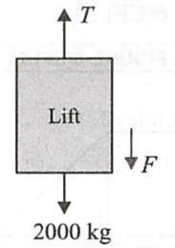
$$u = \sqrt{2g(S-y)} = \sqrt{2g(S - S/4)} = \sqrt{\frac{\mathbf{3gS}}{\mathbf{2}}}$$

39. Here, $P = 60$ HP $= 60 \times 746$ W

$m = 2000$ kg, $F = 4000$ N, $v = ?$

As is clear from Fig. 6(CF).22

FIGURE 6(CF).22



HINTS/EXPLANATIONS

For Difficult Questions

$$T = mg + F = 2000 \times 10 + 4000 = 24000 \text{ N}$$

As power $P = T \times v$

$$\therefore \quad v = \frac{P}{T} = \frac{60 \times 746}{2400} = \mathbf{1{\cdot}9 \ m/s}$$

40. When the bob has fallen through a distance h, decrease in P.E $= mgh$

If v is the speed of the bob,

K.E of translation $= \frac{1}{2}mv^2$, and

K.E of rotation of flywheel $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$$

From the principle of conservation of energy.

Loss in P.E = Gain in K.E

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{4gh}{3}}, \quad \omega = \frac{v}{r} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

41. Applying the principle of conservation of mechanical energy between A and C, we get

$$u_1 = mg \times 2, K_1 = 0$$

$$u_2 = mg \times 1, K_2 = ?$$

As $\quad u_2 + K_2 = u_1 + K_1$

$$\therefore \quad K_2 = u_1 - u_2 = 2\,mg - mg = mg$$

$$= 1 \times 10 = \mathbf{10 \ J}$$

42. Work done in moving the particle from A to B, Fig. 6(CF).9 is

$$W = \int_A^B \vec{F} \cdot \vec{dx} = \int_1^0 -x\,dx + \int_0^1 y\,dy$$

$$= \left[-\frac{x^2}{2}\right]_1^0 + \left[\frac{y^2}{2}\right]_0^1$$

$$W = -\left(0^2 - \frac{1}{2}\right) + \left(\frac{1^2}{2} - 0^2\right) = \frac{1}{2} \times \frac{1}{2} = \mathbf{1 \ J}$$

43. Using the principle of conservation of mechanical energy, $\quad U_i + K_i = U_f + K_f$

$$mg\,\frac{l}{2}\sin 30^\circ + 0 = 0 + \frac{1}{2}Iw^2$$

$$mg \times \frac{1}{2} \times \frac{1}{2} + 0 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\frac{mg}{4} = \frac{ml^2}{6}\omega^2$$

$$\omega^2 = \frac{6}{l^2} \times \frac{g}{4} = \frac{30}{2} = 15$$

$$\omega = \sqrt{15} = \sqrt{n}$$

Here $\quad \mathbf{n = 15}$

44. Here, mass of ball, $m = 0{\cdot}15$ kg

Height, $h = 20$ m

Force $F = ?$, Horizontal distance, $s = 0{\cdot}2$ m

As work done = K.E of ball = P.E acquired

$$\therefore \quad F \times s = \frac{1}{2}mv^2 = mgh$$

$$F \times 0{\cdot}2 = (0{\cdot}15) \times 10 \times 20$$

$$F = \frac{0{\cdot}15 \times 10 \times 20}{0{\cdot}2} = \mathbf{150\,N}$$

45. Here, $m = 5 \text{ g} = 5 \times 10^{-3}$ kg

$v = 210$ m/s

$s = 0{\cdot}030$ cal/g/°C $= 30 \times 4{\cdot}2$ J/kg/°C

Rise in temp. $\theta = ?$

As half of the heat produced goes into the wood, therefore rise in temp. (θ) is given by

$$ms\theta = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$$

$$\theta = \frac{v^2}{4s} = \frac{210 \times 210}{4 \times 30 \times 4{\cdot}2} = \mathbf{87{\cdot}5^\circ C}$$

46. Suppose v' is combined velocity of the three masses after inelastic collision. From the principle of conservation of linear momentum

$$mv = (2m + M)\,v' \quad \text{or} \quad v' = \frac{mv}{(2m + M)}$$

Initial energy, $E_1 = \frac{1}{2}mv^2$

Final energy, $E_2 = \frac{1}{2}(2m + M)v'^2$

$$= \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$$

$$= \frac{m^2v^2}{2(2m + M)}$$

HINTS/EXPLANATIONS
For Difficult Questions

$$\text{Loss of energy} = E_1 - E_2 = \frac{5}{6}\text{ (initial KE)}$$

$$= \frac{1}{2}mv^2 - \frac{m^2v^2}{2(2m+M)}$$

$$= \frac{5}{6}\left(\frac{1}{2}mv^2\right)$$

On solving, we get $\frac{M}{m} = \mathbf{4}$

47. Here, $P = 100 \text{ kW} = 10^5 \text{ W} = 10^5 \text{ J/s}$,

$t = 1 \text{ hr} = 60 \times 60 \text{ sec.}$

Energy radiated $= P \times t = 10^5 \times 60 \times 60 \text{ J}$

$$= \mathbf{36 \times 10^7 \text{ J}}$$

48. Here, $h = 60$ m, $\frac{dm}{dt} = 15$ kg/sec

Incident power on turbine

$$P = \frac{d}{dt}(mgh) = gh\left(\frac{dm}{dt}\right)$$

$$= 10 \times 60 \times 15 = 9000 \text{ W}$$

As losses are 10%, therefore, net power generated

$$= \left(1 - \frac{10}{100}\right)P = \frac{9}{10} \times 9000 \text{ W}$$

$$= \mathbf{8{\cdot}1 \text{ kW}}$$

49. Here, $m_1 = 4$ m ; $m_2 = 2$ m

As the collision is head on elastic, therefore,

$$\text{fractional loss of K.E of body A} = \frac{4m_1m_2}{(m_1+m_2)^2}$$

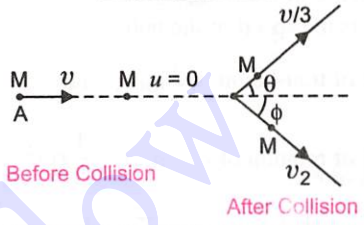
$$= \frac{4 \times 4m \times 2m}{(4m+2m)^2} = \frac{32\text{m}^2}{36\text{m}^2} = \mathbf{\frac{8}{9}}$$

50. Two particles A and B will collide if the direction of relative velocity of particle B with respect to particle $A[=(\vec{v}_2 - \vec{v}_1)/|\vec{v}_2 - \vec{v}_1|]$ is directed towards the direction of relative position of particle A with respect to particle $B[=(\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|]$. Therefore, for collision of A and B

$$\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$$

51. Refer to Fig. 6(CF).23. As the collision is perfectly elastic, hence the laws of conservation of linear momentum and kinetic energy are obeyed.

FIGURE 6(CF).23



According to law of conservation of kinetic energy

Total KE before collision = total KE after collision

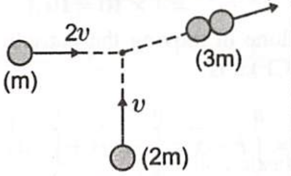
$$\text{i.e., } \frac{1}{2}M\,v^2 + 0 = \frac{1}{2}M\,(v/3)^2 + \frac{1}{2}M\,v_2^2$$

$$\text{or} \quad v^2 = \frac{v^2}{9} + v_2^2 \quad \text{or} \quad v_2^2 = v^2 - \frac{v^2}{9} = \frac{8v^2}{9}$$

$$\text{or} \quad v_2 = \mathbf{\frac{2\sqrt{2}\,v}{3}}$$

52. For perfectly inelastic collision

FIGURE 6(CF).24



$$\therefore \text{Initial momentum} = \sqrt{(m(2v))^2 + \{2m(v)\}^2}$$

$$= 2\sqrt{2}\,mv$$

final momentum $= (2\,m + m)\,V = 3$ mV

As momentum is conserved

final momentum = Initial momentum

$$3\text{mV} = 2\sqrt{2}\,mv$$

$$V = \frac{2\sqrt{2}\,v}{3}$$

HINTS/EXPLANATIONS

For Difficult Questions

$$\text{Initial KE} = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 = 3mv^2$$

$$\text{Final KE} = \frac{1}{2}(3m)V^2 = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2$$

$$= \frac{4}{3}mv^2$$

$$\text{Loss of energy} = \frac{3mv^2 - \frac{4}{3}mv^2}{3mv^2} \times 100$$

$$= \frac{500}{9} = 55{\cdot}5\% \approx \mathbf{56\%}$$

53. Here $m_1 = 0{\cdot}50$ kg, $u = 2{\cdot}00$ m/s, $m_2 = 1$ kg

$u_2 = 0, v_1 = v_2 = v$

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

or $$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$= \frac{0{\cdot}5 \times 2 + 1 \times 0}{0{\cdot}5 + 1} = \frac{1}{1{\cdot}5} = \frac{2}{3}\text{ m/s} = 0{\cdot}67 \text{ m/s}$$

$$\text{Loss of energy} = K_i - K_f$$

$$= \frac{1}{2}(0{\cdot}5)(2)^2 - \frac{1}{2}(1{\cdot}5)(0{\cdot}67)^2$$

$$= 1 - 0{\cdot}33 = \mathbf{0{\cdot}67\ J}$$

54. In an elastic collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

As second ball is initially at rest, $u_2 = 0$

$$\therefore \quad v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

As $$v_1 = \frac{2}{3}u_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

$$\therefore \quad 2m_1 + 2m_2 = 3m_1 - 3m_2$$

or $5m_2 = m_1$ or $\dfrac{m_1}{m_2} = \dfrac{\mathbf{5}}{\mathbf{1}}$

55. Here $m_1 = 5$ g $= 0{\cdot}005$ kg, $u_1 = 600$ m/s,

$m_2 = 5$ kg, $u_2 = 0$

After collision, they will move with the same velocity v.

As momentum is conserved

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$\therefore \quad v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$v = \frac{0{\cdot}005 \times 600 + 5 \times 0}{(5 + 0{\cdot}005)} = 0{\cdot}6 \text{ m/s} = \mathbf{60\ cm/s}$$

56. Here $u_1 = 2$ m/s, $u_2 = 0$, $e = 0{\cdot}5$,

$m_1 = m$, $m_2 = 2$m

As momentum is conserved.

$$\therefore \quad m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m \times 2 + 2m \times 0 = mv_1 + 2mv_2$$

or $$v_1 + 2v_2 = 2 \qquad \textbf{...(}\textbf{\textit{i}}\textbf{)}$$

As coefficient of restitution $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$0{\cdot}5 = \frac{v_2 - v_1}{2 - 0} \text{ or } v_2 - v_1 = 1 \qquad \textbf{...(}\textbf{\textit{ii}}\textbf{)}$$

Adding (*i*) and (*ii*)

$3v_2 = 3$ or $v_2 = \mathbf{1\ m/s}$

$v_1 = \mathbf{0\ m/s}$

57. Elastic P.E. of spring = K.E. of ball

$$\frac{1}{2}Kx^2 = \frac{1}{2}mv^2$$

$$x = \sqrt{\frac{mv^2}{K}} = \sqrt{\frac{4 \times 3^2}{144}} = \frac{1}{2} = 0{\cdot}5 \text{ m}$$

$\therefore$ Length of compressed spring $= l - x = 2 - 0{\cdot}5$
$= \mathbf{1{\cdot}5\ m}$

58. Here, $k = 2{\cdot}0$ N/m, $m_1 = 2$ kg

$m_2 = 1$ kg, $v_2 = 2{\cdot}0$ m/s

Time period of spring,

$$T = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{2}{2}} = 2\pi\, s$$

Time taken by the spring to return to its unstretched position $= \frac{T}{2}$ sec $= \frac{2\pi}{2} = \pi$ sec.

Distance between the two blocks at this time

$$x = \text{combined vol.} \times \text{time}$$

$$= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \times \left(\frac{T}{2}\right)$$

$$= \frac{2 \times 0 + 1 \times 2}{1 + 2} \times \pi = \frac{2\pi}{3} \text{ metre}$$

$$x = \frac{2}{3} \times 3{\cdot}14 = \mathbf{2{\cdot}09\ m}$$

59. Let m be the mass of the shell. Total momentum at the highest point before explosion $= m\,v \cos\theta$. One of the pieces of mass $m/2$ will retrace its path to the cannon only when its velocity is $(-\,v\cos\theta)$. Therefore, its momentum $= -\frac{m}{2} v \cos\theta$. Applying principle of conservation of linear momentum.

$$\frac{m}{2} v' - \frac{m}{2} v\cos\theta = m\,v\cos\theta$$

$$\frac{m}{2} v' = \frac{3}{2} m\,v\cos\theta$$

$$\therefore \quad \mathbf{v' = 3\,v\cos\theta}$$

60. Here, $m_1 = 40$ kg, $u_1 = 4$ m/s
$m_2 = 60$ kg, $u_2 = 2$ m/s

Final vel. after collision, $v = \frac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)}$

$$v = \frac{40 \times 4 + 60 \times 2}{(40 + 60)} = 2{\cdot}8 \text{ m/s}$$

Loss of energy $= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \frac{1}{2}(m_1 + m_2)\, v^2$

$$= \frac{1}{2} \times 40 \times 4^2 + \frac{1}{2} \times 60 \times 2^2 - \frac{1}{2}(40 + 60) \times (2{\cdot}8)^2$$

$= 320 + 120 - 392 =$ **48 joule**

61. Here $m_1 = m_A$, $m_2 = m_B$, $u_1 = u$, $u_2 = 0$,
$v_1 = v$, $v_2 = -\,v$

Applying principle of conservation of linear momentum

$$m_1 u + 0 = m_1 v + m_2(-v) = (m_1 - m_2)\, v \quad \text{...(i)}$$

Applying principle of conservation of K.E.

$$\frac{1}{2} m_1 u^2 = \frac{1}{2}(m_1 + m_2)\, v^2 \quad \text{...(ii)}$$

Dividing (ii) by (i), we get

$$u = \frac{(m_1 + m_2)\, v}{m_1 - m_2} \quad \text{or} \quad \frac{u}{v} = \frac{m_1 + m_2}{m_1 - m_2} \quad \text{...(iii)}$$

From (i), $\frac{u}{v} = \frac{m_1 - m_2}{m_1}$

$\therefore$ from (iii), $\frac{m_1 + m_2}{m_1 - m_2} = \frac{m_1 - m_2}{m_1}$

On solving, we get

$$\frac{m_1}{m_2} = \frac{1}{3}, \text{ i.e., } \frac{m_A}{m_B} = \mathbf{\frac{1}{3}}$$

62. Here, $m_1 = m_2 = m$, $u_1 = u$, $u_2 = 0$. Let v_1, v_2 be their velocities after collision.

According to principle of conservation of linear momentum.

$$m\,u + 0 = m\,(v_1 + v_2) \quad \text{or} \quad v_1 + v_2 = u \quad \text{...(i)}$$

By definition, $e = \frac{v_2 - v_1}{u - 0}$

or $\quad v_2 - v_1 = e\,u \quad$ ...(ii)

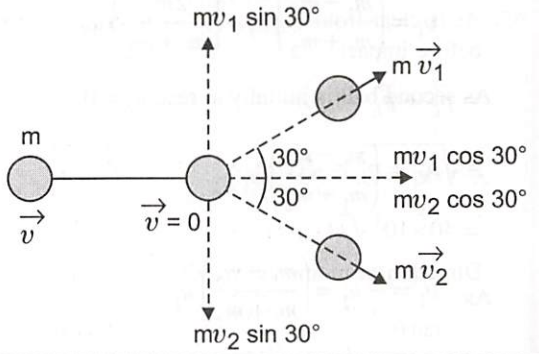
Add (i) and (ii), $v_2 = \frac{u\,(1 + e)}{2}$

Subtract (ii) from (i)

$$v_1 = \frac{(1 - e)\,u}{2} \quad \therefore \quad \frac{v_1}{v_2} = \mathbf{\frac{1 - e}{1 + e}}$$

63. As shown in Fig. 6(CF).25,

FIGURE 6(CF).25



HINTS/EXPLANATIONS
For Difficult Questions

application of the law of conservation of linear momentum perpendicular to initial line of motion, gives

$$0 = mv_1 \sin 30^\circ - mv_2 \sin 30^\circ \quad \therefore \; v_1 = v_2$$

Applying the same principle along the line of motion,

$$m \times v + m \times 0 = mv_1 \cos 30^\circ + mv_2 \cos 30^\circ$$

$$m \times 6 + m \times 0 = 2\, mv_1 \cos 30^\circ = 2mv_1 \frac{\sqrt{3}}{2}$$

$$v_1 = \frac{6}{\sqrt{3}} = \mathbf{2\sqrt{3}\ m/s}$$

64. Ignoring g compared to initial velocity, the bullet and block will meet after $t = \frac{h}{u} = \frac{100}{100} = 1\,\text{sec}$.

During this time, distance travelled by the block

$$s_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times 1^2 = 5\,\text{m}$$

Distance travelled by the bullet,

$$s_2 = 100 - s_1 = 100 - 5 = 95\ \text{m}$$

velocity of bullet just before collision,

$$u_1 = 100 - 10 \times 1 = 90\ \text{m/s, upwards}$$

velocity of block just before collision,

$$u_2 = 10 \times 1 = 10\ \text{m/s, downwards}$$

According to law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, V$$

$$0{\cdot}01\,(-10 + 90) = 0{\cdot}02\ V$$

$$V = 40\ \text{m/s}$$

Maximum height risen by the block

$$= \frac{V^2}{2g} = \frac{40 \times 40}{2 \times 10} = 80\,\text{m}$$

Height reached above the top of the tower

$$= 80 - s_1 = 80 - 5 = \mathbf{75\ m}$$

65. As is clear from Fig. 6(CF).26, total momentum before impact

$$= \vec{p}_C + \vec{p}_L = |\,\vec{p}_C + \vec{p}_L\,|$$

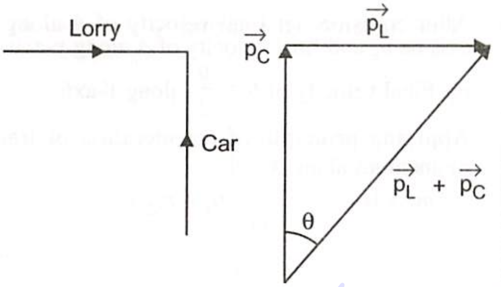
$$= \sqrt{p_C^2 + p_L^2} = \sqrt{(2 \times 10^3 \times 20)^2 + (8 \times 10^3 \times 5)^2}$$

$$= 40 \times 10^3 \sqrt{2}\ \text{kg m/s}$$

Direction of momentum with main road

$$\tan\theta = \frac{p_L}{p_C} = \frac{8 \times 10^3 \times 5}{40 \times 10^3} = 1 \quad \therefore \; \theta = \mathbf{45^\circ}$$

FIGURE 6(CF).26



According to the law of conservation of linear momentum,

$$10^3 \times 40\sqrt{2} = (2 \times 10^3 + 8 \times 10^3) \times V$$

$$V = \mathbf{4\sqrt{2}\ m/s}$$

66. Here, $m = 0{\cdot}18$ kg ; $k = 2$ N/m, $\mu = 0{\cdot}1$, $x = 0{\cdot}06$ m,

According to work energy theorem, we have

$$\frac{1}{2} mv^2 = \mu mg\, x + \frac{1}{2} kx^2$$

$$\therefore \; \frac{1}{2} \times 0{\cdot}18 \times v^2 = 0{\cdot}1 \times 0{\cdot}18 \times 10 \times 0{\cdot}06 + \frac{1}{2} \times 2 \times (0{\cdot}06)^2$$

$$\text{or } v^2 = \frac{144 \times 10^{-4}}{9 \times 10^{-2}} = 16 \times 10^{-2} \text{ or } v = \frac{4}{10}\ \text{m/s}$$

As per question, $\frac{N}{10} = \frac{4}{10}$ or $N = \mathbf{4}$

67. Here, $m = 3$ kg, $v = 4$ m/s
$k = 200\ \text{Nm}^{-1}, x = ?$

The compression will be maximum, when

Gain in PE of spring = Loss in total KE of rolling cylinder

$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{1}{2} mr^2\right)\omega^2 = \frac{1}{2} mv^2 + \frac{1}{4} mv^2$$

$$\frac{1}{2} kx^2 = \frac{3}{4} mv^2$$

$$x = \sqrt{\frac{3}{2}\frac{m}{k}} \times v = \sqrt{\frac{3}{2} \times \frac{3}{200}} \times 4 = \frac{3 \times 4}{20}$$

$$x = \mathbf{0{\cdot}6\ m}$$

68. Here, mass of $A = m_1$; mass of $B = m_2$
Initial velocity of A ; $u_1 = 0$
Initial velocity of B, $u_2 = v$, along x-axis.

HINTS/EXPLANATIONS....

For Difficult Questions

After collision, let final velocity of A along x-axis be v_x and final velocity of A along y-axis be v_y. Final velocity of $B = \dfrac{v}{2}$ along Y-axis.

Applying principle of conservation of linear momentum along X-axis,

$$m_1 \times 0 + m_2\, v = m_1\, v_x + m_2 \times 0$$

$$v_x = \frac{m_2\, v}{m_1} \qquad \text{...}(i)$$

Applying principle of conservation of linear momentum along Y-axis,

$$m_1 \times 0 + m_2 \times 0 = m_1\, v_y + m_2\left(\frac{v}{2}\right)$$

or $$v_y = -\frac{m_2\, v}{2\, m_1} \qquad \text{...}(ii)$$

If mass A moves at angle θ with the X-axis, then

$$\tan\,\theta = \frac{v_y}{v_x} = -\frac{m_2\, v}{2\, m_1} \times \frac{m_1}{m_2\, v} = -\frac{1}{2}$$

$$\therefore \qquad \theta = \tan^{-1}\left(-\frac{1}{2}\right) \textbf{ to the } x\textbf{-axis.}$$

69. Here, $m_1 = m_2 = m$, $m_3 = 4\,m - 2\,m = 2\,m$, $p_1 = p_2 = mv$. As these pieces move perpendicular to eachother, their resultant momentum

$$= p = \sqrt{p_1^2 + p_2^2} = \sqrt{(mv)^2 + (mv)^2} = mv\sqrt{2}$$

Principle of conservation of linear momentum gives $p_3 = m_3 v_3 = p = mv\sqrt{2}$

$$v_3 = \frac{mv\sqrt{2}}{m_3} = \frac{mv\sqrt{2}}{2\,m} = \frac{v}{\sqrt{2}}$$

$$\text{Total K.E.} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} mv^2 + \frac{1}{2}\,(2m)\left(\frac{v}{\sqrt{2}}\right)^2$$

$$E = \frac{3}{2}\, mv^2$$

70. If v' is combined velocity of the three masses after inelastic collision, then from principle of conservation of linear momentum,

$$m\,v = (2\,m + M)\,v' \quad \text{or} \quad v' = \frac{m\,v}{2\,m + M}$$

$$\text{Initial energy, } E_1 = \frac{1}{2}\, mv^2$$

$$\text{Final energy, } E_2 = \frac{1}{2}\,(2\,m + M)\,v'^2$$

$$= \frac{1}{2}\,(2\,m + M)\left(\frac{mv}{2\,m + M}\right)^2 = \frac{m^2\,v^2}{2\,(2\,m + M)}$$

$$\text{Loss of energy} = E_1 - E_2 = \frac{5}{6} \text{ of initial KE}$$

$$\frac{1}{2}\, mv^2 - \frac{m^2\,v^2}{2\,(m + M)} = \frac{5}{6}\left(\frac{1}{2}\, mv^2\right)$$

on solving, we get $\dfrac{M}{m} = \mathbf{4}$

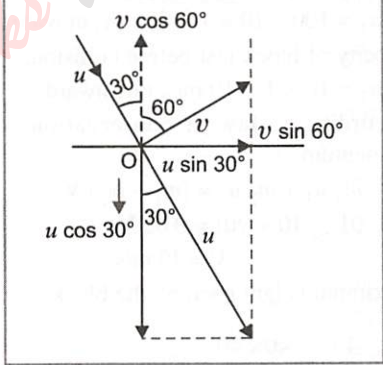
71. Here, $F = ax + bx^2$

$$dW = F\,dx,\; W = \int_0^L F\,dx = \int_0^L (ax + bx^2)\,dx$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^L = \left(\frac{\mathbf{a}L^2}{\mathbf{2}} + \frac{\mathbf{b}L^3}{\mathbf{3}}\right)$$

72. In Fig. 6(CF).27, we have shown initial velocity of hail storm (before impact) = u at 30° with vertical. Its components are u cos 30° along vertical and u sin 30° along the horizontal.

FIGURE 6(CF).27



After impact, velocity of hail storm is v at 60° with vertical. Its components are v cos 60° along vertical and v sin 60° along the horizontal.

As the surface of frozen lake is smooth, the horizontal component remains unaffected

$$\therefore \qquad v \sin 60^\circ = u \sin 30^\circ \qquad \text{...}(i)$$

The vertical component changes from u cos 30° to v cos 60°. If e is coefficient of restitution, then

$$v \cos 60^\circ = e\, u \cos 30^\circ \qquad \text{...}(ii)$$

Divide (ii) by (i),

$$\frac{e\,u \cos 30^\circ}{u \sin 30^\circ} = \frac{v \cos 60^\circ}{v \sin 60^\circ} \quad \text{or} \quad e \cot 30^\circ = \cot 60^\circ$$

$$e = \frac{\cot 60^\circ}{\cot 30^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{\mathbf{1}}{\mathbf{3}}$$

HINTS/EXPLANATIONS

For Difficult Questions

73. Here, final total KE is 50% greater than the original KE. Therefore, it is a case of super elastic collision.

If v_1, v_2 are velocities of two particles after collision, then from principle of conservation of linear momentum,

$$m\,v_1 + m\,v_2 = m\,v_0$$

or $$v_1 + v_2 = v_0 \qquad ...(i)$$

Applying principle of conservation of energy,

$$\frac{1}{2}m\,(v_1^2+v_2^2) = \frac{3}{2}\left(\frac{1}{2}m v_0^2\right)$$

$$v_1^2+v_2^2 = \frac{3}{2}v_0^2 \qquad ...(ii)$$

Now, $(v_1+v_2)^2 = v_1^2+v_2^2+2\,v_1\,v_2$

or $$v_0^2 = \frac{3}{2}v_0^2 + 2\,v_1\,v_2$$

or $$2\,v_1\,v_2 = v_0^2 - \frac{3}{2}v_0^2 = -\frac{v_0^2}{2}$$

Again, $(v_1-v_2)^2 = (v_1+v_2)^2 - 4\,v_1\,v_2$

$$= v_0^2 + v_0^2 = 2\,v_0^2$$

$$\therefore \quad v_1 - v_2 = \sqrt{2v_0^2} = v_0\sqrt{2}$$

74. Fractional loss of KE of body A

$$= \frac{4m_1 m_2}{(m_1+m_2)^2} = \frac{4(4\,\text{m})(2\,\text{m})}{(4\,\text{m}+2\,\text{m})^2} = \frac{32\,\text{m}^2}{36\,\text{m}^2} = \frac{8}{9}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

75. Initial linear momentum $= p\,\hat{i} + (-p\,\hat{i}) = 0$.

According to law of conservation of momentum, the final momentum should be zero.

For option (*a*)

$$\vec{p}_1{}' + \vec{p}_2{}' = (a_1+a_2)\,\hat{i} + (b_1+b_2)\,\hat{j} + c_1\,\hat{k}$$

The term $c_1\hat{k}$ cannot be zero

For option (*d*) $\vec{p}_1{}' + \vec{p}_2{}' = (a_1+a_2)\,\hat{i} + 2b_1\,\hat{j}$

Here, **term $2b_1\,\hat{j}$ cannot be zero.**

Therefore, **choices (*a*) and (*d*) are not allowed.**

76. According to work energy theorem

Work $= K_f - K_i$, when work done is positive.

This shows $K_f > K_i$. Hence, **option (*a*).**

When KE increases it means $v > u$. So $p_f > p_i$

Hence, **option (*d*).**

77. As force is always perpendicular to velocity, type of **motion is circular motion.** Also, as in uniform circular motion speed is always constant, therefore, KE is constant.

78. As work done $W = \vec{F}.\vec{S}, \vec{S}$ does not depend upon path, so work done does not depend on the path taken by the suitcase. Also, it does not depend on the time.

79. Here, $\theta = 90°$, $W = Fs\cos\theta = Fs\cos 90° = 0$

So work done is zero.

In uniform circular motion, **speed is always a constant** but velocity changes as direction changes.

If speed is constant, **then KE is also constant.**

80. Refer to Fig. 6(CF).28,

total momentum before collision

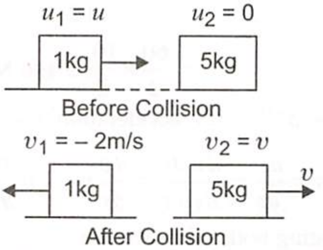
$= m_1\,u_1 + m_2\,u_2 = 1\times u + 5\times 0 = u$

Total momentum after collision

$= m_1\,v_1 + m_2\,v_2 = 1\times(-2) + 5\times v$

$= 5\,v - 2$

FIGURE 6(CF).28



According to law of conservation of linear momentum

$$u = 5\,v - 2 \qquad ...(i)$$

According to law of conservation of K.E.

$$\frac{1}{2}m_1\,u_1^2 + \frac{1}{2}m_2\,u_2^2 = \frac{1}{2}m_1\,v_1^2 + \frac{1}{2}m_2\,v_2^2$$

or $$\frac{1}{2}\times 1\times u^2 + \frac{1}{2}\times 5\times 0 = \frac{1}{2}\times 1\times(-2)^2 + \frac{1}{2}\times 5\,v^2$$

or $$u^2 = 4 + 5\,v^2 \qquad ...(ii)$$

HINTS/EXPLANATIONS

For Difficult Questions

Solving (*i*) and (*ii*), we get $v = 1$ m/s
and $u = 3$ m/s.

Total momentum of the system before collision

$$= \mathbf{3\ kg\ m/s.}$$

Momentum of 5 kg mass after collision $= m_2 v_2$

$$= 5 \times 1 = 5 \text{ kg ms}^{-1}$$

$$\text{Now, } v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{1(-2) + 5(1)}{1+5} = \frac{1}{2} \text{ m/s}$$

$$\text{K.E. of c.m} = \frac{1}{2}(m_1 + m_2) v_{cm}^2$$

$$= \frac{1}{2}(1+5)\left(\frac{1}{2}\right)^2 = \mathbf{0{\cdot}75\ J}$$

$$\text{Total K.E. of the system} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} \times 1 \times (-2)^2 + \frac{1}{2} \times 5 \times 1^2$$

$$= 2 + 2{\cdot}5 = 4{\cdot}5 \text{ J}$$

Choices (*a*) and (*c*) are correct.

81. As is clear from Fig. 6(CF).16, components of momentum perpendicular to wall cancel out, and components along the wall add. As

$F \times t$ = change in momentum

$$= 2\, m\, v \cos\theta$$

$$\therefore\ F = \frac{2 m v \cos\theta}{t} = \frac{2 \times 3 \times 10 \cos 60^\circ}{0{\cdot}2}$$

$$= \frac{60}{2} \times \frac{10}{2} = \mathbf{150\ N}$$

82. As $F \propto S^{-1/3}$ $\therefore$ acceleration, $a \propto S^{-1/3}$

$$\text{Now } a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds} v \quad \therefore\ v\frac{dv}{ds} \propto S^{-1/3}$$

Integrating both sides, we find

$$v^2 \propto S^{2/3} \quad \text{or} \quad v \propto S^{1/3}$$

As $P = F v$ $\therefore$ $P \propto (S^{-1/3})(S^{1/3}) = \mathbf{S^0}$

i.e. power is independent of *S*.

83. When a particle undergoes normal collision with a floor or a wall, with coefficient of restitution *e*, the speed after collision is *e* times the speed before collision. Therefore, change in momentum after first impact $= ep - (-p) = p(1+e)$

After the second impact, change in momentum would be $e(ep) - (-ep) = ep(1+e)$ and so on.

Therefore, total change in momentum of ball

= momentum imparted to floor

$$= p(1+e)[1 + e + e^2 + \ldots]$$

$$= p(1+e)\left[\frac{1}{1-e}\right] = \mathbf{\frac{p(1+e)}{(1-e)}}$$

84. $W = F s \cos\theta = 0$, when either $s = 0$ or $\theta = 90^\circ$ *i.e.* when object is stationary but the point of application of the force moves on the object or object moves in such a way that point of application of force remains fixed ; or force is at 90° to the acceleration. **Choices (*a*), (*b*), (*d*) are correct.**

85. When force of constant magnitude is perpendicular to the velocity of the particle, motion is **along a circular path.** Direction of velocity/acc. goes on changing, but **KE** $= \frac{1}{2} m v^2$ **= constant.**

Choices (*a*) and (*d*) are correct.

86. Refer to Fig. 6(CF).29. In the journey over the upper half of incline, $v^2 - u^2 = 2as$

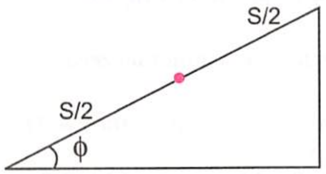
$$v^2 - 0 = 2(g \sin\phi)\frac{s}{2} = g \sin\phi \,.\, s$$

In the journey over the lower half of incline

$$v^2 - u^2 = 2 \text{ as}$$

$$0 - g \sin\phi \,.\, s = 2g(\sin\phi - \mu\cos\phi)\frac{s}{2}$$

FIGURE 6(CF).29



$$-\sin\phi = \sin\phi - \mu\cos\phi$$

$$\mu = \frac{2\sin\phi}{\cos\phi} = \mathbf{2\tan\phi}$$

87. From $s = ut + \frac{1}{2}at^2$, on smooth incline,

$$d = 0 + \frac{1}{2}(g \sin 45^\circ)\, t^2 \qquad \ldots(i)$$

On rough incline,

$$d = 0 + \frac{1}{2} g\,(\sin 45^\circ - \mu_k \cos 45^\circ)(nt)^2$$

From (*i*) and (*ii*),

$$\frac{1}{2} g(\sin 45^\circ - \mu_k \cos 45^\circ) n^2 t^2 = \frac{1}{2} g \sin 45^\circ\, t^2$$

$$(1 - \mu_k)\frac{n^2}{\sqrt{2}} = \frac{1}{\sqrt{2}} \qquad \mu_k = \mathbf{1 - \frac{1}{n^2}}$$

HINTS/EXPLANATIONS

For Difficult Questions

Note that μ_s is coefficient of static friction, μ_k is coefficient of kinetic (sliding) friction.

88. When a body collides with a finite body at rest, both the bodies cannot come to rest. Neither the stationary body can remain stationary as momentum of the two bodies is shared and conserved. **Choices (*a*) and (*d*) are correct.**

89. Here, $\frac{dK}{dt} = \gamma t$

As $K = \frac{1}{2} m v^2$

$$\therefore \frac{dK}{dt} = \frac{1}{2} m (2v) \frac{dv}{dt} = m v \frac{dv}{dt} = \gamma t$$

$$\therefore m \int_0^v v \, dv = \int_0^t \gamma t \, dt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2} \quad \text{or} \quad v = \sqrt{\frac{\gamma}{m}}\, t$$

As acceleration, $a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} = \text{const.}$

$$F = ma = m \sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{const.}$$

Choices (*a*, *b*, *d*) are correct.

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

90. Resisting force opposes the applied force. Box moves at 180° to the resisting force.

$\therefore$ $W = F \cos\theta = 100 \times 4 \cos 180° = \mathbf{-400\ J}$

91. Force applied = tension in coupling = 2000 N

As $P = F v \cos\theta$

$\therefore$ $P = 2000 \times 20 \cos 0° = 40000\ \text{W} = \mathbf{40\ kW}$

92. Here, $m = 1$ kg $u = 0$, $t = 5$ s, $p = ?$

$v = u + at$

$v = 0 + 10 \times 5 = 50$ m/s

$p = m v = 1 \times 50 = \mathbf{50\ kg\ m\ s^{-1}}$

93. $m\,gh$ = K.E. = 1250

$$h = \frac{1250}{m g} = \frac{1250}{1 \times 10} = \mathbf{125\ m}$$

94. When h is half the original height, P.E. = $m\,gh$ also becomes half. The other half of total energy appears as kinetic energy. Therefore, **PE/KE = 1**

IV. Matching Type Questions

95. $W = F s \cos\theta$ = max., when $\theta = 0°$

KE $= \frac{1}{2} mv^2$, becomes 4 times when v is doubled.

As $k \propto \frac{1}{l}$, therefore k becomes twice when l is halved.

PE $= \frac{1}{2} kx^2$ = +, when x is positive or negative.

Choice (*a*) is correct.

96. As K.E. $= \frac{1}{2} mv^2$, therefore, it can never be negative. In uniform circular motion, K.E. $= \frac{1}{2} mv^2$ = constant.

When a shot fired from a gun explodes in air, its linear momentum remains constant, but its K.E. increases. When $p \to 2p$, K.E. $= \frac{p^2}{2m}$ becomes 4 times *i.e.* K.E. increases by 300%. **Choice (*d*) is correct.**

V. Matrix-Match Type Questions

97. (A-*p*,*q*,*r* ; B-*p*,*q*,*r* ; C-*q*,*r* ; D-*p*,*r*)

Magnetic potential energy $= \frac{\mu_0}{4\pi} \times \frac{m_1 m_2}{r}$

It may be negative, positive or even zero.

Electrostatic potential energy $= \frac{q_1 q_2}{4\pi\epsilon_0 r}$

It can also be positive, negative or zero.

Elastic potential energy $= \frac{1}{2} Kx^2$

It can be positive or zero.

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Gravitational potential energy $= -\frac{G m_1 m_2}{r}$

It can be negative or zero.

98. (A-*r* ; B-*p,s* ; C-*p,q* ; D-*p,q*)

In a perfectly elastic collision, coeff. of restitution, $e = 1$.

In a perfectly inelastic collision, $e = 0$ and some K.E. is lost.

In both, partially elastic or partially inelastic collision ; some K.E. is lost, and $0 < e < 1$.

99. (*A-p,q,r,t ; B-q,s ; C-p,q,r,s ; D-p,r,s*)

(A), $U = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$

$$F = -\frac{dU}{dx} = -\frac{U_0}{2}.2\left[1-\left(\frac{x}{a}\right)^2\right]\times\left(-\frac{2x}{a^2}\right)$$

$$= \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]\frac{2x}{a^2}$$

If $x = 0$, $F = 0$;

If $1-\left(\frac{x}{a}\right)^2 = 0$, $\left(\frac{x}{a}\right)^2 = 1$ or $\frac{x}{a} = \pm 1$ or $x = \pm a$

When $U_0 = \frac{U_0}{4}$ then $F = \frac{U_0/4}{2}\left[1-\left(\frac{x}{a}\right)^2\right]\frac{2x}{a^2}$

$F \propto x$ and if $x = -a$, then F is directed towards equilibrium position, *i.e.*, the particle will oscillate. Hence option T is true. Therefore, (A) → *p, q, r, t* is satisfied

(B) $U = \frac{U_0}{2}\left[\frac{x}{a}\right]^2$;

$$F = -\frac{dU}{dx} = -\frac{U_0}{2a^2}\,.\,2x = -\frac{U_0}{a^2}x$$

If $x = 0$, $F = 0$.

In the above relation of force the negative sign shows that the force is attractive. Thus, option (B) → *q, s* is satisfied.

(C) $U = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 e^{-\left(\frac{x}{a}\right)^2}$

$$F = -\frac{dU}{dx} = \left[\frac{U_0}{2}\left(\frac{x}{a}\right)^2 e^{-\left(\frac{x}{a}\right)^2}\times\left(-\frac{2x}{a^2}\right) + e^{-\left(\frac{x}{a}\right)^2}\times\frac{U_0}{2}\times\frac{2x}{a^2}\right]$$

$$= -\frac{U_0}{2}\frac{2x}{a^2}\left[-\left(\frac{x}{a}\right)^2 e^{-\left(\frac{x}{a}\right)^2} + e^{-\left(\frac{x}{a}\right)^2}\right]$$

$$= -U_0\frac{x}{a^2}\left[1-\left(\frac{x}{a}\right)^2\right]e^{-\left(\frac{x}{a}\right)^2}$$

$$= -U_0\frac{x}{a^2}e^{-\left(\frac{x}{a}\right)^2}\left[1-\left(\frac{x}{a}\right)^2\right]$$

Here, $F = 0$, when $x = 0$, $x = \pm a$. Also F is negative for $x < a$. Thus, option (C) → ***p, q, r, s* is satisfied.**

(D) $U = \frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$

$$F = -\frac{dU}{dx} = -\frac{U_0}{2}\left[\frac{1}{a}-\frac{1}{3}\frac{3x^2}{a^3}\right]$$

$$= -\frac{U_0}{2a}\left[1-\frac{x^2}{a^2}\right]$$

Here, $F = 0$, When $x = \pm a$. Also F is negative for $x < a$. Thus, option (D) → ***p, r, s* is satisfied.**

VI. Integer Type Questions

100. Let m be the mass of bullet and v be its initial velocity. If F is constant resistance to motion, then

(*i*) Work done = loss in KE = force × distance

$$= \frac{1}{2}m\left(v^2-\frac{v^2}{4}\right) = F\times 3$$

or $$\frac{1}{2}m\left(\frac{3v^2}{4}\right) = F\times 3 \qquad \textbf{...(i)}$$

(*ii*) Work done = loss in KE = force × distance

$$= \frac{1}{2}m\left(\frac{v^2}{4}-0\right) = F\times s \qquad \textbf{...(ii)}$$

Dividing (*ii*) by (*i*), we get

$$\frac{1}{3} = \frac{s}{3}$$

∴ **s = 1 cm**

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101. When speed is maximum, KE is maximum. PE is minimum.

As $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J = minimum,

$$\therefore \quad \frac{dV(x)}{dx} = 0$$

$$4\frac{x^3}{4} - \frac{2x}{2} = 0 \quad \therefore x = 0 \text{ or } x = \pm 1$$

$$V_{\min} \text{ (at } x = \pm 1) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} = -0{\cdot}25 \text{ J}$$

As $K_{\max} + V_{\min}$ = Mechanical energy

$$\therefore \quad K_{\max} = ME - V_{\min}$$

$$\frac{1}{2}mv_{\max}^2 = 0{\cdot}75 - (-0{\cdot}25) = 1 \text{ J}$$

$$v_{\max} = \sqrt{\frac{2\times 1}{m}} = \sqrt{\frac{2}{2}} = \mathbf{1\,m/s}$$

102. Force of friction between the two blocks, Fig. 6(CF).17,

$$f = \mu R = \mu mg = 0{\cdot}1 \times 1 \times 10 = 1 \text{ N}$$

As applied force $F = 0{\cdot}5 \text{ N} < f$.

∴ The two blocks move together.

Common acceleration,

$$a = \frac{F}{1+2} = \frac{0{\cdot}5}{3} = \frac{1}{6} \text{ m/s}^2$$

∴ Force applied by upper block on lower block

$$F' = ma = 2a = 2\times\frac{1}{6} = \frac{1}{3}\text{N}$$

Work done by upper block on lower block for displacement of 3 m = $F\,3' \times s$

$$= \frac{1}{3}\times 3 = \mathbf{1\,J}$$

103. As the given graph is linear, Fig. 6(CF).18,

∴ $a = \alpha x$, where α is slope of graph

$$\alpha = \frac{a}{x} = \frac{20}{8} = 2{\cdot}5\,\text{s}^{-2}$$

Now, $F = ma = m\,\alpha x$

$$W = \int_0^8 F\,dx = \int_0^8 m\,\alpha x\,dx = m\,\alpha\left[\frac{x^2}{2}\right]_0^8$$

$$= \frac{2{\cdot}5}{10}\left(\frac{64-0}{2}\right) = \mathbf{8\,J}$$

104. Here, $m_1 = m$, $m_2 = 4$ m

If v_1, v_2 are their respective velocities, then as

$$p_1 = p_2$$

$$m_1 v_1 = m_2 v_2$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = 4\ ;$$

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}m_1 v_1^2}{\frac{1}{2}m_2 v_2^2} = \frac{1}{4}\left(\frac{4}{1}\right)^2 = \mathbf{4}$$

105. As is clear from Fig. 6(CF).30,

$$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

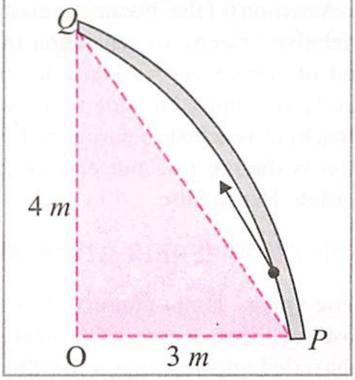
Work done in taking the block from P to Q along elliptically shaped rail,

$$W = F \times PQ = 18 \times 5 = 90 \text{ J}$$

Potential energy of block at $Q = +\,mgh$

$$= +\,1 \times 10 \times 4 = +\,40 \text{ J}$$

FIGURE 6(CF).30



∴ K.E. at $Q = W -$ P.E. $= 90 - 40 = \mathbf{50\ J} = 10\,n$

∴ $\quad n = \mathbf{5}$

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VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

106. Both, the Assertion and Reason are true. The Reason is correct explanation of the Assertion.

107. Here, Assertion is true, but the reason is false.

According to law of conservation of mechanical energy (If no external forces do work on a system), for internal conservative forces, $K + U =$ constant or $\Delta K + \Delta U = 0$ or $\Delta K = -\Delta U$

108. In a quick collision, time t is small. As $F \times t =$ constant, therefore, force involved is large, *i.e.*, collision is more violent. Both, the assertion and reason are true and the latter is correct explanation of the former.

109. Frictional force always opposes relative motion. Therefore, work done by the frictional force is negative. Assertion is true, reason is false.

110. Both, the assertion and reason are true and the latter is correct explanation of the former.

111. Both, assertion and reason are true, but the latter is not a correct explanation of the former.

112. Work done on the spring is stored in the spring in the form of potential energy as restoring force is always opposite to applied force.

113. When a cell supplies current, chemical energy of cell is converted into electrical energy. Conversion to heat is partial. Choice (*d*) is correct.

114. Here Assertion is false, because in elastic collision the relative velocity of separation (not relative speed of separation) is equal to the relative velocity of approach (not relative speed of approach) of two bodies during collision. Hence reason is though true but cannot explain the Assertion. Hence false.

FOR ENGINEERING STUDENTS

115. Kinetic energy is not conserved in a perfectly inelastic collision, as some of the total energy may be converted into heat energy etc. Statement-1 is false, but statement-2 is true.

116. For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision, $e = 0$. Therefore, e lies between 0 and 1. Choice (*a*) is correct.

117. Statement-1 is false. Negative energy means,

$$K + U = \text{negative}$$

$$\therefore \quad K \neq 0.$$

Therefore, the body can have linear momentum. Statement-2 is false. Choice (*c*) is correct.

118. From $E = mc^2$,

when $m = 1$ microgram $= 10^{-6}$ g $= 10^{-9}$ kg

$$E = 10^{-9}(3 \times 10^8)^2$$
$$= 9 \times 10^7 \text{ J}$$

The statement-1 is true, but the statement-2 is certainly false. Choice (*c*) is correct.

119. Let x_1 be extension produced in S_1 and x_2 be extension produced in S_2, when two springs are stretched by the same force.

$$\therefore \quad F = K_1 x_1 = K_2 x_2$$

$$W_1 = F x_1 \quad \text{and} \quad W_2 = F x_2$$

As $W_1 > W_2$, $F x_1 > F x_2$ or $x_1 > x_2$

$$\therefore \quad K_1 < K_2$$

Statement 2 is true.

When two springs are stretched by the same amount, *i.e.*, $x_1 = x_2 = x$

$$W_1 = \frac{1}{2} K_1 x^2 \quad \text{and} \quad W_2 = \frac{1}{2} K_2 x^2$$

As $K_1 < K_2$, $W_1 < W_2$.

Statement 1 is false.

120. Both the statements are true. But statement 2 is not a correct explanation of statement 1.

121. Statement-2 is true, from the knowledge of theory.

$$\text{Now, maximum energy loss} = \frac{p^2}{2m} - \frac{p^2}{2(M+m)}$$

$$= \frac{p^2 M + p^2 m - p^2 m}{2m(M+m)} = M \frac{p^2}{2m(M+m)}$$

$$= \left(\frac{1}{2} m v^2\right) \frac{M}{(M+m)} = f\left(\frac{1}{2} m v^2\right),$$

$$\text{where} \quad f = \frac{M}{M+m}$$

$\therefore$ Statement-1 is false.

Choice (*d*) is correct.

UNIT-5

SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

CHAPTER 7.

SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

7

SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

7.1. INTRODUCTION

So far, we have studied the motion of a single particle, treated as a point mass having practically no size. We may define **a particle** *as an object which has a finite mass, but whose size and internal structure can be ignored.*

Any real body which we come across in daily life has a finite size. Such an extended body consists of a system of particles. We may define **a system** as a collection of very large number of particles, which mutually interact with one another.

For the sake of simplicity, we shall consider the extended bodies as *rigid bodies.* ***Ideally, a rigid body is that which has a definite size and definite shape. This is because distances between different pairs of particles of such a body do not change on applying any force on it.*** However, no real body is truely rigid as real bodies deform under the action of forces applied on them. But in many situations, the deformations are often negligible.

For describing the motion of rigid bodies, we shall introduce the key concept of **'centre of mass'.** This concept enables us to understand how we can apply justifiably the Newton's laws of motion, in essentially the same form to objects of large size including even the astronomical objects like the planets and the stars.

7.2. THE CONCEPT OF A SYSTEM, INTERNAL FORCES AND EXTERNAL FORCES

Every body of finite size is made up of a very large number of particles. It is almost impossible to describe the positions and velocities of all the particles individually. We have, therefore, to devise a method for describing the overall motion of the body. As stated already *collection of any number of particles interacting with one another is said to form a system.* Note that 'particles interacting with one another' means that the particles are exerting forces on one another. These forces alone enable them to form a well defined system. Thus *any object of finite size can be regarded as a system.*

All the forces exerted by various particles of the system on one another are called **internal forces.** Note that internal forces between any two particles of the system are mutual *i.e.* internal forces between a pair of particles are equal and opposite. Hence *such forces cancel out in pairs.*

To move or stop an object of finite size, we have to apply a force on the object from outside. This force exerted on a given system by the agencies outside the system is called an **external force.** The overall motion of a body is affected by external forces only.

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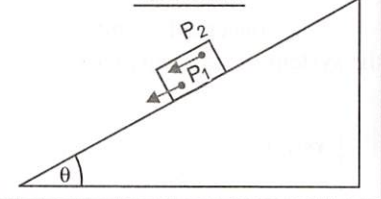
7.3. KINDS OF MOTION OF A RIGID BODY

A rigid body may have three kinds of motion :

1. Pure Translational Motion
2. Pure Rotational Motion
3. Combination of Translational and Rotational Motion.

FIGURE 7.1



(*a*) **Pure Translation Motion**

In such a motion, every particle of the body travels the same linear distance in a given time interval. Fig. 7.1 shows pure translational motion of a block down an inclined plane.

Further, if paths of particles are straight lines, the motion is said to be *rectilinear translation*, and if the paths are curved lines, the motion is said to be *curvilinear translation.*

(*b*) **Pure Rotational Motion**

In such a motion, *a rigid body rotates about a fixed axis. Every particle of the body moves in a circle, which lies in a plane perpendicular to the axis, and has its centre on the axis.*

FIGURE 7.2

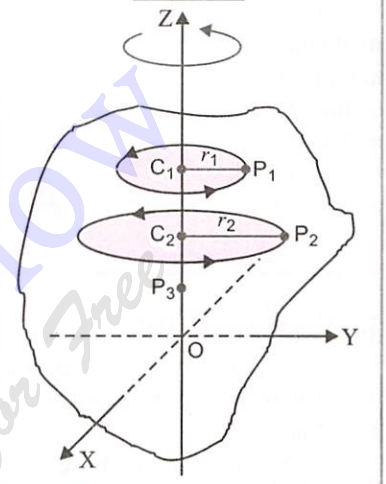


Fig. 7.2, shows the rotational motion of a rigid body about a fixed axis, the Z-axis. The body is in XY plane. A particle P_1 of the body describes a circle of radius r_1 with centre at C_1 on the Z-axis. Another particle P_2 of the body describes a circle of radius r_2 with centre at C_2 lying on the same Z-axis. The circles described by P_1 and P_2 may lie in different planes, but both these planes are perpendicular to the axis of rotation. For any particle P_3 on the axis of rotation, $r = 0$. Therefore, such a particle remains stationary, while the body rotates. *Note that angular velocity* ω *of all the particles is the same irrespective of their distance from the axis of rotation.* This is because the body being rigid, all particles complete together one revolution about the given axis. However, their *linear velocities are different* ($v = r\,\omega$) depending on their distance (r) from the axis.

Some of the examples of pure rotation about an axis are : the motion of a ceiling fan, a potter's wheel, a giant wheel in a circus, a merry-go-round and so on.

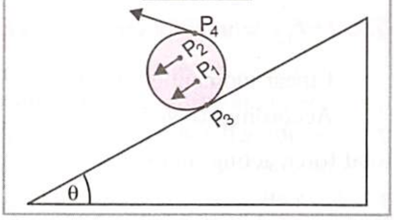
In *more general cases of rotation, one point and not one line* of the rigid body is fixed.

For example, in an oscillating table fan or pedestal fan, the axis of rotation is horizontal. This axis has an oscillating sideways movement in a horizontal plane about the vertical through the point at which the axis is pivoted.

(*c*) **Combination of translational and rotational motion**

For example, when a cylinder rolls down an inclined plane, its motion is a combination of translation and rotation about a fixed axis. As the cylinder shifts from top to the bottom of inclined plane, the points P_1, P_2, P_3, P_4 on the rolling cylinder have different velocities at a particular instant of time, Fig. 7.3. If the cylinder were to roll without slipping, the velocity of the point of contact P_3 would be zero, at any instant of time.

FIGURE 7.3



From the above discussion, we conclude that : (*i*) *The motion of a rigid body, which is not pivoted or fixed in some way is either a pure translation or a combination of translation and rotation.*

(*ii*) *The motion of a rigid body, which is pivoted or fixed in some way is rotation.*

The rotation may be about an axis which is fixed *e.g.* in a ceiling fan or about an axis which is moving *e.g.*, in an oscillating table fan. We shall confine ourselves to rotation about a fixed axis.

DO YOU KNOW ?

The combination of rotational motion and the spin motion about a given axis of rotation is called **Precession.** For example, the motion of a spinning top represents precession.

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7.4. CONCEPT OF CENTRE OF MASS

The concept of centre of mass of a system enables us to discuss overall motion of the system by replacing the system by an *equivalent single point object*. We may define

> ***Centre of mass** of a body or a system of bodies as a point at which the entire mass of the body/system of bodies, is supposed to be concentrated. If all the external forces acting on the body/system of bodies were to be applied at the centre of mass, the state of rest/motion of the body/system of bodies, shall remain unaffected.*

It should be clearly understood that the centre of mass of the given system is only a point defined mathematically for the sake of convenience. *It is not at all necessary that the total mass of the system be actually present at the centre of mass.* For example, centre of mass of a uniform circular ring is at the centre of the ring, where there is no mass. Similarly, there is no dough at the centre of mass of a doughnut and no iron at the centre of mass of a horse shoe. The position of the centre of mass of the system at any time is calculated using the usual Newtonian type of equations of motion. This will be clear from the articles that follow.

7.5. CENTRE OF MASS OF A TWO PARTICLE SYSTEM

Let O be the origin of a rectangular co-ordinate system XYZ, Fig. 7.4. Consider a system of two particles of masses m_1 and m_2 located at A and B respectively,

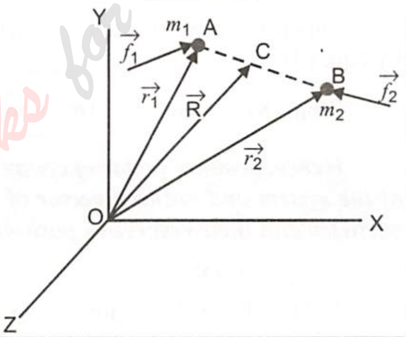
where $\overrightarrow{OA} = \vec{r_1}$ and $\overrightarrow{OB} = \vec{r_2}$

Let C be the position of centre of mass of the system of two particles. It would lie on the line joining A and B. Let $\overrightarrow{OC} = \vec{R}$ be the position vector of centre of mass.

To evaluate $\vec{R}$, suppose $\vec{v_1}, \vec{v_2}$ are velocities of particles m_1 and m_2 respectively at any instant t. Then

$$\vec{v_1} = \frac{d\vec{r_1}}{dt} \quad \text{and} \quad \vec{v_2} = \frac{d\vec{r_2}}{dt} \qquad ...(1)$$

FIGURE 7.4



Let $\vec{f_1}$ = external force applied on particle of mass m_1, $\vec{f_2}$ = external force applied on particle of mass m_2

$\vec{F_{12}}$ = internal force on m_1 due to m_2, $\vec{F_{21}}$ = internal force on m_2 due to m_1

Linear momentum of particle m_1 is $\vec{p_1} = m_1 \vec{v_1}$ ...(2)

According to Newton's second law, the rate of change of linear momentum of this particle is equal to total force acting on this particle, which is $(\vec{f_1} + \vec{F_{12}})$

$$\therefore \qquad \frac{d\vec{p_1}}{dt} = \vec{f_1} + \vec{F_{12}}$$

Using (2), $\frac{d}{dt}(m_1 \vec{v_1}) = \vec{f_1} + \vec{F_{12}}$ ...(3)

Similarly, the equation of motion of second particle may be written as $\frac{d}{dt}(m_2 \vec{v_2}) = \vec{f_2} + \vec{F_{21}}$ ...(4)

Adding (3) and (4), we get $\frac{d}{dt}(m_1 \vec{v_1}) + \frac{d}{dt}(m_2 \vec{v_2}) = \vec{f_1} + \vec{F_{12}} + \vec{f_2} + \vec{F_{21}}$

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As $\vec{F}_{12} = -\vec{F}_{21}$ $\therefore$ $\frac{d}{dt}[(m_1 \vec{v}_1) + (m_2 \vec{v}_2)] = \vec{f}_1 + \vec{f}_2 = \vec{f}$ ...(5)

where $\vec{f}$ = total external force on the system of two particles.

Using (1), we get

$$\frac{d}{dt}\left[m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt}\right] = \vec{f} \quad \text{or} \quad \frac{d}{dt}\left[\frac{d}{dt}(m_1 \vec{r}_1 + m_2 \vec{r}_2)\right] = \vec{f} \quad \text{or} \quad \frac{d^2}{dt^2}(m_1 \vec{r}_1 + m_2 \vec{r}_2) = \vec{f}$$

Multiplying numerator and denominator of left side by $(m_1 + m_2)$, we get

$$(m_1 + m_2)\frac{d^2}{dt^2}\frac{(m_1 \vec{r}_1 + m_2 \vec{r}_2)}{(m_1 + m_2)} = \vec{f} \qquad \text{...(6)}$$

When whole mass is concentrated at C, whose position vector is $\vec{R}$, its eqn. of motion can be written as

$$(m_1 + m_2)\frac{d^2 \vec{R}}{dt^2} = \vec{f} \qquad \text{...(7)}$$

Comparing (6) and (7), we get $\boxed{\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}}$ ...(8)

This is the position vector of centre of mass of the two particle system, which is given mathematically by eqn. (8).

From (8), we get $\boxed{(m_1 + m_2)\vec{R} = m_1 \vec{r}_1 + m_2 \vec{r}_2}$ ...(9)

Hence, position vector of centre of mass of a two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to sum of the products of masses of the two particles and their respective position vectors.

DISCUSSION

1. The above derivation shows that with the concept of centre of mass, motion of total mass of the system is described under the effect of external forces only. The internal forces between the particles forming the system cancel out in pairs.

2. Equation (7) describes the motion of total mass $(m_1 + m_2)$ supposed to be concentrated at a single point called the centre of mass of the system. Hence we conclude that motion of the system can be studied either by applying Newton's second law of motion to individual particles of the system *or* by applying Newton's second law of motion to total mass $(m_1 + m_2)$ located at the centre of mass.

3. Equation (8) shows that *position vector of centre of mass can be taken as the weighted average of the position vectors of the two particles, the weights attached being m_1 and m_2 at position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively.*

4. If the centre of mass of two particles of the system were at the origin

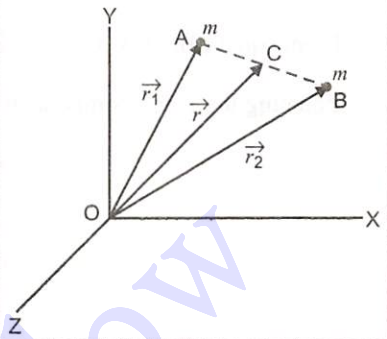
i.e., $\vec{r} = 0$, then from (8), $m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$ or $m_2 \vec{r}_2 = -m_1 \vec{r}_1$

$$\boxed{\vec{r}_2 = -\frac{m_1}{m_2}\vec{r}_1} \qquad \text{...(10)}$$

Eqn. (10) shows *that if $\vec{r}_1$ is negative, $\vec{r}_2$ is positive. It means, if m_1 lies on the left of the origin (c.m), then m_2 lies on the right of the origin (c.m).* ***Hence centre of mass of a system of two particles lies always on the straight line joining these particles.***

Further, if $m_1 > m_2$, then $\vec{r_1} < \vec{r_2}$, *i.e., centre of mass of two particle system lies closer to the heavier particle.* In general, ***the centre of mass divides internally the line joining the two particles in the inverse ratio of masses.***

FIGURE 7.5



5. When the two particles are of equal masses *i.e.,* $m_1 = m_2 = m$ say, then from (9),

$$(m+m)\ \vec{r} = m\vec{r_1} + m\vec{r_2} = m\ (\vec{r_1} + \vec{r_2})$$

$$\vec{r} = \frac{m\ (\vec{r_1} + \vec{r_2})}{2m} = \frac{\vec{r_1} + \vec{r_2}}{2}$$

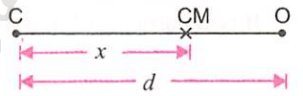
$$\boxed{\vec{r} = \frac{\vec{r_1} + \vec{r_2}}{2}} \qquad \text{...(11)}$$

***i.e., centre of mass of two particles of equal masses lies exactly midway between them as shown in* Fig. 7.5.**

Sample Problem **Find the centre of mass of a carbon monoxide molecule of bond length *d*.**

Sol. We know that $\dfrac{m_c}{m_0} = \dfrac{12}{16} = \dfrac{3}{4}$

FIGURE 7.6



Let the centre of mass of the molecule be at a distance *x* from carbon atom as shown in Fig. 7.6.

As $m_c \times x = m_0\ (d-x)$ $\quad \therefore \quad \dfrac{m_c}{m_0} = \dfrac{d-x}{x} = \dfrac{3}{4}$

$$3\,x = 4\,d - 4x \text{ or } x = \frac{4}{7}\,d$$

7.6. CENTRE OF MASS OF A SYSTEM OF *n* PARTICLES

The results obtained in the previous article are generalised easily to a system of *n* particles–not necessarily lying in a plane, but distributed in space.

If $\vec{r_i}$ be the position vector of *i*th particle and m_i be the mass of the *i*th particle, then generalising eqn. (8), the position vector $\vec{R}$ of the centre of mass would be given as

$$\boxed{\vec{R} = \frac{\sum\limits_{i=1}^{i=n} m_i\,\vec{r_i}}{\sum\limits_{i=1}^{i=n} m_i}} \qquad \text{...(12)}$$

The symbol Σ (the Greek letter, sigma) denotes summation, in this case over *n* particles. The index *i* runs from 1 to *n*. The sum $\sum\limits_{i=1}^{i=n} m_i$ is the total mass of the system of *n* particles. Therefore, from (12),

$$\vec{R} = \frac{\sum\limits_{i=1}^{i=n} m_i\,\vec{r_i}}{M} \qquad \text{...(13)}$$

Note that the sum on the right hand side is a vector sum.

Let x_i, y_i, z_i be the *co-ordinates* of the point, where *i*th particle of the system is located.

$$\therefore \qquad \vec{r}_i = \hat{i}\,x_i + \hat{j}\,y_i + \hat{k}\,z_i \qquad \text{...(14)}$$

If X, Y, Z be the co-ordinates of the centre of mass of the system, we may write

$$\vec{R} = \hat{i}\,X + \hat{j}\,Y + \hat{k}\,Z$$

From eqn. (13), $\hat{i}\,X + \hat{j}\,Y + \hat{k}\,Z = \dfrac{1}{M}\sum_{i=1}^{i=n} m_i\,(\hat{i}\,x_i + \hat{j}\,y_i + \hat{k}\,z_i)$

Equating the x, y, z components on either side, we obtain

$$X = \frac{1}{M}\sum_{i=1}^{i=n} m_i x_i \qquad \text{...(15)}$$

$$Y = \frac{1}{M}\sum_{i=1}^{i=n} m_i y_i \qquad \text{...(16)}$$

$$Z = \frac{1}{M}\sum_{i=1}^{i=n} m_i z_i \qquad \text{...(17)}$$

IMPORTANT NOTE

If two particles of a system lie along X-axis, then $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$; $y_{\text{cm}} = 0$.

If two particles of a system lie along Y-axis, then $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$; $x_{\text{cm}} = 0$.

The number of particles (atoms or molecules) in a rigid body of finite size is so large that it is impossible to carry out the summations over individual particles in these equations.

As the spacing of the particles is small, we can treat the body as a *continuous distribution of mass*. Let us subdivide the body into n small elements of masses Δm_1, Δm_2, ... Δm_n. If the ith element of mass Δm_i is located about the point (x_i, y_i, z_i), the co-ordinates of the centre of mass are approximately given by

$$X = \frac{1}{M}\sum_{i=1}^{i=n} (\Delta m_i)\,x_i \qquad \text{...(18)}$$

$$Y = \frac{1}{M}\sum_{i=1}^{i=n} (\Delta m_i)\,y_i \qquad \text{...(19)}$$

$$Z = \frac{1}{M}\sum_{i=1}^{i=n} (\Delta m_i)\,z_i \qquad \text{...(20)}$$

As we make n bigger and bigger and each Δm_i smaller and smaller, these expressions become exact. In that case, we denote the sums over i by integrals. Thus,

$$\sum (\Delta m_i)\,x_i = \int x\,dm \qquad \text{...(21)}$$

$$\sum (\Delta m_i)\,y_i = \int y\,dm \qquad \text{...(22)}$$

$$\sum (\Delta m_i)\,z_i = \int z\,dm \qquad \text{...(23)}$$

The co-ordinates of the centre of mass now are

$$X = \frac{1}{M}\int x\,dm \qquad ...(24)$$

$$Y = \frac{1}{M}\int y\,dm \qquad ...(25)$$

$$Z = \frac{1}{M}\int z\,dm \qquad ...(26)$$

The vector expression equivalent to these three scalar expressions is

$$\boxed{\vec{R} = \frac{1}{M}\int \vec{r}\,dm} \qquad ...(27)$$

If centre of mass lies at the origin of the co-ordinate system, then

$$\vec{R}\,(x, y, z) = 0$$

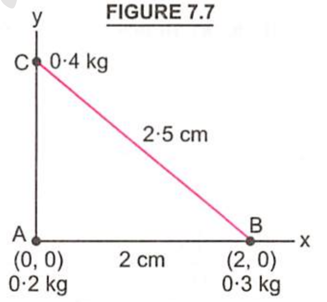
$$\therefore \qquad \int \vec{r}\,dm = 0$$

$$\text{or} \qquad \int x\,dm = \int y\,dm = \int z\,dm = 0 \qquad ...(28)$$

Sample Problem **Three particles of masses 0·2 kg, 0·3 kg and 0·4 kg are situated at the vertices *A*, *B* and *C* of a right angled triangle *ABC* with ∠*A* = 90° ; *AB* = 2 cm along *X*-axis and *BC* = 2·5 cm. Find the distance of centre of mass from *A*.**

Sol. In Fig. 7.7, *A* is taken as origin (0, 0) of co-ordinate system, *AB* is *X*-axis.

FIGURE 7.7



$m_1 = 0{\cdot}2$ kg at *A*, $m_2 = 0{\cdot}3$ kg at *B*, $m_3 = 0{\cdot}4$ kg at *C*

As $AC = \sqrt{BC^2 - AB^2} = \sqrt{(2{\cdot}5)^2 - 2^2} = 1{\cdot}5$ cm

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{0{\cdot}2\times 0 + 0{\cdot}3\times 2 + 0{\cdot}4\,(0)}{0{\cdot}2 + 0{\cdot}3 + 0{\cdot}4} = \frac{2}{3}\text{ cm}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{0{\cdot}2\times 0 + 0{\cdot}3\times 0 + 0{\cdot}4\times 1{\cdot}5}{0{\cdot}2 + 0{\cdot}3 + 0{\cdot}4}$$

$$= \frac{0{\cdot}4\times 1{\cdot}5}{0{\cdot}9} = \frac{2}{3}\text{ cm}$$

The distance of cm from origin *A* is $z_{cm} = \sqrt{x_{cm}^2 + y_{cm}^2} = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \mathbf{\frac{2}{3}\sqrt{2}\ cm}$

7.7. THE CENTRE OF MASS OF RIGID BODIES

A rigid body is composed of many particles. It does not show any change in its configuration on applying a force on it. If a rigid body has an irregular shape or non uniform distribution of mass, its centre of mass can be calculated with the help of integral calculus.

If a body has a symmetrical shape, then the position of its centre of mass lies at the centre of symmetry or on the axis of symmetry. For example :

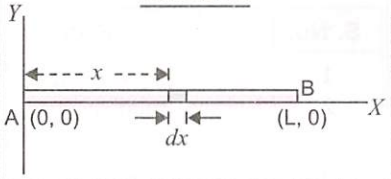
(*i*) in case of a sphere/disc/parallelopiped, the centre of mass lies at the centre of sphere/disc/parallelopiped.

(*ii*) in case of a uniform circular ring, the centre of mass lies at the centre of the ring.

Infact, for all homogeneous rigid bodies the centre of mass coincides with their geometrical centre.

(*a*) Centre of mass of a uniform thin rod.

FIGURE 7.8



Consider a uniform thin rod *AB* of mass *M* and length *L*. It is held along *x*-axis with its end *A* at the origin, where $x = 0$, $y = 0$. Fig. 7.8. Let the rod be made of elements placed side by side. One such element of length *dx* of rod is shown at distance *x* from the origin. Mass of the element,

$$dm = \frac{M}{L}\,dx$$

Let (x_{cm}, y_{cm}) be the coordinates of centre of mass of thin rod from the origin. Then

$$x_{cm} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x\left(\frac{M}{L}\,dx\right) = \frac{1}{L}\int_0^L x\,dx = \frac{1}{L}\left(\frac{x^2}{2}\right)_0^L = \frac{L}{2}$$

and $$y_{cm} = \frac{1}{M}\int y\,dm = 0 \qquad \text{(as } y = 0 \text{ for thin rod)}$$

It means the centre of mass of thin rod *AB* lies at point $(L/2, 0)$, *i.e.*, **midway between its ends *A* and *B*.**

(*b*) Centre of mass of circular ring, disc and sphere

The circular ring, disc and sphere are symmetrical bodies. Each body has a symmetry about a point which is the centre of symmetry, *i.e.*, the centre of mass of respective bodies.

If *r* is the radius of circular ring or disc or sphere as shown in Fig. 7.9(a), the centre of mass lies at *C*, whose coordinates are (*r*; *r*).

If the origin lies at *A* as shown in Fig. 7.9(b), then the centre of mass lies at C_1, whose coordinates are (0, *r*).

If the origin lies at *B*, as shown in Fig. 7.9(c), then the centre of mass lies at C_2, whose coordinates are (*r*, 0).

FIGURE 7.9

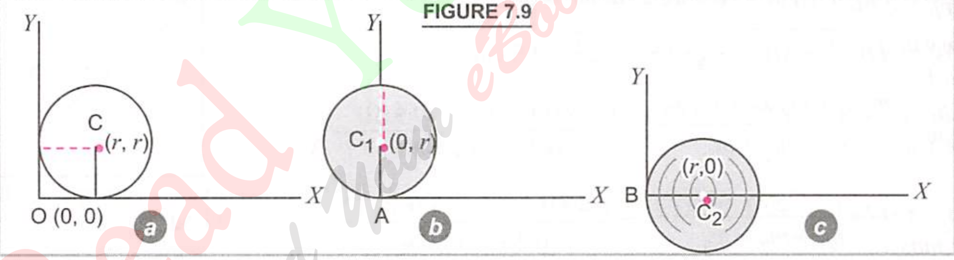
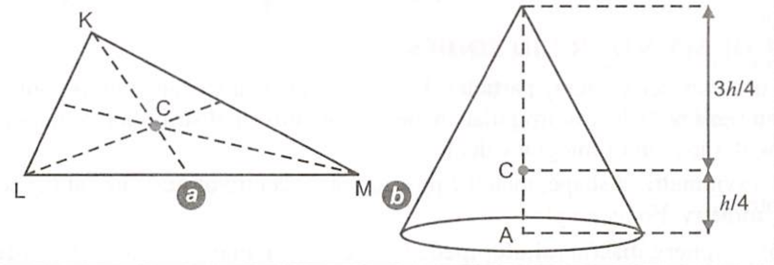


Fig 7.10(a) shows that centre of mass of a uniform triangular lamina is at the centroid* *C* of the triangle, which is the point of intersection of the medians of the triangle.

Fig. 7.10(b) shows the centre of mass *C* of a solid symmetrical cone.

FIGURE 7.10



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*Median is a line joining the vertex of a triangle to the middle point of the opposite side. Every triangle has three medians and their point of intersection is called **'centroid'** of the triangle.

Table 7.1 lists the position of centre of mass of some of the homogeneous bodies of regular shape.

TABLE 7.1. Position of centre of mass of some homogeneous bodies of regular shape

S. No.	Body	Position of Centre of Mass
1.	Uniform hollow sphere	Centre of the sphere
2.	Uniform solid sphere	Centre of the sphere
3.	Uniform circular ring	Centre of the ring
4.	Uniform circular disc.	Centre of the disc
5.	Uniform rod	Centre of the rod
6.	A plane lamina in the form of a square or a rectangle or a parallelogram.	Point of intersection of diagonals.
7.	Triangular plane lamina	Point of intersection of the medians of the triangle., Fig. 7.10(a).
8.	Rectangular or cubical block.	Point of intersection of the diagonals.
9.	Hollow cylinder	Middle point of the axis of the cylinder
10.	Solid cylinder	Middle point of the axis of the cylinder
11.	Cone or pyramid	On the axis of the cone at a point distant 3 $h/4$ from the vertex O where h = OA is height of the cone, Fig. 7.10(b).

Following points regarding centre of mass should be clearly understood.

1. *The position of the centre of mass of a system is independent of the choice of co-ordinate system.*

2. *The position of the centre of mass depends on the shape and size of the body and the distribution of its mass. Hence it may lie within or outside the material of the body.*

3. *The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of a body.*

4. *The concept of centre of mass proves that the laws of mechanics, which are true for a point mass, are equally valid for all macroscopic bodies.*

5. *If we know the centre of mass of different parts of the system and their masses, we can get the combined centre of mass by treating various parts as point objects whose masses are concentrated at their respective centres of masses.*

DO YOU KNOW ?

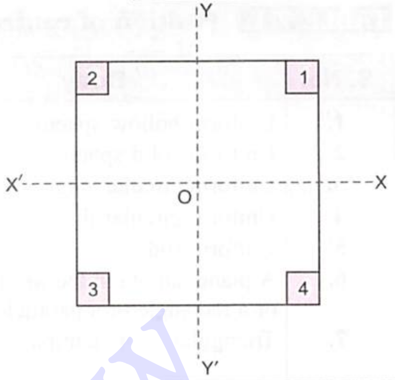
Centre of mass of a body is defined as the point where the entire mass of the body is supposed to be concentrated. In reality, in certain cases, there may be no mass at the centre of mass of the body. For example, there is no dough at the centre of mass of a doughnut, and no iron at the centre of mass of a horse shoe. The centre of a uniform circular ring, a semicircular hoop, a hollow sphere, a bangle, etc. which is its centre of mass, is just empty.

DO YOU KNOW ?

1. The centre of mass of a uniform semicircular ring of radius r are $(x_{cm}, y_{cm}) = \left(0, \dfrac{2r}{\pi}\right)$
2. The centre of mass of a uniform circular disc of radius r are $(x_{cm}, y_{cm}) = (0, 0)$
3. The centre of mass of a uniform semicircular disc of radius r are $(x_{cm}, y_{cm}) = \left(0, \dfrac{4r}{3\pi}\right)$
4. The centre of mass of a uniform quarter disc of radius r are $(x_{cm}, y_{cm}) = \left(0, \dfrac{2r}{3\pi}\right)$
5. The centre of mass of a uniform solid hemisphere of radius r are $(x_{cm}, y_{cm}) = \left(0, \dfrac{3r}{8}\right)$

Sample Problem **Fig. 7.11 shows a uniform square plate from which four identical squares at the corners can be removed. (a) Where is the centre of mass of the plate originally ? (b) Where is it after square 1 is removed ? (c) Where is it after squares 1 and 2 are removed ? (d) Where is c.m after squares 1 and 3 are removed ? (e) Where is c.m after squares 1, 2, 3 are removed ? (f) Where is c.m after all the four squares are removed ? Answer in terms of quadrants and axes.**

FIGURE 7.11



Sol. (*a*) Originally, centre of mass is **at the centre *O*.**

(*b*) After square 1 is removed, *c.m* lies **in quadrant 3.**

(*c*) After squares 1 and 2 are removed, *c.m* lies **on Y-axis** below *O*.

(*d*) When squares 1 and 3 are removed, *c.m* will remain **at *O*.**

(*e*) When squares 1, 2, 3 are removed, *c.m* will **shift to fourth quadrant.**

(*f*) When all the four squares are removed, *c.m* will **shift back to *O*.**

Retain in Memory

1. In case of homogeneous bodies like a uniform circular disc, hollow sphere, solid sphere, marble ball, glass ball, etc. the centre of mass coincides with the geometrical centre.
2. In case of bodies having axis of symmetry like a solid cylinder, hollow cylinder, etc., the centre of mass lies on the axis of symmetry.
3. When a hollow sphere is half filled with water/sand, the centre of mass shifts from centre of the sphere to a point below the centre.

7.8. MOTION OF CENTRE OF MASS

We know that position vector $\vec{R}$ of a system of n particles is given by eqn. (13)

$$\vec{R} = \frac{1}{M} \sum_{i=1}^{i=n} m_i \vec{r}_i$$

or

$$M\vec{R} = \sum_{i=1}^{i=n} m_i \vec{r}_i$$

$$M\vec{R} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots + m_n \vec{r}_n \qquad \ldots(29)$$

Assuming that masses $m_1, m_2, \ldots, m_n$ and total mass M do not change with time ; and differentiating both sides of eqn. (29) with respect to time, we get

$$M\frac{d\vec{R}}{dt} = m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + \ldots m_n \frac{d\vec{r}_n}{dt} \qquad \ldots(30)$$

DO YOU KNOW ?

Although the centre of mass is just a geometrical point, it has a position, a velocity and an acceleration, as if it were a particle.

Now, $\frac{d\vec{r}_1}{dt} = \vec{v}_1$ = velocity of first particle,

$\frac{d\vec{r}_2}{dt} = \vec{v}_2$ = velocity of second particle and so on

$\frac{d\vec{R}}{dt} = \vec{V}$ = velocity of centre of mass of the system of particles

$$\therefore \qquad M\vec{V} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots m_n \vec{v}_n \qquad \ldots(31)$$

Differentiating both sides of (31) again w.r.t. time, we get

$$M\frac{d\vec{V}}{dt} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + ...m_n\frac{d\vec{v}_n}{dt} \qquad ...(32)$$

or $$M\vec{A} = m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_n\vec{a}_n \qquad ...(33)$$

where $\vec{a}_1 = \frac{d\vec{v}_1}{dt}$ = acceleration of first particle

$\vec{a}_2 = \frac{d\vec{v}_2}{dt}$ = acceleration of second particle and so on

$\vec{A} = \frac{d\vec{V}}{dt}$ = acceleration of the centre of mass of the system of particles

According to Newton's second law,

$m_1\vec{a}_1 = \vec{F}_1$ = vector sum of all the forces acting on the first particle.

$m_2\vec{a}_2 = \vec{F}_2$ = vector sum of all forces on second particle and so on.

∴ Eqn. (33) can be written as $M\vec{A} = \vec{F}_1 + \vec{F}_2 + ...\vec{F}_n$ ...(34)

Therefore, total mass of a system of particles, multiplied by acceleration of its centre of mass is equal to vector sum of all the forces acting on the system of particles.

Among these forces, there will be *external forces* exerted by agencies outside the system and also *internal forces* exerted by particles of the system on one another. We know from Newton's third law that internal forces exist in equal and opposite pairs. Therefore, their contribution is zero. Only external forces contribute to eqn. (34), which can therefore be rewritten as

$$\boxed{M\vec{A} = \vec{F}_{ext}} \qquad ...(35)$$

where $\vec{F}_{ext}$ is the vector sum of all external forces acting on the particles of the system.

Eqn. (35) shows that the ***centre of mass of a system of particles moves as if the entire mass of the system were concentrated at the centre of mass and all the external forces were applied at that point.***

Hence we conclude that ***whatever be the system of particles and whatever be the motion of its individual particles, the centre of mass of the system moves according to eqn. (35).*** This is how translational motion of a rigid body is separated even when it is rotating as well. The same is true for a system of particles with all kinds of internal motion.

IMPORTANT NOTE

1. The position of centre of mass is independent of the choice of co-ordinate system.
2. While the position of centre of mass of a body changes in translatory motion, but it remains unchanged in rotatory motion.

7.9. LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

Let us consider a system of *n* particles with masses $m_1, m_2, ... m_n$ moving with velocities $\vec{v}_1, \vec{v}_2, ... \vec{v}_n$ respectively. The particles may be interacting and have external forces acting on them.

The linear momentum of the first particle is $\vec{p}_1 = m_1\vec{v}_1$; of the second particle is $\vec{p}_2 = m_2\vec{v}_2$ and so on.

For the system of *n* particles, the linear momentum $\vec{P}$ of the system is the vector sum of linear momenta of all individual particles of the system, *i.e.*,

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + ... \vec{p}_n$$

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + ... m_n \vec{v}_n \quad ...(36)$$

If M is the total mass of the system of n particles concentrated at the centre of mass and $\vec{V}$ is velocity of the centre of mass, then

$$\vec{P} = M \vec{V} \quad ...(37)$$

From (36) and (37), $\vec{P} = M \vec{V} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + ... m_n \vec{v}_n$

Hence the total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

Differentiating (37) w.r.t. time, we get $\dfrac{d\vec{P}}{dt} = M \dfrac{d\vec{V}}{dt} = M\vec{A} = \vec{F}_{\text{ext}}$ ...(38)

This is the statement of *Newton's second law extended to a system of particles.*

In an isolated system, the vector sum of external forces acting on a system of particles is zero.

$\therefore$ from (38), $\dfrac{d\vec{P}}{dt} = 0$ or $\boxed{\vec{P} = \text{constant}}$ ...(39)

i.e., $M\vec{V} = \text{constant}$ $\therefore$ $\boxed{\vec{V} = \text{constant}}$ ...(40)

i.e., velocity of centre of mass remains constant when total external force on the system is zero.

Note that on account of internal forces i.e., the forces exerted by the particles on one another, the individual particles may have complicated paths. Yet if the total external force acting on the system is zero, the centre of mass moves with a constant velocity, i.e., moves uniformly along a straight line like a free particle. If the centre of mass is at rest, it would remain at rest when total external force acting on the system is zero.

Note that eqn. (39) implies that $P_x = C_1$, $P_y = C_2$ and $P_z = C_3$ where P_x, P_y, P_z are the components of total linear momentum $\vec{P}$ along x, y and z axes and C_1, C_2, C_3 are constants.

7.10. SOME EXAMPLES OF THE MOTION OF CENTRE OF MASS

We have established above that when no external force acts on a system, whose centre of mass is initially at rest, the c.m. would continue to be at rest.

And if the c.m. is moving with some velocity along a straight line, it would continue to do so in the absence of external forces.

It should be clearly understood that the system, as a whole, may be changing shapes or orientations due to internal forces, but it will have no effect on the trajectory of centre of mass of isolated system.

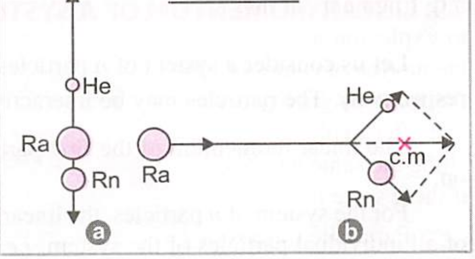
We discuss below some common examples of motion of centre of mass :

1. Consider a radium nucleus disintegrating into a nucleus of radon and an alpha particle. The forces leading to the decay are internal to the system. The external forces on the system are negligible. So the total linear momentum of the system is the same before and after decay.

If the decaying radium nucleus is at rest, the products of decay (Rn and alpha particle) move *back to back* as shown in Fig. 7.12(a) so that their centre of mass remains at rest.

If the decaying radium nucleus is moving along a straight line with some velocity, the two products of decay, Rn nucleus and alpha particle move in different directions in such a way that their centre of mass keeps on moving with the same velocity along the same straight line as the original decaying nucleus was moving [Fig. 7.12(b)].

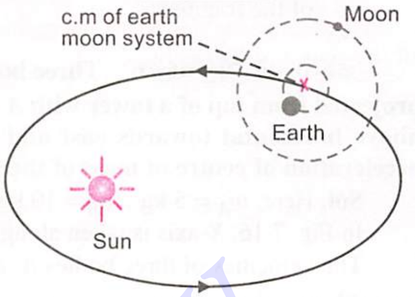
FIGURE 7.12



2. We know that moon revolves around the earth in a circular orbit and the earth revolves around the sun in an elliptical orbit.

The earth and moon exert gravitational forces of attraction on eachother, which are *internal forces* only. To rule out the possibility of earth and moon colliding against each other under the internal gravitational forces, we imagine that *both, the earth and moon are revolving about their common centre of mass, such that they are always on opposite sides of the common centre of mass.* The much larger mass of earth, makes their centre of mass very close to earth. To a first approximation, we can consider the common centre of mass at the earth itself. It is under this approximation that we often talk of only the moon revolving around the earth.

FIGURE 7.13

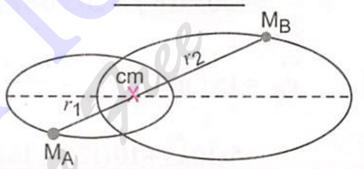


Infact, the earth-moon system moves in such a way that its centre of mass follows the elliptical path around the sun, Fig. 7.13. For earth moon system, force of attraction due to the sun is an external force.

3. Binary stars. As is known, two stars which move in their elliptical orbits around a common centre of mass, are said to form binary stars.

Sirius is an example of a binary star system in our universe. If M_A is mass of star Sirius A, M_B is mass of star Sirius B and $\vec{r_1}$, $\vec{r_2}$ are their respective position vectors from their common centre of mass at any instant, as shown in Fig. 7.14, then by definition

FIGURE 7.14



$$\vec{r}_{cm} = \frac{M_A \vec{r_1} + M_B \vec{r_2}}{M_A + M_B} \qquad ...(41)$$

As centre of mass coincides with the point about which the two stars orbit, therefore, $\vec{r}_{cm} = 0$

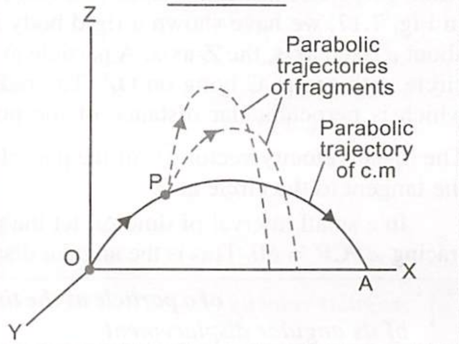
From (41), $$M_A \vec{r_1} + M_B \vec{r_2} = 0 \quad \text{or} \quad \frac{M_A}{M_B} = -\frac{r_2}{r_1} \qquad ...(42)$$

The two stars of binary system exert equal and opposite gravitational forces on each other. Hence the net external force on them is zero. However, the motion of these binary stars is influenced by some other celestial bodies passing close to them. Therefore, the paths of these binary stars are disturbed on account of net external force acting on them. The effect of external force on the movement of stars is known as **wobbling.**

4. We know that *when an object of finite size is thrown with some initial velocity at an angle with the horizontal, it follows a parabolic path.* The centre of mass of such an object also follows the parabolic path, even if the object were to disintegrate in mid-air.

For example, suppose a fire cracker fired from O explodes into fragments in mid air at P [Fig. 7.15]. The forces leading to explosion are internal forces. They contribute nothing to the motion of centre of mass. The total external force, namely, the force of gravity acting on the cracker is the same before and after explosion. The centre of mass, therefore, continues along the same parabolic trajectory as it would have followed, if there were no explosion.

FIGURE 7.15



Parabolic trajectories of individual fragments are also shown in Fig. 7.15.

Retain in Memory

The internal forces of explosion cannot change the original parabolic path of the centre of mass of the fragments.

Sample Problem **Three bodies A, B, C having masses 5 kg, 10 kg and 15 kg respectively are projected from top of a tower with A vertically upwards with 10 m/s ; B with 20 m/s at an angle of 53° above horizontal towards east and C with 15 m/s horizontally towards south. Find velocity and acceleration of centre of mass of the system.**

Sol. Here, $m_A = 5$ kg ; $m_B = 10$ kg ; $m_C = 15$ kg

In Fig. 7.16, X-axis is taken along south, Y-axis is along east and Z-axis is upwards.

The velocities of three bodies A, B, C in vector form can be written as

$$\vec{v}_A = 10\hat{k} \text{ m/s}$$

$$\vec{v}_B = 20\,(\cos 53°\,\hat{j} + \sin 53°\,\hat{k}) \text{ m/s}$$

$$= (20 \times 0{\cdot}6\,\hat{j} + 20 \times 0{\cdot}8\,\hat{k}) \text{ m/s} \;\; = (12\,\hat{j} + 16\,\hat{k}) \text{ m/s}$$

$$\vec{v}_C = 15\,\hat{i} \text{ m/s}, \;\; \vec{v}_{cm} = \frac{m_A \vec{v}_A + m_B \vec{v}_B + m_C \vec{v}_C}{m_A + m_B + m_C}$$

$$= \frac{5(10\hat{k}) + 10(12\,\hat{j} + 16\,\hat{k}) + 15\,(15\,\hat{i})}{5 + 10 + 15}$$

$$= \left(\frac{225}{30}\,\hat{i} + \frac{120}{30}\,\hat{j} + \frac{210}{30}\,\hat{k}\right) \text{m/s}$$

$$\vec{v}_{cm} = \mathbf{(7{\cdot}5\,\hat{i} + 4\,\hat{j} + 7\,\hat{k}) \text{ m/s}}$$

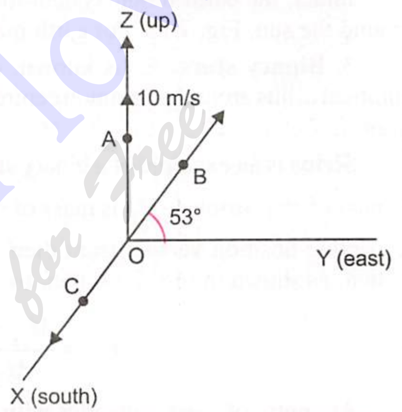


FIGURE 7.16

Now, $\vec{a}_A = -9{\cdot}8\,\hat{k}$ m/s^2, $\vec{a}_B = -9{\cdot}8\,\hat{k}$ m/s^2, $\vec{a}_C = -9{\cdot}8\,\hat{k}$ m/s

$$\therefore \; \vec{a}_{cm} = \frac{m_A \vec{a}_A + m_B \vec{a}_B + m_C \vec{a}_C}{m_A + m_B + m_C} = \frac{5(-9{\cdot}8\,\hat{k}) + 10(-9{\cdot}8\,\hat{k}) + 15(-9{\cdot}8\,\hat{k})}{5 + 10 + 15} \qquad \vec{a}_{cm} = \mathbf{-9{\cdot}8\,\hat{k}}$$

7.11. ANGULAR VELOCITY AND ITS RELATION WITH LINEAR VELOCITY

We know that in pure rotation of a rigid body about a fixed axis, *every particle of the body moves in a circle which lies in a plane perpendicular to the axis, and has its centre on the axis.* In Fig. 7.17, we have shown a rigid body in XY plane rotating about a fixed axis, the Z-axis. A particle at a point P describes a circle with centre C lying on OZ. The radius of the circle is r, which is perpendicular distance of the point P from the axis. The linear velocity vector $\vec{v}$ of the particle at P is shown along the tangent to the circle at P.

In a small interval of time Δt, let the particle reach P′ after tracing ∠PCP′ = $\Delta\theta$. This is the angular displacement. We define

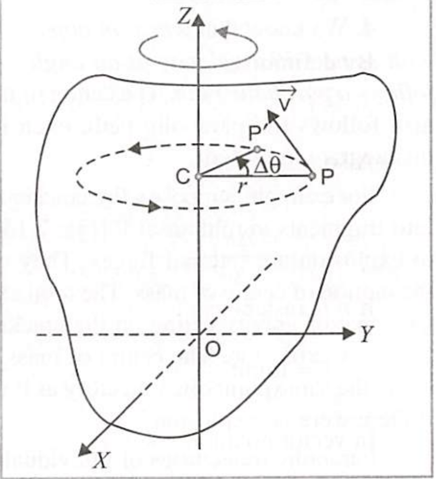


FIGURE 7.17

> ***Angular velocity** of a particle as the time rate of change of its angular displacement.*

$$i.e., \text{ Angular velocity } = \frac{\text{angular displacement}}{\text{time taken}} = \frac{\Delta\theta}{\Delta t}$$

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As Δt tends to zero, the ratio $\frac{\Delta\theta}{\Delta t}$ approaches a limit which is the instantaneous angular velocity $\left(\frac{d\theta}{dt}\right)$ of the particle at P. We represent the instantaneous angular velocity by the Greek letter ω (omega)

$$\therefore \quad \boxed{\omega = \frac{d\theta}{dt}} \qquad \text{...(43)}$$

Angular velocity is measured in radian/sec and its **dimensional formula is [$M^0 L^0 T^{-1}$].**

Angular velocity is a **vector** quantity whose direction is given by **right hand screw rule.**

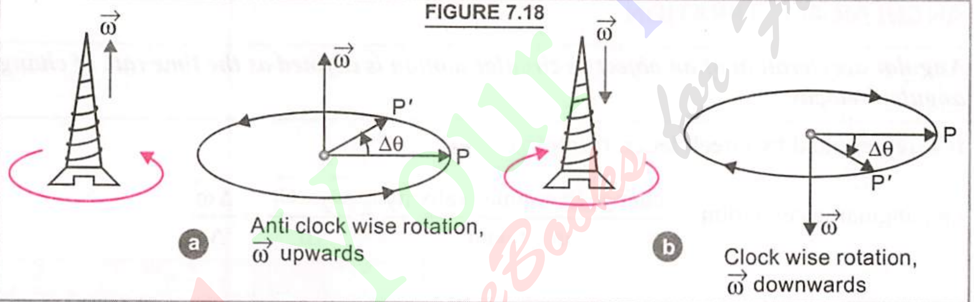
For rotation about a fixed axis, the angular velocity vector lies along the axis of rotation and points out in the direction in which the tip of right hand screw would advance if the head of the screw is rotated with the body.

Retain in Memory

For anticlockwise rotation, the direction of angular velocity $\vec{\omega}$ is along the axis of rotation and *directed upwards,* Fig. 7.18(a).

For clockwise rotation, the direction of angular velocity $\vec{\omega}$ is along the axis of rotation and *directed downwards,* Fig. 7.18(b).

FIGURE 7.18



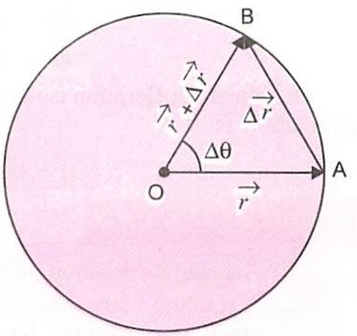
(a) Anti clock wise rotation, $\vec{\omega}$ upwards

(b) Clock wise rotation, $\vec{\omega}$ downwards

Relation between Angular velocity and Linear velocity

Suppose a particle is moving along a circular path of radius r with a uniform angular speed ω, Fig.. 7.19. At any time t, suppose the particle is at A, where $\overrightarrow{OA} = \vec{r}$. In a small time Δt, let the particle reach B tracing $\angle AOB = \Delta\theta$. Let $\overrightarrow{OB} = (\vec{r} + \Delta\vec{r})$ $\therefore$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \Delta\vec{r}$

FIGURE 7.19



By definition, $\omega = \frac{\Delta\theta}{\Delta t}$ or $\Delta\theta = \omega(\Delta t)$.

Also, $\Delta\theta = \frac{\overset{\frown}{AB}}{OA} \simeq \frac{\overrightarrow{AB}}{OA} = \frac{|\Delta\vec{r}|}{r}$, *i.e.,* $\omega(\Delta t) = \frac{|\Delta\vec{r}|}{r}$

$$\frac{|\Delta\vec{r}|}{\Delta t} = \omega r$$

If v is instantaneous velocity of the particle at time t, then

$$v = \underset{\Delta t \to 0}{\text{Limit}} \frac{|\Delta\vec{r}|}{\Delta t} = \underset{\Delta t \to 0}{\text{Limit}} (\omega r) \quad \text{or} \quad \boxed{v = \omega r} \qquad \text{...(44)}$$

In vector notation, the relation is $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$ ...(45)

For particles on the axis, $r = 0$ and hence $v = \omega r = 0$. Thus, *particles on the axis are stationary.* This verifies that the axis is fixed.

Further, as the body is rigid, all the particles of the body complete one revolution about the given axis together, in the same time. Therefore, time period (T) of all the particles is the same. Therefore, $\omega = \frac{2\pi}{T}$ for all the particles is the same.

Thus, just as in pure translation, all particles of the body have the same linear velocity at any instant of time, similarly, in pure rotation, all particles of the body have the same angular velocity at any instant of time.

Eqn. (44) shows that v depends on r, distance of particles from the axis of rotation. Thus, although angular velocity ω of all the particles of rigid body is the same, **the linear velocity v of particles lying at different distances from the axis of rotation is different.**

IMPORTANT NOTE

Note that for rotation about a fixed axis, the direction of angular velocity $\vec{\omega}$ does not change with time. Its magnitude may change from instant to instant, when the rotation is non-uniform.

For more general rotation like that of an oscillating fan, both the magnitude and direction of $\vec{\omega}$ may change from time to time.

7.12. ANGULAR ACCELERATION

Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

It is represented by Greek letter, α (alpha).

i.e., angular acceleration $= \dfrac{\text{change in angular velocity}}{\text{time taken}} = \dfrac{\vec{\omega}_2 - \vec{\omega}_1}{\Delta t} = \dfrac{\Delta\vec{\omega}}{\Delta t}$

The magnitude of angular acceleration will be $\boxed{\alpha = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{\omega}}{\Delta t}\right) = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}}$ ...(46)

SI unit of α is **rad s^{-2}** and its dimensional formula is $[M^0 L^0 T^{-2}]$. Angular acceleration is a vector quantity.

Relation between angular acceleration and linear acceleration.

The relation between linear velocity v and angular velocity ω is $v = \omega r$

Differentiating both sides with respect to time, we get

$$\frac{dv}{dt} = \frac{d}{dt}(\omega r) = \left(\frac{d\omega}{dt}\right) r \quad \text{or} \quad \boxed{a = \alpha\, r} \qquad \text{...(47)}$$

i.e., linear acceleration is the product of angular acceleration and radius. In terms of vectors, we rewrite this relation as

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r}} \qquad \text{...(48)}$$

Retain in Memory

1. Angular acceleration (α) is positive when angular velocity (ω) is increasing ; and vice-versa.
2. A rotating body speeds up when ω and α have the same sign. Obviously, a rotating body slows down when ω and α have opposite sign.
3. When a disc rotates with a uniform angular velocity, its speed of rotation, sense of rotation and orientation of the axis of rotation remain the same. However, the angular acceleration of the disc is zero.

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Sample Problem **The speed of a motor increases from 600 *rpm* to 1200 *rpm* in 20 seconds. What is its angular acceleraion, and how many revolutions does it make during this time ?**

Sol. Here, $n_1 = 600\ rpm = \dfrac{600}{60} = 10\ rps$, $n_2 = 1200\ rpm = \dfrac{1200}{60} = 20\ rps$

$$t = 20\ s,\ \alpha = ?\ n = ?,\ \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(20-10)}{20} = \boldsymbol{\pi\ \text{rad/s}^2}$$

From $2\alpha\,\theta = \omega_2^2 - \omega_1^2$, $\theta = \dfrac{\omega_2^2 - \omega_1^2}{2\alpha} = \dfrac{(40\pi)^2 - (20\pi)^2}{2\pi} = \dfrac{1200\,\pi^2}{2\pi} = 600\,\pi$ radian

$\therefore$ Number of rotations made, $n = \dfrac{\theta}{2\pi} = \dfrac{600\pi}{2\pi} = \mathbf{300}$

7.13. CONCEPT OF TORQUE OR MOMENT OF FORCE

When we switch on a fan, the centre of the fan remains unmoved, while the fan rotates with an angular acceleration. As the centre of mass of the fan remains at rest, the vector sum of external forces acting on the fan must be zero. It means ***one can produce angular acceleration even when resultant external force is zero.*** We also know that we cannot produce angular acceleration without applying an external force. What is then responsible for producing angular acceleration ? The answer is moment of force or torque due to the force.

When an external force acting on a body has a tendency to rotate the body about a fixed point or about a fixed axis, it is said to exert a torque on the body.

> ***The moment of a force or the torque due to a force gives us the turning effect of the force about the fixed point/axis. It is measured by the product of magnitude of force and perpendicular distance of the line of action of force from the axis of rotation.***

Torque is represented by a Greek letter τ. Thus

Moment of force or Torque = force × perpendicular distance

We shall prove that $\boxed{\vec{\tau} = \vec{r} \times \vec{F} = r\,F \sin\theta\,\hat{n}}$ **...(49)**

where θ is smaller angle between $\vec{r}$ and $\vec{F}$; $\hat{n}$ is unit vector along $\vec{\tau}$. The direction of $\vec{\tau}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$, and is determined by **right hand screw rule** or **right hand thumb rule.** The **SI unit** of torque is N-*m,* which is equivalent to joule. The **dimensions** of torque are $[M^1\,L^2\,T^{-2}]$.

To understand further, the moment of force or torque due to force, let us consider a rod OA of length *r*, suspended from O and capable of rotation about O, Fig. 7.20. Let a force F be applied at the free end A of the rod. In Fig. 7.20(a), angle between $\vec{r}$ and $\vec{F}$ is θ = 0°. Therefore, magnitude of moment of force or torque,

$$\tau = r\,F \sin\theta = r\,F \sin 0° = 0 \qquad \textbf{...(50)}$$

i.e., turning effect of the force in this case is zero.

In Fig. 7.20(b), θ = 90°

$\therefore\quad \tau = r\,F\sin\theta = r\,F\sin 90° = r\,F = \max.$

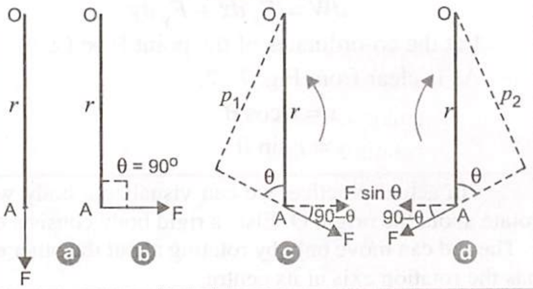
i.e., turning effect of the force in this case, is maximum.

In Fig. 7.20(c),

$$\tau = r\,F\sin\theta = F\,(r\sin\theta) = F \times p_1$$

where $p_1 = r\sin\theta$ = perpendicular distance of line of action of force from O. The rotation is *anticlockwise.*

FIGURE 7.20



In Fig. 7.20(d), $\tau = r\,F \sin\theta = F\,(r \sin\theta) = F \times p_2$

where $p_2 = r \sin\theta$ = perpendicular distance of line of action of force from O. The rotation is *clockwise*.

In Fig. 7.20(c) and (d), we find that component of force F perpendicular to r is $F \sin\theta$. From eq. (49)

$$\boxed{\tau = r\,F \sin\theta = r\,(F \sin\theta) = r\,F_\perp}$$

Again, as component of r perpendicular to F is $r \sin\theta$, we may write $\tau = r\,F \sin\theta = F\,(r \sin\theta) = F\,r_\perp$. Hence

$$\boxed{\tau = r\,F_\perp = F\,r_\perp}$$

By convention, anticlockwise moments are taken as positive and clockwise moments are taken as negative.

DO YOU KNOW ?

In linear motion, force is important. But in rotational motion, it is not the force alone, but how and where the force is applied is important.

7.14. EXPRESSION FOR TORQUE IN CARTESIAN CO-ORDINATES FROM ROTATION OF A PARTICLE IN A PLANE : PHYSICAL MEANING OF TORQUE

The expression for torque can be obtained when we calculate work done by a force in rotating a particle of mass m in a plane.

Consider a particle of mass m rotating in plane XY about a fixed axis OZ (not shown). Let P be the position* of the particle at any instant, where $\overrightarrow{OP} = \vec{r}$ and $\angle XOP = \theta$. Let the rotation occur under the action of a force $\vec{F}$ applied at P, along $\overrightarrow{PA}$, Fig. 7.21.

In a small time dt, let the particle at P reach Q,

where $\overrightarrow{OQ} = (\vec{r} + d\vec{r})$ and $\angle POQ = d\theta$.

In vector triangle OPQ, $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

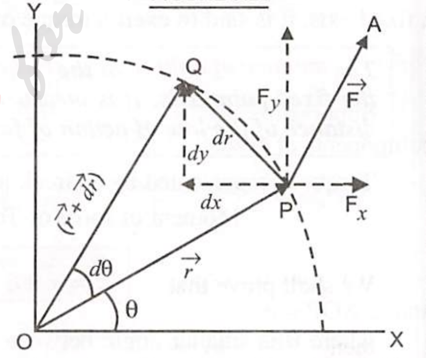
or $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (\vec{r} + d\vec{r}) - \vec{r}$

$\overrightarrow{PQ} = d\vec{r}$

Small amount of work done in rotating the particle from P to Q is

$$dW = \vec{F} \cdot d\vec{r} \qquad \text{...(51)}$$

FIGURE 7.21



If F_x, F_y are rectangular components of force $\vec{F}$ and dx, dy are rectangular components of displacement $d\vec{r}$, then

$$\vec{F} = (\hat{i}\,F_x + \hat{j}\,F_y)$$

$$d\vec{r} = (\hat{i}\,dx + \hat{j}\,dy)$$

From (51), we get

$$dW = (\hat{i}\,F_x + \hat{j}\,F_y) \cdot (\hat{i}\,dx + \hat{j}\,dy)$$

$$dW = F_x\,dx + F_y\,dy \qquad \text{...(52)}$$

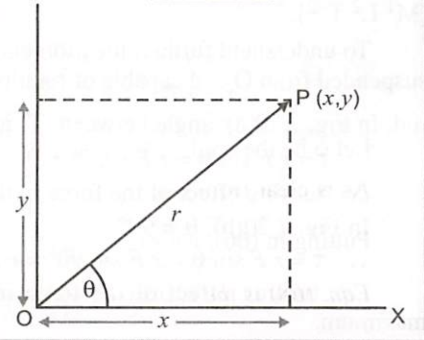
Let the co-ordinates of the point P be (x, y)

As is clear from Fig. 7.22,

$$x = r \cos\theta \qquad \text{...(53)}$$

$$y = r \sin\theta \qquad \text{...(54)}$$

FIGURE 7.22



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*In actual practice, we can visualize a body whose mass (m) is concentrated at P, and the body is free to rotate about the origin O. Else, a rigid body consists of a particle of mass m on one end of a massless rod of length r. The rod can move only by rotating about the other end O. Thus, the particle can move only in a circular path that has the rotation axis at its centre.

Differentiating (53) *w.r.t.* θ, we get

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(r\cos\theta) = r\,\frac{d}{d\theta}(\cos\theta) = r(-\sin\theta) \qquad ...(55)$$

$$\therefore \qquad dx = -\,y\,d\theta \qquad ...(56)$$

Again, differentiating (54) *w.r.t.* θ, we get $\frac{dy}{d\theta} = \frac{d}{d\theta}(r\sin\theta) = r\cos\theta = x$ ...(57)

$$\therefore \qquad dy = x\,d\theta \qquad ...(58)$$

Substituting in (52), we get $dW = F_x(-y\,d\theta) + F_y(x\,d\theta) = x\,F_y\,d\theta - y\,F_x\,d\theta$

$$dW = (x\,F_y - y\,F_x)\,d\theta \qquad ...(59)$$

or $\qquad dW = \tau_z\,(d\theta)$ ...(60)

where $\qquad \boxed{\tau_z = (x\,F_y - y\,F_x)}$ ...(61)

τ_z is called torque. Eqn. (61) is the expression for torque in cartesian co-ordinates.

From (60), We define

Torque τ as a quantity in rotational motion, which when multiplied by a small angular displacement gives us work done in rotational motion. This quantity corresponds to force in linear motion, which when multiplied by a small linear displacement gives us work done in linear motion. This is the physical meaning of torque.

7.15. EXPRESSION FOR TORQUE IN POLAR CO-ORDINATES

Suppose the line of action of force $\vec{F}$ makes an angle α with X-axis, Fig. 7.23. Two rectangular components of *F* are :

$$\therefore \qquad F_x = F\cos\alpha \qquad ...(62)$$

$$F_y = F\sin\alpha \qquad ...(63)$$

If *x*, *y* are the co-ordinates of the point P, where $\overrightarrow{OP} = \vec{r}$ and $\angle XOP = \theta$,

then $\qquad x = r\cos\theta$ ...(64)

and $\qquad y = r\sin\theta$ ...(65)

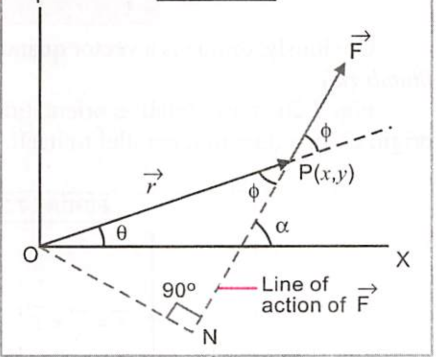
Substituting these values in (61), we get

$$\tau = (r\cos\theta)\,F\sin\alpha - (r\sin\theta)\,(F\cos\alpha)$$

$$= r\,F\,[\sin\alpha\cos\theta - \cos\alpha\sin\theta]$$

$$\tau = r\,F\sin(\alpha - \theta) \qquad ...(66)$$

FIGURE 7.23



Let ϕ be the angle which the line of action of $\vec{F}$ makes with the position vector $\overrightarrow{OP} = \vec{r}$.

As is clear from Fig. 7.22, $\qquad \theta + \phi = \alpha \qquad$ or $\qquad \phi = \alpha - \theta$ ...(67)

Putting in (66), we get, $\qquad \boxed{\tau = r\,F\sin\phi}$ ...(68)

Eqn. (68) is the expression for torque in polar co-ordinates.

Discussion

1. Equation (68) shows that ability of $\vec{F}$ to rotate the body depends not only on the magnitude of $\vec{F}$, but also on just how far from O, the force is applied and in what direction.

In Fig. 7.23, draw ON ⊥ on the line of action of $\vec{F}$.

In ΔOPN, $\quad \sin\phi = \dfrac{ON}{OP} = \dfrac{ON}{r}$

$\therefore \qquad ON = r\sin\phi \qquad$ ...(69)

From (68), $\quad \tau = rF\sin\phi = F(r\sin\phi) = F(ON) \qquad$ ...(70)

Hence, torque due to a force is the product of force and perpendicular distance of line of action of force from the axis of rotation.

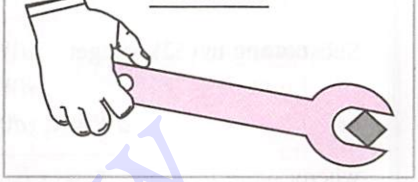


FIGURE 7.24

2. From eqn. (70), we find that (*i*) Torque due to a force is maximum, when r is maximum.

For example, *to unscrew a nut fitted tightly to a bolt, we need a* **wrench with a long arm,** Fig. 7.24.

When the length of arm (r) is long, the force (F) required to produce a given turning effect ($\tau = rF\sin\phi$) *is smaller.*

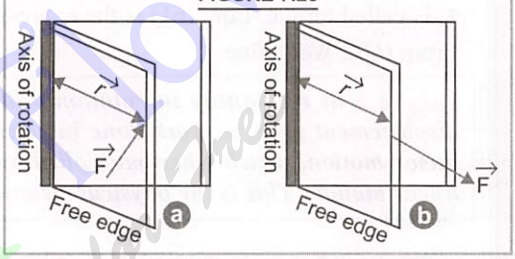


FIGURE 7.25

3. Torque will be maximum, when $\sin\phi$ = max. = + 1. Therefore, $\phi = 90^\circ$, *i.e.*, when force is applied in a direction perpendicular to $\vec{r}$. For example, it is easiest to open or close a door by applying force at the edge of the plank in a direction perpendicular to the plank of the door, as shown in Fig. 7.25.

4. When $\phi = 0^\circ$ or 180°, $\sin\phi = \sin 0^\circ$ or $\sin 180^\circ = 0$. Therefore, $\tau = rF\sin\phi = 0$.

i.e., torque of the force is zero. For example, the door cannot be rotated by applying force in a direction parallel to the plank of the door, as shown in Fig. 7.25(b).

5. Eqn. (68) can be rewritten in vector form as

$$\boxed{\vec{\tau} = \vec{r}\times\vec{F}} \qquad ...(71)$$

Obviously, torque is a vector quantity whose direction is given by *right handed screw rule* or *right hand thumb rule.*

Fig. 7.26 shows relative orientation of $\vec{r}$ and $\vec{F}$. Force $\vec{F}$ acting actually at P has been shifted to origin O, in a direction parallel to itself.

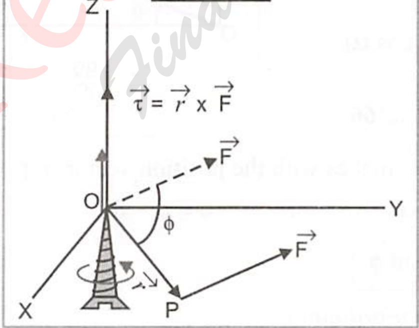


FIGURE 7.26

DO YOU KNOW ?

In the world of pure rotation, a vector defines an axis of rotation and not the direction in which something moves.

When $\vec{F}$ rotates the particle in XY plane in anticlockwise direction, the tip of the right hand screw moves along the positive Z direction, which is the direction of the torque τ.

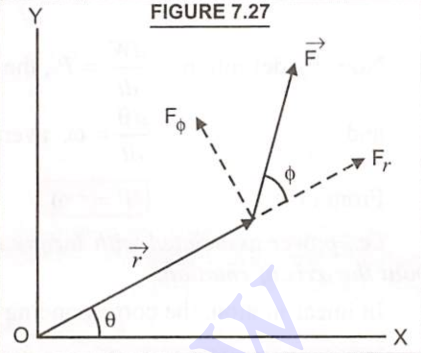
IMPORTANT NOTE

Torque on a door about the axis of rotation, due to the weight of the door is zero.

Retain in Memory

In Fig. 7.27, we have shown two rectangular components of $\vec{F}$: $\vec{F}_r = F \cos \phi$, in the direcion of increasing r. It is called "**radial component** of $\vec{F}$, and $F_\phi = F \sin \phi$, in the direction of increasing θ and perpendicular to F_r. This is called **transverse component** of $\vec{F}$.

FIGURE 7.27



From (68), $\tau = r F \sin \phi = r (F \sin \phi) = r F_\phi$,

i.e., torque of a force is given by the product of transverse component of force and perpendicular distance from the axis of rotation. *Thus torque is due to transverse component only. The radial component of force has no role to play in the torque.*

Sample Problem **Find the torque of a force $(5\hat{i} - 2\hat{j} + 7\hat{k})$ about the origin, which acts on a particle whose position vector is $(2\hat{i} - \hat{j} + \hat{k})$.**

Sol. Here, $\vec{\tau} = ?$; $\vec{F} = (5\hat{i} - 2\hat{j} + 7\hat{k})$, $\vec{r} = (2\hat{i} - \hat{j} + \hat{k})$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 5 & -2 & 7 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}(-7+2) - \hat{j}(14-5) + \hat{k}(-4+5) \quad \text{or} \quad \vec{\tau} = -5\hat{i} - 9\hat{j} + \hat{k}$$

7.16. RECTANGULAR COMPONENTS OF TORQUE

In Art. 7.14, we have obtained an expression for torque due to a force rotating the particle in XY plane. This is only the z-component of torque. The other two components of torque, in this case, are zero i.e. $\tau_y = 0$ and $\tau_x = 0$.

The force applied may rotate a body in three dimensions. In that case, we shall have three rectangular components of torque, which can be obtained easily from eqn. (71) as follows :

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\begin{aligned} \hat{i}\,\tau_x + \hat{j}\,\tau_y + \hat{k}\,\tau_z &= (\hat{i}\,x + \hat{j}\,y + \hat{k}\,z) \times (\hat{i}\,F_x + \hat{j}\,F_y + \hat{k}\,F_z) \\ &= \hat{k}\,x F_y - \hat{j}\,x F_z - \hat{k}\,y F_x + \hat{i}\,y F_z + \hat{j}\,z F_x - \hat{i}\,z F_y \\ &= \hat{i}\,(y F_z - z F_y) + \hat{j}(z F_x - x F_z) + \hat{k}(x F_y - y F_x) \end{aligned}$$

Equating the three rectangular components on two sides, we get

$$\boxed{\begin{aligned} \tau_x &= y F_z - z F_y \\ \tau_y &= z F_x - x F_z \\ \tau_z &= x F_y - y F_x \end{aligned}} \qquad \begin{aligned} &\text{...(72)} \\ &\text{...(73)} \\ &\text{...(74)} \end{aligned}$$

7.17. POWER ASSOCIATED WITH TORQUE

In Art. 7.15, we have obtained work done (dW) in rotating a particle through a small angle ($d\theta$) as

$$dW = \tau\,(d\theta) \qquad \text{...(60)}$$

If this work is done in a small time interval dt, then dividing both sides of (60) by dt, we get

$$\frac{dW}{dt} = \tau\left(\frac{d\theta}{dt}\right) \qquad ...(75)$$

Now, by definition, $\frac{dW}{dt} = P$, the average power associated with torque,

and $\frac{d\theta}{dt} = \omega$, average angular speed of the body in this interval.

From (75), $\boxed{P = \tau\,\omega}$ ...(76)

i.e., power associated with torque is given by the product of torque and angular speed of the body about the axis of rotation.

In linear motion, the corresponding relation for power is $\boxed{P = F\,v}$ ...(77)

Sample Problem **To maintain a rotor at a uniform angular speed of 200 s^{-1}, an engine needs to transmit a torque of 180 N-m. What is the power of the engine required ?**

Sol. Here, $\omega = 200\ s^{-1}$, $\tau = 180$ N-m, $P = ?$

As $P = \tau\,\omega$ $\therefore$ $P = 180 \times 200 = 36000$ watt = **36 kW**

7.18. THE CONCEPT OF ANGULAR MOMENTUM–EXPRESSION FOR ANGULAR MOMENTUM IN CARTESIAN CO-ORDINATES

We know the concept of torque as the rotating effect or turning effect of a force. Torque is regarded as rotational analogue of force. According to Newton's second law of motion, force is equal to rate of change of linear momentum. Therefore, ***if we can represent torque as the rate of change of some quantity, that quantity will be rotational analogue of linear momentum, and it can appropriately be called the angular momentum.***

In order to express torque as the rate of change of some quantity, we rewrite expression for torque rotating a particle in XY plane, (Eqn. 61) :

$$\tau = x\,F_y - y\,F_x \qquad ...(78)$$

If $p_x = m\,v_x$ and $p_y = m\,v_y$ are the x and y components of linear momentum of the body, then

According to Newton's 2nd law of motion,

$$F_x = \frac{dp_x}{dt} = \frac{d}{dt}(mv_x) = m\,\frac{dv_x}{dt} \quad \text{and} \quad F_y = \frac{dp_y}{dt} = \frac{d}{dt}(mv_y) = m\,\frac{dv_y}{dt}$$

Substituting in (78), we get $\tau = x\,m\frac{dv_y}{dt} - y\,m\frac{dv_x}{dt}$ or $\tau = m\left[x\,\frac{dv_y}{dt} - y\,\frac{dv_x}{dt}\right]$ ...(79)

Using rule for differentiation of products, we may write

$$\frac{d}{dt}(xv_y - yv_x) = x\frac{dv_y}{dt} + v_y\frac{dx}{dt} - y\frac{dv_x}{dt} - v_x\frac{dy}{dt} = x\frac{dv_y}{dt} + v_y\,v_x - y\frac{dv_x}{dt} - v_x\,v_y \qquad ...(80)$$

$$= x\frac{dv_y}{dt} - y\frac{dv_x}{dt} \qquad \left[\because \frac{dx}{dt} = v_x\,;\frac{dy}{dt} = v_y\right]$$

Substituting (80) in (79), we obtain

$$\tau = m\,\frac{d}{dt}(x\,v_y - y\,v_x)$$

$$\tau = \frac{d}{dt}(x\,m\,v_y - y\,m\,v_x).$$

As $m\,v_y = p_y$ and $m\,v_x = p_x$ $\therefore$ $\tau = \frac{d}{dt}(x\,p_y - y\,p_x)$ ...(81)

Put $\boxed{(x\,p_y - y\,p_x) = L}$ ...(82)

$\therefore$ $\boxed{\tau = \frac{dL}{dt}}$ ...(83)

Thus we have obtained **torque τ as the rate of change of a quantity L defined by eqn. (82). We call this quantity L as the *angular momentum* of the body.**

Eqn. (82) represents ***an expression for angular momentum in cartesian co-ordinates. Thus basically, angular momentum of a particle/body about a given axis is the moment of linear momentum of the particle/body about that axis.***

As is known, linear momentum is the product of objects' mass and its velocity. Angular momentum of an object about a given axis of rotation is the product of objecs' mass, its velocity and perpendicular distance of line of action of momentum from the centre of rotation. Note that we have to take component of velocity perpendicular to this distance.

7.19. EXPRESSION FOR ANGULAR MOMENTUM IN POLAR CO-ORDINATES

Suppose $K(x, y)$ is position of a particle of mass m and linear momentum $\vec{p}$ rotating in XY plane about Z-axis, Fig. 7.28. Let $\overrightarrow{OK} = \vec{r}$ and $\angle XOK = \theta$.

$\therefore$ $x = r\cos\theta$ and $y = r\sin\theta$ ...(84)

Let the line of action of linear momentum $\vec{p}$ make an angle α with OX and angle ϕ with $\vec{r}$.

$\therefore$ $p_x = p\cos\alpha$

and $p_y = p\sin\alpha$ ...(85)

Putting these values in (82), we get

$$L = (r\cos\theta)(p\sin\alpha) - (r\sin\theta)(p\cos\alpha)$$

$$= p\,r\,[\sin\alpha\cos\theta - \cos\alpha\sin\theta]$$

$$L = p\,r\sin(\alpha - \theta) \quad \text{...(86)}$$

From Fig. 7.28, $\theta + \phi = \alpha \;\therefore\; \phi = \alpha - \theta$

Putting in (86), we get $\boxed{L = p\,r\sin\phi}$ ...(87)

This is the expression for angular momentum of a particle in polar co-ordinates.

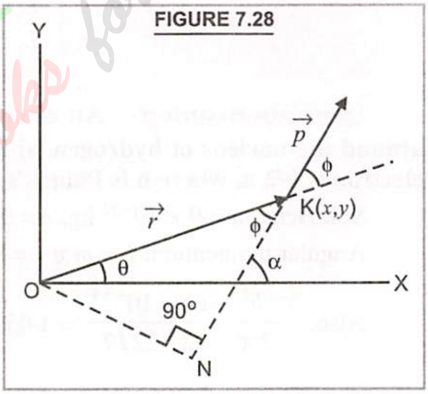


FIGURE 7.28

In Fig. 7.28, draw ON perpendicular to the line of action of $\vec{p}$.

In ΔOKN, $\sin\phi = \frac{ON}{OK} = \frac{ON}{r}$ $\quad ON = r\sin\phi$

Substituting in (87), we get $\boxed{L = p\,(ON) = p\,(r_\perp)}$ ...(88)

> ***Angular momentum** of a body about a given axis is the product of linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation.*

This is the **physical meaning of angular momentum.**

We may rewrite eqn. (87) as

$$L = pr\sin\phi = r\,(p\sin\phi) = r\,p_\perp$$

$\therefore$ $\boxed{L = p\,(r_\perp) = r\,(p_\perp)}$

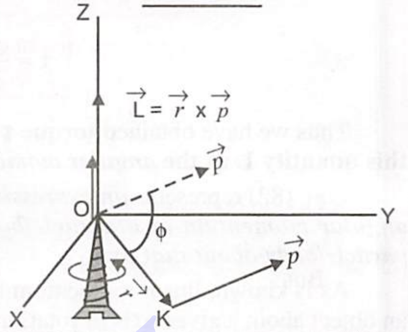
In SI, the units of angular momentum are (kg ms^{-1}) (m) = kg m^{2}s^{-1}. The dimensional formula for angular momentum would be [M^1 L^2 T^{-1}].

Eqn. (87) can be rewritten in vector form as

$$\boxed{\vec{L} = \vec{r} \times \vec{p}} \qquad \text{...(89)}$$

This indicates that angular momentum is a vector quantity and its direction is given by *right hand screw rule* or *right hand thumb rule*. When the centre of mass rotates through smaller angle ϕ in XY plane, say from the direction of $\vec{r}$ to the direction of $\vec{p}$, the angular momentum vector $\vec{L}$ is along OZ *i.e.* the positive Z-direction, as illustrated in Fig. 7.29.

FIGURE 7.29



Retain in Memory

1. Proceeding as in the case of torque, we can show that radial component of linear momentum does not contribute to angular momentum of the particle. It is only the transverse component of linear momentum (perpendicular to position vector $\vec{r}$), which when multiplied by distance from the axis of rotation gives us angular momentum.
2. Again, proceeding as in case of torque, we may write the three rectangular components of angular momentum as

$$\boxed{\begin{aligned} L_x &= y\,p_z - z\,p_y \\ L_y &= z\,p_x - x\,p_z \\ L_z &= x\,p_y - y\,p_x \end{aligned}} \qquad \text{...(90)}$$

Sample Problem **An electron of mass 9×10^{-31} kg revolves in a circle of radius 0·53 Å around the nucleus of hydrogen with a velocity of $2{\cdot}2 \times 10^6$ ms^{-1}. Show that angular momentum of electron is $h/2\,\pi$, where h is Planck's constant.**

Sol. Here, $m = 9 \times 10^{-31}$ kg, $r = 0{\cdot}53$ Å $= 0{\cdot}53 \times 10^{-10}$ m, $v = 2{\cdot}2 \times 10^6$ m/s

Angular momentum, $L = m\,v\,r = 9 \times 10^{-31} \times 2{\cdot}2 \times 10^6 \times 0{\cdot}53 \times 10^{-10} = 1{\cdot}0494 \times 10^{-34}$ Js.

Also, $\dfrac{h}{2\pi} = \dfrac{6{\cdot}6 \times 10^{-34}}{2 \times 22/7} = 1{\cdot}05 \times 10^{-34}$ Js. Hence, $\boldsymbol{L \approx h/2\,\pi}$

7.20. GEOMETRICAL MEANING OF ANGULAR MOMENTUM

To understand the geometrical **meaning** of angular momentum, let us consider a particle rotating in XY plane about an axis OZ, Fig. 7.30. At any time *t*, let $\overrightarrow{OK} = \vec{r}$ be the position vector of the particle.

In a small time *dt*, let the particle at K reach L,

where $\overrightarrow{OL} = (\vec{r} + d\vec{r})$

Join $\overrightarrow{KL}$, which represents displacement of the particle in small time *dt*.

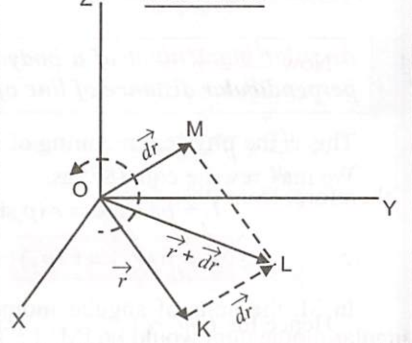
In vector ΔOKL,

$$\overrightarrow{OK} + \overrightarrow{KL} = \overrightarrow{OL}$$

$$\overrightarrow{KL} = \overrightarrow{OL} - \overrightarrow{OK} = (\vec{r} + d\vec{r}) - \vec{r} = d\vec{r} \qquad \text{...(91)}$$

From O, draw $\overrightarrow{OM}$ equal and parallel to $\overrightarrow{KL}$. Join ML.

FIGURE 7.30



Area swept by the position vector in a small time dt is

$$|d\vec{A}| = \text{Area of } \Delta\text{OKL} = \frac{1}{2}\ (\text{area of parallelogram OKLM}) = \frac{1}{2}\,|\overrightarrow{\text{OK}}\times\overrightarrow{\text{OM}}|$$

$$|d\vec{A}| = \frac{1}{2}\,|\vec{r}\times d\vec{r}\,|$$

Divide both sides by dt $\quad \left|\frac{d\vec{A}}{dt}\right| = \frac{1}{2}\left|\vec{r}\times\frac{d\vec{r}}{dt}\right|$

But $\quad \frac{d\vec{r}}{dt} = \vec{v} = \frac{\vec{p}}{m} \qquad \therefore \quad \left|\frac{d\vec{A}}{dt}\right| = \frac{1}{2}\left|\vec{r}\times\frac{\vec{p}}{m}\right|$

$$\left|\frac{d\vec{A}}{dt}\right| = \frac{1}{2m}\,|\vec{L}\,| \qquad (\because\ \vec{r}\times\vec{p} = \vec{L}\,)$$

$$\boxed{|\vec{L}\,| = 2m\left|\frac{d\vec{A}}{dt}\right|} \qquad \text{...(92)}$$

Here $\left|\frac{d\vec{A}}{dt}\right|$ is area swept by the position vector per unit time, and is called *areal velocity* of the position vector of particle.

Equation (92) shows that

> ***angular momentum of a particle about a given axis is twice the product of mass of the particle and areal velocity of position vector of the particle. This is the geometrical meaning of angular momentum.***

DO YOU KNOW ?

Just as linear momentum is the product of mass and linear velocity, angular momentum of a particle about a given axis is twice the product of mass of particle and areal velocity of position vector of the particle.

It follows that ***if mass m of particle and its angular momentum L are constant, the areal velocity of the position vector of particle would be constant.*** This is another way of stating geometrical meaning of angular momentum.

7.21. RELATION BETWEEN ANGULAR MOMENTUM AND TORQUE

The vector relation between angular momentum $\vec{L}$ and linear momentum $\vec{p}$ is $\vec{L} = \vec{r}\times\vec{p}$

Differentiating both sides w.r.t. time, we get $\quad \frac{d\vec{L}}{dt} = \frac{d}{dt}\,(\vec{r}\times\vec{p}\,)$

Applying the product rule for differentiation on the right hand side,

$$\frac{d}{dt}\,(\vec{r}\times\vec{p}\,) = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} \qquad \text{...(93)}$$

Now $\frac{d\vec{r}}{dt} = \vec{v}$ = velocity of the particle and $\vec{p} = m\,\vec{v}$ = linear momentum of the particle

$\therefore\ \frac{d\vec{r}}{dt}\times\vec{p} = \vec{v}\times m\,\vec{v} = 0,$ as the cross product of two parallel vectors is zero. Further, as $\frac{d\vec{p}}{dt} = \vec{F}$, therefore, from (93),

$$\frac{d}{dt}\,(\vec{r}\times\vec{p}\,) = \vec{r}\times\vec{F} = \vec{\tau} \quad \text{or} \quad \boxed{\frac{d}{dt}\,(\vec{L}) = \vec{\tau}} \qquad \text{...(94)}$$

Hence the *time rate of change of angular momentum of a particle is equal to the torque acting on it.*

Eqn. (94) is the rotational analogue of the equation $\frac{d}{dt}(\vec{p}) = \vec{F}$,

which represents Newton's second law for the translational motion of a single particle.

7.22. TORQUE AND ANGULAR MOMENTUM OF A SYSTEM OF PARTICLES

The total angular momentum of a system of particles about a given point is the vector sum of angular momenta of individual particles about the same point. Thus, for a system of n particles,

$$\vec{L} = \vec{L_1} + \vec{L_2} + \ldots \vec{L_n} = \sum_{i=1}^{i=n} \vec{L_i} \qquad \ldots(95)$$

The angular momentum of the ith particle is given by

$$\vec{L_i} = \vec{r_1} \times \vec{p_i} \qquad \therefore \qquad \vec{L} = \sum_{i=1}^{i=n} \vec{L_i} = \sum_{i=1}^{i=n} \vec{r_i} \times \vec{p_i}$$

This is the generalisation of the definition of angular momentum for a system of particles.

Now

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(\Sigma \vec{L_i}) \qquad \ldots(96)$$

$$= \sum_{i=1}^{i=n} \frac{d\vec{L_i}}{dt} = \sum_{i=1}^{i=n} \vec{\tau_i} \qquad \ldots(97)$$

where $\vec{\tau_i}$ is the torque acting on ith particle

$$\vec{\tau_i} = \vec{r_i} \times \vec{F_i} \qquad \ldots(98)$$

The force $\vec{F_i}$ on the *ith particle is the vector sum of external force* $\vec{F_i}^{\text{ext}}$ *acting on the particle, and the internal force* $\vec{F_i}^{\text{int}}$ *exerted on it by the other particles of the system. Therefore, total torque*

$$\vec{\tau} = \sum_{i=1}^{i=n} \vec{\tau_i} = \sum_{i=1}^{i=n} \vec{r_i} \times \vec{F_i}$$

As $\vec{F_i} = \vec{F_i}^{\text{ext}} + \vec{F_i}^{\text{int}}$, therefore, $\vec{\tau}_{\text{ext}} = \sum_{i=1}^{i=n} \vec{r_i} \times \vec{F_i}^{\text{ext}}$, and $\vec{\tau}_{\text{int}} = \sum_{i=1}^{i=n} \vec{r_i} \times \vec{F_i}^{\text{int}}$

According to Newton's third law, the internal forces between any two particles of the system are equal and opposite. Therefore, contribution of internal forces to the total torque on the system is zero

i.e., $\vec{\tau}_{\text{int}} = 0 \quad \therefore \quad \vec{\tau} = \sum_{i=1}^{i=n} \vec{\tau_i} = \vec{\tau}_{\text{ext}}$

From (96),

$$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{\tau}_{\text{ext}}} \qquad \ldots(99)$$

Hence the time rate of change of total angular momentum of a system of particles about a given point is equal to the vector sum of external torques acting on the system, taken about the same point.

This is the generalisation of the single particle case to the system of particles. This eqn. (99) is the rotational analogue of the equation

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{ext}}$$

The results of Art. 7.22 regarding angular momentum and torque of a system of particles apply equally well to rigid bodies.

Curiosity Questions

Q. 1. How do you account for the motion of your arms and legs ?

Ans. The concept of torque accounts for the motion of our arms and legs.

For example, when we bend our arm at the elbow, our forearm rotates about the elbow, which acts as fulcrum. The biceps produce the torque. Muscles, exert forces when they shorten and the tendons carry these forces to the required location.

When we hold a metal block in our hand of the bent arm, the force of gravity on the block tends to straighten our arm by exerting a torque. To keep our arm bent, our biceps must produce an equal torque in the opposite direction. Obviously, the force exerted by the biceps, which is closer to the hinge (elbow) must be greater than the force of gravity on the block, which is farther from the elbow.

Q. 2. What do you know about artificial joints ?

Ans. On account of tremendous stress on the bones and joints of arms and legs, designing and substituting artificial joints for the damaged ones have become really important.

The hip and elbow joints are the most stressed ones. The ball and socket joint in the hip carries most of the body weight and is essential for walking.

The major consideration in artificial joints is their biocompatibility. Many systems fail because the body rejects the cement used to interface the artificial and real body parts.

The most common material used for making artificial joints is *titanium*. Other light weight plastics and materials similar to bone are also being developed.

7.23. EQUILIBRIUM OF RIGID BODIES

A body, in general, can undergo translational as well as rotational motions.

For mechanical equilibrium of a rigid body, *two conditions need to be satisfied.*

1. First condition of Equilibrium

A rigid body is said to be in translational equilibrium, if it remains at rest or moving with a constant velocity in a particular direction. For this, the net external force or the vector sum of all the external forces acting on the body must be zero., i.e.,

$$\vec{F} = 0 \quad \text{or} \quad \vec{F} = \sum_i \vec{F}_i = 0$$

It implies that in translational equilibrium of a rigid body, $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$ *i.e.* all forces along X-axis must balance out. Same is true for forces along Y and Z axes.

From Newton's second law, $\vec{F} = m\,\vec{a} = 0$, for translational equilibrium,

$$\therefore \quad \vec{a} = 0 \quad \text{or} \quad \frac{d\vec{v}}{dt} = 0$$

$$\therefore \quad \vec{v} = \text{constant or zero.}$$

It means when a body is in translational equilibrium, it will be either at rest ($v = 0$) or it will be in uniform motion (v = constant) along a straight line, *i.e.*, its linear momentum will *be constant. The body will* have zero linear acceleration. If the body is at rest, equilibrium is said to be ***static equilibrium.*** If the body is in uniform motion, equilibrium is said to be ***dynamic equilibrium.***

Retain in Memory

For static equilibrium, the linear momentum $\vec{p}$ of the body must be zero.

Further, $F = -\dfrac{dU}{dr}$, where U is potential energy of the body. In translational equilibrium,

$$F = -\frac{dU}{dr} = 0$$

$\therefore$ U = constant *i.e.* in equilibrium, *potential energy of the body is constant (maximum or minimum).*

The *translational static equilibrium* of a body may be of following three types :

(*i*) **Stable equilibrium :** *When a body tries to regain its equilibrium position after being slightly displaced and released, it is said to be in stable equilibrium.* Potential energy of the body in stable equilibrium tends to be minimum. For example, a table/chair lying on ground is in stable equilibrium.

(*ii*) **Unstable equilibrium :** *When a body gets disturbed further after being slightly displaced and released, it is said to be in unstable equilibrium.* Potential energy of the body in unstable equilibrium tends to increase. For example, a book standing on an edge is in unstable equilibrium.

(*iii*) **Neutral equilibrium :** *When a body can stay in equilibrium even after being slightly displaced and released, it is said to be in neutral equilibrium.* Potential energy of the body in neutral equilibrium remains constant *i.e.* there is no change in P.E. of the body. For example, a ball rolling on the ground is in neutral equilibrium.

Note that in stable equilibrium, the centre of mass of a body is raised on being displaced slightly. In unstable equilibrium, the centre of mass of the body is lowered on being displaced slightly ; and in neutral equilibrium, the centre of mass of the body remains at the same height on being displaced slightly.

2. Second condition of Equilibrium

A rigid body is said to be in rotational equilibrium, if the body does not rotate or rotates with constant angular velocity. For this, the net external torque or the vector sum of all the torques acting on the body is zero.

In other words, a body is in rotational equilibrium, when the algebraic sum of moments of all the forces acting on the body about a fixed point is zero.

Infact, a body is in rotational equilibrium, when algebraic sum of moments of all the forces acting on the body, about a fixed point/axis is zero. For example, in case of beam balance or see-saw shown in Fig. 7.31, the system will be in rotational equilibrium, if

$F_1 \times l_1 - F_2 \times l_2 = 0$

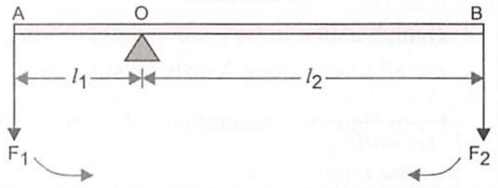
Now, $F_1 \times l_1 = +\tau_1$ (anticlockwise moment)

and $F_2 \times l_2 = -\tau_2$ (clockwise moment)

$\therefore$ $\tau_1 + \tau_2 = 0$

or $\Sigma\,\tau = 0$

FIGURE 7.31



i.e., for rotational equilibrium, *total external torque acting on the body must be zero.*

The equation of motion of a rotating body is given by

$$\vec{\tau}_{\text{ext}} = \sum_{i=1}^{i=n} \vec{\tau}_i = \frac{d\vec{L}}{dt}$$

For the body to be in rotational equilibrium, $\vec{\tau}_{\text{ext}} = 0 \;\therefore\; \dfrac{d\vec{L}}{dt} = 0$ or $\vec{L}$ = constant

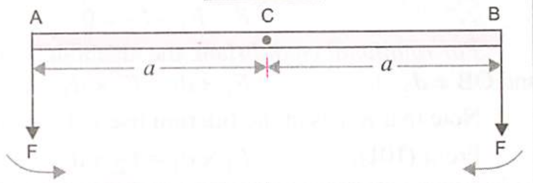
i.e., angular momentum of the body in rotational equilibrium will stay constant.

As $\vec{\tau}_{\text{ext}} = I\,\vec{\alpha} = 0 \;\therefore\; \vec{\alpha} = 0$

i.e., angular acceleration of the body in rotational equilibrium will be zero.

Sometimes, a rigid body may be in **partial equilibrium,** *i.e., it may be in translational equilibrium and not in rotational equilibrium OR the body may be in rotational equilibrium and not in translational equilibrium. For example,* let us consider a light rod AB of negligible mass with centre at C. Two parallel forces, each of magnitude F are applied at the ends perpendicular to the rod as shown in Fig. 7.32.

FIGURE 7.32

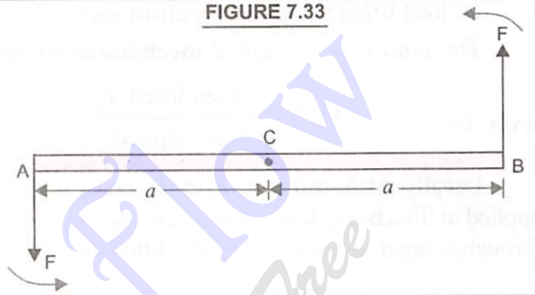


$\therefore$ Net external force $= F + F = 2F \neq 0$

As $\Sigma \vec{F} \neq 0$, therefore, *the rod will not be in translational equilibrium.* However, the moment of forces at A and B about fixed point C will be equal in magnitude (= *a*F), but opposite in sense. Therefore, the net moment of forces on the rod will be zero. Hence *the rod will be in rotational equilibrium.*

Let the force applied at end B of the rod be reversed as shown in Fig. 7.33. Here, the net external force on the rod $= F - F = 0$.

FIGURE 7.33



$\therefore$ *The rod is in translational equilibrium.*

Now, moments of both forces about C are equal (= *a*F), and they act in the same sense and cause anticlockwise rotation of the rod. Thus, the *rod is not in rotational equilibrium.*

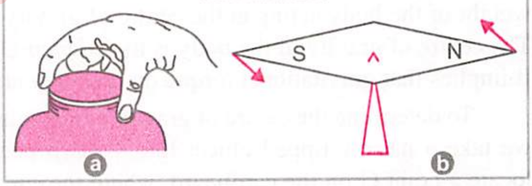
IMPORTANT NOTE

A pair of equal and opposite forces with different lines of action are said to form a **couple.** A couple shown in Fig. 7.33, *produces rotation without translation.*

For example, when we open the lid of a bottle by turning it, our fingers apply a couple on the lid, Fig. 7.34(a).

Again, when a compass needle is held arbitrarily in any direction in earth's magnetic field, a couple acts on the needle. The force on north pole of needle is towards the north and force on south pole of needle is towards the south. The forces are equal in magnitude and opposite in direction. The couple so formed rotates the compass needle and aligns it along north south direction, Fig. 7.34(b).

FIGURE 7.34



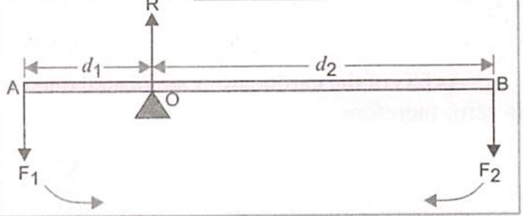
7.24. PRINCIPLE OF MOMENTS

According to the principle of moments, a body will be in rotational equilibrium if algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.

By convention, *we take anticlockwise moments as positive and clockwise moments as negative. It means for rotational equilibrium of a body, sum of anticlockwise moments about a fixed point is equal to the sum of clockwise moments, about the same point.*

To illustrate this principle, let us take an *ideal lever* comprising of a light rod AB of negligible mass pivoted at a point O, Fig. 7.35. This point is called **fulcrum** of the lever. A seasaw on children's playground, the beam of a physical balance are common examples of lever. Two forces F_1 and F_2 parallel to eachother and per- pendicular to the lever rod are applied at the ends A and B as shown. Let R be the reaction of the support at the fulcrum. R is directed opposite to the forces F_1 and F_2.

FIGURE 7.35



The lever is a system in mechanical equilibrium. *For translational equilibrium,* net force = 0.

$$\therefore \quad R - F_1 - F_2 = 0 \quad \text{or} \quad R = F_1 + F_2 \qquad \text{...(100)}$$

For rotational equilibrium, the algebraic sum of moments of forces about O must be zero. If AO = d_1 and OB = d_2, then

$$F_1 \times d_1 - F_2 \times d_2 = 0 \qquad \text{...(101)}$$

Note that R acts at the fulcrum itself. Therefore its moment about the fulcrum is zero.

From (101),

$$F_1 \times d_1 = F_2 \times d_2 \qquad \text{...(102)}$$

In case of the lever, force F_1 is usually some weight to be lifted. It is called **load** ; and its distance from the fulcrum ; AO = d_1 is called the **load arm.** Again, force F_2 is the **effort applied** to lift the load and its distance from the fulcrum ; OB = d_2 is called the **effort arm.** Therefore, from (102), we find that

load lifted × load arm = effort applied × effort arm ...(103)

The ratio F_1/F_2 is called **mechanical advantage** (M.A.) of the lever, *i.e.,*

$$\text{M.A.} = \frac{\text{load lifted, } F_1}{\text{effort applied, } F_2} = \frac{d_2}{d_1}$$

Usually, M.A. > 1, *i.e.,* $F_1 > F_2$ or $F_2 < F_1$, *i.e.,* a small effort is applied to lift a heavy load. In that case, $d_2 > d_1$, *i.e.,* effort is to be applied through a larger distance and load is lifted through a smaller distance.

DO YOU KNOW ?

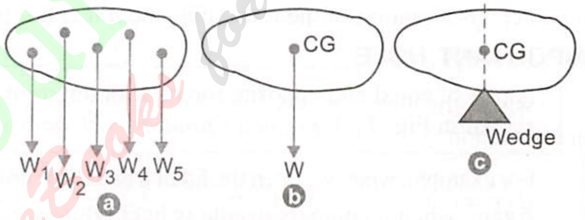
The principle of moments is true even when forces F_1 and F_2 act at some angle to the lever rod. The forces need not only be perpendicular to the rod.

7.25. CENTRE OF GRAVITY

The centre of gravity (CG) of a body is a point where the entire weight of the body (i.e., gravitational force on the body) acts. Obviously, gravitational torque on the body about this point is always zero.

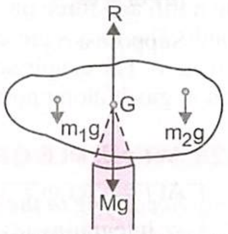
In Fig. 7.36(a), we have shown the weights W_1, W_2, W_3, W_4, W_5 ..... of particles of a body situated at different positions. W is the resultant weight of the body acting at the centre of gravity (CG) of the body, Fig. 7.36(b). The centre of gravity of the body is its *balancing point,* as shown in Fig. 7.36(c). It implies that gravitational torque on the body about CG is zero.

FIGURE 7.36



To determine the centre of gravity of irregular shaped body, say a cardboard, we take a narrow tipped object like a sharp pencil. By trial and error, we can locate a point G on the cardboard, where the cardboard is balanced on the tip of the pencil. This *point of balance* is the centre of gravity of the cardboard. The tip of the pencil at G provides the *reaction R* (in the upward direction) to the *total weight Mg* of the cardboard. The cardboard is in translational equilibrium, as R = Mg, Fig. 7.37.

FIGURE 7.37



Also, the cardboard is in rotational equilibrium-because if it were not so, the cardboard due to an unbalanced torque, would tilt and fall. Forces of gravity like m_1g, m_2g, ... etc. act on individual particles of the cardboard. They make up the torques on the cardboard. For *i*th particle of mass m_i, the force of gravity is $m_i\,(\vec{g})$. If $\vec{r}_i$ is the position vector of this particle from CG of the cardboard, then torque about the CG is

$$\vec{\tau}_i = \vec{r}_i \times m_i\,(\vec{g}) \qquad \text{...(104)}$$

As CG of the cardboard is so located that the total torque on it due to forces of gravity on all the particles is zero, therefore,

$$\vec{\tau} = \sum_{i=1}^{i=n} \vec{\tau}_i = \sum_{i=1}^{i=n} \vec{r}_i \times m_i\,(\vec{g}) = 0 \qquad \text{...(105)}$$

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As $\vec{g}$ is non-zero and same for all the particles of the body, we may rewrite (105) as

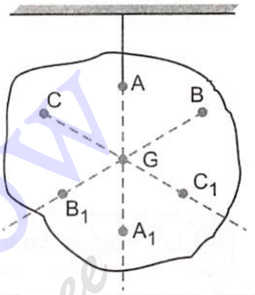
$$\sum_{i=1}^{i=n} m_i \vec{r}_i = 0 \qquad \text{...(106)}$$

This is the condition, when *centre of mass of the body lies at the origin.* As position vectors $\vec{r}_i$ are taken w.r.t. the CG, therefore, ***the centre of gravity of the body coincides with the centre of mass of the body.***

However, ***if the body is so extended that $\vec{g}$ varies from part to part of the body, then the centre of gravity shall not coincide with the centre of mass of the body.***

There is yet another way of determining the CG of a body of irregular shape. Suspend the body from some point like A. Draw the vertical line AA_1. It passes through CG. Similarly, draw vertical lines BB_1 and CC_1 by suspending the body from some other points B and C etc. as shown in Fig. 7.38. The point of intersection G of these verticals gives us the position of CG of the irregular body. If the body is small enough, $\vec{g}$ is same for all the particles of the body. Therefore, centre of mass of the body would coincide with its CG.

FIGURE 7.38



7.26. EQUATIONS OF ROTATIONAL MOTION

Three simple relations between rotational kinematic variables are :

(*i*) $\omega = \omega_0 + \alpha t$ (*ii*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ and (*iii*) $\omega^2 - \omega_0^2 = 2\alpha\theta$,

where the symbols have their usual meaning. They correspond respectively to the three equations of linear motion

(*i*) $v = u + at$ (*ii*) $s = ut + \frac{1}{2}at^2$ (*iii*) $v^2 - u^2 = 2as$

We can deduce rotational kinematic equations as follows :

(*a*) $\omega = \omega_0 + \alpha t$

Suppose a rigid body is rotating about a given axis with a uniform angular acceleration α. We know that

$$\alpha = \frac{d\omega}{dt} \quad \text{or} \quad d\omega = \alpha\, dt \qquad \text{...(107)}$$

At $t = 0$, let $\omega = \omega_0$

At $t = t$, let $\omega = \omega$

∴ Integrating (107) within proper limits, we get $\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt$

$$[\omega]_{\omega_0}^{\omega} = \alpha\,[t]_0^t$$

$$\omega - \omega_0 = \alpha(t - 0) = \alpha t \quad \text{or} \quad \omega = \omega_0 + \alpha t \qquad \text{...(108)}$$

(*b*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

If ω is angular velocity of the rigid body at any time *t*, then we know that

$$\omega = \frac{d\theta}{dt} \quad \therefore \quad d\theta = \omega\, dt \qquad \text{...(109)}$$

At $t = 0$, let $\theta = 0$

At $t = t$, let $\theta = \theta$

∴ Integrating (109) within proper limits, we get

$$\int_0^\theta d\theta = \int_0^t \omega\, dt = \int_0^t (\omega_0 + \alpha t)\, dt \qquad \text{...using (108)}$$

$$\int_0^\theta d\theta = \int_0^t \omega_0\, dt + \int_0^t \alpha t\, dt$$

$$\Big[\theta\Big]_0^\theta = \omega_0 \Big[t\Big]_0^t + \alpha\left[\frac{t^2}{2}\right]_0^t$$

$$\theta - 0 = \omega_0 (t-0) + \frac{\alpha}{2}(t^2 - 0)$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \text{...(110)}$$

$$\boxed{(c)\ \omega^2 - \omega_0^2 = 2\alpha\theta}$$

We know that $\omega = \frac{d\theta}{dt}$ and $\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\left(\frac{d\theta}{dt}\right) = \left(\frac{d\omega}{d\theta}\right)\omega$ or $\omega\, d\omega = \alpha\, d\theta$ ...(111)

When $\theta = 0$, $\omega = \omega_0$, initial angular velocity and when $\theta = \theta$, $\omega = \omega$, final angular velocity

∴ Integrating (111) within proper limits, we get

$$\int_{\omega_0}^{\omega} d\omega = \int_0^\theta \alpha\, d\theta$$

$$\left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \alpha\Big[\theta\Big]_0^\theta$$

$$\frac{\omega^2 - \omega_0^2}{2} = \alpha(\theta - 0) = \alpha\theta \quad \text{or} \quad \omega^2 - \omega_0^2 = 2\alpha\theta \qquad \text{...(112)}$$

Sample Problem **The motor of an engine is rotating about its axis with an angular velocity of 100 rpm. It comes to rest in 15 s, after being switched off. Assuming constant angular deceleration, calculate the number of revolutions made by it before coming to rest.**

Sol. Here, $\omega_0 = 100 \text{ rpm} = \frac{100}{60} \times 2\pi = \frac{10\pi}{3}$ rad/s

$\omega = 0,\ t = 15$ s

From $\omega = \omega_0 + \alpha t$

$$0 = \frac{10\pi}{3} + \alpha \times 15 \quad \text{or} \quad \alpha = \frac{-10\pi}{45}$$

$$\alpha = \frac{-2\pi}{9}\ \text{rad/s}^2$$

Angle traced by the motor in the process is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{10\pi}{3} \times 15 + \frac{1}{2}\left(\frac{-2\pi}{9}\right)(15)^2 = 50\pi - 25\pi = 25\pi$$

∴ No. of revolutions $= \frac{\theta}{2\pi} = \frac{25\pi}{2\pi} = \mathbf{12{\cdot}5}$

7.27. ROTATIONAL INERTIA/MOMENT OF INERTIA

In unit 3, we have studied *inertia of linear motion i.e.* inability of a body to change by itself, its state of rest or state of uniform linear motion. An external force is always required to change the state of rest or the state of uniform linear motion of the body. This force varies directly as the mass of the body. ***Hence mass of a body is a measure of inertia of the body in linear motion.***

Similarly, a body at rest cannot start rotating about an axis on its own ; and a body rotating about a given axis cannot stop on its own, *i.e.* there is inertia of rotational motion as well.

A quantity that measures the inertia of rotational motion of the body is called rotational inertia or moment of inertia of the body.

Retain in Memory

Thus rotational inertia plays the same role in rotational motion as mass plays in linear motion, *i.e. moment of inertia is rotational analogue of mass in linear motion.*

7.28. KINETIC ENERGY OF ROTATION

Kinetic energy of rotation of a body is the energy possessed by the body on account of its rotation about a given axis.

In Fig. 7.39, we have shown a rigid body rotating in *XY* plane about Z-axis, with a uniform angular velocity ω.

Let the body consist of particles of masses $m_1, m_2, m_3, ...$ at perpendicular distances $r_1, r_2, r_3 ......$ respectively from the axis of rotation.

As the body is rigid, all the particles of the body complete one revolution about *OZ* in the same time. Therefore, angular velocity of all the particles is the same *i.e.*, ω. However, as distances of particles from the axis of rotation are different, therefore linear velocities of different particles are different. If $v_1, v_2, v_3 ......$ are the respective linear velocities of the particles, then $v_1 = r_1\,\omega, v_2 = r_2\,\omega, v_3 = r_3\,\omega .....$

Kinetic energy of particle of mass m_1 is $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (r_1\,\omega)^2 = \frac{1}{2} m_1 r_1^2 \omega^2$

Similarly, kinetic energies of other particles of the body are $\frac{1}{2} m_2 r_2^2 \omega^2, \frac{1}{2} m_3 r_3^2 \omega^2, ...$

$\therefore$ K.E. of rotation of the body

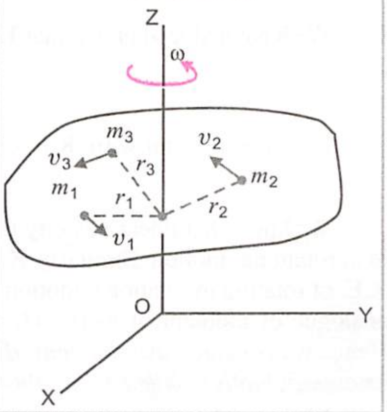
$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + ...$$

$$= \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + m_2 r_3^2 + ...)\,\omega^2$$

$$\text{K.E. of rotation} = \frac{1}{2}\left(\sum_{i=1}^{i=n} m_i r_i^2\right)\omega^2 \qquad ...(113)$$

Put $\boxed{\sum_{i=1}^{i=n} m_i r_i^2 = I}$ ...(114)

FIGURE 7.39



where *I* is called **moment of inertia** of the body about the given axis of rotation.

Clearly, moment of inertia I involves both, the mass of the body and its distribution about the given axis.

$$\therefore \quad \boxed{\text{K.E. of rotation} = \frac{1}{2} I \omega^2} \qquad \text{...(115)}$$

7.29. MOMENT OF INERTIA AND ITS PHYSICAL SIGNIFICANCE

In Art. 7.2, we have represented moment of inertia as $I = \sum_{i=1}^{i=n} m_i r_i^2$. Hence we may define

***Moment of inertia** of a body about a given axis as the sum of the products of masses of all the particles of the body and squares of their respective perpendicular distances from the axis of rotation.*

The value of I shall depend upon (*i*) *position of the axis of rotation,* (*ii*) *orientation of the axis of rotation,* (*iii*) *shape of the body,* (*iv*) *size of the body,* (*v*) *distribution of mass of the body about the axis of rotation.*

When the body does not consist of discrete particles, and has a continuous uniform distribution of mass, the summation in eqn. (114) can be replaced by integration and we may rewrite eqn. (114) as

$$I = \int r^2 \, dm \qquad \text{...(116)}$$

where dm is the mass of a small element of the body at a perpendicular distance r from the axis of rotation. If ρ is density of the body, then the mass of small element of volume dV is $dm = \rho . dV$.

$\therefore$ From (4), $\quad I = \int \rho \, r^2 . dV$

We can also define moment of inertia I from eqn. (115) :

$$\text{K.E. of rotation} = \frac{1}{2} I \omega^2$$

$$\text{If} \quad \omega = 1, \text{ then K.E. of rotation} = \frac{1}{2} I \times 1^2 = \frac{I}{2} \quad \text{or} \quad \boxed{I = 2 \times \text{KE of rotation}} \quad \text{i.e.,} \qquad \text{...(117)}$$

***moment of inertia** of a body about a given axis is equal to twice the K.E. of rotation of the body rotating with unit angular velocity about the given axis.*

Physical significance of Moment of Inertia

We have deduced above that K.E. of rotation of a body $= \frac{1}{2} I \omega^2$ ...(115)

Let us compare it with : K.E. of translation of the body $= \frac{1}{2} m v^2$ ...(118)

We know that linear velocity v is an analogue of angular velocity ω in rotational motion. Similarly, K.E. of translation is an analogue of K.E. of rotation in rotational motion. Hence mass (m) of the body is an analogue of moment of inertia (I) of the body in rotational motion. Hence we conclude that ***moment of inertia (I) plays the same role in rotational motion as mass (m) plays in linear motion.***

We know that mass of a body is regarded as a measure of inertia of the body for linear/ translational motion.

DO YOU KNOW ?

Jerky motion of a vehicle can be reduced by attaching a flywheel with its engine. A flywheel is a circular disc capable of rotating about an axle passing through its centre and perpendicular to its plane. Most of the mass of flywheel is concentrated at its rim (*i.e.*, away from the axis of rotation). Therefore, moment of inertia of flywheel is large. Whenever, there is sudden increase or decrease in the speed of the vehicle, the large moment of inertia of flywheel opposes it. That is how sudden jerks are minimised.

Infact, a body free to rotate about an axis has a tendency to oppose any change in its state of rest or state of uniform rotation, *i.e.*, the body opposes any torque or moment of couple that tends to change its state of uniform rotation. Thus the body has inertia for rotational motion. The nature of rotational inertia must be that of a moment of force. Hence the name moment of inertia. Greater the moment of inertia of a body about a given axis, more difficult it will be for a torque to change the state of rotation of the body. Hence ***moment of inertia of a body can be regarded as a measure of rotational inertia of the body.***

Units of Moment of Inertia

As $\quad I = \text{mass (distance)}^2$

$\therefore$ units of moment of inertia are $kg\ m^2$ or $g\ cm^2$. The *dimensions* of moment of inertia are $[M^1 L^2 T^0]$

As explained already in Unit II, ***moment of inertia of a body is treated as tensor.***

This is because moment of inertia of a body about different axes of rotation has different values. You may recollect that other examples of tensors are density, stress, strain, refractive index, etc.

7.30. RADIUS OF GYRATION

The radius of gyration of a body about a given axis is the perpendicular distance of a point P from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass. This distance is represented by K.

Suppose a rigid body consists of n particles, each of mass m. Let $r_1, r_2, r_3 \ldots\ r_n$ be the perpendicular distances of these particles from the axis of rotation, Fig. 7.40.

By definition, moment of inertia of the body about the given axis is

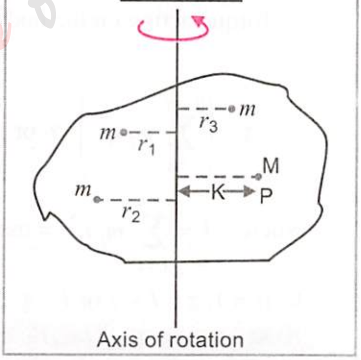
$$I = m r_1^2 + m r_2^2 + m r_3^2 + \ldots\ldots m r_n^2$$

$$= m\,(r_1^2 + r_2^2 + r_3^2 + \ldots r_n^2)$$

$$= m \times n \left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots r_n^2}{n} \right)$$

$$I = M \left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots r_n^2}{n} \right) \qquad \ldots(119)$$

FIGURE 7.40



where $\quad M = m \times n =$ total mass of the body.

If the total mass of the body M were concentrated at P at a perpendicular distance K from the axis, its moment of inertia would be

$$I' = M K^2$$

When K is radius of gyration, $I' = I$

$$\therefore \quad M\ K^2 = M\,\frac{(r_1^2 + r_2^2 + \ldots r_n^2)}{n} \quad \text{or} \quad \boxed{K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots r_n^2}{n}}} \quad \text{Hence} \qquad \ldots(120)$$

Radius of gyration of a body about a given axis is equal to root mean square distance of the constituent particles of the body from the given axis of rotation.

The radius of gyration being a distance is measured in *metre* or *cm*. Its dimensions are $[M^0 L^1 T^0]$.

The value of radius of gyration shall depend upon (i) shape and size of the body, (ii) position and configuration of the axis of rotation, and (iii) distribution of mass of the body w.r.t. the axis of rotation.

Sample Problem **A body of moment of inertia 0·5 kg m^2 is rotating about a given axis at the rate 1 rps. What is kinetic energy of rotation of the body about that axis ?**

Sol. Here, $I = 0{\cdot}5$ kg m^2, $n = 1$ *rps*, $KE = ?$

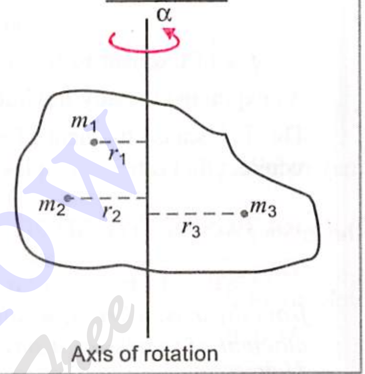
$$KE \text{ of rotation } = \frac{1}{2} I\,\omega^2 = \frac{1}{2} I\,(2\pi n)^2 = 2\,\pi^2 n^2 I = 2 \times \frac{22}{7} \times \frac{22}{7} \times 1^2 \times 0{\cdot}5 = \mathbf{9{\cdot}85\ J}$$

7.31. TORQUE AND MOMENT OF INERTIA

Consider a rigid body rotating about a given axis with a uniform angular acceleration α, under the action of a torque.

Let the body consist of particles of masses m_1, m_2, m_3 ..... m_n at perpendicular distances r_1, r_2, r_3, ... r_n respectively from the axis of rotation, Fig. 7.41.

FIGURE 7.41



As the body is rigid, angular acceleration α of all the particles of the body is the same. However, their linear accelerations are different because of different distances of the particles from the axis.

If a_1, a_2, a_3, ... a_n are the respective linear accelerations of the particles, then $a_1 = r_1\,\alpha,\ a_2 = r_2\,\alpha,\ a_3 = r_3\,\alpha$, ...

Force on particle of mass m_1 is $f_1 = m_1\,a_1 = m_1\,r_1\,\alpha$

Moment of this force about the axis of rotation $= f_1 \times r_1 = (m_1\,r_1\,\alpha) \times r_1 = m_1\,r_1^2\,\alpha$

Similarly, moments of forces on other particles about the axis of rotation $m_2\,r_2^2\,\alpha,\ m_3\,r_3^2\alpha, ...m_n\,r_n^2\alpha$.

$\therefore$ Torque acting on the body, $\tau = m_1\,r_1^2\,\alpha + m_2\,r_2^2\,\alpha + m_3\,r_3^2\,\alpha + ...m_n\,r_n^2\,\alpha$

$$= (m_1\,r_1^2 + m_2\,r_2^2 + m_3\,r_3^2 + ...m_n\,r_n^2)\,\alpha$$

$$\tau = \left(\sum_{i=1}^{i=n} m_i\,r_1^2\right)\alpha \text{ or } \boxed{\tau = I\,\alpha} \qquad ...(121)$$

where $I = \sum_{i=1}^{i=n} m_i\,r_1^2$ = moment of inertia of the body about the given axis of rotation.

If $\alpha = 1,\ \tau = I \times 1$ or $I = \tau$

Hence, moment of inertia of a body about a given axis is numerically equal to torque acting on the body rotating with unit angular acceleration about it.

We may rewrite eqn. (121) in vector form as $\boxed{\vec{\tau} = I\,\vec{\alpha}}$ ...(122)

This equation is called **fundamental equation of rotation or law of rotation.** It corresponds to the relation $\boxed{\vec{F} = m\,\vec{a}}$ which is the fundamental equation of linear motion.

7.32. ANGULAR MOMENTUM AND MOMENT OF INERTIA

Consider a rigid body rotating about a given axis with a uniform angular velocity ω, Fig. 7.42. Let the body consist of n particles of masses m_1, m_2, m_3, ... m_n at perpendicular distances r_1, r_2, r_3, ... r_n respectively from the axis of rotation.

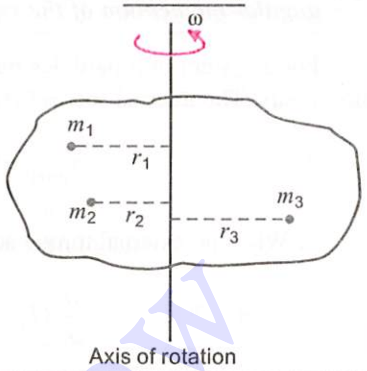
As the body is rigid, angular velocity ω of all the particles is the same. However, as the distances of the particles from the axis of rotation are different, their linear velocities are different. If v_1, v_2, v_3 ..... v_n are the linear velocities of the respective particles, then $v_1 = r_1\omega,\ v_2 = r_2\,\omega,\ v_3 = r_3\,\omega$, ...

The linear momentum of particle of mass m_1 is $p_1 = m_1\,v_1 = m_1\,(r_1\,\omega)$

The angular momentum of this particle about the given axis $= p_1 \times r_1 = (m_1\,r_1\,\omega) \times r_1 = m_1\,r_1^2\,\omega$

Similarly, angular momenta of other particles of the body about the given axis are $m_2 r_2^2 \omega$, $m_3 r_3^2 \omega$, ...$m_n r_n^2 \omega$

FIGURE 7.42



∴ Angular momentum of the body about the given axis,

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + \ldots m_n r_n^2 \omega$$

$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots m_n r_n^2)\,\omega$$

$$L = \left(\sum_{i=1}^{i=n} m_i r_i^2\right)\omega \quad \text{or} \quad \boxed{L = I\,\omega} \qquad \ldots(123)$$

where $I = \sum_{i=1}^{i=n} m_i r_i^2$ = moment of inertia of the body about the given axis.

Thus angular momentum of a body about a given axis is the product of moment of inertia and angular velocity of the body about that axis.

Also, if $\omega = 1$, then from (123), $L = I \times 1$ or $I = L$

Hence moment of inertia of a body about a given axis is numerically equal to angular momentum of the body rotating with unit angular velocity about that axis.

We can rewrite eqn. (123) in vector form as $\boxed{\vec{L} = I\,\vec{\omega}}$ ...(124)

This equation corresponds to the equation $\boxed{\vec{p} = m\,\vec{v}}$ for the linear/rectilinear motion. ...(125)

Sample Problem **Calculate angular momentum of earth rotating about its axis. Take $I = \frac{2}{5}MR^2$, where $M = 6 \times 10^{24}$ kg and $R = 6400$ km.**

Sol. Here, $L = ?$, $T = 24$ hrs $= 24 \times 60 \times 60$ s,

$M = 6 \times 10^{24}$ kg, $R = 6400$ km $= 6{\cdot}4 \times 10^6$ m

$$L = I\,\omega = \left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)$$

$$= \frac{4\pi MR^2}{5T} = \frac{4 \times 22}{5 \times 7} \times \frac{6 \times 10^{24}\,(6{\cdot}4 \times 10^6)^2}{24 \times 60 \times 60} = \mathbf{7{\cdot}15 \times 10^{33}\ kg\ m^2\ s^{-1}}$$

7.33. RELATION BETWEEN TORQUE AND ANGULAR MOMENTUM OF A RIGID BODY

We have already proved in case of a rigid body that

$$L = I\,\omega \qquad \ldots(126)$$

Differentiating both sides *w.r.t. t*, we get $\frac{dL}{dt} = I\left(\frac{d\omega}{dt}\right) = I\,\alpha$ ...(127)

where $\alpha = \frac{d\omega}{dt}$ = angular acceleration of the body

But torque, $\tau = I\,\alpha$ ...(121)

From (127) and (121), we get $\boxed{\tau = \frac{dL}{dt}}$ ...(128)

i.e., torque acting on a body is measured by the rate of change of angular momentum of the body.

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7.34. PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

According to this principle, when no external torque acts on a system of particles, then the total angular momentum of the system remains always a constant.

For a system of *n* particles making up a rigid body, total torque acting on the body is due to external forces only. The internal forces between the particles of the body do not contribute to the torque.

As $$\vec{\tau}_{\text{total}} = \frac{d}{dt}(\vec{L}_{\text{total}}) \qquad \text{...(129)}$$

$\therefore$ When no external torque acts on the system, $\vec{\tau}_{\text{total}} = 0$

$\therefore$ from (129), $$\frac{d}{dt}(\vec{L}_{\text{total}}) = 0$$

or $$\vec{L}_{\text{total}} = \text{constant}$$

i.e., $$\boxed{\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots.. \vec{L}_n = \text{constant (vector)}} \qquad \text{...(130)}$$

This is the principle of conservation of angular momentum.

It means that net angular momentum at some initial time is equal to net angular momentum at some later time, *i.e.,* if net external torque acting on a system is zero, angular momentum *L* of the system remains constant, *no matter what changes take place within the system.*

As eqns. (129) and (130) are vector equations, they are equivalent to three component equations corresponding to the conservation of angular momentum in three mutually perpendicular directions.

DO YOU KNOW ?

The law of conservation of angular momentum, the law of conservation of linear momentum and the law of conservation of energy, all the three hold in the world of subatomic particles, and also when speeds of objects approach the speed of light. No exceptions, whatsoever, have ever been found to these laws.

Retain in Memory

We have stated above that in the absence of an external torque, angular momentum vector remains constant. Hence the direction of $\vec{L}$ also remains unchanged. This suggests a fixed plane for the orbit of every planet. Hence we conclude that every planet revolves around the sun in a fixed elliptical orbit, *i.e., plane of the orbit of a planet can never change on its own.*

7.35. SOME EXAMPLES OF CONSERVATION OF ANGULAR MOMENTUM

1. ***The angular velocity of revolution of a planet around the sun in an elliptical orbit increases, when the planet comes closer to the sun and vice-versa.***

This is because, when the planet comes closer to the sun, its moment of inertia (*I*) about the sun decreases. As $L = I\,\omega$ is constant, therefore, its angular velocity ω increases. The reverse is also true.

2. ***A circus acrobat*** *performs feats involving spin by bringing her arms and legs closer to her body or vice-versa.* On bringing the arms and legs closer to the body, her moment of inertia *I* decreases. Hence her angular velocity ω increases. The reverse is also true.

3. *Exactly on the same basis as above,* ***an ice skater*** *or* ***a ballet dancer*** *performs the feats.*

Suppose a ballet dancer is rotating with her arms and legs stretched outwards, Fig. 7.43. When she folds her arms and brings the stretched leg close to the other leg, her moment of inertia decreases and hence her angular velocity increases as shown in Fig. 7.43.

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FIGURE 7.43

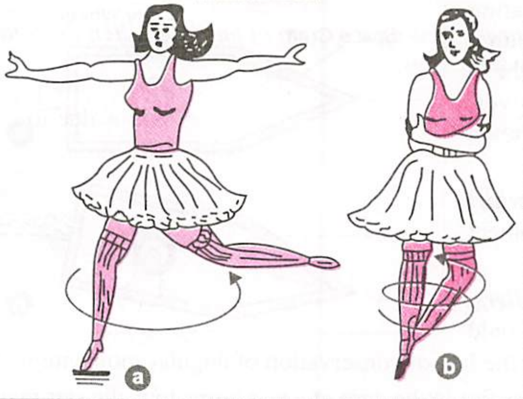
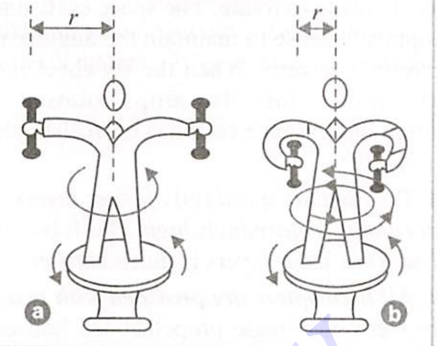


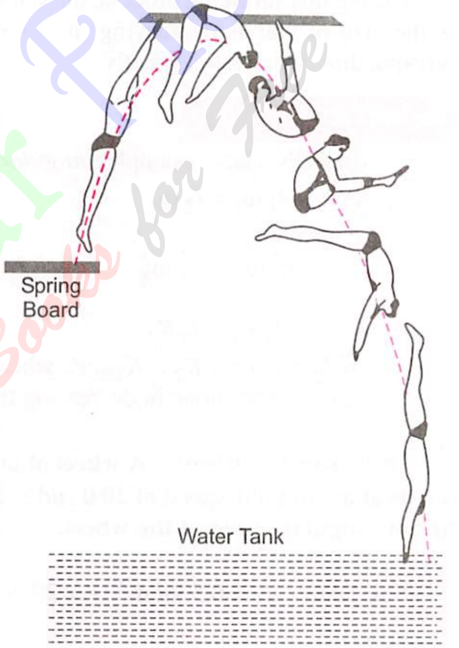
FIGURE 7.44



4. The spinning volunteer. *A person carrying dumbbells in his hands and standing on a rotating platform can change the speed of the platform.* Fig. 7.44(a) shows the dumbbells in arms which are outstretched. When the person suddenly folds his arms, Fig. 7.44(b), his moment of inertia (I) decreases because effective distance r of the dumbbells from the axis of rotation decreases. As no net external torque acts on the system consisting of the person, dumbbells and rotating table, therefore angular momentum ($I \times \omega$) remains constant. Therefore, angular velocity (ω) of his body, which is the angular velocity of the rotating platform increases.

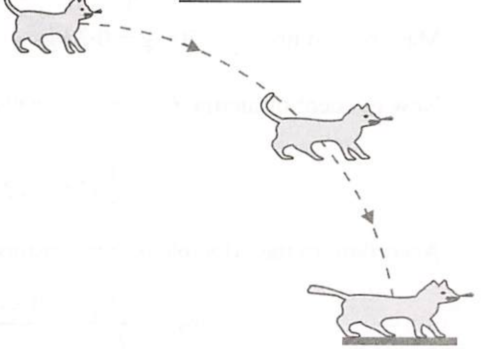
5. A springboard diver. *A diver performs somersaults by jumping from a high diving board keeping his legs and arms out stretched first, and then curling his body.* On doing so, the moment of inertia I of his body decreases. As angular momentum ($I \times \omega$) remains constant, therefore, angular velocity ω of his body increases. He then performs somersaults. As the diver is about to touch the surface of water, he stretches out his limbs. The moment of inertia of his body increases and his angular velocity decreases. He can enter the water with a little splash. All this is shown in Fig. 7.45.

FIGURE 7.45



6. *While jumping, a cat* *stretches its body alongwith its tail so that its moment of inertia (I) increases.* As no external torque is acting. $L = I\ \omega =$ constant. Since I increases, ω decreases and the cat lands gently on its feet, Fig. 7.46.

FIGURE 7.46



7. *Space craft orientation*

A flywheel is mounted rigidly on a space craft for orientation control as shown in Fig. 7.47(a). The space craft and fly wheel form an isolated system. When neither the space craft nor the flywheel is turning, the total angular momentum L of the system is zero.

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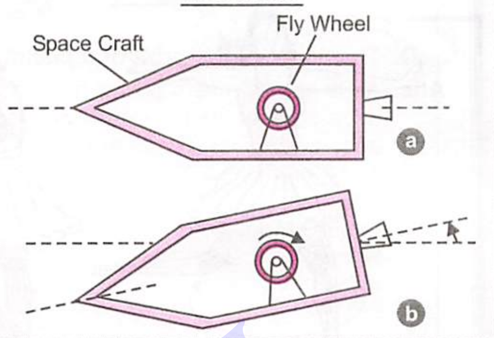
To change the orientation of the space craft, the flywheel is made to rotate. The space craft starts rotating in the opposite sense to maintain the angular momentum of the system at zero. When the flywheel is brought to rest, the space craft also stops rotating. However, orientation of the space craft has been changed as shown in Fig. 7.47(b).

FIGURE 7.47



8. *The angular speed (ω) of inner layers of the* ***whirl wind (tornado)*** *is alarmingly high.* This is because moment of inertia (*I*) of inner layers is much smaller.

9. ***All helicopters are provided with two propellers.*** If there were one single propeller, the helicopter would rotate itself in opposite direction in accordance with the law of conservation of angular momentum.

10. ***The incredible shrinking star.*** When nuclear fire in the core of a star burns low, the star may begin to collapse, ultimately becoming a *neutron star.*

During this shrinking process, the star is an isolated system and its angular momentum *L* cannot change. As the size of star is decreasing, its rotational inertia *I* is decreasing. Therefore, its angular speed is correspondingly increased greatly.

Retain in Memory

In all the above examples, ***angular momentum is conserved, but K.E. of rotation is not conserved.***

As $I_1\,\omega_1 = I_2\,\omega_2$

$$\therefore \quad I_1^2\,\omega_1^2 = I_2^2\,\omega_2^2 \qquad \text{or} \qquad I_1\left(\frac{1}{2}I_1\,\omega_1^2\right) = I_2\left(\frac{1}{2}I_2\,\omega_2^2\right)$$

i.e. $I_1\,K_1 = I_2\,K_2$

If $I_2 < I_1$ then $K_2 > K_1$, *i.e.*, when moment of inertia decreases, rotational KE increases. This is due to work done in decreasing the moment of inertia of the body.

Sample Problem **A wheel of moment of inertia 0·5 kg m² and radius 20 cm is rotating about its axis at an angular speed of 20·0 rad/s. It picks up a stationary particle of mass 200 g at its edge. Find the new angular speed of the wheel.**

Sol. Here, $I_1 = 0{\cdot}5\ \text{kg m}^2$, $r = 20\ \text{cm} = \frac{20}{100}\ \text{m} = \frac{1}{5}\ \text{m}$, $\omega_1 = 20{\cdot}0$ rad/s, $\omega_2 = ?$

As $I_1 = \frac{1}{2} m_1 r_1^2 \quad \therefore \quad m_1 = \frac{2 I_1}{r_1^2} = \frac{2\times 0{\cdot}5}{(1\times 5)^2} = 25$ kg

Mass picked up, $m_2 = 200\ \text{g} = 0{\cdot}2$ kg

New moment of inertia $I_2 = \frac{1}{2} m_1 r^2 + m_2 r^2 = \frac{1}{2}(m_1 + m_2)\,r^2$

$$= \frac{1}{2}\,(25 + 0{\cdot}2)\left(\frac{1}{5}\right)^2 = \frac{25{\cdot}2}{50}\ \text{kg}\ m^2$$

According to the principle of conservation of angular momentum, $I_2\,\omega_2 = I_1\,\omega_1$

$$\omega_2 = \frac{I_1\,\omega_1}{I_2} = \frac{0{\cdot}5\times 20\times 50}{25{\cdot}2} = \mathbf{19{\cdot}84\ rad/s}$$

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Curiosity Question

Q. The plane of the orbit of a planet can never change on its own. Why ?

Ans. As is known, every planet revolves around the sun in an elliptical orbit. The centripetal force necessary for the purpose is provided by gravitational pull of the sun on the planet. As this force acts towards the centre of the sun, external torque due to this force is zero.

According to the principle of conservation of angular momentum, in the absence of external torque, angular momentum $\vec{L}$ of the planet shall remain constant in magnitude and direction. Hence, orbit of every planet will be in a fixed plane. The plane of the orbit cannot change on its own.

7.36. THEOREMS ON MOMENT OF INERTIA

When moment of inertia of a body about any given axis is known, we can calculate moment of inertia of the same body about another axis, using the following two theorems on moment of inertia :

1. Theorem of parallel axes, 2. Theorem of perpendicular axes.

7.37. THEOREM OF PARALLEL AXES

According to the theorem of parallel axes, moment of inertia of a rigid body about any axis AB is equal to moment of inertia of the body about another axis KL passing through centre of mass C of the body in a direction parallel to AB, plus the product of total mass M of the body and square of the perpendicular distance between the two parallel axes, AB and KL.

If h is perpendicular distance between the axes AB and KL, Fig. 7.48, then according to the theorem of parallel axes,

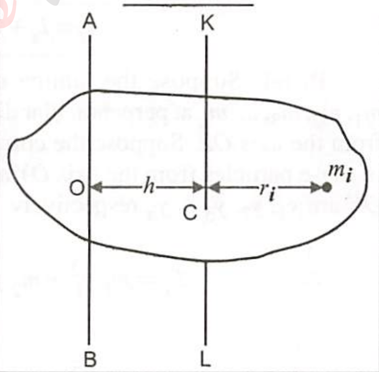
$$\boxed{I_{AB} = I_{KL} + Mh^2} \qquad \text{...(131)}$$

Proof. Suppose the rigid body is made up of n particles of masses $m_1, m_2, m_3 \ldots m_n$ at perpendicular distances $r_1, r_2, r_3, \ldots r_n$ respectively from the axis KL passing through centre of mass C of the body.

If r_i is the perpendicular distance of a particle of mass m_i from KL, then

$$I_{KL} = \sum_{i=1}^{i=n} m_i r_i^2 \qquad \text{...(132)}$$

FIGURE 7.48



The perpendicular distance of ith particle from the axis $AB = (r_i + h)$

$$\therefore \qquad I_{AB} = \sum_{i=1}^{i=n} m_i (r_i + h)^2 = \sum_{i=1}^{i=n} m_i (r_i^2 + h^2 + 2 r_i h)$$

$$I_{AB} = \sum_{i=1}^{i=n} m_i r_i^2 + \sum_{i=1}^{i=n} m_i h^2 + 2h \sum_{i=1}^{i=n} m_i r_i \qquad \text{...(133)}$$

As the body is balanced about the centre of mass, the algebraic sum of the moments of the weights (mg) of all particles about an axis passing through C must be zero

$$i.e., \quad \sum_{i=1}^{i=n} (m_i g) r_i = 0 \quad \text{or} \quad g \sum_{i=1}^{i=n} m_i r_i = 0$$

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As $g \neq 0$, $\therefore \sum_{i=1}^{i=n} m_i r_i = 0$

Also, $\sum_{i=1}^{i=n} m_i = M =$ total mass of the body.

$\therefore$ From eqn. (133) and eqn. (132), $I_{AB} = I_{KL} + M h^2$

This proves the theorem of parallel axes.

7.38. THEOREM OF PERPENDICULAR AXES

> ***According to the theorem of perpendicular axes, the moment of inertia of a plane lamina (i.e., a two dimensional body of any shape/size) about any axis OZ perpendicular to the plane of the lamina is equal to sum of the moments of inertia of the lamina about any two mutually perpendicular axes OX and OY in the plane of the lamina, meeting at a point where the given axis OZ passes through the lamina.***

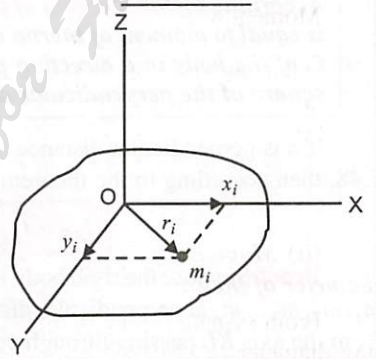
Suppose the lamina is in *XY* plane, Fig. 7.49. Let

I_x = moment of inertia of the lamina about *OX*
I_y = moment of inertia of the lamina about *OY*
I_z = moment of inertia of the lamina about *OZ*.

According to the theorem of perpendicular axes,

$$\boxed{I_z = I_x + I_y} \qquad \text{...(134)}$$

FIGURE 7.49



Proof. Suppose the lamina consists of *n* particles of masses $m_1, m_2, m_3, ... m_n$ at perpendicular distances $r_1, r_2, r_3, ... r_n$ respectively from the axis *OZ*. Suppose the corresponding perpendicular distances of these particles from the axis *OY* are $x_1 x_2, x_3 .... x_n$ and from the axis *OX* are $y_1, y_2, y_3 ... y_n$ respectively.

$$\therefore \qquad I_x = m_1 y_1^2 + m_2 y_2^2 + m_3 y_3^2 + ... m_n y_n^2 = \sum_{i=1}^{i=n} m_i y_i^2 \qquad \text{...(135)}$$

$$I_y = m_1 x_1^2 + m_2 x_2^2 + m_3 x_3^2 + ... m_n x_n^2 = \sum_{i=1}^{i=n} m_i x_i^2 \qquad \text{...(136)}$$

$$\text{and} \qquad I_z = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... m_n r_n^2 = \sum_{i=1}^{i=n} m_i r_i^2 \qquad \text{...(137)}$$

Adding eqns. (135) and (136), we get

$$I_x + I_y = \sum_{i=1}^{i=n} m_i y_i^2 + \sum_{i=1}^{i=n} m_i x_i^2 = \sum_{i=1}^{i=n} m_i (y_i^2 + x_i^2)$$

As is clear from Fig. 7.49, $r_i^2 = x_i^2 + y_i^2 \quad \therefore \quad I_x + I_y = \sum_{i=1}^{i=n} m_i r_i^2 = I_z$

Hence, $I_z = I_x + I_y$, which proves the theorem of perpendicular axes.

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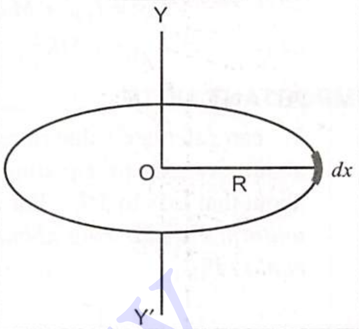
7.39. MOMENT OF INERTIA OF A THIN CIRCULAR RING

Suppose M is the mass of a thin circular ring of radius R with centre O. We have to calculate (*i*) *moment of inertia of the ring about an axis YOY′, perpendicular to the plane of the ring and passing through its centre*, Fig. 7.50.

FIGURE 7.50



Length of the ring = circumference = $2\pi R$

Mass per unit length of the ring = $M/2\pi R$

Consider a small element of the ring of length dx, Fig. 7.12.

Mass of this element $= \dfrac{M}{2\pi R}dx$

Moment of inertia of this element about YOY'

$$= \left(\frac{M}{2\pi R}dx\right)R^2 = \frac{MR}{2\pi}dx$$

As the small element can lie anywhere over the entire length of the ring, *i.e.*, from $x = 0$ to $x = 2\pi R$, therefore,

Moment of inertia of the entire circular ring about YOY'

$$I = \int_{x=0}^{x=2\pi R} \frac{MR}{2\pi}dx = \frac{MR}{2\pi}[x]_0^{2\pi R} = \frac{MR}{2\pi}(2\pi R - 0) = MR^2$$

$$\boxed{I = MR^2 = \text{mass (radius)}^2} \qquad \text{...(138)}$$

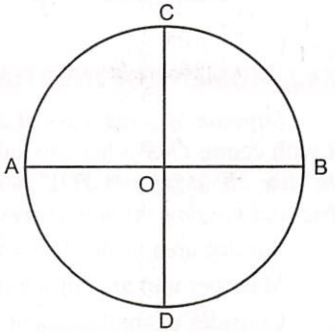
(ii) Moment of inertia of a uniform circular ring about any diameter of the ring.

From symmetry, we find that moment of inertia of the ring about any diameter (d) will be the same. Let us consider any two mutually perpendicular diameters AB and CD, Fig. 7.51. Using the *theorem of perpendicular axes*, we find that

$$I_{AB} + I_{CD} = MR^2$$

$$I_d + I_d = MR^2 \quad \text{or} \quad \boxed{I_d = \frac{1}{2}MR^2} \qquad \text{...(139)}$$

FIGURE 7.51



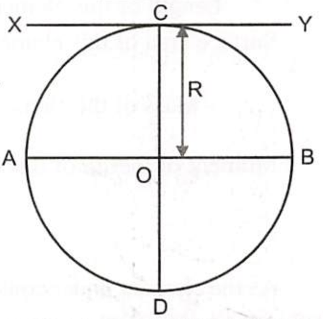
(iii) Moment of inertia of a uniform circular ring about a tangent in the plane of the ring.

In Fig. 7.52, XY is a tangent to the circular ring in the plane of the ring. To find I_{XY}, we use the *theorem of parallel axes*.

$$I_{XY} = I_{AB} + M(R)^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$\boxed{I_{XY} = \frac{3}{2}MR^2} \qquad \text{...(140)}$$

FIGURE 7.52



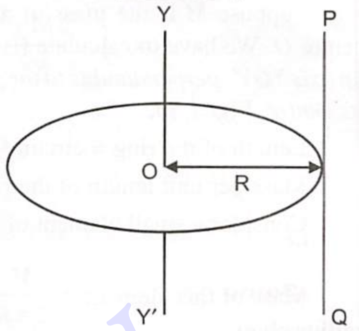
(iv) Moment of inertia of a uniform circular ring about a tangent perpendicular to the plane of the ring.

In Fig. 7.53, PQ is a tangent to the circular ring perpendicular to the plane of the ring. To find I_{PQ}, we use the *theorem of parallel axes.*

$$I_{PQ} = I_{YY'} + M\,(R)^2 = MR^2 + MR^2 = 2\,MR^2,$$

i.e., $$I_{PQ} = 2\,MR^2 \qquad \text{...(141)}$$

FIGURE 7.53



IMPORTANT NOTE

> We can calculate value of radius of gyration (K) of the body about any axis by equating moment of inertia of the body about that axis to MK^2. For example, *radius of gyration of a uniform circular ring about any diameter is obtained from eqn. (139).*
>
> $$I_d = \frac{1}{2}MR^2 = MK^2 \quad \therefore \quad K = \frac{R}{\sqrt{2}}$$

Sample Problem **Calculate moment of inertia of a uniform circular disc of mass 10 kg and diameter 0·5 m about a tangent perpendicular to the plane of the disc.**

Sol. Here, $M = 10$ kg, $R = \frac{0{\cdot}5}{2}\text{ m} = \frac{1}{4}\text{ m}$

About an axis passing through centre of disc perpendicular to its plane, $I = \frac{1}{2}MR^2$. Using theorem of parallel axis, moment of inertia about a tangent perpendicular to plane of disc,

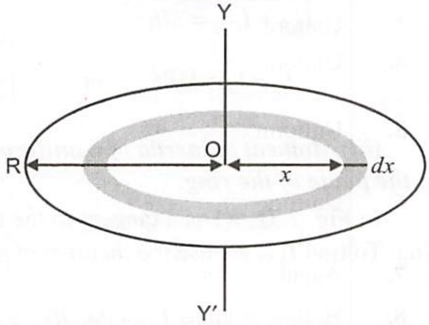
$$I' = I + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$I' = \frac{3}{2}\times 10\left(\frac{1}{4}\right)^2 = \frac{30}{32} = \mathbf{\frac{15}{16}\ kg\ m^2}$$

7.40. MOMENT OF INERTIA OF A UNIFORM CIRCULAR DISC

Suppose M is the mass of a uniform circular disc of radius R with centre O. We have to calculate (*i*) *moment of inertia of the disc about an axis YOY′ perpendicular to the plane of the disc and passing through its centre,* Fig. 7.54.

FIGURE 7.54



Surface area of the disc $= \pi R^2$

Mass per unit area of the disc $= M/\pi R^2$

Consider a small element of the disc, which would be a circular strip of radius x and width dx, say.

Length of this element $= 2\,\pi\,x$

Surface area of this element $= 2\,\pi\,x\,dx$

$$\therefore \quad \text{Mass of the element} = \frac{M}{\pi R^2}(2\,\pi\,x\,dx) = \frac{2\,M\,x\,dx}{R^2}$$

Moment of inertia of this element of the disc about YOY' = mass × (distance)^2

$$= \frac{2\,M\,x\,dx}{R^2}(x^2) = \frac{2\,M\,x^3\,dx}{R^2}$$

As the element under consideration may lie anywhere from the centre of disc ($x = 0$) to periphery of the disc ($x = R$), therefore,

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∴ Moment of inertia of the circular disc about YOY'

$$I = \int_{x=0}^{x=R} \frac{2\,\mathrm{M}\,x^3}{R^2}\,dx = \frac{2\,\mathrm{M}}{R^2}\int_{x=0}^{x=R} x^3\,dx = \frac{2\,\mathrm{M}}{R^2}\left[\frac{x^4}{4}\right]_{x=0}^{x=R} = \frac{2\,\mathrm{M}}{R^2}\,(R^4-0)$$

$$\boxed{I = \frac{1}{2}\mathrm{MR}^2} \qquad \text{...(142)}$$

i.e., $I = \frac{1}{2}$ (mass) (radius)^2

Sample Problem **Show that the radii of gyration of a circular ring and circular disc of the same radius about an axis passing through their centres and perpendicular to their plane are in the ratio $\sqrt{2} : 1$.**

Sol. Here, moment of inertia of circular ring about the given axis is $I_1 = MK_1^2 = MR^2$ ...(*i*)

and moment of inertia of circular disc about the same axis is $I_2 = MK_2^2 = \frac{1}{2}MR^2$ ...(*ii*)

Dividing (*i*) by (*ii*), we get $\frac{K_1^2}{K_2^2} = \frac{2}{1}$ or $\frac{K_1}{K_2} = \frac{\sqrt{2}}{1}$

7.41. MOMENT OF INERTIA OF SOME BODIES OF REGULAR SHAPE

Table 7.2 lists the formulae of moment of inertia of some bodies of regular shape about given axes :

TABLE 7.2. Moment of Inertia of some bodies of regular shape about given axis

S. No.	Body	Axis of Rotation	Moment of Inertia
1.	Uniform rod of length l	perpendicular to rod through its centre	$\frac{1}{12} M\,l^2$
2.	Uniform rectangular lamina of length l and breadth b	perpendicular to lamina and through its centre	$M\left(\frac{l^2+b^2}{12}\right)$
3.	Uniform circular ring of radius R	perpendicular to its plane and through the centre	MR^2
4.	Uniform circular ring of radius R	Diameter	$MR^2/2$
5.	Uniform circular disc of radius R	perpendicular to its plane and through the centre	$\frac{1}{2} MR^2$
6.	Uniform circular disc of radius R	Diameter	$\frac{1}{4} MR^2$
7.	Annular cylinder (or ring)	Central axis	$\frac{1}{2} M\,(R_1^2 + R_2^2)$
8.	Hollow cylinder of radius R	Axis of cylinder	MR^2
9.	Solid cylinder of radius R	Axis of cylinder	$\frac{1}{2} MR^2$
10.	Hollow sphere of radius R	Diameter	$\frac{2}{3} MR^2$
11.	Solid sphere of radius R	Diameter	$\frac{2}{5} MR^2$

7.42. DYNAMICS OF ROTATIONAL MOTION ABOUT A FIXED AXIS

In case of rotational motion about a fixed axis, it should be clearly understood that

(*i*) We need to consider only those forces that lie in planes perpendicular to the axis. The forces which are parallel to the axis of rotation will give torques perpendicular to the axis. They will tend to turn the axis. As the axis is fixed, we shall ignore the torques perpendicular to the axis.

(*ii*) We need to consider only those components of position vectors which are perpendicular to the axis of rotation. Components of position vectors along the axis will result in torques perpendicular to the axis. They need not be taken into account.

Keeping these two simplifications in mind, we have derived in Art. 7.15, small amount of work done by torque τ_1 in turning the body through a small angle $d\theta$

$$dW_1 = \tau_1\, d\theta \qquad \text{...(143)}$$

If there are more than one forces acting on the body, work done by all of them can be added to give total work done on the body. If torques due to different forces are τ_1, τ_2, ... etc., then total work done on the body

$$dW = (\tau_1 + \tau_2 + \tau_3 + ...)\, d\theta \qquad \text{...(144)}$$

Note that angular displacement $d\theta$ for all the particles of the rigid body is the same.

As all the torques are parallel to the fixed axis of rotation, the magnitude τ of the total torque is the algebraic sum of the magnitudes of component torques, *i.e.*, $\tau = \tau_1 + \tau_2 + \tau_3 + ...$

$$\therefore \qquad dW = \tau\, d\theta \qquad \text{...(145)}$$

The corresponding relation for linear motion is $dW = F\,dx$ ...(146)

Dividing both sides of eqn. (145) by time dt, we get $\dfrac{dW}{dt} = \tau \times \dfrac{d\theta}{dt}$ or $\boxed{P = \tau\,\omega}$ ...(147)

This is the instantaneous power associated with torque. This relation corresponds to expression for power in linear motion, *i.e.*,

$$\boxed{P = F\,v} \qquad \text{...(148)}$$

In a perfectly rigid body, as there is no internal motion amongst the particles, therefore, there is no dissipation of energy. The rate at which work is done on the body is equal to rate at which K.E. of rotation of the body increases, *i.e.*,

$$\frac{dW}{dt} = \tau\,\omega = \frac{d}{dt}\left(\frac{1}{2} I\,\omega^2\right)$$

If we assume that I does not change with time, *i.e.*, mass of body, size of body and axis of rotation of body do not change with time, then

$$\frac{dW}{dt} = \tau\,\omega = \frac{1}{2} I\,(2\,\omega)\,\frac{d\omega}{dt} = I\,\omega\,\alpha \qquad \text{...(149)}$$

From (149), $\boxed{\tau = I\,\alpha}$ ...(150)

This corresponds to the famous equation of linear motion $\boxed{F = ma}$

Just as force produces linear acceleration, torque produces angular acceleration $\left(\alpha = \dfrac{\tau}{I}\right)$, which is directly proportional to applied torque and is inversely proportional to moment of inertia of the body.

7.43. ANGULAR MOMENTUM AND TORQUE IN CASE OF ROTATION ABOUT A FIXED AXIS

The general expression for total angular momentum of a system of particles is

$$\vec{L} = \sum_{i=1}^{i=n} \vec{r}_i \times \vec{p}_i = \sum_{i=1}^{i=n} \vec{r}_i \times m_i \times \vec{v}_i = \sum_{i=1}^{i=n} \vec{r}_i \times m_i\,(\vec{\omega} \times \vec{r}_i)$$

Using vector triple product, we get

$$\vec{L} = \sum_{i=1}^{i=n} m_i \, [(\vec{r}_i \cdot \vec{r}_i)\, \vec{\omega} - (\vec{r}_i \cdot \vec{\omega}) \cdot \vec{r}_i] = \sum_{i=1}^{i=n} m_i \, [r_i^2 \, \vec{\omega} - 0] = \sum_{i=1}^{i=n} m_i \, r_i^2 \times \vec{\omega} \quad \text{...(151)}$$

For any individual particle, angular momentum vector and angular velocity vector are not necessarily parallel. This is in contrast to linear motion, where linear momentum vector and linear velocity vector are always parallel to each other.

As $\sum_{i=1}^{i=n} m_i \, r_i^2 = I$, moment of inertia of the body about the axis of rotation – which we have taken as Z-axis, therefore, from (151),

$$\vec{L} = \vec{L}_z = I \, \omega \, \hat{k} \quad \text{...(152)}$$

Differentiating both sides w.r.t. time, taking $\hat{k}$ as a fixed constant vector, we get

$$\frac{d}{dt}(\vec{L}_z) = \frac{d}{dt}(I \, \omega) \, \hat{k} \quad \text{...(153)}$$

For rotation about a fixed axis, $\dfrac{d \vec{L}_z}{dt} = \tau \hat{k}$ ...(154)

From (153) and (154), $\dfrac{d}{dt}(I \, \omega) = \tau$ ...(155)

If moment of inertia I does not change with time, then from (155), $I \dfrac{d\omega}{dt} = \tau$ or $I \alpha = \tau$

This is the relation we have derived earlier.

7.44. ROLLING MOTION

The rolling motion is one of the most common motions observed in daily life. All wheels, for example, used in transportation have rolling motion. We can study rolling motion as a combination of translation of the centre of mass and rotation of rest of the object around the centre of mass. This is shown in Fig. 7.55.

Fig. 7.55(a) shows *pure rotation* as if axis of rotation through the centre of wheel were stationary. Every point on the wheel rotates about its centre with angular speed ω. And every point on the outside edge of the wheel has linear speed $v_{cm} = R\,\omega$, where R is radius of the wheel.

FIGURE 7.55

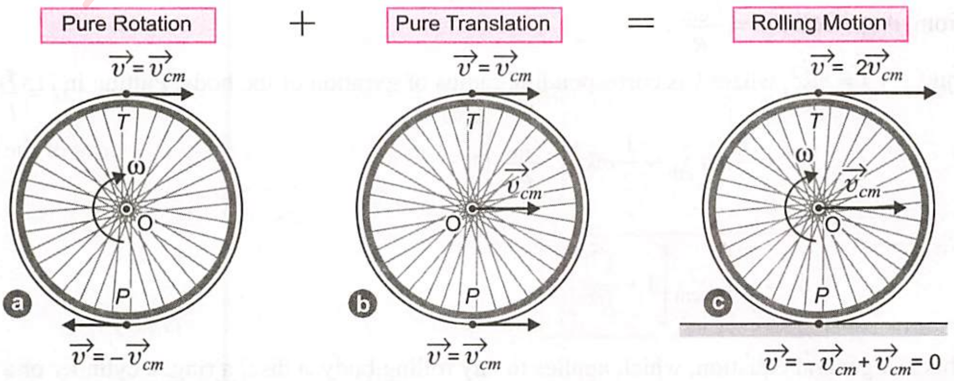
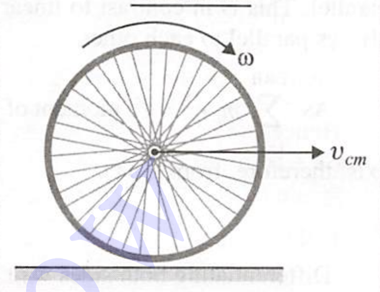


Fig. 7.55(b) shows *pure translation* as if the wheel did not rotate at all. Every point on the outside edge of wheel moves to the right with linear speed v_{cm} and so does the centre of the wheel.

Fig. 7.55(c) is the combination of Figs. 7.55(a) and (b). This represents the actual rolling motion of the wheel. Clearly, the portion of the wheel at the bottom (at P) would be stationary and the portion of the wheel at the top T would be moving at speed = $2\ u_{cm}$, faster than any other portion of the wheel.

Thus the velocity of a spherical body rolling over the horizontal surface without slipping goes on increasing from point of contact ($v = 0$) to the topmost point on the periphery of the body ($v = 2\ \omega R$), Fig. 7.55(c).

FIGURE 7.56



7.45. KINETIC ENERGY OF ROLLING MOTION

Consider a wheel of radius R rolling over a horizontal smooth surface without slipping. The wheel has two types of motion ; Fig. 7.56.

(*i*) the linear motion of the centre of mass with velocity v_{cm} along the horizontal,

(*ii*) rotational motion of the wheel about a horizontal axis passing through its centre, with an angular speed ω.

Therefore, total kinetic energy K of the system = K.E. of translational motion of centre of mass + K.E. of rotational motion (K_R) about the centre of mass

i.e., $$K = K_T + K_R \qquad \text{...(156)}$$

If m is mass of the body and v_{cm} is velocity of centre of mass of the body, then

K.E. of translation, $$K_T = \frac{1}{2} m v_{cm}^2$$

Since motion of rolling body about the centre of mass is rotation, therefore, K_R represents K.E. of rotation of the body

i.e., $$K_R = \frac{1}{2} I \omega^2$$

where I is moment of inertia of the body about the symmetry axis of the rolling body.

DO YOU KNOW ?

When an object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping. See that friction is responsible for such a motion. But no work is done against friction because there is no relative motion between the body and the surface at the point of contact.

Putting in (156), we get

K.E. of rolling body, $$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2 \qquad \text{...(157)}$$

From $v_{cm} = R\omega$, $\omega = \dfrac{v_{cm}}{R}$,

and $I = mk^2$, where k is corresponding radius of gyration of the body. Putting in (157), we get

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} mk^2 \left(\frac{v_{cm}}{R}\right)^2$$

$$\boxed{K = \frac{1}{2} m v_{cm}^2 \left[1 + \frac{k^2}{R^2}\right]} \qquad \text{...(158)}$$

This is a general equation, which applies to any rolling body, a disc, a ring, a cylinder or a sphere.

IMPORTANT NOTE

In eqn. (45), $\frac{1}{2} m v_{cm}^2$ is the kinetic energy associated, with the translational motion of the wheel's centre of mass, and $I = I_{cm}$ = moment of inertia of the body about an axis through its centre of mass $\frac{1}{2} I \omega^2 = \frac{1}{2} I_{cm} \omega^2 = \frac{1}{2} mk^2 \left(\frac{v_{cm}}{R}\right)^2$ is the kinetic energy associated with the rotation of the wheel about an axis through the centre of mass.

Hence, we conclude that *KE* of a rolling object = translational *KE* due to translation of its centre of mass + rotational *KE* due to its rotation about the centre of mass.

Special cases

1. If the rolling body is a ***solid cylinder*** or ***uniform circular disc*** of radius r and mass m, then

$$I = mk^2 = \frac{1}{2} mr^2 \qquad \therefore \qquad \frac{k^2}{r^2} = \frac{1}{2}$$

From (158),

$$K = \frac{1}{2} mv_{\text{cm}}^2 \left(1 + \frac{1}{2}\right) = \frac{3}{4} mv_{\text{cm}}^2$$

2. If the rolling body is a ***hollow sphere*** of radius r and mass m, then

$$I = mk^2 = \frac{2}{3} mr^2 \qquad \therefore \qquad \frac{k^2}{r^2} = \frac{2}{3}$$

From (158), $$K = \frac{1}{2} \text{in}\, v_{\text{cm}}^2 \left(1 + \frac{2}{3}\right) = \frac{5}{6} \text{mv}_{\text{cm}}^2$$

3. If the rolling body is a ***solid sphere*** of radius r and mass m, then

$$I = mk^2 = \frac{2}{5} mr^2 \qquad \therefore \qquad \frac{k^2}{r^2} = \frac{2}{5}$$

From (158), $$K = \frac{1}{2} mv_{\text{cm}}^2 \left(1 + \frac{2}{5}\right) = \frac{7}{10} mv^2$$

4. If the rolling body is a ***hollow cylinder*** or ***a circular ring*** of radius r and mass m, then

$$I = mk^2 = mr^2, \frac{k^2}{r^2} = 1$$

From (158), $$K = \frac{1}{2} mv_{\text{cm}}^2 \, (1 + 1) = mv_{\text{cm}}^2$$

Sample Problem **A thin hollow cylinder, open at both ends and weighing 5 kg (a) slides with a speed of 10 m/s without rotating (b) rolls with the same speed without slipping. Compare the kinetic energies of the cylinder in the two cases.**

Sol. Here, $m = 5$ kg, $v = 10$ m/s

(*a*) When cylinder slides without rotating, its energy is wholly translatory.

$$E_1 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{ J}$$

(*b*) When cylinder rolls without slipping, it has both translational and rotational K.E.

$$E_2 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

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But $I = m\,r^2$ and $r\,\omega = v$

$$\therefore \quad E_2 = \frac{1}{2} m\,v^2 + \frac{1}{2}(m\,r^2)\,\omega^2 = \frac{1}{2} m\,v^2 + \frac{1}{2} m\,v^2 = m\,v^2 = 5 \times 10^2 = 500 \text{ J}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{250}{500} = \frac{1}{2}$$

7.46. MOTION OF A CYLINDER ROLLING WITHOUT SLIPPING ON AN INCLINED PLANE OR CYLINDER ROLLING WITHOUT SLIPPING DOWN A RAMP

Consider a solid cylinder of mass m and radius r rolling down an inclined plane of inclination θ, without slipping.

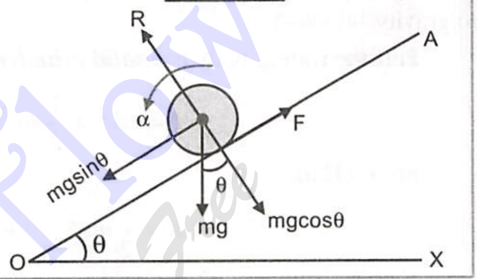
In Fig. 7.57, we have shown cross section of a right circular cylinder.

The various forces involved are : (*i*) weight (mg) of the cylinder acting vertically downwards,

(*ii*) normal reaction (R) of the inclined plane acting in a direction perpendicular to the inclined plane OA

(*iii*) force of friction (F) between the cylinder and the inclined plane. This force acts tangentially upwards on the inclined plane.

FIGURE 7.57



The weight (mg) can be resolved into two rectangular components : $mg \cos\theta$, opposite to R and $mg \sin\theta$, acting down the inclined plane.

As there is no motion in a direction perpendicular to the inclined plane, therefore

$$R = mg\cos\theta \qquad \text{...(159)}$$

If a is linear acceleration of the cylinder rolling down the inclined plane, then net force on the cylinder down the inclined plane is $f = m\,a = mg\sin\theta - F$ **...(160)**

Now, the torque τ acting on the rolling cylinder is only due to the force of friction (F),* which acts tangentially upwards on the surface of the cylinder. The weight mg and normal reaction R do not contribute to the torque, being radial forces.

$$\therefore \qquad \tau = F \times r \qquad \text{...(161)}$$

If I is moment of inertia of the solid cylinder about its axis of symmetry and α is angular acceleration of the cylinder down the inclined plane, then

$$\tau = I\,\alpha = \frac{I\,a}{r} \qquad (\because\ a = r\,\alpha) \qquad \text{...(162)}$$

From (161) and (162), $F \times r = \dfrac{I\,a}{r}$

$$F = I\,a/r^2 \qquad \text{...(163)}$$

Putting in (160), we get $m\,a = mg\sin\theta - I\,a/r^2$

$$a\,(m + I/r^2) = mg\sin\theta$$

$$\boxed{a = \frac{mg\sin\theta}{m + I/r^2}} \qquad \text{...(164)}$$

As $I = mk^2$, therefore, $a = \dfrac{mg\sin\theta}{m + mk^2/r^2} = \dfrac{g\sin\theta}{(1 + k^2/r^2)}$, *i.e.*, $\boxed{a = \dfrac{g\sin\theta}{1 + k^2/r^2}}$ **...(165)**

---

*Out of total force ($mg\sin\theta$), acting on the cylinder down the inclined plane, a part (ma) is spent in producing downward acc. (a). The balance downward force ($mg\sin\theta - ma$) = F (in magnitude) is responsible for producing torque on the cylinder rotating it with an angular acceleration α. This value of friction is just right for the body to roll smoothly down the ramp without sliding.

Now, moment of inertia of the solid cylinder about its own axis of symmetry is

$$I = \frac{1}{2} m r^2$$

$$\therefore \qquad a = \frac{mg\sin\theta}{m + \frac{1}{2} m r^2/r^2} = \frac{mg\sin\theta}{3\,m/2} = \frac{2}{3} g\sin\theta$$

$$\therefore \qquad \boxed{a = \frac{2}{3} g\sin\theta} \qquad \text{...(166)}$$

Clearly, $a < g$ *i.e.* acceleration of the solid cylinder down the inclined plane is less than acceleration due to gravity.

Putting this value of a in (163), we get

$$F = \frac{I}{r^2}\frac{2}{3} g\sin\theta = \left(\frac{1}{2} m r^2\right)\frac{2}{3r^2} g\sin\theta \qquad \left(\because I = \frac{1}{2} m r^2\right)$$

$$\boxed{F = \frac{1}{3} mg\sin\theta} \qquad \text{...(167)}$$

Obviously, $F < mg$, *i.e.* force of friction is less than the weight of the body.

Now, for rolling without slipping, there must be a certain minimum value of coefficient of friction between the cylinder and the inclined plane, *i.e.*,

$$\boxed{\mu = \frac{F}{R} = \frac{1}{3}\frac{mg\sin\theta}{mg\cos\theta} = \frac{1}{3}\tan\theta} \qquad \text{...(168)}$$

Note that in the above example, centre of mass of rolling cylinder is in motion.

Sample Problem **A cylinder of mass 10 kg is rolling perfectly on a plane of inclination 30°. Find the force of friction between the cylinder and the surface of inclined plane.**

Sol. Here, $M = 10$ kg, $\theta = 30°$

When a cylinder rolls smoothly without slipping, force of friction is given by

$$F = \frac{1}{3}\text{mg}\sin\theta = \frac{1}{3}\times 10\times 9{\cdot}8\sin 30° = \frac{98}{3}\times\frac{1}{2} = \mathbf{16{\cdot}3\ N}$$

7.47. LAWS OF ROTATIONAL MOTION

Corresponding to Newton's three laws of translational motion, we can state three laws of rotational motion as follows :

1. *A body continues to be in a state of rest or in a state of uniform rotation about a given axis unless an external torque is applied on the body.*

2. *The rate of change of angular momentum of a body about a given axis is directly proportional to the external torque applied on the body.*

3. *When a rigid body A exerts a torque on another rigid body B in contact with it, then the body B would exert an equal and opposite torque on the body A.*

7.48. ANALOGY BETWEEN LINEAR MOTION AND ROTATIONAL MOTION

Table 7.3 gives a summary of the analogy (similarity) between various quantities which describe linear motion to those which describe rotational motion.

TABLE 7.3. Analogy between linear motion and rotational motion

Linear Motion (Fixed Direction)	Rotational Motion (Fixed Axis)
1. Distance/displacement (s)	**1.** Angle or angular displacement (θ)
2. Linear velocity, $v = \frac{ds}{dt}$	**2.** Angular velocity, $\omega = \frac{d\theta}{dt}$
3. Linear acceleration, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	**3.** Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
4. Mass (m)	**4.** Moment of inertia (I)
5. Linear momentum, $p = m\,v$	**5.** Angular momentum, $L = I\,\omega$
6. Force, $F = m\,a$	**6.** Torque, $\tau = I\,\alpha$
7. Also, force, $F = \frac{dp}{dt}$	**7.** Also, torque $\tau = \frac{dL}{dt}$
8. Translational K.E $= \frac{1}{2} m v^2 = \frac{p^2}{2m}$	**8.** Rotational K.E $= \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$
9. Work done, $W = F\,s$	**9.** Work done, $W = \tau\,\theta$
10. Power, $P = F\,v$	**10.** Power, $P = \tau\,\omega$
11. Linear momentum of a system is conserved when no external force acts on the system. (*Principle of conservation of linear momentum*)	**11.** Angular momentum of a system is conserved when no external torque acts on the system. (*Principle of conservation of angular momentum*)
12. Work-Kinetic Energy Theorem : $W = \Delta K$	**12.** Work-Kinetic Energy Theorem : $W = \Delta K$
13. *Equations of Translational motion* (*i*) $v = u + a\,t$ (*ii*) $s = ut + \frac{1}{2} a t^2$ (*iii*) $v^2 - u^2 = 2\,a\,s$, where the symbols have their usual meaning.	**13.** *Equations of Rotational motion* (*i*) $\omega_2 = \omega_1 + \alpha\,t$ (*ii*) $\theta = \omega_1\,t + \frac{1}{2}\alpha t^2$ (*iii*) $\omega_2^2 - \omega_1^2 = 2\,\alpha\,\theta$, where the symbols have their usual meaning.
14. Distance travelled in nth second, $S_{nth} = u + \frac{a}{2}(2n-1)$	**14.** Angle traced in nth second, $\theta_{nth} = \omega_1 + \frac{\alpha}{2}(2n-1)$

SOLVED EXAMPLES

TYPE I. CENTRE OF MASS

Formulae used.

1. Position vector of centre of mass of n particle system is

$$\vec{r} = \frac{\sum_{i=1}^{i=n} m_i \vec{r}_i}{M}$$

2. For two particle system, position vector of centre of mass is

$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

3. The co-ordinates of centre of mass will be

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \quad y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

and $$z = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

4. If centre of mass lies at the origin,

$$x = y = z = r = 0$$

$$\therefore \quad m_1 x_1 + m_2 x_2 = 0$$

$$\therefore \quad x_2 = -\frac{m_1 x_1}{m_2}$$

Similarly, $$y_2 = -\frac{m_1 y_1}{m_2}$$

and $$z_2 = -\frac{m_1 z_1}{m_2}$$

Clearly, the algebraic sum of the moments of masses of various particles about its centre of mass is zero.

5. For a continuous mass distribution,

$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r}\, dm$$

6. Velocity of c.m. of a system of two particles is

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

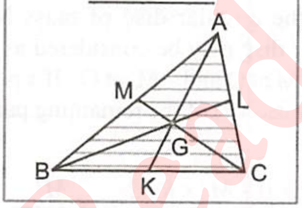
Units used. All masses are in kg, all distances are in metre and all velocities in ms^{-1}

Example 1 **Find the centre of mass of a uniform triangular lamina.**

(NCERT Solved Example)

Solution. In Fig. 7.58, ABC is a uniform triangular lamina. It may be subdivided into ***thin strips,*** each parallel to the base BC as shown.

FIGURE 7.58



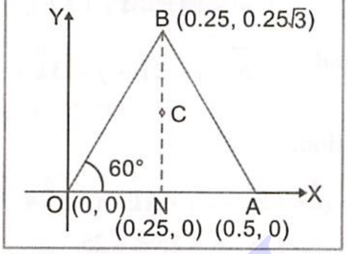
By symmetry, each strip has its centre of mass at its geometrical centre. If these points are joined, we get the median AK. Similarly, we can argue that centre of mass lies on median BL and also on median CM. It means the centre of mass of uniform triangular lamina lies at the point of intersection of the three medians, *i.e.*, on the **centroid G of the triangle.**

Example 2 **Find the centre of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100 g, 150 g and 200 g respectively. Each side of the equilateral triangle is 0.5 m long.**

(NCERT Solved Example)

Solution. Refer to Fig. 7.59,

FIGURE 7.59



In ΔOBN, $\cos 60^\circ = \frac{ON}{OB} = \frac{ON}{0{\cdot}5}$

$$ON = 0{\cdot}5 \cos 60^\circ = 0{\cdot}25 \text{ m}$$

$$\sin 60^\circ = \frac{BN}{OB} = \frac{BN}{0{\cdot}5}$$

$$BN = 0{\cdot}5 \sin 60^\circ = 0{\cdot}5\, \frac{\sqrt{3}}{2} = 0{\cdot}25\sqrt{3}$$

∴ The co-ordinates of O (0, 0), A (0·5, 0) and B (0·25, 0·25 $\sqrt{3}$). The masses of 100 g, 150 g and 200 g are located at the vertices O, A, B of the equilateral triangle OAB. The co-ordinates of the centre of mass are

$$x = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{100 \times 0 + 150 \times 0{\cdot}5 + 200 \times 0{\cdot}25}{(100 + 150 + 200)}$$

$$= \frac{75 + 50}{450} = \frac{125}{450} = \frac{\mathbf{5}}{\mathbf{18}} \textbf{ m}$$

and $$y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{100 \times 0 + 150 \times 0 + 200 \times 0{\cdot}25\sqrt{3}}{100 + 150 + 200}$$

$$= \frac{50\sqrt{3}}{450} = \frac{\sqrt{3}}{9} = \frac{\mathbf{1}}{\mathbf{3\sqrt{3}}} \textbf{ m}$$

Note that the centre of mass C is not the geometric centre of triangle OAB. This is because the three vertices of the triangle are not uniformly loaded.

Example 3 **Find the scalar and vector product of two vectors**

$$\vec{a} = (3\hat{i} - 4\hat{j} + 5\hat{k}),$$

and $$\vec{b} = (-2\hat{i} + \hat{j} - 3\hat{k})$$

(NCERT Solved Example)

Solution.

$$\vec{a}\cdot\vec{b} = (3\hat{i} - 4\hat{j} + 5\hat{k})\cdot(-2\hat{i} + \hat{j} - 3\hat{k})$$
$$= -6 - 4 - 15 = \mathbf{-25}$$

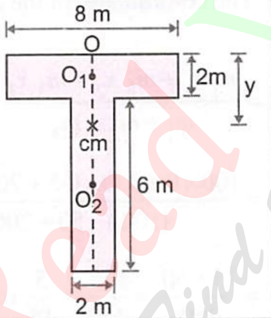
$$\vec{a}\times\vec{b} = (3\hat{i} - 4\hat{j} + 5\hat{k})\times(-2\hat{i} + \hat{j} - 3\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & +1 & -3 \end{vmatrix} = -7\hat{i} + \hat{j} - 5\hat{k}$$

$$\vec{b}\times\vec{a} = -(\vec{a}\times\vec{b}) = 7\hat{i} - \hat{j} + 5\hat{k}$$

Example 4 **Find the position of centre of mass of the T-shaped plate from O, in Fig. 7.60.**

FIGURE 7.60



Solution. Suppose σ is mass per unit area of the plate.

∴ Mass of horizontal portion = 8 × 2 σ = 16 σ

Mass of vertical portion = 6 × 2 σ = 12 σ

Centre of mass O_1 of horizontal portion is such that $OO_1 = 1$ m.

Centre of mass O_2 of vertical portion is such that $OO_2 = 2 + 3 = 5$ m.

If y is distance of cm of T-shaped plate from O, then

$$y = \frac{16\sigma\times1 + 12\sigma\times5}{16\sigma + 12\sigma} = \frac{76\sigma}{28\sigma} = \mathbf{2{\cdot}71\ m}$$

Example 5 **From a uniform circular disc of radius R, a circular disc of radius R/6 and having centre at a distance R/2 from the centre of the disc is removed. Determine the centre of mass of remaining portion of the disc.**

Solution. Suppose mass per unit area of the disc is m. Therefore, mass of original disc

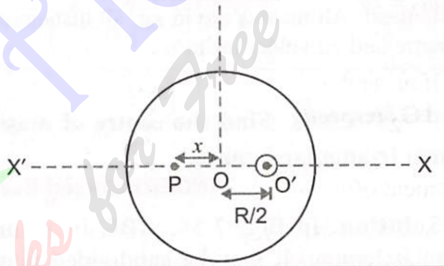
$$M = \pi R^2 \times m$$

Mass of portion removed from the disc

$$M' = \pi\left(\frac{R}{6}\right)^2 \times m = \frac{\pi R^2}{36} m$$

In Fig. 7.61, mass M is concentrated at O and mass M′ was concentrated at O′,

FIGURE 7.61



where $OO' = \frac{R}{2}$.

After the circular disc of mass M′ has been removed, the disc may be considered as a system of two masses, M at O and – M′ at O′. If x is the distance of centre of mass (P) of the remaining part of the disc from O, then

$$x = \frac{M\times0 - M'\times R/2}{M + (-M')} = \frac{M'}{M - M'}\times\frac{R}{2}$$

$$= \frac{\pi R^2 m/36}{\pi R^2 m - \pi R^2 m/36}\times\frac{R}{2} = -\frac{\mathbf{R}}{\mathbf{70}}$$

Negative sign shows that P is to the left of O.

Example 6 **A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate. Find centre of mass of the remaining portion of the disc.**

Solution. Here, radius of ciruclar plate $= \frac{56}{2}$ cm $= 28$ cm with centre O. And radius of cir-

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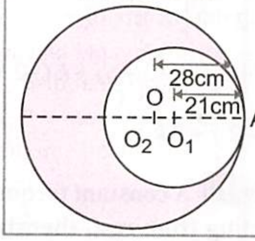
cular portion removed $= \frac{42}{2} = 21\text{ cm}$ with centre O_1, Fig. 7.62.

FIGURE 7.62



If m is mass per unit area of disc, then

$M = \pi\,(28)^2$ m and $m_1 = \pi\,(21)^2$ m

Mass of shaded portion,

$$m_2 = M - m_1 = [\pi\,(28)^2 - \pi\,(21)^2]\text{ m}$$
$$= 343\,\pi\text{ m}$$

If m_1 and m_2 are assumed to be concentrated at O_1 and O_2 respectively, and O is their centre of mass, then

moment of m_2 about O = moment of m_1 about O.

$$m_2 \times O_2O = m_1 \times O_1O$$

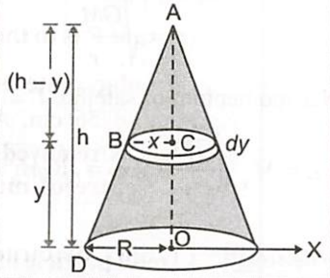
$$O_2O = \frac{m_1}{m_2} \times O_1O = \frac{441\,\pi\text{ m}}{343\,\pi\text{ m}} \times 7\text{ cm} = \mathbf{9\text{ cm}}$$

This is the distance of c.m. of remaining portion of disc from centre of disc.

Example 7 **Determine the coordinates of the centre of mass of a right circular solid cone of base radius *R* and height *h*.**

Solution. Fig. 7.63 shows a uniform solid cone of base radius R and height h. It is supposed to be made up of a large number of circular discs of different radii and masses.

FIGURE 7.63



Consider one such disc of radius $BC = x$ and mass dm at a height y from the base.

Δ^S ABC and ADO are similar,

$$\therefore \quad \frac{AO}{AC} = \frac{DO}{BC} \quad \text{or} \quad \frac{h}{h-y} = \frac{R}{x}$$

$$\text{or} \quad x = \frac{R\,(h-y)}{h} \qquad \text{...}(i)$$

Mass of elementary disc, dm = Volume × density

$$dm = \pi\,x^2 \,.\, dy \times \rho$$
$$= \pi\rho\,\frac{R^2\,(h-y)^2}{h^2}\,dy \qquad \text{...}(ii)$$

∴ Co-ordinates of centre of mass of cone are

$$y_{cm} = \frac{1}{M}\int y\,dm$$
$$= \frac{1}{M}\int y.\frac{\pi\rho\,R^2}{h^2}\,(h-y)^2\,dy$$
$$= \frac{\pi\rho\,R^2}{M\,h^2}\int_0^h (h^2y + y^3 - 2\,hy^2)\,dy$$

$$y_{cm} = \frac{\pi\rho\,R^2}{\frac{1}{3}\pi R^2\,h\rho \times h^2} \times \frac{h^4}{12} = \frac{h}{4}$$

$$\left(\because M = \frac{1}{3}\pi\,r^2\,h\,\rho\right)$$

Similarly $x_{cm} = 0$, $z_{cm} = 0$

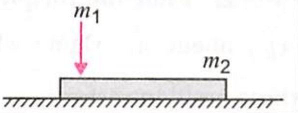
Hence co-ordinates of the centre of mass of solid cone are $\left(\mathbf{0, \frac{h}{4}, 0}\right)$.

Example 8 **A man of mass m_1 is standing on a platform of mass m_2 kept on a smooth horizontal surface. It the man moves a distance *d* w.r.t., the platform, find the displacement of the platform *w.r.t.*, ground.**

Solution. Here, mass of man $= m_1$

mass of platform $= m_2$

FIGURE 7.64



As the surface on which platform is placed is smooth, therefore no external force is acting along the horizontal, *i.e.*,

$F_{ext.} = 0$, Therefore, $\vec{a}_{cm} = 0$ and $\vec{v}_{cm} = \text{const.}$

As $\vec{v}_{cm}$ is zero initially, it would remain zero even after the man moves. Refer to Fig. 7.64.

Let x_1 = displacement of man towards right

x_2 = displacement of platform towards left

$$\therefore \quad m_1 x_1 = m_2 x_2 \quad \text{and} \quad x_1 + x_2 = d$$

$$\frac{m_2 x_2}{m_1} + x_2 = d$$

$$x_2 \left(\frac{m_2}{m_1} + 1 \right) = d$$

$$x_2 = \frac{m_1 d}{m_1 + m_2}$$

TYPE II. TORQUE, POWER OF TORQUE, WORK DONE BY TORQUE AND ANGULAR MOMENTUM

Formula used

1. Torque = force × its perpendicular distance from the axis of rotation

2. Torque, $\tau = rF \sin\theta$ or $\vec{\tau} = \vec{r} \times \vec{F}$

3. Power of torque, $P = \tau\,\omega$

4. Work done by torque, $W = \tau\,\theta$

5. Angular momentum = Linear momentum × its perpendicular distance from the axis of rotation

6. Angular momentum,

$$\vec{L} = rp \sin\theta \quad \text{or} \quad \vec{L} = \vec{r} \times \vec{p}.$$

7. When a particle of mass m is moving with a uniform speed v along a circle of radius r ;

$$L = m\,v\,r.$$

8. Torque = rate of change of angular momentum

$$\tau = dL/dt$$

Units used. Torque is in N-m, work done in joule, power in watt, angle in radian, angular velocity in radian/sec., Mass in kg, linear velocity v in m/s and angular momentum L in kg m^2 s^{-1}.

Example 9 **Find the torque of a force $(5\hat{i} - 2\hat{j} + 3\hat{k})$ about the origin ; which acts on a particle whose position vector is $(\hat{i} - \hat{j} + \hat{k})$.**

(NCERT Solved Example)

Solution. Here,

$$\vec{F} = (5\hat{i} - 2\hat{j} + 3\hat{k}) \quad \text{and} \quad \vec{r} = (\hat{i} - \hat{j} + \hat{k})$$

$$\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j} + \hat{k}) \times (5\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 5 & -2 & 3 \end{vmatrix}$$

$$= \hat{i}(-3+2) - \hat{j}(3-5) + \hat{k}(-2+5)$$

$$= (-\hat{i} + 2\hat{j} + 3\hat{k})$$

Example 10 **A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t seconds, show that the angular acceleration is given by $\alpha = \dfrac{4\pi n}{t^2}$ rad s^{-2}.**

(NCERT Solved Example)

Solution. Here, $\omega_0 = 0$, $\theta = 2\pi n$ radian,

$t = t$, $\alpha = ?$

$$\text{As} \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\therefore \quad 2\pi n = 0 + \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{4\pi n}{t^2} \text{ rad s}^{-2}$$

Example 11 **Show that angular momentum of a satellite of mass M_s revolving around earth of mass M_e in an orbit of radius r is $\sqrt{GM_s^2 M_e r}$.**

Solution. A satellite revolves around earth under gravitational pull which provides the necessary centripetal force,

$$i.e., \qquad M_s \frac{v^2}{r} = \frac{GM_e M_s}{r^2}$$

$$v = \sqrt{\frac{GM_e}{r}}$$

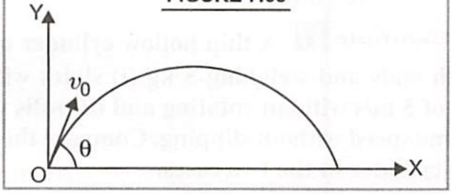
Angular momentum of satellite $L = M_s \cdot v \cdot r$

$$L = M_s \sqrt{\frac{GM_e}{r}} \times r = \sqrt{GM_s^2 M_e r}$$

Example 12 **A small particle of mass m is projected at an angle θ with x-axis with initial velocity v_0 in x-y plane as shown in Fig. 7.65.**

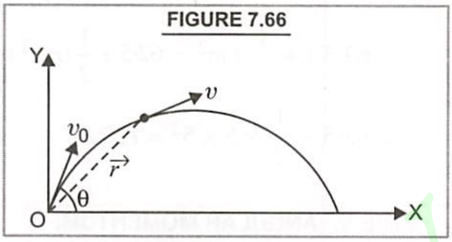
Calculate the angular momentum of the particle at $t < \dfrac{v_0 \sin\theta}{g}$.

FIGURE 7.65



Solution. As time of flight $T = \dfrac{2 v_0 \sin\theta}{g}$, therefore, at $t < \dfrac{v_0 \sin\theta}{g}$, particle has not reached the maximum height. Let $\vec{r}$ be position vector of the particle as shown in Fig. 7.66.

FIGURE 7.66



$$\vec{r} = \vec{u}\, t - \frac{1}{2} \vec{g}\, t^2$$

$$= (v_0 \cos\theta\, \hat{i} + v_0 \sin\theta\, \hat{j})\, t - \frac{1}{2} g t^2\, \hat{j}$$

$$\vec{r} = v_0 \cos\theta\, (t)\, \hat{i} + \left(v_0 \sin\theta\, (t) - \frac{1}{2} g t^2 \right) \hat{j}$$

and $\vec{v} = \vec{u} - \vec{g}\, t$

$$= v_0 \cos\theta\, \hat{i} + v_0 \sin\theta\, \hat{j} - g t\, \hat{j}$$

$$= v_0 \cos\theta\, \hat{i} + (v_0 \sin\theta - g t)\, \hat{j}$$

Angular momentum of particle,

$$\vec{L} = m(\vec{r} \times \vec{v}) \qquad ...(i)$$

Now,

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_0 \cos\theta\, (t) & v_0 \sin\theta\, (t) - \frac{1}{2} g t^2 & 0 \\ v_0 \cos\theta & (v_0 \sin\theta - g t) & 0 \end{vmatrix}$$

$$= \hat{i}\,(0 - 0) - \hat{j}\,(0 - 0) + \hat{k}\, [v_0^2 \cos\theta \sin\theta\, (t) - v_0 \cos\theta\, g t^2 - v_0^2 \cos\theta \sin\theta\, t + \frac{1}{2} v_0 \cos\theta\, g t^2]$$

$$\vec{r} \times \vec{v} = -\frac{1}{2} v_0 \cos\theta\, g t^2\, \hat{k}$$

From (*i*),

$$\vec{L} = -\frac{1}{2} m g v_0 t^2 \cos\theta\, \hat{k}$$

TYPE III. EQUATIONS OF ROTATIONAL MOTION

Formulae used. **1.** $\omega_2 = \omega_1 + \alpha t$

2. $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$ **3.** $\omega_2^2 - \omega_1^2 = 2 \alpha \theta$

4. $v = r\omega$; $\omega = 2\pi n = \dfrac{2\pi}{T}$ **5.** $a = r\alpha$

6. Centripetal acc. $= \dfrac{v^2}{r} = r\omega^2$

Units used. n in rps ; ω_1, ω_2 in rad/s ; α in radian/s^2 ; T, t in sec ; θ in radian ; r in metre ; v in m/s and a in m/s^2.

Example 13 **On application of a constant torque, a wheel is turned from rest through an angle of 200 radian in 8 s. What is its angular acceleration ? If the same torque continues to act, what will be the angular velocity of the wheel 16 s from the start ?**

Solution. Here, $\omega_1 = 0$, $\theta = 200$ rad, $t = 8$ s, $\alpha = ?$, $\omega_2 = ?$, $t = 16$ s

From $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$

$$200 = 0 + \frac{1}{2} \alpha (8)^2 \; ; \; \alpha = \frac{200}{32} = \mathbf{6{\cdot}25\ rad\ s^{-2}}$$

Again, from $\omega_2 = \omega_1 + \alpha t = 0 + 6{\cdot}25 \times 16$
$= \mathbf{100\ rad/s}$

Example 14 **A flywheel rotating at a speed of 600 rpm about its axis is brought to rest by applying a constant torque for 10 seconds. Find the angular deceleration and angular velocity 5 second after the application of the torque.**

Solution. Here, $n_1 = 600$ rpm $= \frac{600}{60}$ rps

$= 10$ rps

$$n_2 = 0,\ t = 10 \text{ seconds},\ \alpha = ?$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$$

$$\alpha = \frac{2\pi(0-10)}{10} = -2\pi \text{ rad/s}^2$$

∴ angular deceleration = $\mathbf{2\pi}$ **rad/s²**

Again, $\omega_2 = \omega_1 + \alpha t \quad \therefore \quad \omega_2 = 2\pi n_1 + \alpha t$

$$\omega_2 = 2\pi \times 10 - 2\pi \times 5 = \mathbf{10\pi \text{ rad/s}}$$

Example 15 **Obtain the equation $\omega = \omega_0 + \alpha t$ from first principles.**

(NCERT Solved Example)

Solution. Refer to Art. 7.26(*a*).

TYPE IV. ENERGY OF ROTATION

Formulae used.

1. K.E. of rotation $= \frac{1}{2} I \omega^2$
2. K.E. of translation $= \frac{1}{2} m v^2$
3. Total K.E. $= \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

Units used. v in ms^{-1}, ω in rad s^{-1}, m in kg, I in kg m^2 ; energy in joule.

Example 16 **An energy of 484 J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm. Calculate moment of inertia of flywheel.**

Solution. Here, energy spent, $W = 484$ J ;

Initial speed, $\omega_1 = 60$ rpm $= \frac{60}{60} \times 2\pi$ rad/sec

$= 2\pi$ rad/sec

Final speed, $\omega_2 = 360$ rpm

$= \frac{360}{60} \times 2\pi$ radian/sec $= 12\pi$ radian/sec ;

$I = ?$

As work done = increase in K.E. of rotation

$$\therefore \quad W = E_2 - E_1 = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$$

$$484 = \frac{1}{2} I [(12\pi)^2 - (2\pi)^2] = \frac{1}{2} I \times 140\pi^2$$

$$484 = 70 I \times \frac{22}{7} \times \frac{22}{7}$$

$$I = \frac{484 \times 49}{70 \times 484} = \mathbf{0{\cdot}7 \text{ kg } m^2}$$

Example 17 **A thin hollow cylinder open at both ends and weighing 5 kg (*i*) slides with a speed of 5 m/s without rotating and (*ii*) rolls with the same speed without slipping. Compare the KE of the cylinder in the two cases.**

Solution. Here, $m = 5$ kg, $v = 5$ m/s

(*i*) Translational K.E. $= \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times 5^2$

$= \mathbf{62{\cdot}5 \text{ J}}$

(*ii*) When the cylinder rolls with the same speed

Total K.E. = K.E. of translation + K.E. of rotation

$$= 62{\cdot}5 \text{ J} + \frac{1}{2} I \omega^2 = 62{\cdot}5 + \frac{1}{2}(mr^2)\frac{v^2}{r^2}$$

$$= 62{\cdot}5 + \frac{1}{2} \times 5 \times 5^2 = \mathbf{125 \text{ J}}$$

TYPE V. ANGULAR MOMENTUM, TORQUE AND MOMENT OF INERTIA

Formulae used.

1. Angular momentum, $L = I\omega$
2. Torque, $\tau = I\alpha$
3. Relation between torque and angular momentum,

$$\tau = \frac{dL}{dt}$$

Units used. T in sec., ω in rad/*s*, α in rad/s², I in kg m² ; L in kg m² s^{-1}, τ in N-m

Example 18 **A flywheel of mass 25 kg has a radius of 0·2 m. It is making 240 rpm. What is the torque necessary to bring it to rest in 20 s ? If the torque is due to a force applied tangentially on the rim of the wheel, what is the magnitude of the force ? Assume that mass of flywheel is concentrated at its rim.**

Solution. Here, $M = 25$ kg, $R = 0{\cdot}2$ m,

$$n_1 = 240 \text{ rpm} = \frac{240}{60} \text{ rps} = 4 \text{ rps},\ n_2 = 0,\ t = 20 \text{ s}$$

Moment of inertia of flywheel,

$I = MR^2 = 25\,(0{\cdot}2)^2 = 1$ kg m²

$$\tau = I\,\alpha = \frac{I\,(\omega_2 - \omega_1)}{t} = I\,\frac{2\pi\,(n_2 - n_1)}{t}$$

$$\tau = \frac{1 \times 2\pi\,(0 - 4)}{20} = -\,0{\cdot}4\,\pi = -\,\mathbf{1{\cdot}257\ N\text{-}m}$$

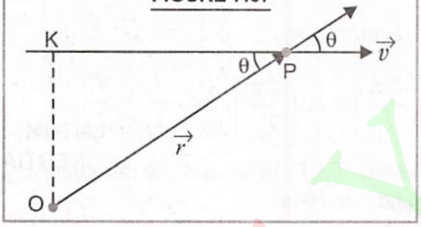
If F is the tangential force applied on the rim, then as $F \times R = \tau$

$$\therefore \quad F = \frac{\tau}{R} = \frac{1{\cdot}257}{0{\cdot}2} = \mathbf{6{\cdot}285\ N}$$

Example 19 **Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion.** **(NCERT Solved Example)**

Solution. Refer to Fig. 7.67. At any time t, suppose the particle moving with velocity $\vec{v}$ is at P. We have to calculate angular momentum of the particle about any arbitrary point O. The angular momentum

FIGURE 7.67



$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\,\vec{v}$$

$$|\,\vec{L}\,| = r\,m\,v \sin\theta$$

where θ is the smaller angle between $\vec{r}$ and $\vec{v}$.

From Fig. 7.20, $r \sin\theta = OK$ = perpendicular distance of O from the line of motion of particle.

As position of particle changes, r and θ both change, but $r \sin\theta$ remains constant.

Therefore, magnitude of $\vec{L} = m\,v\,r \sin\theta = m\,v\,(OK)$ = constant

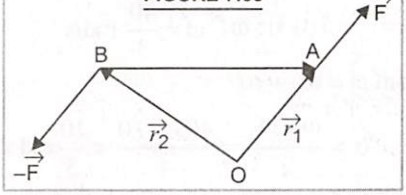
The direction of $\vec{L}$ is ⊥ $\vec{r}$ and $\vec{v}$ and inwards. This also does not change.

Hence $\vec{L}$ ***remains the same in magnitude and direction.***

Example 20 **Show that moment of a couple does not depend on the point about which you take the moments.** **(NCERT Solved Example)**

Solution. Refer to Fig. 7.68. Forces $\vec{F}$ at A and $-\vec{F}$ at B form a couple. Let O be the origin. Let $\overrightarrow{OA} = \vec{r}_1$ and $\overrightarrow{OB} = \vec{r}_2$ be the position vectors of A and B.

FIGURE 7.68



Let us take the moment of the couple about the origin O.

Moment of couple = sum of moments of two forces forming the couple

$$= \vec{r}_1 \times \vec{F} + \vec{r}_2 \times (-\,\vec{F}) = (\vec{r}_1 - \vec{r}_2) \times \vec{F}$$

Using triangle law,

$$\overrightarrow{OB} + \overrightarrow{BA} = \overrightarrow{OA}$$

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \vec{r}_1 - \vec{r}_2$$

$$\therefore \text{ Moment of couple } = \overrightarrow{BA} \times \vec{F}$$

This expression is independent of location of the origin O – about which we took the moment of the couple.

Example 21 **A body whose moment of inertia is 3 kg m^2 is at rest. It is rotated for 20 s with a moment of force 6 N-m. Find angular displacement of the body. What is work done ?**

Solution. Here, $I = 3$ kg m^2, $\omega_1 = 0$, $t = 20$ s

Moment of force, $\tau = 6$ N-m, $\theta = ?$, $W = ?$

As $\tau = I\,\alpha$, $\alpha = \frac{\tau}{I} = \frac{6}{3} = 2$ rad s^{-2}

Angular displacement, $\theta = \omega_1 t + \frac{1}{2}\,\alpha\,t^2$

$$= 0 + \frac{1}{2} \times 2\,(20)^2 = 400 \text{ rad.}$$

Work done, $W = \tau\,\theta = 6 \times 400 = \mathbf{2400\ J}$

Example 22 **A solid cylinder of mass 5 kg and radius 30 cm and free to rotate about its axis, receives an angular impulse of 3 kg m^2 s^{-1} initially, followed by a similar impulse after every 4 second. What is the angular speed of the cylinder 30 sec. after the initial impulse. Given that the cylinder is initially at rest.**

Solution. Here, $m = 5$ kg, $r = 30$ cm $= 0{\cdot}3$ m,

$$\omega_0 = 0,\ \omega = ?$$

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Angular impulse = change in angular momentum

$$3 = I(\omega_2 - \omega_1) = \frac{1}{2}\,mr^2\,(\omega - 0)$$

$$3 = \frac{1}{2}\times 5\,(0{\cdot}3)^2\,\omega; \quad \omega = \frac{40}{3}\ \text{rad/s}$$

From $\omega = \omega_0 + \alpha t$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{40/3 - 0}{4} = \frac{10}{3}\ \text{rad s}^{-2}$$

As angular impulse is imparted after every 4 second, *i.e.*, at $t = 0, 4, 8, 12, 16, 20, 24$ and 28 s. The last impulse continues upto $t = 32$ s

$$\omega = \omega_0 + \alpha\,t = 0 + \frac{10}{3}\times 32 = \mathbf{106{\cdot}7\ rad/s}$$

Example 23 **To speed up a flywheel from 60 rpm to 120 rpm, energy equal to 9000 J is required. Calculate the moment of inertia of flywheel. Also calculate change in angular momentum of flywheel.**

Solution. Here, $v_1 = 60$ rpm = 1 rps

$v_2 = 120$ rpm = 2 rps, $E = 9000$ J

$\omega_1 = 2\,\pi\,v_1 = 2\,\pi \times 1$ rad/s

$\omega_2 = 2\,\pi\,v_2 = 2\,\pi \times 2$ rad/s

As $E = E_2 - E_1 = \frac{1}{2}I\,\omega_2^2 - \frac{1}{2}I\,\omega_1^2$

$$E = \frac{1}{2}\,I\,(16\,\pi^2 - 4\,\pi^2) = 6\,I \times \pi^2$$

$$9000 = 6\,I\,\pi^2, \quad I = \frac{9000}{6\pi^2} = \mathbf{151{\cdot}97\ kg\ m^2}$$

Change in angular momentum

$$\Delta L = I(\omega_2 - \omega_1) = 151{\cdot}97\,(4\,\pi - 2\,\pi)$$

$$= 151{\cdot}97 \times 2\,\pi = \mathbf{954{\cdot}98\ kg\ m^2\ s^{-1}}$$

Example 24 **A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane, the centre of mass of the cylinder has a speed of 5 m/s.**

(a) How far will the cylinder go up the plane ?

(b) How long will it take to return to the bottom ?

Solution. Here, $\theta = 30°$, $v = 5$ m/s

Let the cylinder go up the plane upto a height h.

From $\frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2 = mgh$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = mgh$$

$$\frac{3}{4}m\,v^2 = mgh$$

$$h = \frac{3v^2}{4\,g} = \frac{3\times 5^2}{4\times 9{\cdot}8} = 1{\cdot}913\ \text{m}$$

If s is the distance up the inclined plane, then as

$$\sin\theta = \frac{h}{s}, \quad s = \frac{h}{\sin\theta} = \frac{1{\cdot}913}{\sin 30°} = \mathbf{3{\cdot}826\ m}$$

Time taken to return to the bottom

$$t = \sqrt{\frac{2\,s\,(1 + K^2/r^2)}{g\sin\theta}} = \sqrt{\frac{2\times 3{\cdot}826\,(1 + 1/2)}{9{\cdot}8\sin 30°}}$$

$$= \mathbf{1{\cdot}53\ s}$$

TYPE VI. PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

Formulae used. In the absence of an external torque, *i.e.*, when $\tau = 0$

$$\frac{dL}{dt} = 0 \text{ or } L = \text{constant}, \ i.e., \ L = I\,\omega = \text{constant}$$

$$I_1\,\omega_1 = I_2\,\omega_2$$

$$I_1\left(\frac{2\pi}{T_1}\right) = I_2\left(\frac{2\pi}{T_2}\right), \ i.e., \ \frac{I_1}{I_2} = \frac{T_1}{T_2}$$

Units used. T_1, T_2 in second ; ω in radian/sec, I in kg m^2 and τ in N-m

Example 25 **What will be the duration of the day, if the earth suddenly shrinks to 1/64 th of its original volume, mass remaining unchanged ?**

Solution. Here,

final volume, $V_2 = \frac{1}{64}$ initial volume, V_1

$$i.e., \quad \frac{4}{3}\pi R_2^3 = \frac{1}{64}\times\frac{4}{3}\pi R_1^3$$

$$R_2^3 = \left(\frac{1}{4}R_1\right)^3 \quad \text{or} \quad R_2 = \frac{1}{4}R_1$$

As no external torque is acting,

$$L = I\,\omega = I\left(\frac{2\pi}{T}\right) = \text{constant.}$$

$$i.e., \quad I_2/T_2 = I_1/T_1$$

$$\left(\frac{2}{5}MR_2^2\right)\frac{1}{T_2} = \left(\frac{2}{5}MR_1^2\right)\times\frac{1}{T_1}$$

$$T_2 = \frac{R_2^2}{R_1^2}\times T_1$$

As T_1 = time taken by earth to complete one revolution about its axis.

i.e., normal length of the day = 24 hours

$$\therefore \quad T_2 = \left(\frac{1}{4}\right)^2 \times 24 = \mathbf{1{\cdot}5\ h}$$

Example 26 A horizontal disc rotating about a vertical axis passing through its centre makes 180 rpm. A small piece of wax of mass 10 g falls vertically on the disc and sticks to it at a distance of 8 cm from its axis. If the frequency is thus reduced to 150 rpm, calculate moment of inertia of the disc.

Solution. Here, $n_1 = 180$ rpm = 3 rps ;

$$n_2 = 150 \text{ rpm } = \frac{5}{2} \text{rps}$$

Let I_1 be the moment of inertia of the disc about the given axis.

When mass $m = 10$ g = 10^{-2} kg sticks to disc at $r = 8$ cm = 8×10^{-2} m, then $I_2 = I_1 + mr^2$.

According to principle of conservation of angular momentum $\quad I_2\,\omega_2 = I_1\,\omega_1$

$$\text{or} \quad (I_1 + mr^2)\, 2\,\pi\, n_2 = I_1\, 2\,\pi\, n_1$$

$$\text{or} \quad I_1 + mr^2 = I_1 \frac{n_1}{n_2} = I_1 \times \frac{3}{5/2} = \frac{6}{5} I_1$$

$$\frac{6}{5} I_1 - I_1 = mr^2$$

$$I_1 = 5\, mr^2 = 5 \times 10^{-2}\,(8 \times 10^{-2})^2$$

$$= \mathbf{3{\cdot}2 \times 10^{-4}\ kg\ m^2}$$

Example 27 A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7·6 kg m².

(a) What is his new angular speed ? (Neglect friction.) (b) Is kinetic energy conserved in the process ? If not, from where does the change come about ?

Solution. Here, $I_1 = 7{\cdot}6 + 2 \times 5\,(0{\cdot}9)^2$

$= 15{\cdot}7$ kg m², $\omega_1 = 30$ rpm

$I_2 = 7{\cdot}6 + 2 \times 5\,(0{\cdot}2)^2 = 8{\cdot}0$ kg m², $\quad \omega_2 = ?$

According to the principle of conservation of angular momentum, $\quad I_2\,\omega_2 = I_1\,\omega_1$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{15{\cdot}7 \times 30}{8{\cdot}0} = \mathbf{58{\cdot}88\ rpm}$$

No, kinetic energy is not conserved in the process. Infact, as moment of inertia decreases. K.E. of rotation increases. This change comes about as work is done by the man in bringing his arms closer to his body.

Example 28 A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1·0 m wide and weights 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (Hint. The moment of inertia of the door about the vertical axis at one end is ML²/3.)

Solution. Angular momentum imparted by the bullet $L = mv \times r = (10 \times 10^{-3}) \times 500 \times \frac{1}{2} = 2{\cdot}5$

$$\text{Also,} \quad I = \frac{ML^2}{3} = \frac{12 \times 1{\cdot}0^2}{3} = 4 \text{ kg m}^2$$

$$\text{As} \quad L = I\,\omega \ \therefore \ \omega = \frac{L}{I} = \frac{2{\cdot}5}{4} = \mathbf{0{\cdot}625\ rad/sec.}$$

Example 29 Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre) and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. Calculate (i) angular speed of the two disc system (ii) Show that K.E. of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss of energy ? Take $\omega_1 \neq \omega_2$.

Solution. Here, total initial angular momentum of the two discs $L_1 = I_1\,\omega_1 + I_2\,\omega_2$

Under the given conditions, moment of inertia of the two disc system = $(I_1 + I_2)$

If ω is angular speed of the combined system, the final angular momentum of the system

$$L_2 = (I_1 + I_2)\,\omega$$

As no external torque is involved in this exercise, therefore, $L_2 = L_1$

$$(I_1 + I_2)\,\omega = I_1\,\omega_1 + I_2\,\omega_2$$

$$\omega = \frac{I_1\,\omega_1 + I_2\,\omega_2}{I_1 + I_2} \qquad \text{...(}i\text{)}$$

(*b*) Initial K.E. of two discs

$$E_1 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$$

Final K.E. of the system $E_2 = \frac{1}{2}(I_1 + I_2)\ \omega^2$

using (*i*), $E_2 = \frac{1}{2}(I_1 + I_2) \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{(I_1 + I_2)^2}$

$$= \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{2(I_1 + I_2)}$$

Now,

$$E_1 - E_2 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 - \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{2(I_1 + I_2)}$$

which on simplification, gives us

$$E_1 - E_2 = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)},$$

which is a positive quantity (*i.e.* > 0), involving a perfect square.

$\therefore \quad E_1 - E_2 > 0 \quad$ or $\quad E_1 > E_2 \quad$ or $\quad E_2 < E_1$

Hence there occurs a loss of K.E. in the process. Loss of K.E. = $E_1 - E_2$. This loss must be due to friction in the contact of the two discs.

Note that angular momentum is conserved as torque due to friction is only an internal torque.

TYPE VII. MOMENT OF INERTIA AND RADIUS OF GYRATION

Formulae used.

1. $I = \sum_{i=1}^{i=n} m_i r_i^2 = MK^2$ **2.** $K = \sum_{i=1}^{i=n} \sqrt{r_i^2/n}$

3. According to theorem of parallel axes,

$$I_{AB} = I_C + M h^2$$

4. According to theorem of perpendicular axes,

$$I_z = I_x + I_y$$

5. For a uniform circular ring, moment of inertia about an axis passing through the centre and perpendicular to the plane of the ring, $I = MR^2$

6. For a uniform circular disc, moment of inertia about an axis passing through the centre and perpendicular to plane of disc $I = \frac{1}{2} \mathrm{MR}^2$

7. For a thin uniform rod, moment of inertia about an axis through its centre perpendicular to length of rod, $I = \frac{1}{12} \mathrm{M} l^2$

8. *MI* of a rectangular lamina of sides *l* and *b* about an axis passing through its centre and ⊥ to its plane

$$= \frac{m(l^2 + b^2)}{12}$$

9. Moment of inertia of a hollow cylinder about its axis = MR^2

10. Moment of inertia of a solid cylinder about its axis $= \frac{1}{2} \mathrm{MR}^2$

11. Moment of inertia of a hollow sphere about its diameter $= \frac{2}{3} \mathrm{MR}^2$

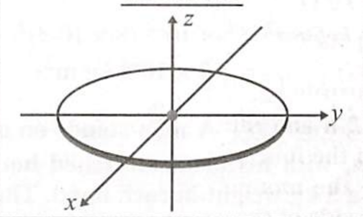
12. Moment of inertia of a solid sphere about its diameter $= \frac{2}{5} \mathrm{MR}^2$

Units used. *m*, *M* in kg ; *l*, *r*, *R* in metre ; *I* in kg m^2

Example 30 **What is the moment of inertia of a circular disc about one of its diameters ?** **(NCERT Solved Example)**

Solution. Moment of inertia of a disc of radius *R* and mass *M*, about an axis perpendicular to its plane and passing through its centre is $I_z = MR^2/2$. Let us take three concurrent axes through the centre *O* of the disc, as *x*, *y*, *z* axes as shown in Fig. 7.69 ; *x* and *y* axes are in the plane of disc and *z* axis is perpendicular to it. According to theorem of ⊥ axes,

FIGURE 7.69



$$I_z = I_x + I_y$$

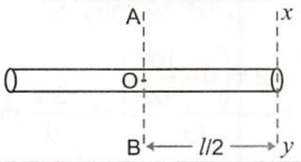
$$\frac{1}{2} \mathrm{MR}^2 = 2 \mathrm{I}_x \qquad (\because I_y = I_x)$$

or $$I_x = \frac{1}{4} \mathrm{MR}^2$$

Example 31 **What is the moment of inertia of a rod of mass *M*, length *l* about an axis perpendicular to it passing through one of its ends ?** **(NCERT Solved Example)**

Solution. For the rod of mass *M* and length *l*, moment of inertia about an axis *AB*, Fig. 7.70, is

FIGURE 7.70



$$I_{AB} = \frac{M\,l^2}{12}$$

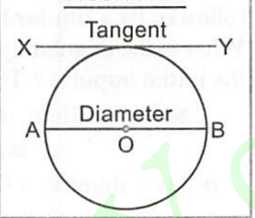
The given axis is XY. Applying theorem of parallel axes, $I_{XY} = I_{AB} + M\,(l/2)^2$

$$I_{XY} = \frac{M\,l^2}{12} + \frac{M\,l^2}{4} = \frac{M\,l^2}{3}$$

Example 32 **What is the moment of inertia of a ring about a tangent to the circle in the plane of the ring ?** (NCERT Solved Example)

Solution. Moment of inertia of a circular ring of mass M and radius R about the diameter AB, Fig. 7.71 is

FIGURE 7.71



$$I_{AB} = \frac{MR^2}{2}$$

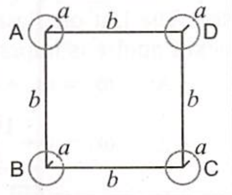
The given axis is XY, tangent to the ring in the plane of ring and parallel to its diameter AB. Using the theorem of parallel axes,

$$I_{XY} = I_{AB} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

Example 33 **Four spheres, each of diameter 2 a and mass M are placed with their centres on the four corners of a square of side b. Calculate the moment of inertia of the system about one side of the square taken as the axis.**

Solution. In Fig. 7.72, $ABCD$ is a square of side b. Four spheres, each of mass M and radius a are placed at the four corners of the square.

FIGURE 7.72



Moment of inertia of the system about any side, say CD is

= MI of sphere at A about CD

+ MI of sphere at B about CD

+ MI of sphere at C about CD

+ MI of sphere at D about CD

$$= \left(\frac{2}{5}\mathrm{M}\,a^2 + M\,b^2\right) + \left(\frac{2}{5}\mathrm{M}\,a^2 + M\,b^2\right) + \left(\frac{2}{5}\mathrm{M}\,a^2\right) + \left(\frac{2}{5}\mathrm{M}\,a^2\right)$$

$$= \frac{8}{5}\mathrm{M}\,a^2 + 2\mathrm{M}\,b^2 = \frac{2}{5}\,(4a^2 + 5b^2)$$

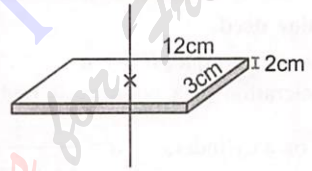
Example 34 **Find the moment of inertia of a rectangular bar magnet about an axis passing through its centre and parallel to its thickness. Mass of magnet is 100 g, its length is 12 cm, breadth is 3 cm and thickness is 2 cm.**

Solution. Here, $I = ?$, $m = 100$ g ;

$l = 12$ cm, $b = 3$ cm, $t = 2$ cm

The axis shown in the Fig. 7.73 is parallel to thickness of rectangular bar.

FIGURE 7.73



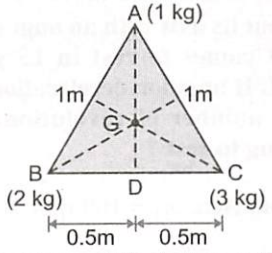
$$I = m\left(\frac{l^2 + b^2}{12}\right) = 100\left(\frac{12^2 + 3^2}{12}\right)$$

$$= 100 \times \frac{153}{12} = \mathbf{1275\ g\ cm^2}$$

Example 35 **Three balls of masses 1 kg, 2 kg and 3 kg are arranged at the corners of an equilateral triangle of side 1 m. What will be the moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle.**

Solution. In Fig. 7.74, G is centroid of equilateral triangle ABC.

FIGURE 7.74



Now, median $AD = \sqrt{AB^2 - BD^2}$

$$= \sqrt{1^2 - (0{\cdot}5)^2} = \sqrt{0{\cdot}75}$$

$$AG = BG = CG = \frac{2}{3} AD = \frac{2}{3} \sqrt{0{\cdot}75}$$

Moment of inertia of the system about an axis through centroid G and perpendicular to plane of triangle ABC is

$$I = 1 \times AG^2 + 2 \times BG^2 + 3 \times CG^2$$

$$= (1 + 2 + 3)\, AG^2 = 6\left(\frac{2}{3}\sqrt{0{\cdot}75}\right)^2$$

$$= 6 \times \frac{4}{9} \times 0{\cdot}75 = \mathbf{2\ kg\ m^2}$$

TYPE VIII. TYPICAL EXAMPLES

Formulae used.

1. Power of a torque, $P = \tau\,\omega$

2. Acceleration of a point mass tied to the string wound on a cylinder, $a = \dfrac{g}{1 + I/mR^2}$

3. Tension in the string, $T = \dfrac{mg}{1 + mR^2/I}$

4. Acceleration of a body rolling down a rough inclined plane of inclination θ is

$$a = \frac{mg\sin\theta}{m + I/r^2} = \frac{g\sin\theta}{(1 + k^2/r^2)}$$

5. Coefficient of friction for rolling of solid cylinder without slipping down the rough inclined plane is

$$\mu = \frac{1}{3}\tan\theta$$

Units used. Mass m in kg, accelerations a and g in ms^{-2}, tension T in newton, moment of inertia I in kg m^2, Power P in watt and torque τ in N-m

Example 36 **The motor of an engine is rotating about its axis with an angular velocity of 100 rpm. It comes to rest in 15 s after being switched off. If angular deceleration is constant, what is the number of revolutions made by it before coming to rest ?**

Solution. Here, $n_1 = 100 \text{ rpm} = \dfrac{100}{60}\text{ rps}$,

$$n_2 = 0,\ t = 15\text{ s},\ \alpha = ?,\ n = ?$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$$

$$= \frac{2\pi\left(0 - \frac{100}{60}\right)}{15} = -\frac{2\pi}{9}\ \text{rad s}^{-2}.$$

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$= 2\pi \times \frac{100}{60} \times 15 + \frac{1}{2}\left(-\frac{2\pi}{9}\right)(15)^2$$

$$= 50\,\pi - 25\,\pi = 25\,\pi$$

Number of rotations made,

$$n = \frac{\theta}{2\pi} = \frac{25\pi}{2\pi} = \mathbf{12{\cdot}5}$$

Example 37 **A cylinder of mass 5 kg and radius 30 cm and free to rotate about its axis receives an angular impulse of 3 kg m^2 s^{-1} initially, followed by a similar impulse after every 4 second. What is the angular speed of the cylinder 30 s after the initial impulse ? The cylinder is at rest initially.**

Solution. Here, $m = 5$ kg, $r = 30$ cm $= 0{\cdot}3$ m

$$\omega_1 = 0,\ \omega_2 = ?,\ t = 30\text{ s}$$

Angular impulse = Change in angular momentum

$$3 = I(\omega_2 - \omega_1) = \frac{1}{2} mr^2(\omega_2 - \omega_1)$$

$$= \frac{1}{2} \times 5 \times \left(\frac{3}{10}\right)^2 (\omega_2 - 0)$$

$$\omega_2 = \frac{3 \times 2 \times 100}{5 \times 9} = \frac{40}{3}$$

As $\omega_2 = \omega_1 + \alpha t$

$$\therefore \quad \frac{40}{3} = 0 + \alpha\, 4 \quad \text{or} \quad \alpha = \frac{10}{3}\ \text{rad/s}^2$$

As impulse is imparted after every 4 seconds, impulses imparted are at $t = 0, 4, 8, 12, 16, 20, 24, 28$ sec. The last one lasts upto $t = 32$ sec. (before the next impulse is imparted).

As $\omega_2 = \omega_1 + \alpha t$

$$\therefore \quad \omega_2 = 0 + \frac{10}{3} \times 32 = \frac{320}{3} = \mathbf{106{\cdot}7\ rad/s}$$

Example 38 **A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0·1 m. If the height of the inclined plane is 4 m, what is its rotational K.E. when it reaches foot of the plane ? Assume that the surfaces are smooth. Take M.I. of solid cylinder about its axis = $mr^2/2$.**

Solution. Here, $m = 2$ kg, $r = 0{\cdot}1$ m

Height of inclined plane, $h = 4$ m.

Rotational K.E. = ?

At the top of inclined plane, the solid cylinder has P.E. = mgh

At the bottom of inclined plane, the cylinder has K.E. of translation $\left(\frac{1}{2}mv^2\right)$

and K.E. of rotation $\left(\frac{1}{2}I\omega^2\right)$

$$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

Now, $v = r\omega$, $I = \frac{1}{2}mr^2$

$$\therefore \quad \frac{1}{2}m(r\omega)^2 + \frac{1}{2}\times\left(\frac{1}{2}mr^2\right)\omega^2 = mgh$$

$$\frac{3}{4}mr^2\omega^2 = mgh$$

$$\omega^2 = \frac{4mgh}{3mr^2} = \frac{4gh}{3r^2}$$

$$\text{K.E. of rotation} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\times\frac{4gh}{3r^2}$$

$$\frac{mgh}{3} = \frac{2\times 9{\cdot}8\times 4}{3} = \mathbf{26{\cdot}13\ J}$$

Example 39 **A 70 kg man stands in contact against the wall of a cylindrical drum of radius 3 m rotating about its vertical axis with 200 rpm. The coefficient of friction between the wall and his clothing is 0·15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?**

Solution. Here, $m = 70$ kg, $r = 3$ m.

$$\omega = 200\text{ rpm} = \frac{200}{60}\text{ rps} = \frac{10}{3}\times 2\pi\ \text{rad/s}$$

In the present case, necessary centripetal force is provided by the horizontal reaction H of the wall on the man, *i.e.*,

$$H = \frac{mv^2}{r} = mr\omega^2 \qquad (\because\ v = r\omega)$$

The frictional force F acts upwards and balances the weight mg of the man.

The man will remain stuck to the wall after the floor is removed, if weight of man ≤ force of friction

or $\quad mg \le \mu H\ (= \mu m r\omega^2)$

or $\quad \frac{g}{\mu r} \le \omega^2 \quad$ or $\quad \omega^2 \ge \frac{g}{\mu r}$

$$\therefore \quad \omega_{min} = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{9{\cdot}8}{0{\cdot}15\times 3}} = \mathbf{4{\cdot}67\ rad/sec}$$

Example 40 **The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. (*i*) What is the angular acceleration, assuming the acceleration to be uniform ? (*ii*) How many revolutions does the engine make during this time ?** **(NCERT Solved Example)**

Solution. Here,

$$n_1 = 1200\text{ rpm} = \frac{1200}{60}\text{ rps} = 20\text{ rps}$$

$$n_2 = 3120\text{ rpm} = \frac{3120}{60}\text{ rps} = 52\text{ rps}$$

$t = 16$ secs

$\alpha = ?\ ;\ \theta = ?$

From $\quad \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$

$$\alpha = \frac{2\pi(52-20)}{16} = \mathbf{4\pi\ rad/s^2}$$

Angular displacement (θ) in time t is calculated from the relation

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 2\pi n_1 t + \frac{1}{2}\alpha t^2$$

$$= 2\pi\times 20\times 16 + \frac{1}{2}\times 4\pi(16)^2$$

$$= 640\pi + 512\pi = 1152\pi\text{ rad}$$

As $\theta = 2\pi$ radian in one revolution, therefore, number of revolutions

$$= \frac{\theta}{2\pi} = \frac{1152\pi}{2\pi} = \mathbf{576}$$

Example 41 **Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity ?** **(NCERT Solved Example)**

Solution. In Art. 7.20, we have obtained acceleration of an object rolling down an inclined plane

$$a = \frac{g \sin\theta}{1 + I/mr^2} = \frac{g \sin\theta}{(1 + k^2/r^2)}$$

For a ring, $I = mr^2$; $k^2 = r^2$

$$\therefore\ a_{\text{ring}} = \frac{g \sin\theta}{1+1} = 0{\cdot}5\ g \sin\theta$$

For a solid cylinder, $I = \frac{1}{2} mr^2$; $k^2 = \frac{1}{2} r^2$

$$\therefore\ a_{\text{cyl.}} = \frac{g \sin\theta}{1 + \frac{1}{2}} = \mathbf{0{\cdot}67\ g \sin\theta}$$

For a solid sphere, $I = \frac{2}{5} mr^2$; $k^2 = \frac{2}{5} r^2$

$$\therefore\ a_{\text{sph}} = \frac{g \sin\theta}{1 + 2/5} = \mathbf{0{\cdot}71\ g \sin\theta}$$

As acceleration of the sphere is maximum, therefore, **the sphere will reach the ground with maximum velocity.**

Example 42 **A body of mass 5 kg is attached to a weightless string wound round a cylinder of mass 8 kg and radius 0·3 m. The body is allowed to fall. Calculate (*i*) tension in the string, (*ii*) acceleration with which the body falls and (*iii*) angular acceleration of the cylinder.**

Solution. Here, $m = 5$ kg, $M = 8$ kg, $R = 0{\cdot}3$ m

Moment of inertia of the cylinder $I = \frac{1}{2} MR^2$

(*i*) Tension in the string, $T = \dfrac{mg}{1 + mR^2/I}$

$$= \frac{mg}{1 + \frac{2\,mR^2}{MR^2}} = \frac{mg}{1 + \frac{2m}{M}} = \frac{5 \times 9{\cdot}8}{1 + \frac{2 \times 5}{8}}$$

$= \mathbf{21{\cdot}78\ N}$

(*ii*) Linear acceleration

$$a = \frac{g}{1 + I/mR^2} = \frac{g}{1 + \frac{M}{2\,m}} = \frac{9{\cdot}8}{1 + \frac{8}{2 \times 5}}$$

$= \mathbf{5{\cdot}44\ m/s^2}$

(*iii*) Angular acceleration,

$$\alpha = \frac{a}{R} = \frac{5{\cdot}44}{0{\cdot}3} = \mathbf{18{\cdot}13\ rad/s^2}$$

REVISION AT A GLANCE

1. **Centre of mass** of a body is a point where the entire mass of the body can be supposed to be concentrated. Infact, nature of motion executed by the body shall remain unaffected if all the forces acting on the body were applied directly at this point.

   Mathematically : If $\vec{r_1}$ and $\vec{r_2}$ are position vectors of the two particles of masses m_1 and m_2 constituting the system, then position vector $\vec{r}$ of the centre of mass is given by

$$\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

2. For a system of n-particles, the centre of mass is given by

$$\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2} + m_3 \vec{r_3} + \ldots.. + m_n \vec{r_n}}{m_1 + m_2 + m_3 + \ldots. + m_n} = \frac{\sum_{i=1}^{i=n} m_i \vec{r_1}}{M}$$

   In symmetrical bodies with uniform distribution of mass, the centre of mass coincides with the geometrical centre of the body.

   The centre of mass of an isolated system is either at rest or it moves with a uniform velocity along a straight line path.

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3. **Torque** $(\vec{\tau})$. *The turning effect of a force about the axis of rotation is called moment of force or torque due to the force.* Torque is measured as the product of the magnitude of the force and the perpendicular distance of the line of action of the force from the axis of rotation.

   Mathematically : If a force $\vec{F}$ acts at a point, whose position vector is $\vec{r}$; the torque due to force

   $$\vec{\tau} = \vec{r} \times \vec{F}$$

   (*a*) *In cartesian coordinates,* $\tau_z = x F_y - y F_x$

   (*b*) *In polar coordinates :* $\tau = r F \sin \phi$, where ϕ is angle between force vector $\vec{F}$ and position vector $\vec{r}$.
   SI unit of torque is Nm.

4. **Angular momentum** $(\vec{L})$. *It is the rotational analogue of linear momentum and is measured as the product of the linear momentum and the perpendicular distance of its line of action from the axis of rotation.*

   Mathematically : If $\vec{p}$ is linear momentum of the particle and $\vec{r}$ its position vector, then angular momentum of the particle, $\vec{L} = \vec{r} \times \vec{p}$

   (*a*) In cartesian coordinates : $L_z = x\, p_y - y\, p_x$
   (*b*) In polar coordinates : $L = r\, p \sin \phi$,

   where ϕ is angle between the linear momentum vector $\vec{p}$ and the position vector $\vec{r}$.
   SI unit of angular momentum is kg $m^2 s^{-1}$.
   Geometrically, angular momentum of a particle is equal to twice the product of mass of the particle and areal velocity of its radius vector about the given axis.

5. **Relation** between torque and angular momentum :

   (*i*) $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

   (*ii*) If the system consists of *n*-particles, then

   $$\vec{\tau}\,(tot) = \frac{d\vec{L_1}}{dt} + \frac{d\vec{L_2}}{dt} + \frac{d\vec{L_3}}{dt} + \ldots\ldots \frac{d\vec{L_n}}{dt}$$

6. **Law of conservation of angular momentum.** If no external torque acts on a system, then the total angular momentum of the system remains conserved.

   Mathematically : $\vec{L_1} + \vec{L_2} + \vec{L_3} + \ldots\ldots + \vec{L_n} = \vec{L}_{\text{total}}$ = *a* constant, when $\tau_{\text{ext.}} = 0$

7. **Moment of inertia (*I*).** *The moment of inertia of a rigid body about a given axis of rotation is the sum of the products of masses of the various particles and squares of their respective perpendicular distances from the axis of rotation.*

   Mathematically : $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots\ldots + m_n r_n^2 = \sum_{i=1}^{i=n} m_i r_i^2$

   SI unit of moment of inertia is kg m^2.
   MI corresponds to mass of the body. However, it depends on shape & size of the body & also on position and configuration of the axis of rotation.

8. **Radius of gyration (*K*).** *It is defined as the distance of a point from the axis of rotation at which, if whole mass of the body were concentrated, the moment of inertia of the body would be same as with the actual distribution of mass of the body.*

Mathematically : $K = \sqrt{\dfrac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}}$ = *rms distance of particles from the axis of rotation.*

SI unit of radius of gyration is *m*. Note that the moment of inertia of a body about a given axis is equal to the product of mass of the body and square of its radius of gyration about that axis, *i.e.,* $I = MK^2$.

9. **Theorem of perpendicular axes.** *It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through the lamina.*

Mathematically : $I_z = I_x + I_y$, where *X* and Y-axes lie in the plane of the Lamina and Z-axis is perpendicular to its plane and passes through the point of intersection of *X* and *Y* axes.

10. **Theorem of parallel axes.** *It states that the moment of inertia of a rigid body about any axis is equal to moment of inertia of the body about a parallel axis through its centre of mass plus the product of mass of the body and the square of the perpendicular distance between the axes.*

Mathematically : $I = I_c + M\,h^2$, where I_c is moment of inertia of the body about an axis through its centre of mass and *h* is the perpendicular distance betwween the two axes.

11. **Moment of inertia** of a few bodies of regular shape :

(*i*) M.I. of a rod about an axis through its c.m. and perpendicular to rod, $I = \dfrac{1}{12}\mathrm{ML}^2$

(*ii*) M.I. of a circular ring about an axis through its centre and perpendicular to its plane, $I = MR^2$

(*iii*) M.I. of a circular disc about an axis through its centre and perpendicular to its plane, $I = \dfrac{1}{2}\mathrm{MR}^2$

(*iv*) M.I. of a right circular solid cylinder about its symmetry axis, $I = \dfrac{1}{2}\mathrm{MR}^2$

(*v*) M.I. of a right circular hollow cylinder about its axis = MR^2

(*vi*) M.I. of a solid sphere about its diameter, $I = \dfrac{2}{5}\mathrm{MR}^2$

(*vii*) M.I. of a spherical shell about its diameter, $I = \dfrac{2}{3}\mathrm{MR}^2$

12. **Moment of inertia and angular momentum.** The moment of inertia of a rigid body about an axis is numerically equal to the angular momentum of the rigid body, when rotating with unit angular velocity about that axis.

Mathematically : $L = I\,\omega$

13. **Moment of inertia and kinetic energy of rotation.** The moment of inertia of a rigid body about an axis of rotation is numerically equal to twice the kinetic energy of rotation of the body, when rotating with unit angular velocity about that axis.

Mathematically : K.E. of rotation $= \dfrac{1}{2}\mathrm{I}\,\omega^2$

14. **Moment of inertia and torque.** The moment of inertia of a rigid body about an axis of rotation is numerically equal to the external torque required to produce a unit angular acceleration in the body about the given axis.

Mathematically : $\tau = I\,\alpha$

15. **Law of conservation of angular momentum.** If no external torque acts on a system, the total angular momentum of the system remains unchanged.

Mathematically : $I\,\omega$ = constant vector, *i.e.*, in magnitude, $I_1\,\omega_1 = I_2\,\omega_2$, provided no external torque acts on the system.

16. For **translational equilibrium** of a rigid body, $\vec{F} = \sum\limits_i \vec{F}_i = 0$

For **rotational equilibrium** of a rigid body, $\vec{\tau} = \sum\limits_i \vec{\tau}_i = 0$

17. The following table gives a summary of the **analogy** between various quantities describing linear motion and rotational motion.

S. No.	Linear motion	S. No.	Rotational motion
1.	Distance/displacement (s)	**1.**	Angle or angular displacement (θ)
2.	Linear velocity, $v = \dfrac{ds}{dt}$	**2.**	Angular velocity, $\omega = \dfrac{d\theta}{dt}$
3.	Linear acceleration, $a = \dfrac{dv}{dt} = \dfrac{d^2r}{dt^2}$	**3.**	Angular acceleration, $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$
4.	Mass (m)	**4.**	Moment of inertia (I)
5.	Linear momentum, $p = m\,v$	**5.**	Angular momentum, $L = I\omega$.
6.	Force, $F = m\,a$	**6.**	Torque, $\tau = I\,\alpha$
7.	Also, force $F = \dfrac{dp}{dt}$	**7.**	Also, torque, $\tau = \dfrac{dL}{dt}$
8.	Translational KE, $K_T = \dfrac{1}{2} m v^2$	**8.**	Rotational K.E., $K_R = \dfrac{1}{2} \mathrm{I}\,\omega^2$
9.	Work done, $W = F\,s$	**9.**	Work done, $W = \tau\,\theta$
10.	Power, $P = F\,v$	**10.**	Power, $P = \tau\,\omega$
11.	Linear momentum of a system is conserved when no external force acts on the system. (Principle of conservation of linear momentum).	**11.**	Angular momentum of a system is conserved when no external torque acts on the system (Principle of conservation of angular momentum).
12.	Equations of translatory motion (*i*) $v = u + at$ (*ii*) $s = ut + \dfrac{1}{2} a t^2$ (*iii*) $v^2 - u^2 = 2\,a\,s$, where the symbols have their usual meaning.	**12.**	Equations of rotational motion (*i*) $\omega_2 = \omega_1 + \alpha\,t$ (*ii*) $\theta = \omega_1\,t + \dfrac{1}{2}\alpha\,t^2$ (*iii*) $\omega_2^2 - \omega_1^2 = 2\,\alpha\,\theta$, where the symbols have their usual meaning.

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. The centre of mass of a body is moving with a uniform velocity of 10 cm/s. Three forces are applied on the body, which are in equilibrium. The velocity of centre of mass would become

(*a*) Zero (*b*) > 10 cm/s
(*c*) < 10 cm/s (*d*) 10 cm/s

2. A fire cracker following a parabolic path explodes in mid air. The centre of mass of all the fragments will follow a path

(*a*) along horizontal
(*b*) along vertical
(*c*) along same parabola
(*d*) along circle

3. The correct relation between linear velocity $\vec{v}$ and angular velocity $\vec{\omega}$ of a particle is

(*a*) $\vec{v} = \vec{\omega} \times \vec{r}$ (*b*) $\vec{\omega} = \vec{v} \times \vec{r}$
(*c*) $\vec{\omega} = \vec{r} \times \vec{v}$ (*d*) $\vec{v} = \vec{r} \times \vec{\omega}$

4. The expression for torque in polar co-ordinates is

(*a*) $\tau = r\,F \sin\phi$ (*b*) $\tau = r\,F \cos\phi$
(*c*) $\tau = r\,F \tan\phi$ (*d*) $\tau = \dfrac{F \sin\phi}{r}$

5. To maintain a rotor at a uniform angular speed of 100 s^{-1}, an engine needs to transmit a torque of 90 N-m. The power of engine must be

(*a*) 9 kW (*b*) 90 kW
(*c*) 9 MW (*d*) 90 MW

6. A rigid body is said to be in translational equilibrium, when its velocity $\vec{v}$ is

(*a*) zero
(*b*) constant
(*c*) constant or zero
(*d*) neither constant nor zero

7. A rigid body is said to be in partial equilibrium, when it is in

(*a*) translational equilibrium only
(*b*) rotational equilibrium only
(*c*) either (*a*) or (*b*)
(*d*) neither (*a*) nor (*b*)

8. Angular velocity of minute's hand of a clock in radian/sec is

(*a*) π/30 (*b*) π/1800
(*c*) 2π/30 (*d*) $\dfrac{2\pi}{1800}$

9. Moment of inertia of a body depends upon

(*a*) mass of body
(*b*) shape and size of body
(*c*) position and orientation of axis of rotation
(*d*) all the above

10. Formation of a neutron star is explained on the basis of

(*a*) conservation of linear momentum
(*b*) conservation of energy
(*c*) conservation of angular momentum
(*d*) none of these

11. Moment of inertia of a uniform circular ring of mass 2 kg and diameter 1 m about its diameter is

(*a*) 0·25 kg m^2 (*b*) 0·5 kg m^2
(*c*) 1 kg m^2 (*d*) 2 kg m^2

12. The radius of gyration of a uniform circular disc of radius *R*, about any diameter of the disc is

(*a*) $K = R$ (*b*) $K = R/2$
(*c*) $K = 2\,R$ (*d*) none of these

13. Moment of inertia of a hollow cylinder of mass *M* and radius *R*, about the axis of cylinder is

(*a*) $\dfrac{1}{2}MR^2$ (*b*) MR^2
(*c*) $\dfrac{2}{3}MR^2$ (*d*) $\dfrac{2}{5}MR^2$

14. Kinetic energy of a body rolling without slipping is

(*a*) $K = \dfrac{1}{2}mv^2$ (*b*) $K = \dfrac{1}{2}I\,\omega^2$
(*c*) $K = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\,\omega^2$ (*d*) $K = \dfrac{1}{2}mv^2 - \dfrac{1}{2}I\,\omega^2$

15. Acceleration of a solid cylinder rolling without slipping down an incline of inclination θ is

(*a*) $\dfrac{2}{3}g\sin\theta$ (*b*) $\dfrac{2}{3}g\cos\theta$
(*c*) $\dfrac{2}{3}g$ (*d*) $\dfrac{2}{3}g\tan\theta$

16. The rotational analogue of force is

(*a*) momentum (*b*) angular momentum

(*c*) torque (*d*) none of these

17. A grindstone of moment of inertia 6 kg m^2 is found to have a speed of 150 rpm, 10 sec. after starting from rest. Torque applied is

(*a*) 6 π Ns (*b*) 3 π Ns

(*c*) 9 π Ns (*d*) 12 π Ns

18. What would be the length of the day, if earth were to shrink suddenly to 1/64th of its original volume ?

(*a*) 24 h (*b*) 12 h

(*c*) 6 h (*d*) 1·5 h

19. Dimensional Formula of torque

(*a*) ML^2T^{-2} (*b*) MLT^{-2}

(*c*) $ML^{-2}T^2$ (*d*) $ML^{-2}LT$

20. The SI unit of Angular Momentum

(*a*) Nm (*b*) kg m^2/s

(*c*) kg m s^{-1} (*d*) kg m s^{-2}

21. The moment of momentum is also called

(*a*) couple (*b*) torque

(*c*) impulse (*d*) angular momentum

22. A coin of mass 0·48 kg and radius one meter is rolling on a horizontal surface without sliding with angular velocity 600 rotation/min. The total K.E. of the coin is

(*a*) 360 J (*b*) 1440 J

(*c*) 4000 J (*d*) 600 J

23. A body of mass *M* while falling vertically downwards under gravity breaks into two parts, a part *B* of mass *M*/3 and body *C* of mass 2 *M*/3. The centre of mass of the bodies *B* and *C* taken together

(*a*) does not shift

(*b*) shifts towards body *B*

(*c*) shifts towards body *C*

(*d*) depends upon height of breaking

24. Two identical particles move towards each other with velocity 2 *v* and *v* respectively. The velocity of their centre of mass is

(*a*) 0 (*b*) *v*

(*c*) $\frac{v}{2}$ (*d*) $\frac{v}{3}$

25. Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the mass centre of particles through a distance *d*, by what distance would the particle of mass m_2 move, so as to keep the mass centre of the particles at the original position ?

(*a*) *d* (*b*) $\frac{m_2 d}{m_1}$

(*c*) $\frac{m_1 d}{m_2}$ (*d*) $\frac{m_1 d}{m_1 + m_2}$

26. A system consists of two identical particles, one of the particles has an acceleration *a* and the other is at rest. The center of mass has an acceleration of

(*a*) 2 *a* (*b*) *a*

(*c*) $\frac{a}{2}$ (*d*) $\frac{a}{4}$

27. A point on the rim of a wheel of diameter 400 cm has a velocity of 16 m-s^{-1}. The angular velocity of the wheel is

(*a*) 2 rad-s^{-1} (*b*) 4 rad-s^{-1}

(*c*) 6 rad-s^{-1} (*d*) 8 rad-s^{-1}

28. An engine flywheel of diameter 1 m rotates with an angular velocity of 600 rev-min^{-1}. The linear velocity of a particle on the periphery of the wheel will be

(*a*) 40 π m - s^{-1} (*b*) 30 π m - s^{-1}

(*c*) 20 π m - s^{-1} (*d*) 10 π m - s^{-1}

29. The radius of gyration of a disc of mass 50 g and radius 2·5 cm, about an axis passing through its centre of gravity and perpendicular to the plane, is

(*a*) 0·52 cm (*b*) 1·76 cm

(*c*) 3·54 cm (*d*) 6·54 cm

30. A cylinder of mass 500 g and radius 10 cm has moment of inertia about its central axis in kg-m^2 equal to

(*a*) $1{\cdot}25 \times 10^{-3}$ (*b*) $2{\cdot}5 \times 10^{-3}$

(*c*) $5{\cdot}5 \times 10^{-3}$ (*d*) $7{\cdot}5 \times 10^{-3}$

31. A solid sphere of mass *M* and radius *R* is rotating about an axis passing through its centre making 600 rev-min^{-1}. Its rotational kinetic energy is

(*a*) $80\,\pi R$ (*b*) $80\,\pi^2 MR^2$

(*c*) $\frac{2}{5}\pi^2 MR$ (*d*) $\frac{2}{5}\pi M^2 R^2$

32. The moment of inertia of a body (initially at rest) about a given axis is 1·2 kg-m^2. In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of 25 rad-s^2 must be applied about that axis for a period of

(*a*) 1 s (*b*) 2 s

(*c*) 4 s (*d*) 10 s

33. If a solid spherical ball is rolling on a horizontal table, the ratio of its rotational kinetic energy to the total energy is

(*a*) $\frac{7}{10}$ (*b*) $\frac{2}{7}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{1}{6}$

34. If the earth were to suddenly contract to half of its present radius, then change in the duration of the day will be (Assume earth to be a perfect solid sphere of moment of inertia $= \frac{2}{5} MR^2$)

(*a*) 6 *h* increase (*b*) 18 *h* increase

(*c*) 6 *h* decrease (*d*) 18 *h* decrease

35. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination is

(*a*) ω (*b*) $\frac{I_1 \omega}{I_1 + I_2}$

(*c*) $\frac{I_2 \omega}{I_1 + I_2}$ (*d*) $\frac{(I_1 + I_2)\omega}{I_2}$

36. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. The four objects each of mass m are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(*a*) $\frac{M\omega}{4m}$ (*b*) $\frac{M\omega}{M + 4m}$

(*c*) $\frac{(M + 4m)\omega}{M}$ (*d*) $\frac{(M - 4m)\omega}{M + 4m}$

37. The angular velocity of a body changes from ω_1 to ω_2 without applying any torque but due to change in its moment of inertia. The ratio of radii of gyration in both the cases is

(*a*) $\sqrt{\omega_2} : \sqrt{\omega_1}$ (*b*) $\sqrt{\omega_1} : \sqrt{\omega_2}$

(*c*) $\omega_1 : \omega_2$ (*d*) $\omega_2 : \omega_1$

38. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

(*a*) $2L$ (*b*) $4L$

(*c*) $\frac{L}{2}$ (*d*) $\frac{L}{4}$

39. The position of a particle in a space is given by :

$$\vec{r} = \vec{i} + 2\vec{j} - \vec{k}$$

and momentum $\vec{p} = 3\vec{i} + 4\vec{j} - 2\vec{k}$.

The angular momentum is perpendicular to

(*a*) *x*-axis (*b*) *y*-axis

(*c*) *z*-axis (*d*) all of these

40. A body of moment of inertia of 3 kg-m^2 rotating with an angular velocity of 2 rad-s^{-1} has the same kinetic energy as a body of mass of 12 kg moving with a velocity of

(*a*) 1 m-s^{-1} (*b*) 2 m-s^{-1}

(*c*) 4 m-s^{-1} (*d*) 8 m-s^{-1}

41. If rotational kinetic energies of two bodies having moments of inertia of 9 kg-m^2 and 1 kg-m^2 are same, then ratio of their angular momenta is

(*a*) 9 : 1 (*b*) 1 : 9

(*c*) 1 : 3 (*d*) 3 : 1

42. A point P on the wheel is in contact with the ground. It rolls on the ground without slipping. The displacement of point P, when the wheel completes half of rotation is (Radius of wheel = 1 m)

(*a*) 1 m (*b*) 2 m

(*c*) $(\sqrt{\pi^2 + 2})m$ (*d*) $(\sqrt{\pi^2 + 4})m$

43. A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom ?

(*a*) $\sqrt{gh}$ (*b*) $\sqrt{2gh}$

(*c*) $\sqrt{\frac{3}{4}gh}$ (*d*) $\sqrt{\frac{4}{3}gh}$

44. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rev-min^{-1} about this axis. The torque, which can stop the wheel's rotation in one minute, would be

(*a*) $\frac{\pi}{18}$ N-m (*b*) $\frac{\pi}{15}$ N-m

(*c*) $\frac{\pi}{12}$ N-m (*d*) $\frac{2\pi}{15}$ N-m

45. A constant torque of 314 N-m is exerted on a pivoted wheel. If angular acceleration of wheel is 4π rad/s^2, then the moment of inertia of the wheel is

(*a*) 25 kg-m^2 (*b*) 35 kg-m^2

(*c*) 45 kg-m^2 (*d*) 55 kg-m^2

46. A flywheel has a moment of inertia 0·5 kg-m^2. What torque is required to increase its velocity from 2π rad – s^{-1} to 15π rad – s^{-1} in 10 s ?

(*a*) 1·0 N-m (*b*) 1·5 N-m

(*c*) 2·0 N-m (*d*) 3·0 N-m

II. Assertion-Reason Type Questions

Direction. For the following questions two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true, but R is a false.

(*d*) A is false, and R is also false.

47. Assertion. The angular speed of hour hand of a clock is more than angular speed of earth about its axis.

Reason. In both the cases, time period is 24 hrs.

48. Assertion. The whole mass of anybody can be supposed to be at the centre of mass of the body.

Reason. Centre of mass and centre of gravity of anybody represent the same point.

49. Assertion. Angular speed of rotation of minutes hand of a watch is less than angular speed of rotation of second's hand of the watch.

Reason. Both, the minutes hand and seconds hand move together only.

50. Assertion. A rigid body is that which has a definite shape and size.

Reason. Distances between different pairs of particles of such a body do not change on applying any force on it.

51. Assertion. Angular speed of second's hand of a clock is $\pi/30$ radian/sec.

Reason. $\omega = \frac{2\pi}{T} = \frac{2\pi}{60}$ radian/sec

$= \pi/30$ rad/sec.

52. Assertion. If earth were to shrink suddenly to 1/64th of its original volume, the length of the day would be 1·5 hour.

Reason. This follows from the principle of conservation of linear momentum.

53. Assertion. A body of mass M, while falling vertically downwards under gravity breaks into two parts of masses $M/3$ and $2\,M/3$. The centre of mass of the two bodies taken together does not shift.

Reason. This is because no external force is involved.

54. Assertion. When two identical particles move towards eachother with velocities v and $2\,v$, respectively, the velocity of their centre of mass is $v/2$.

Reason. $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$$= \frac{m(2v) + m(-v)}{m+m} = \frac{v}{2}$$

55. Assertion. If earth were to suddenly contract to half its present radius, the length of the day would become 6 hours.

Reason. This is based on the principle of conservation of angular momentum.

56. Assertion. A constant torque of 314 N-m is exerted on a pivoted wheel of moment of inertia 25 kg m^2, the angular acceleration of the wheel would be 4 π radian/sec^2.

Reason. From $\tau = I\,\alpha$,

$$\alpha = \frac{\tau}{I} = \frac{314}{25} = \frac{100\,\pi}{25}$$

$$= 4\,\pi \text{ rad/s}^2.$$

ANSWERS

I. Multiple Choice Questions

1. (*d*)	**2.** (*c*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)	**6.** (*c*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*c*)
11. (*a*)	**12.** (*b*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*c*)	**17.** (*b*)	**18.** (*d*)	**19.** (*a*)	**20.** (*b*)
21. (*d*)	**22.** (*b*)	**23.** (*a*)	**24.** (*c*)	**25.** (*c*)	**26.** (*c*)	**27.** (*d*)	**28.** (*d*)	**29.** (*b*)	**30.** (*b*)
31. (*b*)	**32.** (*b*)	**33.** (*b*)	**34.** (*d*)	**35.** (*b*)	**36.** (*b*)	**37.** (*a*)	**38.** (*d*)	**39.** (*a*)	**40.** (*a*)
41. (*d*)	**42.** (*d*)	**43.** (*d*)	**44.** (*b*)	**45.** (*a*)	**46.** (*c*)				

II. Assertion-Reason Type Questions

47. (*c*) **48.** (*c*) **49.** (*c*) **50.** (*a*) **51.** (*a*) **52.** (*c*) **53.** (*a*) **54.** (*a*) **55.** (*b*) **56.** (*a*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. As the applied forces are in equilibrium, velocity of centre of mass remains the same *i.e.* 10 cm/s.

2. As no external force is involved in explosion, the cracker the centre of mass of all the fragments will continue along the same parabola.

4. $\tau = r\,F \sin\phi$

5. $P = \tau\,\omega = 90 \times 100 = 9000\text{ W} = 9\text{ kW}$

7. Partial equilibrium means translational equilibrium or rotational equilibrium and not both.

8. $\omega = \frac{2\pi}{T} = \frac{2\pi}{60\times 60}\text{ rad/s} = \frac{\pi}{1800}\text{ rad/s.}$

10. Formation of neutron star is explained on the basis of conservation of angular momentum.

11. $I = \frac{1}{2}mr^2 = \frac{1}{2}\times 2\left(\frac{1}{2}\right)^2 = 0{\cdot}25\text{ kg m}^2.$

12. $I = MK^2 = \frac{1}{4}MR^2\ ;\ K = R/2$

14. When a body is rolling without slipping,

$$\text{K.E.} = \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2$$

15. From $a = \frac{mg\sin\theta}{m + I/r^2}$

For solid cylinder, $I = \frac{1}{2}mr^2$

$$\therefore\quad a = \frac{mg\sin\theta}{m + m/2} = \frac{2}{3}g\sin\theta$$

16. Torque is rotational analogue of force.

17. $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi\,(150/60) - 0}{10} = 0{\cdot}5\,\pi$

$$\tau = I\,\alpha = 6 \times 0{\cdot}5\,\pi = \mathbf{3\,\pi\ N\text{-}s}$$

18. From principle of conservation of angular momentum.

$$I\,\omega = I \times \frac{2\pi}{T} = \text{constant.}$$

$$\therefore\quad T \propto I$$

when $V_2 = \frac{1}{64}V_1\ ;\ R_2 = \frac{1}{4}R_1\ ;$

$$R_2^2 = \frac{R_1^2}{16},\ I_2 = \frac{I_1}{16}$$

$$T_2 = \frac{T_1}{16} = \frac{24}{16}\text{ hr} = \mathbf{1{\cdot}5\ hr}$$

19. Torque $= F \times r = (MLT^{-2}) \times L = [ML^2\,T^{-2}]$

20. Angular momentum $= I\,\omega = \text{kg m}^2/\text{s}$.

22. Here, $m = 0{\cdot}48$ kg, $r = 1$ m, $n = 600$ rpm $= 10$ rps.

$$\text{Total KE} = \frac{1}{2}mu^2 + \frac{1}{2}I\,\omega^2$$

$$= \frac{1}{2}mr^2\omega^2 + \frac{1}{2}\times\frac{1}{2}mr^2\,\omega^2$$

$$= \frac{3}{4}mr^2\,\omega^2$$

$$E = \frac{3}{4}\times 0{\cdot}48 \times 1^2\ (2\,\pi \times 10)^2 = \mathbf{1440\ J}$$

23. As no external force is involved, therefore centre of mass does not shift.

24. $v_{cm} = \frac{m(2\,v) + m(-v)}{m + m} = \frac{v}{2}$

25. As mass centre of particles is to be kept at original position, therefore,

$$m_1\,d = m_2\,d'$$

or $$d' = \frac{m_1\,d}{m_2}$$

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26. $a' = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

$$a' = \frac{m a + m \times 0}{m + m} = \frac{a}{2}$$

27. Here, $r = \frac{4\omega}{2}$ cm = 2 m. v = 16 m/s

From $v = r\,\omega$

$$\omega = \frac{v}{r} = \frac{16}{2} = \mathbf{8\ rad/s}$$

28. Here, $r = \frac{1}{2}\,m,\ n = 600$ rpm $= \frac{600}{60}$ rps = 10 rps

$$v = r\omega = \frac{1}{2} \times 2\pi\, n = \pi\, n = \pi \times 10 = 10\,\pi \text{ m/s}$$

29. $K = ?\ m = 50$ g, $r = 2{\cdot}5$ cm

$$K = \frac{r}{\sqrt{2}} = \frac{2{\cdot}5}{1{\cdot}414} = \mathbf{1{\cdot}76\ cm}$$

30. Here, $m = 500$ g $= \frac{1}{2}$ kg, $r = 10$ cm $= 0{\cdot}1$ m

$$I = \frac{1}{2}\,mr^2 = \frac{1}{2} \times \frac{1}{2}\,(0{\cdot}1)^2$$

$$= \mathbf{2{\cdot}5 \times 10^{-3}\ kg\ m^2}$$

31. $n = 600$ rpm $= \frac{600}{60}$ rps $= 10$ rps.

$$\text{Rotational KE} = \frac{1}{2}\,I\,\omega^2 = \frac{1}{2}\left(\frac{2}{5}\,MR^2\right)(2\,\pi\,n)^2$$

$$= \frac{4\pi^2}{5} \times 10 \times 10\ MR^2$$

$$= \mathbf{80\,\pi^2\ MR^2}$$

32. Here, $I = 1{\cdot}2$ kg m^2, $\omega_1 = 0$

$$\text{Rotational KE} = \frac{1}{2}\,I\,\omega_2^2 = 1500,\ \alpha = 25 \text{ rad/s}^2$$

$$\omega_2 = \sqrt{\frac{2 \times 1500}{1{\cdot}2}} = 50 \text{ rad/s}$$

From $t = \frac{\omega_2 - \omega_1}{\alpha} = \frac{50 - 0}{25} = \mathbf{2\ sec}$

33. $K_R = \frac{1}{2}\,I\,\omega^2 = \frac{1}{2}\left(\frac{2}{5}\,mr^2\right)\omega^2 = \frac{1}{5}\,mv^2$

$$K_T = \frac{1}{2}\,mv^2$$

Total energy $= K = K_R + K_T$

$$= \frac{1}{5}\,mv^2 + \frac{1}{2}\,mv^2 = \frac{7}{10}\,mv^2$$

$$\frac{K_R}{K} = \frac{1/5\ mv^2}{7/10\ mv^2} = \mathbf{\frac{2}{7}}$$

34. According to the principle of conservation of angular momentum.

$$I\,\omega = I \times \frac{2\pi}{T} = \text{constant}$$

$$\therefore \qquad T \propto I \cdot \left(= \frac{2}{5}\,MR^2\right)$$

when $R \to R/2$, I becomes $I/4$.

$\therefore$ T becomes $T/4$ *i.e.*, 24/4 = 6 hrs.

Decrease in duration of the day = 24 – 6 = **18 h**

35. If ω' is final angular velocity of the combination, then applying conservation of angular momentum principle, we have

$$(I_1 + I_2)\,\omega' = I_1\,\omega$$

$$\omega' = \frac{I_1\,\omega}{I_1 + I_2}$$

36. Applying principle of conservation of angular momentum, we get

$$(M + 4\,m)\,\omega' = M\,\omega$$

$$\omega' = \frac{M\,\omega}{M + 4m}$$

37. If moment of inertia changes from I_1 to I_2

then $\quad I_1\,\omega_1 = I_2\,\omega_2$

$$\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1}$$

$$\frac{m\,K_1^2}{m\,K_2^2} = \frac{\omega_2}{\omega_1}$$

$$\frac{K_1}{K_2} = \sqrt{\omega_2} : \sqrt{\omega_1}$$

38. KE $= \frac{1}{2} I \omega^2$

when ω is doubled and KE is halved, I must become $I/8$.

As $I\omega = L$, therefore when I becomes $I/8$ and ω is doubled, L becomes $L/4$.

39. $\vec{L} = \vec{r} \times \vec{p}$

$$= (\hat{i} + 2\hat{j} - \hat{k}) \times (3\hat{i} + 4\hat{j} - 2\hat{k})$$

$$= 4\,\hat{i} \times \hat{j} - 2\,\hat{i} \times \hat{k} + 6\hat{j} \times \hat{i} - 4\hat{j} \times \hat{k} - 3\hat{k} \times \hat{i} - 4\hat{k} \times \hat{j}$$

$$\vec{L} = -2\,\hat{i} \times \hat{j} - \hat{k} \times \hat{i} = -2\,\hat{k} - \hat{j}$$

Hence $\vec{L}$ lies in $y - z$ plane. It must be perpendicular to **x-axis.**

40. Here, $I_1 = 3$ kg m^2, $\omega_1 = 2$ rad/s

$m_2 = 12$ kg $\quad v_2 = ?$

As $\frac{1}{2} m_2 v_2^2 = \frac{1}{2} I_1 \omega_1^2$

$\therefore \quad 12\, v_2^2 = 3 \times 2^2 = 12$

$v_2^2 = 1, \; v_2 =$ **1 m/s**

41. (*d*) : Here, $I_1 = 9$ kg m^2, $I_2 = 1$ kg m^2

$$\frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_2 \omega_2^2$$

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$$\frac{L_1}{L_2} = \frac{I_1 \omega_1}{I_2 \omega_2} = \frac{9}{1} \times \frac{1}{3} = \frac{3}{1}$$

42. In half the rotation,

$$x = \frac{2\pi r}{2} = \pi\, r = \pi \times 1 \text{ metre}$$

$$y = 2\,r = 2 \times 1\, m = 2 \text{ m.}$$

$\therefore$ Displacement of point $P = \sqrt{x^2 + y^2}$

$$= \sqrt{\pi^2 + 4} \textbf{ metre}$$

43. Here, $K_T + K_R = Mgh$

$$\frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 = Mgh$$

$$\frac{1}{2} M v^2 + \frac{1}{2}\left(\frac{1}{2} MR^2\right)\omega^2 = Mgh$$

$$\frac{3}{4} M v^2 = Mgh$$

$$v = \sqrt{\frac{4}{3} gh}$$

44. Here, $I = 2$ kg m^2, $n_1 = 60$ rpm $= 1$ rps.

$t = 1$ min. $= 60$ s : $n_2 = 0$, $\tau = ?$

$$\sigma = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$$

$$= \frac{2\pi(0-1)}{60} = \frac{-\pi}{30}$$

$$\tau = I\,\alpha = 2\,(-\pi/30) = \frac{-\pi}{15} \textbf{ N-m}$$

45. $\tau = 314$ N-m, $\alpha = 4\,\pi$ rad/s. $I = ?$

From $\tau = I\,\alpha$, $I = \frac{\tau}{\alpha} = \frac{314}{4\pi}$ **= 25 kg m^2**

46. $I = 0{\cdot}5$ kg m^2 $\tau = ?$

$\omega_1 = 2\,\pi$ rad/s, $\omega_2 = 15\,\pi$ rad/s $\quad t = 10$ s.

$$\tau = I\,\alpha = I\,\frac{(\omega_2 - \omega_1)}{t}$$

$$= \frac{0{\cdot}5\,(15\pi - 2\pi)}{10}$$

$$= \frac{6{\cdot}5\pi}{10} = \mathbf{2{\cdot}0 \text{ N-m}}$$

II. Assertion-Reason Questions

47. From $\omega = \frac{2\pi}{T}$, the Assertion is true, but the Reason is false. For hour hand, $T = 12$ hrs and for earth, $T = 24$ hrs.

48. Assertion is true, but the Reason is false.

49. From $\omega = 2\,\pi/T$. The Assertion is true. However, the Reason is false.

50. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

51. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

52. Assertion is true, but the Reason is false. The Assertion follows from the principle of conservation of angular momentum.

53. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

54. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

55. Both, the Assertion and Reason are true. But Reason does not explain the Assertion properly.

56. Both, the Assertion and Reason are true and Reason is correct explanation of the Assertion.

CONCEPTUAL PROBLEMS

I. Centre of Mass

1. Is centre of mass a reality ?

Sol. No. The centre of mass of a system is a hypothetical point, where the entire mass of the body is supposed to be concentrated.

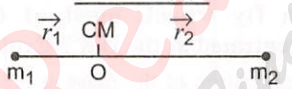
2. What is the advantage of concept of centre of mass ?

Sol. The concept of centre of mass enables us to describe the behaviour of a macroscopic bodies in terms of the laws developed for the microscopic bodies.

3. Prove that the centre of mass of two particles divides the line joining the particles in the inverse ratio of their masses.

Sol. Let $\vec{r}_1, \vec{r}_2$ be the position vectors of two particles of masses m_1 and m_2 situated at *A* and *B* respectively, Fig. 7(Q).1. Let the origin O of the frame of reference coincide with the centre of mass of the two particles.

FIGURE 7(Q).1



$$\therefore \quad m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$$

$$\text{or} \quad \vec{r}_1 = -\frac{m_2}{m_1} \vec{r}_2$$

$$\text{or} \quad |\vec{r}_1| = \frac{m_2}{m_1} |\vec{r}_2| \quad \text{or} \quad \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

which was to be proved.

4. In a stationary railway compartment, there are several passengers. If they start moving in the compartment, will *CM* of the compartment change ? Will the *CM* of the system of compartment and passengers change ?

Sol. The position of *CM* of the compartment will change as the passengers are external bodies for the compartment.

However, the position of *CM* of the system (compartment + passengers) will not change as no external force is acting on the system.

5. An isolated particle of mass *m* is moving in a horizontal plane (*x* – *y*), along the *x*-axis at a certain height above the ground. It explodes suddenly into two fragments of masses *m*/4 and 3m/4. An instant later, the smaller fragment is at *y* = + 15 cm. What is the position of larger fragment at this instant ?

Sol. As isolated particle is moving along *x*-axis at a certain height above the ground, there is no motion along Y-axis. Further, the explosion is under internal forces only. Therefore, centre of mass remains stationary along Y-axis after collision. Let the co-ordinates of centre of mass be (x_{cm}, 0).

$$\text{Now,} \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = 0$$

$$\therefore \quad m_1 y_1 + m_2 y_2 = 0$$

$$\text{or} \quad y_2 = \frac{-m_1 y_1}{m_2} = \frac{-m/4}{3m/4} \times 15 = -5 \text{ cm}$$

∴ Larger fragment will be at *y* = – 5 cm ; along *x*-axis.

II. Rotational Motion

6. Which physical quantities are expressed by the following :

(*i*) rate of change of angular momentum

(*ii*) moment of linear momentum ?

Sol. (*i*) Rate of change of angular momentum represents torque.

(*ii*) Moment of linear momentum represents angular momentum.

7. Torque and work are both equal to force time distance. How do they differ ?

Sol. Work is a scalar quantity, but torque is vector. Work = force × distance moved by the body in the direction of force. Torque is measured by force multiplied by perpendicular distance of the line of action of force from the axis of the rotation.

8. Why is the handle of a screw made wide ?

Sol. Turning moment of a force = force × distance (r) from the axis of rotation.

To produce a given turning moment, force required is smaller, when r is large. This is what happens when handle of the screw is made wide.

9. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its K.E. is halved, what happens to its angular momentum ?

Sol. $L = m\,v\,r$ and $v = r\,\omega = r\,(2\,\pi\,n)$

$$r = \frac{v}{2\pi n}$$

$$\therefore \quad L = m\,v\left(\frac{v}{2\pi n}\right) = \frac{m\,v^2}{2\pi n}$$

As $\text{K.E.} = \frac{1}{2}\,mv^2$, therefore, $L = \frac{\text{K.E.}}{\pi n}$

When K.E. is halved and frequency (n) is doubled, $L' = \frac{\text{K.E.}'}{\pi n'} = \frac{\text{K.E.}/2}{\pi(2n)} = \frac{\text{K.E.}}{4\pi n} = \frac{L}{4}$

i.e., **angular momentum becomes one fourth.**

III. Moment of Inertia

10. Why is moment of inertia called rotational inertia ?

Sol. This is because moment of inertia of a body is a measure of inertia of the body in rotational motion.

11. What is the advantage of a flywheel ?

Sol. In a flywheel, the entire mass is distributed on the rim of the wheel at maximum distance from the axis of rotation. Therefore, moment of inertia of the flywheel is large. It helps in making the motion more smooth or less jerky.

12. Why spokes are provided in a bicycle wheel ?

Sol. By connecting the rim of the wheel to the axle through the spokes, the mass of wheel goes over to the rim. As a result, moment of inertia of the wheel increases. This ensures smooth motion.

13. The moment of inertia of a solid sphere about a tangent is $\frac{5}{3}\,\mathbf{mr^2}$. What would be its moment of inertia about its diameter.

Sol. According to theorem of parallel axis,

$$I_T = I_{cm} + mr^2$$

$$I_{cm} = I_T - mr^2 = \frac{5}{3}mr^2 - mr^2 = \frac{2}{3}\mathbf{mr^2}$$

14. Find radius of gyration of a rod of length l and mass m about an axis perpendicular its length through one end.

Sol. Here, $K = ?$

According to theorem of parallel axes,

$$I_{\text{end}} = I_{cm} + m\,(l/2)^2 = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

$$\therefore\ mK^2 = \frac{ml^2}{3} \quad \text{or} \quad K = \frac{l}{\sqrt{3}}$$

15. For a given mass and size, moment of inertia of a solid disc is smaller than that of a ring. Why ?

Sol. This is because entire mass of ring is at its periphery *i.e.* at maximum distance from the centre. The mass of disc is distributed from the centre to the rim.

16. In a fly wheel, most of the mass is concentrated at the rim ? Explain why ?

Sol. Concentration of the mass at the rim increases moment of inertia (I) of the flywheel. If such a wheel gains or loses some K.E. of rotation, $\left(\frac{1}{2}\text{I}\,\omega^2\right)$, it brings about a relatively smaller change in its angular speed ω, ($\because$ I is large). Hence such a flywheel helps in maintaining uniform rotation.

17. Two satellites of equal masses, which can be considered as particles are orbiting the earth at different heights. Will their moments of inertia be same or different ?

Sol. Moments of inertia of the two satellites will be different. This is because, $I = \text{mass} \times (\text{distance})^2$.

For the satellite revolving at a greater height, distance from the axis of rotation is larger. Therefore, its moment of inertia is larger.

18. How will you distinguish between a hard boiled egg and a raw egg by spinning each on a table top ?

Sol. To distinguish between a hard boiled egg and a raw egg, we spin each on a table top. The egg which spins at a slower rate shall be a raw egg. This is because in a raw egg, liquid matter inside tries to get away from the axis of rotation. Therefore, its moment of inertia I increases. As $\tau = I\,\alpha$ = constant, therefore, α decreases *i.e.* raw egg will spin with smaller angular acceleration. The reverse is true for a hard boiled egg which will rotate more or less like a rigid body.

19. If earth were to shrink suddenly, what would happen to the length of the day ?

Sol. If earth were to shrink suddenly, its radius R would decrease. The moment of inertia of earth

$I = \frac{2}{5}\mathrm{MR}^2$ would decrease. As no external torque is acting on earth, its angular momentum

$L = I\,\omega = I\frac{2\pi}{T}$ remains constant. As I decreases, T must decrease. Hence **the length of the day will decrease.**

20. Two discs of same mass and thickness are made of materials having different densities. Which one of them will have larger moment of inertia ?

Sol. Here, $m_1 = m_2 \quad \therefore \; V_1\rho_1 = V_2\rho_2$

$$(\pi r_1^2)\,t.\rho_1 = (\pi r_2^2)\,t.\rho_2$$

$$\frac{\rho_1}{\rho_2} = \frac{r_2^2}{r_1^2} \qquad \ldots(i)$$

Now, $$\frac{I_1}{I_2} = \frac{\frac{1}{2}m_1 r_1^2}{\frac{1}{2}m_2 r_2^2} = \frac{r_1^2}{r_2^2} \qquad (\because m_1 = m_2)$$

$$\frac{I_1}{I_2} = \frac{\rho_2}{\rho_1} \qquad \ldots\text{using } (i)$$

$$\therefore \quad I \propto \frac{1}{\rho}$$

The disc of material with lower density will have larger moment of inertia.

21. The moments of inertia of two rotating bodies A and B are I_A and I_B. ($I_A > I_B$) and their angular momenta are equal. Which one has greater K.E. ?

Sol. We know, angular momentum $L = I\,\omega$

and K.E. of rotation,

$$K = \frac{1}{2}\mathrm{I}\,\omega^2 = \frac{1}{2}\frac{I^2\,\omega^2}{I} = \frac{1}{2\mathrm{I}}(L^2)$$

When L is constant, $K \propto 1/I$

As $I_A > I_B \therefore K_A < K_B$ or $K_B > K_A$ *i.e.* the body B has greater K.E. of rotation than the body A.

22. If the ice on the polar caps of the earth melts, how will it affect the duration of the day ? Explain.

Sol. Earth rotates about its polar axis. When ice of polar caps of earth melts, mass concentrated near the axis of rotation spreads out. Therefore, moment of inertia I increases.

As no external torque acts,

$$\therefore \quad L = I\,\omega = I\left(\frac{2\pi}{T}\right) = \text{constant.}$$

With increase of I, T will increase *i.e* **length of the day will increase.**

IV. Dynamics of Rotational Motion

23. A planet revolves around a massive star in a highly elliptical orbit. Is its angular momentum constant over the entire orbit ?

Sol. Yes, angular momentum of the planet is constant over the entire orbit. This is because revolution of planet around the star is under the effect of gravitational force between the star and the planet. This is a radial force whose torque is zero. Therefore, angular momentum of the planet is a constant (vector), whatever be the nature of the orbit.

24. If no external torque acts on a body, will its angular velocity remain conserved ?

Sol. When $\tau = 0$, $L = I\,\omega$ = constant.

$\therefore$ Angular velocity ω will be constant so long as I remains constant *i.e.* if moment of inertia of the body about the given axis remains fixed.

25. How does an ice-skater, a ballet dancer or an acrobat take advantage of the principle of conservation of angular momentum ?

Sol. During the course of their performance, an ice skater, a ballet dancer or an acrobat take advantage of the principle of conservation of angular momentum (*i.e.*, $I\,\omega$ = constant), by stretching out arms and legs or vice-versa. On doing so, their moment of inertia increases/decreases. Hence angular velocity ω of their spin motion decreases/increases accordingly.

26. Explain why the speed of a whirl wind in a tornado is alarmingly high ?

Sol. In a whirl wind, air from nearby regions gets concentrated in a small space. Moment of inertia (I) decreases on account of decrease in distance. As $L = I\,\omega$ = constant, therefore, angular speed ω increases to alarmingly high values.

27. Explain how is a cat able to land on its feet after a fall taking advantage of the principle of conservation of angular momentum ?

Sol. While falling, a cat stretches its body alongwith the tail so that its moment of inertia (I) increases. As no external torque is acting, $L = I\,\omega$ = constant. As I increases, ω decreases and it lands gently on its feet.

28. If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved ? Explain.

Sol. Here, $L = I\,\omega$ = constant

K.E. of rotation, $K = \frac{1}{2} I\,\omega^2$

$$K = \frac{1}{2I} I^2\,\omega^2 = \frac{L^2}{2I}$$

As L is constant, $\therefore\ K \propto 1/I$

When moment of inertia (I) decreases, K.E. of rotation (K) increases. Thus K.E. of rotation is not conserved.

29. When there is no external torque acting on a rotating body, which of the following quantities can change ? (*i*) Angular acceleration (*ii*) Angular momentum (*iii*) Angular speed.

Sol. (*i*) The relation between torque and angular acceleration is $\tau = I\,\alpha$ when $\tau = 0$, $\alpha = 0$

$\therefore$ Angular acceleration cannot change.

(*ii*) The relation between torque and angular momentum is $\tau = \frac{dL}{dt}$

when $\tau = 0$, $\frac{dL}{dt} = 0$ or L = constant

$\therefore$ angular momentum of the body does not change when torque is zero.

(*iii*) As $L = I\,\omega$ = constant, when $\tau = 0$, therefore, angular speed (ω) will change if moment of inertia (I) changes.

30. Equal torques are applied on a cylinder and a hollow sphere. Both have same mass and radius. The cylinder rotates about its axis and the sphere rotates about one of its diameters. Which will acquire greater speed ? Explain.

Sol. From $\tau = I\,\alpha$ or $\alpha = \frac{\tau}{I}$

As τ = constant, $\alpha \propto \frac{1}{I}$ $\therefore$ $\frac{\alpha_c}{\alpha_s} = \frac{I_s}{I_c}$

Now moment of inertia of cylinder about its axis is

$$I_c = \frac{1}{2} MR^2$$

and moment of inertia of hollow sphere about its diameter is

$$I_s = \frac{2}{3} MR^2$$

$$\therefore \quad \frac{\alpha_c}{\alpha_s} = \frac{2/3\,MR^2}{1/2\,MR^2} = \frac{4}{3}$$

$$\therefore \quad \alpha_c > \alpha_s$$

Hence, **the cylinder will acquire greater speed compared to the hollow sphere.**

31. A solid sphere is made to roll down from the same height on two inclined planes having different angles of inclination. In which case will it take less time to reach the bottom ?

Sol. As two inclined planes have different inclinations, but *same height*, therefore, velocity on reaching the bottom along two inclines is the same.

From $v = u + at = 0 + at$

$t = v/a$

Now, v is same and $a \propto \sin\theta$ $\therefore$ $t \propto \frac{1}{\sin\theta}$

***i.e.*, time taken will be smaller when the spheres rolls down the plane of higher inclination.**

32. A thin wheel can stay upright on its rim for a considerable length of time when rolled with a considerable velocity, while it falls from its upright position at the slightest disturbance, when stationary. Explain.

Sol. When the thin wheel is rolling upright, it possesses some angular momentum in the horizontal direction along the axis of the wheel. As angular momentum is conserved in magnitude as well as direction (in the absence of external torque) the wheel cannot fall from its upright position. However, when external torque due to friction reduces angular velocity of the wheel to zero, (*i.e.*, when the wheel becomes stationary) it falls from its upright position at the slightest disturbance, on account of moment of force due to its weight.

33. A one kg ball rolling on a smooth horizontal surface at 20 m s^{-1} comes to the bottom of an inclined plane making an angle of 30° with the horizontal. Calculate K.E. of the ball when it is at the bottom of incline. How far up the incline will the ball roll ? Neglect friction.

Sol. As is known from theory,

$$\text{Total KE of the ball} = \frac{7}{10} m v^2$$

$$= \frac{7}{10} \times 1 \times (20)^2 = 280 \text{ J}$$

As work done in stopping the ball = K.E. of ball

$$\therefore \quad (mg \sin\theta) \times s = 280$$

$$s = \frac{280}{mg \sin\theta} = \frac{280}{1 \times 9{\cdot}8 \times \frac{1}{2}} = \mathbf{57{\cdot}14 \text{ m}}$$

34. A very small particle rests on the top of a hemisphere of radius 20 cm. Calculate the smallest horizontal velocity to be given to it if it is to leave the hemisphere without sliding down its surface, take $g = 9{\cdot}8$ m/s^2.

Sol. The particle will leave the hemisphere, when normal reaction R becomes zero. In that event,

$$\frac{mv^2}{r} = mg$$

$$\therefore \quad v = \sqrt{rg} = \sqrt{\frac{20}{100} \times 9{\cdot}8}$$

$$= \sqrt{1{\cdot}96} = \mathbf{1{\cdot}4 \text{ m/s}}$$

35. The angular velocity of earth around the sun increases when it comes closer to the sun. Why ?

Sol. On coming closer, distance of earth from the axis through the sun decreases. $I = MK^2$ decreases. As $I\omega$ is constant, ω increases.

36. A particle performing uniform circular motion has angular momentum *L*. What will be its new angular momentum if its angular frequency is doubled and its KE is halved ?

Sol. Rotational KE, $K = \frac{1}{2} I\omega^2$

$$\therefore \qquad I = \frac{2K}{\omega^2}$$

$$\text{Angular momentum, } L = I\,\omega = \frac{2K}{\omega^2}\,\omega = \frac{2K}{\omega}$$

When ω is doubled and K is halved,

$$\text{angular momentum, } L' = \frac{2(K/2)}{2\omega} = \frac{2K}{4\omega}$$

$$= \frac{L}{4}$$

37. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by

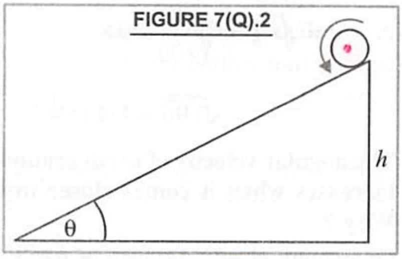
$$v^2 = \frac{2gh}{(1 + k^2/R^2)}$$

using dynamical consideration (*i.e.*, by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Sol. When a body rolls down an incline of height h, we apply the principle of conservation of energy.

K.E. of translation + K.E. of rotation (at the bottom) = P.E. at the top, Fig. 7(Q).2.

FIGURE 7(Q).2



i.e. $$\frac{1}{2}mv^2+\frac{1}{2}I\,\omega^2=mgh$$

$$\frac{1}{2}mv^2+\frac{1}{2}(mk^2)\,\omega^2=mgh$$

As $\omega=\dfrac{v}{R}$

$$\therefore\quad \frac{1}{2}mv^2+\frac{1}{2}m\frac{k^2}{R^2}v^2=mgh$$

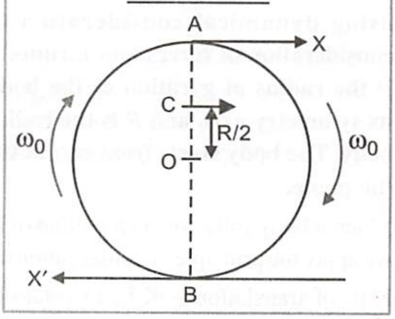
or $$\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)=mgh$$

$$v^2=\frac{2gh}{(1+k^2/R^2)}$$

which was to be proved.

38. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational pull) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig. 7(Q).3 ? Will the disc roll in the direction indicated ?

FIGURE 7(Q).3



Sol. Using the relation $v=r\,\omega$, we get

For point A, $v_A=R\,\omega_0$, along AX

For point B, $v_B=R\,\omega_0$, along BX'

For point C, $v_c=\left(\dfrac{R}{2}\right)\omega_0$ parallel to AX,

The disc *will not rotate*, because it is placed on a *perfectly frictionless table*. Without friction, rolling is not possible.

39. Explain why friction is necessary to make the disc roll (refer to Q. 38) in the direction indicated.

(a) give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.

(b) What is the force of friction after perfect rolling begins ?

Sol. To roll a disc, we require a torque, which can be provided only by a tangential force. As force of friction is the only tangential force in this case, it is necessary.

(*a*) As frictional force at B opposes the velocity of point B, which is to the left, the frictional force must be to the *right*. The sense of frictional torque will be perpendicular to the plane of the disc and outwards.

(*b*) As frictional force at B decreases the velocity of the point of contact B with the surface, the perfect rolling begins only when velocity of point B becomes zero. Also, force of friction would become zero at this stage.

40. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30º. The coefficient of static friction is $\mu_s=0{\cdot}25$.

(a) How much is the force of friction acting on the cylinder ?

(b) What is the work done against friction during rolling ?

(c) If the inclination θ of plane is increased, at what value of θ does the cylinder begin to skid and not roll perfectly ?

Sol. Here, $m=10$ kg , $r=15$ cm $=0{\cdot}15$ m

$\theta=30^\circ$, $\mu_s=0{\cdot}25$

Acceleration of the cylinder down the incline,

$$a=\frac{2}{3}\,g\sin\theta=\frac{2}{3}\times 9{\cdot}8\sin 30^\circ=\frac{9{\cdot}8}{3}\ \text{m/s}^2$$

(*a*) Force of friction,

$$F = mg \sin\theta - ma = m(g \sin\theta - a)$$

$$= 10\left(9{\cdot}8 \sin 30^\circ - \frac{9{\cdot}8}{3}\right) = \mathbf{16{\cdot}4\ N}$$

(*b*) During rolling, the point of contact is at rest. Therefore, work done against friction is **zero.**

(*c*) For rolling without slipping/skidding,

$$\mu = \frac{1}{3}\tan\theta$$

$$\tan\theta = 3\,\mu = 3 \times 0{\cdot}25 = 0{\cdot}75$$

$$\theta = 37^\circ$$

41. Read each statement below carefully and state with reasons, if it is true or false.

(a) During rolling, the force of friction acts in the same direction as the direction of motion of c.m of the body.

(b) The instantaneous speed of the point of contact during rolling is zero.

(c) The instantaneous acceleration of the point of contact during rolling is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling motion).

Sol. (*a*) The statement is false as is explained in Art. 7.46.

(*b*) True. This is because rolling body can be imagined to be rotating about an axis passing through the point of contact of the body with the ground. Hence its instantaneous speed is zero.

(*c*) This is not true. This is because when the body is rotating, its instantaneous acc. is not zero.

(*d*) It is true. This is because once the perfect rolling begins, force of friction becomes zero. Hence work done against friction is zero.

(*e*) The statement is true. This is because rolling occurs only on account of friction which is a tangential force capable of providing torque. When the inclined plane is perfectly smooth, the wheel will simply slip under the effect of its own weight.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Centre of Mass

1. What is the position vector of centre of mass of two particles of equal masses ?

Ans. It is the average of the position vectors of two particles.

2. Does centre of mass of a system of two particles lie on the line joining the particles ?

Ans. Yes, always.

3. Can centre of mass of a body lie where there is absolutely no mass ?

Ans. Yes, it can. For example, centre of mass of a uniform circular ring lies at the centre of the ring, where there is no mass.

4. Does centre of mass of a rigid body lie always on the body ?

Ans. No, it is not necessary. The centre of mass of a rigid body may or may not lie on the body.

5. On what factors does the position of cm of a rigid body depend ?

Ans. The *cm* of a rigid body depends on geometrical shape of the body and distribution of its mass.

6. Where does the centre of mass of a uniform triangular lamina lie ?

Ans. It lies at the centroid of the triangle *i.e.* at the point of intersection of three medians of the triangle.

7. Where does the centre of mass of a uniform cone lie ?

Ans. It lies on the line joining the apex to the centre of the base at a distance equal to $\frac{1}{4}$th of the length of this line from the base.

8. What is nature of motion of cm of an isolated system ?

Ans. The *cm* of an isolated system remains at rest or moves with a uniform velocity as no external force is acting on the isolated system.

9. Where does the centre of mass of a uniform rectangular lamina lie ?

Ans. *CM* lies at the point of intersection of diagonals of the rectangle.

II. Rotational Motion

10. Which physical quantity is represented by the product of the moment of inertia and the angular velocity ?

Ans. Product of I and ω represents angular momentum ($L = I\,\omega$).

11. Name the rotational analogue of force. What are its units ?

Ans. Torque is rotational analogue of force. Its units are N-m.

12. Write an expression for torque. Which rule is used for finding its direction ?

Ans. $\vec{\tau} = \vec{r} \times \vec{F}$. The direction of $\vec{\tau}$ is given by right handed screw rule.

13. Write the dimensional formula of angular momentum. Is it scalar or vecotr ?

Ans. Angular momentum = $[M^1 L^2 T^{-1}]$. It is a vector.

14. Name the physical quantity whose dimensions are same as that of angular momentum.

Ans. Planck's constant (*h*) has the dimensions of angular momentum.

15. Which physical quantity is conserved when a planet revolves around the sun ?

Ans. Angular momentum of plant is conserved.

16. What is angular velocity of earth spinning around its own axis ?

Ans. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{25 \times 60 \times 60}$ rad/s.

III. Moment of Inertia

17. Is moment of inertia a scalar or a vector ?

Ans. Moment of inertia is a scalar quantity.

18. What is the expression for moment of inertia of a hollow cylinder of mass *m*, length *l* and radius *r* about the axis of cylinder ?

Ans. $I = mr^2$.

19. What is moment of inertia of a solid cylinder of mass *m*, length *l* and radius *r* about the axis of the cylinder ?

Ans. $I = \dfrac{1}{2} mr^2$

20. What is radius of gyration of a solid sphere of mass *m* and radius *r* about its diameter ?

Ans. $I = MK^2 = \dfrac{2}{5} MR^2 = 0{\cdot}4\, MR^2$

$K = R\sqrt{0{\cdot}4}$

21. On what factors does the radius of gyration of a body depend ?

Ans. The radius of gyration of a body depends on (*i*) position and orientation of the axis of rotation (*ii*) distribution of mass of the body about the axis of rotation.

22. A ring and a circular disc of different materials have equal masses and equal radii. Which one will have a larger moment of inertia about an axis passing through its centre of mass perpendicular to its plane ?

Ans. A ring has a larger moment of inertia because its entire mass is concentrated at the rim at maximum distance from the axis.

23. A disc of metal is melted and recast in the form of a solid sphere. What will happen to the moment of inertia about a vertical axis passing through the centre ?

Ans. Moment of inertia will decrease, because $I_d = \dfrac{1}{2}\, m\, r^2$ and $I_s = \dfrac{2}{5}\, m\, r^2$, the radius of sphere formed on recasting the disc will also decrease.

24. What are the units and dimensions of moment of inertia ? Is it a vector ?

Ans. The units of M.I are kg m^2 and its dimensional formula is $[M^1 L^2 T^0]$. No, it is not a vector.

25. What is rotational analogue of mass of a body ?

Ans. Rotational analogue of mass of a body is moment of inertia of the body.

26. What are the factors on which moment of inertia of a body depend ?

Ans. Moment of inertia of a body depends on position and orientation of the axis of rotation. It also depends on shape, size of the body and also on the distribution of mass of the body about the given axis.

27. Is radius of gyration of a body constant quantity ?

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Ans. No, radius of gyration of a body depends on axis of rotation and also on distribution of mass of the body about this axis.

28. There are two spheres of same mass and same radius, one is solid and other is hollow. Which of them has a larger moment of inertia about its diameter ?

Ans. The hollow sphere shall have greater moment of inertia, as its entire mass is concentrated at the boundary of the sphere, at maximum distance from the axis.

29. Two solid spheres of the same mass are made of metals of different densities. Which of them has a larger moment of inertia about a diameter ?

Ans. The sphere of metal with smaller density shall be bigger in size & hence it will have larger moment of inertia.

IV. Dynamic of Rotational Motion

30. A ballet-dancer stretches her hands out for slowing down. This is based on principle of conservation of... .

Ans. This is based on the principle of conservation of angular momentum.

31. What is rotational analogue of force ?

Ans. Rotational analogue of force is torque.

32. A cannon ball and a marble ball roll from rest down an incline. Which goes to the bottom first ?

Ans. Both the balls reach the bottom simultaneously. This is because acceleration due to gravity is independent of the mass.

33. Can a body in translatory motion have angular momentum ?

Ans. Yes, a particle in translatory motion always has an angular momentum, unless the point (about which angular momentum is calculated) lies on the line of motion.

34. Why spin angular velocity of a star is greatly enhanced when it collapses under gravitational pull and becomes a neutron star ?

Ans. On collapsing under gravitational pull, size of star decreases. Therefore, its moment of inertia decreases. As angular momentum ($L = I\,\omega$) is conserved, and I decreases, therefore, spin angular velocity ω increases.

35. Complete the statement : Angular momentum of a particle is equal to twice ....

Ans. Angular moment of a particle is equal to twice the product of its mass and areal velocity.

36. Rolling motion can be treated as a combination of .... Fill in the blanks.

Ans. Rolling motion can be treated as a combination of pure rotation and pure translation.

37. Write an expression for kinetic energy of rolling body.

Ans. $K = \frac{1}{2}\,m v_{cm}^2 \left[1 + \frac{K^2}{R^2}\right]$

38. What is the minimum value of coefficient of friction between the cylinder and inclined plane for rolling without slipping ?

Ans. $\mu = \frac{1}{3}\tan\theta$

39. A body is rotating about a given axis with an angular acceleration α. Angle traced by the body in nth second is :

Ans. $\theta_{nth} = \omega_1 + \frac{\alpha}{2}\,(2n - 1)$

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Centre of Mass

1. Where does the centre of mass of a uniform triangular lamina lie ?

Ans. It lies at the centroid of the triangular lamina *i.e.,* where the three medians of the triangular lamina intersect.

2. Two particles of masses m_1 and m_2 move with velocities v_1 and v_2 towards eachother on a smooth horizontal surface. What is the velocity of their centre of mass ?

Ans. $\vec{v}_{cm} = \dfrac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$

3. Two balls of mass m each are placed at the two vertices of an equilateral triangle. Another ball of mass 2 m is placed at the third vertex of the triangle. Locate the centre of mass of the system.

Ans. As is clear from Fig. 7(Q).4 the centre of mass of two balls at A and B lies at the centre D of AB.

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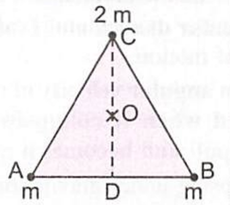
FIGURE 7(Q).4



The centre of mass of mass 2 m at *D* and 2 m at *C* would lie at *O*, the middle point of *CD*.

II. Rotational Motion

4. Can a body in translatory motion have angular momentum ? Explain.

Ans. Yes, a body in translatory motion shall have angular momentum, unless the fixed point about which angular momentum is taken lies on the line of motion of the body. This follows from $|L| = r\,p \sin\phi$. $L = 0$, only when $\phi = 0°$ or $\phi = 180°$.

5. The length of seconds hand of a clock is 10 cm. The speed of the tip of the hand is......... .

Ans. $v = r\omega = r\left(\dfrac{2\pi}{T}\right) = 10 \times \dfrac{2\pi}{60} = \dfrac{\pi}{3}$ **cm/s.**

6. Is torque a scalar or vector ? If it is a vector, what rule is used to determine its direction ?

Ans. Torque is a vector quantity. $\vec{\tau} = \vec{r} \times \vec{F}$. Its direction is determined by right handed screw rule or right hand thumb rule and is perpendicular to $\vec{r}$ and $\vec{F}$.

7. Why do we prefer to use a wrench with a long arm ?

Ans. The turning effect of force, $\vec{\tau} = \vec{r} \times \vec{F}$. When arm of the wrench is long, *r* is larger. Therefore, smaller force (*F*) will produce the same turning effect.

8. What is the dimensional formula of angular momentum and what are its units ? Is it a scalar ?

Ans. The dimensional formula of angular momentum is $[M^1 L^2 T^{-1}]$. Its units are kg m^2 s^{-1}. It is not a scalar, but a vector.

9. Is the angular speed of rotation of hour hand of a watch greater or smaller than the angular speed of earth's rotation about its axis ?

Ans. The hour hand of a watch takes 12 h to complete one rotation *i.e.* $T_1 = 12$ hour. And the earth takes 24 hours to rotate once around its axis, *i.e.*, $T_2 = 24$ hour.

As $\omega = \dfrac{2\pi}{T}$

and $T_1 < T_2 \quad \therefore \quad \omega_1 > \omega_2$

i.e., angular speed of hour hand is greater than the angular speed of earth around its axis.

10. Why are doors provided with handles near the outer edges, far away from the hinges ?

Ans. As $\tau = r\,F \sin\theta$, therefore to produce a given torque, *r* is made large so that force needed is small. Hence the doors can be opened or closed easily.

11. It is difficult to open or close the door by pushing/pulling it at the hinges. Why ?

Ans. This is because by doing so, $r = 0$, therefore, torque $= rF \sin\theta =$ Zero.

12. To open or close a heavy door, why force is applied at right angles to the door.

Ans. When $\theta = 90°$, $\tau = rF \sin 90° = rF =$ maximum.

13. Why is it easier to open a tap with two fingers than with one finger ?

Ans. With two fingers, we are applying a couple whose arm is equal to length of handle of the tap. When we apply force with one finger, an equal and opposite force acts at the axis of rotation. Length of arm of couple is halved. Moment of couple is halved.

14. What is angular impulse ?

Ans. The angular impulse acting on a body is the change in angular momentum of the body about a given axis.

15. Which component of linear momentum does not contribute to angular momentum ?

Ans. The radial component of linear momentum does not contribute to angular momentum.

16. A particle moves in a circular path with decreasing speed. What happens to its angular momentum ?

Ans. From $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\,\vec{v}$

When speed $\vec{v}$ decreases, magnitude of angular momentum decreases, but the direction of angular momentum remains the same.

III. Moment of Inertia

17. About which axis would the moment of inertia of a body be minimum ?

Ans. The moment of inertia of a body is minimum about an axis passing through its centre of mass.

18. About which axis would a uniform cube have minimum rotational inertia ?

Ans. The uniform cube will have minimum rotational inertia about a diagonal.

19. Is radius of gyration of a body a constant quantity ?

Ans. No. It depends upon position and orientation of the axis of rotation of the body.

20. A solid disc is recast into a thin walled cylinder of same radius. Which will have larger moment of inertia ?

Ans. The thin walled hollow cylinder will have larger moment of inertia as its mass is located at comparatively larger distance from the axis of rotation.

21. Two satellites of equal masses are orbiting the earth at different heights. Will their moment of inertia be same ?

Ans. No. Moment of inertia = $M(R+h)^2$. As heights are different, moments of inertia of the two satellites will be different.

22. In which of the following cases shown in Fig. 7(Q).5, it is most difficult to rotate the rod ? Explain why ?

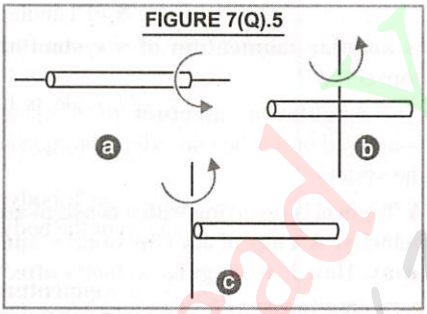


FIGURE 7(Q).5

Ans. In Fig. 7.28, moment of inertia (I) of the rod is maximum in case of (c). Work required to rotate the rod, $W = \frac{1}{2} I \omega^2$ is maximum in case of (c).

23. There is a stick half of which is wooden and half is of steel. It is pivoted at the wooden end and a force is applied at the steel end at right angles to its length. Next, it is pivoted at the steel end and the same force is applied at the wooden end. In which case is angular acceleration more and why ?

Ans. We know that torque,

$$\tau = \text{Force} \times \text{distance} = I\alpha = \text{constant}$$

$$\therefore \quad \alpha = \frac{\tau}{I} \quad i.e \quad \alpha \propto \frac{1}{I}$$

Angular acc. (α) will be more, when I is small, for which lighter material (wood) should be at larger distance from the axis of rotation, *i.e.*, when the stick is pivoted at the steel end.

24. Two circular discs A and B of the same mass and same thickness are made of two different metals whose densities are d_A and d_B ($d_A > d_B$). Their moments of inertia about the axes passing through their centres of gravity and perpendicular to their planes are I_A and I_B. Which is greater, I_A or I_B ?

Ans. Let r_A and r_B be the radii of discs A & B. As their mass (m) and thickness (t) are same, therefore,

$$m = (\pi r_A^2)\, t \times d_A = (\pi r_B^2) \times t \times d_B$$

$$\therefore \quad \frac{r_A^2}{r_B^2} = \frac{d_B}{d_A}$$

$$\text{Now} \quad \frac{I_A}{I_B} = \frac{\frac{1}{2} m r_A^2}{\frac{1}{2} m r_B^2} = \frac{r_A^2}{r_B^2} = \frac{d_B}{d_A}$$

$$\text{As} \quad d_A > d_B, \quad \therefore \quad I_B > I_A$$

25. Calculate radius of gyration of a cylindrical rod of mass m and length L about an axis of rotation perpendicular to its length and passing through the centre.

Ans. K = ? mass = m, length L.

Moment on inertia of the rod about an axis perpendicular to its length and passing through the centre is

$$I = \frac{mL^2}{12}$$

$$\text{Also,} \quad I = mK^2$$

$$\therefore \quad mK^2 = \frac{mL^2}{12} \quad \text{or} \quad K = \frac{L}{\sqrt{12}} = \frac{L}{2\sqrt{3}}$$

IV. Dynamic of Rotational Motion

26. (*i*) A person sits near the edge of a circular platform revolving with a uniform angular speed. What will be the change in the motion of the platform ?

(*ii*) What if the person starts moving from the edge towards the centre of the platform ?

Ans. (*i*) As $L = I\omega$ = constant, and I increases, therefore, ω will decrease.

(*ii*) As the person starts moving from the edge towards the centre of platform, I goes on decreasing. Hence ω goes on increasing.

27. Why there are two propellers in a helicopter ?

Ans. If the helicopter had only one propeller, then due to conservation of angular momentum, the helicopter itself would turn in the opposite direction.

28. A solid wooden sphere rolls down two different inclined planes of the same height but of different inclinations. (*a*) Will it reach the bottom with same speed in each case ? (*b*) Will it take longer to roll down one inclined plane than other ? Explain.

Ans. (*a*) Yes, because speed at the bottom depends only on height and not on slope.

(*b*) Yes, greater the inclination (θ), smaller will be the time of descent, as $t \propto 1/\sin\theta$.

29. Using expressisions for power and kinetic energy of rotational motion, derive the relation $\tau = I\,\alpha$, where letters have their usual meaning.

Ans. We know that power in rotational motion,

$$P = \tau\,\omega \qquad ...(i)$$

and K.E. of rotation, $E = \frac{1}{2} I\omega^2 \qquad ...(ii)$

As power = time rate of doing work in rotational motion, and work is stored in the body in the form of K.E.

$$\therefore \quad P = \frac{d}{dt}\text{ (K.E. of rotation)}$$

$$= \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = \frac{1}{2} I \times 2\omega \left(\frac{d\omega}{dt}\right)$$

$$P = I\,\omega\,\alpha$$

Using (*i*), $P = \tau\,\omega = I\,\omega\,\alpha$ or $\tau = I\,\alpha$, which is the required relation.

30. A thin uniform circular disc of mass *M* and radius *R* is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass *M*/4 is placed gently on the first disc co-axially. Show that angular velocity of the system is $4\omega/5$.

Ans. Initial angular momentum of one disc.

$$L = I\,\omega = \frac{1}{2} MR^2\,\omega$$

When another disc. of mass $\frac{M}{4}$ and radius R is placed gently on it, total moment of inertia of the combination is

$$I' = \frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2 = \frac{5}{8}MR^2$$

As no external torque has been applied, angular momentum is conserved.

$$\therefore\ I'\,\omega' = I\,\omega, \quad \omega' = \frac{I\,\omega}{I'} = \frac{\frac{1}{2}MR^2\,\omega}{\frac{5}{8}MR^2} = \frac{4}{5}\omega$$

31. Is angular momentum of a system always conserved ?

Ans. No. Angular momentum of a system is conserved only when no external torque acts on the system.

32. A flywheel is revolving with a constant angular velocity. A chip of its rim breaks and flies away. How is its angular velocity affected ?

Ans. As a chip breaks off, mass and hence moment of inertia of flywheel decreases. As $I\omega$ = constant, therefore angular velocity ω increases.

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. Explain that torque is only due to transverse component of force. Radial component has nothing to do with torque. **[Art. 7.15]**
2. Show that centre of mass of an isolated system moves with a uniform velocity along a straight line path. **[Art. 7.8]**
3. Locate the centre of mass of uniform triangular lamina and a uniform cone. **[Art. 7.7]**
4. Explain the concepts of torque and angular momentum. **[Arts. 7.13 and 7.19]**
5. Explain what is meant by centre of gravity. **[Art. 7.25]**
6. Why is the speed of whirl wind in a tornado so high ? **[Art. 7.9]**
7. State the two theorems of moment of inertia. **[Arts. 7.11 and 7.12]**
8. Explain the physical significance of moment of inertia and radius of gyration. **[Arts. 7.2 and 7.3]**
9. Obtain expression for K.E. of rolling motion. **[Art. 7.19]**
10. State the laws of rotational motion. **[Art. 7.21]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Obtain an expression for the position vector of centre of mass of a two particle system. [Art. 7.5]
2. Obtain an expression for the position vector of the centre of mass of a system of n particles. [Art. 7.6]
3. Prove that centre of mass of an isolated system moves with a uniform velocity along a straight line path. [Art. 7.8]
4. Discuss atleast three examples of the motion of centre of mass. [Art. 7.10]
5. Briefly discuss the concept of torque or moment of a force. [Art 7.13]
6. Derive an expression for torque in cartesian co-ordinates. [Art. 7.14]
7. Obtain an expression for torque in polar co-ordinates. [Art. 7.15]
8. Explain the concept of angular momentum and obtain an expression for it in cartesian co-ordinates. [Art. 7.18]
9. Discuss the physical meaning of angular momentum. [Art. 7.19]
10. Briefly explain the geometrical meaning of angular momentum. [Art 7.20]
11. Briefly explain equilibrium of a rigid body. When is a body said to be in partial equilibrium ? [Art. 7.23]
12. Obtain an expression for kinetic energy of rotation of a body. Hence define moment of inertia of the body. Explain its physical significance. **(J & K Board 2011)** **[Arts. 7.2 and 7.3]**
13. Establish a relation between torque and moment of inertia of a rigid body. [Art. 7.5]
14. Establish a relation between angular momentum and moment of inertia of a rigid body. [Art. 7.6]
15. State and explain the principle of conservation of angular momentum. Give atleast two examples. [Arts 7.8 & 7.9]
16. State and prove theorem of parallel axes. [Art. 7.11]
17. State and prove theorem of perpendicular axes. [Art. 7.12]
18. Derive an expression for moment of inertia of a thin circular ring about an axis passing through its centre and perpendicular to the plane of the ring. [Art. 7.13]
19. Obtain an expression for moment of inertia of a uniform circular disc about a diameter of the disc. **(J & K Board 2011)** [Art. 7.14]
20. Discuss rolling without slipping of a cylinder down a rough inclined plane and obtain an expression for the coefficient of friction necessary for the same. [Art. 7.20]
21. Draw analogy between rotational motion and translational motion. [Art. 7.22]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. Centre of mass of a body or system of bodies is a point at which the entire mass of the body/system of bodies is supposed to be concentrated. If all the forces acting on the body/system of bodies were to be applied at the centre of mass, the state of rest/motion of the body/system of bodies shall remain unaffected.

Note that this definition of centre of mass is only for the sake of convenience. It is not at all necessary that total mass of the system be actually present at the centre of mass. For example, the centre of mass of a uniform circular ring is at the centre of the ring, where there is no mass.

We can show that position vector $\vec{R}$ of a system of two particles of masses m_1 and m_2 at position vectors $\vec{r_1}, \vec{r_2}$ respectively is

$$\vec{R} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2} \quad \text{...}(i)$$

Thus, position vector of centre of mass of a two particle system is such that the product of the total mass of the system and position vector of centre of mass is equal to sum of the products of masses of two particles and their respective position vectors.

If the centre of mass of two particles were at the origin, *i.e.*, $\vec{r} = 0$, then from (*i*),

$$m_1 \vec{r_1} + m_2 \vec{r_2} = 0 \quad \text{or} \quad \vec{r_2} = \frac{-m_1}{m_2} \vec{r_1}$$

It implies that if m_1 lies on the left of origin, m_2 lies on the right of the origin. Clearly, centre of mass of a system of two particles lies on the straight line joining these particles. The centre of mass divides internally the line joining the two particles in the inverse ratio of masses. When two particles are of equal masses, their centre of mass lies exactly midway between them.

For a system of n particles, we may write

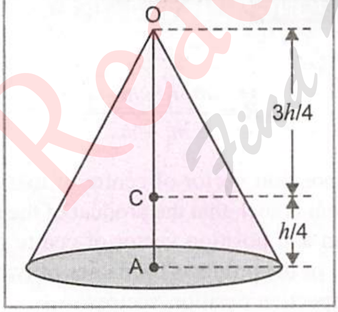
$$\vec{R} = \sum_{i=1}^{i=n} m_i \vec{r}_i \Big/ \sum_{i=1}^{i=n} m_i$$

Read carefully the above passage and answer the following very short answer and short answer questions :

1. What is centre of mass of a body ?
2. Give an example of a body at the centre of mass of which, no mass is present.
3. What is the position vector of centre of mass of a system of two particles ?
4. The centre of mass of two particle system is closer to heavier particle. Justify.

CASE 2. If a body has a symmetrical shape, then the position of centre of mass lies at the centre of symmetry or on the axis of symmetry. In case of a uniform circular ring, sphere, disc, the centre of mass lies at their respective centres. The centre of mass of a uniform triangular lamina is at the **centroid** of the triangle. It is the point of intersection of the medians of the triangle. The centre of mass of a solid symmetrical cone is at a distance $h/4$ from the centre of the base of the cone as shown in Fig. 7(Q).6.

FIGURE 7(Q).6



The position of the centre of mass depends on **shape and size** of the body and the distribution of its mass. Hence it may lie inside or outside the material of the body.

If we know the centres of mass of different parts of the system, and their masses, we can get the combined centre of mass by treating various parts as point objects, whose masses are concentrated at their respective centres of masses.

Read the above passage carefully and answer the following very short and short answer questions :

5. Where does the centre of mass of a uniform triangular lamina lie ?
6. Where does the centre of mass of a solid symmetrical cone of height h lie ?
7. On what factors does the centre of mass of a body depend ?
8. How do we obtain the centre of mass of a body of any shape and size ?

CASE 3. **Torque** due to a force or the moment of force gives us the turning effect of force about a fixed point/axis. It is measured by the product of magnitude of force and perpendicular distance of the line of action of force from the axis of rotation. Thus

Torque = force × perpendicular distance

$$\vec{\tau} = \vec{r} \times \vec{F} = r\,F \sin\theta\, \hat{n}$$

where θ is smaller angle between $\vec{r}$ and $\vec{F}$ and $\hat{n}$ is unit vector along $\vec{\tau}$. The direction of $\vec{\tau}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$, and is determined by **right hand screw rule** or **right hand thumb rule**. SI unit of torque is **N-m** ; which is equivalent to **joule**. The dimensions of torque are $[M^1 L^2 T^{-2}]$. We can prove that in cartesian co-ordinates, $\boxed{\tau_z = (x F_y - y F_x)}$. Thus, we may define torque as a quantity in rotational motion, which when multiplied by a small angular displacement gives us work done in rotational motion.

In polar co-ordinates, we show that $\boxed{\tau = r F \sin\phi}$, where ϕ is the angle which line of action of $\vec{F}$ makes with the position vector $\vec{r}$.

We can show that power associated with torque is $\boxed{P = \tau\,\omega}$

Read the above passage carefully and answer the following very short answer and short answer questions :

9. What is represented by torque due to a force ?
10. What is power associated with torque ?
11. Which rule is used for the direction of torque ?
12. What are the expression for torque in cartesian co-ordinates and in polar co-ordinates ?

ANSWERS

1. Centre of mass of a body is a point at which entire mass of the body is supposed to be concentrated.

2. Centre of mass of a uniform circular ring is at the centre of the ring, where there is no mass.

3. The position vector $\vec{R}$ of a system of two particles of masses m_1, m_2 with position vectors $\vec{r_1}$ and $\vec{r_2}$ respectively is

$$\vec{R} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

4. When centre of mass of two particles lies at the origin

$$m_1 \vec{r_1} + m_2 \vec{r_2} = 0$$

$$\vec{r_2} = -\frac{m_1 \vec{r_1}}{m_2}$$

when $m_2 > m_1$, $r_2 < r_1$

Hence centre of mass of two particle system is closer to heavier particle.

5. The centre of mass of a uniform triangular lamina lies at the centroid of the triangle. It is the point of intersection of the three medians of the triangle.

6. For a solid symmetrical cone of height h, centre of mass lies at a height $h/4$ above the centre of base of the cone.

7. The position of centre of mass depends on shape and size of the body and the distribution of its mass.

8. If we know the centre of mass of different parts of the system and their masses, we can get the combined centre of mass of body, by treating various parts as point objects whose masses are concentrated at their respective centres of masses.

9. Torque due to a force gives us the turning effect of force about a fixed point/axis.

10. Power associated with torque is $P = \tau\,\omega$

where ω is angular velocity of the body.

11. For the direction of torque, we use right hand thumb rule or right hand screw rule.

12. In cartesian co-ordinates, $\tau_z = x\,F_y - y\,F_x$,

and in polar co-ordinates, $\tau = r\,F \sin\phi$

The symbols have standard meaning.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. Angular momentum $(\vec{L})$ is rotational analogue of linear momentum $(\vec{p})$. We can show that

$$L = (x\,p_y - y\,p_x) \quad \text{and} \quad \tau = \frac{dL}{dt}$$

Basically, angular momentum of a particle/body about a given axis is the moment of linear momentum of the particle/body about that axis. It is equal to the product of linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation. This is the physical meaning of angular momentum.

In polar co-ordinates, we put

$$\vec{L} = p\,r\sin\phi = \vec{r} \times \vec{p}$$

where ϕ = angle which line of action of $\vec{p}$ makes with $\vec{r}$. The **geometrical meaning** of angular momentum is : angular momentum of a particle about a given axis is twice the product of mass of the particles and areal velocity of position vector of particle.

The time rate of change of angular momentum of a particle is equal to the torque.

A pair of equal and opposite forces with different lines of action are said to form a **couple**. A couple produces rotation without translation.

According to the **principle of moments,** a body will be in rotational equilibrium if algebraic sum of moments of all forces acting on the body, about a fixed point is zero. By convention, anticlockwise moments are taken as positive and clockwise moments are taken as negative.

Based on the above paragraph, answer questions no. 1 to 4 :

1. Angular momentum is rotational analogue of

(*a*) force (*b*) energy

(*c*) power (*d*) linear momentum

2. The correct expression of angular momentum $(\vec{L})$ in polar co-ordinates is (*a*) $\vec{L} = \vec{r} \times \vec{p}$ (*b*) $L = r\,p$

(*c*) $\vec{L} = \dfrac{\vec{p}}{\vec{r}}$ (*d*) $\vec{L} = \dfrac{\vec{r}}{\vec{p}}$

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

3. **Assertion.** The time rate of change of linear momentum of a particle is equal to the torque acting on it.

   Reason. Linear momentum and angular momentum are the same.

4. **Assertion.** A couple produces rotation without translation.

   Reason. A single force can produce translation as well as rotation

CASE 2. Three simple relations between rotational kinematic variables are

(*i*) $\omega = \omega_0 + \alpha\, t$

(*ii*) $\theta = \omega_0\, t + \frac{1}{2}\alpha\, t^2$

(*iii*) $\omega^2 - \omega_0^2 = 2\,\alpha\,\theta$, where the symbols have their usual meaning. These equations correspond respectively to the three equations of linear motion :

(*i*) $v = u + at$

(*ii*) $s = ut + \frac{1}{2}at^2$

(*iii*) $v^2 - u^2 = 2\,as$

Kinetic energy of rotation of a body is the energy possessed by the body on account of its rotation about a given axis. We can show that

$\boxed{\text{K.E. of rotation} = \frac{1}{2} I\,\omega^2}$, where I is **moment of inertia** of the body about the given axis of rotation. We can show that

$$I = \sum_{i=1}^{i=n} m_i\, r_i^2$$

It implies that moment of inertia of a body about a given axis is the sum of the products of all the particles of the body and square of their respective perpendicular distances from the axis of rotation.

The value of moment of inertia (I) shall depend upon (*i*) position of the axis of rotation (*ii*) orientation of the axis of rotation (*iii*) shape of the body (*iv*) size of the body (*v*) distribution of mass of the body about the axis of rotation.

We can show that moment of inertia of a body about a given axis is equal to, twice the K.E of rotation of the body rotating with unit angular velocity about the given axis.

Further, we can show that moment of inertia (I) plays the same role in rotational motion as mass (m) plays in linear motion. Therefore, moment of inertia (I) of a body can be regarded as measure of rotational inertia of the body.

Based on the above paragraph, answer questions no. 5 to 8 :

5. Choose the correct relation between rotational kinematic variables. Symbols have their usual meaning

   (*a*) $\omega_0 = \omega + \alpha\, t$ (*b*) $\alpha = \dfrac{\omega + \omega_0}{t}$

   (*c*) $\dfrac{\omega_0}{\omega} = \alpha$ (*d*) $\omega - \omega_0 = \alpha\, t$

6. Moment of inertia plays the same role in rotational motion as .... plays in linear motion. Fill in the blanks out of

   (*a*) mass (*b*) velocity

   (*c*) density (*d*) K.E.

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

7. Assertion. With usual meaning of the symbols,

$$\omega^2 - \omega_0^2 = 2\,\alpha\,\theta$$

Reason. $\theta = \omega_0\, t + \frac{1}{2}\,\alpha\, t^2$

8. Assertion. Moment of inertia of a body about a given axis plays the same role in rotational motion as mass plays in linear motion.

Reason. Units of moment of inertia are kg/m^2.

ANSWERS

1. (*d*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*a*) **7.** (*b*) **8.** (*c*)

HINTS/EXPLANATIONS For Difficult Questions

1. Angular momentum is rotational analogue of linear momentum.

2. $\vec{L} = \vec{r} \times \vec{p}$ is the correct relation.

3. Both, the Assertion (A) and Reason (R) are false.

4. Assertion is true, but Reason is false.

5. $\omega = \omega_0 + \alpha\, t$ or $\omega - \omega_0 = \alpha\, t$.

6. *MI* plays the same role in rotational motion as mass plays in linear motion.

7. Both, Assertion and Reason are true, but Reason is not a correct explanation of the Assertion.

8. Assertion (A) is true, but Reason (R) is false.

PROBLEMS FOR PRACTICE

TYPE I. CENTRE OF MASS

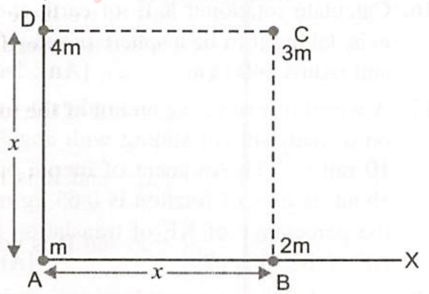
1. Three blocks of uniform thickness and masses *m*, *m* and 2 *m* are placed at the corners of a triangle having co-ordinates (2·5, 1·5), (3·5, 1·5) and (3, 3) respectively. Find the co-ordinates of the centre of mass of the system. **[Ans. (3, 2·25)]**

2. The centre of mass of three particles of masses 1 kg, 2 kg and 3 kg lies at the point (3 m, 3 m, 3 m) where should a fourth particle of mass 4 kg be positioned so that centre of mass of the four particle system lies at (1 m, 1 m, 1 m) ?

[Ans. (– 2 m, – 2 m, – 2 m)]

3. Four particles *A*, *B*, *C* and *D* of masses *m*, 2 *m*, 3 *m* and 4 *m* respectively are placed at corners of a square of side *x* as shown in Fig. 7(Q).7.

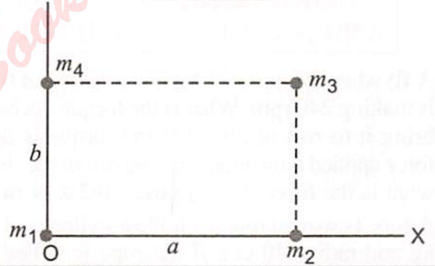
FIGURE 7(Q).7



Locate the centre of mass. $\left[\textbf{Ans.}\ \frac{x}{2}, \frac{7x}{10}\right]$

4. Four particles of masses m_1 = 1 kg, m_2 = 2 kg, m_3 = 3 kg and m_4 = 4 kg are located at the corners of a rectangle as shown in Fig. 7(Q).8.

FIGURE 7(Q).8

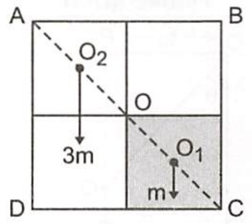


Locate the position of centre of mass.

[Ans. $0{\cdot}5a\,\hat{i} + 0{\cdot}7b\,\hat{j}$]

5. A square of side 4 cm having uniform thickness is divided into four equal squares as shown in Fig. 7(Q).9. If one of the squares is cut off, find

FIGURE 7(Q).9



the position of centre of mass of the remaining portion from the centre O. **[Ans. $\sqrt{2}/3$ cm]**

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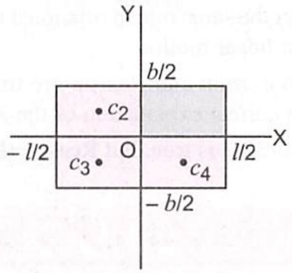
6. From a uniform circular disc of diameter D, a circular hole of diameter $D/6$ and having centre at a distance of $D/4$ from the centre of disc is cut off. Find the centre of mass of the remaining portion. **[Ans. At distance $D/140$ from centre of disc to the left]**

7. A rectangular plate of dimensions $l \times b$ is in $x-y$ plane as shown in Fig. 7(Q).10. If the portion of this plate lying in quadrant I is removed, find the position of centre of mass of remaining part of plate.

FIGURE 7(Q).10



[Ans. $l/12$, $-b/12$]

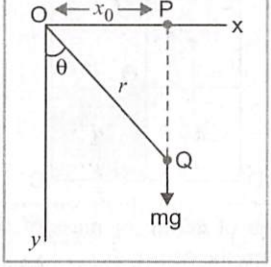
TYPE II. TORQUE, POWER OF TORQUE, WORK DONE BY TORQUE AND ANGULAR MOMENTUM

8. A flywheel of mass 25 kg has a radius of 0·2 m. It is making 240 rpm. What is the torque necessary to bring it to rest in 20 s ? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the force ? **[Ans. – 0·2 π N-m ; π N]**

9. A rope is wound round a hollow cylinder of mass 3 kg and radius 40 cm. If the rope is pulled with a force of 30 N, what is the angular acceleration of the cylinder ? **[Ans. 25 rad s^{-2}]**

10. A particle of mass m is released from rest from point P at $x = x_0$ on X-axis from origin O, and falls vertically along y-axis as shown in Fig. 7(Q).11. What is the magnitude of the torque acting on the particle at time t, when it is at the point Q w.r.t. O ?

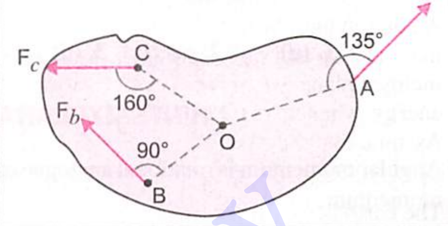
FIGURE 7(Q).11



[Ans. $mg\ x_0$]

11. The body shown in Fig. 7(Q).12 is pivoted at point O. Three forces act on it. F_a = 10 N at point A, 8·0 m from O ; F_b = 16 N at B, 4·0 m from O, and F_c = 19 N at C, 3·0 m from O. What is the net torque about O ?

FIGURE 7(Q).12



[Ans. 12·1 N–m]

TYPE III. EQUATIONS OF ROTATIONAL MOTION

12. The speed of a wheel increases from 600 rpm to 1200 rpm in 20 s. What is its angular acceleration ? How many revolutions will it make during this time ? **[Ans. π rad/s² ; 300]**

13. The spin drier of a washing machine revolving at 15 rps slows down to 5 rps, while making 50 revolutions. Find (*a*) angular acceleration and (*b*) time taken. **[Ans. – 4 π rad/s² ; 5 s]**

14. An automobile travelling at 80 km/*h* has tyres of 75·0 cm diameter. What is the angular speed of the tyres about their axles ? If the car is brought to stop uniformly in 30 complete turns of the tyres without skidding, what is the magnitude of angular acceleration of the wheels ? How far does the car move during the braking ?

[Ans. 59·2 rad/s ; 9·33 rad/s², 70·65 m]

TYPE IV. ENERGY OF ROTATION

15. A wheel is rotating at a speed of 1000 rpm and its KE is 10^6 J. What is moment of inertia of the wheel about its axis of rotation ? **[Ans. 182·56 kg m²]**

16. Calculate rotational K.E. of earth about its own axis, taking it to be a sphere of mass 6×10^{24} kg and radius 6400 km. **[Ans. $2{\cdot}6 \times 10^{29}$ J]**

17. A wheel of mass 5 kg and radius 0·4 m is rolling on a road without sliding with angular velocity 10 rad s^{-1}. The moment of inertia of the wheel about its axis of rotation is 0·65 kg m². What is the percentage of KE of translation in the total KE of the wheel ? **[Ans. 55·2%]**

18. A circular disc of mass M and radius r is set rolling on a table. If ω be its angular velocity, show that its total K.E. is given by (3/4) $M v^2$, where v is its

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linear velocity. M.I. of circular disc = (1/2) mass × $(\text{radius})^2$.

19. A spherical ball rolls on a table without slipping. Determine the percentage of its K.E. which is rotational. Moment of inertia of sphere = (2/5) × mass × $(\text{radius})^2$. **[Ans. 28·57 %]**

20. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0·1 m. If the height of the inclined plane is 4 m, what is its rotational kinetic energy when it reaches the foot of the plane ? Assume that the surfaces are smooth.

[Ans. 26·13 J]

21. The earth has a mass of 6×10^{24} kg and a radius of $6{\cdot}4 \times 10^6$ m. Calculate the amount of work in joules that must be done if its rotation were to be slowed down so that the duration of the day becomes 30 hours instead of 24 hours. Moment of inertia of earth = (2/5) mass × $(\text{radius})^2$.

[Ans. $9{\cdot}36 \times 10^{28}$ J]

22. A bucket of mass 8 kg is supported by a light rope wound around a solid wooden cylinder of mass 12 kg and radius 20 cm, free to rotate about its axis. A man holding the free end of the rope with the bucket and the cylinder at rest initially, lets go the bucket freely downward in a well 50 m deep. Neglecting friction, obtain the speed of bucket and angular speed of the cylinder just before the bucket enters water. Take $g = 10$ m/s^2.

[Ans. 23·9 m/s ; 119·5 rad/s]

TYPE V. ANGULAR MOMENTUM, TORQUE AND MOMENT OF INERTIA

23. A 40 kg flywheel in the form of a uniform circular disc 200 cm. in diameter is making 120 revolutions/minute. Calculate angular momentum. Moment of inertia of disc = (1/2) mass × $(\text{radius})^2$.

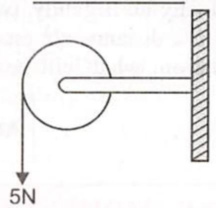
[Ans. 251·4 kg m^2 s^{-1}]

24. A circular ring of diameter 40 cm. and mass 1 kg is rotating about an axis normal to its plane and passing through its centre with a frequency of 10 revolutions/sec. Calculate the angular momentum about its axis of rotation. Take moment of inertia of circular ring about the axis = mass × $(\text{radius})^2$.

[Ans. 2·51 kg. m^2 s^{-1}]

25. A wheel of radius 10 cm can rotate freely about its centre as shown in Fig. 7(Q).13. A string wrapped over its rim is pulled by a force of 5 N. It is found that the torque produces an angular acceleration of 2 rad/s^2 in the wheel. Calculate the moment of inertia of the wheel.

FIGURE 7(Q).13



[Ans. 0·25 kg m^2]

26. A sphere of mass 2 kg and radius 5 cm is rotating at the rate of 300 revolutions per minute. Calculate the torque required to stop it in 6·28 revolutions. [Moment of inertia of sphere about diameter = 2/5 mass × $(\text{radius})^2$]. **[Ans. $2{\cdot}542 \times 10^{-2}$ N–m]**

27. A flywheel of moment of inertia 5·0 kg m^2 is rotated at a speed of 60 rad/s. Because of friction on the axle, it comes to rest in 5 minutes. Find

(*a*) average torque of the friction

(*b*) total work done by the friction

(*c*) Angular momentum of the wheel one minute before it stops rotating.

[Ans. (*a*) – 1 N*m* (*b*) 9 kJ (*c*) 60 kg m^2 s^{-1}]

28. How much tangential force will be needed to stop the earth in one year, if it is rotating with an angular velocity of $7{\cdot}3 \times 10^{-5}$ rad/s ? Given moment of inertia of earth = $9{\cdot}3 \times 10^{37}$ kg m^2 and radius of earth = $6{\cdot}4 \times 10^6$ m. **[Ans. $3{\cdot}364 \times 10^{19}$ N]**

TYPE VI. PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

29. The sun rotates around itself once in 27 days. If it were to expand to twice its present diameter, what would be its new period of revolution ?

[Ans. 108 days]

30. A uniform disc rotating freely about a vertical axis makes 90 rpm. A small piece of wax of mass *m* gram falls vertically on the disc and sticks to it at a distance *r* cm from the axis. If number of rotations per minute reduces to 60, find the moment of inertia of the disc. **[Ans. 2 $m\,r^2$ g cm^2]**

31. A circular disc of moment of inertia I_t is rotating in a horizontal plane about its symmetry axis with a constant angular velocity ω_i. Another disc of moment of inertia I_b is dropped co-axially onto the rotating disc. Initially, the second disc has zero angular speed. Eventually, both the discs rotate with a constant angular speed ω_f. Calculate the energy lost by the initially rotating disc due to friction.

$$\left[\textbf{Ans. } -\frac{1}{2} I_b\, \omega_i\, \omega_f\right]$$

32. A disc of mass M and radius r is rotating with an angular velocity ω. If gently, two masses m each are placed at a distance $r/2$ on either side of the axis of rotation, what will be the new angular velocity ? $\left[\textbf{Ans. } \left(\dfrac{M}{M+m}\right)\omega\right]$

TYPE VII. MOMENT OF INERTIA AND RADIUS OF GYRATION

33. Calculate moment of inertia of a uniform circular disc of mass 700 g and diameter 20 cm about (*i*) an axis through the centre of disc and perpendicular to its plane, (*ii*) a diameter of disc, (*iii*) a tangent in the plane of the disc, (*iv*) a tangent perpendicular to the plane of the disc.

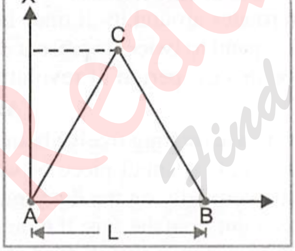
[Ans. $3\cdot5 \times 10^4$ **g cm²** ; $1\cdot75 \times 10^4$ **g cm²** ; $8\cdot75 \times 10^4$ **g cm²** ; $1\cdot05 \times 10^5$ **g cm²]**

34. What is the moment of inertia of a ring of mass 2 kg and diameter 1 m about a transverse axis passing through its centre. How is moment of inertia affected if axis passes through edge of the ring parallel to given axis ?

[Ans. 0·5 kg m² ; 1 kg m²]

35. Three particles, each of mass m, are situated at the vertices of an equilateral triangle ABC of side L. Find the moment of inertia of the system about the line AX perpendicular to AB in the plane of ABC, Fig. 7(Q).14.

FIGURE 7(Q).14



$\left[\textbf{Ans. } \dfrac{5}{4} m L^2\right]$

36. Four particles of masses 4 kg, 2 kg, 3 kg and 5 kg are respectively located at the four corners A, B, C, D of a square of side 1 m. Calculate the moment of inertia of the system about

(*i*) the axis passing through point of intersection of the diagonals and perpendicular to the plane of the square. (*ii*) side AB (*iii*) diagonal BD.

[Ans. 7 kg m² ; 8 kg m² ; 3·5 kg m²]

37. The earth has a mass of 6×10^{24} kg and radius of $6{\cdot}4 \times 10^6$ m. Calculate the amount of work that must be done to slow down its rotation so that duration of day becomes 30 hrs instead of 24 hours. Moment of inertia of earth $= \dfrac{2}{5} MR^2$.

[Ans. $-9\cdot36 \times 10^{28}$ **J]**

38. A uniform rod of length 1 metre has a mass of 500 gram. What is moment of inertia of the rod about an axis passing through the centre of the rod perpendicular to its length. How is moment of inertia changed when the same axis passes through one end of the rod ?

$\left[\textbf{Ans. } \dfrac{1}{24}\ \text{kg m}^2 ;\ \dfrac{1}{6}\ \text{kg m}^2\right]$

TYPE VIII. TYPICAL PROBLEMS

39. Three identical spheres *each* of radius r, and mass m are placed touching eachother on a horizontal floor. Locate the position of centre of mass of the system. **[Ans.** $(r, r/\sqrt{3})$**]**

40. A wheel rotating at an angular speed of 20 rad/s is brought to rest by a constant torque in 4 seconds. If the moment of inertia of the wheel about the axis of rotation is 0·20 kg-m², find the work done by the torque in first two seconds.

[Ans. 30 joule]

41. A flywheel rotating at the rate of 120 rpm slows down at a constant rate of 2 rad. s^{-2}. What time is required to stop the flywheel and how many rotations does it make in the process ?

[Ans. 2 π sec, 2 π rev.]

42. A rod of length L, whose lower end is resting along the horizontal plane, starts to topple from the vertical position. What will be the velocity of the upper end when it hits the ground ? **[Ans.** $\sqrt{3\,g\,L}$ **]**

43. A uniform rod of length 1 m having mass 1 kg rests against a smooth wall at an angle of 30° with the ground. Calculate the force exerted by the ground on the rod. Take $g = 10$ ms^{-2}.

[Ans. 13·22 N]

44. Two small balls A and B, each of mass m, are attached rigidly to the ends of a light rod of length d. The structure rotates about the perpendicular bisector of the rod at an angular speed ω. Calculate the angular momentum of the individual balls and of the system about the axis of rotation.

$\left[\textbf{Ans. } \dfrac{1}{4} m\,\omega\, d^2 ;\ \dfrac{1}{2} m\,\omega\, d^2\right]$

45. A cylinder of mass 5 kg and radius 30 cm is rolling down an inclined plane at an angle of 45° with the horizontal. Calculate (*i*) force of friction (*ii*) acceleration of cylinder (*iii*) minimum value of coeff. of static friction so that cylinder does not slip while rolling down the plane.

[Ans. 11·55 N, 4·62 m/s^2 ; 1/3]

46. A wheel of radius 6 cm is mounted so as to rotate about a horizontal axis through its centre. A string of negligible mass wrapped round its circumference carries a mass of 0·2 kg attached to its free end. When let fall, the mass descends through 1 m in 5 s. Calculate the angular acceleration of the wheel, its moment of inertia and tension in the cord.

[Ans. 1·33 rad/s^2 ; 8·75 × 10^{-2} kg m^2 ; 1·944 N]

HINTS

For Difficult Problems

1. Here, $m_1 = m$, $m_2 = m$, $m_3 = 2\,m$

$x_1 = 2{\cdot}5,\ x_2 = 3{\cdot}5,\ x_3 = 3$;

$y_1 = 1{\cdot}5,\ y_2 = 1{\cdot}5,\ y_3 = 3.$

$$\therefore\ x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m\times 2{\cdot}5 + m\times 3{\cdot}5 + 2m\times 3}{m+m+2m} = \mathbf{3}$$

$$y_{\text{cm}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m\times 1{\cdot}5 + m\times 1{\cdot}5 + 2m\times 3}{m+m+2m} = \mathbf{2{\cdot}25}$$

2. Suppose particle of 4 kg mass is placed at point (*x, y, z*). Then the centre of mass of 6 kg (1 kg + 2 kg + 3 kg) mass and 4 kg mass must lie at the point (1 m, 1 m, 1 m).

$$\therefore\ x_{cm} = \frac{6\times 3 + 4x}{6+4} = 1 \text{ or } x = \mathbf{-2\ m}$$

$$y_{cm} = \frac{6\times 3 + 4y}{6+4} = 1 \text{ or } y = \mathbf{-2m}$$

$$z_{cm} = \frac{6\times 3 + 4z}{6+4} = 1 \text{ or } z = \mathbf{-2\ m}$$

3. Taking A as origin, co-ordinates of A (0, 0), B (*x*, 0), C (*x*, *x*), D (0, *x*)

$$x_{cm} = \frac{m\times 0 + 2m\times x + 3m\times x + 4m\times 0}{m + 2m + 3m + 4m}$$

$$= \frac{x}{2}$$

$$y_{cm} = \frac{m\times 0 + 2m\times 0 + 3m\,(x) + 4m\,(x)}{m+2m+3m+4m}$$

$$= \frac{\mathbf{7}}{\mathbf{10}}\,x$$

4. Here, $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 3$ kg, $m_4 = 4$ kg

As is clear from Fig. 7(Q).8, m_1 is taken at the origin. Therefore, $x_1 = 0$, $y_1 = 0$.

For m_2 ; $x_2 = a\,\hat{i},\ y_2 = 0$

For m_3 ; $x_3 = a\,\hat{i}\ ; y_3 = b\,\hat{j}$

For m_4 ; $x_4 = 0,\ y_4 = b\,\hat{j}$

∴ Co-ordinates of centre of mass of the system

are $x = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$$= \frac{1\times 0 + 2\,(a\,\hat{i}) + 3\,(a\,\hat{i}) + 4\times 0}{1+2+3+4} = 0{\cdot}5\,a\,\hat{i}$$

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1\times 0 + 2\times 0 + 3\times b\,\hat{j} + 4\times b\,\hat{j}}{1+2+3+4} = 0{\cdot}7\,b\,\hat{j}$$

∴ Position of centre of mass of the system is

$[\mathbf{0{\cdot}5\,a\,\hat{i} + 0{\cdot}7\,b\,\hat{j}}]$

5. Let *m* be the mass of each small square. Therefore, total mass of given square = 4m. This acts at the centre O of the square. Let O$_1$ be cm of the cut off square (shown shaded) of mass *m* and O$_2$ be cm of the remaining square of mass 3 m.

Now, $AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ cm

$$OC = \frac{1}{2}AC = \frac{4\sqrt{2}}{2} = 2\sqrt{2}\text{ cm}$$

$$OO_1 = \frac{1}{2}OC = \sqrt{2}\text{ cm}$$

Now, moment of unshaded portion of mass (3 m) about O = moment of shaded portion of mass (m) about O.

$\therefore\ 3\,m \times OO_2 = m \times OO_1$

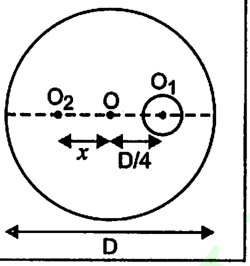
$$OO_2 = \frac{1}{3}(OO_1) = \frac{1}{3} \times \sqrt{2}\,\textbf{cm}$$

6. Let m be the mass per unit area of the disc.

$\therefore$ Mass of disc $M = m \times \frac{\pi D^2}{4}$

It is concentrated at the centre O of the disc, Fig. 7(Q).15.

FIGURE 7(Q).15



Mass of cut off portion,

$$m_1 = m \times \frac{\pi(D/6)^2}{4} = \frac{m\pi D^2}{144} = \frac{M}{36}$$

It is concentrated at O_1, where $OO_1 = D/4$.

Let the centre of mass of remaining portion of disc be at O_2 where $OO_2 = x$

Mass of remaining portion of disc

$$m_2 = m \times \frac{\pi D^2}{4} - m\frac{\pi D^2}{144} = \frac{35M}{36}$$

Clearly, $m_1 \times \frac{D}{4} = m_2\,x$

$$x = \frac{m_1}{m_2}\frac{D}{4}$$

$$x = \frac{M/36}{35M/36} \times \frac{D}{4} = \frac{D}{35 \times 4} = \frac{\mathbf{D}}{\mathbf{140}}$$

$\therefore$ Centre of mass of remaining portion of disc is at $D/140$ from centre O of the disc, to the left side.

7. In Fig. 7(Q).10, we have chosen O as origin, length l of rectangular plate along X-axis and its breadth b along Y-axis. The plate is divided into four equal quadrants each of mass m say. The shaded quadrant is removed. If C_2, C_3, C_4 are the centres of mass of quadrants indicated, then their co-ordinates are :

$C_2\,(-l/4, b/4)$, $C_3\,(-l/4, -b/4)$, $C_4\,(l/4, -b/4)$.

The co-ordinates of centre of mass of L-shaped lamina are

$$x_{cm} = \frac{m(x_1 + x_2 + x_3)}{m_1 + m_2 + m_3} = \frac{x_1 + x_2 + x_3}{3}$$

$$= \frac{-l/4 - l/4 + l/4}{3} = \frac{l}{12}$$

Similarly,

$$y_{cm} = \frac{y_1 + y_2 + y_3}{3} = \frac{b/4 - b/4 - b/4}{3} = \frac{-b}{12}$$

9. Moment of inertia of the hollow cylinder about its axis is

$I = MR^2 = 3{\cdot}0\,(0{\cdot}40)^2 = 0{\cdot}48\text{ kg m}^2$

Torque on the cylinder

τ = force × moment arm = $30 \times 0{\cdot}40 = 12$ Nm

As $\tau = I\,\alpha$

$$\therefore\quad \alpha = \frac{\tau}{I} = \frac{12}{0{\cdot}48} = \mathbf{25\ rad\ s^{-2}}$$

10. Refer to Fig. 7(Q).11.

Magnitude of torque is

$$\tau = r\,F\sin\theta = r \times mg \times \frac{x_0}{r} = \mathbf{mg\,x_0}$$

11. As is clear from Fig. 7(Q).12

$$\vec{\tau} = \vec{\tau}_a + \vec{\tau}_b + \vec{\tau}_c$$

$$= r_a F_a \sin\phi_a - r_b F_b \sin\phi_b + r_c F_c \sin\phi_c$$

$$= (8{\cdot}0)\ 10\sin 135° - (4{\cdot}0)\ 16\sin 90° + 3{\cdot}0 \times 19\sin 160°$$

$$= 80 \times \frac{1}{\sqrt{2}} - 64 \times 1 + 57 \times 0{\cdot}342$$

$$|\vec{\tau}| = 56{\cdot}6 - 64{\cdot}0 + 19{\cdot}5 = \mathbf{12{\cdot}1\ N-m}$$

11. Here, $n_1 = 600$ rpm $= \frac{600}{60} = 10$ rps ;

$$n_2 = \frac{1200}{60} = 20\text{ rps}$$

$t = 20$ s, $\alpha = ?$, $n = ?$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi\,(20 - 10)}{20}$$

$$= \pi\ \mathbf{rad/s^2}$$

Again, $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 2\pi n_1 t + \frac{1}{2}\alpha t^2$

$$= 2\pi \times 10 \times 20 + \frac{\pi}{2}(20)^2 = 600\,\pi \text{ rad}$$

Number of rotations made,

$$n = \frac{\theta}{2\pi} = \frac{600\pi}{2\pi} = \mathbf{300}$$

13. Here, $n_1 = 15$ rps, $n_2 = 5$ rps

Angle traced in 50 revolutions, $\theta = 50 \times 2\,\pi$ rad

As $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

$$\therefore \ \alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{4\pi^2(n_2^2 - n_1^2)}{2\theta}$$

$$\alpha = \frac{2\pi^2(5^2 - 15^2)}{100\pi} = \mathbf{-4\,\pi \text{ rad/s}^2}$$

From $\omega_2 = \omega_1 + \alpha\,t$

$$t = \frac{\omega_2 - \omega_1}{\alpha} = \frac{2\pi(n_2 - n_1)}{\alpha} = \frac{2\pi(5 - 15)}{-4\pi} = 5\text{s}$$

14. Here, $v = 80$ km/h $= \frac{80 \times 1000}{60 \times 60} = 22{\cdot}2$ m/s

This is also the speed of centre of mass of the car

As $\quad r = \frac{75}{2}$ cm $= 37{\cdot}5$ cm $= 0{\cdot}375$ m

$$\therefore \quad \omega_0 = \frac{v_{\text{cm}}}{r} = \frac{22{\cdot}2}{0{\cdot}375} = \mathbf{59{\cdot}2 \text{ rad/s}}$$

For $\omega = 0$, $\theta = 30 \times 2\,\pi$ radian

As $\quad \omega^2 = \omega_0^2 + 2\alpha\,\theta$

$$\therefore \quad \alpha = \frac{\omega^2 - \omega_0^2}{2\theta}$$

$$\alpha = \frac{0 - (59{\cdot}2)^2}{2 \times 30\,(2\pi)} = \mathbf{9{\cdot}33 \text{ rad/s}^2}$$

The distance moved by the car during braking,

$s = 30 \times 2\,\pi\,r = 30 \times 2 \times 3{\cdot}14 \times 0{\cdot}375 = \mathbf{70{\cdot}65 \text{ m}}$

15. $n = 1000$ rpm $= \frac{1000}{60}$ rps $= \frac{50}{3}$ rps,

$E = 10^6$ J, $I = ?$

As $\ E = \frac{1}{2} I\omega^2$,

$$I = \frac{2E}{\omega^2} = \frac{2E}{(2\pi n)^2} = \frac{E}{2\pi^2 n^2}$$

$$= \frac{10^6}{2 \times 3{\cdot}14 \times 3{\cdot}14\,(50/3)^2} = \mathbf{182{\cdot}56 \text{ kg m}^2}$$

16. $E = ?$, $M = 6 \times 10^{24}$ kg,

$R = 6400$ km $= 6{\cdot}4 \times 10^6$ m

Earth completes one rotation about its axis in 1 day.

$$\therefore \ \omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60} \text{rad/s}$$

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{2}{5} MR^2\omega^2$$

$$= \frac{1}{5} \times \frac{6 \times 10^{24}(6{\cdot}4 \times 10^6)^2 \times (2\pi)^2}{(24 \times 60 \times 60)^2}$$

$$= \mathbf{2{\cdot}6 \times 10^{29} \text{ J}}$$

17. Here, $m = 5$ kg, $r = 0{\cdot}4$ m, $\omega = 10$ rad/s,

$I = 0{\cdot}65$ kg m^2 ; $v = r\,\omega = 0{\cdot}4 \times 10 = 4{\cdot}0$ m/s

Translational KE $= \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times (4{\cdot}0)^2 = 40$ J

Rotational KE $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0{\cdot}65\,(10)^2 = 32{\cdot}5$ J

Total KE = Translational KE + Rotational KE
$= 40 + 32{\cdot}5 = 72{\cdot}5$ J

$$\frac{\text{KE of translational}}{\text{Total KE}} = \frac{40}{72{\cdot}5} = 0{\cdot}552 = \mathbf{55{\cdot}2\%}$$

19. Total energy = K.E. of translation + K.E. of rotation

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$E = \frac{1}{2}m\,v^2 + \frac{1}{2}\left(\frac{2}{5}m\,r^2\right)\omega^2$$

$$= \frac{1}{2}m\,v^2 + \frac{1}{5}m\,v^2 = \frac{7}{10}m\,v^2$$

$$\frac{\text{K.E. of rotation}}{\text{total energy}} = \frac{\left(\frac{1}{2}\right)I\,\omega^2}{\left(\frac{7}{10}\right)m\,v^2}$$

$$= \frac{\left(\frac{1}{5}\right)m\,v^2}{\left(\frac{7}{10}\right)m\,v^2} = \frac{2}{7} = \frac{2}{7} \times 100\% = \mathbf{28{\cdot}57\%}$$

20. $K_T + K_R = K$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2 = K$$

But $\quad \frac{1}{2}I\,\omega^2 = \frac{1}{2}\left(\frac{1}{2}m\,r^2\right)\omega^2 = \frac{1}{4}m\,v^2$

$$\therefore \quad \frac{1}{2}mv^2+\frac{1}{4}mv^2=K \text{ or } \frac{3}{4}mv^2=K$$

Rotational KE, $\frac{1}{2}I\omega^2=\frac{1}{4}mv^2=\frac{K}{3}$

$$=\frac{mgh}{3}=\frac{2\times9.8\times4}{3}=\mathbf{26.13\ J}$$

21. Here, $M=6\times10^{24}$ kg ; $R=6.4\times10^6$ m ;
$W=?$, $T_1=24$ h, $T_2=30$ h
Work done = Increase in K.E. of rotation

$$W=E_2-E_1=\frac{1}{2}I(\omega_2^2-\omega_1^2)$$

$$=\frac{1}{2}\times\left(\frac{2}{5}MR^2\right)\left[\left(\frac{2\pi}{T_2}\right)^2-\left(\frac{2\pi}{T_1}\right)^2\right]$$

$$=\frac{4\pi^2}{5}\times MR^2\left[\frac{1}{T_2^2}-\frac{1}{T_1^2}\right]$$

$$W=4\times\frac{22}{7}\times\frac{22}{7\times5}\times6\times10^{24}(6.4\times10^6)^2$$

$$\left[\frac{1}{(30\times60\times60)^2}-\frac{1}{(24\times60\times60)^2}\right]$$

$\mathbf{W=-9.36\times10^{28}\ J}$

Negative sign indicates that work is done against the rotation of earth in order to slow down its speed.

22. Here, mass of bucket, $m_1=8$ kg
mass of cylinder, $m_2=12$ kg
radius of cylinder, $r=20$ cm $=0.2$ m

When the bucket enters water,
P.E. lost by bucket = Translation K.E. of bucket + rotational K.E. of cylinder

$$m_1gh=\frac{1}{2}m_1v^2+\frac{1}{2}I\omega^2$$

$$=\frac{1}{2}m_1v^2+\frac{1}{2}\left(\frac{1}{2}m_2r^2\right)\frac{v^2}{r^2}$$

$$=\frac{1}{2}v^2\left(m_1+\frac{1}{2}m_2\right)$$

$$8\times10\times50=\frac{1}{2}v^2\left(8+\frac{1}{2}\times12\right)=7v^2$$

$$v=\sqrt{\frac{8\times10\times50}{7}}=\sqrt{\frac{4000}{7}}$$

$$=\mathbf{23.9\ m/s}$$

Angular speed of the cylinder before the bucket enters water, $\omega=\frac{v}{r}=\frac{23.9}{0.20}=\mathbf{119.5\ rad/s}$

23. Here, $m=40$ kg, $d=200$ cm,

$$r=\frac{d}{2}=\frac{200}{2}\text{ cm}=100\text{ cm}=1\text{ m}$$

$$n=120\text{ rps}=\frac{120}{60}\text{ rps}=2\text{ rps}, L=?$$

$$L=I\omega=\left(\frac{1}{2}mr^2\right)2\pi n=mr^2\pi n$$

$$=40\times1^2\times\frac{22}{7}\times2\text{ kg m}^2\text{ s}^{-1}$$

$$\mathbf{L=251.4\ kg\ m^2\ s^{-1}}$$

25. Here, $r=10$ cm $=0.1$ m, $F=5.0$ N,
$\alpha=2.0$ rad/s^2, $I=?$, Fig. 7(Q).13

Torques due to weight (W) of wheel and normal reaction (R) at the support are separately zero. The only effective torque is due to force applied.

As $\tau=I\alpha$

$$\therefore \quad I=\frac{\tau}{\alpha}=\frac{F\times r}{\alpha}=\frac{5\times0.1}{2}=\mathbf{0.25\ kg\ m^2}$$

27. Here, $I=5.0$ kg m^2, $\omega_0=60$ rad/s
$\omega=0$, $t=5\times60=300$ s, $\tau=?$

From $\omega=\omega_0+\alpha t$

$$0=60+\alpha\times300,\ \alpha=\frac{-1}{5}\text{rad/s}^2$$

(*a*) Average torque of friction $=I\alpha=5.0\left(-\frac{1}{5}\right)$
$=\mathbf{-1\ N\text{-}m}$

(*b*) Total work done by friction = KE of rotation

$$W=\frac{1}{2}I\omega_0^2=\frac{1}{2}\times5.0(60)^2=9000\text{ J}=\mathbf{9\ kJ}$$

(*c*) From $\omega=\omega_0+\alpha t$

$$=60-\frac{1}{5}\times(5-1)\times60=12\text{ rad/s}$$

$$L=I\omega=5.0\times12=\mathbf{60\ kg\ m^2\ s^{-1}}$$

28. $F=?$ $\omega_2=0$, $t=1$ year $=365\times24\times60\times60$ s
$\omega_1=7.3\times10^{-5}$ rad/s, $I=9.3\times10^{37}$ kg m^2
$R=6.4\times10^6$ m

From $\omega_2-\omega_1=\alpha t$

$$\alpha=\frac{\omega_2-\omega_1}{t}=\frac{0-7.3\times10^{-5}}{365\times24\times60\times60}\text{rad/s}^2$$

$$\tau = I\alpha = 9{\cdot}3\times10^{37}\left(\frac{7{\cdot}3\times10^{-5}}{365\times24\times60\times60}\right)$$

$$= \frac{9{\cdot}3\times7{\cdot}3}{365\times24\times36}\times10^{30}\text{ N-m}$$

As $\tau = F\times r \quad \therefore \quad F = \frac{\tau}{r}$

$$F = \frac{9{\cdot}3\times7{\cdot}3\times10^{30}}{365\times24\times36}\times\frac{1}{6{\cdot}4\times10^{6}}$$

$$\mathbf{F = 3{\cdot}364\times10^{19}\ N}$$

29. Here, $T_1 = 27$ days, $R_2 = 2R_1$, $T_2 = ?$

Applying principle of conservation of angular momentum : $I_2\,\omega_2 = I_1\,\omega_1$

$$\left(\frac{2}{5}MR_2^2\right)\frac{2\pi}{T_2} = \left(\frac{2}{5}MR_1^2\right)\frac{2\pi}{T_1}$$

$$\frac{R_2^2}{T_2} = \frac{R_1^2}{T_1}$$

or $\quad T_2 = \frac{R_2^2}{R_1^2}\times T_1 = (2)^2\times27 = \mathbf{108\ days}$

30. Here, $n_1 = 90$ rpm ; $n_2 = 60$ rpm

Let $\quad I_1$ = moment of inertia of disc.

$\therefore \quad I_2 = I_1 + mr^2$

From the principle of conservation of angular momentum,

$I_2\,\omega_2 = I_1\,\omega_1$ or $(I_1 + mr^2)\,2\pi n_2 = I_1\times2\pi n_1$

or $\quad I_1 + mr^2 = I_1\frac{n_1}{n_2} = I_1\frac{90}{60} = \frac{3I_1}{2}$

or $\quad \frac{3}{2}I_1 - I_1 = mr^2$, $\quad \mathbf{I_1 = 2\,mr^2\ g\ cm^2}$

31. Here, $\quad I_i = I_t$

Let ω_i be the initial angular speed

$$I_f = (I_t + I_b)$$

Let ω_f be the final angular speed of the system.

According to the law of conservation of angular momentum,

$$I_f\,\omega_f = I_i\,\omega_i$$

$$\omega_f = \frac{I_t\,\omega_i}{I_t + I_b} \qquad \text{...(}i\text{)}$$

Energy lost $= K_f - K_i = \frac{1}{2}I_f\,\omega_f^2 - \frac{1}{2}I_i\,\omega_i^2$

$$= \frac{1}{2}(I_t + I_b)\,\omega_f^2 - \frac{1}{2}I_t\,\omega_i^2$$

using (i), we get

$$\text{Energy lost} = \frac{1}{2}(I_t + I_b)\left(\frac{I_t\,\omega_i}{I_t + I_b}\right)^2 - \frac{1}{2}I_t\,\omega_i^2$$

$$= \frac{1}{2}\frac{I_t^2\,\omega_i^2}{(I_t + I_b)} - \frac{1}{2}I_t\,\omega_i^2$$

$$= \frac{1}{2}I_t\,\omega_i^2\left[\frac{I_t}{I_t + I_b} - 1\right] = -\frac{1}{2}\frac{I_b\,I_t\,\omega_i^2}{(I_t + I_b)}$$

$$\text{Energy lost} = -\frac{1}{2}I_b\,\omega_i\,\omega_f \qquad [\text{using } (i)]$$

32. Here, moment of inertia of the disc about the given axis.

$$I = \frac{1}{2}M\,r^2$$

When two masses m each are placed at a distance $r/2$ on either side of the axis, new moment of inertia becomes

$$I' = \frac{1}{2}M\,r^2 + m\,(r/2)^2 + m(r/2)^2$$

$$I' = \frac{1}{2}(M + m)\,r^2$$

If ω' is the new angular velocity, then according to the principle of conservation of angular momentum.

$$I'\,\omega' = I\,\omega$$

$$\omega' = \frac{I}{I'}\,\omega = \frac{\frac{1}{2}M\,r^2}{\frac{1}{2}(M + m)\,r^2}\times\omega$$

$$\omega' = \left(\frac{M}{M + m}\right)\omega$$

33. Here, $M = 700$ g, $\quad R = \frac{20}{2}$ cm $= 10$ cm

(i) $I = \frac{1}{2}MR^2 = \frac{1}{2}\times700\times10^2 = \mathbf{3{\cdot}5\times10^4\ g\ cm^2}$

(ii) $I = \frac{1}{4}MR^2 = \frac{1}{4}\times700\times10^2$

$= \mathbf{1{\cdot}75\times10^4\ g\ cm^2}$

(iii) $I = \frac{5}{4}MR^2 = \frac{5}{4}\times700\times10^2$

$= \mathbf{8{\cdot}75\times10^4\ g\ cm^2}$

(iv) $I = \frac{3}{2}MR^2 = \frac{3}{2}\times700\times10^2$

$= \mathbf{1{\cdot}05\times10^5\ g\ cm^2}$

35. As is clear from Fig. 7(Q).14,
perpendicular distance of A from $AX = 0$
perpendicular distance of B from $AX = L$

and perpendicular distance of C from $AX = \frac{L}{2}$

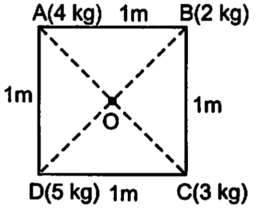
∴ Moment of inertia of three particle system about AX

$$= 0 + mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{5}{4} mL^2$$

36. In Fig. 7(Q).16,

$$AC = BD = \sqrt{1^2 + 1^2} = \sqrt{2}\ \text{m}$$

FIGURE 7(Q).16



$$\therefore\ AO = BO = CO = DO = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}\ \text{m}$$

(*i*) About the axis passing through O and ⊥ to plane of square,

$$I = 4\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{2}}\right)^2 + 5\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 14 \times \frac{1}{2} = \mathbf{7\ kg\ m^2}$$

(*ii*) About AB

$I = 4 \times (0)^2 + 2\,(0)^2 + 3\,(1)^2 + 5\,(1)^2 = \mathbf{8\ kg\ m^2}$

(*iii*) About diagonal BD

$$I = 4\left(\frac{1}{\sqrt{2}}\right)^2 + 2(0)^2 + 3\left(\frac{1}{\sqrt{2}}\right)^2 + 5(0)^2$$

$$= \mathbf{3{\cdot}5\ kg\ m^2}$$

37. Here, $M = 6 \times 10^{24}$ kg, $R = 6{\cdot}4 \times 10^6$ m, $W = ?$
$T_1 = 24$ hrs, $T_2 = 30$ hrs.

$$I = \frac{2}{5} MR^2 = \frac{2}{5} \times 6 \times 10^{24} \times (6{\cdot}4 \times 10^6)^2$$

$$= 9{\cdot}83 \times 10^{37}\ \text{kg m}^2$$

Work done = Decrease in rotational KE

$$W = \frac{1}{2} I\,(\omega_2^2 - \omega_1^2) = \frac{I}{2} \times 4\pi^2 \left[\frac{1}{T_2^2} - \frac{1}{T_1^2}\right]$$

$$= \frac{2}{5} MR^2 \times 2\pi^2 \left(\frac{T_1^2 - T_2^2}{T_1^2 T_2^2}\right)$$

$$= 9{\cdot}83 \times 10^{37} \times 2 \times \frac{22}{7} \times \frac{22}{7}$$

$$\times \left\{\frac{(24 \times 60 \times 60)^2 - (30 \times 60 \times 60)^2}{(24 \times 60 \times 60)^2\,(30 \times 60 \times 60)^2}\right\}$$

$$\mathbf{W = -\,9{\cdot}36 \times 10^{28}\ J}$$

38. Here, $l = 1$ m, $M = 500\ g = \frac{1}{2}$ kg

About an axis passing through the centre of the rod and perpendicular to its length,

$$I = M\frac{l^2}{12} = \frac{1}{2} \times \frac{1^2}{12} = \frac{\mathbf{1}}{\mathbf{24}}\ \mathbf{kg\ m^2}$$

When same axis passes through one end of the rod, $x = \frac{l}{2} = \frac{1}{2}$ m,

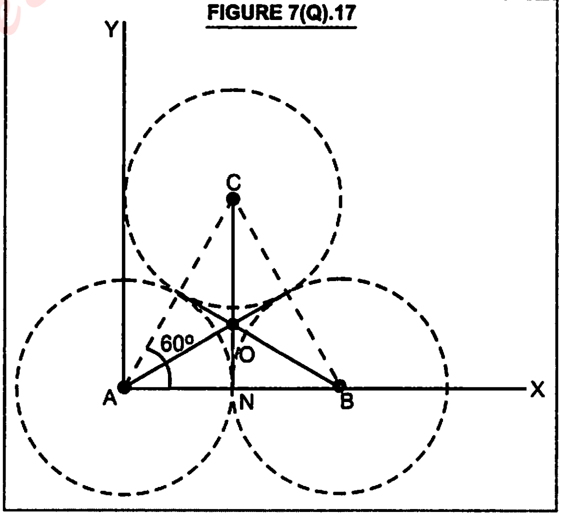
applying theorem of parallel axes,

$$I' = I + Mx^2$$

$$I' = \frac{1}{24} + \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{24} + \frac{1}{8} = \frac{\mathbf{1}}{\mathbf{6}}\ \mathbf{kg\ m^2}$$

39. The centres of three spheres lie at the vertices A, B, C of an equilateral triangle, Fig. 7(Q).17, Centre of mass is at the centroid of ΔABC.

FIGURE 7(Q).17



Taking A as the origin, the co-ordinates of the three vertices of the triangle are :

$A\,(x_1, y_1) = 0, 0\ ;\ B\,(x_2, y_2) = 2\,r, 0$

and $C\,(x_3, y_3) = r,\ r\sqrt{3}$

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∴ Co-ordinates of centre of mass O are

$$x = \frac{m(x_1 + x_2 + x_3)}{3m} = \frac{0 + 2r + r}{3} = r$$

$$y = \frac{m(y_1 + y_2 + y_3)}{3m} = \frac{0 + 0 + r\sqrt{3}}{3} = \frac{r}{\sqrt{3}}$$

40. Here, $\omega_1 = 20$ rad/s, $\omega_2 = 0$, $t = 4$ sec.
$I = 0{\cdot}20$ kg m^2, $W = ?$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0 - 20}{4} = -5 \text{ rad/s}^2$$

Negative sign is for deceleration.

Torque, $\tau = I\,\alpha = 0{\cdot}20\ (5) = 1{\cdot}0$ N-m

Angle traced in first *two* seconds

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 20(2) + \frac{1}{2}(-5)\ (2)^2$$
$$= 30 \text{ radian}$$

∴ Work done by the torque in 1st two seconds,

$W = \tau\,\theta = 1{\cdot}0\ (30) =$ **30 joule**

41. Here, $n_1 = 120$ *r p m* $= 2$ *r p s*.
$\alpha = -2$ rad s^{-2}, $n_2 = 0$; $t = ?$, $\theta = ?$

From, $\omega_2 = \omega_1 + \alpha\,t$

$$t = \frac{\omega_2 - \omega_1}{\alpha} = \frac{2\pi\,(n_2 - n_1)}{\alpha} = \frac{2\pi\,(0 - 2)}{-2}$$
$$= \mathbf{2\,\pi\ sec}$$

From, $\theta = \omega_1\ t + \frac{1}{2}\alpha t^2 = 2\,\pi\,(2)\cdot 2\,\pi + \frac{1}{2}$
$(-2)\ (2\,\pi)^2 = 4\,\pi^2$

This is the angle traced by the flywheel before stopping. As angle traced in one revolution is $2\,\pi$ radian, therefore, number of revolutions completed

$$= \frac{\theta}{2\pi} = \frac{4\pi^2}{2\pi} = \mathbf{2\,\pi}$$

42. When upper end of the rod hits the ground, the distance through which centre of gravity of the rod falls $= L/2$.

∴ Loss of potential energy $= Mg\ (L/2)$, where M is mass of the rod.

Gain in rotational K.E. $= \frac{1}{2} I\,\omega^2$

Where $I = ML^2/3$ and $\omega = v/L$

∴ Gain in rotational K.E.

$$= \frac{1}{2}\left(\frac{M\,L^2}{3}\right)\left(\frac{v}{L}\right)^2 = \frac{M\,v^2}{6}$$

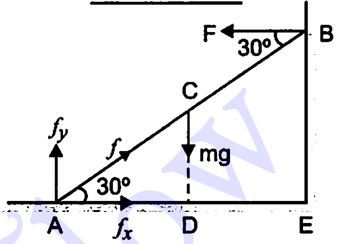
According to the law of conservation of energy

$$\frac{M\,v^2}{6} = Mg\,\frac{L}{2}$$

$$v = \sqrt{3gL}$$

43. In Fig. 7(Q).18, AB is a uniform rod of length 1 m with centre at C, $\angle BAE = 30°$, $AC = 0{\cdot}5$ m

FIGURE 7(Q).18



The various forces involved are :

(*i*) Weight mg of the rod acting vertically downwards at C.

(*ii*) Horizontal force F due to the wall at B.

(*iii*) Reaction of floor f acting along AB. This is the force exerted by the ground on the rod. It is the resultant of normal reaction f_y of floor and force of friction f_x along AE (that prevents sliding away of the rod from the wall).

In equilibrium,

(*i*) net force on the rod along the ground is zero.

$f_x - F = 0$ or $f_x = F$ ...(1)

(*ii*) net force on the rod perpendicular to the ground is zero.

$f_y - mg = 0$ or $f_y = mg = 1 \times 10 = 10$ N

(*iii*) Algebraic sum of moments of forces about A $= 0$

$$F \times BE = mg \times AD$$

$F \times (AB \sin 30°) = mg \times AC \cos 30°$

$$F \times 1 \times \frac{1}{2} = 1 \times 10 \times 0{\cdot}5 \times \frac{\sqrt{3}}{2}$$

$$F = 5\sqrt{3} = 8{\cdot}66 \text{ N}$$

From (1), $f_x = F = 8{\cdot}66$ N

Force exerted by the ground on the rod

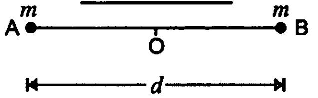
$$f = \sqrt{f_x^2 + f_y^2} = \sqrt{8{\cdot}66^2 + 10^2} = \mathbf{13{\cdot}22\ N}$$

44. As is clear from Fig. 7(Q).19, velocity of each ball *wrt* centre O is $v = \omega\ (d/2)$, where $r = d/2$.

∴ Angular momentum of each ball *w.r.t.* O,

$$L_1 = L_2 = m\,v\,r = m\left(\omega\frac{d}{2}\right)\left(\frac{d}{2}\right) = m\,\omega\frac{d^2}{4}$$

FIGURE 7(Q).19



The angular momentum of the system of two balls.

$$L = L_1 + L_2 = \frac{1}{4}\, m\,\omega\, d^2 + \frac{1}{4}\, m\,\omega\, d^2 = \frac{1}{2} m\,\omega d^2$$

45. Here, $m = 5$ kg, $r = 30$ cm $= 0{\cdot}3$ m, $\theta = 45°$

(*i*) Force of friction,

$$f = \frac{1}{3} mg \sin\theta \;= \frac{1}{3}\times 5\times 9{\cdot}8\times\frac{1}{\sqrt{2}} \;= \mathbf{11{\cdot}55\ N}$$

(*ii*) acceleration, $a = \frac{2}{3} g \sin\theta = \frac{2}{3}\times 9{\cdot}8\times\frac{1}{\sqrt{2}}$

$$= \mathbf{4{\cdot}62\ m/s^2}$$

(*iii*) Minimum value of coeff. of friction

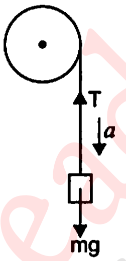
$$\mu_s = \frac{1}{3}\tan\theta = \frac{1}{3}\tan 45° = \frac{1}{3}$$

46. Here, $r = 6$ cm $= 6\times 10^{-2}$ m

$m = 0{\cdot}2$ kg, $s = 1$ m, $t = 5$ s.

If a is acceleration of falling mass, Fig. 7(Q).20.

FIGURE 7(Q).20



from $s = ut + \frac{1}{2} at^2$

$$1 = 0 + \frac{1}{2}\times a\times 5^2,\quad a = \frac{2}{25}\,\text{m/s}^2$$

Angular acc. $\alpha = \frac{a}{r} = \frac{2/25}{6\times 10^{-2}} = \mathbf{1{\cdot}33\ rad/s^2}$

Velocity after 5 s ; $v = u + at$

$$= 0 + \frac{2}{25}\times 5 = 0{\cdot}4\ \text{m/s}$$

Angular velocity $\omega = \frac{v}{r} = \frac{0{\cdot}4}{6\times 10^{-2}} = \frac{20}{3}$ rad/s

According to the law of conservation of energy

Loss of P.E. = Increase in (K.E. of translation + K.E. of rotation)

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\,\omega^2$$

$$0{\cdot}2\times 9{\cdot}8\times 1 = \frac{1}{2}\times 0{\cdot}2(0{\cdot}4)^2 + \frac{1}{2} I\left(\frac{20}{3}\right)^2$$

$$0{\cdot}196 = 0{\cdot}1\times 0{\cdot}16 + I\times\frac{400}{18}$$

$$0{\cdot}180 = I\times\frac{400}{18}$$

or $$I = \frac{0{\cdot}180\times 18}{400} = \mathbf{8{\cdot}75\times 10^{-2}\ kg\ m^2}$$

If T is tension in the string, then from $ma = mg - T$

$$T = mg - ma = m\,(g - a)$$

$$= 0{\cdot}2\,(9{\cdot}8 - 0{\cdot}08) = 0{\cdot}2\times 9{\cdot}72$$

$$T = \mathbf{1{\cdot}944\ N}$$

NCERT EXERCISES WITH SOLUTIONS

Q. 1. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie on the body ?

Sol. In all the four cases, as the mass density is uniform, centre of mass is located at their respective geometrical centres.

No, it is not necessary that the centre of mass of a body should lie on the body. For example, in case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

Q. 2. In the HCl molecule, the separation between the nuclei of the two atoms is about 1·27 Å (1 Å = 10^{-10} m). Find the approximate location of the c.m of the molecule, given that a chlorine atom is about 35·5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus ?

Sol. Let the mass of the *H* atom = *m* unit, mass of the *Cl* atom = 35·5 m units

Let *c.m* be at a distance *x* Å from *H* atom ∴ Distance of *c.m* from *Cl* atom = (1·27 – *x*) Å

If *c.m* is taken at the origin, then $m\,x + (1{\cdot}27 - x)\,35{\cdot}5\text{ m} = 0$

$$mx = -(1{\cdot}27 - x)\,35{\cdot}5\text{ m}$$

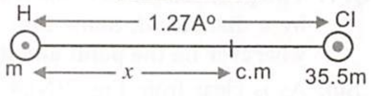
Negative sign indicates that if chlorine atom is on the right side of *c.m* (+), the hydrogen atom is on the left side of c.m, Fig. 7(N).1. So leaving negative sign, we get

$$x + 35{\cdot}5\,x = 1{\cdot}27 \times 35{\cdot}5$$

i.e. $$36{\cdot}5\,x = 45{\cdot}085 = \frac{45{\cdot}085}{36{\cdot}5} = 1{\cdot}235$$

$$x = \mathbf{1{\cdot}235\ Å}$$

FIGURE 7(N).1



Hence *c.m* is located on the line joining centres of *H* and *Cl* atoms at a distance 1·235 Å from *H* atom.

Q. 3. A child sits stationary at one end of a long trolley moving uniformly with a speed *v* on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the c.m of the (trolley + child) system ?

Sol. The speed of the centre of mass of the system (trolley + child) shall remain unchanged, when the child gets up and runs about on the trolley in any manner. This is because forces involved in the exercise are purely internal *i.e.* from within the system. No external force acts on the system and hence there is no change in velocity of centre of mass of the system.

Q. 4. Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$

Sol. Let $\vec{a}$ be represented by $\overrightarrow{OP}$ and $\vec{b}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$, Fig. 7(N).2.

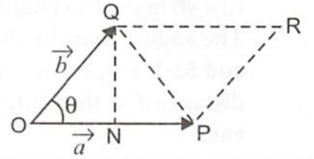
Complete the | | gm *OPRQ*. Join *PQ*.

Draw $QN \perp OP$

In ΔOQN, $\sin\theta = \dfrac{QN}{OQ} = \dfrac{QN}{b}$

$$QN = b \sin\theta$$

FIGURE 7(N).2



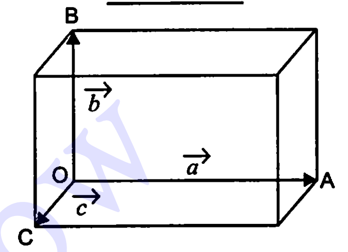
Now, by definition, $|\vec{a} \times \vec{b}| = ab \sin\theta = (OP)(QN) = \frac{2(OP)(QN)}{2} = 2 \times$ area of ΔOPQ

$\therefore$ area of $\Delta OPQ = \frac{1}{2}|\vec{a} \times \vec{b}|$, which was to be proved.

Q. 5. Show that $\vec{a}.(\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelopiped formed on the three vectors, $\vec{a}, \vec{b}$ and $\vec{c}$.

Sol. Let a parallelopiped be formed on the three vectors $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$, Fig. 7(N).3.

FIGURE 7(N).3



Now, $\vec{b} \times \vec{c} = bc \sin 90^\circ \hat{n} = bc\,\hat{n}$

where $\hat{n}$ is unit vector along $\overrightarrow{OA}$ perpendicular to the plane containing $\vec{b}$ and $\vec{c}$.

Now $\vec{a}.(\vec{b} \times \vec{c}) = \vec{a}.bc\,\hat{n} = (a)(bc)\cos 0^\circ = abc$

which is equal in magnitude to the volume of the parallelopiped.

Q. 6. Find the components along the x, y, z axes of the angular momentum $\vec{L}$ of a particle, whose position vector is $\vec{r}$ with components x, y, z and momentum is $\vec{p}$ with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane, the angular momentum has only a z-component.

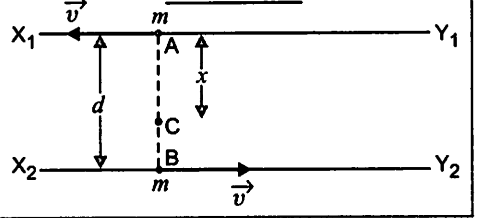
Sol. Refer to Art. 5(a).18.

Q. 7. Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same wherever be the point about which the angular momentum is taken.

Sol. As is clear from Fig. 7(N).4, vector angular momentum of the two particle system about any pt. A on X_1Y_1

$$\vec{L}_A = m\vec{v} \times 0 + m\vec{v} \times d = m\vec{v}\,d$$

FIGURE 7(N).4



Similarly, vector angular momentum of the two particle system about any pt. B on X_2Y_2

$$\vec{L}_B = m\vec{v} \times d + m\vec{v} \times 0 = m\vec{v}\,d$$

Let us consider any other point C on AB, where $AC = x$

$\therefore$ Vector angular momentum of the two particle system about C is

$$\vec{L}_C = m\vec{v}\,(x) + m\vec{v}\,(d - x) = m\vec{v}\,d$$

Clearly, $\vec{L}_A = \vec{L}_B = \vec{L}_C$, which was to be proved.

Q. 8. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. 7(N).5. The angles made by the strings with the vertical are 36·9° and 53·1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.

FIGURE 7(N).5

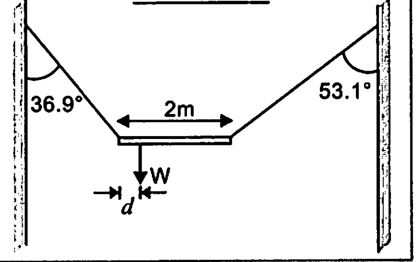


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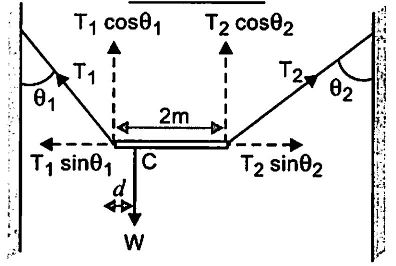
FIGURE 7(N).6



Sol. As is clear from Fig. 7(N).6, $\theta_1 = 36{\cdot}9°$, $\theta_2 = 53{\cdot}1°$.

If T_1, T_2 are the tensions in the two strings, then for equilibrium along the horizontal, $T_1 \sin\theta_1 = T_2 \sin\theta_2$

or
$$\frac{T_1}{T_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 53{\cdot}1°}{\sin 36{\cdot}9°} = \frac{0{\cdot}7407}{0{\cdot}5477} = 1{\cdot}3523$$

Let d be the distance of centre of gravity C of the bar from the left end.

For rotational equilibrium about C,
$$T_1 \cos\theta_1 \times d = T_2 \cos\theta_2 (2-d)$$
$$T_1 \cos 36{\cdot}9° \times d = T_2 \cos 53{\cdot}1° (2-d)$$
$$T_1 \times 0{\cdot}8366\, d = T_2 \times 0{\cdot}6718\,(2-d)$$

Put $T_1 = 1{\cdot}3523\, T_2$ and solve to get $d =$ **0·745 m**

Q. 9. A car weighs 1800 kg. The distance between its front and back axles is 1·8 m. Its centre of gravity is 1·05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Sol. Here, $m = 1800$ kg

Distance between front and back axles = 1·8 m

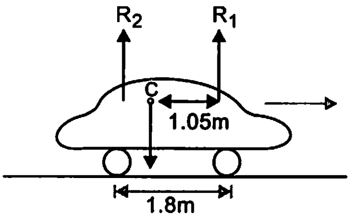
Distance of centre of gravity (C) behind the front axle = 1·05 m.

Let R_1, R_2 be the forces exerted by the level ground on each front wheel and each back wheel. As is clear from Fig. 7(N).7
$$R_1 + R_2 = mg = 1800 \times 9{\cdot}8 \qquad ...(i)$$

For rotational equilibrium about C, $R_1 \times 1{\cdot}05 = R_2(1{\cdot}8 - 1{\cdot}05) = R_2 \times 0{\cdot}75$
$$\frac{R_1}{R_2} = \frac{0{\cdot}75}{1{\cdot}05} = \frac{5}{7}$$

Put in (*i*), $\frac{5}{7}R_2 + R_2 = 1800 \times 9{\cdot}8$
$$R_2 = \frac{7 \times 1800 \times 9{\cdot}8}{12} = \mathbf{10290\ N}$$
$$R_1 = \frac{5}{7}R_2 = \frac{5}{7} \times 10290 = \mathbf{7350\ N}$$

FIGURE 7(N).7



Q. 10. Torques of equal magnitude are applied to hollow cylinder and a solid sphere, both having the same mass and same radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will aquire a greater angular speed after a given time ?

Sol. If M is mass and R is radius of the hollow cylinder and the solid sphere, then

M.I. of hollow cylinder about its axis of symmetry, $I_1 = MR^2$, and

M.I. of solid sphere about an axis through its centre, $I_2 = \frac{2}{5}MR^2$

Torque applied, $\tau = I_1\alpha_1 = I_2\alpha_2 \quad \therefore \quad \frac{\alpha_2}{\alpha_1} = \frac{I_1}{I_2} = \frac{MR^2}{\frac{2}{5}MR^2} = \frac{5}{2} \quad \therefore \quad \alpha_2 > \alpha_1$

From $\omega = \omega_0 + \alpha t$, we find that for given ω_0 and t, $\omega_2 > \omega_1$ *i.e.* **angular speed of solid sphere will be greater** than the angular speed of hollow cylinder.

Q. 11. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 s^{-1}. The radius of the cylinder is 0·25 m. What is the kinetic energy associated with the rotation of the cylinder ? What is the magnitude of angular momentum of the cylinder about its axis ?

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Sol. Here, $M = 20$ kg, $R = 0{\cdot}25$ m, $\omega = 100\ s^{-1}$

Moment of inertia of solid cylinder $= \dfrac{MR^2}{2} = \dfrac{20 \times (0{\cdot}25)^2}{2} = 0{\cdot}625$ kg m^2

$\therefore$ K.E. of rotation $= \dfrac{1}{2} I\omega^2 = \dfrac{1}{2} \times 0{\cdot}625 \times (100)^2 = \mathbf{3125\ J}$

Angular momentum, $L = I\,\omega = 0{\cdot}625 \times 100 = \mathbf{62{\cdot}5\ Js}$

Q. 12. A child stands at the centre of a turn table with his two arms outstretched. The turn table is set rotating with an angular speed of 40 rpm. How much is the angular speed of the child, if he folds his hands back reducing the moment of inertia to (2/5) time the initial value ? Assume that the turn table rotates without friction.

(b) Show that the child's new K.E of rotation is more than the initial K.E. of rotation. How do you account for this increase in K.E. ?

Sol. Here, initial angular speed, $\omega_1 = 40$ rev/min ; $\omega_2 = ?$

final moment of inertia, $I_2 = \dfrac{2}{5} I_1$, initial moment of inertia

As no external torque acts in the process, therefore, L = constant, *i.e.*, $I_2\,\omega_2 = I_1\,\omega_1$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{5}{2} \times 40 = \mathbf{100\ rpm}$$

(*b*) Final K.E. of rotation, $E_2 = \dfrac{1}{2} I_2\,\omega_2^2$; Initial K.E. of rotation, $E_1 = \dfrac{1}{2} I_1\,\omega_1^2$

$$\frac{E_2}{E_1} = \frac{\frac{1}{2} I_2\,\omega_2^2}{\frac{1}{2} I_1\,\omega_1^2} = \left(\frac{I_2}{I_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{2}{5} \times \left(\frac{100}{40}\right)^2 = \frac{5}{2} = 2{\cdot}5 \quad \therefore \quad \mathbf{E_2 = 2{\cdot}5\ E_1}$$

i.e. K.E. of rotation increases. This is because the child spends internal energy in folding back his hands.

Q. 13. A rope of negligible mass is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder, if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ? Assume that there is no slipping.

Sol. Here, $M = 3$ kg, $R = 40$ cm $= 0{\cdot}4$ m

Moment of inertia of the hollow cylinder about its axis.

$$I = MR^2 = 3\,(0{\cdot}4)^2 = 0{\cdot}48\ \text{kg}\ m^2$$

Force applied $F = 30$ N $\therefore$ Torque, $\tau = F \times R = 30 \times 0{\cdot}4 = 12$ N-m

If α is angular acceleration produced, then from $\tau = I\,\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0{\cdot}48} = 25\ \text{rad}\ s^{-2}$$

Linear acceleration, $a = R\,\alpha = 0{\cdot}4 \times 25 = \mathbf{10\ ms^{-2}}$

Q. 14. To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 N-m. What is the power of the engine required ? [Note. Uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque. [Assume that the engine is 100% efficient.]

Sol. Here, $\omega = 200$ rad s^{-1} ; torque, $\tau = 180$ N-m ; Power, $P = ?$

As $P = \tau\,\omega$ $\therefore$ $P = 180 \times 200 = 36000$ watt $= \mathbf{36\ kW}$

Q. 15. From a uniform disc of radius R, a circular section of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locate the centre of mass of the resulting flat body.

Sol. Suppose mass per unit area of the disc $= m$

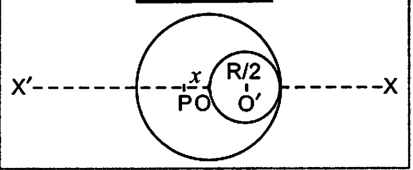
$\therefore$ Mass of original disc, $M = \pi R^2 \times m$

Mass of portion removed from the disc

$$M' = \pi\,(R/2)^2 \times m = \frac{\pi R^2}{4}m = \frac{M}{4}$$

In Fig. 7(N).8, mass M is concentrated at O and mass M' is concentrated at O', where $OO' = R/2$

FIGURE 7(N).8



After the circular disc of mass M' is removed, the remaining portion can be considered as a system of two masses M at O and $-M' = \frac{-M}{4}$ at O'. If x is the distance of centre of mass (P) of the remaining part from the centre O of disc, then

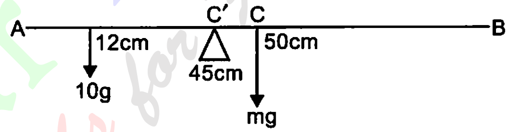
$$x = \frac{M \times 0 - M' \times R/2}{M - M'} = \frac{-\frac{M}{4} \times \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR}{8} \times \frac{4}{3M} = \frac{-R}{6}$$

Negative sign shows that P is to the left of O.

Q. 16. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12·0 cm mark, the stick is found to be balanced at 45·0 cm. What is the mass of the metre stick ?

Sol. Let m be the mass of the stick concentrated at C, the 50 cm mark, Fig. 7(N).9.

FIGURE 7(N).9



For equilibrium about C', the 45 cm mark,

$$10\text{ g}\,(45 - 12) = \text{mg}\,(50 - 45)$$

$$10\text{ g} \times 33 = mg \times 5$$

$$m = \frac{10 \times 33}{5} = \mathbf{66\ gram}$$

Q. 17. The oxygen molecule has a mass of 5·30 × 10^{-26} kg and a moment of inertia of 1·94 × 10^{-46} kg m^2 about an axis through its centre perpendicular to the line joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

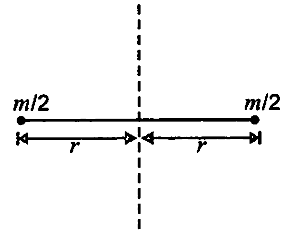
Sol. Here, $m = 5{\cdot}30 \times 10^{-26}$ kg, $I = 1{\cdot}94 \times 10^{-46}$ kg m^2, $v = 500$ m/s

If $\frac{m}{2}$ is mass of each atom of oxygen and $2\,r$ is distance between the two atoms as shown in Fig. 7(N).10, then

$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}} = \sqrt{\frac{1{\cdot}94 \times 10^{-46}}{5{\cdot}3 \times 10^{-26}}} = 0{\cdot}61 \times 10^{-10}\text{ m}$$

FIGURE 7(N).10



As K.E. of rotation $= \frac{2}{3}$ K.E. of translation $\therefore\ \frac{1}{2}\,\text{I}\,\omega^2 = \frac{2}{3} \times \frac{1}{2}mv^2$

$$\frac{1}{2}(mr^2)\,\omega^2 = \frac{1}{3}mv^2$$

$$\omega = \sqrt{\frac{2}{3}}\,\frac{v}{r} = \sqrt{\frac{2}{3}} \times \frac{500}{0{\cdot}61 \times 10^{-10}} = \mathbf{6{\cdot}7 \times 10^{12}\ rad/s}$$

NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

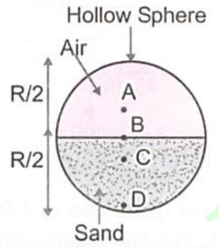
1. For which of the following does the centre of mass lie outside the body ?

(*a*) A pencil (*b*) A shotput

(*c*) A dice (*d*) A bangle

2. Which of the following points is the likely position of the centre of mass of the system shown in Fig. 7(N).11 ?

FIGURE 7(N).11

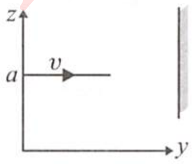


(*a*) A (*b*) B

(*c*) C (*d*) D

3. A particle of mass *m* is moving in *yz*-plane with a uniform velocity *v* with its trajectory running parallel to +ve *y*-axis and intersecting *z*-axis at $z = a$ (Fig. 7(N).12). The change in its angular momentum about the origin as it bounces elastically from a wall at *y* = constant is :

FIGURE 7(N).12



(*a*) $mva\,\hat{e}_x$ (*b*) $2mva\,\hat{e}_x$

(*c*) $ymv\,\hat{e}_x$ (*d*) $2ymv\,\hat{e}_x$

4. When a disc rotates with uniform angular velocity, which of the following is not true ?

(*a*) The sense of rotation remains same.

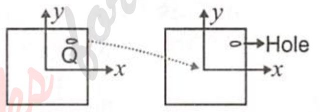
(*b*) The orientation of the axis of rotation remains same.

(*c*) The speed of rotation is non-zero and remains same.

(*d*) The angular acceleration is non-zero and remains same.

5. A uniform square plate has a small piece *Q* of an irregular shape removed and glued to the centre of the plate leaving a hole behind [Fig. 7(N).13]. The moment of inertia about the *z*-axis is then

FIGURE 7(N).13



(*a*) increased (*b*) decreased

(*c*) the same

(*d*) changed in unpredicted manner.

6. In problem 5, the CM of the plate is now in the following quadrant of *x-y* plane.

(*a*) I (*b*) II

(*c*) III (*d*) IV

7. The density of a non-uniform rod of length 1 m is given by $\rho(x) = a(1 + bx^2)$ where *a* and *b* are constants and $0 \leq x \leq 1$. The centre of mass of the rod will be at

(*a*) $\frac{3(2+b)}{4(3+b)}$ (*b*) $\frac{4(2+b)}{3(3+b)}$

(*c*) $\frac{3(3+b)}{4(2+b)}$ (*d*) $\frac{4(3+b)}{3(2+b)}$

8. A Meery-go-round, made of a ring-like platform of radius *R* and mass *M*, is revolving with angular speed ω. A person of mass *M* is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterwards is

(*a*) 2 ω (*b*) ω

(*c*) ω/2 (*d*) 0

MULTIPLE CHOICE QUESTIONS-II

9. Choose the correct alternatives :

(*a*) For a general rotational motion, angular momentum L and angular velocity ω need not be parallel.

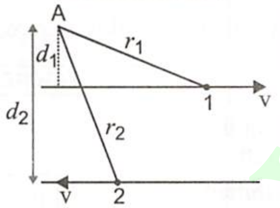
(*b*) For a rotational motion about a fixed axis, angular momentum L and angular velocity ω are always parallel.

(*c*) For a general translational motion, momentum p and velocity v are always parallel.

(*d*) For a general translational motion, acceleration a and velocity v are always parallel.

10. Figure 7(N).14 shows two identical particles 1 and 2, each of mass m, moving in opposite directions with same speed v along parallel lines. At a particular instant, r_1 and r_2 are their respective position vectors drawn from point A which is in the plane of the parallel lines. Choose the correct options :

FIGURE 7(N).14



(*a*) Angular momentum $\vec{L}_1$ of particle 1 about A is $\vec{L}_1 = mv(d_1) \odot$

(*b*) Angular momentum $\vec{L}_2$ of particle 2 about A is $\vec{L}_2 = mvr_2 \odot$

(*c*) Total angular momentum of the system about A is $\vec{L} = mv(r_1 + r_2) \odot$

(*d*) Total angular momentum of the system about A is $\vec{L} = mv(d_2 - d_1) \otimes$

$\odot$ represents a unit vector coming out of the page.

$\otimes$ represents a unit vector going into the page.

11. The net external torque on a system of particles about an axis is zero. Which of the following are compatible with it ?

(*a*) The forces may be acting radially from a point on the axis.

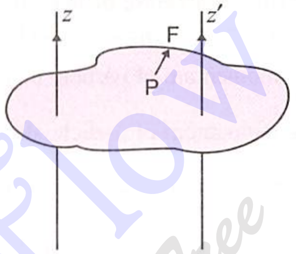
(*b*) The forces may be acting on the axis of rotation.

(*c*) the forces may be acting parallel to the axis of rotation.

(*d*) The torque caused by some forces may be equal and opposite to that caused by other forces.

12. Figure 7(N).15 shows a lamina in x-y plane. Two axes z and z' pass perpendicular to its plane. A force F acts in the plane of lamina at point P as shown. Which of the following are true ? (The point P is closer to z'-axis than the z-axis.)

FIGURE 7(N).15



(*a*) Torque τ caused by F about z-axis is along $-\hat{k}$

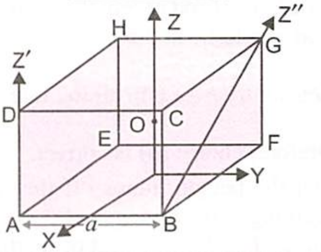
(*b*) Torque τ' caused by F about z' axis is along $-\hat{k}$.

(*c*) Torque τ caused by F about z axis is greater in magnitude than that about z-axis.

(*d*) Total torque is given be $\tau = \tau + \tau'$.

13. With reference to Fig. 7(N).16 of a cube of edge a and mass m, state whether the following are true or false. (O is the centre of the cube.)

FIGURE 7(N).16



(*a*) The moment of inertia of cube about z-axis is, $I_z = I_x + I_y$

(*b*) The moment of inertia of cube about z' is, $I_{z'} = I_z + \frac{ma^2}{2}$

(*c*) The moment of inertia of cube about z'' is $= I_z + \frac{ma^2}{2}$

(*d*) $I_x = I_y$

ANSWERS

1. (*d*) **2.** (*c*) **3.** (*b*) **4.** (*d*) **5.** (*b*) **6.** (*c*) **7.** (*a*) **8.** (*a*)
9. (*a,c*) **10.** (*a,d*) **11.** (*a,b,c,d*) **12.** (*b,c*) **13.** (*b,d*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. Centre of mass of a bangle lies at the centre of the bangle, which is outside the body.

2. The position of centre of mass of the system shown in Fig. 7(N).11 is likely to be at *C*. This is because lower part of the sphere containing sand is heavier than upper part of the sphere containing air.

3. Linear momentum of particle before colliding $= m\vec{v} = mv\hat{e}_y$

Linear momentum of particle after it bounces $= -m\vec{v} = -mv\hat{e}_y$

Change in linear momentum, $\Delta\vec{p} = -m\vec{v} - (m\vec{v}) = -2m\vec{v} = -2mv\,\hat{e}_y$

Change in angular momentum $\Delta\vec{L} = \vec{r}\times\Delta\vec{p}$, where $\vec{r} = (y\hat{e}_y + a\hat{e}_z)$

$$= (y\hat{e}_y + a\hat{e}_z)\times(-2mv\,\hat{e}_y) = 2mva\hat{e}_x\,.$$

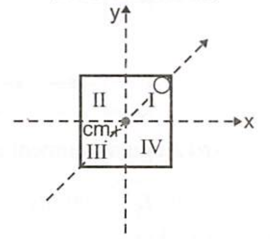
4. When a disc rotates with uniform angular velocity, angular acceleration of the disc is zero. Choice (*d*) is not true.

5. According to the theorem of perpendicular axes, $I_z = I_x + I_y$.

With the hole, I_x and I_y both decrease Gluing the removed piece at the centre of square plate does not affect I_z. Hence, I_z decreases, overall.

6. As piece is removed from quadrant *I*, the CM of the square plate is shifted to quadrant III as shown in Fig. 7(N).17.

FIGURE 7(N).17



7. Here, $\rho(x) = a(1 + bx^2)$

When $b \to 0$, $\rho(x) = a$ = constant

i.e., density of rod of length 1 m is constant. In that event, centre of mass of rod would lie at 0·5 m (*i.e.* at centre of rod).

When we try $b \to 0$ in all the four given options, we find choice (*a*) alone gives $x = \frac{3(2+b)}{4(3+b)} = \frac{6}{12} = 0{\cdot}5$.

Therefore, choice (*a*) is correct.

8. When the person jumps off the round, radially away from the centre, no torque is exerted *i.e.* $\tau = 0$. According to the principle of conservation of angular momentum, $I\times\omega$ = constant. As mass reduces to half (from 2 *M* to *M*), moment of inertia *I* becomes half. Therefore, ω must become twice (= 2 ω).

Multiple Choice Questions – II

9. From the study of theory, we know that for general rotational motion, angular momentum $\vec{L}$ and angular velocity $\vec{\omega}$ need not be parallel.

Again, for a general translational motion, linear momentum $\vec{p}$ and linear velocity $\vec{v}$ are always parallel.

This is because $\vec{p}$ is directed along $\vec{v}$ only.

10. As angular momentum = linear momentum multiplied by perpendicular distance of particle from the axis of rotation, therefore, for particle 1,

$\vec{L}_1 = mv(d_1)\odot$, perpendicular to plane of page and directed **upwards,** and for particle 2.

$\vec{L_2} = mv(d_2)\otimes$, perpendicular to plane of page and directed **inwards.**

$\therefore$ Total angular momentum of the system, $\vec{L} = \vec{L_2} - \vec{L_1} = mv(d_2 - d_1)\otimes$

It would be perpendicular to plane of page and directed inwards.

11. When net external torque on a system of particles about an axis is zero, *i.e.*, $\vec{\tau} = \vec{r}\times\vec{F} = rF\sin\theta\,\hat{\tau}$ = Zero, where, θ is angle between $\vec{r}$ and $\vec{F}$, $\hat{\tau}$ is unit vector along $\vec{\tau}$, then all the four statements (*a*), (*b*), (*c*), (*d*) are compatible.

12. According to right handed screw rule, the direction of torque $\vec{\tau}$ caused by $\vec{F}$ about *Z*-axis is along $\hat{k}$. So choice (*a*) is false.

However choice (*b*) is **true** as direction of torque ($\vec{\tau}'$) caused by $\vec{F}$ about *Z*′ axis is along $-\hat{k}$.

As $\vec{\tau} = \vec{r}\times\vec{F}$ and *P* is closer to *Z*′ axis, therefore $\vec{\tau}$ caused by $\vec{F}$ about *Z*-axis is greater in magnitude than that about *Z*′ axis. Choice (*c*) is true. Choice (*d*) is false as it is meaningless to add torques about different axes.

13. Choice (*a*) is false, as theorem of perpendicular axes applies only to a plane lamina.

Now, *Z* axis is parallel to *Z*′ axis and distance between them $= \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$. Therefore, according to the theorem of parallel axes,

$$I_{z'} = I_z + m\left(\frac{a}{\sqrt{2}}\right)^2 = I_z + \frac{ma^2}{2}$$

Choice (*b*) is true.
Again, choice (*c*) is false as *Z*″ axis is not parallel to *Z*-axis.
Choice (*d*) is true as from symmetry, we find that $I_x = I_y$.

VERY SHORT ANSWER QUESTIONS

14. The centre of gravity of a body on the earth coincides with its centre of mass for a 'small' object whereas for an 'extended' object it may not. What is the qualitative meaning of 'small' and 'extended' in this regard ?

For which of the following the two coincide ? A building, a pond, a lake, a mountain ?

Ans. When vertical height of an object is very small compared to the radius of earth, the object is said to be **small.** Otherwise, the object is said to be **extended.**

A building and a pond are treated as small objects. In their cases, the centre of gravity coincides with the centre of mass.

A deep lake and a mountain are taken as extended objects. In their cases, the centre of gravity may not coincide with the centre of mass.

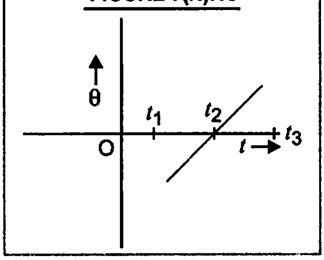
15. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry ?

Ans. By definition, $I = \sum_i m_i r_i^2$

In case of hollow cylinder, entire mass lies at distance *R* from the axis of symmetry. But in case of a solid sphere, the same mass is distributed throughout at distances from zero to *R* from the axis of symmetry. Here, *R* is radius of cylinder/sphere. That is why a solid sphere has smaller moment of inertia than a hollow cylinder of same mass and radius.

16. The variation of angular position θ, of a point on a rotating rigid body, with time *t* is shown in Fig. 7(N).18. Is the body rotating clockwise or anti-clockwise ?

FIGURE 7(N).18



Ans. From Fig. 7(N).18, we find that slope of θ – *t* graph is positive, which by convention, represents **anticlockwise rotation.**

17. A uniform cube of mass *m* and side *a* is placed on a frictionless horizontal surface. A vertical force *F* is applied to the edge as shown in Fig. 7(N).19. Match the following (most appropriate choice) :

(*a*) $mg/4 < F < mg/2$	**(*i*) Cube will move up.**
(*b*) $F > mg/2$	**(*ii*) Cube will not exhibit motion.**
(*c*) $F > mg$	**(*iii*) Cube will begin to rotate and slip at A.**
(*d*) $F = mg/4$	**(*iv*) Normal reaction effectively at *a*/3 from A, no motion.**

FIGURE 7(N).19

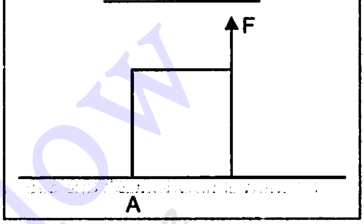
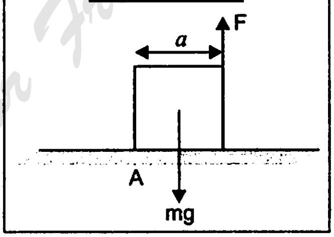


FIGURE 7(N).20



Ans. As is clear from Fig. 7(N).20, torque of force *F* about *A*, $\tau_1 = F \times a$. This is anticlockwise.

Torque of weight *mg* of cube about *A*

$\tau_2 = mg \times \frac{a}{2}$, clockwise. The cube will not exhibit any motion if $\tau_1 = \tau_2$

$$F \times a = mg \times \frac{a}{2} \quad \text{or} \quad F = \frac{mg}{2}$$

The cube will rotate only when $\tau_1 > \tau_2$

$$F \times a > mg\frac{a}{2} \quad \text{or} \quad F > \frac{mg}{2}$$

If we assume that normal reaction is effectively at *a*/3 from *A*, then block would turn if

$$mg \times \frac{a}{3} = F \times a \quad \text{or} \quad F = \frac{mg}{3}.$$

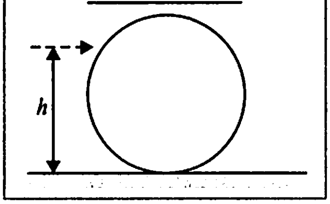
When $F = \frac{mg}{4} < \frac{mg}{3}$, there will be no motion.

Hence, we conclude (*a*) → (*ii*) ; (*b*) → (*iii*) ; (*c*) → (*i*) ; (*d*) → (*iv*).

18. A uniform sphere of mass *m* and radius *R* is placed on a rough horizontal surface [Fig. 7(N).21]. The sphere is struck horizontally at a height *h* from the floor. Match the following :

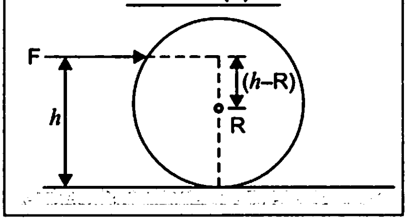
(*a*) $h = R/2$	**(*i*) Sphere rolls without slipping with a constant velocity and no loss of energy.**
(*b*) $h = R$	**(*ii*) Sphere spins clockwise, loses energy by friction.**
(*c*) $h = 3R/2$	**(*iii*) Sphere spins anti-clockwise, loses energy by friction.**
(*d*) $h = 7R/5$	**(*iv*) Sphere has only a translational motion, looses energy by friction.**

FIGURE 7(N).21



Ans. Let the sphere of mass *m* and radius *R* be struck horizontally at a height *h* from the floor, as shown in Fig. 7(N).22.

FIGURE 7(N).22



The sphere will roll without slipping when $\omega = \frac{v}{R}$

Now, angular momentum of sphere, about c.m.

$$mv(h-R) = I\,\omega = \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right) \quad \text{or} \quad mv(h-R) = \frac{2}{5}mvR$$

$$h - R = \frac{2}{5}R \quad \text{or} \quad h = \frac{7}{5}R$$

∴ The sphere will roll without slipping with a constant velocity and no loss of energy when $h = \frac{7}{5}R$. Therefore, $d \rightarrow (i)$.

Torque due to applied force, about c.m., $\tau = F(h - R)$

If $\tau = 0$, $h = R$, sphere will have only translational motion. It would lose energy by friction. Hence, $b \rightarrow (iv)$.

The sphere will spin clockwise, when τ is positive, *i.e.*, $h > R$.

Therefore, $c \rightarrow (ii)$.

Again, the sphere will spin anticlockwise, when τ is negative, *i.e.*, $h < R$. Therefore, $a \rightarrow (iii)$.

SHORT ANSWER QUESTIONS

19. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point ?

Ans. No, not necessary.

We are given that $\sum_i \vec{F}_i \neq 0$.

However, the sum of torques about a certain point O ; $\sum_i \vec{r}_i \times \vec{F}_i = 0$

About any other point O', the sum of torques would be $\sum_i (\vec{r}_i - \vec{a}) \times \vec{F}_i = \sum_i \vec{r}_i \times \vec{F}_i - \vec{a} \times \sum_i \vec{F}_i$

As the second term need not vanish for any value of $\vec{a}$, therefore, sum of all the torques about any arbitrary point need not be zero necessarily.

20. A wheel in uniform motion about an axis passing through its centre and perpendicular to its plane is considered to be in mechanical (translational plus rotational) equilibrium because no net external force or torque is required to sustain its motion. However, the particles that constitute the wheel do experience a centripletal acceleration directed towards the centre. How do you reconcile this fact with the wheel being in equilibrium ?

How would you set a half-wheel into uniform motion about an axis passing through the centre of mass of the wheel and perpendicular to its plane ? Will you require external forces to sustain the motion ?

Ans. A wheel is a rigid body. The centripetal accelerations of the particles of the wheel arise due to the internal elastic forces, which cancel out in pairs.

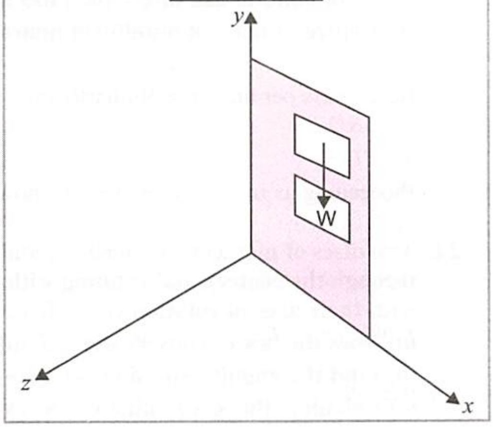
In a half wheel, the distribution of mass about its centre of mass (through which axis of rotation passes) is not symmetrical. Therefore, the direction of angular momentum of the wheel does not coincide with the direction of its angular velocity. Hence, an external torque is required to maintain the motion of the half wheel.

21. A door is hinged at one end and is free to rotate about a vertical axis [Fig. 7(N).23]. Does its weight cause any torque about this axis ? Give reason for your answer.

Ans. No. As $\vec{\tau} = \vec{r} \times \vec{F}$, therefore torque is produced by a force along a direction normal to the force.

If the door is in *xy* plane, the torque produced by its weight can only be along ± *z* direction. It can never be about an axis passing through *x*-direction or *y*-direction.

FIGURE 7(N).23



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22. $(n-1)$ equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector a with respect to the centre of the polygon. Find the position vector of centre of mass.

Ans. Let $\vec{b}$ be the position vector of centre of mass of regular n polygon.

As $(n-1)$ equal point masses each of mass m are placed at $(n-1)$ vertices of regular polygon, therefore,

$$\frac{(n-1)\,mb+ma}{(n-1+1)\,m}=0 \qquad \text{or} \qquad (n-1)\,mb+ma=0 \qquad \text{or} \qquad b=\frac{-a}{(n-1)}$$

LONG ANSWER QUESTIONS

23. Find the centre of mass of a uniform (*a*) half-disc, (*b*) quarter-disc.

Ans. (a) Centre of mass of uniform half disc

FIGURE 7(N).24

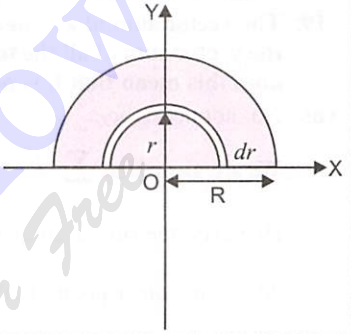


Fig. 7(N).24 shows upper half of uniform circular disc of radius R in XY plane. Let M be total mass of half disc. Mass per unit area of half disc $=\dfrac{M}{\pi R^2/2}=\dfrac{2M}{\pi R^2}$.

The half disc can be supposed to be made up of a large number of semicircular rings, each of mass dm and radii ranging from $r=0$ to $r=R$. Consider one such semicircular ring of radius r and thickness dr as shown. Surface area of this semicircular ring $=(\pi r)\,dr$.

Mass of this element, $dm=(\pi r)\,dr\times\dfrac{2M}{\pi R^2}=\dfrac{2M}{R^2}\,r\,dr$.

If (x, y) are co-ordinates of *c.m* of this element, then $(x,y)=\left(0,\dfrac{2r}{\pi}\right)$, *i.e.,* $x=0,\ y=\dfrac{2r}{\pi}$

Let x_{cm}, y_{cm} be co-ordinates of *c.m* of semicircular disc.

$$\therefore \qquad x_{cm}=\frac{1}{M}\int_0^R x\,dm=\frac{1}{M}\int_0^R 0\,dm=0$$

and
$$y_{cm}=\frac{1}{M}\int_0^R y\,dm=\frac{1}{M}\int_0^R\left(\frac{2r}{\pi}\right)\frac{2M}{R^2}\,r\,dr=\frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R=\frac{4}{\pi R^2}\left[\frac{R^3}{3}-0\right]=\frac{4R}{3\pi}$$

$\therefore$ *c.m* of uniform half disc **= (0, 4 *R*/3 π).**

(b) Centre of mass of a uniform quarter disc

Here, mass per unit area of quarter disc $=\dfrac{M}{\pi R^2/4}=\dfrac{4M}{\pi R^2}$

Proceeding as in (*a*) above, we can show that *c.m* of uniform quarter disc is $\left(\dfrac{4R}{3\pi},\dfrac{4R}{3\pi}\right)$

24. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(*a*) Does the law of conservation of angular momentum apply to the situation ? Why ?

(*b*) Find the angular speed of the two-disc system.

(*c*) Calculate the loss in kinetic energy of the system in the process.

(*d*) Account for this loss.

Ans. (*a*) Yes, the law of conservation of angular momentm applies to the situation. This is because no external torque is involved in bringing the two discs into contact face to face. External forces, gravitation and normal reaction, act through the axis of rotation producing no torque.

(*b*) If ω is angular speed of the two disc system, then from the principle of conservation of angular momentum,

$$(I_1 + I_2)\,\omega = I_1\,\omega_1 + I_2\,\omega_2$$

$$\omega = \frac{I_1\,\omega_1 + I_2\,\omega_2}{I_1 + I_2} \qquad ...(i)$$

(*c*) Initial KE of two discs, $E_i = \frac{1}{2} I_1\,\omega_1^2 + \frac{1}{2} I_2\,\omega_2^2$

Final KE of the system, $E_f = \frac{1}{2}(I_1 + I_2)\,\omega^2$

Using (*i*), $E_f = \frac{1}{2}(I_1 + I_2)\,\frac{(I_1\,\omega_1 + I_2\,\omega_2)^2}{(I_1 + I_2)^2} = \frac{(I_1\,\omega_1 + I_2\,\omega_2)^2}{2(I_1 + I_2)}$

$$\Delta E = E_f - E_i = \frac{(I_1\,\omega_1 + I_2\,\omega_2)^2}{2(I_1 + I_2)} - \frac{I_1\,\omega_1^2 + I_2\,\omega_2^2}{2}$$

$$= \frac{I_1^2\,\omega_1^2 + I_2^2\,\omega_2^2 + 2\,I_1\,\omega_1\,I_2\,\omega_2 - I_1^2\,\omega_1^2 - I_1\,I_2\,\omega_2^2 - I_2\,I_1\,\omega_1^2 - I_2^2\,\omega_2^2}{2(I_1 + I_2)}$$

$= \frac{-I_1\,I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$ which is a negative quantity. Hence, **some KE is always lost.**

(*d*) The loss in KE is due to work done against friction between the two discs.

25. A disc of radius *R* is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k.

(*a*) What was the velocity of its centre of mass before being brought in contact with the table ?

(*b*) What happens to the linear velocity of a point on its rim when placed in contact with the table ?

(*c*) What happens to the linear speed of the centre of mass when disc is placed in contact with the table ?

(*d*) Which force is responsible for the effects in (*b*) and (*c*).

(*e*) What condition should be satisfied for rolling to begin ?

(*f*) Calculate the time taken for the rolling to begin.

Ans. (*a*) Before the disc is brought in contact with the table, it is simply rotating about its horizontal axis. Therefore, its centre of mass is at rest, *i.e.*, v_{cm} = Zero.

(*b*) Linear velocity of any point on the rim of the disc would **decrease.** When disc is placed in contact with the table.

(*c*) When rotating disc is placed in contact with the table, its centre of mass acquires some velocity – which was zero before contact. Hence, velocity of centre of mass increases.

(*d*) In (*b*) above, force of friction between the rotating disc and table is responsible for decreasing linear velocity of any point on the rim of the disc. In (*c*), the disc rolls over the table because of friction. Hence, velocity of centre of mass of disc increases due to friction.

(*e*) Rolling would begin only when $v_{cm} = R\,\omega$.

(*f*) Acceleration produced in centre of mass due to friction $a_{cm} = \frac{F}{m} = \frac{\mu_k\,mg}{m} = \mu_k\,g \qquad ...(i)$

Angular acceleration produced by the torque due to friction, $\alpha = \frac{\tau}{I} = \frac{(\mu_k\,mg)\,R}{I}$

From
$$v_{cm} = u_{cm} + a_{cm}\, t$$
$$v_{cm} = 0 + (\mu_k\, g)\, t = \mu_k\, gt \qquad \text{...}(ii)$$

From
$$\omega = \omega_0 + \alpha\, t = \omega_0 + \frac{(\mu_k\, mg)\, Rt}{I} \qquad \text{...}(iii)$$

For rolling without slipping

$$\frac{v_{cm}}{R} = \omega$$

Using (*ii*) and (*iii*), we get

$$\frac{\mu_k\, gt}{R} = \omega_0 + \frac{(\mu_k\, mg)\, Rt}{I}$$

$$\frac{-(\mu_k\, mg)\, Rt}{I} + \frac{\mu_k\, gt}{R} = \omega_0$$

$$\mu_k\, g\left(-\frac{mR}{I} + \frac{1}{R}\right) t = \omega_0$$

$$\mu_k\, g\left(-\frac{mR^2}{I} + 1\right) \frac{t}{R} = \omega_0$$

$$t = \frac{R\,\omega_0}{\mu_k\, g\left(1 - \dfrac{mR^2}{I}\right)}$$

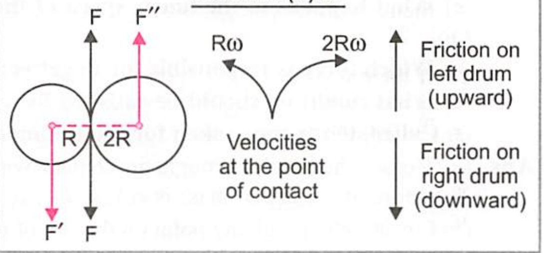
26. Two cylindrical hollow drums of radii *R* and 2*R*, and of a common height *h*, are rotating with angular velocities ω (anti-clockwise) and ω (clockwise), respectively. Their axes, fixed are parallel and in a horizontal plane separated by (3*R* + δ). They are now brought in contact (δ → 0).

(*a*) Show the frictional forces just after contact.

(*b*) Identify forces and torques external to the sytem just after contact.

(*c*) What would be the ratio of final angular velocities when friction ceases ?

FIGURE 7(N).25


F
F″
Rω
2Rω
Friction on left drum (upward)
R
2R
Velocities at the point of contact
Friction on right drum (downward)
F′
F

Ans. (*a*) In Fig. 7(N).25, we have shown the frictional forces on the two drums just after contact.

(*b*) If *F* ′ and *F* ″ are external forces through support on the left and right drums respectively, then

$$F' = F = F''$$

∴ $F_{net} = 0$.

As distance between the centres of two drums = *R* + 2 *R* = 3 *R*, therefore, external torque,

$$\tau = F \times 3\,R \text{ ; anticlockwise.}$$

(*c*) Let ω_1 be final angular velocity of left drum in anticlockwise direction and ω_2 be final angular velocity of right drum in clockwise direction. As there will be no friction, finally.

∴ $R\,(\omega_1) = 2\,R\,(\omega_2)$ or $\dfrac{\omega_1}{\omega_2} = \dfrac{2}{1}$

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27. A uniform square plate S (side c) and a uniform rectangular plate R (sides b, a) have identical areas and masses [Fig. 7(N).26].

Show that

(*i*) $I_{xR}/I_{xS} < 1$; (*ii*) $I_{yR}/I_{yS} > 1$; (*iii*) $I_{zR}/I_{zS} > 1$.

Ans. As area of square plate S = area of rectangular plate R

$\therefore \quad c^2 = a \times b \quad$ ...(*i*)

Now, by definition

(*i*) $\quad \dfrac{I_{xR}}{I_{xS}} = \dfrac{b^2}{c^2}$

As $\quad b < c, \quad \dfrac{I_{xR}}{I_{xS}} < 1$

(*ii*) $\quad \dfrac{I_{yR}}{I_{yS}} = \dfrac{a^2}{c^2}$

As $\quad a > c, \quad \dfrac{I_{yR}}{I_{yS}} > 1.$

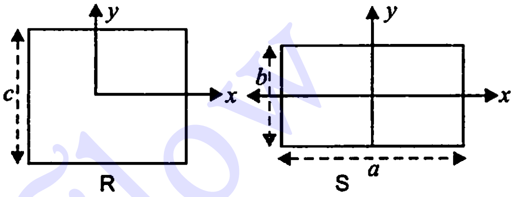
FIGURE 7(N).26

(*iii*) $(I_{zR} - I_{zS}) \propto (a^2 + b^2 - 2\,c^2)$

$\propto (a^2 + b^2 - 2\,ab)$

$\propto (a - b)^2$, which is +

$\therefore \quad (I_{zR} - I_{zS}) > 0 \;$ or $\; \dfrac{I_{zR}}{I_{zS}} > 1$

28. A uniform disc of radius R, is resting on a table on its rim. The coefficient of friction between disc and table is μ, Fig. 7(N).27. Now the disc is pulled with a force F as shown in the Fig. 7(N).28. What is the maximum value of F for which the disc rolls without slipping ?

Ans. Let f be the force of friction, when force applied is F.

If a is acceleration of the centre of mass of disc,

then $\quad Ma = F - f \quad$ ...(*i*)

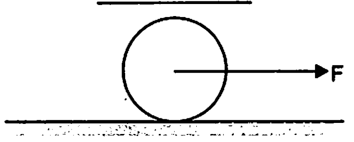
FIGURE 7(N).27

If there is no slipping, angular acceleration of disc, $\alpha = \dfrac{a}{R}$.

Torque due to frictional force

$$f \times R = I\,\alpha = \left(\frac{1}{2}MR^2\right)\frac{a}{R} = \frac{Ma\,R}{2} \quad \text{or} \quad Ma = 2\,f$$

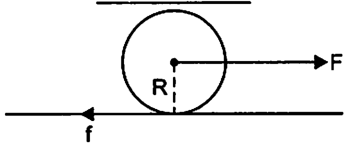
FIGURE 7(N).28

From (*i*), $\quad 2f = F - f \quad$ or $\quad 3f = F \quad$ or $\quad f = \dfrac{F}{3}$.

As there is no sliding, $\quad f \leq \mu\, Mg$

$\therefore \quad \dfrac{F}{3} \leq \mu Mg \quad$ or $\quad F \leq 3\,\mu\, Mg$

Hence, $\quad F_{\max} = 3\,\mu\, Mg$

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I. Multiple Choice Questions (with One Correct Answer)

I. Centre of Mass

1. Distance of the centre of mass of a solid uniform cone from its vertex is Z_0. If the radius of its base is R and its height is h, then Z_0 is equal to

(a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$

(c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

(JEE Main 2015)

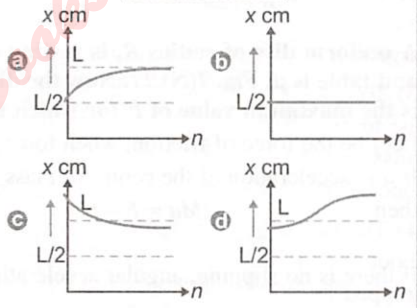
2. A circular disc of radius R is removed from a bigger circular disc of radius 2 R, such that the circumferences of the discs coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$ **(AIEEE 2007)**

3. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive integer. If position x_{cm} of centre of mass of the rod is plotted against n, which of the following graphs, Fig. 7(CF).1 best approximates the dependence of x_{cm} on n ?

FIGURE 7(CF).1



(AIEEE 2008)

4. From a circular disc of radius R and mass $9\,M$, a small disc of mass M and radius $\frac{R}{3}$ is removed concentrically. The moment of intertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is

(a) $\frac{40}{9} MR^2$ (b) MR^2

(c) $4\,MR^2$ (d) $\frac{4}{9} MR^2$

(AIPMT Main 2010)

ANSWERS

1. (b) 2. (b) 3. (a) 4. (a)

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5. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of remaining part of the disc about a perpendicular axis, passing through the centre ?

(*a*) $\frac{13}{32}MR^2$ (*b*) $\frac{11}{32}MR^2$

(*c*) $\frac{9}{32}MR^2$ (*d*) $\frac{15}{32}MR^2$

(NEET 2016)

6. A disc and a solid sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first ?

(*a*) solid sphere

(*b*) both reach at the same time

(*c*) depends on their masses

(*d*) disc **(NEET 2016)**

7. (1) Centre of gravity (C.G.) of a body is the point at which the weight of the body acts,

(2) Centre of mass coincides with the centre of gravity if the earth is assumed to have infinitely large radius,

(3) To evaluate the gravitational field intensity due to any body at an external point, the entire mass of the body can be considered to be concentrated at its C.G.,

(4) The radius of gyration of any body rotating about an axis is the length of the perpendicular dropped from the C.G. of the body to the axis. Which one of the following pairs of statements is correct ?

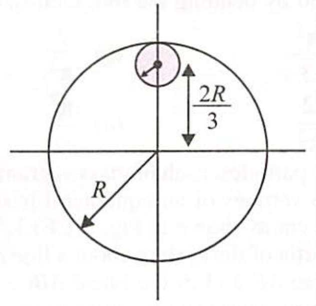
(*a*) (4) and (1) (*b*) (1) and (2)

(*c*) (2) and (3) (*d*) (3) and (4)

(AIPMT Main 2010)

8. From a uniform circular disc of radius R and mass $9\,M$, a small disc of radius $\frac{R}{3}$ is removed as shown in Fig. 7(CF).2. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

FIGURE 7(CF).2



(*a*) $4\,MR^2$ (*b*) $\frac{40}{9}MR^2$

(*c*) $10\,MR^2$ (*d*) $\frac{37}{9}MR^2$

(JEE Main 2018)

II. Moment of Inertia

9. The densities of two solid spheres A and B of same radii R vary with radial distance r as $\rho_A(r) = K\left(\frac{r}{R}\right)$ and $\rho_B(r) = K\left[\frac{r}{R}\right]^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

(*a*) 6 (*b*) 10

(c) 16 (*d*) 7

(JEE Advanced 2015)

10. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

(*a*) $\frac{MR^2}{32\sqrt{2}\,\pi}$ (*b*) $\frac{MR^2}{16\sqrt{2}\,\pi}$

(*c*) $\frac{4\,MR^2}{9\sqrt{3}\,\pi}$ (*d*) $\frac{4\,MR^2}{3\sqrt{3}\,\pi}$

(JEE Main 2015)

11. If I_1 is moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass, and I_2 is the moment of inertia of the ring about an axis perpendicular

ANSWERS

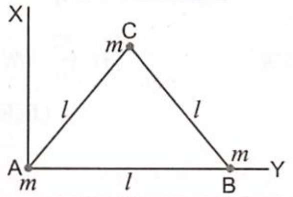
5. (*a*) **6.** (*a*) **7.** (*a*) **8.** (*a*) **9.** (*a*) **10.** (*c*) to plane of ring and passing through its centre formed by bending the rod, then I_1/I_2 is

(*a*) $\frac{\pi^2}{3}$ (*b*) $\frac{3}{\pi^2}$

(*c*) $\frac{2}{\pi^2}$ (*d*) $\frac{\pi^2}{2}$

12. Three particles, each of mass *m* gram, are situated at the vertices of an equilateral triangle *ABC* of side *l* cm as shown in Fig. 7(CF).3. The moment of inertia of the system about a line *AX* perpendicular to *AB* and in the plane *ABC*, in gram-cm^2 will be

FIGURE 7(CF).3



(*a*) $\frac{5}{4}ml^2$ (*b*) $\frac{3}{2}ml^2$

(*c*) $\frac{3}{4}ml^2$ (*d*) $2\ ml^2$

13. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

(*a*) $\sqrt{5}:\sqrt{6}$ (*b*) $1:\sqrt{2}$

(*c*) 2 : 3 (*d*) 2 : 1

14. Three identical rods, each of length *L*, are joined to form a rigid equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to plane of triangle is

(*a*) $\frac{L}{\sqrt{2}}$ (*b*) $\frac{L}{\sqrt{3}}$

(*c*) $\frac{L}{2}$ (*d*) $\frac{L}{3}$

15. Four identical thin rods each of mass *M* and length *l*, form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is

(*a*) $\frac{1}{3}Ml^2$ (*b*) $\frac{4}{3}Ml^2$

(*c*) $\frac{2}{3}Ml^2$ (*d*) $\frac{13}{3}Ml^2$

(AIPMT 2009)

16. A circular disc of moment of inertia I_t is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed $(i)_i$. Another disk of moment of inertia I_b is dropped coaxially onto the rotating disc. Initially, the second disc has zero angular speed. Eventually, both the discs rotate with a constant angular speed $(i)_f$. The energy lost by the initially rotating disc to friction is

(*a*) $\frac{1}{2}\frac{I_t I_b}{(I_t+I_b)}(i)_i^2$ (*b*) $\frac{1}{2}\frac{I_t^2}{(I_t+I_b)}(i)_i^2$

(*c*) $\frac{I_b-I_t}{(I_t+I_b)}(i)_i^2$ (*d*) $\frac{1}{2}\frac{I_b-I_t}{(I_t+I_b)}(i)_i^2$

(AIPMT 2010)

17. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then *N* is

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) 6 **(IIT 2011)**

18. A pulley of radius 2 m is rotated about its axis by a force $F = (20\,t - 5\,t^2)$ newton (where *t* is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion is reversed, is

(*a*) more than 3 but less than 6

(*b*) more than 6 but less than 9

(*c*) more than 9

(*d*) less than 3 **(AIEEE 2011)**

19. The moment of inertia of a uniform cylinder of length *l* and radius *R* about its perpendicular bisector is *I*. What is the ratio *l*/*R* such that the moment of inertia is minimum ?

(*a*) 1 (*b*) $\frac{3}{\sqrt{2}}$

(*c*) $\sqrt{\frac{3}{2}}$ (*d*) $\frac{\sqrt{3}}{2}$

(JEE Main 2017)

ANSWERS

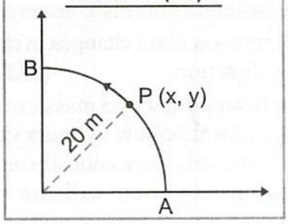
11. (*a*) **12.** (*a*) **13.** (*a*) **14.** (*a*) **15.** (*b*) **16.** (*a*) **17.** (*c*) **18.** (*a*) **19.** (*c*)

20. When a ceiling fan is switched off, its angular velocity falls to half while it makes 36 rotations. How many more rotations will it make before coming to rest ?

(*a*) 24 (*b*) 36
(*c*) 18 (*d*) 12

21. A point P moves in counter clockwise direction on a circular path as shown in Fig. 7(CF).4. The movement of P is such that it sweeps out a length $S = t^3 + 5$, where S is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ S is nearly.

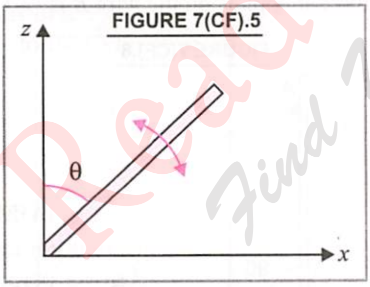
FIGURE 7(CF).4



(*a*) 14 m/s^2 (*b*) 13 m/s^2
(*c*) 12 m/s^2 (*d*) 7·2 m/s^2

22. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane, Fig. 7(CF).5. There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is

FIGURE 7(CF).5



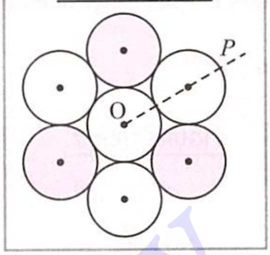
(*a*) $\frac{3g}{2l}\cos\theta$ (*b*) $\frac{2g}{3l}\cos\theta$

(*c*) $\frac{3g}{2l}\sin\theta$ (*d*) $\frac{2g}{3l}\sin\theta$

(JEE Main 2017)

23. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is

FIGURE 7(CF).6



(*a*) $\frac{19}{2}MR^2$ (*b*) $\frac{55}{2}MR^2$

(*c*) $\frac{73}{2}MR^2$ (*d*) $\frac{181}{2}MR^2$

(JEE Main 2018)

III. Rotational Motion

24. The ratio of the accelerations for a solid sphere (mass m, and radius R) rolling down an incline of angle θ without slipping, and slipping down the incline without rolling is

(*a*) 5 : 7 (*b*) 2 : 3
(*c*) 2 : 5 (*d*) 7 : 5 **(AIPMT 2014)**

25. A rod of length L is hinged from one end. It is brought to a horizontal position and released. The angular velocity of the rod, when it is in vertical position is

(*a*) $\sqrt{\frac{2g}{L}}$ (*b*) $\sqrt{\frac{3g}{L}}$

(*c*) $\sqrt{\frac{g}{2L}}$ (*d*) $\sqrt{\frac{g}{L}}$

(AIIMS 2014)

26. A uniform disc is acted by two equal forces of magnitude F. One of them, acts tangentially to the disc, while other one is acting at the central point of the disc. The friction between disc surface and ground surface is nF. If r be the radius of the disc, then the value of n would be (in N)

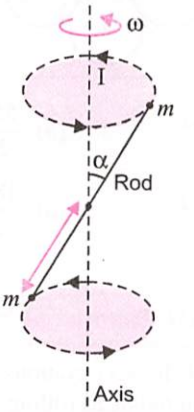
(*a*) 0 (*b*) 1·2
(*c*) 2·0 (*d*) 3·2 **(AIIMS 2015)**

ANSWERS

20. (*d*) 21. (*a*) 22. (*c*) 23. (*d*) 24. (*a*) 25. (*b*) 26. (*a*)

27. A massless rod S having length $2l$ has point masses attached to its two ends as shown in Fig. 7(CF).7. The rod is rotating about an axis passing through its centre and making angle α with the axis. The magnitude of rate of change of momentum of rod, *i.e.*, $\left|\frac{dL}{dt}\right|$ equals

FIGURE 7(CF).7



(*a*) $2\ ml^3\omega^2 \sin\alpha \cdot \cos\alpha$

(*b*) $ml^2\ \omega^2 \sin 2\alpha$

(*c*) $ml^2 \sin 2\alpha$

(*d*) $m^{1/2}\ l^{1/2}\ \omega \sin\alpha \cdot \cos\alpha$ **(AIIMS 2015)**

28. A hemispherical bowl of radius r is set rotating about its axis of symmetry in vertical. A small block kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the block with the vertical is θ, then find the angular speed at which the ball is rotating.

(*a*) $\omega = \sqrt{rg\sin\theta}$ (*b*) $\omega = \sqrt{g/r\cos\theta}$

(*c*) $\omega = \sqrt{\dfrac{gr}{\cos\theta}}$ (*d*) $\omega = \sqrt{\dfrac{gr}{\tan\theta}}$

(AIIMS 2015)

29. A uniform circular disc of radius 50 cm at rest is free to turn about an axis, which is perpendicular to the plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2·0 rad/s^2. Its net acceleration in m/s^2 at the end of 2·0 s is approximately

(*a*) 7·0 (*b*) 6·0

(*c*) 3·0 (*d*) 8·0 **(NEET 2016)**

30. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω radian/sec about the vertical. About the point of suspension,

(*a*) angular momentum changes in direction but not in magnitude,

(*b*) angular momentum changes both in magnitude and direction,

(*c*) angular momentum is conserved,

(*d*) angular momentum changes in magnitude, but not in direction. **(JEE Main 2014)**

31. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ?

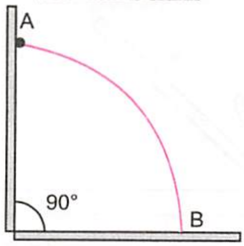
(*a*) 5 g/6 (*b*) g

(*c*) 2 g/3 (*d*) g/2

(JEE Main 2014)

32. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in Fig. 7(CF).8. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire B is

FIGURE 7(CF).8



(*a*) always radially outwards

(*b*) always radially inwards

(*c*) radially outwards initially and radially inwards later

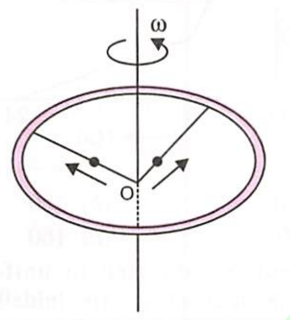
(*d*) radially inwards initially and radially outwards later. **(JEE Advanced 2014)**

ANSWERS

27. (*b*) **28.** (*b*) **29.** (*d*) **30.** (*a*) **31.** (*d*) **32.** (*d*)

33. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in Fig. 7(CF).9. At some instant, the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant, the distance of the other mass from O is

FIGURE 7(CF).9



(a) $\frac{2}{3}R$
(b) $\frac{1}{3}R$
(c) $\frac{3}{5}R$
(d) $\frac{4}{5}R$

(JEE Advanced 2015)

34. A uniform circular disc of radius 50 cm at rest is free to turn about an axis, which is perpendicular to the plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2·0 rad/s^2. Its net acceleration in m/s^2 at the end of 2·0 s is approximately

(a) 7·0
(b) 6·0
(c) 3·0
(d) 8·0 **(NEET 2016)**

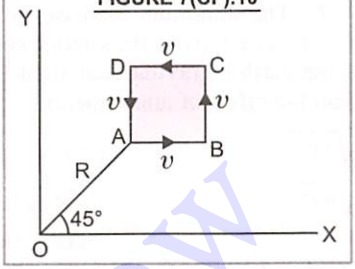
35. A particle of mass 10 gram moves along a circle of radius 6·4 cm, with a constant tangential acceleration. What is the magnitude of this acceleration, if K.E. of the particle becomes equal to 8×10^{-4} J by the end of second revolution after the beginning of the motion.

(a) 0·15 m/s^2
(b) 0·18 m/s^2
(c) 0·20 m/s^2
(d) 0·10 m/s^2

(NEET 2016)

36. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in XY plane as shown in Fig. 7(CF).10. Which of the following statements is false for the angular momentum $\vec{L}$ about the origin ?

FIGURE 7(CF).10



(a) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$, when particle is moving from A to B

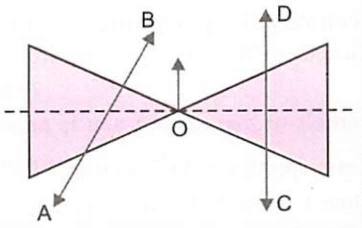
(b) $\vec{L} = mv\left(\frac{R}{\sqrt{2}} - a\right)\hat{k}$, when particle is moving from C to D

(c) $\vec{L} = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{k}$, when particle is moving from B to C

(d) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$, when particle is moving from D to A **(JEE Mains 2016)**

37. A roller is made by joining together two corners at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically, Fig. 7(CF).11, with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD. It is given a light push, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to

FIGURE 7(CF).11



ANSWERS

33. (d) 34. (d) 35. (d) 36. (b,d)

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(*a*) turn left (*b*) turn right

(*c*) go straight

(*d*) turn left and right alternately

(JEE Mains 2016)

38. A satellite is revolving in a circular orbit at a height h from the earth's surface (radius of earth R and $h << R$). The minimum increase in its orbital velocity required so that the satellite could escape from the earth's gravitational field is close to (neglect the effect of atmosphere).

(*a*) $\sqrt{2gR}$ (*b*) $\sqrt{gR}$

(*c*) $\sqrt{gR/2}$ (*d*) $\sqrt{gR}(\sqrt{2}-1)$

(JEE Mains 2016)

39. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed

(*a*) $\sqrt{2}\,v_0$ (*b*) $2\,v_0$

(*c*) v_0 (*d*) zero

40. A disc and a solid sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first ?

(*a*) solid sphere

(*b*) both reach at the same time

(*c*) depends on their masses

(*d*) disc **(NEET 2016)**

41. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$, where ω is a constant. Which of the following is true ?

(*a*) velocity and acceleration both are perpendicular to $\vec{r}$

(*b*) velocity and acceleration both are parallel to $\vec{r}$

(*c*) velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin

(*d*) velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin

(NEET 2016)

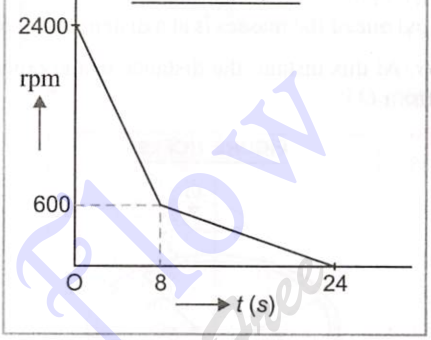
42. A particle of mass $m = 5$ unit is moving with a uniform speed $v = 3\sqrt{2}$ unit in XY plane along the line $Y = X + 4$. The magnitude of the angular momentum about origin is

(*a*) $40\sqrt{2}$ unit (*b*) 7·5 unit

(*c*) zero (*d*) 60 unit

43. A table fan rotating at a speed of 2400 rpm is switched off and the resulting variation of the revolution/minute with time is shown in Fig. 7(CF).12. The total number of revolutions of the fan before it comes to rest is

FIGURE 7(CF).12



(*a*) 280 (*b*) 380

(*c*) 420 (*d*) 160

44. A wheel is subjected to uniform angular acceleration about its axis. Initially, its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1, in the next 2 sec, it rotates through an angle θ_2. The ratio of θ_2/θ_1 is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 5

45. A solid cylinder and a hollow cylinder, both of the same mass and same external diameter are released from the same height at the same time on an inclined plane. Both roll down without slipping. Which one will reach the bottom first ?

(*a*) Solid cylinder

(*b*) Both together

(*c*) One with higher density

(*d*) Hollow cylinder

46. A wheel has moment of inertia 5×10^{-3} kg m^2 and is making 20 rev s^{-1}. The torque needed to stop it in 10 s is...... $\times 10^{-2}$ N -m

(*a*) 2π (*b*) $2{\cdot}5\,\pi$

(*c*) 4π (*d*) $4{\cdot}5\,\pi$

47. A uniform rod of length l and mass m is free to rotate in a vertical plane about A, Fig. 7(CF).13. The rod initially in horizontal position is released.

ANSWERS

37. (*a*) **38.** (*d*) **39.** (*a*) **40.** (*a*) **41.** (*c*) **42.** (*d*) **43.** (*a*) **44.** (*c*) **45.** (*a*) **46.** (*a*)

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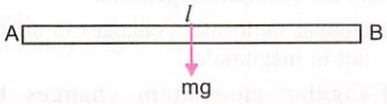
The initial angular acceleration of the rod is (*MI* of rod about *A* is $\frac{ml^2}{3}$)

FIGURE 7(CF).13



(*a*) $mgl/2$ (*b*) $3\,g/2\,l$

(*c*) $2\,l/3\,g$ (*d*) $3\,g/2\,l^2$

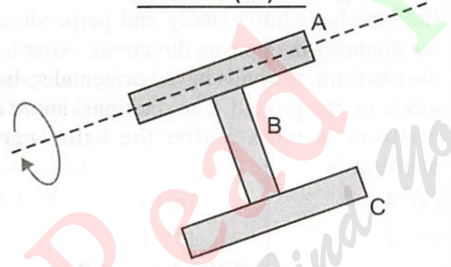
48. A rod of length l whose lower end is fixed along the horizontal plane starts from the vertical position. The velocity of the upper end of the rod when it hits the ground is

(*a*) $\sqrt{3\,gl}$ (*b*) $\sqrt{2\,gl}$

(*c*) $2\sqrt{gl}$ (*d*) $\sqrt{gl}$

49. A rigid body is made of three identical thin rods, each of length *L*, fastened together in the form of letter *H*, Fig. 7(CF).14. The body is free to rotate about a horizontal axis that runs along the length of one of legs of *H*. The body is allowed to fall from rest from a position in which plane of *H* is horizontal. The angular speed of body when plane of *H* is vertical is

FIGURE 7(CF).14

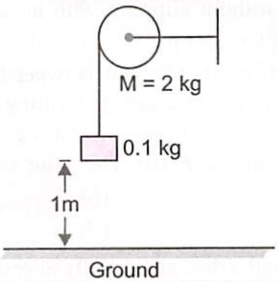


(*a*) $\sqrt{g/L}$ (*b*) $\frac{1}{2}\sqrt{g/L}$

(*c*) $2\sqrt{g/L}$ (*d*) $\frac{3}{2}\sqrt{g/L}$

50. A body of mass 0·1 kg is suspended at a height of 1 m above the ground by a weightless string which passes over a frictionless pulley, Fig. 7(CF).15. The velocity with which the body strikes the ground is

FIGURE 7(CF).15



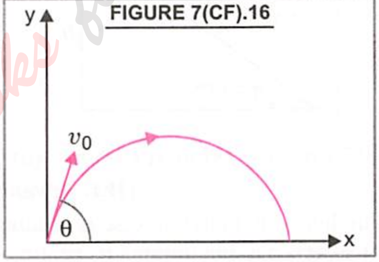
(*a*) $\frac{11}{20}\,\text{ms}^{-1}$ (*b*) $\frac{20}{11}$ m/s

(*c*) $\frac{10}{11}\,\text{ms}^{-1}$ (*d*) $\frac{11}{10}\,\text{ms}^{-1}$

(Meghalaya 2007)

51. A small particle of mass *m* is projected at an angle θ with the *x*-axis with an initial velocity v_0 in the *x-y* plane as shown in Fig. 7(CF).16. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is

FIGURE 7(CF).16



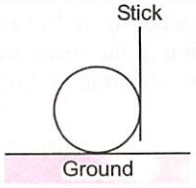
(*a*) $mg\,v_0\,t\cos\theta\;\hat{k}$ (*b*) $-\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{k}$

(*c*) $\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{i}$ (*d*) $-\,mg\,v_0\,t^2\cos\theta\;\hat{j}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along *x*, *y* and *z*-axis respectively. **(AIEEE 2010)**

52. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the Fig. 7(CF).17.

FIGURE 7(CF).17



ANSWERS

47. (*b*) **48.** (*a*) **49.** (*d*) **50.** (*b*) **51.** (*b*)

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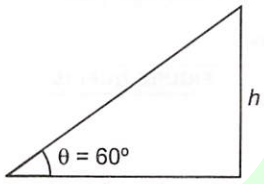
The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2.

The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6 **(IIT 2011)**

53. A ring and a disc are initially at rest, side by side, at the top of an inclined plane, which makes an angle of 60° with the horizontal, Fig. 7(CF).18. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $\frac{(2-\sqrt{3})}{\sqrt{10}}$ sec., then the height of the top of the inclined plane in metres is...... . Take $g = 10$ m/s^2.

FIGURE 7(CF).18



(*a*) 0·75 m (*b*) 1·0 m (*c*) 1·5 m (*d*) 2·0 m
(JEE Advanced 2018)

54. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

(*a*) continuously decreases
(*b*) continuously increases
(*c*) first increases and then decreases
(*d*) remains unchanged **(AIEEE 2011)**

55. A solid cylinder of mass 50 kg and radius 0·5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other end hanging freely. Tension in the string required to produce an angular acceleration of 2 revolution s^{-2} is

(*a*) 25 N (*b*) 50 N
(*c*) 78·5 N (*d*) 157 N
(AIPMT 2014)

56. A bob of mass *m* attached to an inextensible string of length *l* is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω radian/sec about the vertical. About the point of suspension,

(*a*) angular momentum changes in direction but not in magnitude,
(*b*) angular momentum changes both in magnitude and direction,
(*c*) angular momentum is conserved,
(*d*) angular momentum changes in magnitude, but not in direction. **(JEE Main 2014)**

57. A uniform circular disc of mass 1·5 kg and radius 0·5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0{\cdot}5$ N are applied simultaneously along the three sides of an equilateral triangle *xyz* with its vertices on the perimeter of the disc. One second after applying the forces, the angular speed of the disc in rad/s is :

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
(JEE Advanced Paper 1, 2014)

58. A horizontal circular platform of radius 0·5 m and mass 0·45 kg is free to rotate about its axis. Two massless spring toy guns, each carrying a steel ball of mass 0·05 kg are attached to the platform at a distance 0·25 m from the centre on its either side along its diameter. Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 m/s w.r.t. the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1
(JEE Advanced Paper 1, 2014)

59. A thin uniform rod, pivoted at *O*, is rotating in the horizontal plane with constant angular speed ω, as shown in the Fig.7(CF).19. At time $t = 0$, a small insect starts from *O* and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) on the system about *O*, as a function of time is best represented by which plot in Fig. 7(CF).20 ?

ANSWERS

52. (*b*) **53.** (*a*) **54.** (*c*) **55.** (*d*) **56.** (*a*) **57.** (*b*) **58.** (*a*)

FIGURE 7(CF).19

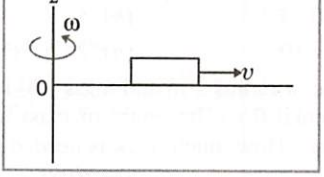
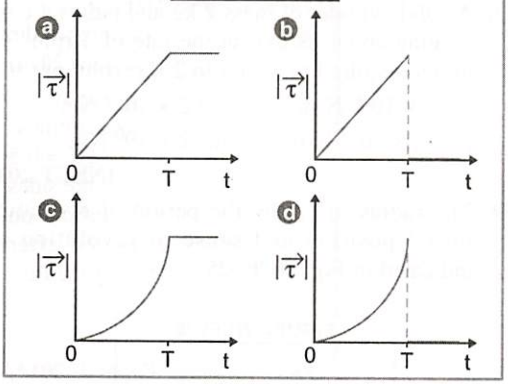


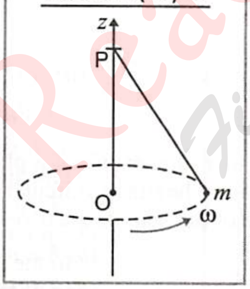
FIGURE 7(CF).20



(IIT 2012)

60. A small mass m is attached to a massless string whose other end is fixed at P as shown in Fig.7(CF).21. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P and denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then

FIGURE 7(CF).21



(*a*) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(*b*) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(*c*) $\vec{L}_O$ remains constant, while $\vec{L}_P$ varies with time

(*d*) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

(IIT 2012)

61. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in Fig.7(CF).22. The relative speed between the two points P and Q is v_r. As a function of time, it is best represented by which plot in Fig.7(CF).23?

FIGURE 7(CF).22

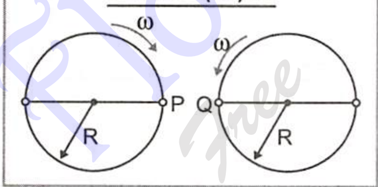
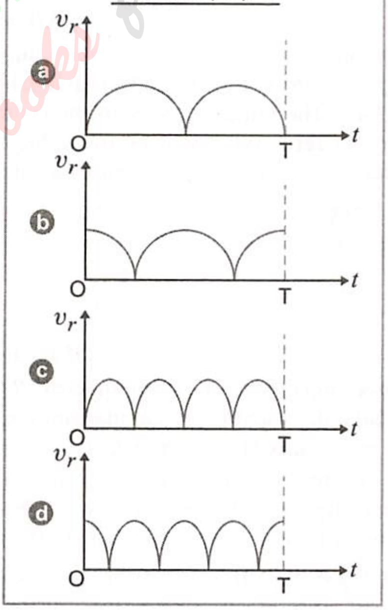


FIGURE 7(CF).23



(IIT 2012)

62. A circular platform is mounted on a frictionless vertical axle. Its radius $R = 2$ m and its moment of inertia about the axle is 200 kg m^2. It is initially at rest. A 50 kg man stands on the edge of the

ANSWERS

59. (*b*) **60.** (*c*) **61.** (*a*) platform and begins to walk along the edge at the speed of 1 ms^{-1} relative to the ground. Time taken by the man to complete one revolution is :

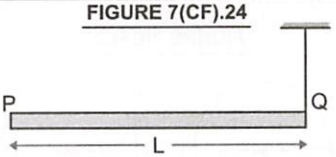
(*a*) π sec. (*b*) $\frac{3\pi}{2}$ sec.

(*c*) 2 π sec. (*d*) $\frac{\pi}{2}$ sec.

[AIPMT (Main) 2012]

63. A rod *PQ* of mass *M* and length *L* is hinged at end *P*. The rod is kept horizontal by a massless string tied to point *Q* as shown in Fig. 7(CF).24. When string is cut, the initial angular acceleration of the rod is :

FIGURE 7(CF).24



(*a*) $\frac{2g}{3L}$ (*b*) $\frac{3g}{2L}$

(*c*) *g/L* (*d*) 2 *g/L*

(AIPMT 2013)

64. A hoop of radius *r* and mass *m* rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ?

(*a*) $\frac{r\omega_0}{4}$ (*b*) $\frac{r\omega_0}{3}$

(*c*) $\frac{r\omega_0}{2}$ (*d*) $r\omega_0$

(JEE Main 2013)

65. Three objects, *A* : (a solid sphere), *B* : (a thin circular disc) and *C* : (a circular ring), each have the same mass *M* and radius *R*. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (*W*) required to bring them to rest, would satisfy the relation

(*a*) $W_B > W_A > W_C$ (*b*) $W_A > W_B > W_C$

(*c*) $W_C > W_B > W_A$ (*d*) $W_A > W_C > W_B$

(NEET 2018)

66. A solid sphere is in rolling motion. In rolling motion, a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is

(*a*) 10 : 7 (*b*) 5 : 7

(*c*) 7 : 10 (*d*) 2 : 5 **(NEET 2018)**

67. A disc of radius 2 m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s. How much work is needed to stop it ?

(*a*) 3 J (*b*) 30 kJ

(*c*) 2 J (*d*) 1 J **(NEET 2019)**

68. A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop it in 2 π revolutions is

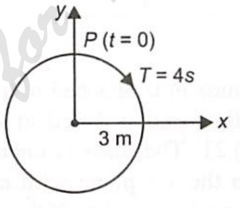
(*a*) 2×10^{-6} N-m (*b*) 2×10^{-3} Nm

(*c*) 12×10^{-4} N-m (*d*) 2×10^{6} Nm

(NEET 2019)

69. The radius of circle, the period of revolution, initial position and sense of revolution are indicated in Fig. 7(CF).25.

FIGURE 7(CF).25



y-projection of the radius vector of rotating particle *P* is

(*a*) $y(t) = -3\cos 2\pi t$ (*b*) $y(t) = 4\sin\frac{\pi t}{2}$

(*c*) $y(t) = 3\cos\left(\frac{3\pi t}{2}\right)$ (*d*) $y(t) = 3\cos\left(\frac{\pi t}{2}\right)$

(NEET 2019)

70. If the angular momentum of a planet of mass *m*, moving round the sun in a circular orbit is *L*, about the centre of the sun, its areal velocity is

(*a*) *L/m* (*b*) 4 *L/m*

(*c*) *L/2 m* (*d*) 2 *L/m*

(JEE Main 2019)

71. An *L* shaped object, made of thin rods of uniform mass density is suspended with a string as shown in Fig. 7(CF).26. If *AB* = *BC*, and the angle made by *AB* with downward vertical is θ, then

ANSWERS

62. (*c*) 63. (*b*) 64. (*c*) 65. (*c*) 66. (*b*) 67. (*a*) 68. (*a*) 69. (*d*) 70. (*c*)

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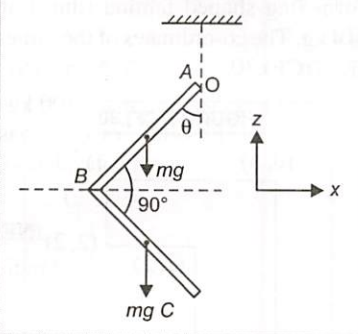
FIGURE 7(CF).26



(*a*) $\tan\theta = \frac{1}{2\sqrt{3}}$

(*b*) $\tan\theta = \frac{1}{2}$

(*c*) $\tan\theta = \frac{2}{\sqrt{3}}$

(*d*) $\tan\theta = \frac{1}{3}$ **(JEE Main 2019)**

72. Find the torque about the origin, when a force of $3\hat{j}$ N acts on a particle whose position vector is $2\hat{k}$ m

(*a*) $6\hat{j}$ N-m (*b*) $-6\hat{i}$ N-m

(*c*) $6\hat{k}$ N-m (*d*) $6\hat{i}$ N-m

(NEET 2020)

73. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle is nearly at a distance of

(*a*) 50 cm (*b*) 67 cm

(*c*) 80 cm (*d*) 33 cm **(NEET 2020)**

74. The angular speed of the wheel of a vehicle is increased from 360 rpm to 1200 rpm in 14 seconds. Its angular acceleration is

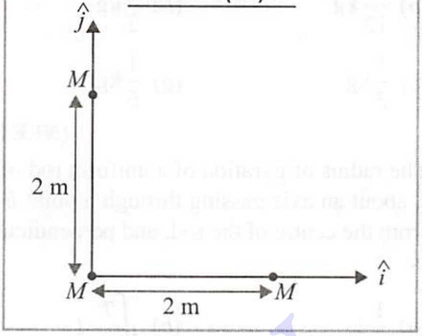
(*a*) 1 rad/sec^2 (*b*) 2 π rad/s^2

(*c*) 28 π rad/s^2 (*d*) 130 π rad/s^2

(NEET 2020)

75. Three identical spheres each of mass *M* are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 2 m. Taking the point of intersection of the two mutually perpendicular sides as the origin, find the position vector of centre of mass.

FIGURE 7(CF).27



(*a*) $\frac{4}{3}(\hat{i}+\hat{j})$ (*b*) $2(\hat{i}+\hat{j})$

(*c*) $(\hat{i}+\hat{j})$ (*d*) $\frac{2}{3}(\hat{i}+\hat{j})$

(NEET 2020)

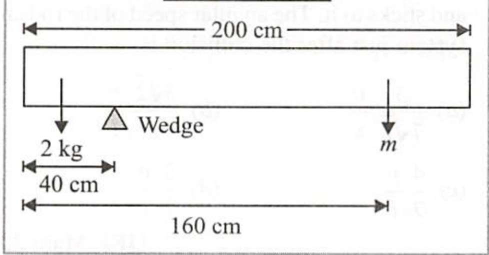
76. From a circular ring of mass *M* and radius *R*, an arc corresponding to a 90° sector is removed. The moment of inertia of remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is *K* times MR^2. The value of *K* is

(*a*) $\frac{1}{8}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{7}{8}$ (*d*) $\frac{1}{4}$ **(NEET 2021)**

77. A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass *m* is suspended from the rod at 160 cm mark as shown in Fig. 7(CF).28. Find the value of *m* such that the rod is in equilibrium ($g = 10$ m/s^2)

FIGURE 7(CF).28



ANSWERS

71. (*d*) **72.** (*b*) **73.** (*b*) **74.** (*b*) **75.** (*d*) **76.** (*b*) (*a*) $\frac{1}{12}$ kg (*b*) $\frac{1}{2}$ kg

(*c*) $\frac{1}{3}$ kg (*d*) $\frac{1}{6}$ kg

(NEET 2021)

78. The radius of gyration of a uniform rod of length l, about an axis passing through a point $l/4$ away from the centre of the rod, and perpendicular to it is

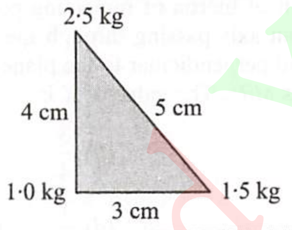
(*a*) $\frac{1}{8}l$ (*b*) $\sqrt{\frac{7}{48}}\,l$

(*c*) $\sqrt{\frac{3}{8}}\,l$ (*d*) $\frac{1}{4}l$

(JEE Main 2020)

79. Three point particles of masses 1·0 kg, 1·5 kg and 2·5 kg are placed at the three corners of a right angled triangle of sides 4 cm, 3 cm and 5 cm as shown in Fig. 7(CF).29. The centre of mass of the system is at a point

FIGURE 7(CF).29



(*a*) 1·5 cm right and 1·2 cm above 1 kg mass

(*b*) 0·9 cm right and 2·0 cm above 1 kg mass

(*c*) 0·6 cm right and 2·0 above 1 kg mass

(*d*) 2·0 cm right and 0·9 cm above 1 kg mass

80. Consider a uniform rod of mass $M = 4$ m and length l pivoted about its centre. A mass m moving with velocity v making an angle $\theta = \pi/4$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is

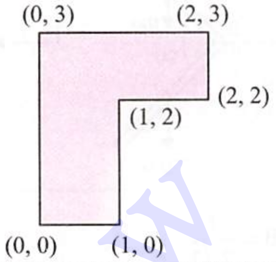
(*a*) $\frac{3}{7\sqrt{2}}\frac{v}{s}$ (*b*) $\frac{3\sqrt{2}}{7}\frac{v}{l}$

(*c*) $\frac{4}{7}\frac{v}{l}$ (*d*) $\frac{3}{7}\frac{v}{l}$

(JEE Main 2020)

81. What are the co-ordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. The co-ordinates of the same are shown in Fig. 7(CF).30.

FIGURE 7(CF).30



(*a*) 0·75 m, 1·75 m (*b*) 1 m, 1·75 m

(*c*) 1·25 m, 1·50 m (*d*) 0·75 m, 0·75 m

82. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ meter. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ meter will be $\sqrt{x}$ N-m. The value of x is

(*a*) 195 (*b*) 205

(*c*) 295 (*d*) 395

(JEE Main 2020)

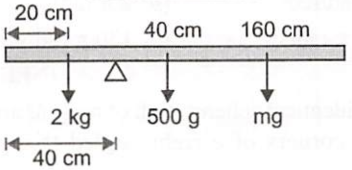
83. From a circular ring of mass M and radius R, an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is K times MR^2. Then the value of K is

(*a*) 1/8 (*b*) 3/4

(*c*) 7/8 (*d*) 1/4 **(NEET 2021)**

84. A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass 'm' is suspended from the rod at 160 cm mark as shown in Fig. 7(CF).31. Find the value of m, such that the rod is in equilibrium ($g = 10$ m/s^2)

FIGURE 7(CF).31



ANSWERS

77. (*a*) **78.** (*b*) **79.** (*a*) **80.** (*b*) **81.** (*a*) **82.** (*a*) **83.** (*b*) (*a*) $\frac{1}{12}$ kg (*b*) $\frac{1}{2}$ kg

(*c*) $\frac{1}{3}$ kg (*d*) $\frac{1}{6}$ kg **(NEET 2021)**

85. The ratio of radius of gyration of a thin uniform disc about an axis passing through its centre and normal to its plane to the radius of gyration of the disc about its diameter is

(*a*) 2 : 1 (*b*) $\sqrt{2}:1$

(*c*) 4 : 1 (*d*) $1:\sqrt{2}$ **(NEET 2022)**

86. Two objects of mass 10 kg and 20 kg respectively are connected to the two ends of a rigid rod of length 10 m with negligible mass. The distance of the centre of mass of the system from 10 kg mass is

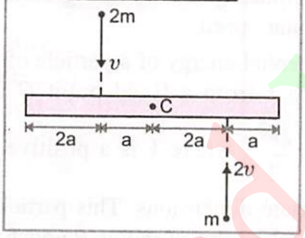
(*a*) $\frac{10}{3}$ m (*b*) $\frac{20}{3}$ m

(*c*) 10 m (*d*) 5 m **(NEET 2022)**

II. Multiple Choice Questions (with One or More than One Correct Answers)

87. A uniform bar of length 6 *a* and mass 8 *m* lies on a smooth horizontal table. Two point masses *m* and 2 *m* moving in the same horizontal plane with speeds 2 *v* and *v* respectively strike the bar [Fig. 7(CF).32] and stick to the bar after collision. Denoting angular velocity (about centre of mass), total energy and velocity of centre of mass by ω, *E* and v_0 respectively, we have after the collision

FIGURE 7(CF).32

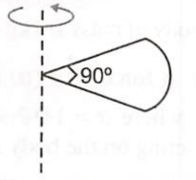


(*a*) $v_0 = 0$ (*b*) $\omega = (3\,v/5\,a)$

(*c*) $\omega = (v/5\,a)$ (*d*) $E = (3\,mv^2/5)$

88. One quarter sector is cut from a uniform circular disc of mass *M* and radius *R*, Fig. 7(CF).33. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is

FIGURE 7(CF).33



(*a*) $\frac{1}{2}$MR^2 (*b*) $\frac{1}{4}$ MR^2

(*c*) $\frac{1}{8}$MR^2 (*d*) $\sqrt{2}$ MR^2

89. A simple pendulum of length *L* and mass (bob) *M* is oscillating in a plane about a vertical line between angular limits – ϕ and ϕ. For an angular displacement θ, [|θ| < ϕ], the tension in the string and velocity of the bob are *T* and *v* respectively. The following relations hold good under the above conditions

(*a*) $T\cos\theta = Mg$

(*b*) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(*c*) Tangential acceleration = $g\sin\theta$

(*d*) $T = Mg\cos\theta$

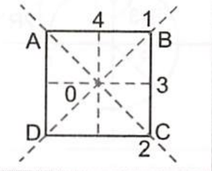
90. The moment of inertia of a thin square plate *ABCD* of uniform thickness about an axis passing through the centre *O* and perpendi- cular to the plate Fig. 7(CF).34 is

(*a*) $I_1 + I_2$ (*b*) $I_3 + I_4$

(*c*) $I_1 + I_3$ (*d*) $I_1 + I_2 + I_3 + I_4$

where I_1, I_2, I_3, I_4 are moments of inertia about axes 1, 2, 3 and 4, which are in the plane of the plate.

FIGURE 7(CF).34



ANSWERS

84. (*a*) **85.** (*b*) **86.** (*b*) **87.** (*a,c,d*) **88.** (*c*) **89.** (*b,c*) **90.** (*a,b,c*)

91. A circular disc X of radius R is made from an iron plate of thickness t, and another plate Y of radius 4 R is made from an iron plate of thickness $t/4$. The ratio between moment of inertia I_Y/I_X is

(*a*) 32 (*b*) 16

(*c*) 1 (*d*) 64

92. Two spherical bodies of mass M and 5 M and radii R and 2 R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(*a*) 2·5 R (*b*) 4·5 R

(*c*) 7·5 R (*d*) 1·5 R

93. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

(*a*) zero (*b*) $m\,v^3/(4\sqrt{2})\,g$

(*c*) $m\,v^3/(\sqrt{2})\,g$ (*d*) $m\sqrt{2\,gh^3}$

94. A non-zero external force acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be v_0 and a_0 at an instant t. It is possible that

(*a*) $v_0 = 0, a_0 = 0$ (*b*) $v_0 \neq 0, a_0 = 0$

(*c*) $v_0 = 0, a_0 \neq 0$ (*d*) $v_0 \neq 0, a_0 \neq 0$

95. A sphere is rolled on a rough horizontal surface. It gradually slows down and stops. The force of friction tries to

(*a*) increase the angular velocity

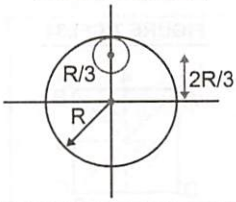
(*b*) decrease the angular velocity

(*c*) increase the linear momentum

(*d*) decrease the linear velocity

96. From a circular disc of radius R and mass 9 M, a small disc of radius $R/3$ is removed, Fig. 7(CF).35. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is

FIGURE 7(CF).35



(*a*) 4 MR^2 (*b*) $\frac{40}{9}$ MR^2

(*c*) 10 MR^2 (*d*) $\frac{37}{9}$ MR^2

97. If there is no external force acting on a non-rigid body, which of the following quantities must remain constant ?

(*a*) linear momentum (*b*) moment of inertia

(*c*) angular momentum (*d*) kinetic energy

98. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is (are) correct ?

(*a*) Both cylinders P and Q reach the ground at the same time

(*b*) Cylinder P has larger linear acceleration than cylinder Q

(*c*) Both cylinders reach the ground with same translational kinetic energy

(*d*) Cylinder Q reaches the ground with larger angular speed. **(IIT 2012)**

99. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = \frac{kr^2}{2}$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is speed of the particle and L is magnitude of its angular momentum about O, which of the following statements is (are) true ?

(*a*) $v = \sqrt{\frac{k}{2m}}\,R$ (*b*) $v = \sqrt{\frac{k}{m}}\,R$

(*c*) $L = \sqrt{mk}\,R^2$ (*d*) $L = \sqrt{\frac{mk}{2}}\,R^2$

(JEE Advanced 2018)

100. Consider a body of mass 1·0 kg at rest at the origin at time $t = 0$. A force $\vec{F} = (\alpha t\,\hat{i} + \beta\,\hat{j})$ is applied on the body where $\alpha = 1{\cdot}0$ Ns^{-1} and $\beta = 1{\cdot}0$ N. The torque acting on the body about the origin at

ANSWERS

91. (*d*) **92.** (*c*) **93.** (*b,d*) **94.** (*c,d*) **95.** (*a,d*) **96.** (*a*) **97.** (*a,c*)
98. (*d*) **99.** (*b,c*)

time $t = 1{\cdot}0$ s is $\vec{\tau}$. Which of the following statement(s) is (are) true ?

(*a*) $|\vec{\tau}| = \frac{1}{3}$ Nm

(*b*) The torque $\vec{\tau}$ is in the direction of unit vector $+\hat{k}$

(*c*) The velocity of the body at t = 1s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\,\text{ms}^{-1}$

(*d*) The magnitude of displacement of the body at t = 1s (one sec) is $\frac{1}{6}$ m

(JEE Advanced 2018)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1 **The centre of mass of a body is a point at which the entire mass of the body is supposed to be concentrated. The position vector $\vec{r}$ of c.m. of the system of two particles of masses m_1 and m_2 with position vectors $\vec{r_1}$ and $\vec{r_2}$ is given by**

$$\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

For an isolated system, where no external force is acting, $\vec{v}_{cm}$ = constant

Under no circumstances, the velocity of centre of mass of an isolated system can undergo a change

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions :

101. Two bodies of masses 1 kg and 2 kg are located at (1, 2) and (–1, 3) respectively. The co-ordinates of the centre of mass are :

(*a*) (–1, 3) (*b*) (1, 2)

(*c*) $\left(-\frac{1}{3}, \frac{8}{3}\right)$ (*d*) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

102. A bomb dropped from an aeroplane in level flight explodes in the middle. The centre of mass of the fragments

(*a*) is at rest

(*b*) moves vertically downwards

(*c*) moves vertically upwards

(*d*) continues to follow the same parabolic path which it would have followed if there was no explosion.

Comprehension 2 **Moment of inertia of a body about a given axis is the rotational inertia of the body about that axis. It is represented by $I = MK^2$, where M is mass of body and K is radius of gyration of the body about that axis. It is a scalar quantity, which is measured in kg m^2. When a body rotates about a given axis, and the axis of rotation also moves, then total K.E. of body = K.E. of translation + K.E. of rotation**

$$E = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions :

103. Moment of inertia of a body depends on

(*i*) mass of body

(*ii*) size and shape of body

(*iii*) axis of rotation of body

(*iv*) all the above

(*a*) (*i*) and (*ii*) (*b*) (*i*) and (*iii*)

(*c*) (*ii*) and (*iii*) (*d*) (*iv*)

104. Kinetic energy of rotation of the flywheel in the above case is

(*a*) 20 J (*b*) 20 J

(*c*) 395 J (*d*) 80 J

ANSWERS

100. (*a*,*c*) **101.** (*c*) **102.** (*d*) **103.** (*d*) **104.** (*c*)

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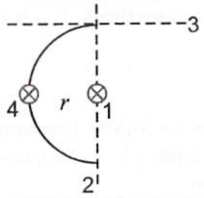
IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices :

105. A semicircular ring of mass m and radius r has moment of inertia I_1, I_2, I_3 and I_4 about the four axes 1, 2, 3, 4 respectively as shown in Fig. 7(CF).36.

FIGURE 7(CF).36



	Column I		Column II
(A)	I_1	(p)	$2\,mr^2$
(B)	I_2	(q)	$\frac{3}{2}mr^2$
(C)	I_3	(r)	mr^2
(D)	I_4	(s)	$\frac{1}{2}mr^2$

(a) A-s, B-r, C-q, D-p
(b) A-p, B-q, C-r, D-s
(c) A-p, B-r, C-q, D-s
(d) A-r, B-s, C-q, D-p

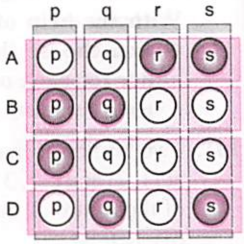
106. The following bodies are rolling without slipping with a velocity v. Each has a mass m. Their total K.E. (of translation and rotation) is

	Column I		Column II
(A)	Hollow cylinder	(p)	$\frac{7}{10}mv^2$
(B)	Solid cylinder	(q)	$\frac{5}{6}mv^2$
(C)	Hollow sphere	(r)	mv^2
(D)	Solid sphere	(s)	$\frac{3}{4}mv^2$

(a) A-r, B-s, C-q, D-p
(b) A-p, B-q, C-r, D-s
(c) A-s, B-r, C-q, D-p
(d) A-p, B-r, C-q, D-s

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*r*, *s*, B-*p*, *q*, C-*p* ; D-*q*, *s* then the correctly bubbled matrix will look like the one shown here :



	p	q	r	s
A	○	○	●	●
B	●	●	○	○
C	●	○	○	○
D	○	●	○	●

107.

	Column I		Column II
(A)	Moment of inertia	(p)	mass
(B)	Radius of gyration	(q)	size and shape
(C)	Angular momentum	(r)	angular velocity
(D)	Kinetic energy of rotation	(s)	position of axis of rotation

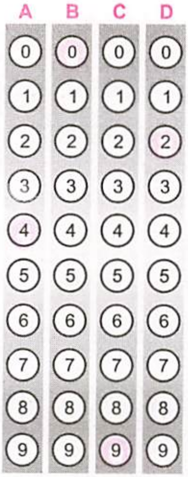
108.

	Column I		Column II
(A)	Angular impulse	(p)	$\vec{r} \cdot \vec{F}$
(B)	Torque	(q)	$\vec{r} \times \vec{p}$
(C)	Angular momentum	(r)	$I\alpha$
(D)	Work done	(s)	$I\omega$

ANSWERS

105. (*d*) **106.** (*a*) **107.** A-*p*, *q*, *s* ; B-*q*, *s* ; C-*p*, *q*, *r*, *s* ; D-*p*, *q*, *r*, *s* **108.** A-*q*, *s* ; B-*r* ; C-*q*, *s* ; D-*p*

VI. Integer Type Questions



DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

109. Two homogeneous spheres A and B of masses m and $2\,m$ having radii $2\,a$ and a respectively are placed in contact. The ratio of distance of $c.m$ from first sphere to the distance of $c.m$ from second sphere is :

110. A non-uniform thin rod of length L is placed along X-axis so that one of its ends is at the origin. The linear mass density of rod is $\lambda = \lambda_0\, x$. The centre of mass of rod divides the length of the rod in the ratio :

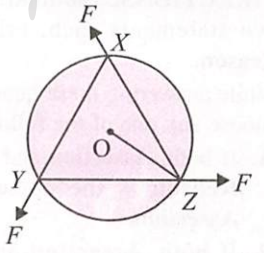
111. A particle performing uniform circular motion has angular momentum L. When its angular velocity is doubled and KE is also doubled, the new angular momentum becomes x times. What is x ?

112. If K_1 is radius of gyration of a circular disc about a tangent perpendicular to plane of disc and K_2 is radius of gyration of a circular ring of same size as disc about any diameter, what is the value of K_1^2 / K_2^2 ?

113. An annular ring with inner and outer radii $R_1 = 1{\cdot}2$ cm and $R_2 = 4{\cdot}8$ cm is rolling without slipping with a uniform angular speed. What is the ratio of the forces experienced by the two particles of same mass situated on the outer and inner parts of the annular ring ?

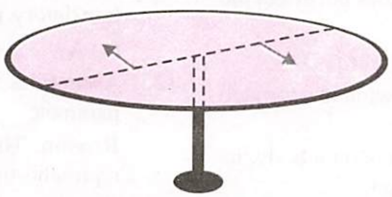
114. A uniform circular disc of mass 1·5 kg and radius 0·5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0{\cdot}5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc, Fig. 7(CF).37. One second after applying the forces, the angular speed of the disc in rad s^{-1} is **(JEE Advanced 2014)**

FIGURE 7(CF).37



115. A horizontal circular platform of radius 0·5 m and mass 0·45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0·05 kg are attached to the platform at a distance 0·25 m from the centre on its either sides along its diameter, Fig. 7(CF).38. Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **(JEE Advanced 2014)**

FIGURE 7(CF).38



116. Two identical uniform discs roll without slipping on two different surfaces AB and CD, Fig. 7(CF).39, starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10$ m/s^2)

ANSWERS

109. (2) **110.** (2/1) **111.** (1) **112.** (3) **113.** (4) **114.** (2) **115.** (4)
116. (7)

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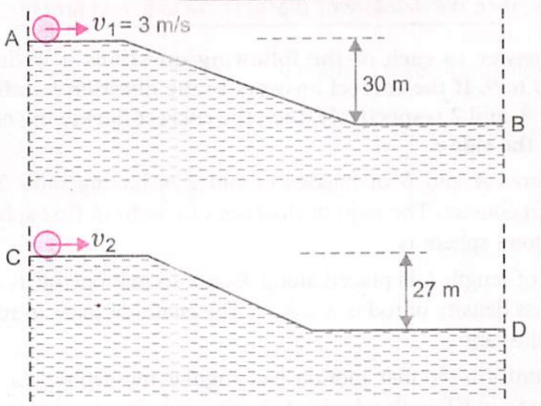
FIGURE 7(CF).39



(JEE Advanced 2015)

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. The following questions consist of two statements each, printed as **Assertion** and **Reason.**

While answering these questions you are required to choose any one of the following four responses.

A. If both, **Assertion** and **Reason** are true and the **Reason** is the correct explanation of the **Assertion.**

B. If both, **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

C. If **Assertion** is true but the **Reason** is false.

D. If both, **Assertion** and **Reason** are false.

117. Assertion. In rolling, all points of a rigid body have the same linear velocity.

Reason. The rotational motion does not affect the linear velocity.

(*a*) A (*b*) B (*c*) C (*d*) D

118. Assertion. The speed of a whirl wind in a tornado is alarmingly high.

Reason. If no external torque acts on a body, its angular velocity remains constant.

(*a*) A (*b*) B (*c*) C (*d*) D

119. Assertion. The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared to when it is rolling down the same plane.

Reason. In rolling down, a body acquires both kinetic energy of translation and rotation.

(*a*) A (*b*) B (*c*) C (*d*) D

120. Assertion. The moment of inertia of a rigid body reduces to its minimum value, when the axis of rotation passes through its centre of gravity.

Reason. The weight of a rigid body always acts through its centre of gravity.

(*a*) A (*b*) B (*c*) C (*d*) D

121. Assertion. At the centre of earth, a body has centre of mass, but no centre of gravity.

Reason. This is because $g = 0$ at the centre of earth.

(*a*) A (*b*) B (*c*) C (*d*) D

122. Assertion. Power associated with torque is product of torque and angular speed of the body about the axis of rotation.

Reason. It is rotational analogue of power in translatory motion.

(*a*) A (*b*) B (*c*) C (*d*) D

123. Assertion. Torque is time rate of change of a parameter, called angular momentum.

Reason. This is because in linear motion, force represents time rate of change of linear momentum.

(*a*) A (*b*) B (*c*) C (*d*) D

124. Assertion. There are two propellers in a helicopter.

Reason. Angular momentum is conserved.

(*a*) A (*b*) B (*c*) C (*d*) D

ANSWERS

117. (*d*) **118.** (*c*) **119.** (*b*) **120.** (*a*) **121.** (*a*) **122.** (*a*) **123.** (*a*) **124.** (*b*)

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125. **Assertion.** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

Reason. The linear momentum of an isolated system remains constant.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and check if

(A) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation of statement-1.

(B) Statement-1 is true ; statement-2 is true but statement-2 is not a correct explanation of statement-1.

(C) Statement-1 is true, but statement-2 is false.

(D) Statement-1 is false, but statement-2 is true.

126. **Statement-1.** In the formation of a neutron star, spin angular velocity increases tremendously.

Statement-2. Conservation of rotational kinetic energy

(*a*) A (*b*) B (*c*) C (*d*) D

127. **Statement-1.** A wheel moving down a perfectly frictionless inclined plane shall undergo slipping (not rolling).

Statement-2. For rolling, torque is required, which is provided by tangential frictional force.

(*a*) A (*b*) B (*c*) C (*d*) D

128. **Statement-1.** The centre of mass of a body may lie where there is no mass.

Statement-2. The centre of mass has nothing to do with the mass.

(*a*) A (*b*) B (*c*) C (*d*) D

129. **Statement-1.** Moment of inertia of a body is same, whatever be the axis of rotation.

Statement-2. Moment of inertia depends on mass and size and axis of rotation of the body.

(*a*) A (*b*) B (*c*) C (*d*) D

130. **Statement-1.** When ice on polar caps of earth melts, duration of the day increases.

Statement-2. $L = I\omega = I\left(\frac{2\pi}{T}\right)$ = constant.

(*a*) A (*b*) B (*c*) C (*d*) D

131. **Statement-1.** A hollow cylinder of diameter 0·5 m has a mass of 10 kg. Its moment of inertia about its axis of symmetry is 0·625 kg m^2.

Statement-2. $I = MR^2$

(*a*) A (*b*) B (*c*) C (*d*) D

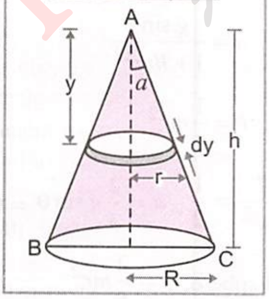
HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Let distance of c.m. from the vertex, $Z_0 = y_{cm}$

FIGURE 7(CF).40



Let density of cone = ρ

$$Z_0 = y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$= \frac{\int_0^h y\pi r^2\,dy\rho}{\frac{1}{3}\pi R^2 h\rho} = \frac{\int_0^h r^2\,y\,dy}{\frac{1}{3}R^2 h} \quad \text{...(i)}$$

As $\frac{r}{R} = \frac{y}{h}$, $\therefore\ r = \frac{y}{h}R$, putting value of y in (*i*)

ANSWERS

125. (*d*) 126. (*c*) 127. (*a*) 128. (*c*) 129. (*d*) 130. (*a*) 131. (*a*)

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$$Z_0 = y_{cm} = \frac{\int_0^h 3\,y^3\,dy}{h^3} = \frac{3\left[\frac{y^4}{4}\right]_0^h}{h^3} = \mathbf{\frac{3}{4}\,h}$$

2. In Fig. 7(CF).41, O is the centre of circular disc of radius 2 R and mass M. C_1 is centre of disc of radius R, which is removed. If ρ is mass per unit area of disc, then $M = \pi\,(2\,R)^2\,\rho$

Mass of disc removed, $M_1 = \pi\,(R^2)\,\rho = \frac{1}{4}M$

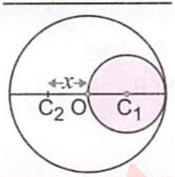
Mass of remaining disc, $M_2 = M - M_1$

$$= M - \frac{1}{4}M = \frac{3}{4}M$$

Let centre of mass of remaining disc be at C_2 where $OC_2 = x$

As $M_1 \times OC_1 = M_2 \times OC_2$

FIGURE 7(CF).41



$$\therefore \quad \frac{M}{4} \times R = \frac{3\text{M}}{4}\,x$$

$$x = \frac{R}{3} = \alpha R \quad \therefore \quad \alpha = \mathbf{\frac{1}{3}}$$

3. Linear density ρ (= mass/length) $= k\left(\frac{x}{L}\right)^n$

When $n = 0$, ρ = constant. The *c.m* will be at the centre of the rod, *i.e.*, at $x = L/2$.

As n increases to 1, 2, 3..., linear density goes on increasing. The *c.m* shifts beyond $x = L/2$, towards $x = L$. For sufficiently large values of n, the *c.m* tends to be at the other end $x = L$ of the rod.

Graph (*a*) in Fig. 7(CF).1 is the best approximation of dependence of x_{cm} on n.

4. Initial moment of inertia of the complete disc about an axis passing through its centre and perpendicular to its plane is

$$I = \frac{1}{2}\ \text{mass} \times (\text{radius})^2 = \frac{1}{2} \times 9M \times R^2 = \frac{9}{2}MR^2$$

Moment of inertia of disc removed, about an axis passing through its centre and perpendicular to its plane is $I' = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{MR^2}{18}$

Moment of inertia of the remaining portion of the disc $= I - I'$

$$= \frac{9}{2}MR^2 - \frac{MR^2}{18} = \frac{80}{18}MR^2 = \mathbf{\frac{40MR^2}{9}}$$

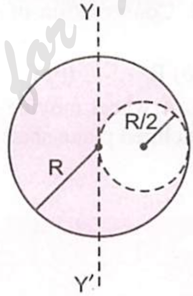
5. As is clear from Fig., 7(CF).42

$$I_{\text{disc}} = \frac{1}{2}MR^2$$

$$I_{\text{hole}} = \frac{1}{2}\left(\frac{M}{4}\right)(R/2)^2 + \frac{M}{4}(R/2)^2$$

$$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3\,MR^2}{32}$$

FIGURE 7(CF).42



$$\therefore \quad I_{\text{remaining}} = I_{\text{disc}} - I_{\text{hole}}$$

$$= \frac{1}{2}MR^2 - \frac{3}{32}MR^2 = \mathbf{\frac{13}{32}MR^2}$$

6. The acceleration of an object rolling down an inclined plane is given by

$$a = \frac{g\sin\theta}{1 + I/mr^2}$$

For disc, $I = \frac{1}{2}mr^2$

$$\therefore \quad \frac{I}{mr^2} = \frac{1}{2},\ a = \frac{2}{3}g\sin\theta = 0{\cdot}67\,g\sin\theta$$

For solid sphere, $I = \frac{2}{5}mr^2$

$$\therefore \quad \frac{I}{mr^2} = \frac{2}{5},\ a = \frac{5}{7}g\sin\theta = 0{\cdot}71\,g\sin\theta$$

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Clearly, $a_{\text{solid sphere}} > a_{\text{disc}} \;\therefore\; t_{\text{solid sphere}} < t_{\text{disc}}$
Hence **solid sphere gets to the bottom of the plane first.**

7. The statement (1) and (4) are true.
Therefore, **option (*a*) is correct.**

8. Here, moment of inertia of the original uniform circular disc about an axis passing through centre of disc and perpendicular to the plane of disc is

$$I_1 = \frac{(9\,\text{M})\,R^2}{2}$$

Mass of small disc removed (of radius $R/3$)

$$m = \frac{9\,\text{M}}{9} = M$$

Moment of inertia of the disc removed about the central axis.

$$I_2 = \frac{M \times (R/3)^2}{2} + M \times \left(\frac{2\text{R}}{3}\right)^2$$

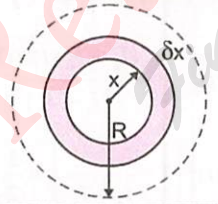
$$= \frac{1}{2}\,\text{MR}^2$$

$$\therefore I_{\text{required}} = I_1 - I_2$$

$$= \frac{9}{2}\,\text{MR}^2 - \frac{1}{2}\,\text{MR}^2 = \mathbf{4\,MR^2}$$

9. Let density of sphere of radius x be $\rho\,(x)$
Here $\delta V = 4\,\pi\,x^2\,.\,\delta x$, Fig. 7(CF).43.

FIGURE 7(CF).43



$$\therefore \quad \delta m = 4\,\pi\,x^2\,.\,\delta x\,.\,\rho\,(x)$$

Moment of Inertia, $\delta I = \dfrac{2}{3}\,(\delta m)\,x^2$

$$= \frac{2}{3} \times 4\pi x^2\,\delta x\,\rho(x).x^2$$

or $\quad I = \int \delta I = \int_0^R \dfrac{2}{3} 4\pi x^4\,\delta x.\rho(x)$

or we can say that $I \propto \int_0^R x^4\,\delta x.\rho(x)$

Using $\rho_A\,(r) = K\left(\dfrac{r}{R}\right)$ and

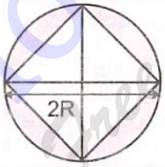
$\rho_B\,(r) = K\,(r/R)^5$ and solving,

we get, $I_B/I_A = \dfrac{6}{10} \quad \therefore\; \boldsymbol{n = 6}$

10. Let ρ be the density of sphere diagonal of cube

$$d = 2\,R = a\sqrt{3} \quad \therefore \quad a = \frac{2}{\sqrt{3}}\,R$$

FIGURE 7(CF).44



Mass of sphere $(M) = \dfrac{4}{3}\pi R^3\,\rho$

Mass of cube $(M') = \left(\dfrac{2}{\sqrt{3}}R\right)^3 \rho$

$$\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3\,\rho}{\left(\frac{2}{\sqrt{3}}R\right)^3 \rho} = \frac{\sqrt{3}}{2}\pi$$

or $\quad M' = \dfrac{2}{\sqrt{3}}\,\dfrac{M}{\pi},$

$\therefore$ Moment of Inertia $I = \dfrac{M'a^2}{6}$

$$= \frac{2M}{\sqrt{3}\,\pi} \times \frac{4}{3}R^2 \times \frac{1}{6} \;=\; \mathbf{\frac{4MR^2}{9\sqrt{3}\,\pi}}$$

11. $\quad I_1 = \dfrac{M\,l^2}{12}$

If r is radius of ring, then
$l = 2\,\pi\,r,\; r = l/2\,\pi$

$$I_2 = M\,r^2 = M\,(l/2\pi)^2 = \frac{M\,l^2}{4\pi^2}$$

$$\frac{I_1}{I_2} = \frac{M\,l^2}{12}.\frac{4\pi^2}{M\,l^2} = \mathbf{\frac{\pi^2}{3}}$$

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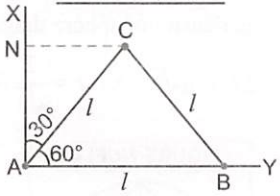
12. From Fig. 7(CF).45,

$$\sin 30^\circ = \frac{CN}{CA} = \frac{CN}{l}$$

$$CN = l \sin 30^\circ = \frac{l}{2}$$

FIGURE 7(CF).45



$$I = m_A (0)^2 + m_B (l)^2 + m_C (CN)^2$$

$$= 0 + ml^2 + m (l/2)^2 = \frac{5}{4} ml^2$$

13. Moment of inertia of a circular disc about a tangential axis in the plane of disc

$$I_1 = MK_1^2 = \frac{5}{4} MR^2 , \quad K_1 = \sqrt{\frac{5}{4}}\, R$$

Moment of inertia of a circular ring of same radius about a tangential axis in the plane of the ring is

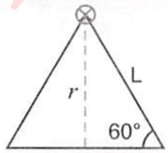
$$I_2 = MK_2^2 = \frac{3}{2} MR^2$$

$$K_2 = \sqrt{\frac{3}{2}}\, R$$

$$\therefore \quad \frac{K_1}{K_2} = \frac{\sqrt{5/4}\, R}{\sqrt{3/2}\, R} = \sqrt{\frac{5}{4} \times \frac{2}{3}} = \sqrt{\frac{5}{6}} = \sqrt{5} : \sqrt{6}$$

14. As is clear from Fig. 7(CF).46,

FIGURE 7(CF).46



$$\sin 60^\circ = \frac{r}{L} \quad \text{or} \quad r = L \sin 60^\circ = \frac{L\sqrt{3}}{2}$$

$$I_x = I_1 + I_2 + I_3$$

$$= \frac{mL^2}{3} + \frac{mL^2}{3} + \left(\frac{mL^2}{12} + mr^2 \right)$$

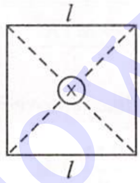
$$MK^2 = \frac{2mL^2}{3} + \frac{mL^2}{12} + m \left(\frac{L\sqrt{3}}{2} \right)^2 = \frac{3}{2} mL^2$$

$$(3m) K^2 = \frac{3}{2} mL^2 , \quad K = \frac{L}{\sqrt{2}}$$

15. Moment of inertia of each rod about an axis through its centre and perpendicular to its length

$$= \frac{M l^2}{12}$$

FIGURE 7(CF).47



Using theorem of parallel axes, Fig. 7(CF).47, M.I of each rod about the given axis

$$= \frac{M l^2}{12} + M \left(\frac{l}{2} \right)^2 = \frac{M l^2}{3}$$

M.I of the square frame about the given axis

$$= \frac{4 M l^2}{3}$$

16. As no external torque is acting, therefore,

$$I_t (i)_i = (I_t + I_b) (i)_f$$

$$(i)_f = \frac{I_t (i)_i}{I_t + I_b}$$

Energy lost $= \Delta K = E_1 - E_2$

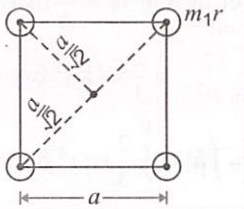
$$\Delta K = \frac{1}{2} I_t (i)_i^2 - \frac{1}{2} (I_t + I_b)(i)_f^2$$

$$= \frac{1}{2} I_t (i)_i^2 - \frac{1}{2} \frac{(I_t + I_b) I_t^2 (i)_i^2}{(I_t + I_b)^2}$$

$$= \frac{1}{2} I_t (i)_i^2 \left[1 - \frac{I_t}{I_t + I_b} \right] = \frac{1}{2} I_t (i)_i^2 \frac{I_b}{(I_t + I_b)}$$

17. Refer to Fig. 7(CF).48.

FIGURE 7(CF).48



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Here, $m = 0{\cdot}5$ kg, $r = \frac{\sqrt{5}}{2}$ cm $= \frac{\sqrt{5}}{2} \times 10^{-2}$ m;

$a = 4$ cm $= 4 \times 10^{-2}$ m

Moment of inertia of the system about the diagonal of the square is

$$I = 2\left[\frac{2}{5}\, mr^2\right] + 2\left[\frac{2}{5} mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right]$$

$$= \frac{8}{5}\, mr^2 + ma^2$$

$$= \frac{8}{5} \times 0{\cdot}5 \times \left(\frac{\sqrt{5}}{2} \times 10^{-2}\right)^2 + 0{\cdot}5 \times (4 \times 10^{-2})^2$$

$= 9 \times 10^{-4}$ kg m^2

As per question, $N \times 10^{-4} = 9 \times 10^{-4}$ or $N = 9$

18. Torque $= r \times F = I\,\alpha$

$\therefore \quad 2\,(20\,t - 5\,t^2) = 10\,\alpha$ or $\alpha = 4\,t - t^2$

or $\quad \frac{d\omega}{dt} = 4\,t - t^2$

or $\quad d\omega = (4\,t - t^2)\, dt$

On integration, we get $\omega = 2t^2 - \frac{t^3}{3}$ ...(*i*)

ω will be zero at $t = 6$ s

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

$$d\theta = \left(2t^2 - \frac{t^3}{3}\right) dt$$

On integration, we get $\theta = \frac{2t^3}{3} - \frac{t^4}{12}$

When $t = 6$ s, $\theta = \frac{2 \times 6^3}{3} - \frac{6^4}{12} = 36$

No. of turns, $n = \frac{\theta}{2\pi} = \frac{36}{2\pi} < 6$

Thus, **option (*a*) is correct.**

19. Moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is

$$I = \frac{ml^2}{12} + \frac{mR^2}{4} = \frac{m}{4}\left(\frac{l^2}{3} + R^2\right) \qquad \text{...}(i)$$

and $\quad m = \pi R^2\, l\,.\,\rho$

$\therefore \quad R^2 = \frac{m}{\pi\, l\, \rho}$

From (*i*), $I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\pi\, l\, \rho}\right)$

I will be minimum, when

$$\frac{dI}{dl} = 0$$

$$\frac{d}{dl}\left[\frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\pi\, l\, \rho}\right)\right] = 0$$

$$\frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\pi\, l^2\, \rho}\right) = 0$$

or $\quad \frac{2l}{3} = \frac{m}{\pi\, l^2\, \rho} = \frac{\pi\, R^2\, l\, \rho}{\pi\, l^2\, \rho} = \frac{R^2}{l}$

or $\quad \frac{l^2}{R^2} = \frac{3}{2}$; $\frac{l}{R} = \sqrt{\frac{3}{2}}$

20. Here $\theta = 36 \times 2\,\pi$, $\omega = \frac{\omega_0}{2}$

As $\omega^2 = \omega_0^2 = 2\,\alpha\,\theta$,

$$\left(\frac{\omega_0}{2}\right)^2 - \omega_0^2 = 2\,\alpha\,(36 \times 2\,\pi)$$

$$-\frac{3\,\omega_0^2}{4} = 144\,\pi\,\alpha \quad \text{or} \quad \alpha = -\frac{\omega_0^2}{192\,\pi}$$

Again, from $2\,\alpha\,\theta = \omega^2 - \omega_0^2$,

As finally fan will stop, So $\omega = 0$

and $\omega_0/2$ = Initial angular velocity

$$2\left(-\frac{\omega_0^2}{192\,\pi}\,\theta\right) = 0 - \left(\frac{\omega_0^2}{2}\right), \quad \boldsymbol{\theta = 24\,\pi}$$

Let no. be rotations be n

$\therefore$ $24\,\pi = 2\,\pi\, n$ or $\boldsymbol{n = 12}$

21. Here, $s = t^3 + 5$, $v = \frac{ds}{dt} = 3t^2$, $r = 20$ m

Tangential acceleration,

$$a_T = \frac{dv}{dt} = \frac{d}{dt}\,(3t^2) = 6t$$

at $\quad t = 2$ sec, $a_T = 6 \times 2 = 12$ m/s^2

and $v = 3\,(2)^2 = 12$ m/s

Centripetal acceleration

$$a_c = \frac{v^2}{r} = \frac{(12)^2}{20} = \frac{144}{20} \text{ m/s}^2$$

Total acceleration,

$$a = \sqrt{a_T^2 + a_C^2} = \sqrt{(12)^2 + \left(\frac{144}{20}\right)^2} \approx \mathbf{14\ m/s^2}$$

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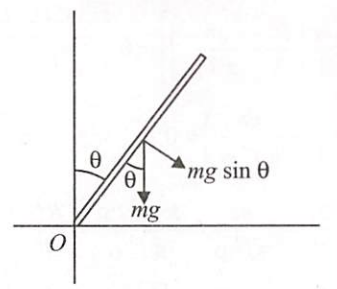
22. As is clear from Fig. 7(CF).49,
torque about pivot = 0, $\tau = I\,\alpha$

FIGURE 7(CF).49



$$(mg\sin\theta)\,\frac{l}{2} = \frac{ml^2}{3}\,\alpha$$

$$\alpha = \mathbf{\frac{3g}{2l}\sin\theta}$$

23. As shown in Fig. 7(CF).6, moment of inertia of the seven discs about an axis passing through O and perpendicular to the plane of discs,

$$I_O = \frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M\,(2\mathrm{R})^2\right]$$

$$= \frac{MR^2}{2} + 27\,\mathrm{MR}^2 = \frac{55}{2}\,\mathrm{MR}^2$$

Applying theorem of parallel axis, moment of inertia of the given system about an axis passing through P and perpendicular to the plane of discs is

$$I_P = I_O + 7\,\mathrm{M}\,(3\,\mathrm{R})^2$$

$$= \frac{55}{2}\,\mathrm{MR}^2 + 63\,\mathrm{MR}^2 = \mathbf{\frac{181}{2}\,MR^2}$$

24. $a_{\text{slipping}} = g\sin\theta$ (without rolling)

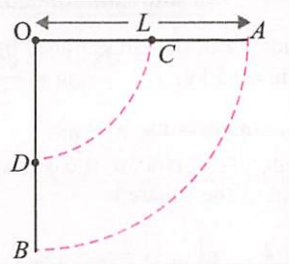
$$a_{\text{rolling}} = \frac{g\sin\theta}{1 + K^2/R^2} \quad \text{(without slipping)}$$

$$\therefore \quad \frac{a_{\text{slipping}}}{a_{\text{rolling}}} = 1 + \frac{K^2}{R^2} = 1 + \frac{2/5\,R^2}{R^2} = \frac{7}{5}$$

Hence, $a_{\text{rolling}} : a_{\text{slipping}} = \mathbf{5 : 7}$

25. Refer to Fig. 7(CF).50, when the rod goes from position OA to OB, then loss in PE of rod is equal to the gain in KE of rotation of rod.

FIGURE 7(CF).50



So $$mg\left(\frac{L}{2}\right) = \frac{1}{2}\,I\,\omega^2 = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2$$

or $$\omega = \sqrt{\frac{3g}{L}}$$

26. Let f be the frictional force between disc surface and ground surface. Let a be the linear acceleration of the disc. For the motion of the disc, we can write

$$2F - f = ma \quad \text{...(i)}$$

and $$(F + f)\,r = I\,\alpha$$

where $$I = \frac{1}{2}mr^2 \text{ and } \alpha = \frac{a}{r}$$

$$\therefore \quad (F + f)\,r = \frac{1}{2}mr^2 \times \frac{a}{r} = \frac{1}{2}mra \quad \text{...(ii)}$$

Putting value of ma from (*i*) in (*ii*), we get

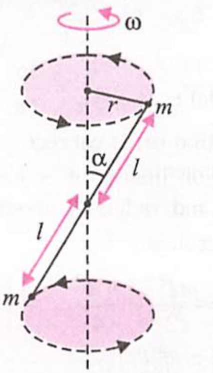
$$(F + f)\,r = \frac{1}{2}\,(2F - f)\,r$$

or $F + f = F - f/2$ or $f = 0 = n\,F$ (Given)

So $n = \mathbf{0}$

27. Refer to Fig. 7(CF).51,

FIGURE 7(CF).51



Axis is of rotation

HINTS/EXPLANATIONS

For Difficult Questions

The radius r of the circle traced by the masses is $r = l \sin \alpha$

Angular momentum $\vec{L} = \vec{r} \times m\vec{v}$

$$\text{or} \quad |\vec{L}| = r \times m\omega r = m\omega r^2$$

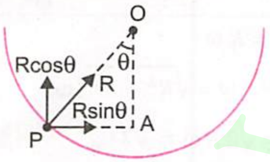
$$= m\omega (l \sin \alpha)^2 = m\omega l^2 \sin^2 \alpha$$

$$\frac{dL}{dt} = m\omega l^2 \, 2 \sin \alpha \cos \alpha \frac{d\alpha}{dt}$$

$$= m\omega l^2 (\sin 2\alpha)\, \omega = \boldsymbol{m\omega^2 l^2 \sin 2\alpha}$$

28. Let at an instant, the block kept in the bowl be at location P when bowl is rotating with angular velocity ω and block does not slip in bowl, Fig. 7(CF).52. Let R be the normal reaction of bowl on block. It is acting along PO. If m is the mass of the block, then in equilibrium position, the horizontal component of R will provide centripetal force and vertical component of R will balance weight of the block. Thus

FIGURE 7(CF).52



$$R \sin \theta = m\omega^2 (r \sin \theta) \quad \text{or} \quad R = m\omega^2 r \qquad \textbf{...(i)}$$

$$\text{and} \quad R \cos \theta = mg \quad \text{or} \quad m\omega^2 r (\cos \theta) = mg$$

[from (*i*)]

$$\text{or} \quad \omega = \sqrt{\frac{g}{r \cos \theta}}$$

29. $r = 50 \text{ cm} = 0{\cdot}5 \text{ m}, \quad \alpha = 2{\cdot}0 \text{ rad/s}^2$

$t = 2{\cdot}0 \text{ s}\ ; \ a = ?$

Angular speed, $\omega = \alpha t = 2{\cdot}0 \times 2{\cdot}0 = 4 \text{ rad/s}$

Centripetal acceleration, $a_c = \omega^2 r = 4^2 \times 0{\cdot}5 = 8 \text{ m/s}^2$

Linear/tangential acceleration at the end of 2 s

$$a_t = \alpha r = 2{\cdot}0 \times 0{\cdot}5 = 1 \text{ m/s}^2$$

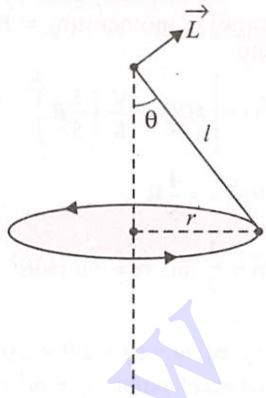
$\therefore$ Net acceleration at the end of 2 second

$$a = \sqrt{a_c^2 + a_t^2}$$

$$a = \sqrt{8^2 + 1^2} = \sqrt{65} \simeq \mathbf{8{\cdot}0\ m/s^2}$$

30. Refer to Fig. 7(CF).53, angular momentum of the bob about the point of suspension,

FIGURE 7(CF).53



$$L = I\omega = (m r^2)\,\omega = m\,(l \sin \theta)^2\, \omega$$

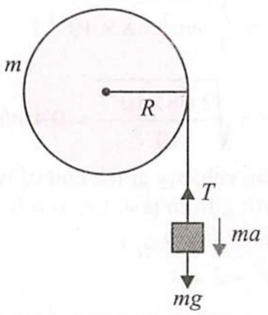
$$= (m\, l^2 \sin^2 \theta)\, \omega$$

As m, l, θ and ω are constants, therefore $\vec{L}$ is constant in magnitude. However, its direction keeps on changing being perpendicular to l, always.

31. If a is acceleration of mass on release, then as is clear from Fig. 7(CF).54,

$$ma = mg - T \qquad \textbf{...(i)}$$

FIGURE 7(CF).54



For the cylinder,

$$\text{Torque} = T \times R = I\alpha = mR^2 \cdot \left(\frac{a}{R}\right)$$

$$\therefore \quad T = ma$$

From (*i*), $ma = mg - ma$

$$2\,ma = mg \quad \text{or} \quad \boldsymbol{a = g/2}$$

32. When bead is at A, the force it applies on the wire is **radially inwards.** When it reaches at B, it strikes the surface and pushed outwards. Thus, force is **radially outwards.**

33. Finally, let the other mass be at a distance x from the centre.

Apply law of conservation of angular momentum about the given axis of rotation, we have

HINTS/EXPLANATIONS
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Initial angular momentum = Final angular momentum

$$MR^2\omega = \left[MR^2 + \frac{M}{8}\left(\frac{3}{5}R\right)^2 + \frac{M}{8}x^2\right]\times\frac{8}{9}\omega$$

On solving, $x = \frac{4}{5}\mathbf{R}$

34. $r = 50\text{ cm} = \frac{1}{2}\text{ m},\ \alpha = 2{\cdot}0\text{ rad/s}^2$

$t = 2{\cdot}0\text{ s};\ a = ?$

Angular speed, $\omega = \alpha t = 2{\cdot}0 \times 2{\cdot}0 = 4\text{ rad/s}$

Centripetal acceleration, $a_c = \omega^2 r = 4^2 \times 0{\cdot}5 = 8\text{ m/s}^2$

Linear/tangential acceleration at the end of 2 s

$$a_t = \alpha r = 2{\cdot}0 \times 0{\cdot}5 = 1\text{ m/s}^2$$

∴ Net acceleration at the end of 2 second

$$a = \sqrt{a_c^2 + a_t^2}$$

$$a = \sqrt{8^2 + 1^2} = \sqrt{65} \simeq \mathbf{8\ m/s^2}$$

35. Here, $m = 10\text{ g} = 10^{-2}\text{ kg}$

$r = 6{\cdot}4\text{ cm} = 6{\cdot}4 \times 10^{-2}\text{ m}$

$a_t = ?$

$$\text{K.E.} = \frac{1}{2}mv^2 = 8 \times 10^{-4}\text{ J}$$

$$v = \sqrt{\frac{2\times8\times10^{-4}}{10^{-2}}} = 0{\cdot}4\text{ m/s}$$

This is the velocity at the end of two revolutions after starting from rest, *i.e.*, $u = 0$, $s = 2 \times 2\pi r$

From $v^2 = u^2 + 2a_t s$

$v^2 = 2a_t s$ $(\because u = 0)$

$$a_t = \frac{v^2}{2s} = \frac{0{\cdot}4\times0{\cdot}4}{2\times2\times2\pi r}$$

$$a_t = \frac{16\times10^{-2}}{8\times3{\cdot}14\times6{\cdot}4\times10^{-2}} = \mathbf{0{\cdot}10\ m/s^2}$$

36. In option (*a*), co-ordinates of *A* are $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}\right)$

$$\therefore\ \vec{r} = \left(\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right) \text{ and } \vec{v} = v\,\hat{i}$$

$$\vec{L} = m(\vec{r}\times\vec{v}) = m\left(\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right)\times v\,\hat{i}$$

$$\vec{L} = -\frac{mR}{\sqrt{2}}v\,\hat{k}$$

∴ Option (*a*) is correct. Similarly, option (*c*) is correct. ***Statements (b) and (d) are false.***

37. Under the given conditions, it can be shown that the cone will **turn left.**

38. From the knowledge of theory,

orbital velocity of satellite, $v_0 = \sqrt{gR}$

velocity required to escape from earth,

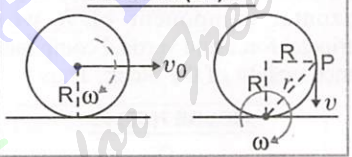
$$v_e = \sqrt{2gR}$$

Increase in orbital velocity $= v_e - v_0$

$$= \sqrt{2gR} - \sqrt{gR} = \mathbf{\sqrt{gR}\,(\sqrt{2}-1)}$$

39. In Fig. 7(CF).55,

FIGURE 7(CF).55



$$v_0 = R\,\omega$$

At *P*, $v = r\,\omega = \sqrt{R^2 + R^2}\,(\omega)$

$$= \sqrt{2}\,R\,\omega = \sqrt{2}\,v_0$$

40. The acceleration of an object rolling down an inclined plane is given by

$$a = \frac{g\sin\theta}{1 + I/mr^2}$$

For disc, $I = \frac{1}{2}mr^2$

$$\therefore\ \frac{I}{mr^2} = \frac{1}{2},\ a = \frac{2}{3}g\sin\theta = 0{\cdot}67\,g\sin\theta$$

For solid sphere, $I = \frac{2}{5}mr^2$

$$\therefore\ \frac{I}{mr^2} = \frac{2}{5},\ a = \frac{5}{7}g\sin\theta = 0{\cdot}71\,g\sin\theta$$

Clearly, $a_{\text{solid sphere}} > a_{\text{disc}}\ \therefore\ t_{\text{solid sphere}} < t_{\text{disc}}$

Hence **solid sphere gets to the bottom of the plane first.**

41. Here, $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$ ...(*i*)

velocity, $\vec{v} = \frac{d\vec{r}}{dt} = -\omega\sin\omega t\,\hat{x} + \omega\cos\omega t\,\hat{y}$ ...(*ii*)

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Acceleration, $\vec{a} = -\omega^2 \cos\omega t\,\hat{x} - \omega^2 \sin\omega t\,\hat{y}$

$$= -\omega^2\,\vec{r} \qquad \text{...(iii)}$$

From (*iii*), we note that **acceleration is directed towards the origin** because $\vec{a}$ is in opposite direction to $\vec{r}$.

Here, $\vec{v}\,.\,\vec{r} = (-\omega \sin\omega t\,\hat{x} + \omega\cos\omega t\,\hat{y}).$

$$(\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y})$$

$= -\,\omega\sin\omega t\cos\omega t + \omega\sin\omega t\cos\omega t$

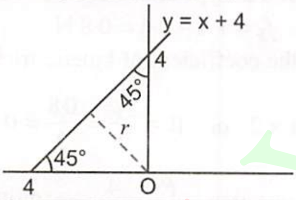
$= 0$

or $\quad v\,r\cos\theta = 0 \quad$ or $\quad \cos\theta = 0 \quad$ or $\quad \theta = 90°$

i.e., **$\vec{v}$ and $\vec{r}$ are perpendicular to each other.**

42. In Fig. 7(CF).56, we have shown the line $Y = X + 4$ along which the particle is moving. Perpendicular distance of this line from the origin,

FIGURE 7(CF).56



$$r = 4\sin 45° = \frac{4}{\sqrt{2}}$$

Angular momentum about origin

$$L = m\,v\,r = 5 \times 3\sqrt{2} \times \frac{4}{\sqrt{2}} = \mathbf{60\,unit}$$

43. Total number of revolutions = area under *n-t* graph

$$= \frac{1}{2}\times 8 \times \frac{1800}{60} + 8 \times \frac{600}{60} + \frac{1}{2}\times 16 \times \frac{600}{60}$$

$$= 120 + 80 + 80 = \mathbf{280}$$

44. From $\theta = \omega_1 t + \dfrac{1}{2}\alpha t^2$,

$$\theta_1 = 0 + \frac{1}{2}\alpha(2)^2 = 2$$

$$\theta_1 + \theta_2 = 0 + \frac{1}{2}\alpha(4)^2 = 8\,\alpha$$

$$\theta_2 = 8\,\alpha - \theta_1 = 8\,\alpha - 2\,\alpha = 6\,\alpha$$

$$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = \mathbf{3}$$

45. Solid cylinder reaches bottom first because for solid cylinder, $\dfrac{K^2}{R^2} = \dfrac{1}{2}$ and for hollow cylinder, $\dfrac{K^2}{R^2} = 1$. Acceleration down the inclined plane is proportional to $\dfrac{1}{K^2/R^2}$. **Solid cylinder has greater acceleration.** It reaches the bottom first.

46. Here, $I = 5 \times 10^{-3}$ kg m^2, $n_1 = 20$ rps

$\tau = ?$, $n_2 = 0$, $t = 10$ s

$$\tau = I\,\alpha = \frac{I\,(\omega_2 - \omega_1)}{t} = \frac{I \times 2\pi\,(n_2 - n_1)}{t}$$

$$= \frac{5\times 10^{-3} \times 2\pi\,(0 - 20)}{10}$$

$$= \mathbf{2\,\pi \times 10^{-2}\ N\text{-}m}$$

47. Torque on rod

= moment of weight of the rod about *A*

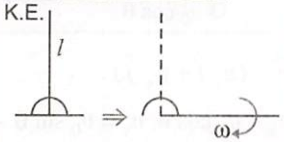
$$\tau = mg \times \frac{l}{2}$$

As $\quad \tau = I\,\alpha = mg\,\dfrac{l}{2} \quad \therefore \quad \dfrac{ml^2}{3}\,\alpha = mg\,\dfrac{l}{2}$

$$\alpha = \mathbf{\frac{3\,g}{2\,l}}$$

48. As shown in Fig. 7(CF).57,

FIGURE 7(CF).57



Decrease in gr. P.E. = Increase in rot. K.E.

$$mg\left(\frac{l}{2}\right) = \frac{1}{2}I\,\omega^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3\,g}{l}}$$

Now, $\quad v = r\,\omega = l\sqrt{\dfrac{3\,g}{l}} = \sqrt{3\,gl}$

49. Moment of inertia of the system of three rods about the given axis, $I = 0 + \dfrac{m\,L^2}{3} + m\,L^2 = \dfrac{4}{3}m\,L^2$

As decrease in gr. P.E. = increase in rot. K.E.

$$\therefore\ mg \times \frac{L}{2} + mg\,(L) = \frac{1}{2}I\,\omega^2$$

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$$\frac{3}{2}\,mg\,L = \frac{1}{2}\left(\frac{4}{3}\,m\,L^2\right)\omega^2$$

$$\omega = \frac{3}{2}\sqrt{\frac{g}{L}}$$

50. Applying the law of conservation of energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2$$

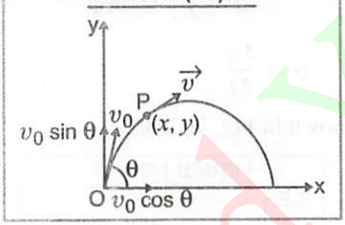
$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

$$= \frac{1}{2}\left(m + \frac{M}{2}\right)v^2$$

$$v = \frac{2\,gh}{(1+M/2\,m)} = \frac{2\times10\times1}{(1+2/0{\cdot}2)} = \frac{20}{11}\,\text{ms}^{-1}$$

51. In Fig. 7(CF).58, the particle projected from O, reaches P (x, y) in t sec. with velocity

FIGURE 7(CF).58



$$\vec{v} = (v_x\,\hat{i} + v_y\,\hat{j}).$$

$$\therefore \quad v_x = v_0\cos\theta,\; v_y = v_0\sin\theta - gt$$

and $\quad x = (v_0\cos\theta)\,t,\;\; y = (v_0\sin\theta)\,t - \frac{1}{2}gt^2$

$$\vec{r} = (x\,\hat{i} + y\,\hat{j})$$

Momentum of particle at P

$$= m\,\vec{v} = m(v_x\,\hat{i} + v_y\,\hat{j})$$

Angular momentum of particle at P about origin O is

$$\vec{L} = \vec{r}\times\vec{p} = (x\,\hat{i} + y\,\hat{j})\times m\,(v_x\,\hat{i} + v_y\,\hat{j})$$

$$= m\,y\,v_x(-\hat{k}) + m\,x\,v_y(\hat{k})$$

$$= m\left((v_0\sin\theta)\,t - \frac{1}{2}gt^2\right)v_0\cos\theta(-\hat{k})$$

$$+\,m\,(v_0\cos\theta)\,t\,(v_0\sin\theta - gt)\;(\hat{k})$$

$$= (-mv_0^2\cos\theta\sin\theta)\,t\,\hat{k} + mv_0\cos\theta\times\frac{1}{2}gt^2\,\hat{k}$$

$$+(mv_0^2\sin\theta\cos\theta)\,t\,\hat{k} - mv_0\cos\theta\,gt^2\,\hat{k}$$

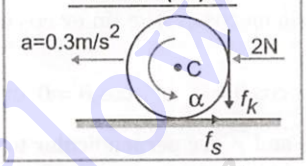
$$= -\frac{1}{2}mv_0\,gt^2\cos\theta\,\hat{k}$$

52. Refer to Fig. 7(CF).59.

$$2 - f_s = Ma$$

or $\quad f_s = 2 - Ma = 2 - 2\times0{\cdot}3 = 1{\cdot}4\,N$

FIGURE 7(CF).59



Taking torque about C, we have

$$f_s\,R - f_k\,R = I.\alpha = MR^2\times\frac{a}{R} = MRa$$

or $\quad 1{\cdot}4\times0{\cdot}5 - f_k\times0{\cdot}5 = 2\times0{\cdot}5\times0{\cdot}3$

or $\quad f_k = 1{\cdot}4 - 0{\cdot}6 = 0{\cdot}8$ N

If μ is the coefficient of kinetic friction, then

$$f_k = \mu\times2 \quad\text{or}\quad \mu = \frac{f_k}{2} = \frac{0{\cdot}8}{2} = 0{\cdot}4 = \frac{4}{10}$$

As per question, $\dfrac{P}{10} = \dfrac{4}{10}$ or $P = 4$

53. As is known, for a body rolling down an incline without slipping,

$$\text{acceleration} = \frac{g\sin\theta}{1 + I/MR^2}$$

For a ring, $\quad I = MR^2$,

$$\therefore \quad a_{\text{ring}} = \frac{g\sin\theta}{2}$$

For a disc, $\quad I = \frac{1}{2}MR^2$

$$\therefore \quad a_{\text{disc}} = \frac{2}{3}g\sin\theta$$

The length l of the incline is

$$l = \frac{h}{\sin\theta} = \frac{1}{2}a_r\times t_1^2 = \frac{1}{2}\times\frac{g\sin\theta}{2}\,t_1^2$$

$$t_1 = \sqrt{\frac{4\,h}{g\sin^2\theta}} = \sqrt{\frac{4\,h}{g\,(\sin 60^\circ)^2}} = \sqrt{\frac{16\,h}{3\,g}}$$

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$$\text{Again, } l = \frac{h}{\sin\theta} = \frac{1}{2} a_{\text{disc}} \times t_2^2 = \frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2$$

$$t_2 = \sqrt{\frac{3h}{g\sin^2\theta}} = \sqrt{\frac{3h}{g(\sin 60^\circ)^2}} = \sqrt{\frac{4h}{g}}$$

$$t_1 - t_2 = \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}}$$

$$\frac{2-\sqrt{3}}{\sqrt{10}} = \frac{\sqrt{h}}{\sqrt{g}}\left(\frac{4}{\sqrt{3}} - 2\right) = \frac{\sqrt{h}}{\sqrt{10}}\left(\frac{4-2\sqrt{3}}{\sqrt{3}}\right)$$

$$(2-\sqrt{3})\sqrt{3} = \sqrt{h}(4-2\sqrt{3})$$

$$\sqrt{h} = \frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})} = \frac{\sqrt{3}}{2}$$

$$h = \frac{3}{4}\text{m} = \mathbf{0{\cdot}75\ m}$$

54. As external torque is zero, *i.e.*, $\tau = 0$, so angular momentum is conserved.

$$\therefore \quad I\,\omega = \text{a constant} = C$$

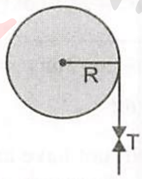
$$\text{or} \quad \omega = \frac{C}{I} \quad \text{or} \quad \omega \propto \frac{1}{I}$$

When insect moves towards centre, I decreases, hence ω increases. As insect moves away from centre towards diametrically opposite point, I increases. Then ω decreases. Thus **ω first increases and then decreases.**

55. Here, $m = 50$ kg, $R = 0{\cdot}5$ m, $T = ?$

$$\alpha = 2\ \text{revol/s}^2 = 2 \times 2\pi\ \text{radian/s}^2$$

FIGURE 7(CF).60



As torque, $\tau = T \times R$

$$T = \frac{\tau}{R} = \frac{I\alpha}{R} = \frac{mR^2}{2} \times \frac{\alpha}{R}$$

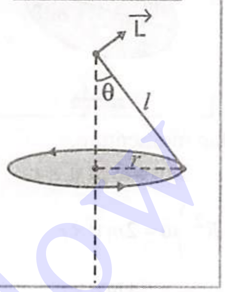
$$= \frac{mR\alpha}{2} = \frac{50 \times 0{\cdot}5 \times 4\pi}{2}$$

$$T = 50\,\pi = \mathbf{157\ N}$$

56. Angular momentum of the bob about the point of suspension, Fig. 7(CF).61,

$$\vec{L} = I\,\omega = (mr^2)\omega = m\,(l\sin\theta)^2\,\omega$$
$$= (m\,l^2\sin^2\theta)\,\omega$$

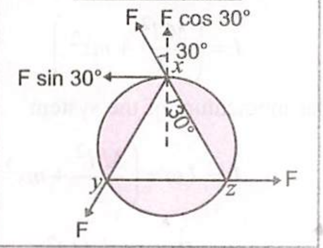
FIGURE 7(CF).61



As m, l, θ and ω are constants, therefore $\vec{L}$ is **constant in magnitude. However, its direction keeps on changing being perpendicular to l, always.**

57. As is clear from Fig. 7(CF).62, torque is produced by three component forces each $= F\sin 30^\circ$ at x, y and z.

FIGURE 7(CF).62



$$\therefore \quad \text{Total torque, } \tau = 3 \times F\sin 30^\circ \times r$$

$$= 3 \times 0{\cdot}5 \times \frac{1}{2} \times 0{\cdot}5 = \frac{3}{8}\text{Nm}$$

$$I = \frac{mr^2}{2} = \frac{1{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5}{2} = \frac{3}{16}\ \text{kg m}^2$$

$$\text{From} \quad \tau = I\,\alpha, \quad \alpha = \frac{\tau}{I} = \frac{3/8}{3/16} = 2\ \text{rad/s}^2$$

$$\text{From} \quad \omega = \omega_0 + \alpha t = 0 + 2 \times 1 = 2\ \text{rad/sec}$$

58. Here, $R = 0{\cdot}5$ m, $M = 0{\cdot}45$ kg

$m = 0{\cdot}05$ kg, $r = 0{\cdot}25$ m

$v = 9$ m/s, $\omega = ?$

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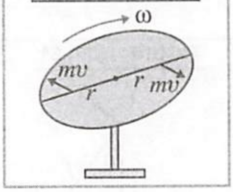
As is clear from Fig. 7(CF).63,

FIGURE 7(CF).63



Angular momentum,

$$I\,\omega = 2\,m\,v \times r$$

$$\left(\frac{1}{2} M\,R^2\right)\omega = 2 m v \times r$$

$$\omega = \frac{4 m v \times r}{M\,R^2}$$

$$\omega = \frac{4 \times 0{\cdot}05 \times 9 \times 0{\cdot}25}{0{\cdot}45 \times (0{\cdot}5)^2} = \mathbf{4\ rad/sec.}$$

59. If M is mass of the rod, l is length of the rod, its moment of inertia about the end O of the rod $= M\,l^2/3$. If m is mass of the insect and at any time t, the insect is at a distance x from O, its moment of inertia about $O = mx^2$

Therefore, moment of inertia of the system about

O is $$I = \left(\frac{M\,l^2}{3} + mx^2\right)$$

Angular momentum of the system

$$L = I\,\omega = \left(\frac{M\,l^2}{3} + mx^2\right)\omega$$

Torque, $$\tau = \frac{dL}{dt} = \frac{d}{dt}\left(\frac{M\,l^2}{3} + mx^2\right)\omega$$

$$= m\,\omega(2\,x)\frac{dx}{dt} = 2\,m\,\omega\,x.v$$

But $x = v\,t$. Therefore,

$$\tau = m\,\omega\,(v\,t)\,v = 2\,m\,\omega\,v^2\,t$$

Clearly, $\tau \propto t$

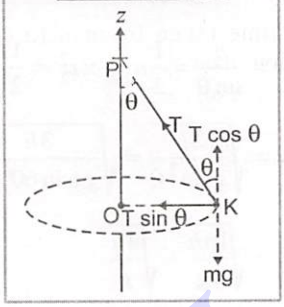
At $t = T$, $v = 0$. Therefore, $\tau = 0$.

Hence, the graph between τ and t is a straight line upto $t = T$. For $t > T$, $\tau = 0$.

Choice (*b*) is correct.

60. In Fig. 7(CF).64, weight mg of the mass acts vertically downwards and tension T in the string acts along KP. Two rectangular components of T are $T\cos\theta$ opposite to mg and $T\sin\theta$ along KO.

FIGURE 7(CF).64



About O, torques due to $T\cos\theta$ and mg cancel out. Torque due to $T\sin\theta$ is zero. Therefore, net torque about O is zero.

As $\tau_0 = \frac{dL_O}{dt}$, therefore, $\vec{L_O}$ = constant

i.e., $\vec{L_O}$ does not vary with time.

About P, torque due to T is zero, but torque due to $mg \neq 0$

As $\tau_P = \frac{dL_P}{dt} \neq 0$

$\therefore$ $\vec{L_P}$ is not constant. It varies with time.

Choice (*c*) is correct.

61. At an instant, speed of $P = v$, going in clockwise direction

Speed of $Q = v$, going in anticlockwise direction

Relative angular velocity of P w.r.t. $Q = \omega - (-\omega)$
$= 2\,\omega$

Relative angular separation of P and Q in time t
$\theta = 2\,\omega t$.

Relative speed between the points P and Q at time t

$$|\vec{v}_r| = \sqrt{v^2 + v^2 - 2vv\cos(2\,\omega t)}$$

$$= \sqrt{2v^2(1 - \cos 2\,\omega t)} = \sqrt{2v^2 \times 2\sin^2 \omega t}$$

$$= 2\,v\,\sin\omega t$$

Since $|\vec{v}_r|$ will not have any negative value so the lower part of the sine wave will come upper side. Hence **option (*a*) is true.**

62. Here, $R = 2$ m, $I = 200$ kg m^2, $u = 0$, $M = 50$ kg
$v = 1$ m/s, $t = ?$

Using principle of conservation of angular momentum, $L_t = L_0$

$$M\,v\,R - I\,\omega = 0,\ \omega = \frac{M\,v\,R}{I} = \frac{50 \times 1 \times 2}{200} = \frac{1}{2}$$

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If t is time taken by man to complete one revolution, then

$$(v + R\,\omega)\,t = 2\,\pi\,R$$

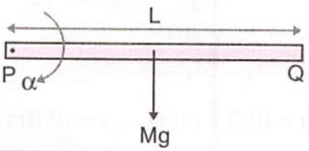
$$\left(1 + 2 \times \frac{1}{2}\right) t = 2\pi \times 2$$

$$\boldsymbol{t = 2\,\pi \text{ sec}}$$

63. If I is moment of inertia of the rod about the end P and α is initial angular acceleration of the rod, then as is clear from Fig. 7(CF).65,

$$\tau = I\,\alpha = Mg \times \frac{L}{2}$$

FIGURE 7(CF).65



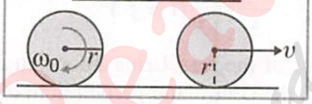
But $\quad I = \dfrac{ML^2}{3}$

$$\therefore \quad \frac{ML^2}{3}\alpha = Mg\,\frac{L}{2}$$

$$\alpha = \boldsymbol{\frac{3g}{2L}}$$

64. Refer to Fig. 7(CF).66.

FIGURE 7(CF).66



Let v be the velocity of the centre of the hoop, when it ceases to slip.

Applying the principle of conservation of angular momentum, we get

$$(m\,r^2)\,\omega_0 = m\,v \times r + mr^2\,\omega$$

$$= m\,v \times r + m\,r^2\left(\frac{v}{r}\right) = 2\,m\,v\,r$$

$$v = \frac{(m\,r^2)\,\omega_0}{2\,m\,r} = \boldsymbol{\frac{\omega_0\,r}{2}}$$

65. Work required to bring the spinning bodies to rest

$$\Delta W = \Delta \text{ K.E.} = \frac{1}{2}\;I\,\omega^2$$

$\therefore\;\; \Delta W \propto I$, as ω is same.

$$\therefore\; W_A : W_B : W_C = I_{\text{solid sphere}} : I_{\text{solid disc}} : I_{\text{circular ring}}$$

$$= \frac{2}{5}MR^2 : \frac{1}{2}MR^2 : MR^2$$

$$= \frac{2}{5} : \frac{1}{2} : 1 = \boldsymbol{4 : 5 : 10}$$

$$\therefore\;\; \boldsymbol{W_C > W_B > W_A}.$$

66. Translational K.E., $K_t = \dfrac{1}{2}mv^2$

Rotational K.E., $K_r = \dfrac{1}{2}I\,\omega^2$

$$= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{5}mv^2$$

$$\therefore \qquad K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$\text{So} \qquad \frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \boldsymbol{\frac{5}{7}}$$

67. Here, $r = 2$ m, $m = 100$ kg, $v = 20$ m/s $= 0{\cdot}2$ m/s

Work required = change in K.E.

= final K.E. – initial K.E.

$$= 0 - \left(\frac{1}{2}\,mv^2 + \frac{1}{2}\,I\,\omega^2\right)$$

$$= -\left(\frac{1}{2}\,m\,v^2 + \frac{1}{2} \times \frac{1}{2}\,m\,r^2\,\omega^2\right)$$

$$= -\left(\frac{1}{2}\,m\,v^2 + \frac{1}{4}\,m\,v^2\right) = \frac{-3}{4}\,m\,v^2$$

$$W = \frac{-3}{4} \times 100\,(0{\cdot}2)^2 = \boldsymbol{-3\text{ J}}$$

68. Here, $m = 2$ kg, $r = 4$ cm $= 4 \times 10^{-2}$ m

$$\omega_1 = 3 \text{ rpm} = 3 \times \frac{2\pi}{60} \text{ rad/s}, \; \omega_2 = 0$$

$$\theta = 2\,\pi \text{ revols} = 2\,\pi \times 2\,\pi \text{ rad}$$

According to work energy theorem,

$$W = \frac{1}{2}\,I\,(\omega_2^2 - \omega_1^2)$$

$$\tau \cdot \theta = \frac{1}{2} \times \frac{1}{2}\,m\,r^2\left(0 - \left[3 \times \frac{2\pi}{60}\right]^2\right)$$

HINTS/EXPLANATIONS
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$$\tau = \frac{1}{4}\frac{mr^2}{\theta} \times \frac{\pi^2}{100} \text{ (leaving – sign)}$$

$$= \frac{1}{4} \times \frac{2 \times (4 \times 10^{-2})^2}{2\pi \times 2\pi} \times \frac{\pi^2}{100}$$

$$\boldsymbol{\tau = 2 \times 10^{-6}\ \text{N-m}}$$

69. At $t = 0$, y-displacement is maximum in Fig. 7(CF).25. Therefore, the required equation must be cosine function.

Now, $T = 4$ s $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

The y-projection of radius vector of particle P is

$$y = a \cos \omega t$$

$$y = 3 \cos \frac{\pi}{2} t$$

70. From geometrical interpretation of angular momentum, $\quad L = 2m\frac{dA}{dt}$

$$\therefore \quad \frac{dA}{dt} = \frac{\boldsymbol{L}}{\boldsymbol{2m}}$$

71. Suppose m is mass of each rod of length a.

For stable equilibrium, torques about 0 must balance

$$\therefore \quad mg\,\frac{a}{2}\sin\theta = mg\left(\frac{a}{2}\cos\theta - a\sin\theta\right)$$

$$mg\left(\frac{a}{2} + a\right)\sin\theta = mg\,\frac{a}{2}\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{3}$$

$$\tan\theta = \frac{1}{3}$$

$$\boldsymbol{\theta = \tan^{-1}(1/3)}$$

72. Here, $\vec{F} = 3\hat{j}$ N $\qquad \vec{r} = 2\hat{k}\,m$, $\vec{\tau} = ?$

As $\quad \vec{\tau} = \vec{r} \times \vec{F}$

$$\therefore \ \vec{\tau} = 2\hat{k} \times 3\hat{j} = 6\hat{k} \times \hat{j} = -6\,\hat{j} \times \hat{k}$$

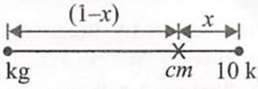
$$\boldsymbol{\tau = -6\hat{i}\ \text{N-m}}$$

73. Let centre of mass be at a distance x from 10 kg mass. From the property of centre of mass

$5x = 10\,(1 - x)$

$$15x = 10,\ x = \frac{10}{15}\text{ m} = \frac{2}{3}\text{ m} = \frac{200}{3}\text{ cm} = \mathbf{67\ cm}$$

FIGURE 7(CF).67



74. Here, $n_1 = 360$ rpm $= \frac{360}{60} = 6$ rps

$$n_2 = 1200 \text{ rpm} = \frac{1200}{60} = 20 \text{ rps}$$

$$t = 14 \text{ sec.}$$

Angular acceleration, $\alpha = \frac{\omega_2 - \omega_1}{t}$

$$= \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(20 - 6)}{14}$$

$$\boldsymbol{= 2\pi\ \text{rad/s}^2}$$

75. Let $M_1 = M_2 = M_3 = M$

$(x_1, y_1) = (0, 2\hat{j})$; $(x_2, y_2) = (0, 0)$;

$(x_3, y_3) = (2\hat{i}, 0)$;

Position vector of centre of mass is

$$x_{cm} = \frac{M \times 0 + M \times 0 + M \times 2\hat{i}}{M + M + M} = \frac{2\hat{i}}{3}$$

$$y_{cm} = \frac{M \times 2\hat{j} + M \times 0 + M \times 0}{M + M + M} = \frac{2\hat{j}}{3}$$

Hence position vector of cm is

$$\frac{2\hat{i}}{3}, \frac{2\hat{j}}{3} = \boldsymbol{\frac{2}{3}}(\hat{i} + \hat{j})$$

76. Moment of inertia of ring about an axis passing through the centre and perpendicular to the plane of the ring, $\quad I = MR^2$

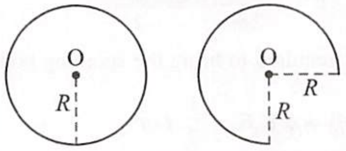
Mass removed $= M/4$

Remaining mass $M' = M - M/4 = 3\,M/4$

Moment of inertia of the remaining part of ring

$$I' = \frac{3M}{4} \times R^2 = KMR^2 \quad \therefore \quad \boldsymbol{K = \frac{3}{4}}$$

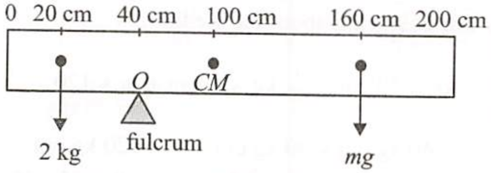
FIGURE 7(CF).68



HINTS/EXPLANATIONS
For Difficult Questions

77. The various forces acting on the rod are shown in Fig. 7(CF).69. Three forces involved are

FIGURE 7(CF).69



(*i*) 2 kg mass at 20 cm from the fulcrum.

(*ii*) unknown mass (*m*) at 160 cm from 0 or at 120 cm from the fulcrum.

(*iii*) weight of rod = *mg* = 500 g at cm at a distance of 60 cm from fulcrum.

For equilibrium of the rod, applying the principle of moments, we get

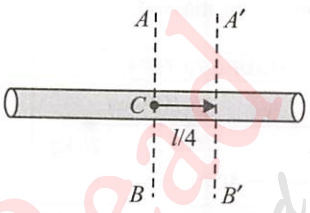
$$m \times 1{\cdot}2 + 0{\cdot}5 \times 0{\cdot}6 = 2 \times \frac{1}{5}$$

$$m \times 1{\cdot}2 = 0{\cdot}4 - 0{\cdot}3 = 0{\cdot}1$$

$$m = \frac{1}{12}\ \text{kg}$$

78. Refer to Fig. 7(CF).70

FIGURE 7(CF).70



Moment of inertia of the rod about an axis *AB* through the centre of the rod is $I = \frac{ml^2}{12}$

About an axis $A'B' \parallel AB$ at a distance ($l/4$) from the centre *c*, moment of inertia $I' = \frac{ml^2}{12} + m\,(l/4)^2$

$$mk^2 = ml^2\left(\frac{1}{12} + \frac{1}{16}\right) = ml^2\left(\frac{7}{48}\right)$$

$$k = \sqrt{\frac{7l^2}{48}} = l\sqrt{\frac{7}{48}}$$

79. In Fig. 7(CF).29, 1·0 kg mass is taken at the origin *O* and *x*, *y* are two mutually perpendicular axes. 1·5 kg mass is at (3, 0) and 2·5 kg mass is at (0, 4)

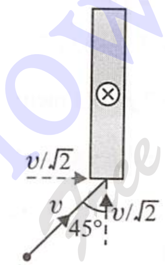
$$\therefore\ x_{cm} = \frac{1 \times 0 + 1{\cdot}5 \times 3 + 2{\cdot}5 \times 0}{1 + 1{\cdot}5 + 2{\cdot}5} = \frac{4{\cdot}5}{5} = \mathbf{0{\cdot}9\,cm}$$

$$y_{cm} = \frac{1 \times 0 + 1{\cdot}5 \times 0 + 2{\cdot}5 \times 4}{1 + 1{\cdot}5 + 2{\cdot}5} = \frac{10{\cdot}0}{5{\cdot}0} = \mathbf{2\,cm}$$

80. Suppose angular velocity of the system after collision is ω.

Applying the principle of conser- vation of angular momentum about the hinge, Fig. 7(CF).71, we get

FIGURE 7(CF).71



$$m\left(\frac{v}{\sqrt{2}}\right)\frac{l}{2} = \left(\frac{ml^2}{3} + \frac{ml^2}{4}\right)\omega$$

$$\frac{mvl}{2\sqrt{2}} = \frac{7}{12}ml^2\omega$$

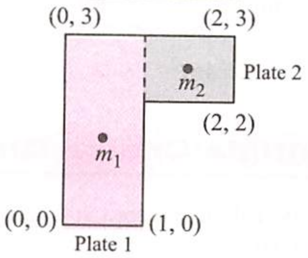
$$\frac{v}{2\sqrt{2}} = \frac{7}{12}l\omega$$

$$\omega = \frac{v}{7l} \times \frac{12}{2\sqrt{2}} = \frac{v}{7l}\,\frac{6}{\sqrt{2}} = \mathbf{\frac{3\sqrt{2}\,v}{7l}}$$

81. Here, total mass of flag shaped lamina = 4 kg

As is clear from Fig. 7(CF).72,

FIGURE 7(CF).72



Mass of plate 1 = m_1 = 3 kg

Mass of plate 2, m_2 = 1 kg

As is evident from Fig. 7(CF).72,

m_1 is assumed to be concentrated at (0·5, 1·5) and m_2 is assumed to be concentrated at (1·5, 2·5)

HINTS/EXPLANATIONS

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$$\therefore \quad x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{3 \times 0{\cdot}5 + 1 \times 1{\cdot}5}{3+1} = \frac{3}{4} = \mathbf{0{\cdot}75\,m}$$

$$\text{and} \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{3 \times 1{\cdot}5 + 1 \times 2{\cdot}5}{3+1} = \frac{7}{4} = \mathbf{1{\cdot}75\,m}$$

82. Here, $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N

$$\vec{r_1} = (\hat{i} + 2\hat{j} + \hat{k}) \text{ metre}$$

$$\vec{r_2} = (4\hat{i} + 3\hat{j} - \hat{k}) \text{ metre}$$

$$\tau = (\vec{r_2} - \vec{r_1}) \times \vec{F} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}\,(3+4) - \hat{j}\,(9+2) + \hat{k}(6-1)$$

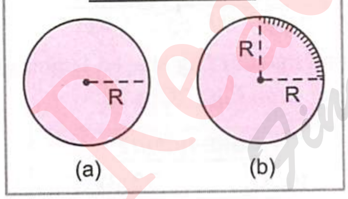
$$\vec{\tau} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$|\vec{\tau}| = \sqrt{7^2 + 11^2 + 5^2} = \sqrt{195} = \sqrt{x}$$

Hence $x = \mathbf{195}$

83. For the original circular ring

FIGURE 7(CF).73



$$I = MR^2$$

when an arc corresponding to 90° sector is removed, moment of inertia about the given axis

$$\text{is} \quad I' = \left(\frac{3M}{4}\right) R^2 = \frac{3}{4}\,(MR^2) = K\,(MR^2)$$

$$\therefore \quad K = 3/4$$

84. For balancing about the wedge

$$2 \text{ kg} \times 20 \text{ cm} = \frac{1}{2} \text{ kg} \times 60\text{cm} + m \times 120$$

$$40 \text{ kg cm} = 30 \text{ kg cm} + m \times 120 \text{ kg cm}$$

85. Let M be the mass and R be the radius of uniform thin disc.

About an axis passing through its centre and normal to its plane,

$$I_1 = \frac{MR^2}{2} = M K_1^2 \quad \therefore \quad K_1 = \frac{R}{\sqrt{2}}$$

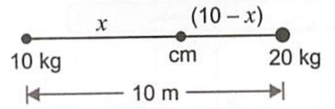
Moment of inertia of disc about its diameter

$$I_2 = \frac{MR^2}{4} = M K_2^2 \quad \therefore \quad K_2 = \frac{R}{2}$$

$$\therefore \quad \frac{K_1}{K_2} = \frac{R/\sqrt{2}}{R/2} = \mathbf{\sqrt{2} : 1}$$

86. Refer to Fig. 7(CF)74. Let x be the distance of c.m. from 10 kg mass.

FIGURE 7(CF).74



$$\therefore \quad 10\,x = 20\,(10 - x)$$

$$30\,x = 200$$

$$x = \frac{200}{30} = \mathbf{\frac{20}{3}\,m}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

87. (*a*) As external force is zero, so linear momentum is conserved.

i.e. $-2\,m \times v + m \times 2\,v + 0 = (2\,m + m + 8\,m)\,v_0$

$\therefore \quad v_0 = \mathbf{0}$

(*c*) As external torque is also zero, so angular momentum is conserved, *i.e.*,

$$m_1 v_1 r_1 + m_2 v_2 r_2 = (I_1 + I_2 + I_{\text{bar}})\,\omega$$

$$(2\,m)\,v\,a + m\,(2\,v)\,(2\,a)$$

$$= \left[2m(a)^2 + m(2a)^2 + \frac{8\,m \times (6\,a)^2}{12}\right] \omega$$

$$6\,m\,v\,a = \left[6ma^2 + \frac{288\,ma^2}{12}\right] \omega$$

$$6\,m\,v\,a = 30\,ma^2\,\omega$$

$$\omega = \frac{v}{5a}$$

(d) $$E = \frac{1}{2}I\omega^2 = \frac{1}{2}(30\,ma^2)\left(\frac{v}{5a}\right)^2$$

$$= \mathbf{\frac{3mv^2}{5}} \qquad [\because I = 30\,ma^2]$$

88. As mass of quarter sector of disc is $M/4$, therefore, its moment of inertia about the given axis is

$$I = \frac{1}{2}\left(\frac{M}{4}\right)R^2 = \mathbf{\frac{1}{8}MR^2}$$

89. From the dynamics of non uniform circular motion, $\mathbf{T - Mg\cos\theta = Mv^2/L}$. Also, tangential acceleration = $\mathbf{g\sin\theta}$

90. According to theorem of perpendicular axes,

$$I = I_1 + I_2 \quad \text{and} \quad I = I_3 + I_4$$

As $$I_2 = I_3 \qquad \therefore \qquad I = I_1 + I_3$$

91. $$I_X = \frac{1}{2}\,(\text{mass})\,(\text{radius})^2 = \frac{1}{2}[(\pi R^2)\,t.\rho]\,R^2$$

$$I_Y = \frac{1}{2}\left[(\pi\,16\,R^2)\frac{t.\rho}{4}\right](4\,R)^2 = \mathbf{64\,I_X}$$

or $$\frac{I_Y}{I_X} = \mathbf{64}$$

92. Effective distance to be travelled by the two bodies before they collide = 12 R – (R + 2 R) = 9 R.

As gravitational pull on both the bodies is the same $\left(= \frac{G\,m_1 . m_2}{r^2}\right)$, therefore, if $m_1 = M$ and $m_2 = 5\,M$ then $\frac{a_1}{a_2} = \frac{5}{1}$ is the ratio of their accelerations. As distance travelled $s \propto a$

$$\therefore \qquad \frac{s_1}{s_2} = \frac{5}{1}$$

Dividing total distance 9 R in the ratio 5 : 1, we get $s_1 = \frac{9\,R}{6} \times 5 = \mathbf{7{\cdot}5\,R}$

93. As $$h = \frac{(v\sin 45°)^2}{2g} = \frac{v^2}{4g}$$

and $L = m\,v\,r\sin\theta = m\,(v\cos 45°)\,h$

$$= \frac{mv}{\sqrt{2}}\cdot\left(\frac{v^2}{4g}\right) = \mathbf{\frac{m\,v^3}{(4\sqrt{2})g}}$$

Also, $L = m\,v\,r\sin\theta = m\,(v\cos 45°)\,h = \frac{mv}{\sqrt{2}}\,.\,h$

and $$v = \sqrt{4gh}$$

$$\therefore \qquad L = \frac{m}{\sqrt{2}}h.\sqrt{4gh} = \mathbf{m\sqrt{2gh^3}}$$

94. When external force is non zero, acc. cannot be zero. However, the centre of mass may be at rest or may be moving with some velocity. Therefore, $\mathbf{v_0 = 0}$ **or** $\mathbf{v_0 \neq 0}$**, but** $\mathbf{a_0 \neq 0}$**.**

95. Force of friction produces the torque for rotation. Therefore, it tries to **increase the angular velocity.** However, friction opposes the motion. Therefore, it tries to **decrease the linear velocity.**

96. Mass per unit area of disc $= \frac{9\,M}{\pi R^2}$

Mass of removed portion of disc

$$= \frac{9\,M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$$

Moment of inertia of removed portion about an axis passing through centre of disc and perpendicular to the plane of disc, using theorem of parallel axis is

$$I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2}MR^2$$

when portion of disc would not have been removed, then the moment of inertia of complete disc about the given axis is $I_2 = \frac{1}{2}MR^2$

So moment of inertia of the disc with removed portion, about the given axis is

$$I = I_2 - I_1 = \frac{9}{2}\,MR^2 - \frac{1}{2}\,MR^2 = \mathbf{4\,MR^2}$$

97. When no external force is acting, the **linear momentum and angular momentum of the body remain constant.** As the body is non rigid, its moment of inertia may change and hence its K.E. may also change.

98. As is known from theory, acceleration of a mass rolling down a fixed inclined plane of inclination θ is

$$a = \frac{g\sin\theta}{1 + K^2/R^2} \qquad \ldots(i)$$

HINTS/EXPLANATIONS

For Difficult Questions

As cylinder P has most of its mass concentrated near its surface, while cylinder Q has most of its mass concentrated near the axis, therefore, radius of gyration, $K_P > K_Q$

From (*i*), $a_P < a_Q$

Therefore, at bottom of incline, $V_P < V_Q$

or $(\omega_P R) < (\omega_Q R)$ or $\omega_P < \omega_Q$ or $\omega_Q > \omega_P$

i.e., cylinder Q reaches the ground with larger angular speed. **Choice (*d*) is correct.**

99. Here, $V = \dfrac{kr^2}{2}$

$$F = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{kr^2}{2}\right)$$

$$F = -kr$$

Now, F = centripetal force $= \dfrac{mv^2}{r}$

$\therefore$ at $r = R$

$$F = \frac{mv^2}{R} = -kR$$

$$v = \sqrt{\frac{k}{m}}\,R \qquad \text{(leaving neg. sign)}$$

Angular momentum, $L = I\omega = (mR^2)\dfrac{v}{R} = mvR$

$$= mR\sqrt{\frac{k}{m}}\,R$$

$$L = m\sqrt{\frac{k}{m}}\,R^2 = \sqrt{mk}\,R^2$$

Choices (*b*) and (*c*) are correct.

100. Here, $\vec{F} = (\alpha t\,\hat{i} + \beta\,\hat{j})$ ...(*i*)

As mass is at rest at the origin at time $t = 0$.

$\therefore$ at $t = 0$, $\vec{v} = 0$ and $\vec{r} = 0$

$\therefore$ $\alpha = 1$; $\beta = 1$

From (*i*), $\vec{F} = (t\,\hat{i} + \hat{j})$ ...(*ii*)

$$m\frac{d\vec{v}}{dt} = t\,\hat{i} + \hat{j}$$

Integrating both sides, we get

$$m\vec{v} = \frac{t^2}{2}\hat{i} + t\,\hat{j}$$

As $m = 1{\cdot}0$ kg, therefore,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\,\hat{j} \qquad \text{...(iii)}$$

Integrating both sides, we get

$$\vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$$

At $t = 1$ s, $\vec{\tau} = \vec{r} \times \vec{F}$

Using (*ii*), $\vec{\tau} = \left(\dfrac{1}{6}\hat{i} + \dfrac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$

$$\vec{\tau} = \frac{1}{6}\hat{k} - \frac{1}{2}\hat{k} = -\frac{1}{3}\hat{k}$$

From (*iii*), $\vec{v} = \dfrac{t^2}{2}\hat{i} + t\,\hat{j}$

At $t = 1$ s, $\vec{v} = \left(\dfrac{1}{2}\hat{i} + \hat{j}\right) = \dfrac{1}{2}(\hat{i} + 2\hat{j})$ **m/s**

At $t = 1$ s, $\vec{s} = \vec{r}_1 - \vec{r}_0$

$$= \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) - (\vec{0}) = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}$$

$$\therefore \quad |\vec{s}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{10}}{6}\text{ m}$$

Hence **choices (*a*) and (*c*) are correct.**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

101. Here, $m_1 = 1$ kg, $m_2 = 2$ kg

$x_1 = 1, y_1 = 2$;

$x_2 = -1, y_2 = 3$

Co-ordinates of the centre of mass are

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1\,(1) + 2\,(-1)}{1 + 2} = -\frac{1}{3}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{1\,(2) + 2\,(3)}{1 + 2} = \frac{8}{3}$$

102. As explosion is due to internal forces only, the centre of mass of fragments **continues to follow the same parabolic path,** which it would have followed, if there was no explosion.

103. Moment of inertia of a body about a given axis depends upon **all the three factors.**

104. K.E. of rotation $= \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}I\,(2\pi n)^2$

$$= \frac{1}{2} \times 5\left(2\pi \times \frac{120}{60}\right)^2 = \mathbf{395{\cdot}1\ J}$$

HINTS/EXPLANATIONS

For Difficult Questions

IV. Matching Type Questions

105. $I_1 = \frac{1}{2} \times$ moment of inertia of full hollow ring about an axis passing through the centre and perpendicular to plane of ring

$$= \frac{1}{2}(2m)r^2 = mr^2$$

$I_2 = MI$ about the diameter $= \frac{1}{2}mr^2$

$I_3 = MI$ about tangent in plane of ring

$$= \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

$I_4 = MI$ about tangent ⊥ to plane of ring

$$= mr^2 + mr^2 = 2\,mr^2$$

Choice (*d*) is correct.

106. Total K.E. = K.E. of translation + K.E. of rotation

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2$$

For hollow cylinder, $I = mr^2$

$\therefore \quad E = mv^2$

For solid cylinder, $I = \frac{1}{2}mr^2$

$\therefore \quad E = \frac{3}{4}mv^2$

For hollow sphere, $I = \frac{2}{3}mr^2$

$\therefore \quad E = \frac{5}{6}mv^2$

for solid sphere, $I = \frac{2}{5}mr^2$

$\therefore \quad E = \frac{7}{10}mv^2$

Choice (*a*) is correct.

V. Matrix-Match Type Questions

107. Moment of inertia of a body depends on mass ; size, shape and position of axis of rotation. Radius of gyration depends on size, shape of body and axis of rotation. Angular momentum = $I\,\omega$ depends on all the four factors listed in col. II, and K.E. of rotation = $1/2\,I\,\omega^2$; also depends on all the four factors.

108. Angular impulse = change in angular momentum

$$= \vec{r} \times \vec{p} = I\,\omega\,.$$

Torque = $I\,\alpha$.

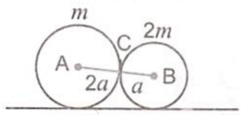
Work done $= \vec{r} \cdot \vec{F}$

VI. Integer Type Questions

109. Assuming that masses of the two spheres are concentrated at their centres *A* and *B* and taking $x = 0$ at *A*, Fig. 7(CF).75, we get

$$x_{cm} = \frac{m \times 0 + 2m \times 3a}{(m + 2m)} = 2a = AC$$

FIGURE 7(CF).75



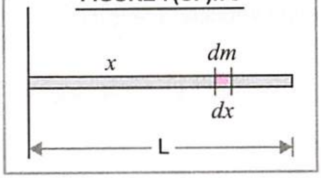
HINTS/EXPLANATIONS....

For Difficult Questions

$$CB = a \quad \therefore \quad \frac{AC}{CB} = \frac{2a}{a} = \mathbf{2}$$

110. As shown in Fig. 7(CF).76, mass of small element of length dx is

FIGURE 7(CF).76



$$dm = \lambda\, dx = \lambda_0\, x\, dx$$

$$\therefore \quad x_{cm} = \frac{\int_0^L x\, dm}{\int dm} = \frac{\int_0^L x(\lambda_0\, x\, dx)}{\int_0^L \lambda_0\, x\, dx}$$

$$x_{cm} = \frac{[x^3/3]_0^L}{[x^2/2]_0^L} = \frac{L^3/3}{L^2/2} = \frac{2L}{3}$$

If $\quad x_1 = \frac{2L}{3}, \quad x_2 = L - \frac{2L}{3} = \frac{L}{3}$

$$\frac{x_1}{x_2} = \frac{2L/3}{L/3} = \frac{\mathbf{2}}{\mathbf{1}}$$

111. Here, $L_1 = I_1\,\omega_1$, When $\omega_2 = 2\,\omega_1$,

and $\quad \text{KE} = \frac{1}{2} I_2\,\omega_2^2 = 2\left(\frac{1}{2} I_1\,\omega_1^2\right)$

$$\frac{1}{2} I_2 (2\omega_1)^2 = 2 \times \frac{1}{2} I_1\,\omega_1^2 \;;\quad I_2 = \frac{1}{2} I_1$$

$$\therefore \quad L_2 = I_2\,\omega_2 = \frac{1}{2} I_1 \times 2\,\omega_1 = I_1\,\omega_1 = L_1$$

$$x = \frac{L_2}{L_1} = \mathbf{1}$$

112. Moment of inertia of circular disc about a tangent perpendicular to the plane of disc

$$I_1 = MK_1^2 = \frac{3}{2} MR^2 \;;\quad K_1^2 = \frac{3}{2} R^2$$

Again, moment of inertia of circular ring of same size as disc about any diameter,

$$I_2 = M'K_2^2 = \frac{1}{2} M'R^2$$

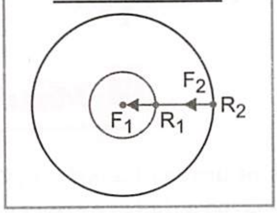
$$K_2^2 = \frac{1}{2} R^2$$

$$\therefore \quad \frac{K_1^2}{K_2^2} = \frac{3/2\,R^2}{1/2\,R^2} = \mathbf{3}$$

113. As the annular ring is rolling without slipping, therefore, its angular velocity ω is constant. So is its linear velocity v. Therefore, no net force or net torque is acting on the ring.

The force experienced by the two particles – one on outer part and other on inner part is only centripetal force directed towards the centre of ring, Fig. 7(CF).77.

FIGURE 7(CF).77



$$\frac{F_2}{F_1} = \frac{m R_2\,\omega^2}{m R_1\,\omega^2} = \frac{R_2}{R_1} = \frac{4{\cdot}8}{1{\cdot}2} = \mathbf{4}$$

114. Torque on disc due to all the three forces is

$$\tau = 3 \times F\, r \sin 30^\circ = 3 \times 0{\cdot}5 \times 0{\cdot}5 \times 1/2$$

$$= \frac{3}{8}$$

Moment of inertia of disc about an axis passing through its centre and perpendicular to the plane

$$I = \frac{M\,r^2}{2} = \frac{1{\cdot}5 \times (0{\cdot}5)^2}{2} = \frac{3}{16}$$

HINTS/EXPLANATIONS

For Difficult Questions

If α is the angular acceleration of the disc due to torque, then

$$\tau = I\alpha \quad \text{or} \quad \alpha = \frac{\tau}{I} = \frac{3/8}{3/16} = 2 \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t = 0 + 2 \times 1 = \mathbf{2 \text{ rad/s}}$$

115. *For circular platform,*

$$R = 0{\cdot}5 \text{ m} \; ; \; M = 0{\cdot}45 \text{ kg}$$

For toy :

$$m = 0{\cdot}05 \text{ kg}, r = 0{\cdot}25 \text{ m} \; ; v = 9 \text{ ms}^{-1}$$

According to law of conservation of angular momentum

$$2 \times mvr = I\,\omega = \frac{MR^2}{2}\omega$$

or $$\omega = \frac{4mvr}{MR^2} = \frac{4 \times 0{\cdot}05 \times 9 \times 0{\cdot}25}{0{\cdot}45 \times (0{\cdot}5)^2} = \mathbf{4 \text{ rad/s}}$$

116. Since final KEs are equal so

$$(\text{KE of rotation + PE})_A = (\text{KE of rotation + PE})_C$$

or $$\frac{1}{2}\left[\frac{3}{2}mR^2\right]\omega_1^2 + mgh_1 = \frac{1}{2}\left[\frac{3}{2}mR^2\right]\omega_2^2 + mgh_2$$

or $$\frac{3}{4}v_1^2 + gh_1 = \frac{3}{4}v_2^2 + gh_2$$

or $$\frac{3}{4}(3)^2 + 10 \times 30 = \frac{3}{4}v_2^2 + 10 \times 27$$

On solving ; $v^2 = 49$ or $v = \mathbf{7 \text{ ms}^{-1}}$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

117. Here, both the assertion and reason are false because in case of a rolling body, all points on it have same angular speed but different linear speed. Linear speed is different at different places.

It is max. at highest point and minimum at lowest point.

118. Here, assertion is true but reason is false speed of whirlwind is a tornado is alarmingly high as *MI* is less so angular velocity is more as per the law of conservation of angular momentum. Also,

$$\tau = \frac{dL}{dt}, \text{ when } \tau = 0, \frac{dL}{dt} = 0 \text{ or } L = \text{constant. So}$$

angular momentum remains constant, not angular velocity.

119. Here, both assertion and reason are true but reason is not correct explanation of Assertion.

When a body slides, its PE is converted into KE of translation, but when it rolls down, some part of its PE is converted into its KE of rotation, thereby, reducing its KE of translation and hence its linear velocity.

120. Both, the assertion and reason are true and the latter is correct explanation of the former. This follows from the theorem of parallel axes on moment of inertia.

121. At the centre of earth, $g = 0$. Therefore, a body has no weight at the centre of earth ; and hence no centre of gravity. But centre of mass of a body has nothing to do with gravity. Therefore, centre of mass exists.

122. In translatory motion, $P = F \times v$

$\therefore$ In rotational motion ; $P = \tau \times \omega$

Choice (*a*) is correct.

123. Angular momentum is rotational analogue of linear momentum, and torque is rotational analogue of force. Choice (*a*) is correct.

124. Both, the assertion and reason are true, but the reason is not a correct explanation of the assertion. Infact, if helicopter had only one propeller, the helicopter itself will turn in opposite direction to conserve the angular momentum.

125. Here Assertion is false because in the absence of external torque, the angular momentum of the system of rotation is constant and not the velocity of the centre of mass. Thus velocity will change. Here Reason is true but cannot explain the Assertion.

HINTS/EXPLANATIONS

For Difficult Questions

FOR ENGINEERING STUDENTS

126. Statement-1 is true, but the statement-2 is false. Infact, in the formation of a neutron star, a heavy contraction occurs on account of tremendous gravitational pull. Moment of inertia I decreases. As $L = I\,\omega$ = constant, therefore, ω increases *i.e.* spin angular velocity increases.

127. Both the statements are true and statement-2 is correct explanation of the statement-1.

128. Statement-1 is true, but the statement-2 is not true. Infact, the centre of mass is related to the distribution of mass of the body.

129. Statement-1 is false. But statement-2 is true.

130. Both the statements are true and statement-2 is a correct explanation of statement-1. Infact, as ice on polar caps of earth melts, mass near the polar axis spreads out, Moment of inertia I increases. Therefore, T increases *i.e.* duration of the day increases.

131. From $I = MR^2 = 10\left(\frac{0.5}{2}\right)^2 = 0.625 \text{ kg } m^2$

Both the statements are true and statement-2 is correct explanation of statement-1.